Neuro-Symbolic Integration Brings Causal and Reliable Reasoning Proofs

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Abstract

Though prompting LLMs with various reasoning structures produces intermediate reasoning steps along with answers, these steps are not ensured to be causal and reliable due to the inherent defects of LLMs. Tracking such deficiencies, we present a neuro-symbolic integration framework, in which a neural LLM is used to represent the knowledge of the problem while an LLM-free symbolic solver is adopted to do deliberate reasoning using the knowledge. Specifically, customized meta-interpreters are implemented to generate intermediate reasoning proofs and to support various search strategies. These reasoning proofs are ensured to be causal and reliable because of the deterministic executing nature of the symbolic solvers. We conduct experiments on two logical reasoning and one arithmetic reasoning datasets. On ProofWriter, our method surpasses the CoT baseline by nearly double in reasoning accuracy and more than triple in reasoning proof similarity. On GSM8K, our method also shows accuracy improvements and nearly doubled proof similarity. Our code is released at `https://anonymous.4open.science/r/CaRing-477B`.

1 Introduction

Large language models (LLMs), like LLaMA-2 (Touvron et al., 2023) and GPT-4 (OpenAI, 2023), are shown to be effective on several reasoning tasks but still struggle with structurally complex reasoning problems, such as logical reasoning (Tafjord et al., 2021) and arithmetic reasoning (Cobbe et al., 2021; Ribeiro et al., 2023). To tap into the potential of LLMs for better complex reasoning, existing works primarily focus on iteratively prompting LLMs to search over reasoning structures such as chains (e.g., CoT) (Wei et al., 2022; Wang et al., 2023; Zhou et al., 2023), trees (e.g., Tree-of-Thoughts, RAP) (Yao et al., 2023; Long, 2023; Hao et al., 2023), and graphs (e.g., Graph-of-Thoughts) (Besta et al., 2023; Zhang et al., 2023; Sun et al., 2023).

Despite the effectiveness of such methods over various complex reasoning problems, it is observed that they often give correct results with erroneous intermediate steps (Ye and Durrett, 2022; Saparov and He, 2023; Ribeiro et al., 2023). For example, Ribeiro et al. (2023) showed that even though prompting GPT-3 given structured intermediate steps as demonstrations yields an average accuracy of 33.84% on five complex reasoning datasets, the average similarity between the predicted and the gold reasoning proofs is merely 0.72%. This discrepancy between reasoning accuracy and reasoning proof similarity raises pressing concerns about the reliability and causality of the underlying reasoning process in LLMs, as shown in Figure 1.

The discrepancies identified in the reasoning capabilities of LLMs underscore their limitations in emulating human-like deliberate reasoning. One natural solution could be adopting an LLM-free deliberate reasoning engine. Inspired by the seminal work of Kowalski (1979), which argued that a problem-solving algorithm benefits from separating the *Logic* component (i.e., the knowledge which can be used to solve the problem) and the *Control* component (i.e., the problem-solving strategy with which the knowledge can be used), we propose a neuro-symbolic integration approach consisting of two components: (1) LLM-based symbolic representation generator (SymGen; §3.1), which translates natural languages into formal knowledge representations that can be used for symbolic inference; (2) LLM-free symbolic inference engine (SymInfer; §3.2), which performs deliberate reasoning by executing the symbolic representations-The execution strategy is implemented with our customized meta-interpreters, allowing (i) tracing of the reasoning process (§3.2.1); (ii) adoption of various search strategies (§3.2.2). Most importantly, by putting LLMs under quarantine during deliberate reasoning, our approach produces reasoning traces that are strictly causal and immune from hallucinations.

To demonstrate its effectiveness in producing better reasoning proofs, we evaluate our CaRing (Causal and Reliable Reasoning) framework on three reasoning datasets that contain reasoning proof annotations, including two logical reasoning datasets, ProofWriter (Tafjord et al., 2021) and PrOntoQA (Saparov and He, 2023), and one arithmetic reasoning dataset, GSM8K (Ribeiro et al., 2023). CaRing consistently outperforms the CoT baseline and existing methods in terms of answer accuracy, reasoning proof similarity, and reasoning proof accuracy. On the challenging ProofWriter dataset, CaRing using Code-LLaMA-34B yields an answer accuracy of 96.5% and a reasoning proof similarity of 81.0%, while previous SoTA achieved an answer accuracy of 79.7%. Further analysis indicates CaRing remains robust when the reasoning problem becomes more complex.

Overall, our contributions in this paper include:

- As far as we know, CaRing is the first LLM-based neuro-symbolic integration approach that customizes symbolic interpreters to generate reasoning proofs.
- We present an implementation using Prolog representations and conduct experiments on three datasets. Empirically, our framework gains significant improvements over strong baselines and existing methods with both final answers and reasoning proofs.

2 Related Work

2.1 Explainable Complex Reasoning

The Chain-of-Thought prompting method, which found out reasoning with LLMs benefits from generating intermediate steps, has sparked a recent trend in how to better do reasoning while remaining explainable. Several works investigated using other reasoning structures, such as trees (Yao et al., 2023; Long, 2023) and graphs (Besta et al., 2023; Zhang et al., 2023). These approaches have shown improved performance, particularly in complex reasoning tasks where the processes involved are often more intricate than simple linear chains. However, despite the alignment of their reasoning proof structures with the gold-standard proofs, these methods still face challenges in ensuring causality and reliability. This limitation stems from their reliance on LLMs for deliberate reasoning, which are prone to hallucinations and may compromise causality.

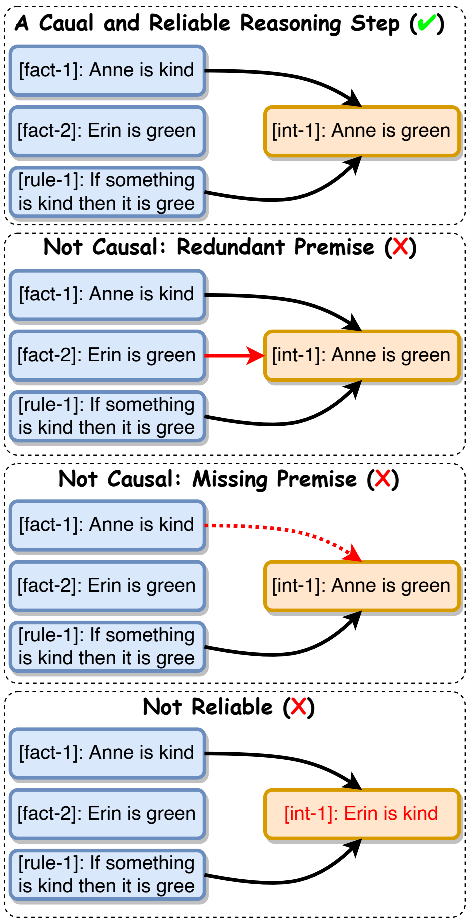


Figure 1: Illustrations of how causality and reliability play important roles in reasoning. LLMs may be (i) non-causal by selecting redundant premises or ignoring relevant ones and (ii) non-reliable by hallucinating incorrect contents during inference.

Some other recent works adopted a less structured manner (Tafjord et al., 2022; Creswell et al., 2023; Kazemi et al., 2023). For example, Selection-Inference (Creswell et al., 2023) divides the reasoning process into two phases: (1) the Selection phase for selecting the premises that might be relevant for the next round of inference, and (2) the Inference phase for conducting a single reasoning step with the selected knowledge fragments.

2.2 Neuro-symbolic Reasoning

Neuro-symbolic systems attempt to leverage the strengths of both neural networks and symbolic reasoning (Andreas et al., 2016; Neelakantan et al., 2017; Hudson and Manning, 2019; Gupta et al., 2020; Nye et al., 2021). This includes the use of neural networks for pattern recognition and learning from unstructured data, integrated with sym-

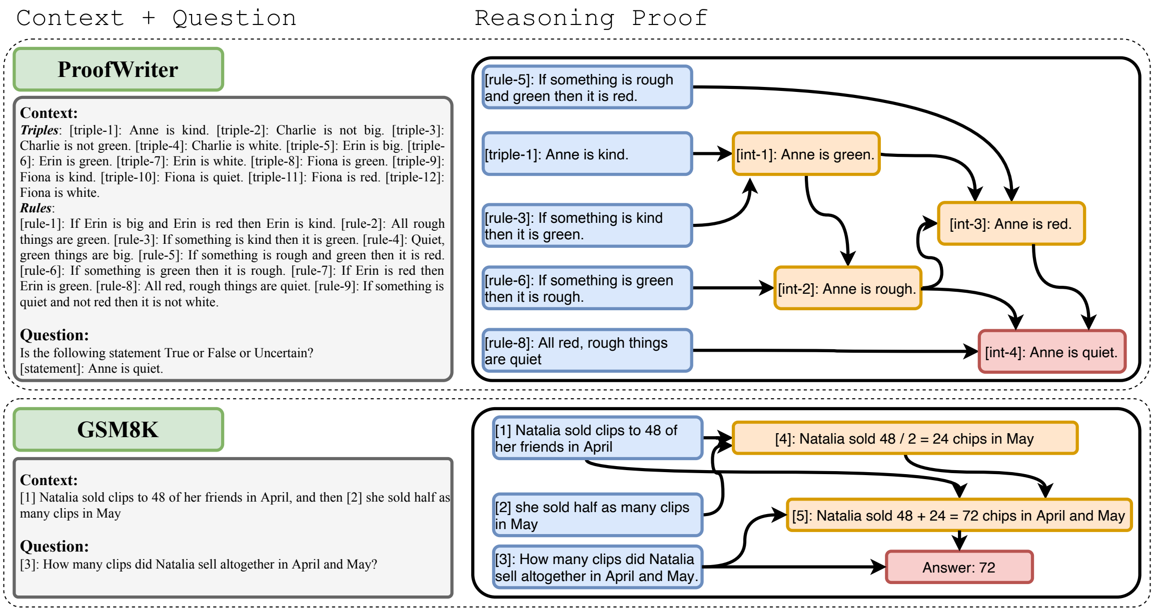


Figure 2: Two examples of complex/structured reasoning problems from ProofWriter and GSM8K, respectively. The reasoning proofs in such problems formulate directed acyclic graphs (DAGs) in a multi-step and multi-premise manner.

bolic systems for rule-based reasoning and knowledge representation. Despite significant progress, neuro-symbolic reasoning faces challenges, notably in scalability and the efficient integration of learning and reasoning components.

Recent advancements in neuro-symbolic research, particularly in reasoning over text, have utilized LLMs to encapsulate knowledge from unstructured human languages, as noted in Lyu et al. (2023); Pan et al. (2023). These methods typically translate natural language into symbolic representations for subsequent execution-based reasoning. However, they have not fully explored the capabilities of symbolic solvers in generating detailed reasoning proofs. In contrast, our approach leverages customized meta-interpreters in conjunction with symbolic solvers to uncover and articulate the underlying reasoning proofs. This not only enhances the transparency of automatic reasoning systems but also simplifies the process for humans to verify their correctness and safety.

3 CaRing

The problems we focus on are featured with structured or complex reasoning. As depicted in Figure 2, these problems typically necessitate multi-step and multi-premise reasoning over a directed acyclic graph (DAG), where individual nodes signify distinct knowledge fragments and directed edges denote reasoning steps. Each reasoning step uses existing knowledge to infer new relevant knowledge. Numerous knowledge fragments are often aggregated to infer a new one, which we denote as "multi-premise". The solver usually performs multiple such steps to reach an ultimate goal, which we denote as "multi-step". This entire reasoning process naturally composes a DAG.

We are interested in providing accurate answers along with causal and reliable explanations for such reasoning problems. This motivates our investigation of LLM-free deliberate reasoning engines. The seminal work of Kowalski (1979) proposed that *Algorithm = Logic + Control*, where *logic* refers to the knowledge which can be used to solve the problem and *control* refers to the problem-solving strategy in which the knowledge can be used. They further proved that an algorithm benefits from separating the *logic* component and the *control* component. Inspired by this, we present CaRing (Causal and Reliable Reasoning), a modular approach consisting of two components:

- SymGen: LLM-based symbolic representation generator (§3.1), which translates natural languages into formal symbolic knowledge representations that can be used for symbolic inference. A major difference between previous work and our method is that we only use LLMs to represent knowledge but not to do deliberate reasoning.

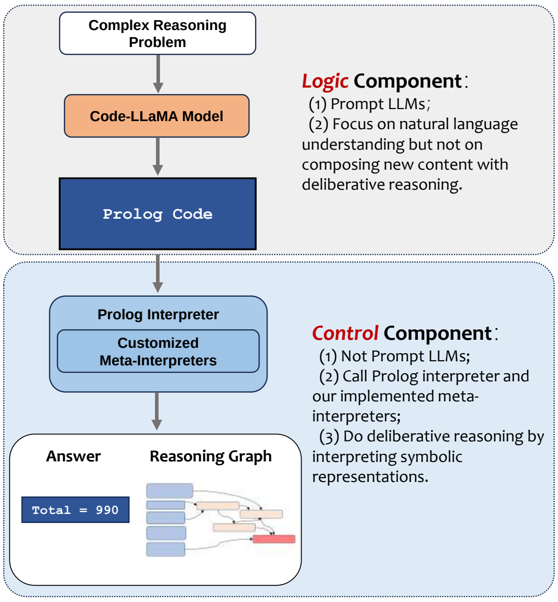


Figure 3: Illustration of our CARING framework, consisting of a *Logic* component and a *Control* component.

Natural Language	Prolog Code
Fiona is green.	`green(fiona).`
All red, rough things are quiet.	`quiet(X) :-` `    red(X), rough(X).`
Tina makes $18.00 an hour.	`wage(18.00).`

(she is eligible for overtime,) which is paid by your hourly wage + 1/2 your hourly wage.	`overtime_wage(W) :-`
	`    wage(W1),`
	`    W is 1.5 * W1.`

Table 1: Examples of natural languages and their Prolog representations. It can be seen that the Prolog code is highly declarative, so the LLM in SYMGEN is only required to do straightforward natural language understanding and translation but not reasoning. In other words, the LLM does not need to infer new knowledge, thus avoiding hallucination as much as possible.

- SYMINFER: LLM-free symbolic inference engine (§3.2), which performs deliberate reasoning by executing the symbolic representations provided by SYMGEN. By implementing customized meta-interpreters, SYMINFER supports (i) causal and reliable tracing of the reasoning process (§3.2.1); (ii) various search strategies, such as Depth-First Search (DFS) and Iterative Deepening Search (IDS) (§3.2.2).

The execution-based tracing approach of SYMINFER guarantees both causality and reliability. Under the principle of **Causality**, the inference of a new knowledge piece is strictly linked to those existing fragments that are relevant, ensuring precise and limited attribution. This implies that a causal relationship is established only when the preceding event (at the base of the edge) directly influences the subsequent event (at the apex). Regarding **Reliability**, the content within each newly inferred node is the result of a deterministic process, safeguarding it from the kinds of erroneous hallucinations often encountered in outputs from LLMs.

3.1 SYMGEN: Symbolic Representation Generator

To represent *logic* (i.e., the knowledge which can be used to solve the problem), we adopt a popular logic programming language, Prolog (Colmerauer and Roussel, 1996). Prolog is a declarative programming language, in which *logic* is expressed as relations (called Facts and Rules), with several examples shown in Table 1. A computation is initiated by running a query over these relations. We will delve into the computation of Prolog in §3.2.

Though LLMs are prone to hallucinate erroneous facts when composing new knowledge, they are shown to be powerful at understanding natural languages and directly translating them into other formats (Ye and Durrett, 2022; Saparov and He, 2023). To utilize such a strong point while avoiding the defect, we only use LLMs to translate natural languages into Prolog representations but not to do deliberate reasoning. Specifically, we few-shot prompt LLMs with several human-written in-context demonstrations, each containing a problem and corresponding Prolog representations, which are later used for symbolic inference.

3.2 SYMINFER: Symbolic Inference Engine

We use SYMINFER to produce answers and reasoning traces by executing the aforementioned symbolic representations. Since we adopt Prolog to represent knowledge, our symbolic inference engine is naturally instantiated with Prolog interpreters. By default, the SWI-Prolog (Wielemaker et al., 2012) interpreter adopts the Depth-First Search (DFS) backtracking strategy and does not yield reasoning proofs. We implement customized Prolog-based meta-interpreters to achieve two goals: (i) To produce reasoning proofs; (ii) To adopt better search algorithms other than DFS.

3.2.1 Reasoning Tracer

We implement a Prolog meta-interpreter to show the reasoning proofs:

```
% Define the operator for proofs
:- op(750, xfy, =>).

% Proof tree generation
mi_tree(true, true).
mi_tree((A,B), (TA,TB)) :-
        mi_tree(A, TA),
        mi_tree(B, TB).
mi_tree(G, builtin(G)) :-
        predicate_property(G, built-in
    ),
        !,
        call(G).
mi_tree(g(G), TBody => G) :-
        mi_clause(G, Body),
        mi_tree(Body, TBody).
```

We showcase how a reasoning trace is induced using the example below. Given a knowledge base like:

```
parent_of(X, Y) :- mother_of(X, Y).
parent_of(X, Y) :- father_of(X, Y).
grandparent_of(X, Y) :-
    parent_of(X, Z), parent_of(Z, Y).
mother_of(morty, beth).
father_of(beth, rick).
```

and a query:

```
?- mi_tree(g(grandparent_of(morty, Who
    )), Proof).
```

the output would be

```
Who=rick,
Proof=(((true=>mother_of(morty, beth))
    =>parent_of(morty, beth), (true=>
    father_of(beth, rick))=>parent_of(
    beth, rick))=>grandparent_of(morty,
     rick)).
```

The output proof is ensured to be causal and reliable since a symbolic approach generates it.

3.2.2 Search Strategy

The default search strategy of Prolog is DFS, which may lead to infinite loops. For example, given the knowledge base:

```
parent_of(X, Y) :- offspring_of(Y, X).
offspring_of(X, Y) :- parent_of(Y, X).
parent_of(X, Y) :- mother_of(X, Y).
parent_of(X, Y) :- father_of(X, Y).
mother_of(jack, anna).
```

and a query `?- parent_of(jack, Who).` , the backtracking process would repeat over the first two lines without resorting to other lines due to DFS. To address this issue, we adopt Iterative Deepening Search (IDS), in which the backtracking process performs a series of depth-limited searches, each with an increasing depth limit. This leverages the strengths of both Breadth-First Search (BFS) and DFS. Our Prolog meta-interpreter for IDS is implemented as:

```
% Depth-limited meta-interpreter with
    proof tree generation
mi_limit(true, true, N, N).
mi_limit((A,B), (TA,TB), N0, N) :-
        mi_limit(A, TA, N0, N1),
        mi_limit(B, TB, N1, N).
mi_limit(g(G), TBody => G, N0, N) :-
        N0 #> 0,
        N1 #= N0 - 1,
        mi_clause(G, Body),
        mi_limit(Body, TBody, N1, N).

% Iterative deepening with proof tree
    generation
mi_id(Goal, Proof) :-
        length(_, N),
        mi_limit(Goal, Proof, N, _).

% Iterative deepening with maximum
    depth with proof tree generation
mi_id_limit(Goal, Proof, MaxDepth) :-
        between(1, MaxDepth, N),
        mi_limit(Goal, Proof, N, _).
```

In practice, other search strategies can also be implemented according to the nature of the target problems, such as Uniform-Cost Search (UCS) and Beam Search.

4 Experiments

We briefly introduce our experimental settings in §4.1 and show the experiment results in §4.2.

4.1 Experimental Settings

We present our experimental settings in this section, including our implementation details of the two components (§4.1.1), a brief introduction of the adopted datasets (§4.1.2) and the baselines (§4.1.4).

4.1.1 Implementation

SymGen We adopt the Code-LLaMA (Rozière et al., 2023) family as the base LLMs to translate natural languages into Prolog representations. Our prompting paradigm is in a pure few-shot in-context-learning (ICL) prompting style, without detailed human-written instructions. Each ICL demonstration comprises a question and a piece of Prolog code.

SymInfer We adopt SWI-Prolog (Wielemaker et al., 2012) and PySwip[1] packages to implement the symbolic inference engine. We set the maximum depth to be 20 for Iterative Deepening Search and the number of generated reasoning paths to 20.

[1] https://github.com/yuce/pyswip

4.1.2 Datasets

We evaluate CARING on three popular complex reasoning datasets, including two logical reasoning datasets (ProofWriter (Tafjord et al., 2021) and PrOntoQA (Saparov and He, 2023)) and one arithmetic dataset (GSM8K (Cobbe et al., 2021; Ribeiro et al., 2023)).

ProofWriter ProofWriter (Tafjord et al., 2021) is a commonly-used logical reasoning dataset. It contains many small rulebases of facts and rules, expressed in English. Each rulebase has a set of questions (English statements) that can either be proven true or false using proofs of various depths, or the answer is "Unknown" (in open-world setting, OWA). The proofs can naturally be represented as directed acyclic graphs (DAGs). The dataset is divided into several sub-sets according to maximum proof depth, namely $\{0, \leq 1, \leq 2, \leq 3, \leq 5\}$. We follow previous work (Pan et al., 2023) to use a 600-instance subset sampled from the most difficult depth-5 test set. We also report additional results on the full depth-5 test set in Appendix §A.1.1.

PrOntoQA PrOntoQA (Saparov and He, 2023) is a synthetic question answering dataset designed for diagnosing the logical reasoning ability of LLMs. Each example aims to validate the feasibility of a statement given a context. We report results on two subsets so we can compare CARING with previous methods. As for the results reported in Table 4, we follow Pan et al. (2023) to adopt the most difficult depth-5 *fictional characters* sub-set, which contains 500 statement-context pairs. As for the results in Table 3, we use the subset adopted by Hao et al. (2023). Similar to ProofWriter, the proofs provided by the dataset can be naturally represented as DAGs.

GSM8K GSM8K (Cobbe et al., 2021) is a multi-step arithmetic reasoning dataset composed of high-quality grade school math word problems. The original GSM8K dataset contains reasoning explanations written in natural language, which raises difficulties in evaluating intermediate steps automatically. Recently, Ribeiro et al. (2023) released a subset that contains 270 questions annotated with structured reasoning proofs in the format of DAGs. We adopt this subset to enable the evaluation of reasoning proofs.

		Method	Acc (%)	Proof Sim (%) All	Proof Sim (%) Correct
GPT-4*		CoT	67.41	–	–
		ToT	70.33	–	–
		CR	71.67	–	–
		DetermLR	79.17	–	–
		Logic-LM	79.66	–	–
Code-LLaMA	**7B**	CoT	46.33	9.69	14.95
		Ours	91.00	72.91	84.39
	13B	CoT	46.50	15.69	25.86
		Ours	95.67	80.65	86.00
	34B	CoT	52.00	15.76	27.74
		Ours	**96.50**	**81.02**	**86.12**

Table 2: Results on the subset of ProofWriter adopted by Pan et al. (2023). The default setting is 2-shot. "All": on all instances. "Correct": on correctly-predicted instances. *All GPT-4 numbers are from Sun et al. (2023).

4.1.3 Evaluation Metrics

Ribeiro et al. (2023) proposed two novel metrics to evaluate the quality of the generated reasoning proofs in addition to the prevalent answer accuracy metric. Similar to them, we adopt the following metrics to evaluate both the answers and the generated reasoning proofs.

Answer Accuracy Answer accuracy measures a model's ability to predict the correct answer. A prediction is deemed correct if it is (i) the same as the gold option for multi-choice problems and (ii) the same integer as the gold answer for arithmetic reasoning problems. This metric is the upper bound for other metrics since a reasoning graph would be marked as incorrect without evaluation if the answer is marked as incorrect. We report this metric for all datasets.

Reasoning Proof Similarity As shown in Figure 2, the problems that we are interested in naturally compose reasoning proofs in the format of directed acyclic graphs (DAGs). Reasoning proof similarity $\mathrm{sim}(\mathcal{G}_g, \mathcal{G}_p)$ measures the graph similarity between the gold and the predicted reasoning graphs. We follow Ribeiro et al. (2023) to adopt the graph edit distance function $\delta(\mathcal{G}_g, \mathcal{G}_p)$. This function quantifies the graph edit distance by determining the minimum number of operations required over nodes and edges to transform one graph into the other, thereby enabling a comparison of $\mathcal{G}_g$ and $\mathcal{G}_p$ based on their structural similarities. The reasoning graph similarity is normalized to $[0, 1]$

Base LLM	#Param	#Shot	Method	Acc (%)	Proof Acc (%)	
					All	Correct
LLaMA-1*	33B	8-shot	CoT	87.8	64.8	–
			RAP	94.2	78.8	–
Code-LLaMA	13B	2-shot	CoT	80.2	52.4	53.4
			Ours	**99.0**	**98.2**	**99.2**

Table 3: Results on the PrOntoQA subset that was adopted by RAP (Hao et al., 2023) for comparison with their method. The results marked with * are from their paper.

		Method	Acc (%)	Proof Acc (%)	
			All	Correct	
GPT4*		CoT	98.8	–	–
		Logic-LM	83.2	–	–
Code-LLaMA	7B	CoT	52.0	24.8	28.5
		Ours	98.8	98.4	99.6
	13B	CoT	61.0	32.2	35.9
		Ours	99.4	98.8	99.4
	34B	CoT	82.8	41.0	41.0
		Ours	**100.0**	**100.0**	**100.0**

Table 4: Results on the depth-5 subset of PrOntoQA. The default setting is 2-shot. Results marked with * are GPT-4 results reported by Logic-LM (Pan et al., 2023).

	Method	Acc	Proof Sim (%)	
			All	Correct
7B	CoT	13.70	4.99	36.39
	Ours	12.22	6.57	53.72
13B	CoT	15.56	5.76	37.03
	Ours	21.48	11.66	**54.26**
34B	CoT	35.19	13.04	37.07
	Ours	**42.22**	**22.91**	54.25

Table 5: Results on GSM8K. The default setting is 5-shot.

as:

$$\text{sim}(\mathcal{G}_p, \mathcal{G}_g) = 1 - \frac{\delta(\mathcal{G}_p, \mathcal{G}_g)}{\max\{|N_p| + |E_p|, |N_g| + |E_g|\}} \quad (1)$$

where $|N_p|$ and $|E_p|$ denote the count of nodes and edges, respectively, within the predicted reasoning graph. A similar notation applies to $|N_g|$ and $|E_g|$, which represent the number of nodes and edges in the gold graph. Note that the reasoning graph similarity is set to zero if the predicted answer is incorrect. We report this metric for ProofWriter and GSM8K.

Reasoning Proof Accuracy This metric evaluates the exact match between the gold and the predicted reasoning proofs in terms of both reasoning graph structures and textual contents[2]. The reasoning proof accuracy is either 1 or 0 for a single instance, making it a discrete version of reasoning proof similarity. Since this metric requires the dataset to have structured content to enable automatic evaluation, we can only apply it to PrOntoQA, which is specifically designed for easy parsing of the proofs.

[2]Note that our implementation here is simpler than that of Ribeiro et al. (2023) because we only apply this metric to PrOntoQA, which is easy to get evaluated.

4.1.4 Baselines

All baselines prompt the LLMs with few-shot in-context-learning (ICL) demonstrations.

Chain-of-Thought (CoT) CoT prompting (Wei et al., 2022) prompts LLMs with ICL demonstrations that contain both intermediate reasoning steps and answers. It serves as a popular and strong baseline for prompting LLMs to solve problems.

Logic-LM Logic-LM (Pan et al., 2023) is a neuro-symblic method that adopts symbolic solvers for logical reasoning problems. The main difference between Logic-LM and CARING is: Logic-LM adopts various solvers for multiple datasets and only focuses on answer accuracy, while CARING universally uses one solver (i.e., SWI-Prolog); and more importantly, CARING showcases how SWI-Prolog interpreters can be customized to generate intermediate reasoning proofs and to adopt various search (i.e., problem-solving) strategies.

Search-based Methods We include several search-based methods as baselines. We directly adopt the released results in their papers, since it is too time-consuming to implement these methods on our own. For ProofWriter, we compare our method with GPT-4 based Tree-of-Thoughts (ToT; (Yao et al., 2023)), Cumulative Reasoning (CR; (Zhang et al., 2023)), and DetermLR (Sun et al., 2023). We cannot make comparisons on reasoning proofs because these methods only reported reasoning accuracy. For PrOntoQA, we compare our method with RAP (Hao et al., 2023), in terms of both reasoning accuracy and reasoning proof accuracy.

4.2 Main Results

Tables 2, 3, 4 and 5 show the experimental results on our adopted datasets.

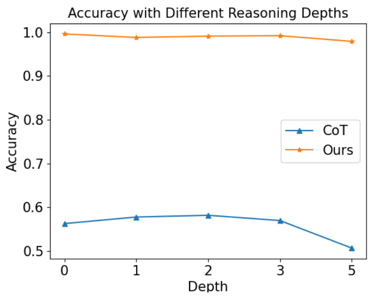
ProofWriter The results on ProofWriter are presented in Table 2. CARING demonstrates notable improvements over existing baselines, particularly in terms of reasoning proof similarity. Utilizing Code-LLaMA-34B, CARING achieves a remarkable answer accuracy of 96.50% and a reasoning proof similarity of 81.02%, significantly surpassing the most powerful method using GPT-4 that obtains an accuracy of 79.66%.

PrOntoQA The results on PrOntoQA are presented in Tables 3 and 4. CARING achieves almost full accuracy with the 13B model. Comparing with RAP, CARING obtains better results in terms of both answer accuracy and proof accuracy even using a smaller base LLM and fewer ICL demonstrations. CARING also outperforms CoT and Logic-LM that use GPT-4.

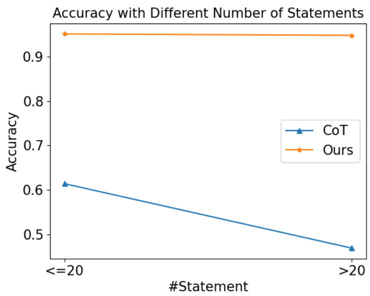
GSM8K The results on GSM8K are presented in Table 5. This dataset is more challenging than the previous two logical reasoning datasets for CARING, since it is generally believed that symbolic languages are restricted by their limited expressiveness and cannot properly handle the ambiguity in real-world human languages. Surprisingly, with the 34B model, CARING outperforms the strong CoT baseline by a large margin and almost doubles the reasoning proof similarity (22.91% vs. 13.04%). We attribute such improvements to increasingly powerful LLMs, which can correctly translate ambiguous human languages into formal symbolic representations.

4.3 When Reasoning Becomes More Complex

A key difficulty confronted by reasoning systems is the rapid expansion of possible states as the reasoning process becomes more complex, such as when additional statements are considered or the depth of inference is greater. To investigate how our method handles more complex reasoning problems, we conduct experiments under two controlled settings: (1)



(a) Answer accuracy with different reasoning depths.



(b) Answer accuracy with different number of statements.

Figure 4: Answer accuracy when reasoning problems become more complex.

#Depth ↑: How does the answer accuracy change with #Depth being $<= 0$, $<= 1$, $<= 2$, $<= 3$ and $<= 5$, respectively; (2) **#Statements** ↑: How does the answer accuracy change with #Statements being $<= 20$ and > 20, respectively.

As shown in Figures 4a and 4b, with increasing levels of reasoning intricacy, the answer accuracy of CARING remains steady. In contrast, CoT sees significant decreases in answer accuracy under both settings. This verifies the robustness of CARING against complex reasoning.

5 Conclusion

This paper presents a framework to address the erroneous reasoning proof problem of LLM-based reasoning systems. Specifically, we develop a neuro-symbolic method called CARING, which produces high-quality reasoning proofs for complex reasoning problems. By implementing customized meta-interpreters for executing Prolog representations and putting LLMs under quarantine during the reasoning phase, CARING ensures the reasoning proofs to be causal and reliable. We conduct experiments on two logical reasoning datasets and one arithmetic reasoning dataset. Experimental results demonstrate our method achieves significant improvements with both final answers and intermediate reasoning proofs. Further analysis indicates CARING remains robust when the reasoning problems become more complex.

Limitations

We observe two limitations regarding our framework:

- The generalization ability of CARING is restricted by the expressiveness of the concerning symbolic representations. In this paper, we showcase a Prolog-based implementation. Other symbolic representations could be explored to generalize CARING to more reasoning tasks.
- CARING requires powerful LLMs as symbolic representation generators, which is suggested by the results on GSM8K. This dependence might prevent it from being applied to productions.

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		Acc	Proof Sim	
			All	Correct
7B	Direct	41.78	–	–
	Direct (3-Shot)	43.32	–	–
	CoT	40.95	11.27	17.20
	CoT (3-shot)	42.58	11.52	21.58
	Ours	92.43	75.85	**86.68**
13B	Direct	43.44	–	–
	Direct (3-shot)	44.31	–	–
	CoT	45.88	16.16	27.32
	CoT (3-shot)	54.70	23.18	32.48
	Ours	96.16	80.74	86.34
34B	Direct	44.00	–	–
	Direct (3-shot)	45.93	–	–
	CoT	52.32	15.08	26.30
	CoT (3-shot)	56.50	24.12	34.61
	Ours	**98.11**	**83.17**	85.65

Table 6: Results on ProofWriter. "All" and "Correct" refer to "on all instances" and "on correctly-predicted instances", respectively. "Proof Sim" refers to "Proof Graph Similarity" while "Proof EM" means "Proof Graph Exact Match". The default setting is 2-shot. We additionally conduct 3-shot experiments for baselines to include all types of labels in the in-context demonstrations because this dataset contains three labels: {`true`, `false`, `uncertain`}. We do not conduct 3-shot experiments for our method because it is not sensitive to the number of labels due to its reasoning-by-execution nature.

A Appendix

A.1 Additional Results

A.1.1 Results on ProofWriter

Table 6 shows the results from our implementation on the depth-5 test set of ProofWriter.