OpenPRM: Building Open-domain Process-based Reward Models with Preference Trees

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Abstract

Scaling inference-time computation is increasingly seen as the next frontier in scaling laws for large language models. Previous work in mathematics and coding has demonstrated the remarkable potential for inference-time scaling. During such scaling, fine-grained supervision through process-based reward models (PRMs) is essential for enhancement. However, exploration of inference-time scaling and PRMs in open-domain problems remains limited, where lacking exact answers and obtaining process supervision prove challenging. In this paper, we explore the construction of PRMs for open-domain tasks, specifically for instruction-following tasks. Utilizing existing outcome-based reward models (ORMs), we develop sentence-level preference trees based on the prefix similarity of parallel sampled candidates from datasets like UltraFeedback. This setup allows us to derive weak supervision for processes via back-propagation from outcome-level rewards. Subsequently, we integrate ORMs and PRMs under the same pairwise ranking objectives, resulting in our newly developed reward models, named OpenPRM. This approach significantly enhances the scalability of process-level supervision in open domains at minimal cost. We assess the performance of OpenPRM across various reward benchmarks, demonstrating its competitive edge over traditional ORMs in open domains and PRMs in specialized domains. Additionally, we investigate the scalability of inference-time computation for open-domain instructions. Our results highlight the limitations of ORMs' scalability, while OpenPRM shows superior performance in scaled settings. Despite these advances, achieving automatic fine-grained supervision for open-domain inference-time scaling remains a substantial challenge. We hope these findings will spur further development of process supervision reward models in open-domain scenarios.

1 Introduction

Large language models (LLMs) such as GPT-4 (Achiam et al., 2023), Llama (Touvron et al., 2023; Dubey et al., 2024), and Gemini (Team et al., 2023; Reid et al., 2024) have garnered interest across various fields due to their robust performance in numerous tasks and domains. The development of LLMs involves an official process that includes pre-training on a large-scale unlabeled corpus (Brown, 2020) followed by post-training using labeled instructions derived from real-world applications. The post-training phase is further categorized into supervised fine-tuning and reinforcement learning from human or model feedback (Ouyang et al., 2022), a process known as alignment (Ji et al., 2023; Shen et al., 2023). During this phase, reward models are crucial as they act as human proxies, providing feedback on model behavior and adjusting the models to better align with human values (Bai et al., 2022). Although current popular alignment algorithms like Direct Preference Optimization (DPO) (Rafailov et al., 2024) implicitly incorporate the rewarding process within the loss function, reward models still play a significant role in ensuring long-term alignment through methods such as online and iterative DPO (Xiong et al., 2023; Guo et al., 2024; Pang et al., 2024) and rejected sampling (Dong et al., 2023; Liu et al., 2023; Khaki et al., 2024).

In addition to the training process, reward models are crucial for enhancing the performance of LLMs during inference-time (Khanov et al., 2024; Deng & Raffel, 2023). Unlike the scaling laws applied to compute during pre-training (Kaplan et al., 2020; Hoffmann et al., 2022), there is a trend towards scaling inference-time compute through extensive searches in the decoding space (Snell et al., 2024; Brown et al., 2024). Reward models play a significant role in pruning the search space and ultimately selecting the most accurate answers (Welleck et al., 2024).

Recent studies indicate that outcome-level reward models fail to apply scaling laws during repeated sampling (Brown et al., 2024), particularly due to their coarse granularity on challenging tasks. Consequently, many researchers are exploring the use of more fine-grained reward models, such as process-level (Uesato et al., 2022; Lightman et al., 2023) or token-level (Deng & Raffel, 2023), to enhance search performance in specialized domains like mathematics, coding, and reasoning tasks (Wang et al., 2024b; Havrilla et al., 2024; Xin et al., 2024; Yuan et al., 2024; Chen et al., 2024a; Setlur et al., 2024; Xie et al., 2024). These efforts often follow the paradigm established by AlphaGo (Silver et al., 2016), integrating LLMs with Monte Carlo Tree Search (Coulom, 2006; Kocsis & Szepesvári, 2006) and a value function, analogous to a reward model. However, process-level reward models are typically tailored for specific tasks and exhibit limited generalizability in open-domain applications such as writing and chat. Moreover, there is scant research on developing process-level reward models for open-domain contexts, primarily due to the high cost of annotation.

Currently, outcome-level reward models (ORMs) are evolving rapidly (Wang et al., 2024a; Cai et al., 2024; Wang et al., 2024e;c; Vu et al., 2024), prompted by the emergence of datasets and benchmarks such as Ultrafeedback (Cui et al., 2023) and RewardBench (Lambert et al., 2024), where open-source reward models trend to outperform proprietary ones. ***This development raises the question of whether process-level reward models (PRMs) can be constructed from instance-level rewards using a weak-to-strong framework*** (Burns et al., 2023).

In this paper, we propose the development of PRMs in the open domain, leveraging existing ORMs to provide fine-grained supervision. Our contributions are summarized as follows:

- We analyze the potential for extending open-domain ORMs to PRMs, elucidating the characteristic and relationship between them. This analysis inspires our proposal to develop PRMs with outcome-level supervision through building preference trees with key process.
- We integrate the modeling of PRMs and ORMs under a unified objective and develop Open-PRM. By leveraging only existing ORMs and employing repeated sampling on prompts, we enhance the performance of ORMs, achieving a 3$\sim$5% improvement on RewardBench.
- We further evaluate OpenPRM across various downstream applications, including different inference-time scaling settings. Our findings show that ORMs struggle to provide effective supervision, while our proposed OpenPRM outperforms previous RMs under these scaling conditions. We also observe that there is a significant journey ahead to fully realize the potential of RMs in open-domain tasks under the inference-time scaling law.

2 Preliminary

2.1 Reward Modeling

Reward models play a crucial role in large language models by aligning model outputs with desired human preferences (Wang et al., 2023). There are primarily two types of reward models based on the granularity of the supervision signal: outcome-level and process-level reward models. We will introduce the development of these two methods as follows.

Outcome-level Reward Model (ORM) ORMs are commonly used for preference learning, particularly after supervised fine-tuning in InstructGPT, where they serve as a proxy for human feedback to model generations (Ouyang et al., 2022; Lee et al., 2023). Although many studies explore reward model-free preference learning, such as direct preference optimization (DPO) (Rafailov et al., 2024), which implicitly models the reward within the policy model training, ORMs continue to be instrumental in further model improvements. This includes applications in online or iterative DPO (Guo et al., 2024; Pang et al., 2024) and rejection sampling (Liu et al., 2023; Dong et al., 2024).

The primary methods for obtaining ORMs involve preparing pairwise responses with preferences (e.g., chosen and rejected) and fine-tuning instructed models using ranking loss (Ouyang et al., 2022; Dong et al., 2024). Some studies also consider DPO models as reward models (Lambert et al., 2024), though the effectiveness of these models in iterative optimization still requires further exploration. Additionally, self-play learning (Chen et al., 2024b; Tao et al., 2024) has recently been applied to

Table 1: A comparison with the most related works on process-level reward models.

Name	Data Acquisition					Training & Inference			Release	
	Domain	Task	Backbone	Annotation	Size	Labeling	Objective	Search	Data	Model
DeepMind PRM (Uesato et al., 2022)	Math	GSM8K	N/A	Human	10k	0/1	N/A	BoN	✗	✗
OpenAI PRM (Lightman et al., 2023)	Math	MATH	GPT-4	Human	800k	-1/0/1	CE Loss	BoN	✓	✗
TS-LLM (Feng et al., 2023)	Math Decision	GSM8K,GAME24, ProntoQA,RLHF, Chess Endgame	LLaMA2-7B	Golden Ans.	$\sim$150k	0$\sim$1	MSE Loss	MCTS-α +Rollout 100$\times$17 leaf	✗	✓
Math-Shepherd (Wang et al., 2024b)	Math	GSM8K	LLemma-7B	Golden Ans.	445k	0/1	CE Loss	MCTS 16$\times$64 leaf	✓	✓
GLoRe (Havrilla et al., 2024)	Math	GSM8K	Llama2-7B	Golden Ans.	N/A	0/1	CE Loss	BoN	✗	✗
MiPS (Wang et al., 2024f)	Math/Code	GSM8K, MATH MBPP	PaLM 2-S/L	Golden Ans.	$\sim$14k	0$\sim$1	MSE Loss	MCTS 32$\times$32 leaf	✗	✗
MCTS-DPO (Xie et al., 2024)	Math Common -sense	GSM8K,MATH ARC,AI2Science, OpenBookQA, CommonSenseQA	Mistral-7B	Golden Ans.	$\sim$24k	0$\sim$1	MSE Loss	MCTS 4/5 leaf	✗	✗
Super MARIO (Chen et al., 2024a)	Math	GSM8K,MATH	DeepSeek-MathBase-7B	Golden Ans.	15k	0$\sim$1	MSE Loss	MCTS 5 leaf	✓	✗
Step-DPO (Lai et al., 2024)	Math	MetaMath, MMIQC	Qwen2-7B/72B	LLM	10k	0/1	Ranking Loss	BoN	✓	✗
OmegaPRM (Luo et al., 2024)	Math	MATH	Gemini Pro	Golden Ans.	1.5m	0$\sim$1	MSE Loss	MCTS	✗	✗
ReST-MCTS* (Zhang et al., 2024)	Math	MATH	Mistral-7B	Golden Ans.	$\sim$700k	0$\sim$1	MSE Loss	MCTS 3 leaf	✓	✗

continually improve reward models, presenting a promising method to enhance the capabilities of ORMs autonomously (Wang et al., 2024c). Our approach on OpenPRM can also be considered a method for continuously improving ORMs through their own annotations.

Process-level Reward Model (PRM) The primary challenge with ORMs is their coarse-grained nature of rewards; even if the final answer is correct, errors may still exist within the solution steps. To address this, there is a growing trend to develop more fine-grained, process-level RMs. The main challenge in developing PRMs lies in obtaining accurate supervision signals for each process within a solution. There are three main approaches: 1) Human Annotation: This method requires experts to annotate each process step as neutral, bad, or good. While human annotation can provide precise process supervision, it is difficult to scale and very costly (Uesato et al., 2022; Lightman et al., 2023). 2) Golden Answer.: For mathematical or coding problems, accurate final answers or feedback from exact matching or interpreters are available. Common methods involve computing the probability of nodes along the path toward the final, accurate answers, integrating with Monte Carlo Search methods (Wang et al., 2024b; Havrilla et al., 2024; Luo et al., 2024). 3) Model-based Judgment or Reward Models: The final approach involves obtaining rewards from model-based judgments or reward models (Lai et al., 2024). Some research utilizes outcome-level rewards to estimate process rewards (Lu et al., 2024), reducing the high costs associated with extensive sampling.

For training PRMs, two main methods are used, depending on the data format required: 1) Single Sample: Each step in the solution process is labeled, and losses such as Cross-Entropy (CE) (Lightman et al., 2023; Wang et al., 2024b; Havrilla et al., 2024) and Mean Squared Error (MSE) (Feng et al., 2023; Wang et al., 2024f) loss are typically used. 2) Pair Sample: Each question is associated with chosen and rejected processes, and pairwise ranking loss is employed (Lai et al., 2024). This method is typically used in the training ORMs (Dong et al., 2024). We provide a detailed survey in Table 1. Currently, many ORMs in open domains benefit from the development of benchmarks (Lambert et al., 2024; Wang et al., 2024e). However, the true effectiveness of these ORMs and the feasibility of developing PRMs from ORMs are still subjects of ongoing exploration.

2.2 Derivation of PRM from ORM

ORM is typically trained to predict the quality of the final outcome, while PRM supervises the intermediate steps of the process. The modeling of ORM can be formalized with a pairwise ranking loss function based on the outcome-level feedback (Ouyang et al., 2022):

$$\mathcal{L}_{\text{ORM}}(\theta) = -\mathbb{E}_{(x,y_c,y_r)\sim D}\left[\log\left(\sigma\left(r_\theta(x,y_c) - r_\theta(x,y_r)\right)\right)\right] \tag{1}$$

Here, $r_\theta(x, y_c)$ and $r_\theta(x, y_r)$ are the model's scores for the chosen and rejected outcomes, respectively. While ORM performs well in outcome-based supervision, it has limitations when applied to process-level supervision, due to the cumulative error effect. This effect arises because ORM focuses on the final result, neglecting errors in intermediate steps that can propagate through the sequence and affect the final outcome (Lightman et al., 2023).

Previous works (Uesato et al., 2022; Havrilla et al., 2024) have shown that in cases where the base model for response generation is sufficiently strong and the task is relatively simple, the cumulative error may be negligible. In such scenarios, ORM can be effectively used as a substitute for PRM.

However, for more complex tasks, the cumulative error can be significant, necessitating additional process-level supervision. To address this issue, we propose a joint modeling approach that integrates both ORM and PRM by introducing supervision at key process of answer. Specifically, we identify the most divergent process between chosen and rejected outcomes, denoted as p_c and p_r, and introduce additional supervision at these critical points. The loss function can be defined as:

$$\mathcal{L}(\theta) = \mathcal{L}_{\text{ORM}}(\theta) + \lambda \mathcal{L}_{\text{PRM}}(\theta) \tag{2}$$

where λ is a hyper-parameter that balances the outcome-based and process-based losses. The process-based loss $\mathcal{L}_{\text{PRM}}$ supervises the divergent steps p_c and p_r, and can be expressed as:

$$\mathcal{L}_{\text{PRM}}(\theta) = -\log\left(\sigma\left(r_\theta(x, p_c) - r_\theta(x, p_r)\right)\right) \tag{3}$$

Here, $r_\theta(x, p_c)$ and $r_\theta(x, p_r)$ represent the scores for the critical steps in the chosen and rejected sequences, respectively. By focusing on these key steps, the cumulative error is mitigated as early errors are corrected at critical junctures. We provide more details in Appendix A. In conclusion, the above analysis leads to the following theorem:

Theorem 1 *Given a dataset D consisting of pairs of responses (y_c, y_r) with outcome-based preferences $y_c > y_r$, and a learned outcome-based reward model $r_\theta(x, y)$, the cumulative error of process supervision can be significantly reduced. This is achieved by identifying the key divergent steps (p_c, p_r), such that $\Delta(p_c, p_r)$ is maximized, and incorporating these steps into a joint modeling framework. Thus, under this framework, the cumulative error in process supervision decreases as the discrepancy $\Delta(p_c, p_r)$, supervision strength S, and model sensitivity γ increase.*

2.3 Empirical Evaluation of Open-Domain ORM in Process Assessment

In this section, we conduct an empirical evaluation of open-domain ORMs as process evaluators and examine some unique phenomena associated with their performance. Specifically, we assess popular reward models such as FsfirX (Wang et al., 2024a), InternRM (Cai et al., 2024), and UltraFeedback (Cui et al., 2023) (trained on the Llama-3 8B model (Dubey et al., 2024)) using RewardBench (named RB) (Lambert et al., 2024). We focus on the primary categories of RB, including Chat, Chat-Hard, Reasoning, and Safety tasks. We provide more details about experiments in § 4.1.

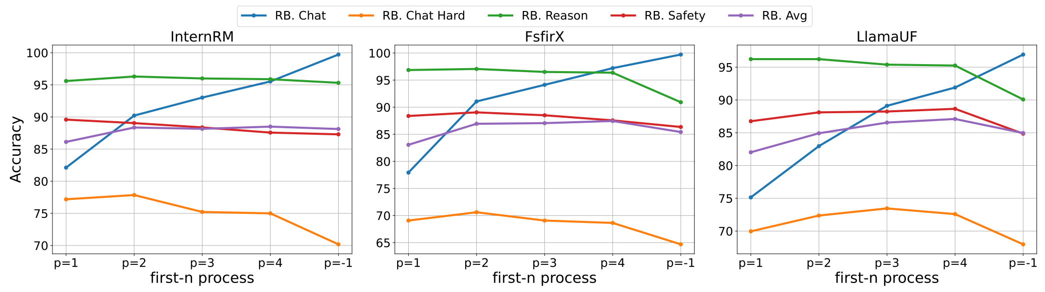


Figure 1: Results of reward models based on rewarding processes of varying lengths within the evaluated content. The results indicate that the initial segments of responses are particularly critical for challenging tasks such as Chat-Hard and Reasoning, where longer content holds length bias.

As illustrated in Figure 1, a consistent trend is observed across various reward models, indicating that RMs perform better on simpler Chat tasks as the evaluated process lengthens. However, this effect reverses in more challenging Chat and Reasoning tasks, where accuracy first increases and then decreases as the length of the evaluated process extends. In conclusion, these results support our Theorem 1, which posits that ORMs can function as PRMs, but their performance deteriorates due to cumulative errors, particularly in harder tasks.

3 METHODOLOGY

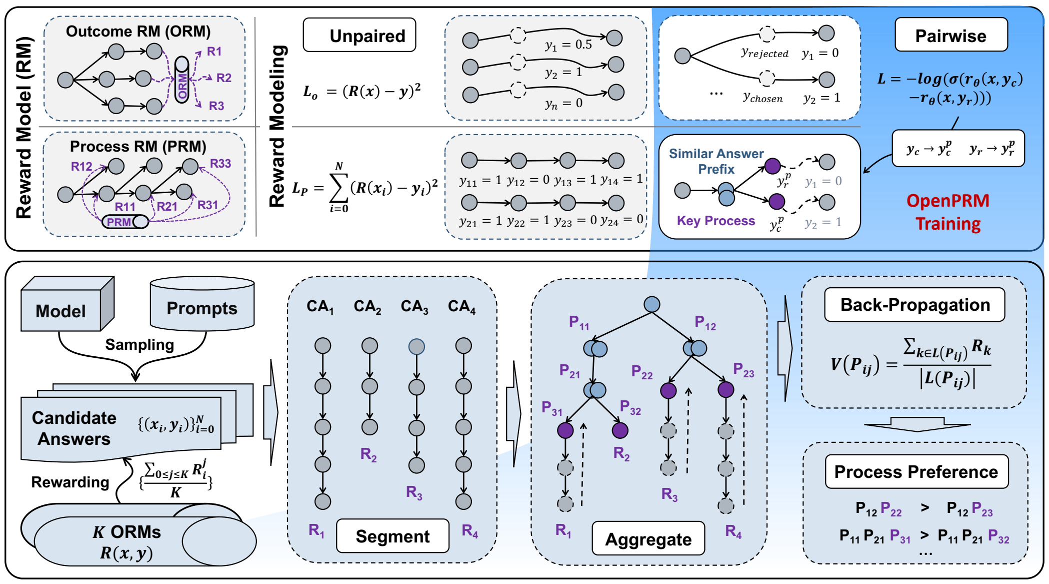


Figure 2: This figure illustrates the differences between outcome-level and process-level reward models in the top left, including their common training strategies with paired and unpaired data. The training of OpenPRM is depicted on the right and below (mainly blue area).

3.1 PROCESS-LEVEL PREFERENCE TREE

To obtain process-level rewards, we construct process-level preference trees using readily available outcome reward models. The pipeline for building preference trees consists of three steps:

Step 1. Repeated Sampling on Prompts We initially prompt open-source language models to generate a large number of parallel candidate responses through repeated sampling. To ensure broad representation, we primarily include models at the 7B and 70B parameter levels.

Step 2. Aggregation on Sentences For each output, we segment it into a collection of sentences and construct a tree using depth-first search algorithms. We calculate the edit distance (Ristad & Yianilos, 1998) between sentences from different outputs and merge sentences into a single node based on a predefined threshold. This helps us reduce the cost of building prefix trees.

Step 3. Backpropagation on Rewards Once outputs with their respective rewards are segmented into sentence collections, which serve as nodes in the preference tree, we designate the outcome rewards for the leaf nodes. For each process node, we compute the process-level rewards using backpropagation. Given the rewards of the leaf nodes, R_k (outcome-level rewards), we can compute the rewards of the inner nodes, P_{ij}, using backpropagation, as detailed in Monte Carlo Tree Search. Notably, the rewards of the inner nodes, denoted as $V(P_{ij})$, can be calculated using the formula

$$V(P_{ij}) = \frac{\sum_{k \in L(P_{ij})} R_k}{|L(P_{ij})|},$$

where $L(P_{ij})$ represents the set of all leaf nodes descending from P_{ij}.

About Rationality of Process Aggregation Unlike previous works in mathematics and coding that reuse partial answers for subsequent answer generations (Lightman et al., 2023; Luo et al., 2024), our method involves directly sampling a large number of candidates and merging identical sentences, akin to state aggregation in Monte-carlo algorithms (Hostetler et al., 2014; Jang et al., 2021). This approach enables the RMs to learn high-level actions and logic within the shared sentences. We provide a real example of question-answering for reference in the Appendix B.3.

3.2 Process-level Reward Modeling

During the development of OpenPRMs, we enhance the models by integrating rewards and domains, aiming to create more robust process-level reward models from the following two perspectives:

Mixture of Rewards Considering the completeness of outputs, output-level rewards serve as specific instances of process-level rewards, where the output encompasses the entire process. Therefore, we blend rewards from both the process and output levels to develop more robust reward models.

Mixture of Domains Existing process-level reward models predominantly focus on domains such as mathematics, and reasoning tasks, which provide certain answers for supervision. To leverage the strengths of these domains, we also integrate them with general domain preferences to enhance the versatility and applicability of OpenPRMs. We provide details about dataset in Appendix B.1.

At the training stage, we treat all preference data as a pairwise ranking task. This involves using the input prompt along with chosen and rejected completions (including both process and output). Using this unified format, we train the PRM with the Bradley-Terry objective, as defined in Equations 1 and 3. This formulation ensures consistent training across both process- and outcome-level datasets.

3.3 Application of Process-level Reward Models

Best-of-N Sampling At inference time, we can generate a large number of candidate answers for given questions. Subsequently, we can determine the final answer through a majority vote (James, 1998) (referred to as *Vote@N*); however, this method is primarily applicable to questions that require exact answers, such as those found in mathematics and reasoning tasks. For open-ended questions, it is more common to apply reward models to all answers and select the one with the highest rewards, a method known as best-of-N sampling (Stiennon et al., 2020) (*BoN@N*). When implementing process-level reward models in the BoN context, there are two approaches to computing rewards: one approach treats the outcome as a special process and computes rewards directly on the outcome, while the other calculates rewards for each process and selects the minimal one (Lightman et al., 2023; Wang et al., 2024b) to derive the outcome rewards.

Process-level Decoding Another significant application of PRMs is in the decoding phase. By evaluating the generated process, we can expand the beam search strategy (Sutskever et al., 2014) from token-level to process-level. As a result, we maintain N sentences at each step and reward each sentence during generation until the completion of the answer, a technique termed process-level beam search (*PBS@N*). Additionally, we can integrate advanced operations akin to those employed in Monte Carlo Tree Search (MCTS) (Browne et al., 2012), such as simulation, retrospection, and memory functions. However, these operations may extend the required processing time and lead to increased inference costs. Previous research (Chen et al., 2024a; Snell et al., 2024) has indicated that *PBS@N* can achieve performance comparable to MCTS but at a reduced cost.

4 Experimental Setup

4.1 Dataset

In developing OpenPRM, we first construct extensive preference trees based on open-domain instruction dataset, as described in § 3.1. This construction utilizes the UltraFeedback (Cui et al., 2023) and ScienceQA (Lu et al., 2022) datasets, which provide a highly diverse and high-quality range of instructions. Additionally, we incorporate the MATH (Hendrycks et al., 2021) dataset to further enhance the math reasoning capabilities of our reward system. For each prompt within this instruction pool, we sample 64 candidate responses from Llama-3 models (Dubey et al., 2024).

4.2 Models

Reward Models. We compare our reward models with state-of-the-art (SOTA) open-source reward models. Due to concerns with inference efficiency, we primarily evaluate classifier-based models, which perform comparably to generative models but are more scalable. We compare our models with ORMs, such as FsfairX (Dong et al., 2024), Eurus (Yuan et al., 2024), and UltraRM (Cui et al., 2023), and PRMs like TS-LLM (Chen et al., 2024a), MathShepherd (Wang et al., 2024b).

Chat Models. We assess the effectiveness of our reward models using state-of-the-art open-sourced chat models, including Llama-3.1-8B-Instruct and Llama-3.1-70B-Instruct (Dubey et al., 2024), and Mistral-Nemo-Instruct-2407 [1]. The latter can be regarded as an out-of-distribution evaluation.

4.3 Evaluations

Evaluation of Reward Models. Given the lack of established benchmarks for evaluating process-based reward models, we primarily compare our process-based models against established outcome-based reward benchmarks, such as UltraFeedback (Cui et al., 2023) and RewardBench (Lambert et al., 2024). RewardBench is designed to assess the capabilities and safety of reward models across four categories: Chat, Hard Chat, Reasoning, and Safety. We employ the primary dataset from RewardBench to evaluate the out-of-domain generalization capabilities of our reward models. We evaluate the effectiveness of process supervision of reward models solely on the test set of PRM800k (Lightman et al., 2023), which features high-quality human annotations. Especially, we evaluate PRMs using specific aggregation strategies, such as selecting the minimal reward across steps. Detailed descriptions of the different aggregation strategies are provided in the Appendix D.1.

Evaluation of Chat Models. To comprehensively evaluate the impact of reward models on chat models, we test the chat models across a variety of benchmarks, primarily referencing the Open LLM Leaderboard [2]. This evaluation includes benchmarks in: 1) Instruction following tasks such as Alpaca Eval 2 (Dubois et al., 2024) and IFEval (Zhou et al., 2023); 2) General domain tasks such as MixEval Hard (Ni et al., 2024), MMLU-Pro (Wang et al., 2024d), and GPQA (Rein et al., 2023); 3) Specific math domain tasks like MATH500 (Lightman et al., 2023). Additional details about these evaluation tasks and methodology are provided in Appendix C.

4.4 Settings for Inference

During inference, we primarily evaluate two methods: majority vote and best-of-N. For the best-of-N method, we adhere to the following protocol (Chen et al., 2021): initially, we sample N responses, where N is set to 128. We then sample K responses from these, repeating the process M times to average the results. The values for K range from 1, 2, 4, 8, 16, 32, 64, to 128, and M is set to 5. This approach allows us to reduce inference costs and achieve robust results through multiple averaging. For process-based beam search, we set the beam size to $\sqrt{N}$ to maintain an approximately equivalent decoding cost with best-of-N, as described in (Snell et al., 2024).

5 Experimental Results

5.1 Results of Reward Benchmarks

As shown in Table 2, we compare OpenPRMs with standard ORMs and specialized PRMs across both general outcome-based and specific process-reward benchmarks. Based on the results, we can draw the following conclusions:

OpenPRMs outperform ORMs Utilizing off-the-shelf ORMs and corresponding preference datasets, we have developed advanced reward models that demonstrate superior performance on RewardBench, particularly in the Chat Hard and Reasoning tasks. Additionally, the process-based preferences built upon our method consistently enhance the performance of the base reward models,

[1] https://mistral.ai/news/mistral-nemo/

[2] https://hf.co/spaces/open-llm-leaderboard/open_llm_leaderboard

Table 2: Results of outcome-level and process-level reward models on instance-level reward benchmarks. Models marked with an asterisk (*) were trained using data compiled by our team.

Model / Task	**Training Data**	**UltraFeedback Test**	**RewardBench**					**PRM800k Test**
			Overall	**Chat**	**Chat Hard**	**Safety**	**Reasoning**	
			Open-Source ORMs					
UltraRM-13B	UltraFeedback (UF)	74.8	68.5	96.4	55.5	59.9	62.4	50.8
Llama-3-8B*	UF Binarized	**77.8**	84.8	97.5	66.9	85.5	89.2	51.8
Eurus-RM-7B	UltraInteract	73.5	82.8	98.0	65.6	81.4	86.3	60.6
FsfairX-7B	Mixture Preference	74.5	84.4	**99.4**	65.1	86.8	86.4	53.3
Intern2-7B-RM	Unknown	77.4	**87.6**	99.2	69.5	**87.2**	**94.5**	**61.0**
Llama-3-8B*	HelpSteer (HS) 2	71.8	86.8	95.3	**76.8**	85.9	89.2	53.7
			Open-Source PRMs (Merely Math Domain)					
MS-7B-PRM	Math-Shepherd	53.5	56.6	62.3	51.3	39.6	73.2	56.9
TS-LLM	Unknown	52.7	57.6	66.8	50.0	55.3	58.4	57.7
Llama-3-8B*	Math-Step-DPO	**70.2**	**73.2**	**98.0**	**58.8**	**59.9**	**76.1**	**61.3**
OpenPRM (FsfairX)	UF Tree + HS 2	72.8	89.4	95.5	81.1	88.7	92.1	64.3
OpenPRM (InternRM)	UF Tree + HS 2	**78.5**	**91.1**	**98.0**	**81.6**	**89.5**	**95.1**	**68.1**

showcasing their generalization capabilities. These findings validate the effectiveness of our preference tree construction strategy discussed in § 3.1. Moreover, the results substantiate our ability to enhance weaker existing models, achieving weak-to-strong generalization (Burns et al., 2023).

OpenPRMs outperform specific PRMs Beyond outcome-level reward benchmarks, we also compare OpenPRMs with publicly available PRMs, which predominantly originate from the math domain, as many previous PRMs are not available. We present some results evaluated using Math-Shepherd (Wang et al., 2024b), TS-LLM (Feng et al., 2023) and Llama-3 trained on (Lai et al., 2024). Due to the domain gap, these math-specific PRMs underperform in open-domain benchmarks, whereas OpenPRMs demonstrate superior performance even on tasks like PRM800k.

5.2 Results of Applications in Decoding

To validate the effectiveness of PRMs, we evaluated OpenPRM under various decoding settings across multiple popular open-domain tasks, comparing strategies such as majority vote, best-of-N, and process-level beam search. We summarize the experimental results of OpenPRM as follows:

OpenPRM Performs Effectively with BoN and PBS As illustrated in Table 3, OpenPRM achieves superior performance in both BoN@16 and PBS@4 compared to Vote@16 with the Llama-3.1-8B and 70B models across nearly all tasks. These results confirm the effectiveness of OpenPRM. Additionally, even the out-of-distribution models, such as Mistral-Nemo (compared to the Llama series), validate the advantages of OpenPRM. We also observed that beam search algorithms outperform BoN, benefiting from the fine-grained evaluation of processes. However, further exploration of decoding strategies (like MCTS) in open-domain settings will be necessary in the future.

Scaling Inference-Time Achieves a High Upper Boundary We further analyze the results of scaling inference-time by progressively increasing sampling times from 1 (2^0) to 128 (2^7). The results depicted in Figures 3 demonstrate that the models can achieve exceptional performance with optimal reward models (refer to coverage@N and pass@N settings (Chen et al., 2021)). The coverage accuracies nearly reach 100% on most open-domain tasks. These findings in open-domain tasks are consistent with prior studies in mathematical and coding domains (Brown et al., 2024; Bansal et al., 2024), suggesting the emergence of a new scaling law at inference time in open-domain as well. We also include the sampling curves for the Llama 70B and Mistral model in Appendix D.4.

OpenPRMs Optimize Inference-Time Utilization Compared to the coverage accuracy depicted by the red curve, nearly all the reward models struggle to scale as inference-time increases. This indicates that significant advancements are still required to develop more effective reward models

Table 3: Results of majority vote, best-of-N sampling, and process-level search on open-domain tasks. The findings from BoN@16 and PBS@16 demonstrate the effectiveness of OpenPRMs. Notably, we have reproduced part of the results, taking into account differences in dataset usage.

Model / Task		Alpaca Eval 2		MixEval	IFEval	GPQA	MMLU-P*	MATH*
		LC%	WC%	Acc	Avg.	Acc	Acc	Acc
GPT-4o (0806)		**57.5**	**51.3**	64.7	85.6	**75.9**	74.68	53.1
GPT-4-Turbo		55.0	46.1	62.6	84.4	73.4	63.71	49.3
Claude-3.5-Sonnet		52.4	40.6	**68.1**	**88.0**	71.1	**76.12**	**59.4**
Llama-3.1-8B Instruct	Reported	20.9	21.8	45.6	79.5	**45.0**	40.8	23.7
	Reproduced	34.6	33.3	39.7	76.5	24.3	37.2	45.6
	Vote@16	—	—	—	—	26.8	43.4	56.4
	BoN@16	35.8	35.1	46.5	**80.7**	30.8	**50.5**	**58.8**
	PBS@4	**39.9**	**42.2**	**47.2**	75.59	31.3	47.8	52.8
Mistral-Nemo Instruct-202407	Reproduced	45.0	37.5	35.7	64.2	31.9	36.6	31.4
	Vote@16	—	—	—	—	35.2	44.4	40.7
	BoN@16	48.4	39.0	36.6	**69.5**	35.5	**48.2**	**52.3**
	PBS@4	**53.2**	**49.8**	**42.0**	63.4	**37.9**	44.0	47.8
Llama-3.1-70B Instruct	Reported	38.1	29.9	55.9	85.8	**65.8**	55.0	41.9
	Reproduced	42.4	41.6	59.1	85.4	45.7	55.4	63.6
	Vote@16	—	—	—	—	51.4	60.3	70.7
	BoN@16	**44.4**	**42.9**	**61.9**	**87.7**	49.8	**67.1**	**72.0**

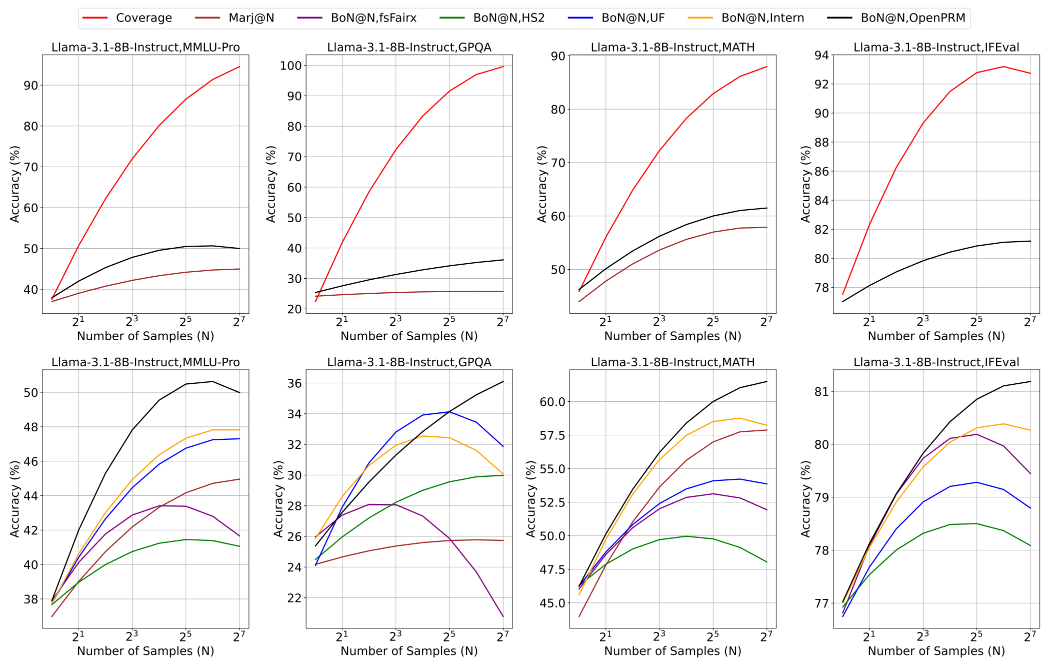


Figure 3: Results of scaling inference-time for Llama-3.1-8B-Instruct on open-domain tasks. These results illustrate the effectiveness of OpenPRMs relative to existing reward models, yet they also highlight the distance to the upper boundary of coverage accuracy (red curve).

for inference-time scaling. Among these previous models, InternRM performs best on most tasks on average, notably on MMLU-Pro and IFEval, though it still lags significantly behind the coverage curve. In contrast, our proposed OpenPRM outperforms InternRM, showing promising results in scaling up the best-of-N sampling. However, achieving scaling comparable to the coverage curve remains a substantial challenge. We will release all of these sampling data to the public to encourage further study on process and outcome reward models for inference-time scaling.

6 Discussion

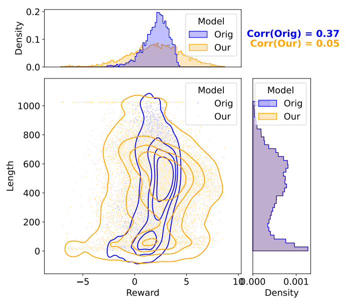
6.1 Ablation Study of PRM Training

We conducted an ablation study on OpenPRM training, focusing on data sources and model configurations. As illustrated in Table 4a, we compared the effects of continuously fine-tuning InternRM and FsfairX using process pairs built upon preference trees and outcome pairs. The results indicate that process pairs yield superior outcomes, thus validating the effectiveness of our method described in Section 3.1. Additionally, the performance of InternRM, when using a shared prefix, was inferior to configurations using distinct prefixes for chosen and rejected pairs, emphasizing the importance of semantic consistency. Furthermore, while using only UltraFeedback data showed promising results in Chat tasks, maintaining diversity and generalization for open-domain applications is crucial. Therefore, we opted to integrate additional reasoning and STEM questions.

6.2 Rewards Shape and Length Bias

Model	Data	RB Avg.
InternRM	PrefTree Pairs	91.1
	w/ Shared Prefix	90.8
FsfairX	PrefTree Pairs	89.4
Llama-3-8B-It	PrefTree Pairs	87.2
InternRM	Outcome Pairs	88.4
FsfairX	Outcome Pairs	87.7
Llama-3-8B-It	Outcome Pairs	86.8

(a) Ablation Study of OpenPRM.



(b) Rewards VS. Length

Figure 4: Ablation Study of OpenPRM

We compare the reward shapes of OpenPRM with other reward models in Figure 6. We analyze the rewards of chosen and rejected candidates for each instruction in RewardBench and observe that while all reward models can generally distinguish between chosen and rejected candidates, indicated by a shift in distributions, there remains some overlap. However, OpenPRM exhibits the minimal overlap among them, similar to the parent reward model (i.e., InternRM).

Language model-based judgers often suffer from length bias, typically awarding higher rewards to longer responses. We address this issue in our analysis of OpenPRM, visualizing the correlation between OpenPRM and InterRM in Figure 4b. The results indicate that OpenPRM maintains a correlation of 0.05, compared to 0.37 for InterRM, suggesting that process-level modeling effectively reduces length bias. This also demonstrates the effectiveness and necessity of developing PRMs.

7 Conclusion

In this paper, we explore the development of process-based reward models (PRMs) in the open domain. We begin by generalizing rewards from outcome-level to process-level, significantly reducing data annotation costs. We then propose the construction of preference trees with parallel candidates for open-domain instructions, from which we derive process pairs using back-propagation. Leveraging this data, our trained OpenPRM achieves excellent results on reward benchmarks and performs well under scaling inference-time search conditions. However, our findings also highlight that there is still considerable progress to be made in building open-domain PRMs to achieve high coverage accuracy. In conclusion, we try to unify ORMs and PRMs in the open domain, paving a new path for PRM development that diverges from domains such as mathematics and coding. We hope that OpenPRM will spark new insights into this topic and stimulate further research.

REFERENCES

Josh Achiam, Steven Adler, Sandhini Agarwal, Lama Ahmad, Ilge Akkaya, Florencia Leoni Aleman, Diogo Almeida, Janko Altenschmidt, Sam Altman, Shyamal Anadkat, et al. Gpt-4 technical report. *arXiv preprint arXiv:2303.08774*, 2023.

Yuntao Bai, Andy Jones, Kamal Ndousse, Amanda Askell, Anna Chen, Nova DasSarma, Dawn Drain, Stanislav Fort, Deep Ganguli, Tom Henighan, et al. Training a helpful and harmless assistant with reinforcement learning from human feedback. *arXiv preprint arXiv:2204.05862*, 2022.

Hritik Bansal, Arian Hosseini, Rishabh Agarwal, Vinh Q Tran, and Mehran Kazemi. Smaller, weaker, yet better: Training llm reasoners via compute-optimal sampling. *arXiv preprint arXiv:2408.16737*, 2024.

Bradley Brown, Jordan Juravsky, Ryan Ehrlich, Ronald Clark, Quoc V Le, Christopher Ré, and Azalia Mirhoseini. Large language monkeys: Scaling inference compute with repeated sampling. *arXiv preprint arXiv:2407.21787*, 2024.

Tom B Brown. Language models are few-shot learners. *arXiv preprint arXiv:2005.14165*, 2020.

Cameron B Browne, Edward Powley, Daniel Whitehouse, Simon M Lucas, Peter I Cowling, Philipp Rohlfshagen, Stephen Tavener, Diego Perez, Spyridon Samothrakis, and Simon Colton. A survey of monte carlo tree search methods. *IEEE Transactions on Computational Intelligence and AI in games*, 4(1):1–43, 2012.

Collin Burns, Pavel Izmailov, Jan Hendrik Kirchner, Bowen Baker, Leo Gao, Leopold Aschenbrenner, Yining Chen, Adrien Ecoffet, Manas Joglekar, Jan Leike, et al. Weak-to-strong generalization: Eliciting strong capabilities with weak supervision. *arXiv preprint arXiv:2312.09390*, 2023.

Zheng Cai, Maosong Cao, Haojiong Chen, Kai Chen, Keyu Chen, Xin Chen, Xun Chen, Zehui Chen, Zhi Chen, Pei Chu, Xiaoyi Dong, Haodong Duan, Qi Fan, Zhaoye Fei, Yang Gao, Jiaye Ge, Chenya Gu, Yuzhe Gu, Tao Gui, Aijia Guo, Qipeng Guo, Conghui He, Yingfan Hu, Ting Huang, Tao Jiang, Penglong Jiao, Zhenjiang Jin, Zhikai Lei, Jiaxing Li, Jingwen Li, Linyang Li, Shuaibin Li, Wei Li, Yining Li, Hongwei Liu, Jiangning Liu, Jiawei Hong, Kaiwen Liu, Kuikun Liu, Xiaoran Liu, Chengqi Lv, Haijun Lv, Kai Lv, Li Ma, Runyuan Ma, Zerun Ma, Wenchang Ning, Linke Ouyang, Jiantao Qiu, Yuan Qu, Fukai Shang, Yunfan Shao, Demin Song, Zifan Song, Zhihao Sui, Peng Sun, Yu Sun, Huanze Tang, Bin Wang, Guoteng Wang, Jiaqi Wang, Jiayu Wang, Rui Wang, Yudong Wang, Ziyi Wang, Xingjian Wei, Qizhen Weng, Fan Wu, Yingtong Xiong, Chao Xu, Ruiliang Xu, Hang Yan, Yirong Yan, Xiaogui Yang, Haochen Ye, Huaiyuan Ying, Jia Yu, Jing Yu, Yuhang Zang, Chuyu Zhang, Li Zhang, Pan Zhang, Peng Zhang, Ruijie Zhang, Shuo Zhang, Songyang Zhang, Wenjian Zhang, Wenwei Zhang, Xingcheng Zhang, Xinyue Zhang, Hui Zhao, Qian Zhao, Xiaomeng Zhao, Fengzhe Zhou, Zaida Zhou, Jingming Zhuo, Yicheng Zou, Xipeng Qiu, Yu Qiao, and Dahua Lin. Internlm2 technical report, 2024.

Guoxin Chen, Minpeng Liao, Chengxi Li, and Kai Fan. Alphamath almost zero: process supervision without process. *arXiv preprint arXiv:2405.03553*, 2024a.

Mark Chen, Jerry Tworek, Heewoo Jun, Qiming Yuan, Henrique Ponde De Oliveira Pinto, Jared Kaplan, Harri Edwards, Yuri Burda, Nicholas Joseph, Greg Brockman, et al. Evaluating large language models trained on code. *arXiv preprint arXiv:2107.03374*, 2021.

Zixiang Chen, Yihe Deng, Huizhuo Yuan, Kaixuan Ji, and Quanquan Gu. Self-play fine-tuning converts weak language models to strong language models. *arXiv preprint arXiv:2401.01335*, 2024b.

Thomas Coste, Usman Anwar, Robert Kirk, and David Krueger. Reward model ensembles help mitigate overoptimization. *arXiv preprint arXiv:2310.02743*, 2023.

Rémi Coulom. Efficient selectivity and backup operators in monte-carlo tree search. In *International conference on computers and games*, pp. 72–83. Springer, 2006.

Ganqu Cui, Lifan Yuan, Ning Ding, Guanming Yao, Wei Zhu, Yuan Ni, Guotong Xie, Zhiyuan Liu, and Maosong Sun. Ultrafeedback: Boosting language models with high-quality feedback. *arXiv preprint arXiv:2310.01377*, 2023.

Haikang Deng and Colin Raffel. Reward-augmented decoding: Efficient controlled text generation with a unidirectional reward model. *arXiv preprint arXiv:2310.09520*, 2023.

Hanze Dong, Wei Xiong, Deepanshu Goyal, Yihan Zhang, Winnie Chow, Rui Pan, Shizhe Diao, Jipeng Zhang, Kashun Shum, and Tong Zhang. Raft: Reward ranked finetuning for generative foundation model alignment. *arXiv preprint arXiv:2304.06767*, 2023.

Hanze Dong, Wei Xiong, Bo Pang, Haoxiang Wang, Han Zhao, Yingbo Zhou, Nan Jiang, Doyen Sahoo, Caiming Xiong, and Tong Zhang. Rlhf workflow: From reward modeling to online rlhf. *arXiv preprint arXiv:2405.07863*, 2024.

Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Amy Yang, Angela Fan, et al. The llama 3 herd of models. *arXiv preprint arXiv:2407.21783*, 2024.

Yann Dubois, Balázs Galambosi, Percy Liang, and Tatsunori B Hashimoto. Length-controlled alpacaeval: A simple way to debias automatic evaluators. *arXiv preprint arXiv:2404.04475*, 2024.

Xidong Feng, Ziyu Wan, Muning Wen, Ying Wen, Weinan Zhang, and Jun Wang. Alphazero-like tree-search can guide large language model decoding and training. *arXiv preprint arXiv:2309.17179*, 2023.

Shangmin Guo, Biao Zhang, Tianlin Liu, Tianqi Liu, Misha Khalman, Felipe Llinares, Alexandre Rame, Thomas Mesnard, Yao Zhao, Bilal Piot, et al. Direct language model alignment from online ai feedback. *arXiv preprint arXiv:2402.04792*, 2024.

Alex Havrilla, Sharath Raparthy, Christoforus Nalmpantis, Jane Dwivedi-Yu, Maksym Zhuravinskyi, Eric Hambro, and Roberta Railneau. Glore: When, where, and how to improve llm reasoning via global and local refinements. *arXiv preprint arXiv:2402.10963*, 2024.

Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. Measuring mathematical problem solving with the math dataset. *NeurIPS*, 2021.

Jordan Hoffmann, Sebastian Borgeaud, Arthur Mensch, Elena Buchatskaya, Trevor Cai, Eliza Rutherford, Diego de Las Casas, Lisa Anne Hendricks, Johannes Welbl, Aidan Clark, et al. Training compute-optimal large language models. *arXiv preprint arXiv:2203.15556*, 2022.

Jesse Hostetler, Alan Fern, and Tom Dietterich. State aggregation in monte carlo tree search. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 28, 2014.

Gareth Michael James. *Majority vote classifiers: theory and applications*. Stanford University, 1998.

Youngsoo Jang, Seokin Seo, Jongmin Lee, and Kee-Eung Kim. Monte-carlo planning and learning with language action value estimates. In *International Conference on Learning Representations*, 2021.

Jiaming Ji, Tianyi Qiu, Boyuan Chen, Borong Zhang, Hantao Lou, Kaile Wang, Yawen Duan, Zhonghao He, Jiayi Zhou, Zhaowei Zhang, et al. Ai alignment: A comprehensive survey. *arXiv preprint arXiv:2310.19852*, 2023.

Jared Kaplan, Sam McCandlish, Tom Henighan, Tom B Brown, Benjamin Chess, Rewon Child, Scott Gray, Alec Radford, Jeffrey Wu, and Dario Amodei. Scaling laws for neural language models. *arXiv preprint arXiv:2001.08361*, 2020.

Saeed Khaki, JinJin Li, Lan Ma, Liu Yang, and Prathap Ramachandra. Rs-dpo: A hybrid rejection sampling and direct preference optimization method for alignment of large language models. *arXiv preprint arXiv:2402.10038*, 2024.

Maxim Khanov, Jirayu Burapacheep, and Yixuan Li. Args: Alignment as reward-guided search. *arXiv preprint arXiv:2402.01694*, 2024.

Levente Kocsis and Csaba Szepesvári. Bandit based monte-carlo planning. In *European conference on machine learning*, pp. 282–293. Springer, 2006.

Xin Lai, Zhuotao Tian, Yukang Chen, Senqiao Yang, Xiangru Peng, and Jiaya Jia. Step-dpo: Step-wise preference optimization for long-chain reasoning of llms. *arXiv preprint arXiv:2406.18629*, 2024.

Nathan Lambert, Valentina Pyatkin, Jacob Morrison, LJ Miranda, Bill Yuchen Lin, Khyathi Chandu, Nouha Dziri, Sachin Kumar, Tom Zick, Yejin Choi, Noah A. Smith, and Hannaneh Hajishirzi. Rewardbench: Evaluating reward models for language modeling, 2024.

Harrison Lee, Samrat Phatale, Hassan Mansoor, Thomas Mesnard, Johan Ferret, Kellie Lu, Colton Bishop, Ethan Hall, Victor Carbune, Abhinav Rastogi, et al. Rlaif: Scaling reinforcement learning from human feedback with ai feedback. *arXiv preprint arXiv:2309.00267*, 2023.

Hunter Lightman, Vineet Kosaraju, Yura Burda, Harri Edwards, Bowen Baker, Teddy Lee, Jan Leike, John Schulman, Ilya Sutskever, and Karl Cobbe. Let's verify step by step. *arXiv preprint arXiv:2305.20050*, 2023.

Tianqi Liu, Yao Zhao, Rishabh Joshi, Misha Khalman, Mohammad Saleh, Peter J Liu, and Jialu Liu. Statistical rejection sampling improves preference optimization. *arXiv preprint arXiv:2309.06657*, 2023.

Jianqiao Lu, Zhiyang Dou, Hongru Wang, Zeyu Cao, Jianbo Dai, Yingjia Wan, Yinya Huang, and Zhijiang Guo. Autocv: Empowering reasoning with automated process labeling via confidence variation. *arXiv preprint arXiv:2405.16802*, 2024.

Pan Lu, Swaroop Mishra, Tony Xia, Liang Qiu, Kai-Wei Chang, Song-Chun Zhu, Oyvind Tafjord, Peter Clark, and Ashwin Kalyan. Learn to explain: Multimodal reasoning via thought chains for science question answering. In *The 36th Conference on Neural Information Processing Systems (NeurIPS)*, 2022.

Liangchen Luo, Yinxiao Liu, Rosanne Liu, Samrat Phatale, Harsh Lara, Yunxuan Li, Lei Shu, Yun Zhu, Lei Meng, Jiao Sun, et al. Improve mathematical reasoning in language models by automated process supervision. *arXiv preprint arXiv:2406.06592*, 2024.

Jinjie Ni, Fuzhao Xue, Xiang Yue, Yuntian Deng, Mahir Shah, Kabir Jain, Graham Neubig, and Yang You. Mixeval: Deriving wisdom of the crowd from llm benchmark mixtures. *arXiv preprint arXiv:2406.06565*, 2024.

Long Ouyang, Jeffrey Wu, Xu Jiang, Diogo Almeida, Carroll Wainwright, Pamela Mishkin, Chong Zhang, Sandhini Agarwal, Katarina Slama, Alex Ray, et al. Training language models to follow instructions with human feedback. *Advances in neural information processing systems*, 35: 27730–27744, 2022.

Richard Yuanzhe Pang, Weizhe Yuan, Kyunghyun Cho, He He, Sainbayar Sukhbaatar, and Jason Weston. Iterative reasoning preference optimization. *arXiv preprint arXiv:2404.19733*, 2024.

Junsoo Park, Seungyeon Jwa, Meiying Ren, Daeyoung Kim, and Sanghyuk Choi. Offsetbias: Leveraging debiased data for tuning evaluators. *arXiv preprint arXiv:2407.06551*, 2024.

Rafael Rafailov, Archit Sharma, Eric Mitchell, Christopher D Manning, Stefano Ermon, and Chelsea Finn. Direct preference optimization: Your language model is secretly a reward model. *Advances in Neural Information Processing Systems*, 36, 2024.

Machel Reid, Nikolay Savinov, Denis Teplyashin, Dmitry Lepikhin, Timothy Lillicrap, Jean-baptiste Alayrac, Radu Soricut, Angeliki Lazaridou, Orhan Firat, Julian Schrittwieser, et al. Gemini 1.5: Unlocking multimodal understanding across millions of tokens of context. *arXiv preprint arXiv:2403.05530*, 2024.

David Rein, Betty Li Hou, Asa Cooper Stickland, Jackson Petty, Richard Yuanzhe Pang, Julien Dirani, Julian Michael, and Samuel R Bowman. Gpqa: A graduate-level google-proof q&a benchmark. *arXiv preprint arXiv:2311.12022*, 2023.

Eric Sven Ristad and Peter N Yianilos. Learning string-edit distance. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 20(5):522–532, 1998.

Amrith Setlur, Saurabh Garg, Xinyang Geng, Naman Garg, Virginia Smith, and Aviral Kumar. Rl on incorrect synthetic data scales the efficiency of llm math reasoning by eight-fold. *arXiv preprint arXiv:2406.14532*, 2024.

Tianhao Shen, Renren Jin, Yufei Huang, Chuang Liu, Weilong Dong, Zishan Guo, Xinwei Wu, Yan Liu, and Deyi Xiong. Large language model alignment: A survey. *arXiv preprint arXiv:2309.15025*, 2023.

David Silver, Aja Huang, Chris J Maddison, Arthur Guez, Laurent Sifre, George Van Den Driessche, Julian Schrittwieser, Ioannis Antonoglou, Veda Panneershelvam, Marc Lanctot, et al. Mastering the game of go with deep neural networks and tree search. *nature*, 529(7587):484–489, 2016.

Charlie Snell, Jaehoon Lee, Kelvin Xu, and Aviral Kumar. Scaling llm test-time compute optimally can be more effective than scaling model parameters. *arXiv preprint arXiv:2408.03314*, 2024.

Nisan Stiennon, Long Ouyang, Jeffrey Wu, Daniel Ziegler, Ryan Lowe, Chelsea Voss, Alec Radford, Dario Amodei, and Paul F Christiano. Learning to summarize with human feedback. *Advances in Neural Information Processing Systems*, 33:3008–3021, 2020.

Ilya Sutskever, Oriol Vinyals, and Quoc V Le. Sequence to sequence learning with neural networks. In Z. Ghahramani, M. Welling, C. Cortes, N. Lawrence, and K.Q. Weinberger (eds.), *Advances in Neural Information Processing Systems*, volume 27. Curran Associates, Inc., 2014. URL https://proceedings.neurips.cc/paper_files/paper/2014/file/a14ac55a4f27472c5d894ec1c3c743d2-Paper.pdf.

Zhengwei Tao, Ting-En Lin, Xiancai Chen, Hangyu Li, Yuchuan Wu, Yongbin Li, Zhi Jin, Fei Huang, Dacheng Tao, and Jingren Zhou. A survey on self-evolution of large language models. *arXiv preprint arXiv:2404.14387*, 2024.

Gemini Team, Rohan Anil, Sebastian Borgeaud, Yonghui Wu, Jean-Baptiste Alayrac, Jiahui Yu, Radu Soricut, Johan Schalkwyk, Andrew M Dai, Anja Hauth, et al. Gemini: a family of highly capable multimodal models. *arXiv preprint arXiv:2312.11805*, 2023.

Hugo Touvron, Thibaut Lavril, Gautier Izacard, Xavier Martinet, Marie-Anne Lachaux, Timothée Lacroix, Baptiste Rozière, Naman Goyal, Eric Hambro, Faisal Azhar, et al. Llama: Open and efficient foundation language models. *arXiv preprint arXiv:2302.13971*, 2023.

Jonathan Uesato, Nate Kushman, Ramana Kumar, Francis Song, Noah Siegel, Lisa Wang, Antonia Creswell, Geoffrey Irving, and Irina Higgins. Solving math word problems with process-and outcome-based feedback. *arXiv preprint arXiv:2211.14275*, 2022.

Tu Vu, Kalpesh Krishna, Salaheddin Alzubi, Chris Tar, Manaal Faruqui, and Yun-Hsuan Sung. Foundational autoraters: Taming large language models for better automatic evaluation. *arXiv preprint arXiv:2407.10817*, 2024.

Haoxiang Wang, Yong Lin, Wei Xiong, Rui Yang, Shizhe Diao, Shuang Qiu, Han Zhao, and Tong Zhang. Arithmetic control of llms for diverse user preferences: Directional preference alignment with multi-objective rewards. *arXiv preprint arXiv:2402.18571*, 2024a.

Peiyi Wang, Lei Li, Zhihong Shao, Runxin Xu, Damai Dai, Yifei Li, Deli Chen, Yu Wu, and Zhifang Sui. Math-shepherd: Verify and reinforce llms step-by-step without human annotations. In *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 9426–9439, 2024b.

Tianlu Wang, Ilia Kulikov, Olga Golovneva, Ping Yu, Weizhe Yuan, Jane Dwivedi-Yu, Richard Yuanzhe Pang, Maryam Fazel-Zarandi, Jason Weston, and Xian Li. Self-taught evaluators. *arXiv preprint arXiv:2408.02666*, 2024c.

Yubo Wang, Xueguang Ma, Ge Zhang, Yuansheng Ni, Abhranil Chandra, Shiguang Guo, Weiming Ren, Aaran Arulraj, Xuan He, Ziyan Jiang, Tianle Li, Max Ku, Kai Wang, Alex Zhuang, Rongqi Fan, Xiang Yue, and Wenhu Chen. Mmlu-pro: A more robust and challenging multi-task language understanding benchmark, 2024d.

Yufei Wang, Wanjun Zhong, Liangyou Li, Fei Mi, Xingshan Zeng, Wenyong Huang, Lifeng Shang, Xin Jiang, and Qun Liu. Aligning large language models with human: A survey. *arXiv preprint arXiv:2307.12966*, 2023.

Zhilin Wang, Yi Dong, Olivier Delalleau, Jiaqi Zeng, Gerald Shen, Daniel Egert, Jimmy J Zhang, Makesh Narsimhan Sreedhar, and Oleksii Kuchaiev. Helpsteer2: Open-source dataset for training top-performing reward models. *arXiv preprint arXiv:2406.08673*, 2024e.

Zihan Wang, Yunxuan Li, Yuexin Wu, Liangchen Luo, Le Hou, Hongkun Yu, and Jingbo Shang. Multi-step problem solving through a verifier: An empirical analysis on model-induced process supervision. *arXiv preprint arXiv:2402.02658*, 2024f.

Sean Welleck, Amanda Bertsch, Matthew Finlayson, Hailey Schoelkopf, Alex Xie, Graham Neubig, Ilia Kulikov, and Zaid Harchaoui. From decoding to meta-generation: Inference-time algorithms for large language models. *arXiv preprint arXiv:2406.16838*, 2024.

Yuxi Xie, Anirudh Goyal, Wenyue Zheng, Min-Yen Kan, Timothy P Lillicrap, Kenji Kawaguchi, and Michael Shieh. Monte carlo tree search boosts reasoning via iterative preference learning. *arXiv preprint arXiv:2405.00451*, 2024.

Huajian Xin, ZZ Ren, Junxiao Song, Zhihong Shao, Wanjia Zhao, Haocheng Wang, Bo Liu, Liyue Zhang, Xuan Lu, Qiushi Du, et al. Deepseek-prover-v1. 5: Harnessing proof assistant feedback for reinforcement learning and monte-carlo tree search. *arXiv preprint arXiv:2408.08152*, 2024.

Wei Xiong, Hanze Dong, Chenlu Ye, Han Zhong, Nan Jiang, and Tong Zhang. Gibbs sampling from human feedback: A provable kl-constrained framework for rlhf. *arXiv preprint arXiv:2312.11456*, 2023.

Lifan Yuan, Ganqu Cui, Hanbin Wang, Ning Ding, Xingyao Wang, Jia Deng, Boji Shan, Huimin Chen, Ruobing Xie, Yankai Lin, et al. Advancing llm reasoning generalists with preference trees. *arXiv preprint arXiv:2404.02078*, 2024.

Dan Zhang, Sining Zhoubian, Yisong Yue, Yuxiao Dong, and Jie Tang. Rest-mcts*: Llm self-training via process reward guided tree search. *arXiv preprint arXiv:2406.03816*, 2024.

Jeffrey Zhou, Tianjian Lu, Swaroop Mishra, Siddhartha Brahma, Sujoy Basu, Yi Luan, Denny Zhou, and Le Hou. Instruction-following evaluation for large language models. *arXiv preprint arXiv:2311.07911*, 2023.

A Details about Derivation of PRM from ORM

The cumulative error from supervising only the outcome can be expressed as follows:

$$\text{Cumulative Error} = \sum_{i=1}^{T} \epsilon_i \cdot \frac{1-\alpha^{T-i+1}}{1-\alpha} \tag{4}$$

where ϵ_i is the error at step i, and α represents the degree to which errors propagate to subsequent steps.

The reduction in cumulative error can be expressed as:

$$\text{Cumulative Error}^{\text{new}} = \sum_{i=i^*}^{T} \beta \cdot \epsilon_i \cdot \frac{1-\alpha^{T-i+1}}{1-\alpha} \tag{5}$$

where β is a reduction factor that depends on the discrepancy between p_c and p_r, and the strength of the supervision. We define β as follows:

$$\beta = \frac{1}{1+\gamma \cdot \Delta(p_c, p_r) \cdot S} \tag{6}$$

where $\Delta(p_c, p_r)$ is the discrepancy between the key steps, γ is a model-dependent factor representing the model's sensitivity to supervision, and S is the strength of the supervision signal. As $\Delta(p_c, p_r)$ and S increase, the factor β decreases, leading to a significant reduction in cumulative error.

B Details about OpenPRM Training

B.1 Datasets for Preference Tree Building

As introduced in Section 4.1, we construct preference tree data primarily using UltraFeedback, which consists of a mixture of instructions in the open domain. Additionally, we incorporate instructions from the Math and STEM domains to enhance the generalization and reasoning capabilities of open-domain models. Beyond process pairs built with preference trees, we also include some outcome-level preference pairs to maintain the capabilities of ORMs, which can be seen as a specific case of PRMs. We provide all the statistics of the datasets used in Table 4.

Dataset	Orig	Retain	Outcome?	Process?
UltraFeedback	59,876	N/A	✓	✗
+PTS	N/A	70,068	✗	✓
ScienceQA	6,508	N/A	✓	✗
+PTS	N/A	12,958	✗	✓
MATH	7,500	N/A	✓	✗
+PTS	N/A	19,913	✗	✓
HelpSteer 2	N/A	7,221	✓	✗

Table 4: Statistics of training datasets. PTS is preference tree sampling strategy proposed in § 3.1.

Training Data Format. When preparing the training data for the PRM, we reformat all process-level and outcome-level pairs into a unified format: $[Q, C, P_c, P_r]$, where P_c and P_l represent the chosen (preferred) and rejected (non-preferred) answers, respectively, based on the same context C. For outcome-level pairs, $[C, P_c]$ and $[C, P_r]$ represent the complete answers. For process-level pairs, these concatenations represent partial answers.

B.2 Hyper-Parameters for Training

For building preference trees, we compute the edit distance on segments of different response candidates, splitting the entire responses using ".\n". The threshold for segment aggregation is set at 0.88 for all tasks based on the distribution of similarity as shown in Figure 5, and the rewards gap threshold between process pairs is 1.0 for UltraFeedback and 0.2 for Math and ScienceQA. For both Llama-3.1-8B-Instruct and Llama-3.1-70B-Instruct, we sample 64 candidate responses for each instruction. Due to constraints on inference cost, we randomly sample 10,000 instructions from UltraFeedback for the Llama-3.1-70B-Instruct model. We set the temperature to 0.5 and top-p to 1.0 for repeated sampling with vLLM engine [3].

For reproducing the UltraFeedback and HelpSteer2 reward models, we finetune Llama-3-8B-Instruct using a learning rate 5×10^{-6} over 1 epoch. Meanwhile, we finetune InternRM and FsfairX on process pairs using a learning rate 1×10^{-6} over 1 epoch. All models are finetuned with a batch size of 64 and a maximum sequence length of 2048.

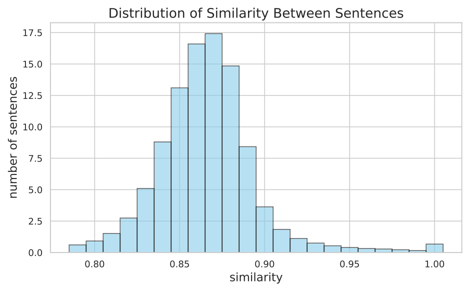


Figure 5: Distribution of Similarity.

B.3 TREE BUILDING EXAMPLE

As illustrated in Figure 2 (mainly segment and aggregate), we first split each answer into sentences and then merge similar sentences across all answers sequentially. For each merging operation, the candidate sentences are sourced from the same parent node and indexed consistently across their respective answers. To clarify, we provide a toy example along with the corresponding tree for merging three answers from ScienceQA in Table 5. We also provide an example of a sentence-level preference tree in Figure 13, which includes rewards for parallel samples at each node.

Table 5: Example for similarity-based sentence merging for preference building.

Node(Depth0): "Question:\nWhich logical fallacy is used in the text?\nBefore I refute my opponent's argument, I would like to draw attention to the fact that he is sweating and clearly does not have much experience on the debate team.\nOptions:\nA. ad hominem: an attack against the person making the argument, rather than the argument itself\nB. bandwagon fallacy: the assumption that the popular choice is automatically correct\nC. appeal to nature: the assumption that natural things are always good\nAnswer: Let's think step by step." — Reward: [] — Reply Indices: []

Node(Depth1): "To identify the logical fallacy used in the text, let's analyze the situation step by step" — Reward: [0, 1] — Reply Indices: [0, 2]
Similar sentences: "To identify the logical fallacy used in the text, let's analyze the situation step by step" ——— "To identify the logical fallacy in the text, let's break down the reasoning process step by step"

Node(Depth2): "The text states that the speaker wants to refute their opponent's argument, but instead of addressing the argument itself, they choose to comment on their opponent's physical appearance (sweating) and a supposed lack of experience on the debate team. This kind of comment is meant to discredit the opponent rather than engaging with the argument they presented" — Reward: [0] — Reply Indices: [0]
Node(Depth3): "The key phrase here is 'attack against the person making the argument, rather than the argument itself,' which is the definition of the ad hominem fallacy. \n\nTherefore, the correct answer is:\nA. ad hominem: an attack against the person making the argument, rather than the argument itself." — Reward: [0] — Reply Indices: [0]

Node(Depth2): "The text states, 'Before I refute my opponentś argument, I would like to draw attention to the fact that he is sweating and clearly does not have much experience on the debate team.' This statement does not address the opponentś argument itself but instead makes a personal attack on the opponentś appearance (sweating) and experience, implying that these characteristics make his argument weak" — Reward: [1] — Reply Indices: [2]

[3] https://github.com/vllm-project/vllm

Node(Depth3): "Now, let's analyze the options:\n\nA. ad hominem (A): This type of fallacy involves attacking the person making the argument rather than addressing the argument itself. In this case, the statement attacks the opponent's person (by mentioning their sweating and lack of experience) rather than addressing the merits of their argument" — Reward: [1] — Reply Indices: [2]

Node(Depth4): "B. bandwagon fallacy (B): This fallacy occurs when it is assumed that something is correct or good because it is popular. There is no indication in the text that the opponent's argument is being rejected because it is unpopular, but rather because of personal characteristics" — Reward: [1] — Reply Indices: [2]

Node(Depth5): "C. appeal to nature (C): This fallacy involves assuming that something is good or true because it is natural. The text does not mention anything about what is natural or unnatural, so this option does not apply" — Reward: [1] — Reply Indices: [2]

Node(Depth6): "Given the analysis, the correct answer is A. ad hominem, as the text attacks the opponent's person rather than addressing the argument itself." — Reward: [1] — Reply Indices: [2]

Node(Depth1): "To identify the logical fallacy in the text, let's break it down step by step:\n\n1. **Identify the claim and the counterargument**: The text claims that the opponent's argument is flawed but before refuting it, it mentions that the opponent is sweating and lacks experience on the debate team. This is an attempt to undermine the opponent's credibility without addressing the argument itself" — Reward: [1] — Reply Indices: [7]

Node(Depth2): "2. **Analyze the nature of the attack**: The attack is not on the argument's merits but on the opponent's character (sweating, lack of experience). Sweating can be a sign of nervousness, which isn't inherently indicative of the validity of one's argument. Lack of experience is also not directly related to the quality of the argument unless it's the first time the opponent is making an argument, which isn't specified" — Reward: [1] — Reply Indices: [7]

Node(Depth3): "3. **Classify the fallacy based on the analysis**: The attack is focused on undermining the opponent personally rather than addressing the argument. This matches the description of the "ad hominem" fallacy, which involves attacking the person making an argument rather than addressing the argument itself" — Reward: [1] — Reply Indices: [7]

Node(Depth4): "Therefore, the logical fallacy used in the text is **ad hominem**" — Reward: [1] — Reply Indices: [7]

Node(Depth5): "The final answer is: A" — Reward: [1] — Reply Indices: [7]

B.4 Results of Reward Shape

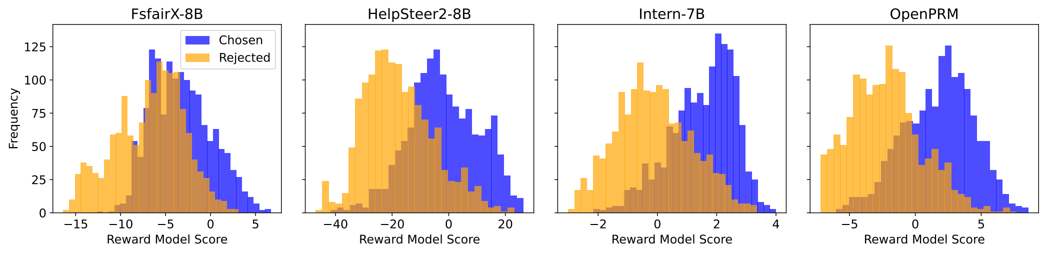


Figure 6: Rewards on chosen and rejected content of various RMs.

C Details about Evaluations

We summarize the open-domain datasets used in our evaluations as follows:

- **Alpaca Eval 2** (Dubois et al., 2024): A popular benchmark for evaluating instruction-based language models using automatic evaluators such as GPT-4. It features approximately 800 open-domain prompts. Given the length bias in GPT-4 evaluations, Alpaca Eval 2 employs length-controlled win-rates.

- **MixEval Hard** (Ni et al., 2024): A ground-truth-based dynamic benchmark derived from established benchmark mixtures. It evaluates LLMs using a highly capable model ranking system. MixEval Hard includes both free-form and multiple-choice questions, each category containing 500 questions.
- **MATH500** (Lightman et al., 2023): A subset of the MATH test dataset from OpenAI, featuring 12,500 challenging competition mathematics problems. We use the MATH500 version, which contains 500 samples that maintain IID consistency with the original test dataset, to evaluate the mathematics abilities of LLMs under scaled inference-time settings.
- **MMLU-PRO** (Wang et al., 2024d): An enhanced benchmark designed to evaluate models across broader and more challenging tasks. Built upon the MMLU dataset, MMLU-PRO integrates more challenging, reasoning-focused questions and increases the number of answer choices per question from four to ten, significantly raising the difficulty and reducing the chance of success through random guessing. We randomly sample 500 questions from the test data for our evaluations.
- **GPQA** (Rein et al., 2023): Consists of PhD-level STEM questions generated by experts in biology, physics, and chemistry. The original GPQA dataset is divided into main, diamond, and extended parts. We utilize the diamond split to align with OpenAI results, which includes about 200 questions.
- **IFEval** (Zhou et al., 2023): Designed to evaluate the instruction-following abilities of chat models. It focuses on a set of verifiable instructions and includes over 500 prompts with tasks such as "write an article with more than 800 words" and "wrap your response with double quotation marks."

During implementation, we use zero-shot chain-of-thought promtps for MATH and GPQA datasets based on prompt code in openai `simple-evals` repository [4]. For Alpaca Eval 2, MixEval, IFEval and MMLU-Pro dataset, we use the evaluation code from the official GitHub repository [5] [6] [7]. Specifically, we found that Mistral-Nemo struggles to adhere to answer format instructions; therefore, we opted to use few-shot Chain of Thought (CoT) examples instead of zero-shot CoT.

D ADDITIONAL RESULTS

D.1 DIFFERENT AGGREGATION STRATEGIES

For applying PRM to outcome-level pairs, we explored several aggregation strategies for calculating step-based rewards, as detailed below:

- Last Step: Use the reward of the final step as the overall reward, similar to ORM.
- Max/Min Step: Select the maximum or minimum reward among all steps.
- Simple Average: Calculate the average reward across all steps.
- Weighted Average: Apply a positional weight, giving later steps higher importance. The formula is: $r = \frac{1}{N}\sum_{i=1}^{N}\frac{i}{N}r_i$.
- Dynamic Aggregation: Utilize uncertainty-weighted optimization (UWO) (Coste et al., 2023), which dynamically adjusts weights based on intra-ensemble variance. This penalizes policies generating outputs with high disagreement across steps.
- Max/Min Delta: Inspired by recent research, we also compute the delta (difference) between step rewards and use the maximum or minimum delta as the final reward.

Different Performance on Chat Category. As shown in Table 2 and 7, OpenPRM's scores in the Chat category decrease, while performance in the Chat Hard category improves. This reflects the inherent trade-off between optimizing for simpler conversational tasks (Chat Easy) and more complex reasoning tasks (Chat Hard), which is a known challenge in reward model optimization, as

[4]https://github.com/openai/simple-evals/
[5]https://github.com/tatsu-lab/alpaca_eval
[6]https://github.com/Psycoy/MixEval
[7]https://github.com/TIGER-AI-Lab/MMLU-Pro

Table 6: Overall score of RewardBench (including Chat, Chat Hard, Safety and Reasoning) and PRM800k Test Results with different aggregation strategies.

Models	Aggregation	Overall	Chat	Chat Hard	Safety	Reasoning	PRM800k Test
intern2-7b-RM	ORM	87.6	99.2	69.5	87.2	94.5	61.0
intern2-7b-RM + FT	ORM	89.6	87.7	84.2	92.4	94.1	63.4
OpenPRM (intern)	Last Step	91.1	98.0	81.6	89.5	95.1	68.1
	Min Step	91.9	96.7	83.6	91.6	95.7	68.1
	Max Step	88.7	95.0	80.5	88.2	91.1	68.1
	Simple Avg	91.3	97.2	83.3	89.9	94.8	68.1
	Weight Avg	91.4	98.0	82.7	89.7	95.0	68.1
	Dynamic	91.6	97.8	83.3	90.4	95.0	68.1
	Min Delta	77.8	64.3	75.7	76.4	95.1	68.1
	Max Delta	77.7	77.4	69.5	74.3	89.5	68.1

also noted in RewardBench (Lambert et al., 2024). The distinction between these categories lies in their data sources: the Chat category includes datasets like AlpacaEval and MT Bench, while Chat Hard is derived from MT Bench and LLMBar. The decreased scores in the Chat category are largely due to AlpacaEval, as there is a distributional shift between this dataset and our training data, which is sourced from UltraFeedback.

D.2 Evaluate RMs with OffsetBias

In addition to benchmarks like RewardBench, UltraFeedback, and PRM800k, we evaluated our reward models using OffsetBias (Park et al., 2024), which provides a more granular assessment of bias in reward models. The results are summarized in the table below, showcasing the effectiveness of OpenPRM across various bias metrics in Table 7.

Table 7: OffsetBias Evaluation Results

Models	Concreteness	Content/ Continuation	Empty Reference	Familiar Knowledge Preference	Length Bias	Nested Instruction	Overall
Eurus-RM-7B	71.4	66.7	84.6	33.3	41.2	66.7	60
FsfairX-LLaMA3-RM	100	91.7	53.8	91.7	41.2	58.3	71.3
FsfairX-LLaMA3-RM + OffsetBias	92.9	100	46.2	58.3	82.4	83.3	77.5
LLaMA3-8B-Instruct + OffsetBias	100	95.8	92.3	83.3	85.3	50	85
Intern2-7b-RM	1	1	1	58.3	58.8	91.7	84.8
Intern2-7b-RM + OpenPRM (Our)	92.86	83.3	1	83.3	88.2	91.7	89.9

D.3 Comparison with MCTS methods

We compared the computational efficiency of our method with MCTS-based approaches under similar sampling budgets. The experimental setup and results are as follows:

- OpenPRM: We start from 60k samples, with 64 responses per sample, producing approximately 3.84M question-answer pairs in 24 hours. From this, 90k pairs were selected for training.
- MCTS: We start from 10k samples, with 4 responses per sample. Responses were split into sentences, resulting in 240k partial outputs. Sampling 8 full paths for each pair of partial outputs produced 3.84M question-answer pairs in 24 hours. Of these, 97k pairs were used for training.

The results of PRM trained with OpenPRM and MCTS are shown in Table 8. In fact, the MCTS method remains a powerful baseline but is significantly more resource-intensive, requiring up to 10 times the computational cost of our approach. With a larger sampling budget, the MCTS method could still be effectively utilized. However, our method offers a more efficient alternative and can be further optimized with more accurate similarity computation techniques, such as embedding-based methods. While these approaches may increase the time required to construct the tree, they hold great potential for improving performance. We plan to explore these optimizations in future work to further enhance the efficiency and accuracy of our method.

Table 8: Comparison with MCTS methods

Models	Aggregation	Overall	Chat	Chat Hard	Safety	Reasoning	PRM800k Test
intern2-7b-RM	ORM	87.6	99.2	69.5	87.2	94.5	61
intern2-7b-RM + outcome labels	ORM	89.6	87.71	84.21	92.43	94.06	63.4
OpenPRM (intern)	Min Step	91.9	96.7	83.6	91.6	95.7	68.1
MCTS (intern)	Min Step	91.4	95.5	81.6	93.2	95.4	68.2

D.4 Results on More Language Models

Scaling Effect of PBS. We have analyzed the scaling effects of Process Beam Search (PBS), where beam search is conducted at the sentence level, selecting the top N generated outputs based on PRM rewards. As presented in Figure 7, the results indicate that OpenPRM consistently outperforms Bag of N-grams (BoN) in PBS settings, showcasing its effectiveness and reliability in these scenarios. However, we also observed significant variance in the scaling effect of PBS across different tasks, in contrast to the more consistent scaling effect seen with best-of-N methods. This variance can likely be attributed to differences in the data distributions across tasks, which highlight the need for further investigation into handling data mixtures effectively. We plan to continue exploring this issue in future work to better understand and address these challenges.

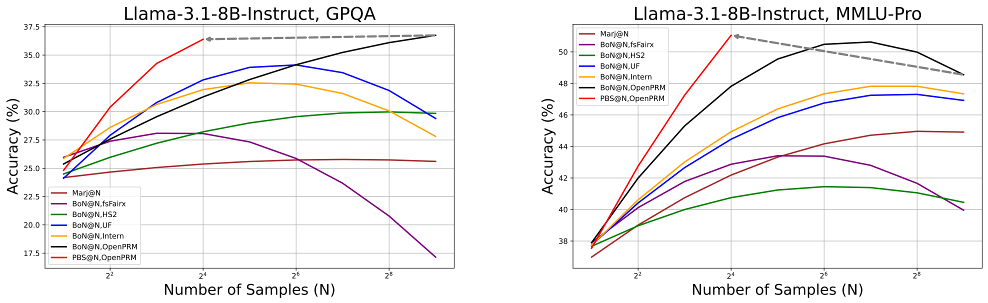


Figure 7: Scaling Effect of Process Beam Search

Scaling Effect of BoN. We present the remaining tasks, in addition to those shown in Figure 3, in Figure 8. We provide additional results for the Llama-3.1-70B-Instruct and Mistral-Nemo model regarding inference-time scaling in Figures 9, 10, 11 and 12, which support the same conclusions as those drawn from the Llama-3.1-8B-Instruct in Figure 3.

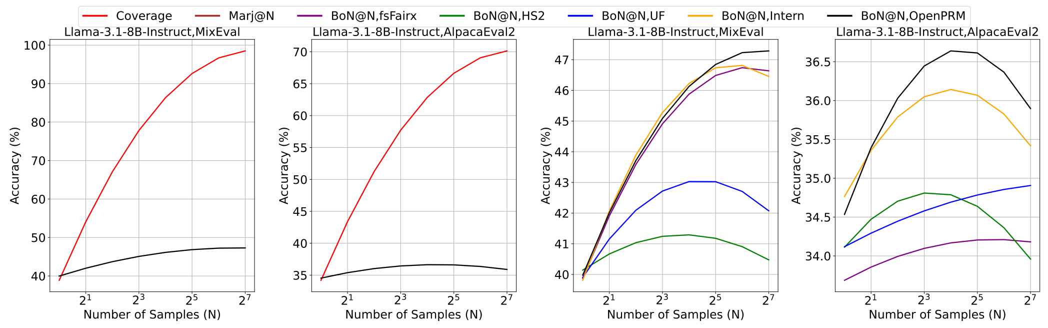


Figure 8: Results of scaling inference-time for Llama-3.1-8B-Instruct on the rest tasks of Figure 3.

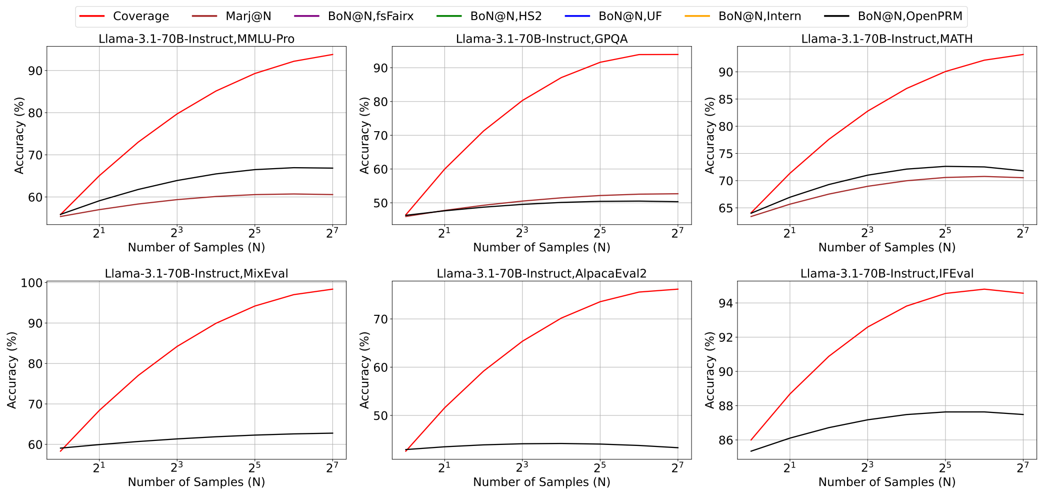


Figure 9: Results of scaling inference-time for Llama-3.1-70B-Instruct on open-domain tasks.

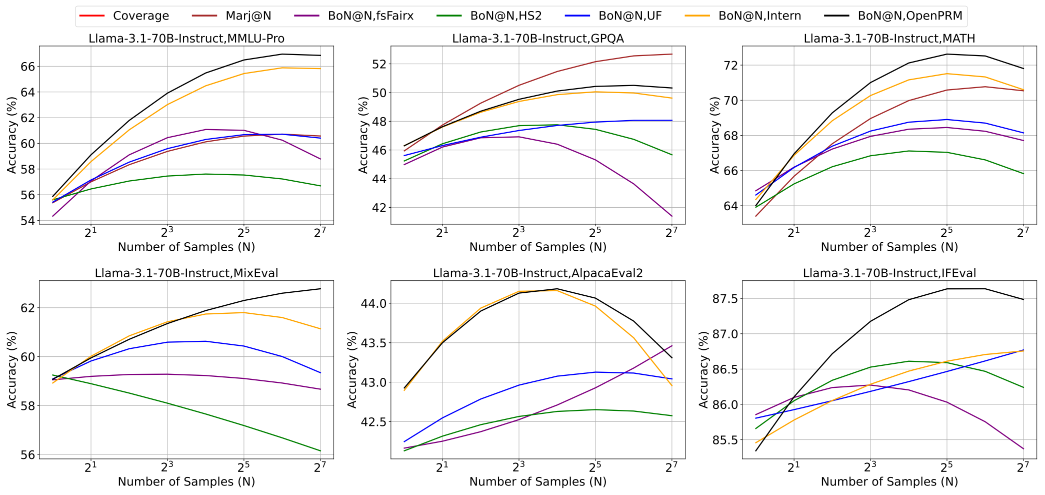


Figure 10: Results of scaling inference-time for Llama-3.1-70B-Instruct on open-domain tasks.

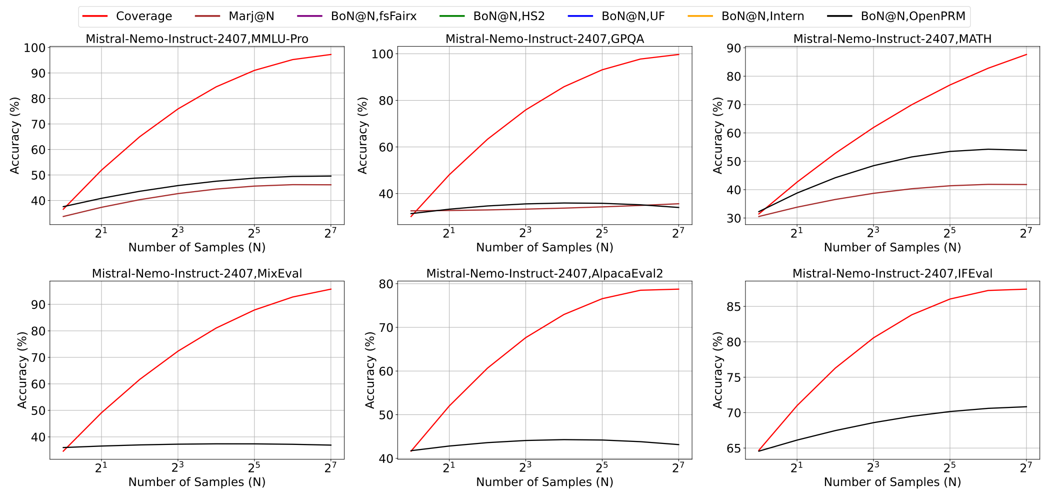


Figure 11: Results of scaling inference-time for Mistral-Nemo on open-domain tasks.

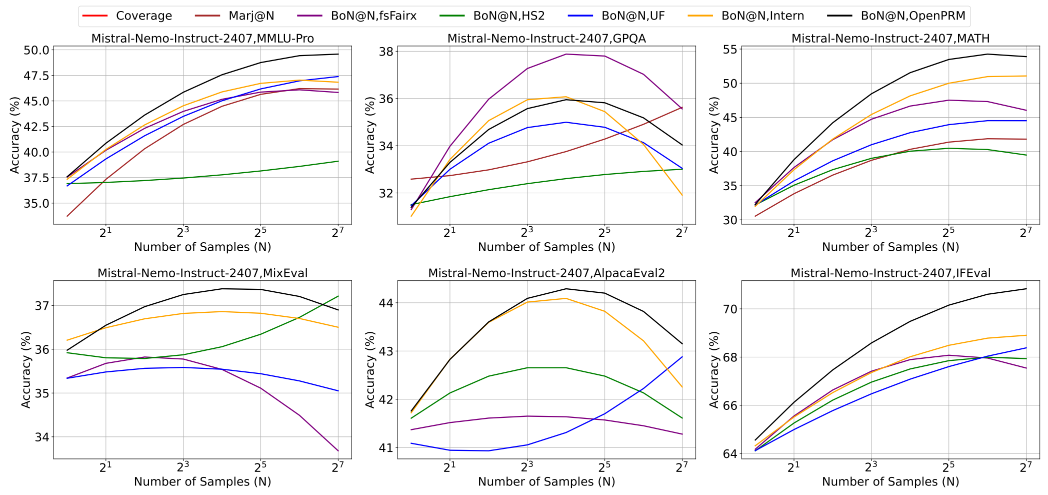


Figure 12: Results of scaling inference-time for Mistral-Nemo on open-domain tasks.

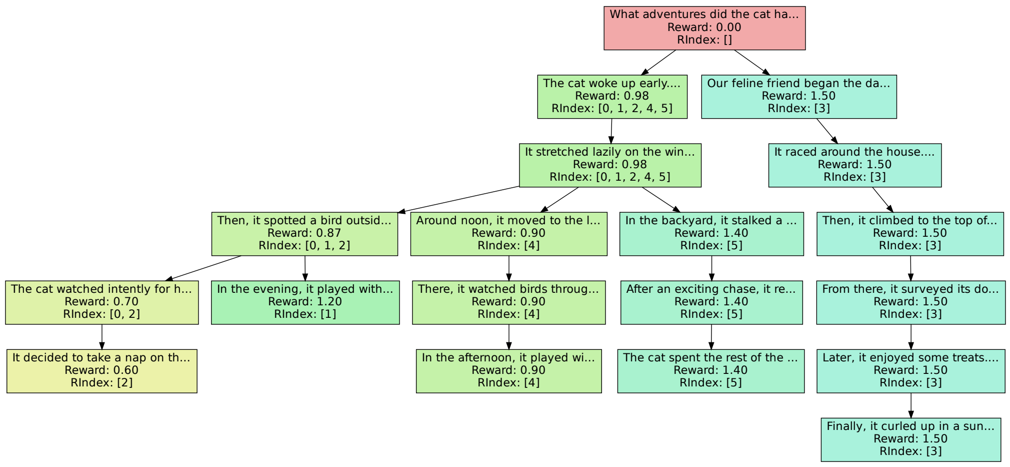

What adventures did the cat ha...
Reward: 0.00
RIndex: []
The cat woke up early....
Reward: 0.98
RIndex: [0, 1, 2, 4, 5]
Our feline friend began the da...
Reward: 1.50
RIndex: [3]
It stretched lazily on the win...
Reward: 0.98
RIndex: [0, 1, 2, 4, 5]
It raced around the house....
Reward: 1.50
RIndex: [3]
Then, it spotted a bird outsid...
Reward: 0.87
RIndex: [0, 1, 2]
Around noon, it moved to the l...
Reward: 0.90
RIndex: [4]
In the backyard, it stalked a ...
Reward: 1.40
RIndex: [5]
Then, it climbed to the top of...
Reward: 1.50
RIndex: [3]
The cat watched intently for h...
Reward: 0.70
RIndex: [0, 2]
In the evening, it played with...
Reward: 1.20
RIndex: [1]
There, it watched birds throug...
Reward: 0.90
RIndex: [4]
After an exciting chase, it re...
Reward: 1.40
RIndex: [5]
From there, it surveyed its do...
Reward: 1.50
RIndex: [3]
It decided to take a nap on th...
Reward: 0.60
RIndex: [2]
In the afternoon, it played wi...
Reward: 0.90
RIndex: [4]
The cat spent the rest of the ...
Reward: 1.40
RIndex: [5]
Later, it enjoyed some treats....
Reward: 1.50
RIndex: [3]
Finally, it curled up in a sun...
Reward: 1.50
RIndex: [3]

Figure 13: Example of sentence-level preference tree for process reward modeling.