A Survey of Mathematical Reasoning in the Era of Multimodal Large Language Model: Benchmark, Method & Challenges

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Abstract

001 Mathematical reasoning, a core aspect of human cognition, is vital across many domains, from educational problem-solving to scientific 004 advancements. As artificial general intelligence (AGI) progresses, integrating large language models (LLMs) with mathematical rea-007 soning tasks is becoming increasingly significant. This survey provides the first comprehensive analysis of mathematical reasoning in the era of multimodal large language models (MLLMs). We review over 200 studies pub-011 lished since 2021, and examine the state-of-theart developments in Math-LLMs, with a focus on multimodal settings. We categorize the field 015 into three dimensions: benchmarks, methodologies, and challenges. In particular, we explore 017 multimodal mathematical reasoning pipeline, as well as the role of (M)LLMs and the associated methodologies. Finally, we identify five 019 major challenges hindering the realization of AGI in this domain, offering insights into the future direction for enhancing multimodal reasoning capabilities. This survey serves as a critical resource for the research community in advancing the capabilities of LLMs to tackle complex multimodal reasoning tasks.

1 Introduction

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Mathematical reasoning is a critical aspect of human cognitive ability, involving the process of deriving conclusions from a set of premises through logical and systematic thinking (Jonsson et al., 2022; Yu et al., 2024b). It plays an essential role in a wide range of applications, from problem-solving in education to advanced scientific discoveries. As artificial general intelligence (AGI) continues to advance (Zhong et al., 2024), the integration of large language models (LLMs) with mathematical reasoning tasks becomes increasingly significant. These models, with their impressive capabilities in language understanding, have the potential to simulate complex reasoning processes that were once

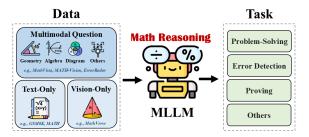


Figure 1: The illustration of our research scope (*i.e.*, investigating the MLLM's math reasoning capability).

Survey	Venue & Year	Scope	Multimodal	LLM	
(Lu et al., 2022b)	ACL'22	DL4Math			
(Li et al., 2023a)	arXiv'23	LLM4Edu		~	
(Liu et al., 2023b)	arXiv'23	LLM4Edu		~	
(Li et al., 2024f)	COLM'24	DL4TP			
(Ahn et al., 2024)	EACL'24	LLM4Math		~	
(Xu et al., 2024a)	IJMLC'24	LLM4Edu		~	
(Wang et al., 2024d)	arXiv'24	LLM4Edu		~	
Ours	-	MLLM4Math	~	~	

Table 1: Comparisons between relevant surveys & ours.

thought to be inherently human. In recent years, both academia and industry have placed increasing emphasis on this direction (Wang et al., 2024d; Xu et al., 2024a; Lu et al., 2022b).

The inputs for mathematical reasoning tasks are diverse, extending beyond traditional text-only to multimodal settings, as illustrated in Figure 1. Mathematical problems often involve not only textual information but also visual elements, such as diagrams, graphs, or equations, which provide essential context for solving the problem (Wang et al., 2024e; Yin et al., 2024). In the past year, multimodal mathematical reasoning has emerged as a key focus for multimodal large language models (MLLMs) (Zhang et al., 2024c; Bai et al., 2024; Wu et al., 2023a). This shift is driven by the recognition that reasoning tasks in fields like mathematics require models capable of integrating and processing multiple modalities simultaneously to achieve human-like performance. However, multimodal mathematical reasoning poses significant

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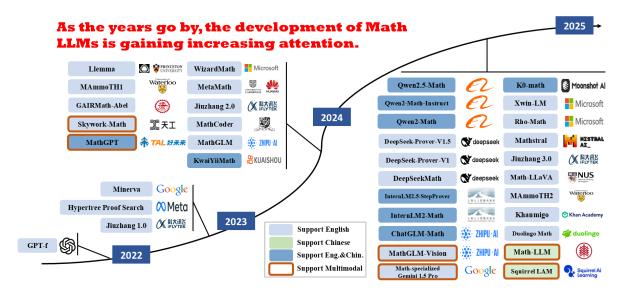


Figure 2: The release timeline of Math-LLMs in recent years.

challenges due to the complex interaction between different modalities, the need for deep semantic understanding, and the importance of context preservation across modalities (Liang et al., 2024a; Song et al., 2023; Fu et al., 2024b). These challenges are central to the realization of AGI, where models must integrate diverse forms of knowledge seamlessly to perform sophisticated reasoning tasks.

Math-LLM Progress. Figure 2 illustrates that, driven by the rapid development of LLMs since 2021, the number of math-specific LLMs (Math-LLMs) has grown steadily, alongside enhanced support for multilingual and multimodal capabilities (More details in Appendix A). The landscape was marked by the introduction of models like GPT-f (Polu and Sutskever, 2021) and Minerva (Lewkowycz et al., 2022), with Hypertree Proof Search (Lample et al., 2022) and Jiuzhang 1.0 (Zhao et al., 2022) highlighting advancements in theorem proving and mathematical question understanding capabilities, respectively. Year 2023 saw a surge in diversity and specialization, alongside multimodal support from models like Skywork-Math (Zeng et al., 2024). In year 2024, there was a clear focus on enhancing mathematical instruction (e.g., Qwen2.5-Math (Yang et al., 2024a)) and proof (e.g., DeepSeek-Proof (Xin et al., 2024a)) capabilities. The year also witnessed the emergence of Math-LLMs with a vision component, such as MathGLM-Vision (Yang et al., 2024b).

Scope. Previous surveys have not fully captured the progress and challenges of mathematical reasoning in the age of MLLMs. As indicated in Table 1, some works have concentrated on the application of deep learning techniques to mathematical reasoning (Lu et al., 2022b) or specific domains such as theorem proving (Li et al., 2024f), but they have overlooked the rapid advancements brought about by the rise of LLMs. Others have broadened the scope to include the role of LLMs in education (Wang et al., 2024d; Xu et al., 2024a; Li et al., 2023a) or mathematical fields (Ahn et al., 2024; Liu et al., 2023b), but have failed to explore the development and challenges of mathematical reasoning in multimodal settings in depth. Therefore, this survey aims to fill this gap by providing the first-ever comprehensive analysis of the current state of mathematical reasoning in the era of MLLMs, focusing on three key dimensions: benchmark, methodology, and challenges.

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Structure. In this paper, we survey over 200 publications from the AI community since 2021 related to (M)LLM-based mathematical reasoning, and summarize the progress of Math-LLMs. We first approach the field from the benchmark perspective, analyzing the LLM-based mathematical reasoning task through three key aspects: dataset, task, and evaluation (Section 2). Subsequently, we explore the roles that (M)LLMs play in mathematical reasoning, categorizing them as enhancers, reasoners, and planners (Section 3). Finally, we identify five core challenges that the mathematical reasoning faces in the era of MLLMs (Section 4). This survey aims to provide the community with comprehensive insights for advancing the multimodal complex reasoning capabilities of LLMs.

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2 Benchmark Perspective

2.1 Overview

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Benchmarking for mathematical reasoning plays a crucial role in advancing LLM research, as it provides standardized, reproducible pipeline for assessing the performance on reasoning tasks. While previous benchmarks such as GSM8K (Cobbe et al., 2021) and MathQA (Amini et al., 2019) were instrumental in the pre-LLM era, our scope is centered on those relevant to (M)LLMs. In this section, we present a comprehensive analysis of recent benchmarks for mathematical reasoning in the context of (M)LLMs (Shown in Table 2). The section is organized into three subsections: Datasets (Sec.2.2), Tasks (Sec.2.3) & Evaluation (Sec.2.4).

2.2 Dataset

Basic Format. In a math reasoning task (taking prblem-solving as a basic setting), the goal is to solve a mathematical problem given a specific format of input and output. The input consists of a statement that describes the problem to be solved. As illustrated in Figure 3, this can be presented in either a textual format (text-only) or a multimodal format (text accompanied by visual elements, such as figures or diagrams). The output is the correct or predicted solution to the mathematical problem, often represented as a numerical or symbolic result.

Language & Size. The majority of benchmarks are available in English, with a few exceptions like Chinese (Li et al., 2024h) or Romanian (Cosma et al., 2024) datasets. This predominance of English datasets underscores the challenges of multilingual representation in the mathematical reasoning domain, suggesting an opportunity for future work to diversify datasets across languages, especially those in underrepresented regions. Moreover, the size of these datasets varies widely, from smaller sets (e.g., QRData (Liu et al., 2024d) with 411 questions) to massive corpora (e.g., OpenMathInstruct-1 (Toshniwal et al., 2024) with 1.8 million problem-solution pairs). Larger datasets are more likely to support robust model training and evaluation, but their size can also present challenges in terms of computational requirements and quality control.

Source. The sources of datasets predominantly consist of public (*i.e.*, derived from public repositories or datasets) and private sources. The private datasets typically offer specialized problem types and tasks, and may present unique challenges, such

(a) <u>Text-only</u> Math Reasoning Setting

[Qns] Find the distance between the two endpoints using the distance formula. The two end points of the line are (-3, 4) and (5, 2), respectively.

[Ans] 8.246

(b) Multimodal Math Reasoning Setting

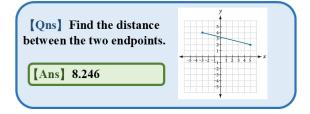


Figure 3: Typical data format of math reasoning task for text-only & multimodal settings. Examples are derived from MathVerse (Zhang et al., 2024f), which assess whether and how much MLLMs can truly understand the visual diagrams for mathematical reasoning.

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as restricted access or ethical considerations. On the other hand, public datasets foster wider community collaboration, though they may suffer from limitations in diversity and task coverage. Some works have also leveraged LLMs to generate the datasets tailored to specific needs. For instance, GeomVerse constructs synthetic datasets to evaluate the multi-hop reasoning abilities required in geometric math problems (Kazemi et al., 2023).

Educational Level. The benchmarks span various educational levels, ranging from elementary school to university-level problems. Besides, there has also been a surge in datasets focused on competition-level problems (Tsoukalas et al.), offering insights into the current limitations of LLMs in comparison to the upper bound of human cognitive abilities. Future directions could involve more focused datasets targeting specific educational levels to enable models to specialize in handling particular age groups or skill sets.

2.3 Task

Model Choice. The choice of models in these benchmarks spans open-source and closed-source models, with a growing interest in Math-LLMs. This trend indicates an increasing recognition of the need for models tailored to mathematical reasoning, which often require specialized training and handling of structured knowledge. Additionally, with the recent release of GPT-40 (OpenAI, 2024) and Gemini-Pro-1.5 (Reid et al., 2024), which have

Benchmarks	Venue	Language	Size	Source	Level(s)	Evaluation	Model(s)	Task(s)
GSM-Plus (Li et al., 2024d)	ACL'24	English	10,552	P	BMHU	Generative	Closed/Open/Math	S
MuggleMath (Li et al., 2024c)	ACL'24	English	37,365	P	E	Discriminative	Open	S
Olympiadbench (He et al., 2024) 🛪	ACL'24	English/Chinese	8,476	S		Generative	Closed/Open/Math	S
MathBench (Liu et al., 2024b)	ACL Findings'24	English/Chinese	3,709	PS	EMHU	Generative	Closed/Open/Math	S
GeoEval (Zhang et al., 2024d) 🛠	ACL Findings'24	English	5,050	PG	EMH	Discriminative	Closed/Open/Math	S
QRData (Liu et al., 2024d)	ACL Findings'24	English	411	S	U	Discriminative	Closed/Open/Math	S
EIC-Math (Li et al., 2024e)	ACL Findings'24	English	1,800	P	EMH	Discriminative	Closed/Open	DO S S
Srivastava et al. (2024)	ACL Findings'24	English	-	P	U	Discriminative	Closed/Open	S
CHAMP (Mao et al., 2024)	ACL Findings'24	English	270	S	<u>.</u>	Generative	Closed/Open	S
IMO-AG-30 (Trinh et al., 2024)	Nature'24	English	30	<u>S</u>	Q	Discriminative	Closed	P
PutnamBench (Tsoukalas et al.)	NeurIPS'24	English	1,697	<u> </u>	9	Generative	Closed	SP
MATH-Vision (Wang et al., 2024a) ☆ CARP (Zhang et al., 2024a)	NeurIPS'24 NeurIPS'24	English Chinese	3,040 4,886	S	EMHU C	Discriminative Discriminative	Closed/Open Closed	S
SMART-840 (Cherian et al., 2024) 🖈	NeurIPS'24	English	840	S	EMA	Discriminative	Closed/Open	Ś
OpenMathInstruct-1 (Toshniwal et al., 2024)	NeurIPS'24	English	1,800,000	P	EMHC	Generative	Closed/Open/Math	S
Didolkar et al. (2024)	NeurIPS'24	English	8,600	P	E	Discriminative	Closed	SO
Scibench (Wang et al., 2023b) 🖈	ICML'24	English	869	Š	Ō	Discriminative	Closed/Open	S
GeomVerse (Kazemi et al., 2023) 🖈	ICML workshop'24	English	1,000	Ğ	ŏ	Discriminative	Closed	8 8 8
MathVista (Lu et al., 2023) 🖄	ICLR'24	English	6,141	S P	BMAD	Discriminative	Closed/Open	ă
MMMU _{math} (Yue et al., 2024a) ★	CVPR'24	English	540			Discriminative	Closed/Open	× ×
MathVerse (Zhang et al., 2024f)		U		~			1	
Mathador-LM (Kurtic et al., 2024)	ECCV'24 EMNLP'24	English English	2,612	G	B	Generative Both	Closed/Open Closed/Open	SD
MM-MATH (Sun et al., 2024a) 🖄	EMNLP Findings'24	English	5,929	S	MH	Discriminative	Closed/Open	SD
Scieval (Sun et al., 2024b)	AAAI'24	English	15,901	SP	Ð	Both	Closed/Open	S
ArqMATH (Satpute et al., 2024)	SIGIR'24	English	450	P	U	Generative	Closed/Open/Math	S
IsoBench (Fu et al., 2024a) 🖈	COLM'24	English	1,887	S	EMHU	Discriminative	Closed/Open	S
MMMU-Pro _{math} (Yue et al., 2024b) 🖈	arXiv'24	English	60	Ś	0	Discriminative	Closed/Open	Ś
MathOdyssey (Fang et al., 2024)	arXiv'24	English	387	S	HŪC	Both	Closed/Open/Math	S
MathScape (Zhou et al., 2024b) 🖈	arXiv'24	Chinese	1,325	Š	B M H	Generative	Closed/Open	8 8 8
U-Math (Chernyshev et al., 2024) 🖈	arXiv'24	English	1,100	ă	-	Discriminative	Closed/Open/Math	S D
MathHay (Wang et al., 2024b)	arXiv'24	English	673	SP	Ö	Both	Closed/Open	S
MathCheck (Zhou et al., 2024d) 🖈	arXiv'24	English/Chinese	4,536	P	EMHU	Discriminative	Closed/Open/Math	S
ErrorRador (Yan et al., 2024a) 🛣	arXiv'24	English	2,500	Š	EMA	Discriminative	Closed/Open	Ď
FaultyMath (Rahman et al., 2024) 🖈	arXiv'24	English	363	Ğ	B M B	Discriminative	Closed/Open/Math	ŏ
MathChat (Liang et al., 2024c)	arXiv'24	English	1.319	P	B	Both	Closed/Open/Math	SDO
E-GSM (Xu et al., 2024e)	arXiv'24	Chinese	4,500	P	ē	Both	Closed/Open/Math	S.O
Tangram (Tang et al., 2024) ★ GSM-Symbolic (Mirzadeh et al., 2024)	arXiv'24 arXiv'24	English English	4,320 5,000	Ś	B M IIC	Discriminative Discriminative	Closed/Open Closed/Open	Q
CMM-Math (Liu et al., 2024c)		Chinese		X		Both		×
CMM-Math (Liu et al., 2024c) 🖄 CMMaTH (Li et al., 2024h) 🖄	arXiv'24		28,069	2			Closed/Open/Math	
	arXiv'24	English/Chinese	23,856	2	EMH	Both	Closed/Open/Math	
EAGLE (Li et al., 2024g) 🛪	arXiv'24	English	170,000	<u> </u>		Discriminative	Closed/Open/Math	S
VisAidMath (Ma et al., 2024) 🛪	arXiv'24	English	1,200	S	MHC	Discriminative	Closed/Open	000000000000000000000000000000000000000
AutoGeo (Huang et al., 2024d) 🖈	arXiv'24	English	100,000	S	EMHU	Both	Closed/Open	0
NTKEval (Guo et al., 2024a)	arXiv'24	English	1,860	PG	Ð	Discriminative	Open	S
Mamo (Huang et al., 2024b)	arXiv'24	English	1,209	SG	U	Generative	Closed/Open/Math	0
RoMath (Cosma et al., 2024)	arXiv'24	Romanian	70,000	S	MHC	Discriminative	Closed/Open/Math	S
MaTT (Davoodi et al., 2024)	arXiv'24	English	1,958	S	U	Discriminative	Closed/Open	S
Li et al. (2024a)	arXiv'24	English	15,000	P	EMU	Generative	Closed/Open/Math	S
DynaMath (Zou et al., 2024) 🖄	arXiv'24	English	5,010	SPG	EMHU	Both	Closed/Open	S
Polymath (Gupta et al., 2024) 🖈	arXiv'24	English	5,000	S	MHU	Discriminative	Closed/Open	S
SuperCLUE-Math6 (Xu et al., 2024b)	arXiv'24	English/Chinese	2,144	S	Ē	Generative	Closed/Open	S
TheoremQA (Chen et al., 2023)	EMNLP'23	English	800	S	Ū	Discriminative	Closed/Open	S
LILA (Mishra et al., 2022)	EMNLP'22	English	133,815	P	Ð	Discriminative	Closed	S
MATH (Hendrycks et al., 2021)	NeurIPS'21	English	12,500	S	C	Discriminative	Closed	S

Table 2: **Overview of LLM-based benchmarks for mathematical reasoning**. Frefers to those designed to evaluate the multimodal mathematical setting. Different colors indicate different types for the following columns: **Source:** S = Self-Sourced, P = Collected from Public Dataset, G = Generated by LLM **Level:** P = Elementary, M = Middle School, <math>P = High School, P = University, C = Competition, <math>P = Hybrid**Task:** S = Problem-Solving, D = Error Detection, P = Proving, <math>O = Others

demonstrated significant advancements in multimodal reasoning capabilities, the latest benchmarks have begun to include them in the evaluations. For example, ErrorRadar, in its initial formulation of multimodal error detection setting, incorporates these state-of-the-art MLLMs to highlight the realworld performance gap between AI systems and human-level reasoning (Yan et al., 2024a).

217**Reasoning Task.** Problem-solving tasks typi-218cally dominate, reflecting the emphasis on students'219ability to apply knowledge and reasoning skills in220real-world contexts. This also serves as the core221objective of current Math-LLMs. In addition, a222growing proportion of error detection tasks suggests an increasing focus on helping students rec-

ognize and correct mistakes (Li et al., 2024e; Yan et al., 2024a; Kurtic et al., 2024). Meanwhile, proving tasks, often associated with higher-order thinking, highlight a shift towards cultivating logical reasoning and systematic problem-solving abilities (Tsoukalas et al.). Moreover, a smaller portion of work has addressed tasks that align with real-world educational needs but lack systematic formulation. For instance, Li et al. (2024e) further introduces error correction (which goes beyond simple error detection); Didolkar et al. (2024) explores automated skill discovery for problem-solving; and MathChat (Liang et al., 2024c) focuses on reasoning in multiturn settings (such as follow-up QA and problem generation). Given the higher demands on rea224

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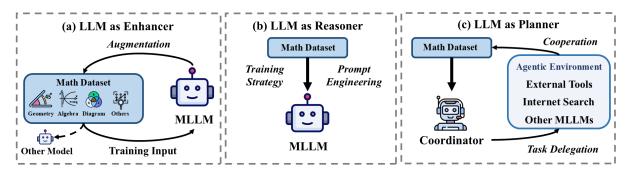


Figure 4: The illustration of the comparisons among three paradigms of (M)LLM-based mathematical reasoning.

soning capabilities in multimodal settings, many studies have also evaluated the aforementioned reasoning tasks in image-text problem settings. These efforts aim to provide the LLM community with more diverse, real-world task scenarios, catering to the needs of multimodal learning environments.

2.4 Evaluation

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Discriminative Evaluation is a common approach, focusing on the ability of M(LLM)s to correctly classify or choose the correct answer (Hendrycks et al., 2021; Mishra et al., 2022; Li et al., 2024c). Based on specific motivations, some works also build their metrics upon accuracy for further expansion. For example, GSM-PLUS, a new adversarial benchmark for evaluating the robustness of LLMs in mathematical reasoning, develops performance drop rate (PDR) to measure the relative decline in performance on question variations compared to the original questions (Li et al., 2024d). Error-Radar uses error step accuracy and error category accuracy together to evaluate the multimodal error detection of MLLMs (Yan et al., 2024a).

Generative Evaluation, on the other hand, measures a M(LLM)'s ability to produce detailed explanations or solve problems from scratch. This evaluation type is gaining traction, particularly for complex mathematical tasks where step-by-step solutions are required. For instance, MathVerse, which modifies problems with varying degrees of information content in multi-modality, employs GPT-4 to score each key step in the reasoning process generated by MLLMs (Zhang et al., 2024f). CHAMP proposes a solution evaluation pipeline where GPT-4 is utilized as a grader for the answer summary, given the ground truth answer (Mao et al., 2024).

Due to page limit, more details of both types of evaluation metrics can be seen in Appendix B.

3 Methodology Perspective

3.1 Overview & Findings

MLLMs have been leveraged in various ways to tackle the broad spectrum of mathematical reasoning tasks. Based on our comprehensive review of recent methodologies (summarized in Table 3), we classify the works into three distinct paradigms: LLM as Enhancer (Sec.3.2), LLM as Reasoner (Sec.3.3), and LLM as Planner (Sec.3.4). 276

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Findings. First, single-modality settings dominate the current landscape of method-oriented research, with the majority focusing solely on algebraic tasks. However, since 2024, multimodal approaches have been increasingly incorporated, expanding the scope of mathematical reasoning to include geometry, diagrams, and even broader mathematical concepts. This shift signals a growing interest in enhancing model robustness through multimodal learning, which can address the diverse nature of mathematical problems. Second, regarding the evaluated tasks, problem-solving and proving are gaining prominence, while some research also focuses on error detection or others (e.g., RefAug includes error correction and follow-up QA as evaluation tasks (Zhang et al., 2024i)). Finally, in terms of the role of LLMs, Reasoner is the most common role, followed by Enhancer, while Planner remains less explored but holds promise due to recent advancements in multi-agent intelligence.

3.2 LLM as Enhancer

Definition. In the *Enhancer* paradigm, M(LLM)s are primarily used to augment data, thereby enabling improvements in mathematical reasoning, as illustrated in Figure 4 (a). This can be achieved by synthesizing new training data, refining existing datasets, or introducing new variations that target specific problem-solving abilities (Li et al., 2022). Data augmentation can include paraphrasing math-

Methods	Venue	Evaluated Math Dataset(s)	Task(s)	Scope(s)	LLM as Enhancer	LLM as Reasoner	LLM as Planner
AlphaGeometry (Trinh et al., 2024) 🖄	Nature'24	IMO-AG-30	SP	G	~	~	
Masked Thought (Chen et al., 2024)	ACL'24	GSM8K/MATH/GSM8K-	S	A	~	~	
		RFT/MetaMathQA/MathInstruct	•	•			
MathGenie (Lu et al., 2024b)	ACL'24 ACL'24	GSM8K/MATH/SVAMP/Simuleq/Mathematics GSM8K/MATH		A	~	~	
MATH-SHEPHERD (Wang et al., 2024c) SEGO (Zhao et al., 2024)	ACL'24 ACL'24	GSM8K/MATH GSM8K/MATH	N.	A	~	~	
Deng et al. (2023)			S.F	A	v	~	
MathCoder (Wang et al., 2023a)	ACL Workshop'24 ICLR'24	GSM8K/SVAMP/MultiArith/MathQA/CSQA GSM8K/MATH	No.	*		~	
ToRA (Gou et al., 2023)	ICLR'24	GSM8K/MATH GSM8K/MATH	8.6	*		~	~
Visual Sketchpad (Hu et al., 2024)	NeurIPS'24	Geometry3K/ IsoBench		Ä		•	
Minimo (Poesia et al., 2024)	NeurIPS'24	Geometry3K/ IsoBench	8	8			v
Sinha et al. (2024)	NeurIPS Workshop'24	- IMO-AG-30		~		~	
		GSM8K	S.P			·	
SBIRAG (Dixit and Oates, 2024) VerityMath (Han et al., 2023)	NeurIPS Workshop'24 ICML Workshop'24	GSM8K	2	A			
RefAug (Zhang et al., 2024i)	EMNLP'24	GSM8K/MATH/Mathematics/MAWPS/ SVAMP/MMLU-Math/SAT-Math/MathChat- FQA/MathChat-EC/MINI-Math	oğg	X	v	,	
Math-LLaVA (Shi et al., 2024) 🖈	EMNLP Findings'24	MathVista/Math-V	90		~	~	
COPRA (Thakur et al., 2024)	COLM'24	miniF2F-test	S.	Ä	•	•	~
PRP (Wu et al., 2024b)	AAAI'24	MAWPS/ASDivA/Math23k/SVAMP/Un-	Š	Ä		~	•
		biasedMWP		•			
PERC (Jin et al., 2024)	L@S'24	PERC	S	A		~	
Math-PUMA (Zhuang et al., 2024) 🕏	arXiv'24	MathVerse/MathVista/WE-MATH	60	M		~	
MultiMath (Peng et al., 2024) 🖈	arXiv'24	MathVista/MathVerse/MultiMath-300K	00	X		~	
MathAttack (Zhou et al., 2024) A	arXiv'24 arXiv'24	GSM8K/MultiAirth		*		~	
MinT (Liang et al., 2023b)	arXiv'24	GSM8K/MathQA/CM17k/Ape210k	Š	Ä		~	
DotaMath (Li et al., 2024b)	arXiv'24	GSM8K/MATH/Mathematics/SVAMP/TabMWP/ASDiv	Š	Ä	~	v	
DFE-GPS (Zhang et al., 2024h)	arXiv'24	FORMALGEO7k	Š	ă	~	~	
JiuZhang 3.0 (Zhou et al., 2024a)	arXiv'24	GSM8K/MATH/SVAMP/ASDiv/MAWPS/CARP	SP.	A	~	~	
PGPSNet-v2 (Zhang et al., 2024e) *	arXiv'24	Geometry3K/PGPS9K	6	an	~	V	
LLaMA-Berry (Zhang et al., 2024b)	arXiv'24	GSM8K/MATH/GaoKao2023En/OlympiadBench/College Math/MMLU STEM	S O	A	·	v	
Skywork-Math (Zeng et al., 2024) 🕸	arXiv'24	GSM8K/MATH	SP	A	~	~	
SIaM (Yu et al., 2024a)	arXiv'24	GSM8K/CMATH	SP	A		~	
InternLM-Math (Ying et al., 2024)	arXiv'24	GSM8K/MATH	SP	A		~	
MathGLM-Vision (Yang et al., 2024b) 🖈	arXiv'24	MathVista/MathVerse/MathVision	SP	M	~	~	
Qwen2.5-Math (Yang et al., 2024a) 🖄	arXiv'24	GSM8K/MATH/MMLU-STEM/CMATH/GaoKao- Math-Cloze/GaoKao-Math-QA	Ö Ø	Ā	~	V	
S3c-Math (Yan et al., 2024c)	arXiv'24	GSM8K/MATH/SVAMP/Mathematics	SP	A	~	~	
BMA-MTIPL (Xiong et al., 2024)	arXiv'24	GSM8K/MATH	SP	A		~	
SIRP (Wu et al., 2024a)	arXiv'24	CSQA/GSM8K/MATH/MBPP	S P	A		~	
AIPS (Wei et al., 2024)	arXiv'24	MO-INT-20	S	G		~	
DART-Math (Tong et al., 2024) DeepSeekMath (Shao et al., 2024)	arXiv'24 arXiv'24	MATH/GSM8K/College/DM/Olympiad/Theorem GSM8K/MATH/OCW/SAT/MMLU STEM/CMATH/- Gaokao MathCloze/Gaokao MathQA	SP S	Å	V	~	
MMIQC (Liu et al., 2024a)	arXiv'24	MATH/MMIQC	8	A	~	~	
LANS (Li et al., 2023c) 🖈	arXiv'24	Geometry3K/PGPS9K	ă	an		~	
VCAR (Jia et al., 2024) *	arXiv'24	MathVista/MathVerse	ă			~	
KPDDS (Huang et al., 2024c)	arXiv'24 arXiv'24	GSM8k/MATH/SVAMP/TabMWP/ASDiv/MAWPS	~	*	~		
HGR (Huang et al., 2024c)	arXiv'24	Geometry3K	00000	Ä		~	
InfiMM-Math (Han et al., 2024)		GSM8K/MMLU/MathVerse/We-Math	×	×		v	
	arXiv'24			A	V	~	
CoSC (Han et al., 2024) Math-TSMC (Feng et al., 2024)	arXiv'24 arXiv'24	GSM8K/MATH GSM8K/MATH500		*	~	~	
SICCV (Liang et al., 2024)	arXiv'24 arXiv'24	GSM8K/MATH500 GSM8k/MATH500		*		~	
BEATS (Sun et al., 2024c)	arXiv'24 arXiv'24	GSM8K/MATH/SVAMP/SimulEq/NumGLUE		*		~	
MindStar (Kang et al., 2024c)	arXiv'24 arXiv'24	GSM8K/MATH	8.6	*			
UMM (Zhang et al., 2024g)	arXiv'24	MMLU/GSM8K-COT/GSM8K-Coding/MATH- COT/MATH-Coding/HumanEval/InfiBench	õõ	Ă		~	
STIC (Deng et al., 2024) ★	arXiv'24	ScienceQA/TextVQA/ChartQA/LLaVA- Bench/MMBench/MM-Vet/MathVista	S	M		~	
SPMWPs (Zhang et al., 2023)	ACL'23	GSM8K	S	A		~	
CoRe (Zhu et al., 2022)	ACL'23	GSM8K/ASDiv-A/SingleOp/SinlgeEq/MultiArith	S	A		~	
TabMWP (Lu et al., 2022a)	ICLR'23	TabMWP	S	AD		~	
Chameleon (Lu et al., 2024a) 🖈	NeurIPS'23	ScienceQA/TabMWP	S	AD			~
ATHENA (Kim et al., 2023)	EMNLP'23	MAWPS/ASDivA/Math23k/SVAMP/Un- biasedMWP	SP	A		~	
UniMath (Liang et al., 2023a) 🛪	EMNLP'23	SVAMP/GeoQA/TabMWP/MathQA/UniGeo- Proving	60			~	
Jiuzhang 2.0 (Zhao et al., 2023)	KDD'23	MCQ/BFQ/CAG/BAG/KPC/QRC/JCAG/JBAG	S	A		~	
TCDP (Qin et al., 2023)	TNNLS'23	Math23k/CM17K	S	<u>A</u>		~	
UniGeo (Chen et al., 2022)	ACL'22	GeoQA	S	g		~	
LogicSolver (Yang et al., 2022)	EMNLP Findings'22	InterMWP/Math23K	S	A	V	~	
Jiuzhang (Zhao et al., 2022)	KDD'22	KPC/QRC/QAM/SQR/QAR/MCQ/BFQ/CAG/BAG	2	A		V	
MWP-BERT (Liang et al., 2021) Inter-GPS (Lu et al., 2021)	NAACL'22 ACL'21	Math23k/MathQA/Ape-210k	No.			~	
		Geometry3K/GEOS					

Table 3: **Overview of LLM-based methods for mathematical reasoning**. \bigstar refers to those specifically designed to tackle the multimodal mathematical setting. Different colors indicate different types for the following columns: **Task: S** = Problem-Solving, **D** = Error Detection, **P** = Proving, **O** = Others **Scope: G** = Geometry, **A** = Algebra, **D** = Diagram, **M** = General Math

ematical problems, adding noise to mathematical expressions, or generating problem variants for underrepresented cases.

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Examples. A typical example of a singlemodality enhancement approach is Masked Thought, which introduces perturbations to the input and randomly masks tokens within the chain of thought during training (Chen et al., 2024). Math-Genie, which aims to generate diverse and reliable math problems and solution from a small-scale dataset, leverages a solution augmentation model to iteratively create new solutions from existing ones (Lu et al., 2024b). For multimodal methods, AlphaGeometry proves most olympiad-level mathematical theorems, via trained from scratch on large-scale synthetic data guiding the symbolic deuction (Trinh et al., 2024); LogicSolver introduces interpretable formula-based tree-structure for each solution equation (Yang et al., 2022); InfiMM-Math achieves the exceptional performance as it is trained on a large-scale multimodal interleaved math dataset developed and validated by LLMs such as LLaMA3-70B-Instruct (Han et al., 2024); DFE-GPS constructs its synthetic training 326

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set, which integrates visual features and geometric formal language (Zhang et al., 2024h).

Summary & Outlook. This paradigm offers substantial performance improvements by enriching the training set. However, challenges remain in ensuring the diversity and relevance of the generated data. Moreover, while text-based augmentation methods have proven effective, the potential for multimodal augmentation is still underexplored. Future research should focus on advancing multimodal data augmentation techniques, especially for tasks that require interaction between visual and textual modalities (Xiao et al., 2023).

3.3 LLM as Reasoner

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Definition. In the *Reasoner* paradigm, M(LLM)s harness their inherent reasoning capabilities to solve mathematical problems, as shown in Figure 4 (b). This can either involve fine-tuning existing LLMs on task-specific datasets or utilizing zeroshot or few-shot learning strategies. These models utilize advanced semantic understanding and reasoning techniques, such as symbolic manipulation, logical deduction, and multi-step reasoning.

Examples. Deng et al. (2023) develops a unified framework for answer calibration that integrates step-level and path-level strategies on multi-step reasoning of LLMs. MATH-SHEPHERD serves as a process-oriented math verifier, which assigns a reward score to each step of the LLM's outputs on math questions (Wang et al., 2024c). As for multimodal approaches, Math-PUMA introduces progressive upward multimodal alignment strategy for reasoning-enhanced training (Zhuang et al., 2024); Math-LLaVA, a LLaVA-1.5-based model, directly bootstraps mathematical reasoning via finetuned on 360K high-quality math QA pairs, which can ensure the depth and breadth of multimodal mathematical problems (Shi et al., 2024); STIC develops a two-stage self-training pipeline (consisting of Image Comprehension Self-Training phase & Description-Infused Fine-Tuning phase) for enhancing visual comprehension (Deng et al., 2024); VCAR emphasizes on the visual-centric supervision, thus proposing a similar two-step training pipleine which handles the visual description generation task first, followed by mathematical rationale generation task (Jia et al., 2024).

Summary & Outlook. This paradigm has shown significant promise, particularly in solving problems requiring multiple steps of reasoning. However, despite improvements, issues with robustness remain, particularly with zero-shot reasoning tasks. Future work should focus on combining reasoning with structured knowledge retrieval systems and enhancing models' ability to reason effectively across diverse domains, especially in multimodal contexts (Fan et al., 2024; Pan et al., 2023).

3.4 LLM as Planner

Definition. In the *Planner* paradigm, M(LLM)s are treated as coordinators that guide the solution of complex mathematical problems by delegating tasks to other models or tools, as illustrated in Figure 4 (c). This includes scenarios where multiple agents or models collaborate to achieve a single objective, thereby enhancing the performance of mathematical problem-solving through cooperative interactions. These models often work in environments with multiple steps or require iterative refinement of solutions.

Examples. A notable tool-integrated agent is ToRA, which plans the sequential use of natural language rationale and program-based tools synergistically to solve mathematical problems in an optimal manner (Gou et al., 2023). Additionally, COPRA simulates a single agent-like reasoning mechanism where GPT-4 proposes tactic applications within a stateful backtracking search, leveraging feedback from the proof environment (Thakur et al., 2024). This can also extend to multimodal scenarios, as seen in Chameleon, which serves as an AI system that augments MLLMs with plug-andplay modules for compositional reasoning, leveraging an LLM-based planner to assemble tools for complex tasks (Lu et al., 2024a). Furthermore, Visual Sketchpad presents the concept of sketching as a ubiquitous tool used by humans for communication, ideation, and problem-solving. Hence, MLLMs can enable external tools (*e.g.*, matplotlib) to generate intermediate sketches to aid in reasoning, which includes an iterative interaction process with an environment (Hu et al., 2024). Although there has been much work on Compositional Visual Reasoning in the past (Gupta and Kembhavi, 2023; Surís et al., 2023; Yao et al., 2022), Visual Sketchpad is the first work that integrates the planning capabilities of MLLMs with the real gap of mathematical reasoning settings (i.e., sketch-based reasoning involving visuo-spaital concepts).

Summary & Outlook. While the Planner paradigm introduces significant improvements, particularly for complex tasks that require multiagent collaboration, it remains a relatively under-

explored area (Xi et al., 2023; Guo et al., 2024b). There is potential for further improvement in task decomposition, agent cooperation strategies, and integration of diverse computational tools. Future work will likely focus on refining these planning strategies, especially for multimodal systems that can jointly leverage visual and textual knowledge to solve more intricate problems (Xie et al., 2024; Durante et al., 2024; Li et al., 2023b).

4 Challenges

In the realm of MLLMs for mathematical reasoning, the following key challenges persist that hinder their full potential. Addressing these challenges is essential for advancing MLLMs toward more robust and flexible systems that can better support mathematical reasoning in real-world settings.

• Insufficient Visual Reasoning. Many math problems require extracting and reasoning over visual content, such as charts, tables, or geometric diagrams. Current models struggle with intricate visual details, such as interpreting three-dimensional geometry or analyzing irregularly structured tables (Zhang et al., 2024f). Hence, it may be beneficial to introduce enhanced visual feature extraction modules and integrate scene graph representations for better reasoning over complex visual elements (Ibrahim et al., 2024; Guo et al., 2024c).

2 Reasoning Beyond Text and Vision. While the current research focus on the combination of text and vision, mathematical reasoning in realworld applications often extends beyond these two modalities. For instance, audio explanations, interactive problem-solving environments, or dynamic simulations might play a role in some tasks. Current models are not well-equipped to handle such diverse inputs (Abrahamson et al., 2020; Jusslin et al., 2022). To address this, datasets should be expanded to include more diverse modalities, such as audio, video, and interactive tools. MLLMs should also be designed with flexible architectures capable of processing and reasoning over multiple types of inputs, allowing for a richer representation of mathematical problems (Dasgupta et al., 2023).

Dimited Domain Generalization. Mathematical reasoning spans many domains, such as algebra, geometry, diagram and commonsense, each with its own specific requirements for problemsolving (Liu et al., 2023b; Lu et al., 2022b). Math-LLMs that perform well in one domain often fail to generalize across others, which can limit their

utility. By pretraining and fine-tuning Math-LLMs on a wide array of problem types, models may handle cross-domain tasks more effectively, improving their ability to generalize across different mathematical topics and problem-solving strategies.

• Error Feedback Limitations. Mathematical reasoning involves various types of errors, such as calculation mistakes, logical inconsistencies, and misinterpretations of the problem. Currently, MLLMs lack mechanisms to detect, categorize, and correct these errors effectively, which can result in compounding mistakes throughout the reasoning process (Yan et al., 2024a; Li et al., 2024e). A potential solution is to integrate error detection and classification modules that can identify errors at each step of the reasoning process. Besides, multi-agent collaboration mechanism could be introduced, via involving multiple agents collaborating by exchanging feedback and collectively refining the reasoning process (Xu et al., 2024d).

Integration with Real-world Educational Needs. Existing benchmarks and models often overlook real-world educational contexts, such as how students use draft work, like handwritten notes or diagrams, to solve problems (Xu et al., 2024c; Wang et al., 2024d). These real-world elements are crucial for understanding how humans approach mathematical reasoning (Mouchere et al., 2011; Gervais et al., 2024). By incorporating draft notes, handwritten calculations, and dynamic problemsolving workflows into the training data, MLLMs can be tailored to provide more accurate and contextually relevant feedback for students.

5 Conclusion

In this survey, we have provided a comprehensive overview of the progress and challenges in mathematical reasoning within the context of MLLMs. We highlighted the significant advances in the development of Math-LLMs and the growing importance of multimodal integration for solving complex reasoning tasks. We identified five key challenges that are crucial for the continued development of AGI systems capable of performing sophisticated mathematical reasoning tasks. As research continues to advance, it is essential to focus on these challenges to unlock the full potential of LLMs in multimodal settings. We hope this survey provides insights to guide future LLM research, ultimately leading to more effective and human-like mathematical reasoning capabilities in AI systems.

540 Limitations

Despite our best efforts to ensure comprehensive 541 coverage of the published works, it is possible that 542 some relevant studies were overlooked. Addition-543 ally, human errors could have occurred during the 544 categorization or referencing of papers in the sur-545 vey. To minimize such errors, we made a con-546 certed effort to gather studies from multiple sources 547 and performed a multiple-round checking process. 548 While minor inconsistencies or omissions may still 549 exist, we believe this survey represents the most 550 comprehensive review of MLLM-based mathemat-551 ical reasoning to date, effectively capturing key re-552 search trends and highlighting ongoing challenges. 553

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A Details of Math-LLMs' Progress

The rapid development of general-purpose LLMs has made significant advancements in natural language processing tasks. However, the development of domain-specific models remains a core requirement, as they are better equipped to handle specialized tasks that general models may not address effectively. This is particularly true in fields such as healthcare (Liu et al., 2023a; Nazi and Peng, 2024), law (Cui et al., 2023; Zhou et al., 2024c; Wang et al., 2023c), finance (Wu et al., 2023b; Yang et al., 2023a; Zhang and Yang, 2023), and urban science (Yan et al., 2024b; Zou et al., 2025; Yan and Lee, 2024), where domain-specific knowledge is critical for high accuracy and performance.

In the case of mathematical reasoning, general models may struggle with tasks that require deep understanding of complex mathematical concepts, structures, and problem-solving steps. Therefore, the development of math-specific LLMs is of paramount importance, as these models are designed to enhance performance in mathematical reasoning, theorem proving, equation solving, and other math-intensive tasks.

Therefore, Table 4 provides a detailed overview of various math-specific LLMs (*i.e.*, Math-(LLMs), sorted by their release date. It includes information about the organization behind each model, the release date, publication details, language(s) supported, parameter size, evaluation benchmarks, and whether the model is open source.

Key findings are summarized as follows:

- 1. **Release Trends:** The models started emerging in 2020, with a significant increase in the number of releases from 2022 onward, indicating a growing interest in developing mathspecific LLMs.
- 2. **Parameter Sizes:** There is a noticeable trend towards larger parameter sizes, with some models offering up to 130B parameters, reflecting the increasing computational capacity for handling complex mathematical tasks.
- 3. Evaluation Benchmarks: Many models are evaluated on popular benchmarks like GSM8K, MATH, and MMLU, highlighting the focus on improving performance across well-established mathematical reasoning datasets.

4. Multilingual Support: While most models1547are focused on English, a few (e.g., MathGPT1548& Math-LLM) also support Chinese, showing1549a trend towards multilingual capabilities.1550

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5. **Open Source:** A significant number of models are open-source, allowing broader access and fostering further research and development in the field.

In summary, the table reflects the rapid development of specialized Math-LLMs, with an increasing trend towards larger models, comprehensive evaluation benchmarks, and support for multilingual applications.

B Details of Metrics

B.1 Discriminative Metrics

Discriminative tasks refer to evaluation processes where the outputs are typically binary, such as "Yes" or "No". These tasks often include multiplechoice questions, fill-in-the-blank problems, or judgment assessments. The evaluation metrics focus on LLM's accuracy in specific task types and its ability to control biases.

Accuracy (ACC): It measures the proportion of correctly predicted outcomes. The value should be as high as possible.

$$ACC = \frac{\sum_{1,m} x_i}{\sum_{1,n} y_j}$$
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Where x_i represents the correct output for the *i*-th instance, y_j represents the *j*-th instance, *m* is the number of the correct instances and *n* is the number of the total instances.

Exact match: It evaluates the congruence between the answers generated by LLM and the correct ones. Specifically, in cases where the answer produced LLM coincides with the reference answer, a score of 1 point will be assigned. Conversely, if there is any discrepancy between them except for bias, a score of 0 point will be given.

 F_1 score: It combines two crucial aspects, namely precision and recall, in order to comprehensively assess the accuracy of LLM. It is calculated as :

$$F_1 = 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$
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The value of the F_1 score ranges from 0 to 1. A higher value of the F_1 score indicates better overall performance of LLM in terms of both precision and recall.

Math (M)LLMs	Organization	Release Date	Publication	Language	Parameter Size	Evaluation Benchmarks OI	pen Source
GPT-f (Polu and Sutskever, 2021)	OpenAI	Sep 2020	-	English	160M/400M/700M		~
Hypertree Proof Search (Lample et al., 2022)	Meta	Nov 2022	NeurIPS'22	English		miniF2F/Metamath	-
Minerva (Lewkowycz et al., 2022)	Google	Jun 2022	NeurIPS'22	English	8B/62B/540B	MATH/MMLU-STEM/GSM8k	-
JiuZhang 1.0 (Zhao et al., 2022)	RUC & iFLYTEK	Jun 2022	KDD'22	English	145M		~
GAIRMath-Abel (Chern et al., 2023)	Shanghai Jiaotong University	2023	-	English	7B/13B/70B	GSM8K/MATH/MMLU/SVAMP/SCQ5K-English/MathQA	~
JiuZhang 2.0 (Zhao et al., 2023)	RUC & iFLYTEK	2023	KDD ADS'23	English		JCAG/JBAG (MathBERT/DART/JiuZhang)	~
KwaiYiiMath (Fu et al., 2023)	Kuaishou	Jan 2023	-	English/Chinese	13B	GSM8K/CMath/KMath	-
MathCoder (Wang et al., 2023a)	CUHK	Jan 2023	ICLR'24	English	7B/13B	GSM8K/MATH	~
Llemma (Azerbayev et al., 2023)	Princeton University & Eleuther AI	Jan 2023	-	English	7B/34B	MATH/GSM8k/MMLU-STEM/SAT/OCWCourse	~
kywork-13B-Math (Zeng et al., 2024) 🖈	SkyworkAI	Jan 2023	-	English	7B/13B	GSM8K/CMATH/MATH	~
MathGPT (TALEducation, 2023)★	TAL Education Group	Aug 2023	-	English/Chinese	130B	CEval-Math/AGIEval-Math/APE5K/CMMLU-Math/GAOKAO- Math/Math401	-
WizardMath (Luo et al., 2023)	Microsoft	Aug 2023	-	English	7B/70B	GSM8K/MATH	~
MAmmoTH1 (Yue et al., 2023)	UWaterloo	Sep 2023	ICLR'24	English	7B/13B/70B	GSM/MATH/MMLU-STEM/AQuA/NumGLUE	~
dathGLM (Yang et al., 2023b)	Tsinghua & Zhipu.AI	Sep 2023	-	English	10M/100M/500M/2B(Arith.)&335M/6B/10B (MWP)	BIG-bench/ Ape210K	~
AetaMath (Yu et al., 2023)	Cambridge & Huawei	Sep 2023	-	English	7B/13B/70B	GSM8k/MATH	~
DeepSeekMath (Shao et al., 2024)	DeepSeek AI	Jan 2024		English	7B	GSM8K/MATH/OCW/SAT/MMLU-STEM/CMATH/Gaokao- MathCloze/Gaokao-MathQA	~
nternLM2.5-StepProver (Wu et al., 2024c)	Shanghai AI Lab	Jan 2024	-	English/Chinese	7B	miniF2F/Lean-Workbook-Plus/ProofNet/Putnam	~
ChatGLM-Math (Xu et al., 2024f)	Zhipu.AI	Apr 2024	-	English/Chinese	32B	MathUserEval/Ape210k/CMath/GSM8k/MATH/Hungarian	-
tho-Math (Lin et al., 2024)	Microsoft	Apr 2024	-	English	1B/7B	GSM8K/MATH/MMLU-STEM/SAT/SVAMP/ASDiv/MAWPS/TAB/MQ/	Α 🖌
DeepSeekProver-V1 (Xin et al., 2024b)	DeepSeek AI	May 2024	-	English	7B	miniF2F/FIMO	-
internLM2-Math (Wu et al., 2024c)	Shanghai AI Lab	May 2024	-	English/Chinese	1.8B/7B/20B/8x22B	MiniF2F-test/MATH/MATH-Python/GSM8K/MathBench- A/Hungary/	~
iuZhang 3.0 (Zhou et al., 2024a)	RUC & iFLYTEK	May 2024	-	English	7B/8B	GSM8k/MATH/G-Hard/SVAMP/MAWPS/ASDiv/TabMWP	~
AmmoTH2 (Yue et al., 2024c)	UWaterloo	May 2024	-	English	7B/8B	TheoremQA/MATH/GSM8K/GPQA/MMLU-STEM/BBH	~
Math-LLaVA (Shi et al., 2024)	NUS	Jun 2024	EMNLP Finding'24	English	13B	MMMU/MATH-V/MathVista	~
Aathstral (MistralAI, 2024)	Mistral AI	Jul 2024	-	English	7B	MATH/GSM8K/GREMath/AMC2023/AIME2024/MathOdyssey	-
DeepSeek-Prover-V1.5 (Xin et al., 2024a)	DeepSeek AI	Aug 2024	-	English	7B	miniF2F-test/ProofNet	~
wen2-Math (Qwen, 2024)	Alibaba	Aug 2024		English/Chinese	1.5B/7B/72B	GSM8K/Math/MMLU-STEM/CMATH/GaoKaoMath Cloze/- GaoKao Math QA	~
Qwen2-Math-Instruct (Qwen, 2024)	Alibaba	Aug 2024	-	English/Chinese	1.5B/7B/72B	GSM8K/MATH/Minerva Math/GaoKao2023 En/Olympiad Bench/College Math/MMLU STEM/Gaokao/CMATH/CNMiddle School 24/AIME24/AMC23	~
MathGLM-Vision (Yang et al., 2024b) 🖈	Tsinghua & Zhipu.AI	Sep 2024		English	9B/19B/32B	MathVista/MathVista(GPS)/MathVerse/Math- Vision/MMMU/MathVL	-
Math-LLM (Liu et al., 2024c) 🖈	East China Normal University	Sep 2024	-	Chinese	8.26B/7B/72B	CMM-Math/MathVista/Math-V	-
Qwen2.5-Math (Yang et al., 2024a)	Alibaba	Sep 2024	-	English/Chinese	1.5B/7B/72B	GSM8K/MATH/MMLU-STEM/CMATH/GaoKao Math	~
Kwin-LM (Ni et al., 2024)	Microsoft	May 2024	-	English	7B/13B/70B	GSM8K/MATH	~
nath-specialized Gemini 1.5 Pro 🖈	Google	Not launched yet	-	English		MATH/AIME2024/Math Odyssey/HiddenMath/IMO Bench	-
0-math (MoonshotAI, 2024)	Moonshot AI	Nov 2024		English/Chinese		KAOYAN/MATH/AIME/OMNI- MATH/GAOKAO/ZHONGKAO	-
Duolingo Math (Duolingo, 2024)	Duolingo	2024		English			-
Khanmigo (KhanAcademy, 2024)	Khan Academy	2024		English			-
Squirrel LAM (SquirrelAiLearning, 2024) 🖈	Souirrel Ai Learning	2024		Chinese			

Table 4: **Overview of math-specific LLMs (sort by release date).** * refers to those designed to support the multimodal mathematical setting.

Macro- F_1 score: It calculates the F_1 score for each category separately and then takes the average of the F_1 scores of all categories, so as to obtain the overall performance of LLM on all categories.

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Round-r accuracy: It is the proportion of correct answers given by a model on the question set Qr in round r. It is calculated as follows:

$$ACC_r(M) = \frac{\sum_{q \in Q_r} I[M(q) = g_t(q)]}{|Q_r|}$$

Here, $ACC_r(M)$ represents the accuracy of LLM M on question set Q_r in round r. I is an indicator function. When the answer M(q) given by M for question q is consistent with the true answer $g_t(q)$ of the question, the value of I is 1; otherwise, it is 0. The symbol $\sum_{q \in Q_r}$ means summing over all questions in question set Q_r . $|Q_r|$ indicates the number of questions in question set Q_r .

 ACC_{step} : It is used to evaluate LLM's ability to identify the first step where an error occurs. The accuracy for identifying the first erroneous step is calculated as follows:

$$ACC_{step} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}(S_{step,i} = G_{step,i})$$

1614Here, N is the total number of samples. For the *i*-th1615sample, $S_{step,i}$ is the predicted step where the error1616occurs, and $G_{step,i}$ is the ground truth label for the1617first erroneous step. The indicator function $\mathbb{I}(\cdot)$ 1618returns 1 if the predicted step matches the ground1619truth and 0 otherwise.

 ACC_{cate} : It is for assessing LLM's performance in categorizing the type of error. The accuracy for error categorization is defined by 1622

$$ACC_{cate} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}(C_{error,i} = G_{error,i})$$
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Here, N is the total number of samples. For the *i*-th sample, $C_{error,i}$ is the predicted error category, and $G_{error,i}$ is the ground truth label for the error category. The indicator function $\mathbb{I}(\cdot)$ has the same meaning as in the previous metric, returning 1 if the predicted error category matches the ground truth and 0 otherwise.

The skill success rate: It measures the proportion of a model correctly applying major skills in problem-solving. It's calculated by analyzing test questions and determining correct use of major skills, then finding the ratio to total questions. For example, in triangle area calculation, checking use of the area formula. Similarly, the secondary skill success rate focuses on the proportion of correct application of secondary skills like understanding graphic properties and unit conversion, calculated by analyzing problem-solving and finding the ratio to total questions.

The False Positive Rate (FPR): It is the proportion of cases where the evaluation LLM misjudges an incorrect answer as a correct one. A low FPR indicates that LLM rarely misjudges incorrect student answers as correct.

The False Negative Rate (FNR): It is the proportion of cases where the evaluation LLM mis-

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judges a correct answer as an incorrect one. A low FNR indicates that LLM is relatively accurate in correctly determining whether a student's answer is correct.

Mean Squared Error (MSE): It is a metric that measures the average of the squares of the differences between the LLM's predicted values and the actual true values. It is calculated as:

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Here, *n* represents the number of samples. For the *i*-th sample, y_i is the true value and \hat{y}_i is the predicted value by LLM. The summation symbol $\sum_{i=1}^{n}$ means summing up the squared differences for all n samples. Dividing by n gives the average squared difference, which is the MSE. MSE should be as low as possible.

Average-Case Accuracy (A_{avg}) : This metric evaluates the average accuracy of LLM across all variants of a seed question. It is calculated as the proportion of correct answers across all variants and seed questions. The formula is:

$$A_{avg} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{M} \sum_{j=1}^{M} I[\operatorname{Ans}(i,j) = \operatorname{GT}(i,j)]$$

where N is the total number of seed questions, M is the number of variants per seed question, and I[Ans(i, j) = GT(i, j)] checks if the answer matches the ground truth.

Worst-Case Accuracy (A_{wst}) : This evaluates the worst-case performance by considering the minimum accuracy across all variants of a seed question. It reflects the robustness of LLM against challenging variations. The formula is:

$$A_{wst} = \frac{1}{N} \sum_{i=1}^{N} \min_{j \in [1,M]} I[\operatorname{Ans}(i,j) = \operatorname{GT}(i,j)]$$

B.2 Generative Metrics

Generative tasks involve evaluating the content generated by LLM, typically encompassing free-form answers and responses to open-ended questions. These tasks focus primarily on assessing the extent of hallucinations in the generated content, especially when the content is not faithful to the given images. Evaluating generative tasks often requires more complex metrics, such as CHAIR and Faithscore, which measure hallucinations across different categories, including objects, attributes, and relationships within the generated content. These

metrics provide a nuanced understanding of the fidelity and reliability of MLLMs in producing content aligned with the visual and textual inputs.

Reasoning Robustness (RR): This metric measures the relative robustness of LLM by comparing the worst-case performance to the average-case performance. The formula is:

$$RR = \frac{A_{wst}}{A_{avg}}$$
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Repetition Consistency (RC): This evaluates the consistency of LLM's responses across repeated queries for the same question variant. It helps distinguish between variability due to randomness and systematic errors. The formula is:

$$RC(i,j) = \frac{1}{K} \sum_{k=1}^{K} I[\operatorname{Ans}_k(i,j) = \operatorname{Ans}(i,j)]$$
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where K is the number of repetitions.

OpenCompass Scoring: It is a comprehensive evaluation framework that leverages the OpenCompass platform to assess the generative capabilities of LLM across multiple dimensions. Perplexity (PPL) evaluates the naturalness and fluency of generated text, with lower scores indicating greater model confidence and the ability to produce contextually coherent sequences. Simultaneously, CircularEval assesses the robustness and consistency of LLM in multiple-choice scenarios by evaluating its performance across N random permutations of the options in an N-option question. A question is deemed correctly answered only if LLM provides the correct response for all permutations, highlighting its ability to handle randomized inputs reliably.

Bilingual Evaluation Understudy (BLEU): It evaluates the quality of text generation by measuring n-gram overlap between generated and reference texts, focusing on precision and brevity. Its formula is:

BLEU = BP · exp
$$\left(\sum_{n=1}^{N} w_n \log p_n\right)$$
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where BP is the brevity penalty, calculated as 1 1730 if c > r, or $\exp(1 - r/c)$ if $c \le r$, with c and 1731 r representing the lengths of the generated and 1732 reference texts, respectively. w_n denotes n-gram 1733 weights (typically uniform), and p_n is the precision 1734 of n-grams of size n. BLEU scores range from 0 to 1735 1 (often expressed as percentages, 0-100%), with 1736 higher scores indicating greater similarity between 1737 the generated and reference texts. 1738 1739Recall-Oriented Understudy for Gisting1740Evaluation-L (ROUGE-L): It evaluates the qual-1741ity of generated text by measuring its similarity to1742reference text, focusing on sequence alignment and1743structural consistency through the Longest Com-1744mon Subsequence (LCS). It calculates recall as the1745proportion of the LCS length relative to the refer-1746ence text length. The formula of recall is:

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$$R = \frac{\text{LCS}(\text{Generated}, \text{Reference})}{\text{Length}(\text{Reference})}$$

1748It also calculates precision as the proportion of the1749LCS length relative to the generated text length.1750The formula is:

$$P = \frac{\text{LCS}(\text{Generated}, \text{Reference})}{\text{Length}(\text{Generated})}$$

The F_1 score is a harmonic mean of precision and recall, expressed as:

$$F_1 = \frac{(1+\beta^2) \cdot P \cdot R}{\beta^2 \cdot P + R}$$

where β (commonly set to 1) controls the weighting of recall and precision. ROUGE-L scores range from 0 to 1, with higher scores indicating greater similarity between the generated and reference texts.

Consensus-based Image Description Evaluation (CIDEr): It is designed for image description tasks, measuring the semantic relevance of generated descriptions by calculating the TF-IDF weighted n-gram similarity with reference descriptions. The formula is:

$$CIDEr_n(c_i, S_i) = \frac{1}{m} \sum_{j=1}^m \frac{g^n(c_i) \cdot g^n(s_{ij})}{\|g^n(c_i)\| \cdot \|g^n(s_{ij})\|}$$

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$$CIDEr(c_i, S_i) = \sum_{n=1}^{N} w_n CIDEr_n(c_i, S_i)$$

Here, c_i is the candidate description, $S_i = \{s_{i1}, s_{i2}, \ldots, s_{im}\}$ is the set of reference descriptions, and m is the number of references. $g^n(c_i)$ and $g^n(s_{ij})$ are the TF-IDF weighted n-gram vectors for the candidate and reference descriptions, with $||g^n(c_i)||$ and $||g^n(s_{ij})||$ being their magnitudes. w_n is the weight for n-grams of different lengths, usually $w_n = 1/N$, where N is the maximum n-gram length. Scores range from 0 to 10, with higher scores indicating stronger alignment between candidate and reference descriptions.

Mathematical Symbol Similarity: This metric measures the similarity between the correct steps in a reasoning process and the steps generated by LLM, using symbolic computation software to perform the evaluation.

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GPT Scoring: This metric evaluates the generated content based on scores assigned by GPT or other language models, focusing on the linguistic coherence and logical consistency of the text.

Context Length Generalization Efficacy (**CoLeG-E**): It is a metric used to measure LLM's consistency in answering variations of the same question across different context lengths. It is defined as:

$$CoLeG - E(M) = \frac{\sum_{q \in Q_R} \left[\bigwedge_{r=1}^R I[M(q^r) = gt(q^r)] \right]}{|Q_R|}$$

where Q_R represents the set of all questions under evaluation, and q^r refers to the *r*-th variation of a question *q*, corresponding to a specific context length. $M(q^r)$ is LLM's predicted answer for the *r*-th variation, while $M(q^r)$ denotes the ground truth answer. The indicator function $I[\cdot]$ equals 1 if LLM's answer matches the ground truth, and 0 otherwise. The logical AND operator $\Lambda_{r=1}^R$ ensures that the model must answer all variations of a question correctly for that question to be considered correctly answered.

Context Length Generalization Robustness (**CoLeG-R**): It measures LLM's robustness to context length expansion by quantifying the relative drop in accuracy from initial to extended questions. It is defined as:

$$CoLeG-R(M) = 1 - \frac{ACC_0(M) - ACC_R(M)}{ACC_0(M)}$$

Here, $ACC_0(M)$ is the LLM's accuracy on the initial set of shorter-context questions Q_0 , and $ACC_R(M)$ is its accuracy on the extended longer-context questions Q_R . Higher CoLeG-R values indicate better robustness, with less performance degradation across context lengths.

Performance Drop Rate (PDR): This metric measures the relative decline in model performance when transitioning from the original dataset to the perturbed dataset. It is defined as:

$$PDR = 1 - \frac{\sum_{(x,y)\in D_a} I[LLM(x), y]/|D_a|}{\sum_{(x,y)\in D} I[LLM(x), y]/|D|}$$
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where D is the original dataset and D_a is the perturbed dataset. I[LLM(x), y] is an indicator function that checks if the LLM's output matches the ground truth y.

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 $AP(q) = \frac{1}{m} \sum_{k=1}^{m} P(k)$

 $P(k) = \frac{\text{\# relevant ans retrieved up to position } k}{k}$

Accurately Solved Pairs (ASP): ASP measures

the percentage of seed questions and their perturbed

variations that are both correctly answered by LLM.

 $ASP = \frac{\sum_{x,y;x',y'} I[LLM(x), y] \cdot I[LLM(x', y')]}{N \cdot |D|}$

where x and x' are a seed question and its variation,

respectively. N is the number of perturbations per

question. |D| is the total number of seed questions.

swers higher in its output list for a given query. It

 $mAP = \frac{1}{|Q|} \sum_{q \in Q} AP(q)$

Mean Average Precision (mAP): It is a metric that evaluates LLM's ability to rank relevant an-

It is defined as:

is defined as:

Here, Q represents the set of all queries in the dataset. AP(q) is the Average Precision for query q, calculated as the mean of the precision values P(k) at ranks where relevant answers appear. P(k) is the precision at rank k, representing the proportion of relevant answers retrieved up to position k. m is the total number of relevant answers for query q.

Training Set Coverage (TSC): It measures how effectively LLM has learned to generate correct solutions for tasks similar to those in its training set. TSC is particularly useful in cross-domain or cross-modal tasks, where it assesses LLM's ability to generalize learned patterns to problems aligned with its training data. Higher TSC scores indicate better learning and consistency, while lower scores suggest insufficient training or overfitting.

Pass@N: This metric measures the likelihood of LLM generating at least one correct solution within N attempts for a given problem. Formally:

 $Pass@N = \mathbb{E}_{Problems}[min(c, 1)]$

1865where c represents the number of correct answers1866out of N responses. A higher Pass@N indicates1867a greater chance of producing a correct answer1868in multiple attempts, reflecting LLM's potential1869capability.

PassRatio@N: This metric calculates the pro-
portion of correct answers among N generated re-
sponses for a given problem. It is defined as:1870
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$$PassRatio@N = \mathbb{E}_{\text{Problems}}\left[\frac{c}{N}\right]$$
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where c is the count of correct answers. This metric1874reflects LLM's stability in consistently generating1875correct answers. It can be considered analogous to1876Pass@1 but offers reduced variance.1877