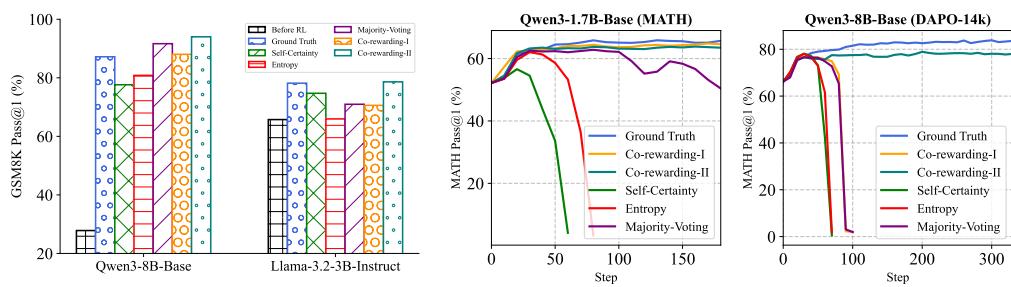


000 001 002 003 004 005 CO-REWARDING: STABLE SELF-SUPERVISED RL FOR 006 ELICITING REASONING IN LARGE LANGUAGE MODELS 007 008 009

010 **Anonymous authors**
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ABSTRACT

011 While reinforcement learning with verifiable rewards (RLVR) is effective to im-
012 prove the reasoning ability of large language models (LLMs), its reliance on human-
013 annotated labels leads to the scaling up dilemma, especially for complex tasks.
014 Recent self-rewarding methods investigate a label-free alternative to unlock the
015 reasoning capabilities of LLMs, yet they frequently encounter the non-negligible
016 training collapse issue, as the single-view supervision signal easily forms the
017 self-consistent illusion, yielding the reward hacking. Inspired by the success of
018 self-supervised learning, we propose *Co-rewarding*, a novel self-supervised RL
019 framework that improves training stability by seeking complementary supervision
020 from another views. Specifically, we instantiate Co-rewarding in two ways: (1) *Co-*
021 *rewarding-I* is a data-side instantiation that derives reward signals from contrastive
022 agreement across semantically analogous questions; and (2) *Co-rewarding-II* is a
023 model-side instantiation that maintains a slowly-updated reference teacher with
024 pseudo labels to realize self-distillation. Intuitively, such instantiations introduce
025 different levels of discrepancy to increase the difficulty of training collapse on
026 trivial reasoning solutions. **We also explore their orthogonally combined version to**
027 **further boost the performance.** Empirically, Co-rewarding exhibits stable training
028 across various setups, and outperforms other self-rewarding baselines by +3.31%
029 improvements on average on multiple mathematical reasoning benchmarks, es-
030 pecially by +7.49% on Llama-3.2-3B-Instruct. Notably, Co-rewarding reaches
031 or even surpasses RLVR with ground-truth (GT) label in several cases, such as a
Pass@1 of 94.01% on GSM8K with Qwen3-8B-Base remarkably higher than GT.



044 1 INTRODUCTION 045

046 Large language models (LLMs) (Achiam et al., 2023; Dubey et al., 2024; Qwen et al., 2025) have
047 demonstrated remarkable general-purpose capabilities in a wide range of linguistic tasks (Hendrycks
048 et al.). To further elicit their reasoning ability in complex scenarios, reinforcement learning with
049 verifiable rewards (RLVR) (Shao et al., 2024; Yu et al., 2025) is developed for post-training with
050 externally verifiable signals like program execution results (Luo et al., 2025) or mathematical
051 equivalence (Shao et al., 2024). Despite the impressive improvement, the reliance on high-quality
052 ground-truth (GT) labels of RLVR remains as a major bottleneck (Ouyang et al., 2022; Bai et al.,
053 2022) in the spirit of the scaling law, which subsequently motivates the emerging exploration of
self-rewarding methods with unlabeled data (Zhao et al., 2025b; Zuo et al., 2025; Zhang et al., 2025b).

054 One prominent line of such label-free methods leverages the internal signals (e.g., entropy (Zhang
 055 et al., 2025c; Prabhudesai et al., 2025) and self-certainty (Zhao et al., 2025b)) to strengthen the
 056 confidence of the model in reasoning. Another critical line seeks the answer-level consensus (Zuo
 057 et al., 2025; Shafayat et al., 2025) to construct pseudo labels as reward basis. While effective initially,
 058 these self-rewarding approaches frequently exhibit non-negligible training collapse (Zhang et al.,
 059 2025d) (indicated as right of Figure 1), which limits the scalability of such label-free training manners.

060 The collapse phenomenon stems from reward hacking (Laidlaw et al., 2025) under self-consistent
 061 illusion: the reward signal is internally produced by the policy model from a single-view data
 062 perspective, which is easily trapped by trivial solutions along with training (see Figure 7). Specifically,
 063 for entropy- or certainty-based rewards, the policy model may concentrate probability mass on a small
 064 set of tokens and produce repetitive strings that minimize entropy or maximize self-certainty (Zhang
 065 et al., 2025d). And for consensus-based rewards, the policy model can converge to a consistent yet
 066 incorrect answer that attains high consensus across rollouts (Shafayat et al., 2025). Overall, the policy
 067 model continually reduces uncertainty without sustained gains in correctness, inflating the reward but
 068 eroding exploration and diversity. It ultimately collapses when a persistent hacking strategy emerges.

069 To this end, we introduce *Co-rewarding*, a self-supervised RL framework that seeks complementary
 070 supervision from another views, inspired by self-supervised learning (Chen et al., 2020; Grill et al.,
 071 2020; Caron et al., 2021). Conceptually, one fundamental characteristic of self-rewarding methods
 072 lies on that supervision intertwined with current policy on single-view outputs, for which we propose
 073 to seek reasoning invariance across different views (see Figure 2). Specifically, we investigate two
 074 initiations of Co-rewarding: (1) *Co-rewarding-I*: a data-side initiation that constructs rewards via
 075 contrastive agreement across semantically analogous questions, each providing pseudo labels for
 076 the other; and (2) *Co-rewarding-II*: a model-side initiation that introduces an extra teacher with
 077 dynamically updated policy and provides stable pseudo-labels insulated from current online policy.
 078 Additionally, we also explore the combined instantiation, *Co-rewarding-III*, which integrates data-side
 079 cross-supervision with model-side self-distillation to further boost the performance.

080 By introducing cross-view supervision on data and decoupling the reward signal from the current
 081 policy, Co-rewarding effectively mitigates training collapse and yields stable self-supervised RL
 082 training. Extensive experiments across multiple datasets validate the stability and superiority of
 083 Co-rewarding, compared to several recent baselines across several LLM families including Qwen3/2.5
 084 and Llama. Notably, both Co-rewarding-I and -II reach or exceed training with ground-truth labels in
 085 several settings, such as achieving up to 94.01% Pass@1 on GSM8K. Our main contributions are

- 086 • We introduce a new perspective, from self-supervised learning, to elicit reasoning capability via
 087 another views of supervision, which prevents the model from training collapse (Section 3.1).
- 088 • We propose Co-rewarding, a novel self-supervised RL framework that is initiated by the data and
 089 model sides to construct self-generate rewards to promote stably reasoning elicitation (Section 3.2).
- 090 • We empirically demonstrate the general effectiveness of Co-rewarding to achieve superior reasoning
 091 performance on LLMs, and also present various ablation studies and further analyses (Section 4).

093 2 PRELIMINARY

095 **Problem Setups.** Given a LLM π_θ parameterized by θ and a dataset \mathcal{D} of question-answer pairs
 096 (x, a) , the model generates a response $y \sim \pi_\theta(\cdot | x)$ autoregressively. Let $y = (y_1, \dots, y_n)$, where
 097 each token is sampled as $y_t \sim \pi_\theta(\cdot | x, y_{<t})$ given the generated prefix $y_{<t}$. We consider the LLM
 098 outputs a stepbystep reasoning trace and a final answer. A verifiable reward function $r(a, y)$ compares
 099 the extracted answer $\text{ans}(y)$ with the ground truth a as follows:

$$100 \quad r(a, y) = \begin{cases} 1 & \text{If } \text{ans}(y) \text{ is correct with answer } a, \\ 101 & 0 \quad \text{If } \text{ans}(y) \text{ is incorrect with answer } a. \end{cases} \quad (1)$$

102 Then, the general objective of training LLM for reasoning via RLVR (Shao et al., 2024; Yu et al.,
 103 2025) can be formulated with the policy model π_θ as follows:

$$104 \quad \max_{\pi_\theta} \mathbb{E}_{(x, a) \in \mathcal{D}, y \sim \pi_\theta(x)} [r(a, y) - \beta \cdot \text{KL}[\pi_\theta(y|x) || \pi_{\text{ref}}(y|x)]], \quad (2)$$

106 where π_{ref} is an initial reference policy, and β is a coefficient controlling the KL divergence to prevent
 107 excessive deviation from the reference model. Intuitively, the training target is to maximize the
 108 reward in passing specific reasoning questions while maintaining the general capability of LLM.

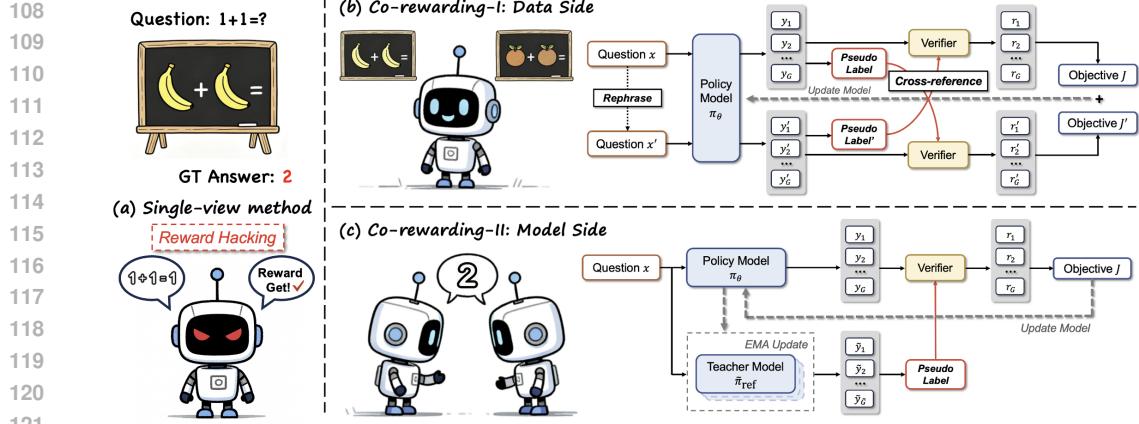


Figure 2: **Illustration of Co-rewarding framework:** Unlike single-view methods that rely only on internal reward signal on original question (a), Co-rewarding introduces complementary supervision. On the data side (b), paraphrased questions yield pseudo-labels for cross-reference. On the model side (c), teacher model isolated from current policy provides stabilized pseudo-labels for updates.

Group Relative Policy Optimization (GRPO). In practice, we adopt GRPO (Shao et al., 2024), a widely used and representative optimization method for objective Eq. (2) that estimates the advantage by normalizing the reward across multiple sampled outputs for the same question. Specifically, for a given question x , GRPO samples G outputs from the old policy π_{old} as $\{y_i\}_{i=1}^G \sim \pi_{\text{old}}(\cdot|x)$. It then computes a reward for each output y_i via a deterministic reward function, forming a group of rewards $\{r(a, y_i)\}_{i=1}^G$ to estimate the advantage \hat{A}_i as follows:

$$\hat{A}_i = \frac{r(a, y_i) - \text{mean}(\{r(a, y_i)\}_{i=1}^G)}{\text{std}(\{r(a, y_i)\}_{i=1}^G)}. \quad (3)$$

Then, the target policy is optimized by maximizing the advantage while ensuring the policy model remains close to the reference policy:

$$\mathcal{J}_{\text{GRPO}}(\theta) = \mathbb{E}_{(x, a) \in \mathcal{D}, \{y_i\}_{i=1}^G \sim \pi_{\text{old}}(\cdot|x)} \underbrace{\frac{1}{G} \sum_{i=1}^G \frac{1}{|y_i|} \sum_{t=1}^{|y_i|} \left(\min \left[c_{i,t}(\theta) \hat{A}_{i,t}, \text{clip}(c_{i,t}(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_{i,t} \right] - \beta \mathbb{D}_{\text{KL}}(\pi_{\theta} || \pi_{\text{ref}}) \right)}_{\mathcal{R}_{\theta}(\hat{A})}, \quad (4)$$

where

$$c_{i,t}(\theta) = \frac{\pi_{\theta}(y_{i,t}|x, y_{i,< t})}{\pi_{\theta_{\text{old}}}(y_{i,t}|x, y_{i,< t})}, \quad \mathbb{D}_{\text{KL}}(\pi_{\theta} || \pi_{\text{ref}}) = \frac{\pi_{\theta}(y_{i,t}|x, y_{i,< t})}{\pi_{\text{ref}}(y_{i,t}|x, y_{i,< t})} - \log \frac{\pi_{\text{ref}}(y_{i,t}|x, y_{i,< t})}{\pi_{\theta}(y_{i,t}|x, y_{i,< t})} - 1. \quad (5)$$

Note that the $\text{clip}(\cdot, 1 - \epsilon, 1 + \epsilon)$ in Eq. (4) is used to ensure that updates do not deviate excessively from the old policy by bounding the policy ratio between $1 - \epsilon$ and $1 + \epsilon$ in a risk function $\mathcal{R}(\hat{A})$. We also provide a comprehensive discussion on additional training variants for RLVR, such as DAPO (Yu et al., 2025) and Dr. GRPO (Liu et al., 2025a), which we leave in Appendix A due to space limits.

3 CO-REWARDING

In the following, we present Co-rewarding in detail, a novel self-supervised RL framework for LLM to elicit the latent reasoning capability through the intuition of seeking complementary supervision.

3.1 CONCEPTUAL PHILOSOPHY: INVARIANCE BEYOND THE SINGLE-VIEW

At the core of self-rewarding methods lies a fundamental tension: the model derives supervisory signals from its own outputs, inevitably intertwining supervision with policy and risks collapse. True reasoning competence, however, cannot be reduced to the mere correctness of isolated answers. It

162 should instead reflect invariance that extends beyond the single-view output for consistency. This calls
 163 for training signals that remain valid across different data views or persist throughout the temporal
 164 evolution of the model, providing a more reliable basis on which self-supervised RL can rely. In this
 165 aspect, stability arises from invariance that prevents reasoning against superficial variations in data
 166 and guides the model towards increasingly valid reasoning trajectories throughout training.

167 This philosophy yields our Co-rewarding framework, whose core idea is to ground self-supervised
 168 RL in invariance rather than the suspicious single-view feedback. We instantiate it in two orthogonal
 169 ways and one combined version: by enforcing analogy-invariance on the data side (Co-rewarding-I),
 170 by disentangling supervision through temporal invariance on the model side (Co-rewarding-II), and
 171 by integrating both mechanisms in a unified instantiation (Co-rewarding-III).

173 3.2 TWO INITIATIONS OF CO-REWARDING FRAMEWORK

174 **Co-rewarding-I: on the Data Side.** Inspired by contrastive learning, such as SimCLR (Chen et al.,
 175 2020) and InfoNCE (Oord et al., 2018), where two views of the same data are encouraged to have
 176 similar representations, we hypothesize an analogy-invariance inductive property of LLMs in eliciting
 177 reasoning capacity: questions that share the same mathematical essence but differ in surface form (e.g.,
 178 via paraphrasing, background substitution, or reformatting) should elicit the comparably valid and
 179 similar reasoning results. This forms the foundation for a self-referential training signal: contrastive
 180 agreement among different question variants can serve as an optimization proxy. Co-rewarding-I
 181 defines contrastive agreement as a principle that aligns model reasoning outputs, treating consistent
 182 inter-view agreement as a signal for valid inference. This complements single-view self-rewarding
 183 strategies by introducing a form of collective validity verification with broader input consideration.

184 Building upon the discussed contrastive agreement, we initiate our *Co-rewarding-I* as illustrated in
 185 Figure 2. Formally, its learning objective can be formulated as follows based on GRPO:

$$187 \mathcal{J}_{\text{Co-rewarding-I}}(\theta) = \underbrace{\mathbb{E}_{\substack{\mathbf{x} \in \mathcal{D}, \{\mathbf{y}_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot|\mathbf{x})}} \mathcal{R}_{\theta}(\hat{A})}_{\mathcal{J}_{\text{original}}(\theta)} + \underbrace{\mathbb{E}_{\substack{\mathbf{x}' \in \mathcal{D}', \{\mathbf{y}'_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot|\mathbf{x}')}} \mathcal{R}_{\theta}(\hat{A}')}_{\mathcal{J}_{\text{rephrased}}(\theta)}, \quad (6)$$

190 where the relative advantages are estimated by the *cross-refereed* supervision as follows:

$$192 \hat{A}_i = \frac{r(\mathbf{y}'_v, \mathbf{y}_i) - \text{mean}(\{r(\mathbf{y}'_v, \mathbf{y}_i)\}_{i=1}^G)}{\text{std}(\{r(\mathbf{y}'_v, \mathbf{y}_i)\}_{i=1}^G)}, \quad \hat{A}'_i = \frac{r(\mathbf{y}_v, \mathbf{y}'_i) - \text{mean}(\{r(\mathbf{y}_v, \mathbf{y}'_i)\}_{i=1}^G)}{\text{std}(\{r(\mathbf{y}_v, \mathbf{y}'_i)\}_{i=1}^G)}. \quad (7)$$

194 Specifically, given a set of original questions, we utilize the rephrased version that keeps the semantical
 195 equivalence for the model to respond, and then collect the self-generated pseudo-labels based on the
 196 majority voting mechanism (Shafayat et al., 2025) as follows to supervise learning on the counterparts,

$$198 \mathbf{y}_v \leftarrow \arg \max_{y^*} \sum_{i=1}^G 1[\text{ans}(\mathbf{y}_i) = \text{ans}(y^*)], \quad \mathbf{y}'_v \leftarrow \arg \max_{y^*} \sum_{i=1}^G 1[\text{ans}(\mathbf{y}'_i) = \text{ans}(y^*)]. \quad (8)$$

201 The overall pipeline can be viewed as a dual-path structure with cross-reference in the reward shaping
 202 process, it may also be compatible with other self-generated feedbacks (Wang et al., 2022) on the
 203 output-side information due to the generality of the core idea. While in the current version, we choose
 204 the majority voting mechanism in the implementation for the empirical effectiveness and simplicity.

205 We summarize the pseudo code of Co-rewarding-I in Algorithm 1. Our contrastive objective operates
 206 on self-generated reasoning answers, encouraging the model to align its reasoning results to different
 207 questions that share the similar semantic intent. Formally, for each input question, the signal of
 208 Co-rewarding-I increases when the model’s output is consistent with the majority answer obtained
 209 from its analogical counterparts, and decreases when it diverges. This contrastive agreement promotes
 210 semantic invariance, implicitly increasing the difficulty of reaching trivial solutions to obtain the
 211 reward (e.g., achieving the arbitrary answers but consistent on single input) by involving data-side
 212 analogy. We leave a more intuitive case study in the Appendix D.12 to present the rephrased questions.

213 **Co-rewarding-II: on the Model Side.** On the data side, our Co-rewarding-I provides complementary
 214 supervision by involving question analogy, while its pseudo-labels are still generated by the current
 215 online policy and may depend on rephrasing quality; consequently, supervision remains partially
 216 entangled with the policy. Inspired by self- or weakly supervised methods like the representative

216 BYOL (Grill et al., 2020), DINO (Caron et al., 2021), and Co-teaching (Han et al., 2018), which share
 217 the common intuition of introducing an auxiliary network to provide supervision beyond the current
 218 model, we initiate *Co-rewarding-II* from another view of complementary supervision: a model-side
 219 strategy that sources pseudo-labels from a teacher reference, which disentangle the self-supervision
 220 reward from the online policy. To avoid the heavy cost of adding and maintaining another LLM in
 221 training, Co-rewarding-II reuses the GRPO reference model as the teacher to generate the rollouts and
 222 produce pseudo-labels. In particular, the teacher is dynamically updated as an exponential moving
 223 average (EMA) of the student policy to ensure pseudo-label quality improving as the policy improves.

224 Intuitively, we illustrate *Co-rewarding-II* in Figure 2. Its learning objective can be formulated as:

$$226 \quad \mathcal{J}_{\text{Co-rewarding-II}}^{(k)}(\theta) = \mathbb{E}_{x \in \mathcal{D}, \underbrace{\{y_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}^{(k)}(\cdot|x)}_{\text{policy student rollout}}, \underbrace{\{\tilde{y}_j\}_{j=1}^{\tilde{G}} \sim \tilde{\pi}_{\text{ref}}^{(k)}(\cdot|x)}_{\text{reference teacher rollout}}} \mathcal{R}_{\theta}(\hat{A}^{(k)}), \quad (9)$$

229 where $\{y_i\}_{i=1}^G$ are policy rollouts and $\{\tilde{y}_j\}_{j=1}^{\tilde{G}}$ are reference teacher rollouts at the k -th training step,
 230 and the estimated advantage $\mathcal{R}(\hat{A}^{(k)})$ is computed as follows:

$$232 \quad \hat{A}_i^{(k)} = \frac{r(\tilde{y}_v^{(k)}, y_i) - \text{mean}(\{r(\tilde{y}_v^{(k)}, y_i)\}_{i=1}^G)}{\text{std}(\{r(\tilde{y}_v^{(k)}, y_i)\}_{i=1}^G)}, \quad \tilde{y}_v^{(k)} = \arg \max_{y^*} \sum_{j=1}^{\tilde{G}} \mathbf{1}[\text{ans}(\tilde{y}_j) = \text{ans}(y^*)], \quad (10)$$

235 where the pseudo label $\tilde{y}_v^{(k)}$ is obtained via majority voting from reference rollouts, and the reference
 236 model is updated via an EMA with the policy to play a role of a slowly updated teacher:

$$238 \quad \tilde{\pi}_{\text{ref}}^{(k)} \leftarrow \alpha^{(k)} \cdot \tilde{\pi}_{\text{ref}}^{(k-1)} + (1 - \alpha^{(k)}) \cdot \pi_{\theta_{\text{old}}}^{(k)}, \quad \alpha^{(k)} = 1 - \frac{(\alpha_{\text{end}} - \alpha_{\text{start}})}{2} \left(1 + \cos \left(\frac{\pi k}{K} \right) \right) \quad (11)$$

240 where $\alpha^{(k)} \in (0, 1)$ is the EMA weight, updated according to a cosine annealing schedule from
 241 α_{start} to α_{end} , such that the teacher is updated rapidly at the beginning and progressively more slowly,
 242 thereby evolving smoothly and remaining temporally decoupled from the current online policy.

244 We summarize the pseudo code of Co-rewarding-II in Algorithm 2. This design can be interpreted
 245 as a kind of self-distillation, in which a slowly updated teacher supervises a faster-moving student.
 246 Such a paradigm breaks the single-step on-policy feedback loop inherent in existing self-rewarding
 247 methods (Zhao et al., 2025b; Prabhudesai et al., 2025; Shafayat et al., 2025), raises the cost of
 248 exploiting trivial low-entropy shortcuts or spurious consensus, and offers a stable reward source
 249 without introducing an additional LLM or optimizer. In this way, it effectively overcomes reward
 250 hacking and prevents training collapse by implicitly seeking a temporal invariance for true reasoning.

251 **Co-rewarding-III: Data-side + Model-side.** Given that Co-rewarding-I and Co-rewarding-II provide
 252 two complementary perspectives for constructing stable self-supervised signals, a natural exploration
 253 is to integrate both data-side cross-supervision and model-side self-distillation into a unified instantiation.
 254 We introduce *Co-rewarding-III*, which leverages analogy-invariance between each original
 255 question and its rephrased counterparts while producing pseudo-labels from the EMA-updated reference
 256 teacher. Specifically, the teacher generates rollouts for both original and rephrased questions,
 257 and the resulting pseudo-label from one side is used to supervise the other. This combination further
 258 boosts the resistance of the reward signal to hacking, promoting more stable training dynamics.

259 Formally, its learning objective can be formulated as:

$$260 \quad \mathcal{J}_{\text{Co-rewarding-III}}^{(k)}(\theta) = \mathbb{E}_{x \in \mathcal{D}, \underbrace{\{y_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}^{(k)}(\cdot|x)}_{\text{policy student rollout from original question}}, \underbrace{\{\tilde{y}'_j\}_{j=1}^{\tilde{G}} \sim \tilde{\pi}_{\text{ref}}^{(k)}(\cdot|x')}_{\text{reference teacher rollout from rephrased question}} \mathcal{R}_{\theta}(\hat{A}^{(k)})$$

$$263 \quad + \mathbb{E}_{x' \in \mathcal{D}', \underbrace{\{y'_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}^{(k)}(\cdot|x')}_{\text{policy student rollout from rephrased question}}, \underbrace{\{\tilde{y}_j\}_{j=1}^{\tilde{G}} \sim \tilde{\pi}_{\text{ref}}^{(k)}(\cdot|x)}_{\text{reference teacher rollout from original question}} \mathcal{R}_{\theta}(\hat{A}'^{(k)}), \quad (12)$$

267 where the first term supervises the original question via pseudo labels generated from its rephrased
 268 counterpart, and the second term, symmetrically, supervises the rephrased question via pseudo
 269 labels generated from the original question. The estimated advantages $\mathcal{R}_{\theta}(\hat{A}^{(k)})$ and $\mathcal{R}_{\theta}(\hat{A}'^{(k)})$ are
 computed in the similar way as in Co-rewarding-I and Co-rewarding-II. The reference teacher is

270 Table 1: **Main Results (%) of Co-rewarding and baselines trained on MATH.** Cell background
 271 colors indicate relative performance: darker colors denote better results within each model group.
 272 Additional results of Qwen2.5-3B/7B and Qwen3-1.7B-Base trained on MATH refer to Table 7.

Training Set: MATH	Mathematics				Code	Instruction	Multi-Task	
Methods	MATH500	GSM8K	AMC	AIME24	LiveCode	CRUX	IFEval	MMLU-Pro
<i>Qwen3-8B-Base</i>								
Before RL	72.4	27.82	20.93	3.75	23.41	54.75	50.89	52.92
- GT-Reward (Shao et al., 2024)	82.6	87.26	54.22	17.15	30.52	63.25	52.78	57.11
- Self-Certainty (Zhao et al., 2025b)	80.2	80.74	50.75	15.73	27.20	64.38	50.98	54.17
- Entropy (Prabhudesai et al., 2025)	80.2	87.19	49.54	15.63	29.38	62.00	51.81	54.86
- Majority-Voting (Shafayat et al., 2025)	79.8	89.76	49.09	15.83	30.52	63.38	51.80	56.93
- Co-rewarding-I (Ours)	81.2	93.70	51.20	15.10	30.81	66.00	55.79	59.95
- Co-rewarding-II (Ours)	80.8	92.42	53.46	14.48	30.23	62.83	60.70	57.50
- Co-rewarding-III (Ours)	81.4	90.98	54.07	13.33	30.71	63.75	53.69	59.10
<i>Qwen3-4B-Base</i>								
Before RL	71.2	26.15	21.08	4.58	11.00	38.88	46.43	47.23
- GT-Reward (Shao et al., 2024)	78.6	89.76	51.20	15.00	26.07	55.38	47.80	53.96
- Self-Certainty (Zhao et al., 2025b)	71.6	71.79	38.86	11.67	22.37	57.00	48.15	48.93
- Entropy (Prabhudesai et al., 2025)	77.0	88.10	47.44	10.94	25.59	52.88	50.44	49.90
- Majority-Voting (Shafayat et al., 2025)	77.4	90.07	45.33	10.10	26.54	57.50	48.78	54.35
- Co-rewarding-I (Ours)	78.8	91.28	46.08	13.85	26.64	56.50	50.35	53.26
- Co-rewarding-II (Ours)	78.0	88.86	45.93	12.17	26.25	55.00	51.30	53.88
- Co-rewarding-III (Ours)	78.6	90.75	48.80	12.71	26.16	56.00	49.23	53.08
<i>Llama-3.2-3B-Instruct</i>								
Before RL	39.2	65.73	10.54	3.75	9.86	25.37	57.32	31.14
- GT-Reward (Shao et al., 2024)	47.0	77.94	22.14	11.67	9.57	31.87	47.51	34.32
- Self-Certainty (Zhao et al., 2025b)	43.4	74.91	18.83	6.88	9.95	25.87	54.88	33.34
- Entropy (Prabhudesai et al., 2025)	43.4	66.19	20.18	6.56	11.66	24.62	54.70	33.52
- Majority-Voting (Shafayat et al., 2025)	46.8	78.77	20.48	9.27	11.00	31.25	47.96	33.18
- Co-rewarding-I (Ours)	50.2	79.45	23.80	10.00	11.28	29.88	48.89	33.77
- Co-rewarding-II (Ours)	49.8	79.30	22.59	10.73	10.80	30.63	49.90	33.61
- Co-rewarding-III (Ours)	51.6	79.91	25.45	10.42	10.43	32.50	46.37	34.50

295 also updated via EMA, as Eq. (11) in Co-rewarding-II. The other formulations and pseudo code of
 296 Co-rewarding-III are supplemented in Appendix B.1 and Algorithm 3.

297 **Remark 1.** Overall, the two instantiations of Co-rewarding embody our core idea from different
 298 perspectives: I leverages data-side analogy-invariance to provide cross supervision, while II employs
 299 model-side self-distillation to stabilize learning. Together, they reflect that stable self-supervised
 300 reasoning elicitation can emerge from both the diversity of data perspectives and the disentanglement
 301 of supervision signals. Co-rewarding-III further explores an orthogonally combined instantiation
 302 of these two sides. Moreover, Co-rewarding offers a flexible framework, in which key components,
 303 such as pseudo-labeling strategies, data rephrasing techniques, teacher model update rules, and policy
 304 optimization, can be seamlessly substituted with other advanced approaches (Yu et al., 2025).

306 4 EXPERIMENTS

309 4.1 SETUPS

311 **Backbone Models and Baselines.** We employ a diverse set of LLMs from different families and
 312 scales in our experiments, including the Qwen2.5 series (Qwen2.5-3B/7B) (Qwen et al., 2025),
 313 the Qwen3 series (Qwen3-1.7B/4B/8B-Base) (Yang et al., 2025), and the Llama3 series (Llama-
 314 3.2-3B-Instruct) (Meta, 2024). Beyond the vanilla GRPO that utilized the GT label for rewarding,
 315 we compare our Co-rewarding against several recent state-of-the-art (SoTA) self-reward reasoning
 316 approaches, denoted as Self-Certainty (Zhao et al., 2025b), Entropy (Prabhudesai et al., 2025) and
 317 Majority Voting (Shafayat et al., 2025). The details of all baselines are summarized in Appendix C.1.

318 **Implementation Details.** We implement our algorithms based on the VeRL framework (Sheng
 319 et al., 2024), and experiments are conducted on $4 \times$ H100-80GB GPUs. For our experiments, we
 320 totally use three training sets: MATH (Hendrycks et al., 2021) (7,500 questions), DAPO-14k (Yu
 321 et al., 2025) (en-version of DAPO-Math-17k, about 14.1k questions), and OpenRS (Dang & Ngo,
 322 2025) (7,000 questions). During RL training, we use a global batch size of 128, set the number of
 323 rollouts to $G = \tilde{G} = 8$ per question for Co-rewarding-I, II and III, and adopt AdamW with a learning
 rate of 3×10^{-6} . In Co-rewarding-I and III, question rephrasing is performed by the open-source

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Table 2: **Main Results (%) of Co-rewarding and baselines trained on DAPO-14k.** Cell background colors indicate relative performance: darker colors denote better results within each model group. Additional Results of Qwen3-8B-Base and Qwen3-4B-Base trained on OpenRS refer to Table 8.

Training Set: DAPO-14k	Mathematics				Code	Instruction	Multi-Task	
Methods	MATH500	GSM8K	AMC	AIME24	LiveCode	CRUX	IFEval	MMLU-Pro
<i>Qwen3-8B-Base</i>								
Before RL	72.4	27.82	20.93	3.75	23.41	54.75	50.89	52.92
- GT-Reward (Shao et al., 2024)	86.6	87.19	61.75	24.58	30.52	63.75	53.11	60.27
- Self-Certainty (Zhao et al., 2025b)	82.0	77.63	49.85	15.00	27.77	60.75	50.58	54.24
- Entropy (Prabhudesai et al., 2025)	79.4	80.82	45.48	15.00	30.14	62.00	51.56	54.57
- Majority-Voting (Shafayat et al., 2025)	78.6	91.66	50.00	11.25	30.33	61.62	51.54	55.65
- Co-rewarding-I (Ours)	78.4	88.02	51.20	11.88	29.38	62.50	50.17	55.39
- Co-rewarding-II (Ours)	80.6	94.01	54.37	16.35	31.66	67.12	53.31	59.83
- Co-rewarding-III (Ours)	81.6	92.27	53.77	17.71	32.70	66.75	55.85	60.02
<i>Qwen3-4B-Base</i>								
Before RL	71.2	26.15	21.08	4.58	11.00	38.88	46.43	47.23
- GT-Reward (Shao et al., 2024)	83.6	85.14	52.86	20.63	18.58	56.88	47.70	55.35
- Self-Certainty (Zhao et al., 2025b)	68.4	44.81	35.39	8.85	25.88	50.12	45.58	48.84
- Entropy (Prabhudesai et al., 2025)	76.6	82.79	43.37	12.81	26.35	50.75	48.20	50.22
- Majority-Voting (Shafayat et al., 2025)	73.4	64.06	40.81	9.17	26.16	53.00	48.91	51.06
- Co-rewarding-I (Ours)	73.8	75.89	43.83	10.63	26.25	50.12	46.84	51.51
- Co-rewarding-II (Ours)	77.8	91.89	48.49	14.27	26.64	54.87	48.90	52.83
- Co-rewarding-III (Ours)	79.2	90.45	48.95	15.10	27.58	54.87	50.30	54.79
<i>Llama-3.2-3B-Instruct</i>								
Before RL	39.2	65.73	10.54	3.75	9.86	25.37	57.32	31.14
- GT-Reward (Shao et al., 2024)	49.4	78.17	25.90	9.17	10.33	31.37	53.10	33.83
- Self-Certainty (Zhao et al., 2025b)	42.4	74.71	17.32	4.79	11.18	28.38	54.50	33.51
- Entropy (Prabhudesai et al., 2025)	44.0	65.85	17.32	6.56	9.95	25.00	55.78	31.95
- Majority-Voting (Shafayat et al., 2025)	42.8	70.96	17.62	8.74	10.14	29.50	54.07	32.95
- Co-rewarding-I (Ours)	46.0	70.58	20.93	7.08	9.57	27.25	53.04	32.61
- Co-rewarding-II (Ours)	49.8	78.62	19.73	8.02	10.43	32.25	51.92	34.46
- Co-rewarding-III (Ours)	48.6	76.95	21.84	8.13	9.86	30.50	49.92	34.01

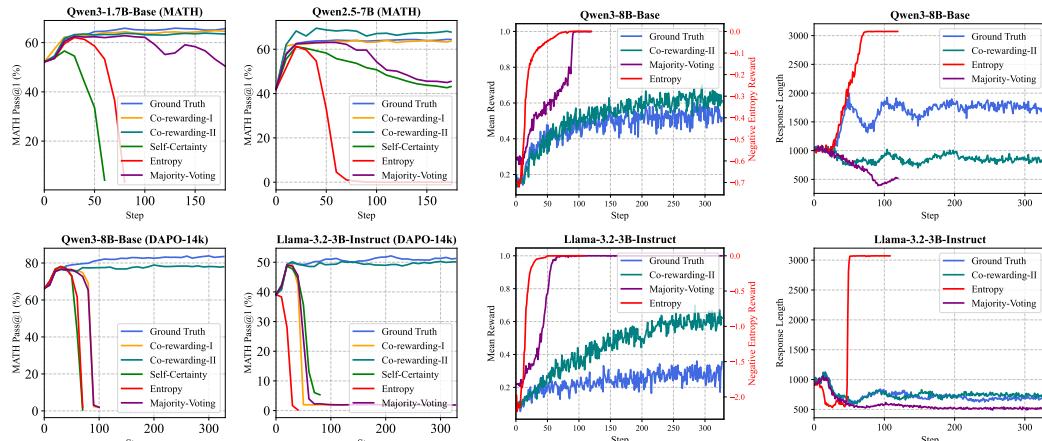


Figure 3: **Performance curves comparison** on validation set. **Top:** Qwen3-1.7B-Base and Qwen2.5-7B trained on the MATH set, and Qwen3-8B-Base and Llama-3.2-3B-Instruct trained on DAPO-14k. **Bottom:** Qwen3-8B-Base and Llama-3.2-3B-Instruct trained on the DAPO-14k set. Figure 4: **Reward** (left) and **response length** (right) of methods. Entropy reward is plotted on the right y-axis of left panels, where the reward is the negative entropy.

Qwen3-32B model. In Co-rewarding-II and III, the EMA weight is scheduled from $\alpha_{\text{start}} = 0.99$ to $\alpha_{\text{end}} = 0.9999$ using cosine annealing. More implementation details are reported in Appendix C.2.

Evaluation Details. To provide a comprehensive evaluation of model capabilities, we utilize a diverse set of benchmarks spanning mathematical reasoning, code generation, instruction-following, and general multi-task abilities. Specifically: (1) Mathematical reasoning: MATH500 (Lightman et al., 2024), GSM8K (Cobbe et al., 2021), AMC (Li et al., 2024a), and AIME24 (Zhang & Math-AI, 2024). (2) Code generation: LiveCodeBench (Jain et al., 2025) release_v6 and CRUX (Gu et al., 2024). (3) Instruction-following and multi-task abilities: IFEval (Zhou et al., 2023b) and MMLU-Pro (Wang et al., 2024). Additional evaluation details are provided in Appendix C.3.

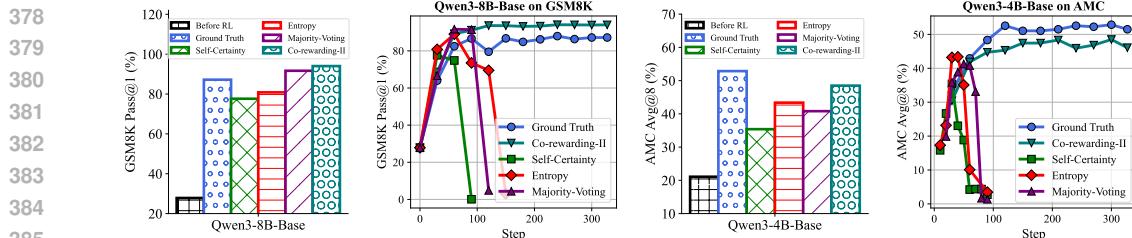


Figure 5: **Performance and Stability on GSM8K and AMC.** The gains of Co-rewarding arise from its training stability, which supports continuous improvements throughout learning.

Table 3: **Ablation study of Co-rewarding.** For Co-rewarding-I, ablations train only on original or rephrased data. For Co-rewarding-II, ablation removes EMA updates of the reference teacher.

Training Set	Methods	MATH500	GSM8K	AMC	AIME24	LiveCode	CRUX	IFEval	MMLU-Pro
<i>Qwen3-8B-Base</i>									
MATH	Co-rewarding-I	81.2	93.70	51.20	15.10	30.81	66.00	55.79	59.95
	- Majority-Voting w/ Union	80.2	93.48	49.70	15.63	31.94	64.88	54.25	59.51
	- Majority-Voting w/ Original	79.8	89.76	49.09	15.83	30.52	63.38	51.80	56.93
	- Majority-Voting w/ Rephrased	79.2	91.51	50.75	14.17	31.66	60.38	52.24	57.26
MATH	Co-rewarding-II	80.8	92.42	53.46	14.48	30.23	62.83	60.70	57.50
	- w/o Updating Reference	79.2	89.46	51.51	13.96	30.62	61.75	56.93	51.85
<i>Llama-3.2-3B-Instruct</i>									
Co-rewarding-I	50.2	79.45	23.80	10.00	11.28	29.88	48.89	33.77	
DAPo-14k	- Majority-Voting w/ Union	48.0	80.52	21.84	9.69	10.14	30.00	43.35	34.05
	- Majority-Voting w/ Original	46.8	78.77	20.48	9.27	11.00	31.25	47.96	33.18
	- Majority-Voting w/ Rephrased	44.0	78.85	21.23	8.85	10.04	17.25	47.84	33.72
	Co-rewarding-II	49.8	79.30	22.59	10.73	10.80	30.63	49.90	33.61
	- w/o Updating Reference	47.0	78.92	22.29	9.06	5.50	31.25	47.88	33.32
<i>Qwen3-8B-Base</i>									
DAPo-14k	Co-rewarding-II	80.6	94.01	54.37	16.35	31.66	67.12	53.31	59.83
	- w/o Updating Reference	78.0	88.40	51.66	15.94	30.62	63.75	52.48	58.01
<i>Llama-3.2-3B-Instruct</i>									
DAPo-14k	Co-rewarding-II	49.8	78.62	19.73	8.02	10.43	32.25	51.92	34.46
	- w/o Updating Reference	45.0	76.72	17.92	8.02	10.05	30.63	51.33	33.94

4.2 EXPERIMENTAL RESULTS

4.2.1 MAIN PERFORMANCE OF CO-REWARDING

Superior Performance of Co-rewarding over self-rewarding baselines. Table 1 and Table 2 report the experimental results trained on MATH and DAPO-14k, respectively. We observe that all three Co-rewarding instantiations (I, II, and III) occupy more darker cells in the tables, demonstrating stronger performance than other self-rewarding SoTA baselines. Specifically, Co-rewarding-I achieves an average relative performance gain of +4.42% over the best baselines across four mathematical benchmarks and models in Table 1, while Co-rewarding-II achieves a larger average relative gain of +12.90% in Table 2. Moreover, Co-rewarding-III achieves improvements on average of +7.11% and 1.72% over Co-rewarding-I and Co-rewarding-II, respectively, suggesting that integrating data-side cross-supervision with model-side self-distillation can further boost performance. Additional results on other training sets and LLMs are provided in Appendix D.1.

Surpassing GT-Reward on certain benchmarks. Surprisingly, we observe that both all three Co-rewarding instantiations (I, II, and III) outperform GT-Reward in certain cases. For example, on GSM8K, they together achieve an average improvement of +2.77% over GT-Reward in Table 1, while Co-rewarding-II further delivers a larger gain of +5.44% in Table 2. Notably, Co-rewarding-II reaches a remarkably high Pass@1 of 94.01% with Qwen3-8B-Base. This may be because GSM8K is a relatively easier benchmark, where self-supervised RL is sufficient to elicit the latent reasoning abilities of base models without relying on GT labels. Additionally, Co-rewarding also shows advantages on the coding benchmark CRUX in several cases. This may be attributed to the distribution difference between the training data and the evaluation benchmarks. Such distribution mismatch may offer opportunities for self-supervised methods to generalize on par with, or even surpass GT-supervised methods in some cases. These findings highlight the potential of self-supervised RL to elicit reasoning capabilities, particularly with Co-rewarding mitigating training collapse.

Code generalization with preserved general performance. Although trained solely on math-oriented datasets, the models show improvements on coding benchmarks, suggesting a cross-domain generalization from math to code in self-supervised reasoning elicitation. Moreover, Co-rewarding

Table 4: **Detailed performance of MMLU-Pro with Qwen3-8B-Base trained on DAPO-14k.** More results refer to Appendix D.10.

MMLU-Pro (Qwen3-8B-Base)							
Methods	biology	business	chemistry	computer sci.	economics	health	history
- GT-Reward	77.96	70.85	60.42	61.95	71.33	59.79	51.44
- Self-Certainty	75.73	58.05	50.53	56.83	69.31	54.77	50.40
- Entropy	74.76	59.70	51.33	56.10	67.90	55.87	48.04
- Majority-Voting	75.32	61.47	54.24	58.29	69.67	58.20	49.34
- Co-rewarding-I	76.85	61.22	53.45	59.02	66.82	55.62	48.29
- Co-rewarding-II	76.71	68.69	64.58	61.71	68.25	56.85	51.71
Methods	law	math	other	philosophy	physics	psychology	engineering
- GT-Reward	31.52	73.28	56.28	52.71	61.97	67.30	46.14
- Self-Certainty	30.43	63.06	51.63	46.29	51.73	66.42	41.07
- Entropy	28.97	63.96	51.51	48.90	53.04	66.80	42.32
- Majority-Voting	31.16	64.62	52.27	48.90	53.27	66.92	40.97
- Co-rewarding-I	30.34	66.17	51.73	48.90	55.19	66.42	39.63
- Co-rewarding-II	31.16	72.17	52.49	52.10	63.21	68.17	47.16

Question: $\frac{m}{n}$ is the Irreducible fraction value of $3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$, what is the value of $m + n$?

To find the value of ..., we need simplify the expression step by step.

First, let's simplify the innermost fraction:

$$3 + \frac{1}{3} = 3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$$

Now, substitute this back into the expression:

.....

Next, simplify the fraction:

Now, substitute this back into the expression:

To add these, we need a common denominator:

So, the irreducible fraction value of the expression is $\frac{109}{33}$. Here, $m = 109$ and $n = 33$.

Therefore, the value of $m + n$ is:

$$m + n = 109 + 33 = 142$$

Thus, the value of $m + n$ is 142

Majority-Voting X

To find the value of $\frac{m}{n}$

Let's simplify the expression step by step.

1. Simplify the innermost fraction:

$$3 + \frac{1}{3} \dots$$

.....

2. Simplify the next fraction:

$$3 + \frac{1}{3 + \frac{1}{3}} = 3 + \frac{1}{\frac{10}{3}} = \frac{3}{3} + \frac{1}{\frac{10}{3}} = \frac{1}{3} + \frac{1}{\frac{10}{3}} = \frac{1}{3} + \frac{3}{10} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

.....

5. Simplify the next fraction:

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}} \dots$$

.....

Entropy (repeat until max response) X

To solve the problem, we need to simplify the given expression step by step and then find the irreducible fraction form.

First, we simplify the innermost fraction:

.....

Now substitute this back into the expression:

.....

Next, simplify the fraction inside the denominator:

.....

Therefore, $m = 109$ and $n = 33$, and the value of $(m + n)$ is: $109 + 33 = 142$.

So, the final answer is 142

Co-rewarding-I ✓

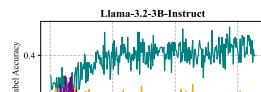
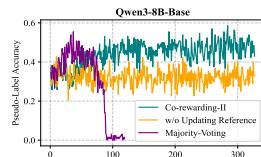


Figure 6: **Pseudo label accuracy comparison.**

Correct Answer: 142

To solve the given expression, we need to simplify it step by step.

First, simplify the innermost fraction:

Next, substitute this back into the next fraction:

Now, simplify the expression $3 + \frac{1}{33}$

The fraction $\frac{109}{33}$ is already in its simplest form because

Therefore, $(m + n) = 109 + 33 = 142$.

The value of $(m + n)$ is 142

Co-rewarding-II ✓

Figure 7: **Case study:** An example comparing the generations from *Majority-Voting*, *Entropy*, and our proposed *Co-rewarding*. The results clearly reveal the reward hacking behavior exhibited by *Majority-Voting* and *Entropy*, while ours generate the correct answer. Full results refer to Appendix D.13.

preserves general instruction-following and multi-task ability on MMLU-Pro and IFEval. As shown in Table 4, Co-rewarding-II outperforms other self-rewarding baselines in 12 of 14 MMLU-Pro categories, demonstrating that its gains do not come at the expense of broader general-domain performance. More detailed results of MMLU-Pro and IFEval refer to Appendix D.10 and D.11.

Importance of stability for performance gain. As shown in Table 2, self-rewarding baselines exhibit noticeably limited performance gain in certain cases, such as Self-Certainty with Qwen3-4B-Base on GSM8K. Figure 5 further reflects this by showing that baselines improve quickly at the beginning but soon collapse on GSM8K and AMC, whereas Co-rewarding sustains steady progress. This collapse restricts the baselines to effective training on only a small portion of the data, preventing further improvements with continued training. These observations underscore the importance of avoiding training collapse in self-supervised RL to unlock further performance gains.

4.2.2 FURTHER ANALYSIS

Co-rewarding alleviates collapse and provides stable self-supervised RL. We use 5,000 questions from the MATH test split as a validation set to monitor training process. Figure 3 shows that all three self-rewarding baselines collapse on both MATH and DAPO-14k. Co-rewarding-I remains stable on MATH but still collapses on DAPO-14k, suggesting that its stability depends on the property of training data. A plausible explanation is that the questions in MATH may provide favorable conditions for promoting diverse rephrasing variability, which is beneficial for the effectiveness of contrastive agreement in Co-rewarding-I. More discussions are provided in Appendix D.7. In contrast, Co-rewarding-II consistently maintains stability across datasets, as its design decouples supervision from the online policy and thus breaks the entanglement between supervision and the policy itself.

Co-rewarding attempts to balance exploration-exploitation. Figure 4 shows reward and response length curves. Entropy and Majority-Voting quickly reach the highest reward, indicating reward hacking rather than genuine reasoning improvement. In contrast, GT-Reward and Co-rewarding exhibit smoother, gradually increasing rewards, reflecting stable training. The response length curves further illustrate this difference: GT-Reward lengthens responses to explore correct reasoning paths; Majority-Voting collapses to short outputs, restricting exploration; and Entropy collapses

486 its probability mass onto a small set of tokens, repeatedly generating them until truncation. Co-
 487 rewarding instead maintains moderate response lengths throughout training, suggesting a balanced
 488 exploration-exploitation trade-off. Additional curves for other LLMs are provided in Appendix D.2.
 489

490 **Each part contributes to Co-rewarding.** Table 3 summarizes the ablations across two training sets.
 491 For Co-rewarding-I, we observe that it typically outperforms all three variants of Majority-Voting:
 492 models trained only on original questions, only on rephrased questions, or on their union. This
 493 indicates that the cross-supervision between original and rephrased questions plays a key role in
 494 mitigating training collapse, whereas simply adding more data does not resolve the inherent instability
 495 of single-view self-rewarding methods. Notably, training only on the original or rephrased data
 496 yields comparable results, reflecting that the quality of original and rephrased data is similar. For
 497 Co-rewarding-II, removing the EMA update of the reference teacher model causes clear degradation,
 498 highlighting the necessity of teacher updates for improving pseudo-label quality.

499 **EMA is essential in Co-rewarding-II for improving pseudo-label quality.** Figure 6 compares
 500 pseudo-label accuracy across Co-rewarding-II, “w/o Updating Reference”, and Majority-Voting.
 501 Co-rewarding-II steadily improves accuracy as training progresses, while “w/o Updating Reference”
 502 remains nearly flat, underscoring the role of EMA updates in allowing the teacher to co-evolve with
 503 the policy and generate higher-quality pseudo labels. By contrast, Majority-Voting briefly improves
 504 but then collapses to near zero, evidencing reward hacking through consistent yet incorrect outputs.

505 **Case Study of the model reasoning with different learning methods.** Figure 7 provides a concrete
 506 example to illustrate the qualitative difference between self-rewarding baselines and our Co-rewarding.
 507 Majority-Voting exhibits reward hacking by boxing an incorrect answer “0” to pursue consensus,
 508 even though the reasoning steps are correct. Entropy produces repetitive outputs as its decoding
 509 probability distribution collapses onto a narrow set of tokens during entropy minimization. In contrast,
 510 Co-rewarding generates coherent step-by-step reasoning and correctly boxes the final answer, showing
 511 its capacity to overcome reward hacking and elicit genuine reasoning. Full results are provided in
 512 Appendix D.13 and additional case studies on code benchmark are discussed in Appendix D.14.

5 RELATED WORK

513 Reinforcement learning with verifiable reward (RLVR) has recently become a mainstream post-
 514 training paradigm for eliciting strong reasoning abilities in LLMs (Guo et al., 2025), achieving
 515 remarkably encouraging success particularly on mathematical (Shao et al., 2024) and coding (Luo
 516 et al., 2025) tasks. However, RLVR fundamentally depends on high-quality and annotated GT
 517 labels to supervise reward signals, which remains a major bottleneck for scalability under the spirit
 518 of the scaling laws. To break this limitation, recent efforts have explored RL without external
 519 reward from multiple perspectives. For instance, methods such as TTRL (Zuo et al., 2025) and
 520 SRT (Shafayat et al., 2025) pursue self-consistency to generate pseudo labels for rewards, where
 521 agreement among multiple rollouts is treated as optimization objective. Additionally, another technical
 522 line such as EMPO (Zhang et al., 2025c), Intuit (Zhao et al., 2025b) and RENT (Prabhudesai et al.,
 523 2025), enhances the LLM confidence by optimizing internal signals of reasoning, such as entropy
 524 minimization or self-certainty maximization. Different from these studies, Co-rewarding focuses
 525 on mitigating inherent training collapse in existing methods and enables stable self-supervised RL
 526 training. More detailed discussions of related work are in Appendix A.

6 CONCLUSION

527 In this work, we introduced Co-rewarding, a self-supervised RL framework that elicits the reasoning
 528 capability of LLMs through complementary supervision. Unlike prior self-rewarding methods that
 529 entangle rewards with single-view outputs and risk collapse, Co-rewarding establishes stability by
 530 decoupling the reward signal from the current online policy with the single-view output. Specifically,
 531 Co-rewarding-I leverages contrastive agreement across semantically analogous questions;
 532 Co-rewarding-II employs a dynamically updated teacher to provide insulated pseudo-labels; and
 533 Co-rewarding-III combines the data-side cross-supervision from Co-rewarding-I and the model-side
 534 teacher-based pseudo labels from Co-rewarding-II to further boost performance. Together, these
 535 designs construct cross-referable reward signals without explicit labels, aligning RL with invariances
 536 in reasoning rather than the mere correctness of isolated outputs. We hope this work will inspire
 537 further exploration into self-supervised RL for reasoning to advance the development.

540 ETHICS STATEMENT
541542 This work complies with the Code of Ethics. It uses only publicly available datasets, involves no
543 human or sensitive data, and raises no foreseeable risks related to privacy, security, or fairness issues.
544 The research is conducted solely for scientific advancement, with no conflicts of interest.
545546 REPRODUCIBILITY STATEMENT
547548 We are committed to ensure the reproducibility of our proposed method. A detailed description of our
549 approach is provided in the Co-rewarding Framework section, and the corresponding source code has
550 been submitted in an anonymous repository at [https://anonymous.4open.science/r/](https://anonymous.4open.science/r/Co-rewarding-ICLR26-submission)
551 [Co-rewarding-ICLR26-submission](https://anonymous.4open.science/r/Co-rewarding-ICLR26-submission). Both backbone models and datasets used in our work
552 are publicly available. Furthermore, all parameters, hyper-parameters, and procedural steps required
553 to reproduce our results are thoroughly recorded in the Implementation Details. We believe that these
554 components provide the community with details necessary to verify and reproduce our work.
555556 REFERENCES
557558 Josh Achiam, Steven Adler, Sandhini Agarwal, Lama Ahmad, Ilge Akkaya, Florencia Leoni Aleman,
559 Diogo Almeida, Janko Altenschmidt, Sam Altman, Shyamal Anadkat, et al. Gpt-4 technical report.
560 *arXiv preprint arXiv:2303.08774*, 2023.561 Shivam Agarwal, Zimin Zhang, Lifan Yuan, Jiawei Han, and Hao Peng. The unreasonable effectiveness
562 of entropy minimization in llm reasoning. *arXiv preprint arXiv:2505.15134*, 2025.563 Yuntao Bai, Andy Jones, Kamal Ndousse, Amanda Askell, Anna Chen, Nova DasSarma, Dawn Drain,
564 Stanislav Fort, Deep Ganguli, Tom Henighan, et al. Training a helpful and harmless assistant with
565 reinforcement learning from human feedback. *arXiv preprint arXiv:2204.05862*, 2022.566 Maciej Besta, Nils Blach, Ales Kubicek, Robert Gerstenberger, Michal Podstawski, Lukas Gianinazzi,
567 Joanna Gajda, Tomasz Lehmann, Hubert Niewiadomski, Piotr Nyczek, et al. Graph of thoughts:
568 Solving elaborate problems with large language models. In *AAAI*, 2024.569 Mathilde Caron, Hugo Touvron, Ishan Misra, Hervé Jégou, Julien Mairal, Piotr Bojanowski, and
570 Armand Joulin. Emerging properties in self-supervised vision transformers. In *Proceedings of the*
571 *IEEE/CVF international conference on computer vision*, pp. 9650–9660, 2021.572 Ting Chen, Simon Kornblith, Mohammad Norouzi, and Geoffrey Hinton. A simple framework for
573 contrastive learning of visual representations. In *International conference on machine learning*, pp.
574 1597–1607. PMLR, 2020.575 Xiangxiang Chu, Hailang Huang, Xiao Zhang, Fei Wei, and Yong Wang. Gpg: A simple and strong
576 reinforcement learning baseline for model reasoning. *arXiv preprint arXiv:2504.02546*, 2025.577 Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser,
578 Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John
579 Schulman. Training verifiers to solve math word problems. *arXiv preprint arXiv:2110.14168*,
580 2021.581 Quy-Anh Dang and Chris Ngo. Reinforcement learning for reasoning in small llms: What works and
582 what doesn't. *arXiv preprint arXiv:2503.16219*, 2025.583 Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha
584 Letman, Akhil Mathur, Alan Schelten, Amy Yang, Angela Fan, et al. The llama 3 herd of models.
585 *arXiv e-prints*, pp. arXiv–2407, 2024.586 Kawin Ethayarajh, Winnie Xu, Niklas Muennighoff, Dan Jurafsky, and Douwe Kiela. Kto: Model
587 alignment as prospect theoretic optimization. *arXiv preprint arXiv:2402.01306*, 2024.588 Wenkai Fang, Shunyu Liu, Yang Zhou, Kongcheng Zhang, Tongya Zheng, Kaixuan Chen, Mingli
589 Song, and Dacheng Tao. Serl: Self-play reinforcement learning for large language models with
590 limited data. *arXiv preprint arXiv:2505.20347*, 2025.

594 Leo Gao, John Schulman, and Jacob Hilton. Scaling laws for reward model overoptimization. In
 595 *International Conference on Machine Learning*, pp. 10835–10866. PMLR, 2023.
 596

597 Jean-Bastien Grill, Florian Strub, Florent Altché, Corentin Tallec, Pierre Richemond, Elena
 598 Buchatskaya, Carl Doersch, Bernardo Avila Pires, Zhaohan Guo, Mohammad Gheshlaghi Azar,
 599 et al. Bootstrap your own latent-a new approach to self-supervised learning. *Advances in neural*
 600 *information processing systems*, 33:21271–21284, 2020.

601 Alex Gu, Baptiste Roziere, Hugh James Leather, Armando Solar-Lezama, Gabriel Synnaeve, and Sida
 602 Wang. Cruxeval: A benchmark for code reasoning, understanding and execution. In *International*
 603 *Conference on Machine Learning*, pp. 16568–16621. PMLR, 2024.
 604

605 Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu,
 606 Shirong Ma, Peiyi Wang, Xiao Bi, et al. Deepseek-r1: Incentivizing reasoning capability in llms
 607 via reinforcement learning. *arXiv preprint arXiv:2501.12948*, 2025.

608 Shangmin Guo, Biao Zhang, Tianlin Liu, Tianqi Liu, Misha Khalman, Felipe Llinares, Alexandre
 609 Rame, Thomas Mesnard, Yao Zhao, Bilal Piot, et al. Direct language model alignment from online
 610 ai feedback. *arXiv preprint arXiv:2402.04792*, 2024.
 611

612 Bo Han, Quanming Yao, Xingrui Yu, Gang Niu, Miao Xu, Weihua Hu, Ivor Tsang, and Masashi
 613 Sugiyama. Co-teaching: Robust training of deep neural networks with extremely noisy labels.
 614 *Advances in neural information processing systems*, 31, 2018.

615 Dan Hendrycks, Collin Burns, Steven Basart, Andy Zou, Mantas Mazeika, Dawn Song, and Jacob
 616 Steinhardt. Measuring massive multitask language understanding. In *International Conference on*
 617 *Learning Representations*.
 618

619 Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song,
 620 and Jacob Steinhardt. Measuring mathematical problem solving with the math dataset. In *NeurIPS*,
 621 2021.

622 Jiwoo Hong, Noah Lee, and James Thorne. Orpo: Monolithic preference optimization without
 623 reference model. *arXiv preprint arXiv:2403.07691*, 2024.

625 Jian Hu. Reinforce++: A simple and efficient approach for aligning large language models. *arXiv*
 626 *preprint arXiv:2501.03262*, 2025.
 627

628 Wenxuan Huang, Bohan Jia, Zijie Zhai, Shaosheng Cao, Zheyu Ye, Fei Zhao, Zhe Xu, Yao Hu, and
 629 Shaohui Lin. Vision-r1: Incentivizing reasoning capability in multimodal large language models.
 630 *arXiv preprint arXiv:2503.06749*, 2025.

631 Naman Jain, King Han, Alex Gu, Wen-Ding Li, Fanjia Yan, Tianjun Zhang, Sida Wang, Armando
 632 Solar-Lezama, Koushik Sen, and Ion Stoica. Livecodebench: Holistic and contamination free
 633 evaluation of large language models for code. In *The Thirteenth International Conference on*
 634 *Learning Representations*, 2025.

635 Seungjae Jung, Gunsoo Han, Daniel Wontae Nam, and Kyoung-Woon On. Binary classifier optimiza-
 636 tion for large language model alignment. *arXiv preprint arXiv:2404.04656*, 2024.

638 Tushar Khot, Harsh Trivedi, Matthew Finlayson, Yao Fu, Kyle Richardson, Peter Clark, and Ashish
 639 Sabharwal. Decomposed prompting: A modular approach for solving complex tasks. In *ICLR*,
 640 2023.

642 Cassidy Laidlaw, Shivam Singhal, and Anca Dragan. Correlated proxies: A new definition and
 643 improved mitigation for reward hacking. In *ICLR*, 2025.

644 Jia Li, Edward Beeching, Lewis Tunstall, Ben Lipkin, Roman Soletskyi, Shengyi Huang, Kashif
 645 Rasul, Longhui Yu, Albert Q Jiang, Ziju Shen, et al. Numinamath: The largest public dataset in
 646 ai4maths with 860k pairs of competition math problems and solutions. *Hugging Face repository*,
 647 13(9):9, 2024a.

648 Pengyi Li, Matvey Skripkin, Alexander Zubrey, Andrey Kuznetsov, and Ivan Oseledets. Confidence
 649 is all you need: Few-shot rl fine-tuning of language models. *arXiv preprint arXiv:2506.06395*,
 650 2025.

651 Ziniu Li, Tian Xu, Yushun Zhang, Zhihang Lin, Yang Yu, Ruoyu Sun, and Zhi-Quan Luo. Remax: A
 652 simple, effective, and efficient reinforcement learning method for aligning large language models.
 653 In *ICML*, 2024b.

654 Hunter Lightman, Vineet Kosaraju, Yuri Burda, Harrison Edwards, Bowen Baker, Teddy Lee, Jan
 655 Leike, John Schulman, Ilya Sutskever, and Karl Cobbe. Let's verify step by step. In *ICLR*, 2024.

656 Yen-Ting Lin, Di Jin, Tengyu Xu, Tianhao Wu, Sainbayar Sukhbaatar, Chen Zhu, Yun He, Yun-Nung
 657 Chen, Jason Weston, Yuandong Tian, et al. Step-kto: Optimizing mathematical reasoning through
 658 stepwise binary feedback. *arXiv preprint arXiv:2501.10799*, 2025a.

659 Zhihang Lin, Mingbao Lin, Yuan Xie, and Rongrong Ji. Cppo: Accelerating the training of group
 660 relative policy optimization-based reasoning models. *arXiv preprint arXiv:2503.22342*, 2025b.

661 Jiawei Liu, Chunqiu Steven Xia, Yuyao Wang, and Lingming Zhang. Is your code generated by
 662 chatgpt really correct? rigorous evaluation of large language models for code generation. *Advances
 663 in Neural Information Processing Systems*, 36:21558–21572, 2023.

664 Zichen Liu, Changyu Chen, Wenjun Li, Penghui Qi, Tianyu Pang, Chao Du, Wee Sun Lee, and Min
 665 Lin. Understanding r1-zero-like training: A critical perspective. *arXiv preprint arXiv:2503.20783*,
 666 2025a.

667 Ziyu Liu, Zeyi Sun, Yuhang Zang, Xiaoyi Dong, Yuhang Cao, Haodong Duan, Dahua Lin, and Jiaqi
 668 Wang. Visual-rft: Visual reinforcement fine-tuning. *arXiv preprint arXiv:2503.01785*, 2025b.

669 Michael Luo, Sijun Tan, Roy Huang, Ameen Patel, Alpay Ariyak, Qingyang Wu, Xiaoxiang Shi,
 670 Rachel Xin, Colin Cai, Maurice Weber, Ce Zhang, Li Erran Li, Raluca Ada Popa, and Ion Stoica.
 671 Deepcoder: A fully open-source 14b coder at o3-mini level, 2025.

672 AI Meta. Llama 3.2: Revolutionizing edge ai and vision with open, customizable models. *Meta AI
 673 Blog*. Retrieved December, 20:2024, 2024.

674 Rémi Munos, Michal Valko, Daniele Calandriello, Mohammad Gheshlaghi Azar, Mark Rowland,
 675 Zhaohan Daniel Guo, Yunhao Tang, Matthieu Geist, Thomas Mesnard, Andrea Michi, et al. Nash
 676 learning from human feedback. In *ICML*, 2024.

677 Aaron van den Oord, Yazhe Li, and Oriol Vinyals. Representation learning with contrastive predictive
 678 coding. *arXiv preprint arXiv:1807.03748*, 2018.

679 Long Ouyang, Jeffrey Wu, Xu Jiang, Diogo Almeida, Carroll Wainwright, Pamela Mishkin, Chong
 680 Zhang, Sandhini Agarwal, Katarina Slama, Alex Ray, et al. Training language models to follow
 681 instructions with human feedback. In *NeurIPS*, 2022.

682 Jing-Cheng Pang, Pengyuan Wang, Kaiyuan Li, Xiong-Hui Chen, Jiacheng Xu, Zongzhang Zhang,
 683 and Yang Yu. Language model self-improvement by reinforcement learning contemplation. In *The
 684 Twelfth International Conference on Learning Representations*, 2024.

685 Mihir Prabhudesai, Lili Chen, Alex Ippoliti, Katerina Fragkiadaki, Hao Liu, and Deepak Pathak.
 686 Maximizing confidence alone improves reasoning. *arXiv preprint arXiv:2505.22660*, 2025.

687 Qwen, ;, An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan
 688 Li, Dayiheng Liu, Fei Huang, et al. Qwen2.5 technical report, 2025. URL <https://arxiv.org/abs/2412.15115>.

689 Rafael Rafailov, Archit Sharma, Eric Mitchell, Christopher D Manning, Stefano Ermon, and Chelsea
 690 Finn. Direct preference optimization: Your language model is secretly a reward model. In *NeurIPS*,
 691 2023.

692 Sheikh Shafayat, Fahim Tajwar, Ruslan Salakhutdinov, Jeff Schneider, and Andrea Zanette. Can
 693 large reasoning models self-train? *arXiv preprint arXiv:2505.21444*, 2025.

702 Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang,
 703 Mingchuan Zhang, YK Li, Yang Wu, et al. Deepseekmath: Pushing the limits of mathemat-
 704 ical reasoning in open language models. *arXiv preprint arXiv:2402.03300*, 2024.

705

706 Guangming Sheng, Chi Zhang, Zilingfeng Ye, Xibin Wu, Wang Zhang, Ru Zhang, Yanghua Peng,
 707 Haibin Lin, and Chuan Wu. Hybridflow: A flexible and efficient rlhf framework. *arXiv preprint*
 708 *arXiv: 2409.19256*, 2024.

709

710 Charlie Snell, Jaehoon Lee, Kelvin Xu, and Aviral Kumar. Scaling llm test-time compute optimally
 711 can be more effective than scaling model parameters. *arXiv preprint arXiv:2408.03314*, 2024.

712

713 Jonathan Uesato, Nate Kushman, Ramana Kumar, Francis Song, Noah Siegel, Lisa Wang, Antonia
 714 Creswell, Geoffrey Irving, and Irina Higgins. Solving math word problems with process-and
 715 outcome-based feedback. *arXiv preprint arXiv:2211.14275*, 2022.

716

717 Haoran Wang, Thaleia Zariphopoulou, and Xunyu Zhou. Exploration versus exploitation in reinforce-
 718 ment learning: A stochastic control approach. *arXiv preprint arXiv:1812.01552*, 2018.

719

720 Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc Le, Ed Chi, Sharan Narang, Aakanksha Chowdh-
 721 ery, and Denny Zhou. Self-consistency improves chain of thought reasoning in language models.
 722 *arXiv preprint arXiv:2203.11171*, 2022.

723

724 Yiping Wang, Qing Yang, Zhiyuan Zeng, Liliang Ren, Liyuan Liu, Baolin Peng, Hao Cheng, Xuehai
 725 He, Kuan Wang, Jianfeng Gao, et al. Reinforcement learning for reasoning in large language
 726 models with one training example. *arXiv preprint arXiv:2504.20571*, 2025.

727

728 Yubo Wang, Xueguang Ma, Ge Zhang, Yuansheng Ni, Abhranil Chandra, Shiguang Guo, Weiming
 729 Ren, Aaran Arulraj, Xuan He, Ziyan Jiang, et al. Mmlu-pro: A more robust and challenging multi-
 730 task language understanding benchmark. *Advances in Neural Information Processing Systems*, 37:
 731 95266–95290, 2024.

732

733 Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, brian ichter, Fei Xia, Ed H. Chi,
 734 Quoc V Le, and Denny Zhou. Chain of thought prompting elicits reasoning in large language
 735 models. In *NeurIPS*, 2022.

736

737 Tengyang Xie, Dylan J Foster, Akshay Krishnamurthy, Corby Rosset, Ahmed Awadallah, and
 738 Alexander Rakhlin. Exploratory preference optimization: Harnessing implicit q^* -approximation
 739 for sample-efficient rlhf. *arXiv preprint arXiv:2405.21046*, 2024.

740

741 Tian Xie, Zitian Gao, Qingnan Ren, Haoming Luo, Yuqian Hong, Bryan Dai, Joey Zhou, Kai Qiu,
 742 Zhirong Wu, and Chong Luo. Logic-rl: Unleashing llm reasoning with rule-based reinforcement
 743 learning. *arXiv preprint arXiv:2502.14768*, 2025.

744

745 Haoran Xu, Amr Sharaf, Yunmo Chen, Weiting Tan, Lingfeng Shen, Benjamin Van Durme, Kenton
 746 Murray, and Young Jin Kim. Contrastive preference optimization: Pushing the boundaries of llm
 747 performance in machine translation. *arXiv preprint arXiv:2401.08417*, 2024.

748

749 An Yang, Anfeng Li, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chang
 750 Gao, Chengan Huang, Chenxu Lv, et al. Qwen3 technical report. *arXiv preprint arXiv:2505.09388*,
 751 2025.

752

753 Ling Yang, Zhaochen Yu, Tianjun Zhang, Shiyi Cao, Minkai Xu, Wentao Zhang, Joseph E Gonzalez,
 754 and Bin Cui. Buffer of thoughts: Thought-augmented reasoning with large language models. In
 755 *NeurIPS*, 2024.

756

757 Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Thomas L. Griffiths, Yuan Cao, and Karthik R
 758 Narasimhan. Tree of thoughts: Deliberate problem solving with large language models. In *NeurIPS*,
 759 2023a.

760

761 Shunyu Yao, Jeffrey Zhao, Dian Yu, Nan Du, Izhak Shafran, Karthik R Narasimhan, and Yuan Cao.
 762 React: Synergizing reasoning and acting in language models. In *ICLR*, 2023b.

756 Qiying Yu, Zheng Zhang, Ruofei Zhu, Yufeng Yuan, Xiaochen Zuo, Yu Yue, Tiantian Fan, Gaohong
 757 Liu, Lingjun Liu, Xin Liu, et al. Dapo: An open-source llm reinforcement learning system at scale.
 758 *arXiv preprint arXiv:2503.14476*, 2025.

759 Eric Zelikman, Yuhuai Wu, Jesse Mu, and Noah Goodman. Star: Bootstrapping reasoning with
 760 reasoning. *Advances in Neural Information Processing Systems*, 35:15476–15488, 2022.

762 Jingyi Zhang, Jiaxing Huang, Huanjin Yao, Shunyu Liu, Xikun Zhang, Shijian Lu, and Dacheng Tao.
 763 R1-vl: Learning to reason with multimodal large language models via step-wise group relative
 764 policy optimization. *arXiv preprint arXiv:2503.12937*, 2025a.

765 Kongcheng Zhang, Qi Yao, Shunyu Liu, Yingjie Wang, Baisheng Lai, Jieping Ye, Mingli Song,
 766 and Dacheng Tao. Consistent paths lead to truth: Self-rewarding reinforcement learning for llm
 767 reasoning. *arXiv preprint arXiv:2506.08745*, 2025b.

769 Qingyang Zhang, Haitao Wu, Changqing Zhang, Peilin Zhao, and Yatao Bian. Right question
 770 is already half the answer: Fully unsupervised llm reasoning incentivization. *arXiv preprint
 771 arXiv:2504.05812*, 2025c.

772 Yanzhi Zhang, Zhaoxi Zhang, Haoxiang Guan, Yilin Cheng, Yitong Duan, Chen Wang, Yue Wang,
 773 Shuxin Zheng, and Jiyan He. No free lunch: Rethinking internal feedback for llm reasoning. *arXiv
 774 preprint arXiv:2506.17219*, 2025d.

776 Yifan Zhang and Team Math-AI. American invitational mathematics examination (aime) 2024, 2024.

778 Andrew Zhao, Yiran Wu, Yang Yue, Tong Wu, Quentin Xu, Matthieu Lin, Shenzhi Wang, Qingyun
 779 Wu, Zilong Zheng, and Gao Huang. Absolute zero: Reinforced self-play reasoning with zero data.
 780 *arXiv preprint arXiv:2505.03335*, 2025a.

781 Xuandong Zhao, Zhewei Kang, Aosong Feng, Sergey Levine, and Dawn Song. Learning to reason
 782 without external rewards. *arXiv preprint arXiv:2505.19590*, 2025b.

783 Denny Zhou, Nathanael Schärli, Le Hou, Jason Wei, Nathan Scales, Xuezhi Wang, Dale Schuurmans,
 784 Claire Cui, Olivier Bousquet, Quoc Le, et al. Least-to-most prompting enables complex reasoning
 785 in large language models. In *ICLR*, 2023a.

787 Hengguang Zhou, Xirui Li, Ruochen Wang, Minhao Cheng, Tianyi Zhou, and Cho-Jui Hsieh. R1-
 788 zero’s” aha moment” in visual reasoning on a 2b non-sft model. *arXiv preprint arXiv:2503.05132*,
 789 2025.

790 Jeffrey Zhou, Tianjian Lu, Swaroop Mishra, Siddhartha Brahma, Sujoy Basu, Yi Luan, Denny
 791 Zhou, and Le Hou. Instruction-following evaluation for large language models. *arXiv preprint
 792 arXiv:2311.07911*, 2023b.

793 Yuxin Zuo, Kaiyan Zhang, Li Sheng, Shang Qu, Ganqu Cui, Xuekai Zhu, Haozhan Li, Yuchen
 794 Zhang, Xinwei Long, Ermo Hua, et al. Ttrl: Test-time reinforcement learning. *arXiv preprint
 795 arXiv:2504.16084*, 2025.

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810 LLM USAGE STATEMENT
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815 Here we clarify the usage of Large Language Models (LLMs) in this work. For the preparation of
816 this paper, LLMs are limited to the role of a general-purpose writing assistant and are not used for
817 research ideation or core content generation. For research methodology, LLM is a core component of
818 our proposed method. Specifically, we utilize the Qwen3-32B model to perform question rephrasing
819 in Co-rewarding-I, which is thoroughly detailed in the Implementation Details section of the main
820 paper. The authors take full responsibility for all content written under their name.

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822 A RELATED WORK
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830 **Large Language Model Reasoning.** LLMs have shown impressive performance on vast tasks that
831 require reasoning, including solving mathematical problems, writing code, and answering logical
832 questions. One of the key techniques that has improved LLM reasoning is Chain-of-Thought (CoT)
833 prompting (Wei et al., 2022). CoT encourages the model to generate intermediate reasoning steps
834 before producing the final answer, which has been shown to enhance performance on tasks like
835 arithmetic, commonsense reasoning, and symbolic reasoning. Subsequent work has extended CoT
836 by integrating it with various strategies, including compositional generalization (Zhou et al., 2023a;
837 Khot et al., 2023) and employing structural reasoning approaches (Yao et al., 2023a; Besta et al.,
838 2024; Yang et al., 2024). In addition, CoT serves as a fundamental framework for techniques like
839 fine-tuning (Zelikman et al., 2022), argentic workflow (Yao et al., 2023b), and paving the way for
840 inference-time scaling (Snell et al., 2024).

841 **RL for Large Language Models.** Several RL algorithms have been developed primarily for
842 alignment tasks. Specifically, DPO (Rafailov et al., 2023), CPO (Xu et al., 2024), and their variants (Li
843 et al., 2024b; Guo et al., 2024; Munos et al., 2024; Hong et al., 2024; Xie et al., 2024) rely on pairs of
844 outputs labeled by human preference. In contrast, KTO (Ethayarajh et al., 2024) and BCO (Jung et al.,
845 2024) require only a single binary label (like or dislike) for each output. Besides, the PRM (Uesato
846 et al., 2022; Lightman et al., 2024) and Step-KTO (Lin et al., 2025a) offer step-by-step guidance
847 by incorporating feedback at each reasoning step rather than focusing solely on the final outputs.
848 Recently, the follow-up work of GRPO improves the optimization objective, *e.g.*, DAPO (Yu et al.,
849 2025), Dr. GRPO (Liu et al., 2025a), REINFORCE++ (Hu, 2025), CPPO (Lin et al., 2025b), and
850 GPG (Chu et al., 2025). Another line of research generalizes GRPO to broader applications such
851 as multimodal reasoning (Zhou et al., 2025; Huang et al., 2025; Chu et al., 2025; Liu et al., 2025b;
852 Zhang et al., 2025a) and logical reasoning (Xie et al., 2025).

853 **RL without External Reward.** RL methods have shown promising scaling capabilities to enhance
854 the reasoning abilities of LLMs (Guo et al., 2025), yet they are often limited by the availability of
855 training data for reward signals (Gao et al., 2023; Liu et al., 2023). Notably, Wang et al. (Wang
856 et al., 2025) demonstrate that RL can effectively bootstrap LLM reasoning with as little as a single
857 training example, highlighting the potential to minimize or even eliminate reliance on external reward
858 signals during training. Recent efforts leverage distinct strategies for reward assignment. For instance,
859 SIRLC (Pang et al., 2024) and AZR (Zhao et al., 2025a) utilize an LLM-as-the-judge approach to
860 assign rewards. In contrast, methods like SRT, TTRL, and their variants (Shafayat et al., 2025; Zuo
861 et al., 2025; Fang et al., 2025; Zhang et al., 2025b) employ self-consistency (Wang et al., 2022) to
862 generate pseudo-rewards, reducing dependence on external annotations. Meanwhile, INTUITOR,
863 RLSC, and RENT (Zhao et al., 2025b; Li et al., 2025; Prabhudesai et al., 2025) harness the internal
864 confidence scores of LLMs as intrinsic reward signals. Additionally, EMPO and its variants (Zhang
865 et al., 2025c; Agarwal et al., 2025) promote reasoning by minimizing entropy during the reasoning
866 process, further diversifying the approaches to incentivize robust LLM performance.

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Algorithm 1 *Co-rewarding-I*

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1: **Input:** policy model π_θ , learning rate η , training dataset \mathcal{D} , rephrased training dataset \mathcal{D}' , total iterations K .
 2: **Output:** trained policy model π_θ .
 3: **for** all iteration $k = 1, \dots, K$ **do**
 4: Sample mini-batch inputs $\mathcal{B} \subseteq \mathcal{D}$ and $\mathcal{B}' \subseteq \mathcal{D}'$.
 5: **for all** input question $x \in \mathcal{B}$ and $x' \in \mathcal{B}'$ **do**
 6: Sample rollouts $\{y_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot | x)$.
 7: Sample rollouts $\{y'_i\}_{i=1}^{G'} \sim \pi_{\theta_{\text{old}}}(\cdot | x')$.
 8: Obtain pseudo labels by Eq. (8).
 9: Estimate relative advantages by Eq. (7).
 10: Compute the objective by Eq. (6).
 11: Update $\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{J}_{\text{Co-rewarding-I}}(\theta)$.
 12: **end for**
 13: **end for**

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Algorithm 2 *Co-rewarding-II*

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1: **Input:** policy model π_θ , learning rate η , training dataset \mathcal{D} , total iterations K .
 2: **Output:** trained policy model π_θ .
 3: **for** iteration $k = 1, \dots, K$ **do**
 4: Sample mini-batch $\mathcal{B} \subseteq \mathcal{D}$.
 5: **for all** $x \in \mathcal{B}$ **do**
 6: Sample rollouts $\{y_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}^{(k)}(\cdot | x)$.
 7: Update the reference teacher by Eq. (11).
 8: Sample rollouts $\{\tilde{y}_j\}_{j=1}^{\tilde{G}} \sim \tilde{\pi}_{\text{ref}}^{(k)}(\cdot | x)$.
 9: Obtain pseudo label from $\{\tilde{y}_j\}_{j=1}^{\tilde{G}}$ by Eq. (10).
 10: Estimate the relative advantage by Eq. (10).
 11: Compute the objective by Eq. (9).
 12: Update $\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{J}_{\text{Co-rewarding-II}}^{(k)}(\theta)$.
 13: **end for**
 14: **end for**

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Algorithm 3 *Co-rewarding-III*

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1: **Input:** policy model π_θ , learning rate η , original training dataset \mathcal{D} , rephrased training dataset \mathcal{D}' , total iterations K .
 2: **Output:** trained policy model π_θ .
 3: **for** iteration $k = 1, \dots, K$ **do**
 4: Sample mini-batch inputs $\mathcal{B} \subseteq \mathcal{D}$ and $\mathcal{B}' \subseteq \mathcal{D}'$.
 5: **for all** $x \in \mathcal{B}$ and $x' \in \mathcal{B}'$ **do**
 6: Sample rollouts $\{y_i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}^{(k)}(\cdot | x)$ and $\{y'_i\}_{i=1}^{G'} \sim \pi_{\theta_{\text{old}}}^{(k)}(\cdot | x')$.
 7: Update the reference teacher by Eq. (11).
 8: Sample rollouts $\{\tilde{y}_j\}_{j=1}^{\tilde{G}} \sim \tilde{\pi}_{\text{ref}}^{(k)}(\cdot | x)$ and $\{\tilde{y}'_j\}_{j=1}^{\tilde{G}} \sim \tilde{\pi}_{\text{ref}}^{(k)}(\cdot | x')$.
 9: Obtain pseudo label from $\{\tilde{y}'_j\}_{j=1}^{\tilde{G}}$ and $\{\tilde{y}_j\}_{j=1}^{\tilde{G}}$ by Eq. (13) and Eq. (14).
 10: Estimate the relative advantages by Eq. (13) and Eq. (14).
 11: Compute the objective by Eq. (12).
 12: Update $\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{J}_{\text{Co-rewarding-III}}^{(k)}(\theta)$.
 13: **end for**
 14: **end for**

918 **B PSEUDO CODE OF CO-REWARDING**919 **B.1 FORMULATION OF CO-REWARDING-III**920 The relative advantages $\mathcal{R}_\theta(\hat{A}^{(k)})$ and $\mathcal{R}_\theta(\hat{A}'^{(k)})$ are computed as:

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$$\hat{A}_i^{(k)} = \frac{r(\tilde{y}_v'^{(k)}, y_i) - \text{mean}(\{r(\tilde{y}_v'^{(k)}, y_i)\}_{i=1}^G)}{\text{std}(\{r(\tilde{y}_v'^{(k)}, y_i)\}_{i=1}^G)}, \tilde{y}_v'^{(k)} = \arg \max_{y^*} \sum_{j=1}^{\tilde{G}} \mathbf{1}[\text{ans}(\tilde{y}_j') = \text{ans}(y^*)], \quad (13)$$

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$$\hat{A}'_i^{(k)} = \frac{r(\tilde{y}_v^{(k)}, y_i) - \text{mean}(\{r(\tilde{y}_v^{(k)}, y_i)\}_{i=1}^G)}{\text{std}(\{r(\tilde{y}_v^{(k)}, y_i)\}_{i=1}^G)}, \tilde{y}_v^{(k)} = \arg \max_{y^*} \sum_{j=1}^{\tilde{G}} \mathbf{1}[\text{ans}(\tilde{y}_j) = \text{ans}(y^*)], \quad (14)$$

923 where the pseudo label $\tilde{y}_v'^{(k)}$ is the majority-vote pseudo label obtained from reference rollouts on the
924 rephrased question, and $\tilde{y}_v^{(k)}$ is the corresponding pseudo label obtained from reference rollouts on
925 original question. The reference model is slowly updated via EMA as in Eq. (11).926 **B.2 PSEUDO CODE**927 To intuitively present the pipeline of Co-rewarding, we summarize the pseudo codes of Co-rewarding-
928 I, Co-rewarding-II and Co-rewarding-III in Algorithm 1, Algorithm 2 and Algorithm 3, respectively.929 **C ADDITIONAL EXPERIMENTAL DETAILS**930 **C.1 DETAILS OF BASELINES**931 We compare our proposed Co-rewarding-I and II against GT-reward and several recent state-of-the-art
932 (SoTA) self-reward approaches:933

- **GT-Reward** (Shao et al., 2024): Originally introduced by DeepSeek-R1 (Guo et al., 2025),
934 GT-Reward supervises training using ground-truth (GT) answers, determining whether model
935 rollouts are correct or not, to guide RL optimization.
- **Self-Certainty** (Zhao et al., 2025b): This method maximizes *self-certainty*, defined as the KL-
936 divergence between the uniform distribution and the model’s decoding distribution, serving as
937 reward to encourage more confident predictions.
- **Entropy** (Prabhudesai et al., 2025): This method minimizes the entropy of the model’s rollout
938 distribution, using negative entropy as reward to maximize model confidence.
- **Majority-Voting** (Shafayat et al., 2025): By generating multiple rollouts per question, Majority-
939 Voting selects the most frequent answer as a pseudo-label to supervise training.

940 For all methods, we adopt the widely used GRPO as the policy optimization algorithm.

941 **C.2 MORE IMPLEMENTATION DETAILS**942 The detailed training configurations are summarized in Table 5, and all baseline methods are trained
943 under the same setup for fairness. For the training system prompt, we adopt the official default prompt
944 provided by VerL¹, shown below:

945 Let’s think step by step and output the final answer within \boxed{}.

946 In addition, the semantically analogical questions used in Co-rewarding-I are generated by Qwen3-
947 32B through a rewriting prompt. The exact rewriting instruction is provided as follows:948 You are given a math problem. Please rewrite it using different wording
949 and a different real-world scenario, while keeping the underlying
950 mathematical meaning and answer exactly the same.951 ¹<https://github.com/volcengine/verl>

Table 5: Detailed training settings.

Settings	Co-rewarding-I	Co-rewarding-II
Batch Size	128	128
Max Prompt Length	512	512
Max Response Length	3072	3072
Train Steps	170-220	300-330
Learning Rate	3e-6	3e-6
# Policy Rollout G	8	8
# Reference Rollout \tilde{G}	-	8
Clip Ratio	0.2	0.2
Warmup Style	Cosine	Cosine
Warmup Steps Ratio	0.1	0.1
KL Loss Coefficient	0.005	0.001
Optimizer	AdamW ($\beta_1 = 0.9, \beta_2 = 0.999, \epsilon = 10^{-8}$)	
Training Temperature	1.0	1.0
Evaluation Temperature	0.8	0.8
EMA α_{start}	-	0.99
EMA α_{end}	-	0.9999

Table 6: **Statistics and usages of datasets** used in our experiments.

Dataset Name	# Data Size	Usage
MATH-Train (Hendrycks et al., 2021)	7,500	Training Set
MATH-Test (Hendrycks et al., 2021)	5,000	Validation Set
DAPO-14k (Yu et al., 2025)	14,109	Training Set
Open-RS (Dang & Ngo, 2025)	7,000	Training Set
MATH500 (Lightman et al., 2024)	500	Evaluation Benchmark
GSM8K (Cobbe et al., 2021)	1,319	Evaluation Benchmark
AMC (Li et al., 2024a)	83	Evaluation Benchmark
LiveCodeBench (Jain et al., 2025)	1,055	Evaluation Benchmark
CRUX (Gu et al., 2024)	800	Evaluation Benchmark
MMLU-Pro (Wang et al., 2024)	12,032	Evaluation Benchmark
IFEval (Zhou et al., 2023b)	541	Evaluation Benchmark

Guidelines:

1. Do not change the math logic or the final answer.
2. Use different words and a new context to make it look like a different problem.
3. Avoid copying phrases or sentence structures from the original.
4. Make sure the rewritten question is natural, clear, and solvable.
5. Output ONLY between the following markers, and strictly in this format (no extra explanation):

RESULT START

ORTGNTAL:

<original question>

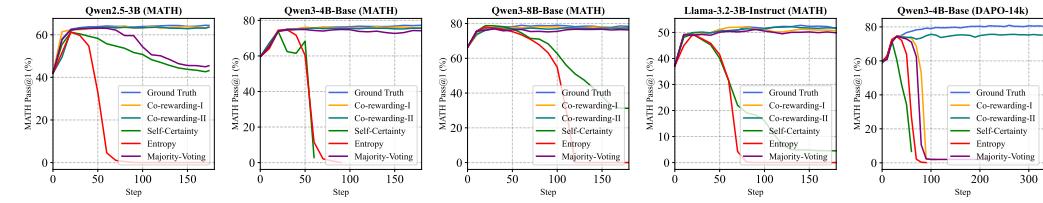
REWRITE:

<rewritten question>

RESULT_END

1026 **Table 7: Supplement Results (%) of Co-rewarding and baselines trained on MATH.** Cell
 1027 background colors: darker colors denote better results within each model group.
 1028

1029 Training Set: MATH	1030 Mathematics				1031 Code		1032 Instruction	1033 Multi-Task
1034 Methods	1035 MATH500	1036 GSM8K	1037 AMC	1038 AIME24	1039 LiveCode	1040 CRUX	1041 IFEval	1042 MMLU-Pro
<i>Qwen2.5-3B</i>								
Before RL	53.6	19.48	10.69	0.52	9.95	18.50	29.83	32.50
- GT-Reward (Shao et al., 2024)	65.4	82.18	32.98	6.77	13.93	32.12	33.66	36.74
- Self-Certainty (Zhao et al., 2025b)	64.2	80.52	28.92	5.00	10.90	29.00	32.22	33.88
- Entropy (Prabhudesai et al., 2025)	63.2	80.44	29.67	5.94	9.05	29.00	32.94	35.35
- Majority-Voting (Shafayat et al., 2025)	64.6	82.41	33.13	5.10	14.03	36.38	35.19	35.50
- Co-rewarding-I (Ours)	65.4	84.53	30.57	5.31	16.40	36.88	33.86	36.38
- Co-rewarding-II (Ours)	65.2	81.72	32.38	4.47	22.25	40.25	32.74	30.79
<i>Qwen2.5-7B</i>								
Before RL	69.4	24.71	15.81	2.81	3.79	26.38	38.19	44.76
- GT-Reward (Shao et al., 2024)	76.4	88.02	45.63	14.06	15.92	45.12	41.49	41.12
- Self-Certainty (Zhao et al., 2025b)	72.8	84.31	38.55	8.75	12.04	54.12	37.24	43.30
- Entropy (Prabhudesai et al., 2025)	72.2	81.43	39.61	10.73	16.49	51.88	40.33	42.79
- Majority-Voting (Shafayat et al., 2025)	74.4	84.53	40.96	11.04	15.45	51.00	38.60	43.35
- Co-rewarding-I (Ours)	74.6	89.61	41.27	10.73	15.73	55.58	42.86	40.51
- Co-rewarding-II (Ours)	73.6	89.31	42.77	11.98	8.25	47.50	41.82	37.45
<i>Qwen3-1.7B-Base</i>								
Before RL	57.0	19.56	8.43	1.15	4.45	7.50	33.65	33.00
- GT-Reward (Shao et al., 2024)	69.6	81.57	35.54	8.23	13.74	35.25	36.16	39.12
- Self-Certainty (Zhao et al., 2025b)	58.2	40.25	23.04	3.02	9.86	18.00	32.96	35.13
- Entropy (Prabhudesai et al., 2025)	63.6	71.79	31.63	6.88	13.74	31.37	35.37	36.67
- Majority-Voting (Shafayat et al., 2025)	65.2	81.57	34.78	7.50	13.08	34.25	35.45	36.00
- Co-rewarding-I (Ours)	67.6	83.01	32.22	8.65	13.50	32.38	35.56	35.53
- Co-rewarding-II (Ours)	66.2	80.89	33.28	7.50	14.40	32.88	36.94	37.59



1057 **Figure 8: Performance curves on validation set.** Left to Right: {Qwen2.5-3B, Qwen3-4B-Base,
 1058 Qwen3-8B-Base, Llama-3.2-3B-Instruct} trained on MATH, Qwen3-4B-Base trained on DAPO-14k.
 1059
 1060
 1061

1062 C.3 MORE EVALUATION DETAILS

1063 We conduct the evaluation across a diverse set of benchmarks, spanning mathematical reasoning,
 1064 code generation, instruction-following, and general multi-task abilities. Specifically: (1) Mathematical
 1065 reasoning: We evaluate on MATH500 (Lightman et al., 2024), GSM8K (Cobbe et al., 2021),
 1066 and AMC (Li et al., 2024a). For MATH500 and GSM8K, we report pass@1 accuracy using the
 1067 `lighteval` library². For AMC, we use the `ttr1`³ library and report avg@8 as the metric. (2)
 1068 Code generation: We assess coding ability using LiveCodeBench (Jain et al., 2025) `release_v6`
 1069 and CRUX (Gu et al., 2024). LiveCodeBench is evaluated with its official evaluation library⁴,
 1070 and CRUX is evaluated via the `ZeroEval` library⁵; for both datasets, we report pass@1 accuracy.
 1071 (3) Instruction-following and multi-task abilities: We evaluate on IFEval (Zhou et al., 2023b) and
 1072 MMLU-Pro (Wang et al., 2024), using the `lm-evaluation-harness` library⁶ for both. Overall,
 1073 we summarize the statistics of the datasets used in this paper in Table 6.
 1074
 1075

²<https://github.com/huggingface/lighteval>

³https://github.com/ruixin31/Spurious_Rewards/tree/main/code/ttr1

⁴<https://github.com/LiveCodeBench/LiveCodeBench>

⁵<https://github.com/WildEval/ZeroEval>

⁶<https://github.com/EleutherAI/lm-evaluation-harness>

1080 Table 8: **Supplement Results (%) of Co-rewarding and baselines trained on OpenRS**. Cell
 1081 background colors: darker colors denote better results within each model group.
 1082

Training Set: Open-RS		Mathematics			Code		Instruction	Multi-Task
Methods		MATH500	GSM8K	AMC	LiveCode	CRUX	IFEval	MMLU-Pro
<i>Qwen3-8B-Base</i>								
Before RL		72.40	27.82	20.93	23.41	54.75	50.89	52.92
- GT-Reward (Shao et al., 2024)		80.20	89.76	54.97	39.00	63.00	52.94	55.49
- Self-Certainty (Zhao et al., 2025b)		82.60	85.22	50.00	37.00	64.62	52.12	56.03
- Entropy (Prabhudesai et al., 2025)		80.60	87.41	48.95	38.00	61.25	52.53	56.80
- Majority-Voting (Shafayat et al., 2025)		78.00	84.23	51.96	36.75	58.00	51.13	54.92
- Co-rewarding-I (Ours)		78.20	92.65	50.60	28.91	63.12	53.11	57.21
- Co-rewarding-II (Ours)		80.00	90.90	53.01	39.75	62.75	52.92	56.55
<i>Qwen3-4B-Base</i>								
Before RL		71.20	26.15	21.08	11.00	38.88	46.43	47.23
- GT-Reward (Shao et al., 2024)		78.80	85.22	49.55	33.50	55.12	46.41	50.12
- Self-Certainty (Zhao et al., 2025b)		73.20	33.43	35.84	32.50	49.50	46.47	48.24
- Entropy (Prabhudesai et al., 2025)		76.80	87.57	42.62	35.00	53.87	47.61	52.42
- Majority-Voting (Shafayat et al., 2025)		76.00	64.14	44.58	32.25	50.25	46.35	48.75
- Co-rewarding-I (Ours)		72.80	83.93	39.41	26.54	53.25	48.11	50.82
- Co-rewarding-II (Ours)		76.60	89.23	42.32	34.00	51.50	48.45	51.80

1086 Table 9: **Performance (%) of test-time training (TTT)**. Since self-supervised methods are label-free,
 1087 they can be leveraged during inference for test-time training to further enhance performance.
 1088

LLMs	Methods	AMC							
		avg@8	pass@8	avg@16	pass@16	avg@32	pass@32	avg@64	pass@64
<i>Qwen2.5-7B</i>	Before-TTT	15.81	46.99	17.55	66.27	16.34	74.70	17.32	75.90
	Self-Certainty	41.57	74.70	39.23	74.70	39.68	78.31	39.95	87.95
	Entropy	38.70	56.63	39.76	68.67	39.57	79.52	39.34	81.93
	Majority-Voting	43.67	63.86	43.67	67.47	43.49	78.31	44.35	85.54
	Co-rewarding-I	44.88	60.24	45.33	60.24	45.44	71.08	45.76	73.49
	Co-rewarding-II	43.22	69.88	41.34	75.90	40.36	78.31	41.64	87.95
<i>Qwen3-8B-Base</i>	Before-TTT	20.93	61.45	21.31	73.49	19.58	79.52	20.97	86.75
	Self-Certainty	49.85	78.31	50.68	78.31	50.41	84.34	49.55	89.16
	Entropy	48.64	74.70	49.92	80.72	49.96	87.95	50.23	89.16
	Majority-Voting	50.90	73.49	50.00	72.29	50.60	80.72	51.36	85.54
	Co-rewarding-I	52.86	68.67	53.46	74.70	53.24	81.93	53.58	84.34
	Co-rewarding-II	48.64	72.29	48.19	73.49	50.19	83.13	49.28	91.57

D ADDITIONAL EXPERIMENTAL RESULTS

D.1 MORE RESULTS ON OTHER TRAINING SETS AND LLMs

1119 Table 7 reports additional results of Qwen2.5-3B and Qwen3-1.7B-Base and Qwen3-4B-Base trained
 1120 on MATH, while Table 8 extends the experiments of Qwen3-8B-Base and Qwen3-4B-Base to
 1121 another training set OpenRS (Dang & Ngo, 2025). It can be observed that Co-rewarding occupies
 1122 relatively darker areas. Across models and training sets, Co-rewarding-I and II achieve an average
 1123 relative improvement of +2.23% on GSM8K, with notably high pass@1 scores of 92.65% and
 1124 90.90% for Qwen3-8B-Base trained on OpenRS, respectively. Moreover, thanks to its stability,
 1125 Co-rewarding-II delivers more reliable gains than self-rewarding baselines, which occasionally suffer
 1126 lower performance on certain models or benchmarks, e.g., Self-Certainty on Qwen3-1.7B-Base in
 1127 Table 7 or Majority-Voting on Qwen3-4B-Base in Table 8. These results further demonstrate the
 1128 effectiveness of Co-rewarding.

D.2 MORE CURVES OF REWARD, RESPONSE LENGTH AND PSEUDO LABEL ACCURACY

1131 Figure 9 supplements the reward and response curves of Qwen3-4B-Base trained on DAPO-14k. The
 1132 trends are consistent with Qwen3-8B-Base and Llama-3.2-3B-Instruct in Figure 4: Majority-Voting
 1133 and Entropy rapidly increase rewards at early stage and quickly peak, a clear sign of reward hacking.
 In contrast, GT-Reward and Co-rewarding-II exhibit smoother, steadily rising rewards, indicating gen-

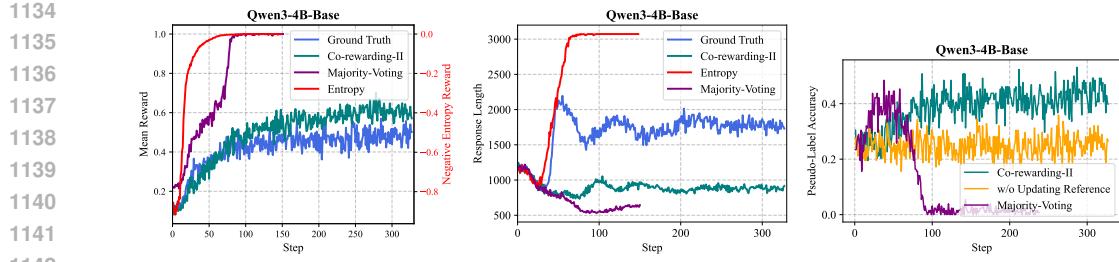


Figure 9: **Curves of reward (Left), response length (Middle), and pseudo label accuracy (Right) of Qwen3-4B-Base** trained on DAPO-14k. Entropy reward is plotted on the right y -axis due to its different reward scale. Note that entropy minimization is to maximizing the negative entropy.

Table 10: **Detailed MMLU-Pro performance on Qwen3-4B-Base and Llama-3.2-3B-Instruct train on DAPO-14k.** Results are reported for each of the 14 categories in MMLU-Pro.

Qwen3-4B-Base							
Methods	biology	business	chemistry	computer sci.	economics	engineering	health
- GT-Reward	73.50	63.49	59.71	56.34	65.05	42.93	50.86
- Self-Certainty	71.41	54.37	45.93	50.73	63.27	35.91	50.12
- Entropy	70.99	56.02	50.44	48.29	63.15	34.37	48.41
- Majority-Voting	70.43	55.77	52.83	53.41	62.79	38.09	50.61
- Co-rewarding-I	73.92	59.82	50.71	54.15	64.93	41.49	49.76
- Co-rewarding-II	72.66	59.95	55.65	53.41	64.10	39.73	50.61
Methods	history	law	math	other	philosophy	physics	psychology
- GT-Reward	44.88	26.34	69.80	48.81	44.69	57.04	65.79
- Self-Certainty	39.63	24.43	59.44	43.94	40.08	47.04	59.65
- Entropy	40.68	26.43	60.99	45.13	43.69	50.89	61.90
- Majority-Voting	40.94	23.43	64.17	43.39	44.09	50.73	63.66
- Co-rewarding-I	40.94	23.25	63.73	44.91	42.69	50.58	60.78
- Co-rewarding-II	42.26	24.79	67.58	44.59	41.88	54.19	62.91
Llama3.2-3B-Instruct							
Methods	biology	business	chemistry	computer sci.	economics	engineering	health
- GT-Reward	54.81	36.25	25.18	33.41	42.65	21.57	39.36
- Self-Certainty	55.23	32.95	27.21	31.95	42.77	20.54	39.12
- Entropy	52.86	31.05	23.94	32.93	41.71	20.43	38.02
- Majority-Voting	56.07	32.95	22.79	30.98	44.19	18.99	39.61
- Co-rewarding-I	51.88	34.22	22.88	34.88	44.67	19.09	38.63
- Co-rewarding-II	56.21	34.35	27.03	35.61	43.01	19.92	40.34
Methods	history	law	math	other	philosophy	physics	psychology
- GT-Reward	30.18	22.71	34.20	34.74	32.06	28.33	50.38
- Self-Certainty	30.45	24.98	33.38	31.60	29.86	28.56	50.50
- Entropy	33.86	21.89	32.35	33.01	29.46	24.25	47.50
- Majority-Voting	32.02	25.25	34.35	34.20	29.86	24.79	48.25
- Co-rewarding-I	33.86	23.25	32.12	33.01	31.86	25.40	48.75
- Co-rewarding-II	32.28	24.34	35.83	36.26	33.27	28.18	49.12

uine learning of reasoning ability. Moreover, Co-rewarding-II maintains moderate response lengths on Qwen3-4B-Base, further demonstrating its generality in balancing the exploration–exploitation trade-off during reasoning training, which is a core principle of RL (Wang et al., 2018).

Additionally, the right panel of Figure 9 presents the pseudo-label accuracy of Qwen3-4B-Base, showing trends consistent with Qwen3-8B-Base and Llama-3.2-3B-Instruct in Figure 6. As training progresses, Co-rewarding-II steadily improves pseudo-label accuracy, while “w/o Updating Reference” remains around 25%. Majority-Voting briefly increases accuracy but soon collapses to zero, clearly indicating reward hacking. This highlights our design philosophy of pairing a fast policy student with a slowly updated teacher, which decouples supervision from the online policy while enabling the teacher to co-evolve with the student, thereby sustaining improvements in pseudo-label quality.

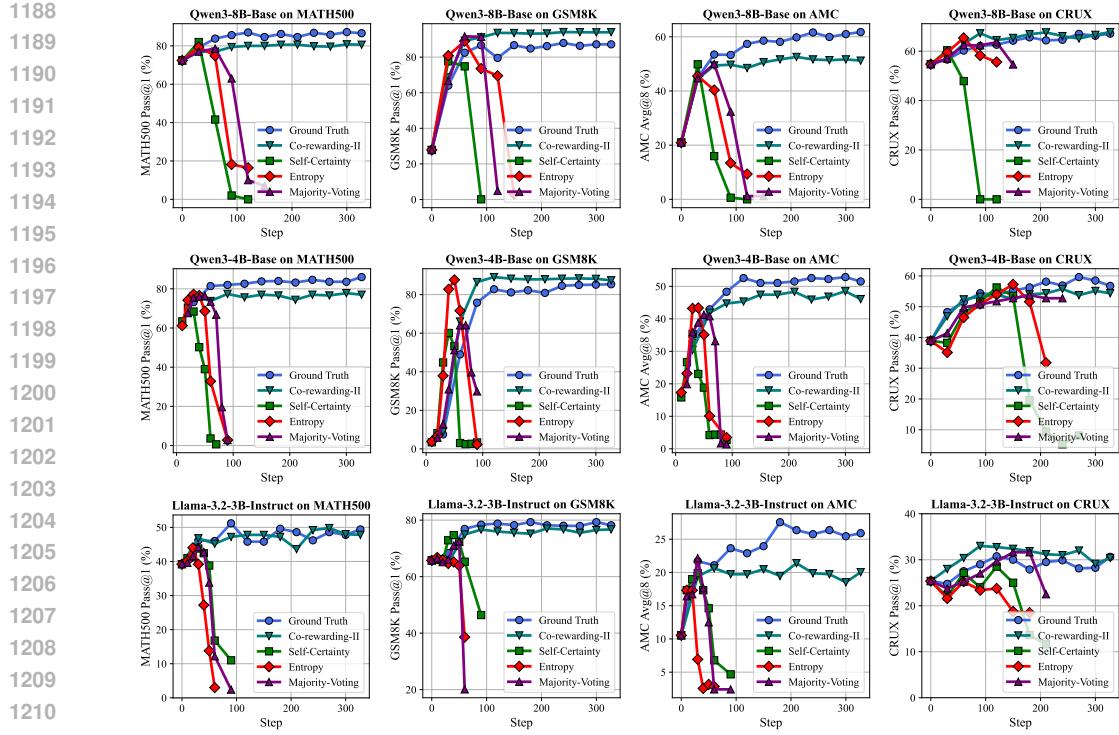


Figure 10: **Performance curves on benchmarks of MATH500, GSM8K, AMC and CRUX** across Qwen3-8B-Base, Qwen3-4B-Base, and Llama-3.2-3B-Instruct trained on DAPO-14k.

D.3 MORE RESULTS OF VALIDATION PERFORMANCE CURVES

As a supplement to Figure 3, Figure 8 presents validation performance curves for Qwen2.5-3B, Qwen3-4B-Base, Qwen3-8B-Base, Llama-3.2-3B-Instruct trained on MATH, as well as Qwen3-4B-Base trained on DAPO-14k. Self-Certainty and Entropy collapse rapidly across all settings, as their supervision signals are tied to internal confidence or entropy and are easily exploited. Majority-Voting also collapses in several cases, reflecting that sampling pseudo labels from outputs cannot prevent hacking. By contrast, Co-rewarding-I maintains stability across MATH-trained models through data-side contrastive agreement, while Co-rewarding-II consistently provides stability across all models and datasets by disentangling supervision with a slowly updated teacher, making hacking substantially harder and optimization more reliable.

D.4 RESULTS OF TEST-TIME TRAINING (TTT)

Thanks to the label-free nature of self-supervised methods, which do not require GT labels, they are naturally compatible with test-time training (TTT), enabling further refinement of the model during inference. Table 9 reports the TTT results on the challenging competition-level benchmark AMC across Co-rewarding and other self-rewarding baselines. We observe that Co-rewarding matches or even surpasses existing methods, achieving the best results on 11 out of 18 metrics. These findings broaden the applicability of self-supervised RL: beyond post-training for reasoning elicitation, it can also be leveraged at inference time to further improve performance on specific benchmarks.

D.5 MORE RESULTS OF BENCHMARK PERFORMANCE CURVES

As a supplement to Figure 3 and Figure 5, Figure 10 presents performance curves on MATH500, GSM8K, AMC, and CRUX with Qwen3-8B-Base, Qwen3-4B-Base, and Llama-3.2-3B-Instruct. Consistent with earlier findings, Self-Certainty, Entropy, and Majority-Voting rapidly collapse across benchmarks and models, while Co-rewarding-II and GT-Reward sustain continued and stable im-

1242 Table 11: **Impact of math training collapse on code and multi-task performance.** Results are
 1243 evaluated on models before and after training collapse.

Qwen3-4B-Base							
Training stage	Methods	Mathematics			Code		Multi-task
		MATH500	GSM8K	AMC	AIME24	LiveCode	MMLU-Pro
Before training collapse	- Self-Certainty	68.4	44.81	35.39	8.85	25.88	50.12
	- Entropy	76.6	82.79	43.37	12.81	26.35	50.75
	- Majority-Voting	73.4	64.06	40.81	9.17	26.16	53.00
After training collapse	- Self-Certainty	2.8	3.34	2.71	0.00	14.22	8.12
	- Entropy	2.8	2.35	3.46	0.00	18.60	31.75
	- Majority-Voting	2.8	4.85	1.36	0.00	24.36	52.75

1253 Table 12: **Difference between original and rephrased questions** from background richness, vocab-
 1254 ularity complexity, and sentence complexity.

Training Set	# Data Size	Background richness	Vocabulary complexity	Sentence complexity
MATH	7,500	+4.91%	+4.79%	+9.05%
DAPO-14k	14,100	+4.65%	1.95%	+4.19%

1261 Table 13: **Success rate of different rephraser LLMs:** MATH training set rephrased by Qwen3-32B,
 1262 Qwen3-8B, and Qwen3-1.7B, respectively.

Rephraser LLM	Training Set	# Original questions	# Rephrased questions	Success rate (%)
Qwen3-32B	MATH	7,500	7,498	99.97%
Qwen3-8B	MATH	7,500	7,477	99.69%
Qwen3-1.7B	MATH	7,500	2,060	27.47%

1271 improvements. These results underscore the link between performance and training stability: stable
 1272 training enables models to continue improving by effectively learning knowledge from more data.

1274 D.6 IMPACT OF MATH TRAINING COLLAPSE ON OTHER TASKS

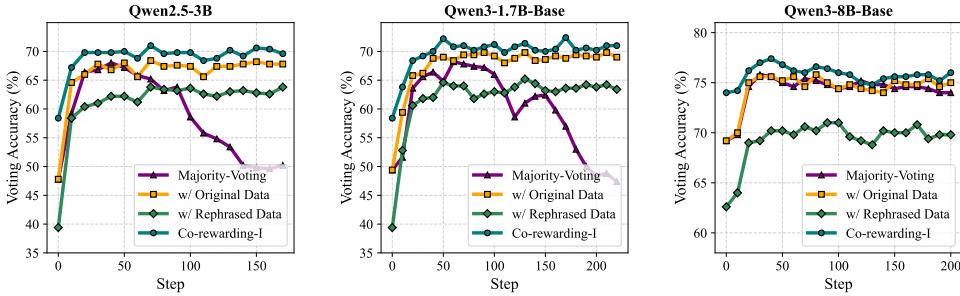
1276 We investigate how training collapse occurring on math-oriented training sets impacts the model’s
 1277 performance on code-generation and multi-task benchmarks. To this end, we evaluate models trained
 1278 with existing self-rewarding methods (Self-Certainty, Entropy, and Majority-Voting) both before
 1279 and after training collapse. Table 11 summarizes the results. We observe that training collapses on
 1280 math-related training sets affect other tasks (LiveCode, CRUX and MMLU-Pro) in different way for
 1281 certainty- or entropy-based methods (Self-Certainty and Entropy) compared with consensus-based
 1282 methods (Majority-Voting). When collapse occurs on math-oriented training sets, all three methods
 1283 show substantial performance degradation on the four math benchmarks (MATH500, GSM8K, AMC,
 1284 and AIME24). However, their impacts on other tasks differ:

1285 For certainty- or entropy-based methods, the performance on LiveCode, CRUX, and MMLU-Pro also
 1286 declines after collapse on math training sets. This arises from their reward objectives: maximizing self-
 1287 certainty or minimizing entropy, result in the decoding probability mass becoming highly concentrated
 1288 on a very subset of tokens. Consequently, the model produces repetitive outputs, and this repetitive
 1289 decoding behavior transfers across tasks, leading to degraded performance beyond the math domain.

1290 For the consensus-based method, Majority-Voting shows similar performance before and after training
 1291 collapse on math-oriented training sets. This may be because its collapses stem from reward hacking
 1292 at the answer format: the model exploits the $\boxed{\cdot}$ structure by consistently inserting an incorrect
 1293 but self-consistent answer to maximize reward. This type of collapse weakly affects the intermediate
 1294 reasoning trace, which largely remains structured. Since code-generation and multi-task benchmarks
 1295 do not rely on boxed-answer extraction, this type of collapse has limited impact on their performance.

1296 Table 14: **Impact of rephraser LLM for Co-rewarding-I.** Train Qwen3-8B-Base using data
 1297 rephrased by Qwen3-32B, Qwen3-8B and Qwen3-1.7B, respectively.

Trained Model	Rephraser LLM	MATH500	GSM8K	AMC	AIME24	LiveCode	CRUX	IFEval	MMLU-Pro
Qwen3-8B-Base	Qwen3-32B	81.2	93.70	51.20	15.10	30.81	66.00	55.79	59.95
	Qwen3-8B	79.2	92.72	51.51	14.58	30.90	63.12	54.73	59.30
	Qwen3-1.7B	78.2	87.41	49.25	12.81	29.57	61.00	53.44	55.85



1303 Figure 11: **Curves of voting accuracy of Majority-Voting, Co-rewarding-I and its ablations with**
 1304 **Qwen2.5-3B, Qwen3-1.7B-Base and Qwen3-8B-Base trained on MATH.**

1314 D.7 DISCUSSION OF MATH AND DAPO-14K

1315 We leverage Qwen3-235B-A22B to score the difference between original and rephrased questions
 1316 from multiple perspectives, including background richness, vocabulary complexity, and sentence
 1317 complexity, for MATH and DAPO-14k. From Table 12, we observe that the rephrasing in MATH
 1318 exhibits larger changes from the original to rephrased questions than DAPO-14k. This suggests that
 1319 the questions in MATH may provide favorable conditions for promoting diverse rephrasing variability,
 1320 which is beneficial for the effectiveness of contrastive agreement in Co-rewarding-I.

1321 D.8 ROBUSTNESS ANALYSIS OF DIFFERENT REPHRASER LLMs

1322 To analyze the impact of different rephraser LLMs for Co-rewarding-I, we conduct additional
 1323 experiments using smaller LLMs instead of Qwen3-32B for rephrasing. To control architectural
 1324 variability in the rephraser LLMs, we employ two smaller LLMs from the same family, i.e., Qwen3-
 1325 8B and Qwen3-1.7B, for rephrasing the MATH training set. Table 13 reports the rephrasing success
 1326 rate. We observe that rephrasing success rates drop as the model size decreases, which is expected:
 1327 rephrasing math questions while preserving the analogical essence is a relatively challenging task,
 1328 and weaker LLMs struggle to achieve this goal. This observation supports our choice of Qwen3-32B
 1329 as the rephraser, as a sufficiently capable LLM is required to produce faithful rephrasing.

1330 We then train Co-rewarding-I on Qwen3-8B-Base using rephrased data generated by Qwen3-32B,
 1331 Qwen3-8B, and Qwen3-1.7B, respectively. The performance is summarized in Table 14. From
 1332 the results, it can be observed that performance gradually degrades as the size of the rephraser
 1333 LLM decreases, but not always significantly. Rephrasing with Qwen3-8B maintains reasonably
 1334 similar performance to using Qwen3-32B, indicating that Co-rewarding-I exhibits a certain degree of
 1335 robustness under moderate reductions in rephrasing quality. Notably, rephrasing with Qwen3-1.7B
 1336 leads to a substantial performance drop. This degradation is largely attributable to the significantly
 1337 lower rephrasing success rate of Qwen3-1.7B, which results in a substantial reduction of usable
 1338 training data and consequently weakens the effectiveness of Co-rewarding-I.

1339 D.9 VOTING ACCURACY ANALYSIS OF CO-REWARDING-I

1340 To demonstrate the stability and efficiency of Co-rewarding-I, we compare its voting accuracy against
 1341 that of Majority-Voting in Figure 11 and Figure 12. These experiments are conducted on Qwen2.5-
 1342 3B, Qwen3-1.7B-Base and Qwen3-8B-Base models, all trained on the MATH dataset. Across all
 1343 settings, the Majority-Voting method exhibits reward hacking, where its performance sharply declines

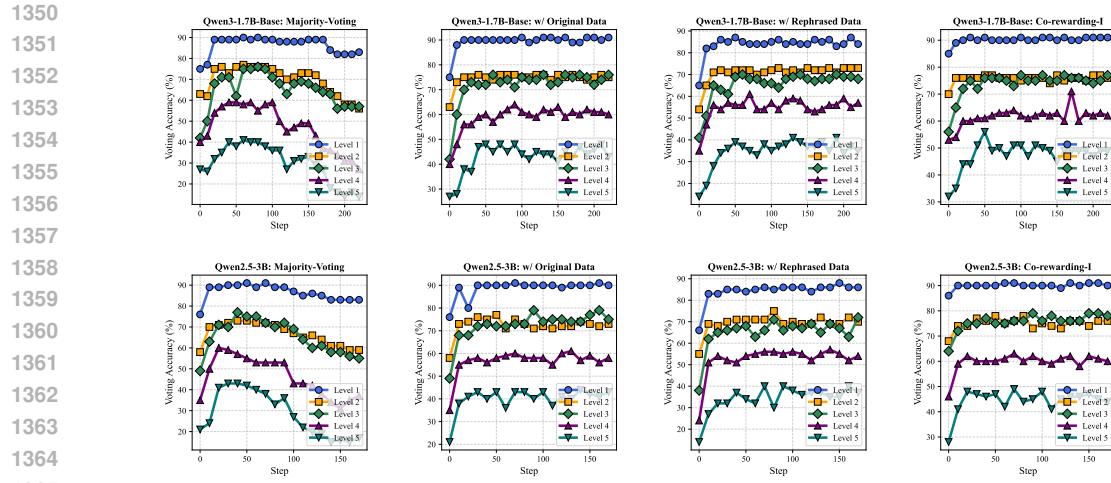


Figure 12: **Voting accuracy of Majority-Voting, Co-rewarding-I and its ablated variants across different difficulty levels of questions** **Top:** Qwen3-1.7B-Base. **Bottom:** Qwen2.5-3B.

after reaching an early peak, particularly on more difficult questions (levels 2 to 5). In contrast, Co-Rewarding-I maintains a stable voting accuracy on both original and rephrased data. Ultimately, it achieves the highest overall voting accuracy across all models and dataset configurations.

Table 15: **Detailed MMLU-Pro performance on Qwen3-8B-Base and Qwen3-4B-Base trained on OpenRS.** Results are reported for each of 14 categories in MMLU-Pro.

Qwen3-4B-Base							
Methods	biology	business	chemistry	computer sci.	economics	engineering	health
- GT-Reward	70.99	59.82	52.30	54.63	65.05	39.01	51.22
- Self-Certainty	69.87	54.50	44.08	49.27	63.63	37.36	50.24
- Entropy	70.71	58.68	49.03	51.22	63.39	37.46	49.63
- Majority-Voting	69.60	55.77	47.17	53.17	63.39	36.02	48.78
- Co-rewarding-I	69.04	55.39	47.79	53.41	63.86	38.39	50.61
- Co-rewarding-II	70.85	58.81	53.27	53.90	66.11	37.15	52.81
Methods	history	law	math	other	philosophy	physics	psychology
- GT-Reward	39.63	24.98	65.58	47.84	40.68	54.50	62.53
- Self-Certainty	39.63	24.25	58.11	46.65	40.88	46.42	61.40
- Entropy	39.90	22.16	62.18	45.02	43.09	50.19	59.90
- Majority-Voting	40.68	22.52	60.25	46.10	41.08	48.42	60.65
- Co-rewarding-I	40.68	24.25	62.18	44.37	44.49	49.58	61.65
- Co-rewarding-II	41.21	25.89	64.91	45.24	39.28	52.27	59.40
Qwen3-8B-Base							
Methods	biology	business	chemistry	computer sci.	economics	engineering	health
- GT-Reward	74.76	63.24	55.48	63.17	68.96	41.38	57.09
- Self-Certainty	75.03	63.62	53.62	55.61	68.96	39.83	57.09
- Entropy	75.73	64.39	54.51	58.29	65.05	41.69	55.87
- Majority-Voting	76.15	60.20	54.15	56.34	69.91	38.91	55.75
- Co-rewarding-I	76.43	65.78	57.07	62.20	69.43	43.14	56.60
- Co-rewarding-II	76.84	64.25	54.68	62.43	68.12	42.00	58.06
Methods	history	law	math	other	philosophy	physics	psychology
- GT-Reward	50.92	30.25	67.58	52.49	51.10	57.20	67.67
- Self-Certainty	49.34	28.88	68.02	51.62	52.10	56.89	66.42
- Entropy	50.39	30.43	65.28	51.41	47.09	54.50	66.67
- Majority-Voting	48.03	28.88	63.43	53.68	48.10	52.50	64.66
- Co-rewarding-I	50.13	29.97	68.54	52.92	50.70	56.66	65.54
- Co-rewarding-II	51.44	30.06	65.80	51.51	52.10	57.58	65.78

1404 D.10 MORE RESULTS OF MMLU-PRO EVALUATION
1405

1406 As a complement to Table 4, Table 10 and Table 15 report detailed MMLU-Pro results for models
1407 trained on DAPO-14k and OpenRS, respectively. We observe that Co-rewarding consistently preserves
1408 general-domain performance across diverse subjects, indicating that though trained on math-oriented
1409 datasets, its improvements do not come at the cost of broader capabilities from other domains.

1410
1411 D.11 MORE RESULTS OF IFEVAL EVALUATION
1412

1413 The aim of IFEval is used to evaluate the instruction-following ability of LLMs. In Table 1, Table 2,
1414 Table 7 and Table 8, we report average IFEval performance due to space constraints. Specifically,
1415 the evaluation of IFEval includes four metrics: {prompt_level.strict_acc, inst_level.strict_acc,
1416 prompt_level.loose_acc and inst_level.loose_acc}, which apply different levels of answer matching.
1417 As a supplement, complete results are provided in Table 16, Table 17, and Table 18. The results show
1418 that Co-rewarding not only preserves the inherent instruction-following ability of base models but
1419 also often surpasses GT-Reward across multiple models. This further confirms that Co-rewarding’s
1420 gains on mathematical and coding benchmarks are achieved without sacrificing general-domain
1421 instruction-following ability.

1422 Table 16: **Detailed IFEval Performance on Qwen2.5-3B/7B, Qwen3-1.7B/4B/8B-Base and Llama-
1423 3.2-3B-Instruct trained on MATH.** Results are reported for loose and strict settings respectively.

Methods	IFEval																	
	Average	Prompt Strict		Prompt Loose		Inst. Strict		Inst. Loose		Average	Prompt Strict		Prompt Loose		Inst. Strict		Inst. Loose	
		Qwen2.5-3B		Qwen2.5-7B		Qwen3-1.7B-Base		Qwen3-4B-Base			Qwen3-8B-Base		Llama3-2-Instruct					
Before RL	29.83	22.55	27.17	31.89	37.70	38.19	29.57	34.57	41.85	46.76	33.66	25.51	31.42	35.85	41.85	39.56	43.65	50.96
- GT-Reward	33.66	25.51	31.42	35.85	41.85	41.49	31.79	39.56	43.65	50.96								
- Self-Certainty	32.22	24.40	29.76	34.65	40.05	37.24	28.47	34.38	40.05	46.04								
- Entropy	32.94	24.77	30.50	35.13	41.37	40.33	30.13	37.87	43.29	50.00								
- Majority-Voting	35.19	26.25	32.72	37.53	44.24	38.60	29.21	35.86	41.61	47.72								
- Co-rewarding-I	33.86	23.84	31.61	36.09	43.88	41.73	32.35	39.37	44.48	50.72								
- Co-rewarding-II	32.74	23.29	29.02	36.33	42.33	41.82	31.79	40.29	43.88	51.31								
		Qwen3-1.7B-Base		Qwen3-4B-Base		Qwen3-8B-Base		Llama3-2-Instruct										
Before RL	33.65	25.69	30.86	36.45	41.60	46.43	36.04	44.18	48.68	56.83								
- GT-Reward	36.16	27.35	31.79	40.64	44.84	47.80	37.34	46.77	49.40	57.67								
- Self-Certainty	32.96	24.58	29.20	36.69	41.36	48.15	39.37	46.76	49.52	56.95								
- Entropy	35.37	26.61	31.42	39.44	44.00	50.44	40.67	48.61	52.52	59.07								
- Majority-Voting	35.45	26.06	32.16	38.72	48.84	48.78	37.89	47.50	50.36	59.65								
- Co-rewarding-I	35.56	27.91	31.23	39.32	43.76	50.35	40.67	49.35	51.56	59.83								
- Co-rewarding-II	36.94	27.17	33.64	40.05	46.88	51.30	41.40	49.54	53.12	61.15								
		Qwen3-1.7B-Base		Qwen3-4B-Base		Qwen3-8B-Base		Llama3-2-Instruct										
Before RL	50.32	40.11	50.27	51.07	59.83	57.32	46.77	55.27	60.19	67.03								
- GT-Reward	52.78	41.96	51.76	54.44	62.95	47.41	37.34	42.88	52.52	57.31								
- Self-Certainty	50.98	39.74	49.54	52.88	61.75	54.88	43.81	52.68	58.15	64.87								
- Entropy	51.81	40.67	51.20	52.76	62.59	54.70	43.81	52.68	57.67	64.63								
- Majority-Voting	51.80	39.74	51.02	53.60	62.83	47.96	37.34	43.44	52.88	58.18								
- Co-rewarding-I	55.79	43.99	57.11	55.63	66.42	49.14	39.37	45.66	53.12	58.39								
- Co-rewarding-II	60.70	55.64	65.59	56.00	65.59	49.90	39.93	45.66	54.68	59.35								

1441
1442 D.12 ORIGINAL QUESTIONS VS. REPHRASED QUESTIONS
1443

1444 To provide an intuitive illustration, we present several examples of original questions with their
1445 rephrased versions in Table 19. We observe that such rephrasings are reasonable and effective, as they
1446 preserve the same underlying mathematical essence while presenting the problems in a substantially
1447 different surface form. This reflects the high quality of our rephrased data and forms the basis of
1448 Co-rewarding-I: by leveraging contrastive agreement across data-invariant variants, the model is
1449 encouraged to elicit more robust reasoning ability.

1450 D.13 COMPLETE CASE STUDY
1451

1452 As a supplement to Figure 7, we present the complete generation outputs of this case study. The
1453 full outputs clearly reveal the reward hacking behaviors of existing self-rewarding baselines. Self-
1454 Certainty and Entropy fall into repetitive outputs—for example, Self-Certainty repeatedly generates
1455 “Understanding,” and Entropy repeatedly produces “Simplify the next fraction” until truncated at
1456 the maximum length. This arises because their decoding probability mass collapses onto a small
1457 subset of tokens, leading the model to loop over them. Majority-Voting shows another form of reward
1458 hacking by boxing an incorrect answer “0” to maximize consensus across rollouts and thereby secure

1458 Table 17: **Detailed IFEval performance on Qwen3-4B/8B-Base and Llama-3.2-3B-Instruct**
 1459 **train on DAPO-14k.** Results are reported for loose and strict settings in IFEval, respectively.

Methods	IFEval				
	Average	Prompt Strict	Prompt Loose	Inst. Strict	Inst. Loose
<i>Qwen3-4B-Base</i>					
Before RL	46.43	36.04	44.18	48.68	56.83
- GT-Reward	47.70	37.52	45.84	49.76	57.67
- Self-Certainty	45.58	35.67	43.99	47.84	54.80
- Entropy	48.20	37.71	46.58	50.48	58.03
- Majority-Voting	48.91	39.19	47.69	50.24	58.51
- Co-rewarding-I	46.84	36.41	45.66	48.80	56.47
- Co-rewarding-II	48.90	39.56	46.21	51.44	58.39
<i>Qwen3-8B-Base</i>					
Before RL	50.32	40.11	50.27	51.07	59.83
- GT-Reward	53.11	41.59	52.13	54.56	64.15
- Self-Certainty	50.58	41.04	49.54	51.68	60.07
- Entropy	51.56	41.59	49.91	53.48	61.27
- Majority-Voting	51.54	41.22	51.02	52.64	61.27
- Co-rewarding-I	50.17	40.67	48.24	52.16	59.59
- Co-rewarding-II	53.31	41.40	53.23	54.20	64.39
<i>Llama3.2-3B-Instruct</i>					
Before RL	57.32	46.77	55.27	60.19	67.03
- GT-Reward	53.10	42.33	49.91	57.19	62.95
- Self-Certainty	54.50	44.55	51.76	58.03	63.67
- Entropy	55.78	45.29	53.23	59.11	65.47
- Majority-Voting	54.07	42.33	52.50	56.83	64.63
- Co-rewarding-I	53.04	42.33	51.02	55.76	63.07
- Co-rewarding-II	51.92	41.59	48.24	56.00	61.87

1484 Table 18: **Detailed IFEval Performance on Qwen3-8B/4B-Base trained on Open-RS.** Results are
 1485 reported for loose and strict settings in IFEval, respectively.

Methods	IFEval					Average	Prompt Strict	Prompt Loose	Inst. Strict	Inst. Loose
	Average	Prompt Strict	Prompt Loose	Inst. Strict	Inst. Loose					
<i>Qwen3-8B-Base</i>										
Before RL	50.32	40.11	50.27	51.07	59.83	46.43	36.04	44.18	48.68	56.83
- GT-Reward	52.53	41.59	51.02	54.56	62.95	47.80	37.34	46.77	49.40	57.67
- Self-Certainty	52.12	41.59	50.83	53.72	62.35	46.47	35.86	44.73	48.56	56.71
- Entropy	52.94	43.25	51.94	53.72	62.83	46.41	36.97	44.73	48.68	55.28
- Majority-Voting	51.13	40.67	49.35	53.36	61.15	46.35	36.41	44.18	48.80	56.00
- Co-rewarding-I	53.11	41.40	53.05	53.95	64.02	46.83	35.86	46.77	47.84	56.83
- Co-rewarding-II	52.92	42.14	52.50	54.08	62.95	48.45	38.26	46.77	50.24	58.51

1494 the highest reward. In contrast, both Co-rewarding and GT-Reward yield the correct final answer
 1495 along with a coherent step-by-step reasoning path.

D.14 CASE STUDY ON CODE BENCHMARK

1500 To understand the behaviors of different methods on other tasks, we supplement an additional case
 1501 study on CRUX. We observe that Self-Certainty and Entropy fail by producing repetitive outputs,
 1502 consistent with the collapse pattern on math-related benchmarks. In contrast, GT-Reward, Majority-
 1503 Voting, and Co-rewarding generate coherent reasoning trace and correct solutions.

1504 Question:

1505 $\frac{m}{n}$ is the Irreducible fraction value of

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

1511 , what is the value of $m + n$?

Table 19: **Case studies:** Original vs. Rephrased Questions

Original Question	Rephrased Question
Sam is hired for a 20-day period. On days that he works, he earns \$60. For each day that he does not work, \$30 is subtracted from his earnings. At the end of the 20-day period, he received \$660. How many days did he not work?	A contractor agrees to a job lasting 20 days. For every day the job is completed on time, the contractor earns \$60. However, for each day the work is delayed, a fine of \$30 is applied. After the 20-day period, the contractor's total earnings are \$660. How many days was the job delayed?
Karen drove continuously from 9:40 a.m. until 1:20 p.m. of the same day and covered a distance of 165 miles. What was her average speed in miles per hour?	A traveler set off at 9:40 a.m. and reached their destination at 1:20 p.m. the same day after traveling a total of 165 miles. What was their average speed during the trip in miles per hour?
Solve for x : $\frac{1}{2} + \frac{1}{x} = \frac{5}{6}$.	A tank is partially filled by two different pipes. One pipe fills half the tank in an hour, and together with another pipe, they fill five-sixths of the tank in the same time. If the second pipe alone fills $\frac{1}{x}$ of the tank in an hour, find the value of x .

Co-rewarding-I Answer:

To solve the problem, we need to simplify the given expression step by step and then find the irreducible fraction form. The expression is:

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

First, we simplify the innermost fraction:

$$3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$$

Now substitute this back into the expression:

$$3 + \frac{1}{\frac{10}{3}} = 3 + \frac{1}{3 + \frac{3}{10}}$$

Next, simplify the fraction inside the denominator:

$$3 + \frac{3}{10} = \frac{30}{10} + \frac{3}{10} = \frac{33}{10}$$

Now substitute this back into the expression:

$$3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33}$$

To add these, we need a common denominator:

$$3 + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}$$

The fraction $\frac{109}{33}$ is already in its simplest form because 109 and 33 have no common factors other than 1. Therefore, $m = 109$ and $n = 33$, and the value of $m + n$ is:

$$109 + 33 = 142$$

So, the final answer is:

142.

1566
1567

Co-rewarding-II Answer:

1568
1569
1570

To solve the given expression $3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$, we need to simplify it step by step.

First, simplify the innermost fraction:

1571
1572
1573

$$3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}.$$

1574

Next, substitute this back into the next fraction:

1575
1576
1577

$$3 + \frac{1}{3 + \frac{1}{10}} = 3 + \frac{1}{3 + \frac{3}{10}} = 3 + \frac{1}{\frac{30}{10} + \frac{3}{10}} = 3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33}.$$

1578
1579
1580
1581

Now, simplify the expression $3 + \frac{10}{33}$:

$$3 + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}.$$

1582
1583

The fraction $\frac{109}{33}$ is already in its simplest form because 109 and 33 are coprime (they have no common factors other than 1).

1584
1585

Thus, the irreducible fraction value of the given expression is $\frac{109}{33}$, where $m = 109$ and $n = 33$. Therefore, $m + n = 109 + 33 = 142$.

1586
1587

The value of $m + n$ is 142.

1588
1589

GT-Reward Answer:

1590
1591

Alright, I have this problem:

The expression is:

1592
1593
1594

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

1595
1596
1597

And I need to find the irreducible fraction $\frac{m}{n}$ that equals this expression, then determine $m + n$.

1598
1599
1600
1601

First, I need to simplify the expression step by step. It looks like a continued fraction, so I'll start by simplifying the innermost part and work my way out.

Let's denote the entire expression as:

$$E = 3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

1602
1603

Step 1: Simplify the innermost fraction

Start with the innermost fraction:

1604
1605

$$3 + \frac{1}{3}$$

1606
1607
1608

Calculate this:

$$3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$$

1609

So now, the expression becomes:

1610
1611
1612

$$E = 3 + \frac{1}{3 + \frac{1}{\frac{10}{3}}}$$

1613
1614
1615
1616

Wait, I need to simplify $\frac{1}{\frac{10}{3}}$:

$$\frac{1}{\frac{10}{3}} = \frac{3}{10}$$

1617
1618
1619

Now, the expression is:

$$E = 3 + \frac{1}{3 + \frac{3}{10}}$$

1620
 1621 ****Step 2: Simplify the next fraction****
 1622 Now, simplify $3 + \frac{3}{10}$:
 1623
$$3 + \frac{3}{10} = \frac{30}{10} + \frac{3}{10} = \frac{33}{10}$$

 1624
 1625 So, the expression now is:
 1626
$$E = 3 + \frac{1}{\frac{33}{10}}$$

 1627
 1628 Simplify $\frac{1}{\frac{33}{10}}$:
 1629
 1630
$$\frac{1}{\frac{33}{10}} = \frac{10}{33}$$

 1631
 1632 Now, the expression is:
 1633
$$E = 3 + \frac{10}{33}$$

 1634
 1635 ****Step 3: Combine the terms****
 1636 Convert 3 to a fraction with denominator 33:
 1637
$$3 = \frac{99}{33}$$

 1638
 1639 Now, add the fractions:
 1640
$$E = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}$$

 1641
 1642 So, the expression simplifies to $\frac{109}{33}$.
 1643 ****Step 4: Ensure the fraction is irreducible****
 1644 Check if 109 and 33 have any common divisors. 109 is a prime number (I think), and 33 is
 1645 3×11 . Since 109 isn't divisible by 3 or 11, the fraction $\frac{109}{33}$ is already in its irreducible form.
 1646
 1647 ****Step 5: Find $m + n$ ****
 1648 Here, $m = 109$ and $n = 33$, so:
 1649
$$m + n = 109 + 33 = 142$$

 1650
 1651 Wait, let's double-check the calculations to ensure no mistakes were made.
 1652 ****Verification:****
 1653 Starting over:
 1654
$$E = 3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

 1655
 1656 Innermost:
 1657
$$3 + \frac{1}{3} = \frac{10}{3}$$

 1658
 1659 Next layer:
 1660
$$3 + \frac{1}{\frac{10}{3}} = 3 + \frac{3}{10} = \frac{33}{10}$$

 1661
 1662 Outermost:
 1663
$$3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}$$

 1664
 1665 Yes, it checks out. Therefore, $m + n = 109 + 33 = 142$.
 1666 142

1668
 1669 **Majority-Voting Answer:**
 1670
 1671 To find the value of $\frac{m}{n}$ as the irreducible fraction of the expression $3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$, we need to
 1672 simplify the expression step by step.
 1673

1674

1675

1676

1677

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1679

First, let's simplify the innermost fraction:

$$3 + \frac{1}{3} = 3 + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{10}{3}$$

Now, substitute this back into the expression:

$$3 + \frac{1}{3 + \frac{1}{3}} = 3 + \frac{1}{3 + \frac{3}{10}}$$

Next, simplify the fraction $3 + \frac{3}{10}$:

$$3 + \frac{3}{10} = \frac{30}{10} + \frac{3}{10} = \frac{33}{10}$$

Now, substitute this back into the expression:

$$3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33}$$

To add these, we need a common denominator:

$$3 + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{109}{33}$$

So, the irreducible fraction value of the expression is $\frac{109}{33}$. Here, $m = 109$ and $n = 33$. Therefore, the value of $m + n$ is:

$$m + n = 109 + 33 = 142$$

Thus, the value of $m + n$ is 0.

Self-Certainty Answer:

Alright, I have this math problem to solve:

Problem: I need to solve a math problem. The problem is about understanding how to solve math problems. I need to understand how to solve this problem so I can understand more about math.

Understanding the Problem

First, I need to understand what the problem is. I need to understand math so I can understand more about numbers and how to solve problems with numbers. Understanding math will help me understand more about how to understand problems.

Understanding Math Problems

After understanding the problem, I need to understand more about math problems. Understanding math problems will help me understand how to understand these problems. Understanding these problems will help me understand more about how to understand math.

Understanding Mathematical Concepts

After understanding math problems, I need to understand more about mathematical concepts. Understanding mathematical concepts will help me understand more about math. Understanding these concepts will help me understand more about how math helps me understand numbers.

Understanding Mathematical Operations

After understanding mathematical concepts, I need to understand more about mathematical operations. Understanding mathematical operations will help me understand more about how to understand these operations. Understanding these operations will help me understand more about how these operations help me understand math.

Understanding Mathematical Operations in Math

After understanding mathematical operations, I need to understand

Understanding Mathematical Methods

1728
 1729 After understanding mathematical operations, I need to understand more about mathematical
 1730 methods.
 1731 **### Understanding How Mathematical Methods Help**
 1732
 1733 **### Understanding How These Methods Help Me Understand Math**
 1734 After understanding more about how these methods help me, I need to understand
 1735 **### Understanding How These Methods Help Me Understand Math Problems**
 1736 After understanding how these methods help me understand math, I need to understand
 1737 **### Understanding How These Methods Help Me Solve Math Problems**
 1738 After understanding how these methods help me understand these math problems, I need to
 1739 understand
 1740 **### Understanding How These Methods Help Me Solve These Problems**
 1741 After understanding how these methods help me solve these problems, I need to understand
 1742 more about
 1743

Entropy Answer:

To find the value of $\frac{m}{n}$ as the irreducible fraction of the expression:

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}$$

Let's simplify the expression step by step.

1. **Simplify the innermost fraction:**

$$\begin{aligned} 3 + \frac{1}{3} &= 3 + \frac{3 \times 3}{3} + \frac{1}{3} = \frac{9}{3} + \frac{1}{3} = \frac{9+1}{3} = \frac{10}{3} \end{aligned}$$

2. **Simplify the next fraction:**

$$\begin{aligned} 3 + \frac{1}{3 + \frac{1}{3}} &= 3 + \frac{1}{\frac{10}{3}} = 3 + \frac{3}{10} \\ 3 + \frac{1}{\frac{10}{3}} &= 3 + \frac{1 \times 3}{10} = 3 + \frac{3}{10} = \frac{3 \times 10}{10} + \frac{3}{10} = \frac{30}{10} + \frac{3}{10} = \frac{30+3}{10} = \frac{33}{10} \end{aligned}$$

3. **Simplify the next fraction:**

$$\begin{aligned} 3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}} &= 3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33} \\ 3 + \frac{1}{\frac{33}{10}} &= 3 + \frac{1 \times 10}{33} = 3 + \frac{10}{33} = \frac{3 \times 33}{33} + \frac{10}{33} = \frac{99}{33} + \frac{10}{33} = \frac{99+10}{33} = \frac{109}{33} \end{aligned}$$

4. **Simplify the next fraction:**

$$\begin{aligned} 3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}} &= 3 + \frac{1}{\frac{109}{33}} = 3 + \frac{33}{109} \\ 3 + \frac{1}{\frac{109}{33}} &= 3 + \frac{1 \times 33}{109} = 3 + \frac{33}{109} = \frac{3 \times 109}{109} + \frac{33}{109} = \frac{327}{109} + \frac{33}{109} = \frac{327+33}{109} = \frac{360}{109} \end{aligned}$$

5. **Simplify the next fraction:**

$$3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}} = 3 + \frac{1}{\frac{360}{109}}$$

Question:

```
1849 ## Question:
1850
1851 You are given a Python function and an assertion containing an
1852 input to the function. Complete the assertion with a literal (no
1853 unsimplified expressions, no function calls) containing the
1854 output when executing the provided code on the given input, even
1855 if the function is incorrect or incomplete.
1856
1857 [PYTHON]
1858 def f(text):
1859     new_text = list(text)
1860     for i in '+':
1861         if i in new_text:
1862             new_text.remove(i)
1863     return ''.join(new_text)
1864 assert f('hbtotfdeiequ') == ???
1865 [/PYTHON]
1866
1867 ## Instruction
1868
1869 Please answer this question by first reasoning and then providing
1870 your answer.
1871 Present your reasoning and solution in the following json format.
1872 Please show your final answer in the 'answer' field, e.g., "answer":"
1873     "42".
1874
1875     '''json
1876     {
1877         "reasoning": "____",
1878         "answer": "____"
1879     }
```

GT-Reward Answer:

```
1880 {  
1881     "reasoning": "The function 'f' is designed to remove all  
1882     occurrences of the character '+' from the input string 'text'.  
1883     The function first converts the string into a list of characters  
1884     , iterates through this list, and removes any '+' characters it  
1885     finds. Finally, it joins the remaining characters back into a  
1886     single string. Given the input string 'hbtofdeiequ', there are  
1887     no '+' characters in the string, so the function will simply  
1888     return the original string unchanged. Therefore, the output of  
1889     the function when given the input 'hbtofdeiequ' is the string '  
1900     hbtofdeiequ'.",  
1901 }
```

```

1890
1891
1892     "answer": "hbtofdeiequ"
1893 }
1894
1895

```

Co-rewarding-I Answer:

```

1896
1897
1898 {
1899     "reasoning": "The function 'f' is designed to remove all
1900     occurrences of the character '+' from the input string 'text'.
1901     The function first converts the string into a list of characters
1902     , then iterates over the character '+' and removes it from the
1903     list if it exists. Finally, it joins the list back into a string
1904     and returns it. Given the input 'hbtofdeiequ', the character
1905     '+' does not appear in the string, so the function will return
1906     the original string unchanged.",
1907     "answer": "hbtofdeiequ"
1908 }
1909
1910

```

Co-rewarding-II Answer:

```

1911
1912 {
1913     "reasoning": "The function 'f' takes a string 'text' and
1914     removes all occurrences of the character '+' from it. The
1915     function then joins the remaining characters back into a string
1916     and returns it. Given the input 'hbtofdeiequ', the function will
1917     remove all '+' characters (though there are none in the input),
1918     so the output will be the same as the input string. Therefore,
1919     the assertion should check if the output of 'f('hbtofdeiequ')'
1920     is 'hbtofdeiequ'.",
1921     "answer": "hbtofdeiequ"
1922 }
1923
1924

```

Majority-Voting Answer:

```

1925
1926 {
1927     "reasoning": "The function 'f' is designed to remove all
1928     occurrences of the character '+' from the input string 'text'.
1929     The input provided to the function is 'hbtofdeiequ'. Since the
1930     character '+' does not appear in this string, the function will
1931     return the original string unchanged. Therefore, the assertion
1932     should compare the output of the function to the original input
1933     string.",
1934     "answer": "'hbtofdeiequ'"
1935 }
1936
1937

```

Self-Certainty Answer:

```

1938     To solve this problem, we need to follow these steps:
1939
1940     1. Understand the given Python function.
1941     2. Determine what the function does with the given input.
1942     3. Complete the assertion with the correct output based on the
1943     function's behavior.

```

```

1944
1945
1946 Let's go through each step one by one:
1947
1948 1. Understand the given Python function.
1949 2. Determine what the function does with the given input.
1950 3. Complete the assertion with the correct output based on the
1951   function's behavior.
1952
1953 We will write the code to understand the function, determine its
1954   behavior with the given input, and then complete the assertion
1955   with the correct output.
1956
1957 Let's write the code step by step.
1958
1959 '''python
# Step 1: Understand the given Python function
# Step 2: Determine what the function does with the given input
# Step 3: Complete the assertion with the correct output based on
#         the function's behavior
1960
1961 # Let's write the code to follow these steps
'''
1962
1963 '''python
# Step 1: Understand the given Python function
# Step 2: Determine what the function does with the given input
# Step 3: Complete the assertion with the correct output based on
#         the function's behavior
1964
1965 # Let's write the code to follow these steps
1966
1967 '''python
# Step 1: Understand the given Python function
# We will define the given function and understand what it does
1968
1969 # Step 2: Determine what the function does with the given input
# We will determine the output of the function for the given input
1970
1971 # Step 3: Complete the assertion with the correct output based on
#         the function's behavior
# We will complete the assertion with the correct output
1972
1973 # Let's write the code
'''
1974
1975
1976
1977
1978
1979
1980
1981 Let's write the code step by step.
1982
1983 '''python
# Step 1: Understand the given Python function
# Step 2: Determine what the function does with the given input
# Step 3: Complete the assertion with the correct output based on
#         the function's behavior
1984
1985
1986
1987 # Let's write the code to follow these steps
'''
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997

```

```
1998
1999
2000      .....
2001      # Let's write the code
2002
2003      # Step 1: Understand the
```

Entropy Answer: