Collaborative Adaptive Labeling with Imperfect Labelers and Selective Expert Escalation

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Abstract

High-quality labels are essential for reliable evaluation of modern machine learning (ML) and artificial intelligence (AI) systems. Increasingly, model evaluation pipelines in practice involve collaborative *"gold-silver"* supervision, where all instances may receive multiple inexpensive, imperfect *silver labels* (e.g., from crowdsourcing platforms or automated AI judges), while a limited number of costly *gold labels* provided by experts can be selectively acquired for difficult cases, such as those with substantial disagreement among silver labels. This setting differs from classical labeling formulations in that an instance can receive multiple silver labels, while expert labeling is applied selectively, and has become increasingly common in the evaluation of modern ML/AI systems.

Yet, a key challenge in this setup is determining when and how to allocate labeling effort across silver and gold labels under a fixed budget, while simultaneously ensuring that the collected labels support reliable model evaluation. Escalating from silver to expert labeling too late may propagate incorrect labels from imperfect annotators, whereas escalating too early wastes scarce expert resources. Moreover, because labeling decisions depend on previously observed labels, the resulting data are adaptively sampled, inducing dependencies between labels and the sampling process. This adaptivity complicates both the design of systematic labeling algorithms and the validity of downstream statistical inference used for ML/AI system evaluation. To address these challenges, we propose a *cost-efficient collaborative adaptive labeling framework* in which each instance may receive multiple imperfect silver labels and, when warranted, an expert-provided gold label. To support valid model evaluation from adaptively collected labels, we propose an estimator that systematically combines expert-provided gold labels and imperfect silver labels, and establish its consistency under mild conditions. Across multiple evaluation datasets, our method substantially improves labeling quality and the reliability of downstream statistical evaluation compared to existing baselines.

1. Introduction

1.1. Motivation and Our Contributions

Reliable evaluation of modern ML and AI systems increasingly depends on large volumes of high-quality labels. Expert annotations remain the gold standard for model evaluation, which we refer to as *gold labels*, but they are costly and limited in scale. At the same time, there is growing access to inexpensive but imperfect labelers, which we refer to as *silver labels*, that in many tasks can produce labels comparable to expert quality. These imperfect sources range from crowdsourced human workers on online platforms to automated evaluation systems such as "LLM-as-a-judge" (Zheng et al., 2023). This shift naturally gives rise to a general problem that we refer to as *collaborative labeling*: how can we effectively combine label sources of different quality and cost to obtain reliable evaluation outcomes under a limited labeling budget? Recent works have explored related ideas under the themes of human–AI collaboration (Ashktorab et al., 2021), learning to defer (Mozannar & Sontag, 2020), and language model routing (Shnitzer et al., 2023).

Despite recent progress, the application of collaborative labeling for ML/AI evaluation is limited by two key practical and theoretical challenges.

First, most existing collaborative labeling frameworks assume that each instance ultimately receives a single label. In practice, however, an instance may receive multiple intermediate labels from inexpensive sources, which can often be combined to produce a higher-quality signal before deciding whether expert annotation is necessary (Dawid & Skene, 1979; Raykar et al., 2010). This substantially complicates

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the labeling decision, as we must determine not only *which* source to query, but also *how many* labels to acquire from each source. At the same time, this multi-label structure creates new opportunities: aggregated silver labels can serve as a diagnostic signal that identifies truly difficult instances and selectively *escalates* only those cases to expert annotation, thereby reserving costly gold labels for instances where they are most informative.

Second, there exists a systematic gap between collaborative labeling and valid post-hoc statistical model evaluation using the collected labels. Because labeling decisions typically depend on previously observed labels and model updates, the resulting data collection process is adaptive and history-dependent. This violates the independence assumptions underlying standard statistical methods designed for i.i.d. datasets (Lakkaraju et al., 2017). A principled framework for multi-source labeling that also guarantees valid post-hoc statistical inference remains largely unexplored.

To address these limitations and opportunities, we propose a *cost-efficient collaborative adaptive labeling (CAL) framework* for reliable model evaluation with imperfect labelers. Our contributions consist of three key components.

First, we introduce a framework that enables collaboration between gold and silver labelers for model evaluation in Section 3. The algorithm operates adaptively: each instance may receive multiple silver labels before a decision is made to selectively escalate it to expert gold labeling. This decision is guided by a one-step-ahead expected uncertainty reduction sampling score that balances evaluation benefit and labeling cost. In addition, to enable valid model evaluation using labels collected under this adaptive process, we develop two post-hoc estimators and construct confidence intervals for the target evaluation quantity in Section 4.

Second, we provide theoretical investigations of both the CAL framework and the proposed post-hoc evaluation estimators in Section 5. We formally establish the *diagnostic role* of silver labels by showing that disagreement between silver labels and model predictions improves the alignment between uncertainty scores and true labeling mistakes, leading to higher expected labeling accuracy under uncertainty sampling. For the post-hoc estimators, we prove unbiasedness, consistency, and asymptotic normality with valid confidence intervals. These results show how statistical theory can guide both adaptive evaluator allocation and valid downstream inference under imperfect and selectively collected labels.

Third, we conduct synthetic experiments to demonstrate the robustness of the diagnostic role of silver labels under heterogeneous error rates, model misspecification, and correlated silver observations in Section 6. We further apply CAL to two real-world evaluation datasets to show that it improves labeling quality while providing statistically consistent and efficient inference for evaluation targets.

1.2. Related Literature

Adaptive label acquisition. A large literature studies adaptive querying under a budget, starting from active learning methods that decide which instance to label to maximize model improvement per cost (Settles, 2009). Extensions consider multiple labelers with heterogeneous cost and quality, jointly selecting instances and annotators (Zhang & Chaudhuri, 2015), and related ideas appear in learning-to-defer and rejection mechanisms that route hard cases to stronger decision-makers (Madras et al., 2018; Mao et al., 2025). Our setting differs in allowing multiple intermediate labels before selective escalation, where silver labels serve as diagnostic signals rather than noisy supervision.

Learning with noisy labels. Learning under label noise has been extensively studied, from classical noise models to modern pipelines involving crowdsourcing and weak supervision. Prior work develops loss correction, noise transition estimation, and data filtering methods to make learning robust to corrupted labels (Natarajan et al., 2013; Patrini et al., 2017; Song et al., 2022). In contrast, we do not treat silver labels as data to be directly learned from; instead, we use them to guide where expert labels should be acquired, turning noise into a diagnostic tool for adaptive sampling.

Statistical inference under adaptive data collection. Recent advances in statistical inference recognize that adaptive sampling induces dependence between data collection and outcomes, invalidating standard estimators. Martingale-based analysis, doubly robust estimators, and anytime-valid inference have been developed to handle such settings (Zhang et al., 2021; Waudby-Smith et al., 2024; Bang & Robins, 2005). Our post-hoc estimators build on these ideas but are tailored to the early stopping and multi-stage escalation mechanism induced by collaborative labeling.

2. Problem Setup

In this section, we introduce the problem setup for collaborative adaptive labeling, as well as the statistical target for evaluating the ML/AI systems based on adaptively collected labels.

Let $X \in \mathcal{X}$ denote the information of an instance to be labeled, such as a pair consisting of a prompt P and an image I generated from a foundation model to be evaluated. Let $Y \in \{0, 1\}$ denote the gold label of the instance. We assume binary labels in the main text for illustration and defer the multiclass extension to the appendix. Due to the high cost of expert labeling, the gold label is *not observed* for all instances. We use $D \in \{0, 1\}$ to denote whether the gold label is observed for an instance ($D = 1$ if observed,

and $D = 0$ otherwise).

In addition to the gold label, we (may) observe *multiple* silver labels for each instance. We first consider the setting where each silver label is an anonymous draw from a population of crowd workers, which reflects the common situation where silver labels are collected from third-party contractors. In this case, the silver labels for an instance are interchangeable, and we denote them as a sequence

$$\mathbf{Z} \in \{0,1\}^m,$$

where the length $m \in \mathbb{Z}_{\geq 0}$ is the number of silver labels, which may differ across instances. In the Appendix, we extend this to a *channel-aware* setting, where silver labels may come from different channels with heterogeneous labeling criteria, such as from different countries.

We now describe the data structure, which resembles the "pool-based" setting in active learning (Settles, 2009). Suppose we observe a historical dataset consisting of N instances with multiple silver labels and possibly a gold label, denoted by

$$\mathcal{D} = \{(X_i, \mathbf{Z}_i, D_i, D_i Y_i)\}_{i=1}^{N}.$$

We then receive a new dataset of n unlabeled instances, denoted by

$$\mathcal{D}^* = \{(X_i)\}_{i=N+1}^{N+n}.$$

In this paper, we pursue two objectives. Our *first objective* is to obtain high-quality labels for the unlabeled dataset $\mathcal{D}^*$ via a *collaborative adaptive labeling* (CAL) algorithm. By *adaptive*, we mean that labeling decisions are made sequentially over T stages, where at each stage the algorithm may take actions on multiple instances based on all labels observed so far. By *collaborative*, we mean that the algorithm jointly leverages inexpensive, imperfect silver labelers and costly expert (gold) labelers, allocating labeling effort across them in a coordinated manner. At each stage and for each currently unlabeled instance, the algorithm selects an action from {GOLD, SILVER, NONE}. The action GOLD escalates the instance to an expert to obtain a gold label, SILVER queries one additional silver label, and NONE defers labeling at the current stage. Let $\hat{Y}_i$ denote the final label assigned to instance i after T stages. Querying a silver label incurs a cost c_s, while obtaining a gold (expert) label incurs a higher cost c_g. The goal of CAL is to design a labeling policy that minimizes overall labeling error under a total budget constraint B.

Our *second objective* is to perform a statistically valid evaluation of an ML/AI system that is aligned with expert (gold) labelers, using all labels-both silver and gold-collected under collaborative adaptive labeling. In practice, expert-provided gold labels define the evaluation metric of interest but are observed for only a small, adaptively selected subset of instances, while inexpensive silver labels are abundant yet potentially biased and noisy. Naively averaging silver labels can therefore lead to biased evaluation, whereas relying exclusively on gold labels can be statistically inefficient. Accordingly, our evaluation target is

$$\mu := \mathbb{E}_{X \sim P_X^*}[Y],$$

where P_X^* denotes the instance distribution underlying the evaluation pool $\mathcal{D}^*$. This quantity represents the expected gold-label evaluation score over the population of instances. Our goal is to construct efficient estimators and valid confidence intervals for μ by leveraging adaptively collected gold labels together with noisy silver labels, thereby enabling statistically sound comparison of ML/AI systems under a fixed labeling budget. Extensions to more general empirical risk minimization targets are deferred to the appendix.

3. Collaborative Adaptive Labeling

This section presents methods for the first objective. We begin with gold-only adaptive labeling (Section 3.1), illustrate the role of silver labels and selective escalation (Section 3.2), and then introduce the proposed collaborative adaptive labeling method (Section 3.3).

3.1. Warm-up: Adaptive Labeling with Gold Labels

We first consider the setting where only gold labels are used. Suppose we train an initial gold-label prediction model $p_0(x) = \hat{\mathbb{P}}(Y = 1 | X = x)$ using the historical data $\mathcal{D}$. The total budget B is divided across T stages, forming $(B_1, \ldots, B_T)$. At each stage t, the model evaluates the prediction uncertainty of all currently unlabeled instances using the score

$$s_{t-1}(X_i) = f\big(p_{t-1}(X_i)\big) \in [0,1],$$

where $f(\cdot)$ can be entropy, margin, or variance. In a greedy labeling strategy, gold labels are assigned to the instances with the largest scores until the stage budget is exhausted. Alternatively, for probabilistic sampling, these scores are converted into softmax sampling weights

$$w_{i,t} = \frac{\exp(s_{t-1}(X_i)/\tau)}{\sum_j \exp(s_{t-1}(X_j)/\tau)},$$

where τ is the temperature parameter controlling the randomness of the sampling. Given stage budget B_t, the sampling probability for querying a gold label is $q_{i,t} = \min\Big(1, \frac{B_t}{c_g} w_{i,t}\Big)$. A gold label is queried according to $\delta_{i,t} \sim \text{Bernoulli}(q_{i,t})$. Once an instance receives a gold label, it is removed from the unlabeled pool. After collecting new gold labels, the model is updated from $p_{t-1}(x)$ to obtain $p_t(x)$.

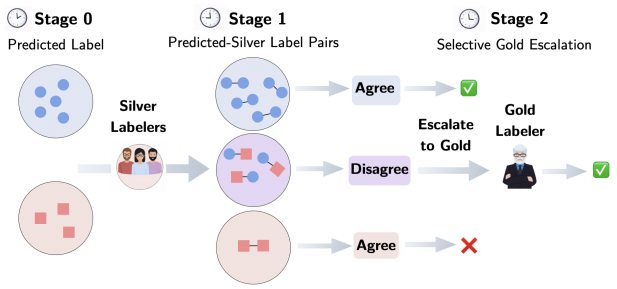


Figure 1. Silver labels diagnose prediction errors and guide selective escalation to gold labeling.

3.2. Motivation: Two Roles of Silver Labels

We first build intuition for how silver labels can substantially improve labeling quality before introducing our collaborative adaptive labeling framework. A formal justification is provided in Section 5.

Diagnostic role. Under limited labeling resources and a pretrained prediction model $p_0(x)$, the Bayes–optimal strategy is to prioritize instances where the model is likely to be wrong, characterized by the *inaccuracy score* $W_i := \mathbf{1}\{\hat{Y}_{i,0} \neq Y_i\}$. Since the gold label Y_i is unobserved, the practical surrogate used in Section 3.1 is an *uncertainty score* s_i, and instances with the largest s_i are labeled. However, the ranking induced by $\{s_i\}$ can be poorly aligned with the true ranking of $\{W_i\}$, causing standard uncertainty sampling to miss instances that genuinely require labeling.

As illustrated in Figure 1, silver labels provide a cheap diagnostic signal for this misalignment. When a silver label disagrees with the model prediction, even if s_i is small, there is a high probability that $W_i = 1$. Elevating the sampling priority of such disagreement cases produces a ranking that more closely approximates the true inaccuracy score, allowing these instances to be selectively escalated to gold labeling. Conversely, consistent agreement between silver labels and the model prediction provides evidence that escalation is unnecessary.

Prediction role. Silver labels can also improve the prediction performance of $p_0(x)$ even without escalation to gold. When the error patterns of diverse silver labelers are not perfectly aligned with those of $p_0(x)$, their information can be aggregated, for example through weighted majority voting or posterior updating, to refine the model's belief and produce more accurate predictions.

3.3. Collaborative Adaptive Labeling Framework

In this section, we present the proposed labeling framework. Before describing the full algorithm, we introduce its key building blocks.

3.3.1. Base Models

The framework relies on two base models that support the diagnostic and prediction roles of silver labels. The first is gold label prediction model (*G-model*) $p(X;\theta) = \mathbb{P}(Y = 1|X)$, which predicts the gold label using only instance information X, without incorporating any silver label information. It plays the similar role as the predictor used in standard uncertainty sampling (Section 3.1).

The second is the item-level difficulty model (*D-model*). For each instance, we introduce a latent item difficulty random variable $e_i \in [0, 1]$, interpreted as the probability that a silver label Z disagrees with the corresponding gold label. To capture heterogeneity across instances, we model $\mathbb{P}[Z \neq Y|X] = e(X;\phi)$. We assume $e_i \leq \bar{e} < 0.5$ almost surely, since otherwise the label can be flipped.

Both models are trained on the historical dataset $\mathcal{D}$, yielding initial estimates $\hat{\theta}_0$ and $\hat{\phi}_0$. The framework does not impose restrictions on model classes, and these models can be implemented using any suitable architecture, such as lightweight heads on top of pre-trained embeddings.

3.3.2. Silver-Guided Prediction Update

We next describe the prediction update rule after collecting silver labels in the T-stage sequential framework. For instance i, suppose that up to stage $t \leq T-1$ we have observed a sequence of silver labels $\mathbf{Z}_{i,t}$ of length $m_{i,t}$, among which $k_{i,t}$ labels take value 1. Combining information from the G-model and the D-model, we refine the prediction of the gold label by

$$p_t(X_i) \propto p(X_i;\hat{\theta})\,(1 - e(X_i;\hat{\phi}))^{k_{i,t}}\, e(X_i;\hat{\phi})^{\,m_{i,t}-k_{i,t}}.$$

In log-odds form, this update can be interpreted as a difficulty-aware weighted majority vote between the G-model prediction and the collected silver labels, where each silver label contributes a weight $\log \frac{1-e(X_i;\hat{\phi})}{e(X_i;\hat{\phi})}$ determined by the estimated item difficulty.

As in Section 3.1, we compute an uncertainty score $s_t(X_i) = f\big(p_t(X_i)\big)$ from the updated prediction. For standard uncertainty measures that attain their maximum at $p_t(X_i) = 0.5$, observing a new silver label that disagrees with the current prediction moves $p_t(X_i)$ toward 0.5 and thus increases the uncertainty score, while agreement decreases it. This behavior naturally reflects the diagnostic role of silver labels.

3.3.3. One-Stage-Ahead Sampling Score

We now define the sampling score for each action $\{\text{GOLD}, \text{SILVER}, \text{NONE}\}$ used in adaptive labeling. To evaluate the benefit of querying an additional silver label,

we compute the one-stage-ahead expected uncertainty score

$$\hat{s}_{t+1}(X_i) = \hat{\mathbb{E}}[f(p_{t+1}(X_i)) \mid X_i, \mathbf{Z}_{i,t}] = \sum_{z\in\{0,1\}} \hat{\mathbb{P}}(Z_{i,t+1} = z \mid X_i, \mathbf{Z}_{i,t})\, f\left(p^{(z)}_{t+1}(X_i)\right),$$

where $p^{(z)}_{t+1}(X_i)$ is the updated prediction after observing a new silver label $z \in \{0, 1\}$. The probability of the next silver label depends on the current prediction and the estimated difficulty $\hat{e}_i = e(X_i; \hat{\phi})$:

$$\hat{\mathbb{P}}(Z_{i,t+1} = 1 \mid X_i, \mathbf{Z}_{i,t}) = \hat{e}_i + (1 - 2\hat{e}_i)\, p_t(X_i),$$

$$\hat{\mathbb{P}}(Z_{i,t+1} = 0 \mid X_i, \mathbf{Z}_{i,t}) = (1 - \hat{e}_i) - (1 - 2\hat{e}_i)\, p_t(X_i).$$

The sampling score for SILVER is defined as the expected uncertainty reduction per unit cost c_s:

$$u_{i,t,\text{SILVER}} = (s_t(X_i) - \hat{s}_{t+1}(X_i))\,/c_s.$$

Although querying a silver label may have multi-stage effects (e.g., increasing uncertainty and triggering future escalation to gold), this long-term impact is difficult to quantify. We therefore adopt a one-stage-ahead approximation.

For GOLD, the uncertainty after labeling becomes zero, so the one-stage uncertainty reduction equals $s_t(X_i)$. Its sampling score is

$$u_{i,t,\text{GOLD}} = s_t(X_i)/c_g.$$

3.3.4. TWO-STEP ADAPTIVE SAMPLING

We now describe a two-step hierarchical probabilistic sampling rule for selecting stage $t+1$ actions while respecting the stage budget B_{t+1}. A greedy version can be obtained by replacing the softmax steps with maximization. For each instance, define the action preference

$$q_{i,t,G} = \frac{\exp(u_{i,t,\text{GOLD}}/\tau)}{\exp(u_{i,t,\text{GOLD}}/\tau) + \exp(u_{i,t,\text{SILVER}}/\tau)},$$

and $q_{i,t,S} = 1 - q_{i,t,G}$. The expected utility and cost of acting on instance i are therefore

$$\tilde{u}_{i,t+1} = q_{i,t,G}\, u_{i,t+1,\text{GOLD}} + q_{i,t,S}\, u_{i,t+1,\text{SILVER}},$$

$$\tilde{c}_{i,t+1} = q_{i,t,G}\, c_g + q_{i,t,S}\, c_s.$$

Compute instance weights $\tilde{w}_{i,t+1} = \frac{\exp(\tilde{u}_{i,t+1}/\tau)}{\sum_j \exp(\tilde{u}_{j,t+1}/\tau)}$, and scale them by $\alpha_{t+1} = \frac{B_t}{\sum_i \tilde{w}_{i,t+1}\, \tilde{c}_{i,t+1}}$. Each instance is selected with probability $\alpha_{t+1}\tilde{w}_{i,t+1}$; conditional on selection, GOLD is chosen with probability $q_{i,t,G}$ and SILVER otherwise. As in Section 3.1, instances receiving gold labels are removed and models are updated using the newly collected data (see Appendix for pseudocode).

4. Statistical Post-Hoc Evaluation

In this section, we present methods for the second objective, using the labels collected from the Section 3.3. We first describe the statistical challenges in estimating $\mu = \mathbb{E}_{X\sim P^*_X}[Y]$ in Section 4.1, and then introduce our proposed estimators in Section 4.2.

4.1. Statistical Challenges

Consistent and efficient estimation of the target μ faces the following challenges:

Adaptive sampling with early stopping. As described in Section 3.3, the probability that an instance receives a gold label depends on the full history of the labeling algorithm, including past labels and model updates. Moreover, once a gold label is collected, the instance leaves the unlabeled set and will never be labeled again. This creates a history-dependent sampling process with early stopping, which invalidates naive sample averages and is not addressed by standard i.i.d. assumptions or classical inverse propensity score weighting (IPW) formulations (Rosenbaum & Rubin, 1983).

Noisy silver labels. Instances that do not receive gold labels typically obtain multiple silver labels that contain useful information about Y. However, these labels are noisy, and directly using them in estimation can introduce bias. A key challenge is how to leverage silver-label information to improve estimation efficiency while preserving consistency and valid inference (Angelopoulos et al., 2023).

4.2. Proposed Estimators

We now introduce two estimators for μ with strong statistical guarantees, whose properties are established in Section 5. Notably, our method allows the instance distribution in the historical dataset to differ from that in the unlabeled evaluation dataset; that is, $P_X \neq P^*_X$.

We first introduce additional notation for the label collection procedure based on the framework in Section 3.3. Define $\delta_{i,t} \in \{0, 1\}$ as the indicator of whether a gold label is collected at stage t for instance i. Let the information collected up to stage $t-1$ be denoted by $\mathcal{F}_{t-1}$. According to the algorithm in Section 3.3, each $\delta_{i,t}$ equals 1 with known sampling probability

$$\pi_{i,t} := \mathbb{P}(\delta_{i,t} = 1 \mid \mathcal{F}_{t-1}) = \alpha_{t+1}\, \tilde{w}_{i,t+1} q_{i,t,G} \in (0, 1),$$

which depends on past information. We further define the stage at which a gold label is collected as τ_i, such that $1 \le \tau_i \le T$ if a gold label is collected within T stages, and $\tau_i = \infty$ otherwise. Once a gold label is collected, the instance is removed from the unlabeled pool, and the probability of collecting a gold label becomes 0, creating

an early stopping mechanism that standard IPW does not directly handle.

To obtain an unbiased estimator of the target, we propose an adjusted IPW estimator, denoted by $\hat{\mu}_{\texttt{IPW,CAL}}$. Define

$$D_i = \frac{1}{T}\Big(\sum_{t \le \min\{\tau_i, T\}} \frac{\delta_{i,t}}{\pi_{i,t}} + \underbrace{(T - \tau_i)_+}_{\text{Early stopping adjustment}} \Big).$$

The adjusted IPW estimator is then given by

$$\hat{\mu}_{\texttt{IPW,CAL}} = \frac{1}{n} \sum_{i=N+1}^{N+n} D_i Y_i.$$

Compared to standard IPW, this adjusted estimator explicitly accounts for the early stopping mechanism by incorporating the deterministic contribution of post-gold stages, thereby avoiding invalid weights and preserving unbiasedness under the adaptive sampling process. We can estimate the variance by

$$\hat{\mathbb{V}}_{\texttt{IPW,CAL}} = \frac{1}{n} \sum_{i=N+1}^{N+n} \left(D_i Y_i - \hat{\mu}_{\texttt{IPW,CAL}}\right)^2,$$

and construct an asymptotic $(1-\alpha)$ confidence interval as $\left[\hat{\mu}_{\texttt{IPW,CAL}} \pm z_{1-\alpha/2}\sqrt{\hat{\mathbb{V}}_{\texttt{IPW,CAL}}/n}\right]$.

To leverage the collected silver labels to further improve asymptotic estimation efficiency without introducing bias, we propose an adjusted augmented IPW estimator:

$$\hat{\mu}_{\texttt{AIPW,CAL}} = \frac{1}{n} \sum_{i=N+1}^{N+n} \psi_i, \quad \hat{\mathbb{V}}_{\texttt{AIPW,CAL}} = \frac{1}{n} \sum_{i=N+1}^{N+n} (\psi_i - \bar{\psi})^2,$$

where

$$\psi_i = D_i Y_i + \underbrace{\frac{1}{T} \sum_{t \le \min\{\tau_i, T\}} \left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right) p_{t-1}(X_i)}_{\text{Sequential prediction augmentation}}.$$

Here, $p_{t-1}(X_i)$ denotes the silver-guided gold prediction introduced in Section 3.3.2. Notably, this construction respects the sequential nature of the adaptive labeling procedure: at each stage, the augmented term is paired with the prediction model available at that time, without using any future information. The theoretical guarantees of both estimators are established in Section 5.

5. Theoretical Investigation

This section provides theoretical support for the methods in Sections 3 and 4. We formalize the role of silver labels and selective gold escalation in Section 5.1, and establish finite-sample and asymptotic properties of the estimators in Section 4.2 in Section 5.2.

5.1. Diagnostic Role of Silver Labels

While a full analysis of the CAL framework is beyond the scope of this paper, we show in a simplified setting when the diagnostic role of silver labels (Section 3.2) can improve labeling quality over standard uncertainty sampling.

We first present a lemma showing that a sampling score more aligned with true labeling mistakes leads to higher labeling accuracy. Consider greedy uncertainty sampling with gold labels only (Section 3.1) under a single stage ($T = 1$), without model updating, and suppose the labeling budget is a proportion $b \in (0, 1)$. Recall the *inaccuracy score* $W_i := \mathbf{1}\{\hat{Y}_{i,0} \neq Y_i\}$ defined in Section 3.2, where $\hat{Y}_{i,0}$ is the predicted label from $p_0(x)$.

For any sampling score $U_i \in \mathbb{R}$ (larger values indicate higher priority for labeling), define its alignment with the true inaccuracy score via the AUC: $\mathrm{AUC}(U, W) = \mathbb{P}(U^+ > U^-) + \frac{1}{2}\mathbb{P}(U^+ = U^-)$, where U^+ is drawn from $U \mid (W = 1)$ and U^- is drawn from $U \mid (W = 0)$. Let $S_b(U)$ denote the top-b fraction of instances ranked by U, and define the labeling gain as $G_b(U) := \mathbb{P}(W = 1 \mid i \in S_b(U))$. This quantity directly determines the improvement in final labeling accuracy, since gold labeling corrects exactly those mistakes.

Lemma 5.1 (Score alignment and accuracy)**.** *Let S and U be two sampling scores. If $\mathrm{AUC}(S, W) > \mathrm{AUC}(U, W)$, then, averaged over all labeling budgets $b \in (0, 1)$,*

$$\int_0^1 G_b(S)\, db > \int_0^1 G_b(U)\, db.$$

In other words, a sampling score that is more aligned with the inaccuracy score (in the AUC sense) leads to higher expected labeling accuracy.

We next show that querying silver labels can produce an updated sampling score that is better aligned with the inaccuracy score under mild conditions. Suppose each instance receives a silver label $S_i \in \{0, 1\}$ with error rate $e_i := \mathbb{P}(S_i \neq Y_i \mid X_i)$. We consider a general sampling score update rule after observing silver, which accommodates but goes beyond the update rule in Section 3.3.2.

Define the agreement indicator $M_i := \mathbf{1}\{S_i = \hat{Y}_i\}$, where $\hat{Y}_i$ is the predicted label. Let the baseline sampling score be U_i. We define the updated sampling score as an additive perturbation of the baseline score $T_i^{\mathrm{sil}} := U_i + \Delta_i$, where the update term Δ_i depends on the agreement indicator and satisfies

$$\Delta_i = \begin{cases} \delta_i^+, & M_i = 0 \quad \text{(disagreement)}, \\ -\delta_i^-, & M_i = 1 \quad \text{(agreement)}, \end{cases}$$

for some nonnegative quantities $\delta_i^+, \delta_i^- \geq 0$, with $\delta_i^+ + \delta_i^- > 0$. That is, disagreement between the silver label and

the model prediction increases the sampling score relative to agreement. We impose the following assumptions.

Assumption 5.2 (Independent silver noise)**.** $S_i \perp \hat{Y}_i \mid (Y_i, X_i)$.

Assumption 5.3 (Silver quality)**.** There exists $\bar{e} < \frac{1}{2}$ such that $e_i \leq \bar{e}$ for all i.

Assumption 5.2 states that the silver label provides an independent noisy observation of the ground truth relative to the model prediction, rather than being derived from the model itself. Assumption 5.3 requires that the silver labeler performs uniformly better than random guessing.

Proposition 5.4 (Silver disagreement improves score alignment)**.** *Under Assumptions 5.2–5.3, the updated score* T^{sil} *satisfies*

$$\text{AUC}(T^{\text{sil}}, W) > \text{AUC}(U, W),$$

unless $\text{AUC}(U, W) = 1$.

Proposition 5.4 shows that the benefit of silver labels does not rely on an exact posterior update. Any update rule that increases the score upon disagreement injects an independent signal that perturbs the ranking in the correct direction, and is *robust* to misspecification of the D-model. Combined with Lemma 5.1, this implies that silver-based score updates lead to higher labeling accuracy on average across labeling budgets whenever the baseline uncertainty score is not already an optimal detector of its own mistakes.

5.2. Statistical Properties of Post-Hoc Estimators

A key advantage of the proposed CAL framework is that, despite the adaptive labeling process with early stopping, valid post-hoc statistical inference remains possible for evaluation. We establish finite-sample unbiasedness together with asymptotic consistency and normality for the estimators introduced in Section 4.2. Notably, these guarantees hold for the *softmax probabilistic sampling* version of CAL, but not for the greedy version. The probabilistic design ensures that, before stopping, the probability of acquiring a gold label for any instance is bounded away from 0, which corresponds to the overlap condition in missing data problems.

Proposition 5.5 (Adjusted IPW estimator)**.** *Under the adaptive labeling design in Section 3.3, for any finite number of stages* T*, the estimator* $\hat{\mu}_{\texttt{IPW,CAL}}$ *is unbiased and consistent for* μ*. Moreover,* $\frac{\sqrt{n}\left(\hat{\mu}_{\texttt{IPW,CAL}} - \mu\right)}{\sqrt{\hat{\mathbb{V}}_{\texttt{IPW,CAL}}}} \Rightarrow \mathcal{N}(0, 1)$.

This result implies that valid confidence intervals for μ can be constructed directly using the estimated variance $\hat{\mathbb{V}}_{\texttt{IPW,CAL}}$, despite the adaptive and sequential nature of the labeling process.

Assumption 5.6 (Stability of Base Models)**.** Suppose the parameters of the D-model and G-model admit probability limits θ^* and ϕ^*, such that $\hat{\theta} \xrightarrow{p} \theta^*$ and $\hat{\phi} \xrightarrow{p} \phi^*$. In addition, the evaluation dataset size and the historical dataset size satisfy $n/N \to c$ for some constant $c \in (0, \infty)$.

Proposition 5.7 (Adjusted AIPW estimator)**.** *Under the adaptive labeling design in Section 3.3, for any finite number of stages* T*, the estimator* $\hat{\mu}_{\texttt{AIPW,CAL}}$ *is unbiased for* μ*. Moreover, under Assumption 5.6, it is consistent for* μ *and asymptotically normal, i.e.,* $\frac{\sqrt{n}\left(\hat{\mu}_{\texttt{AIPW,CAL}} - \mu\right)}{\sqrt{\hat{\mathbb{V}}_{\texttt{AIPW,CAL}}}} \Rightarrow \mathcal{N}(0, 1)$.

The adjusted AIPW estimator further incorporates predictions from the base models to reduce variance while preserving validity under the adaptive labeling framework. Notably, the proposition does not rely on strong modeling assumptions on these base models, making the estimator robust to potential misspecification. This combination of robustness and efficiency is particularly valuable in settings where gold labels are limited and labeling decisions depend heavily on silver labels. Empirical results in Section 6.2 demonstrate the practical efficiency gains of this estimator in realistic ML evaluation scenarios.

6. Experiments

We first present a synthetic study in Section 6.1 to illustrate the diagnostic role of silver labels and how posterior updates improve the reliability of the uncertainty score. We then evaluate the performance of the proposed CAL framework on two real-world datasets in Section 6.2. Detailed implementation settings and additional experiments are deferred to the Appendix.

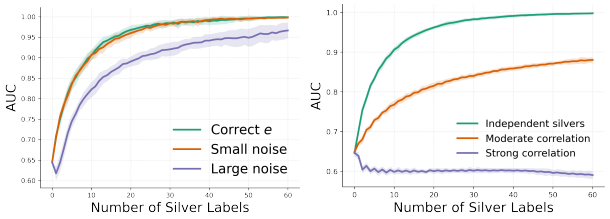
6.1. Synthetic Data

We construct a synthetic experiment to study how silver label updates improve the quality of the uncertainty score produced by an imperfect prediction model under several practical scenarios.

The initial prediction probabilities $p_0(X_i)$, the corresponding uncertainty scores, and the gold labels are randomly generated. Silver labels are generated with heterogeneous, instance-specific error rates. The prediction update rule follows the rule described in Section 3.3.2.

Scenarios. We consider two realistic situations that may weaken the diagnostic role of silver labels. First, we introduce noise to the true silver error rate when performing the posterior update. This mimics the case where the D-model is learned from data and its estimated difficulty variable deviates from the truth. Second, we introduce a latent shared bias term into the silver label generation process. This mimics correlated mistakes across silver labels, for example due to shared misconceptions or information leakage.

Results. Figure 2 illustrates how the alignment between



(a) Noisy error rate *(b)* Correlated silver labels

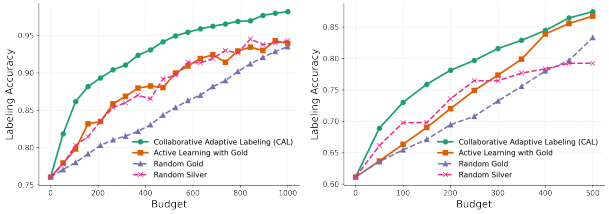
Figure 2. Evolution of the AUC between the uncertainty score and the realized labeling error as the number of silver labels increases under different practical conditions.

the uncertainty score and the realized labeling error evolves as more silver labels are collected. When the error rate used in the update is noisy, the uncertainty score still becomes increasingly predictive of labeling mistakes, demonstrating robustness to misspecification. In contrast, when silver labels are correlated, the alignment improves more slowly and may even deteriorate as additional silver labels are added. This occurs because correlated errors reinforce the same incorrect belief, reducing the effective information gained from each query.

6.2. Real-World Data

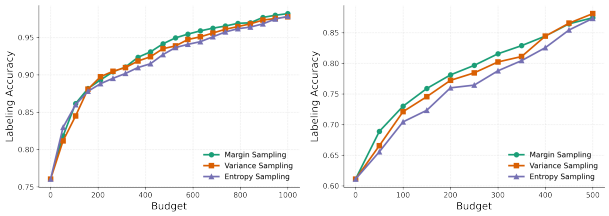
Data and labels. We evaluate CAL on two real-world settings that capture complementary aspects of model evaluation with imperfect labelers. The first is CIFAR-10 (Krizhevsky et al., 2009), a 10-class image classification dataset with 32×32 color images. Although CIFAR-10 is not itself a model evaluation dataset, it provides a controlled analogue of image-model evaluation, where expert-validated labels are available and noisy judgments can be treated as imperfect evaluator outputs. We generate silver labels with class-dependent error rates to mimic heterogeneous annotator quality and class-specific difficulty. The second is the Google Image Caption Quality (GICQ) dataset (Levinboim et al., 2021), which is directly motivated by foundation-model evaluation. In GICQ, AI-generated captions are evaluated by crowd workers with binary quality labels. We sample silver labels without replacement from real crowd annotations to preserve heterogeneity and correlation, and construct gold labels by majority vote with additional noise to reflect imperfect expert annotation. For each run, we subsample 2000 instances, using 30% as historical data and 70% as the evaluation pool. We fix $c_g = 1$ and $c_s = 0.1$.

Base models. For CIFAR-10, we use a convolutional neural network (CNN) trained on the historical dataset as the initial gold prediction model. For GICQ, each instance is an image–caption pair. We obtain pretrained CLIP embeddings for both the image and the caption (Radford et al., 2021), construct interaction features from the embeddings, and train a regularized linear classifier as the base model.



(a) CIFAR-10 *(b)* GICQ

Figure 4. Labeling accuracy versus budget for CAL and baseline methods.



(a) CIFAR-10 *(b)* GICQ

Figure 5. Effect of different uncertainty measures on CAL performance.

Benchmark methods. We compare CAL with three baselines: (i) *Active learning with gold*, which performs uncertainty-based sampling using only gold labels; (ii) *Random gold*, which randomly allocates the gold labeling budget; (iii) *Random silver*, which randomly queries silver labels and applies majority voting.

Uncertainty measures. When comparing CAL with baselines, we use margin score as the default uncertainty measure. We also evaluate entropy and variance scores to examine the robustness of CAL to the choice of uncertainty metric.

Results. Figure 4 shows that CAL consistently outperforms the baselines across budgets, especially when the budget is small. As the budget increases, CAL behaves similarly to active learning with gold on GICQ, since the budget is enough for most instances to be eventually escalated to gold. Figure 5 shows that CAL is robust to the choice of uncertainty measure, with margin sampling performing slightly better in both datasets. Figure 3 demonstrates that the proposed IPW and AIPW estimators are unbiased and consistent for estimating the population average, while the AIPW estimator achieves substantially smaller variance.

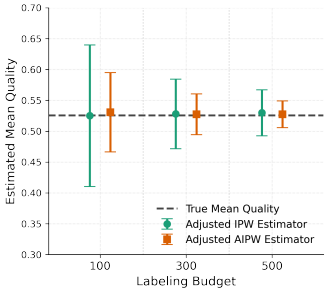


Figure 3. CI performance of IPW and AIPW.

Impact Statement

This paper presents work whose goal is to advance the field of Machine Learning. There are many potential societal consequences of our work, none which we feel must be specifically highlighted here.

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A. Extensions

A.1. Channel-Aware Silver Labeling Extension

The CAL framework naturally extends to the setting where silver labels originate from multiple heterogeneous channels. Suppose there are K silver channels. For each channel k, let $Z_i^{(k)} \in \{0,1\}$ denote a silver label, with channel-specific error model

$$e_k(X;\phi_k) = \mathbb{P}\Big(Z^{(k)} \neq Y \mid X\Big), \qquad e_k(X) \leq \bar{e} < \tfrac{1}{2},$$

and cost $c_s^{(k)}$. If instance i has received $m_{i,t}^{(k)}$ silver labels from channel k, among which $k_{i,t}^{(k)}$ are 1's, the silver-guided prediction update in Section 3.3.2 generalizes to

$$p_t(X_i) \propto p(X_i;\hat{\theta}) \prod_{k=1}^{K} (1 - e_k(X_i;\hat{\phi}_k))^{k_{i,t}^{(k)}} e_k(X_i;\hat{\phi}_k)^{m_{i,t}^{(k)} - k_{i,t}^{(k)}}.$$

Each channel contributes a weight $\log \frac{1-e_k(X_i)}{e_k(X_i)}$ in the log-odds update. The one-step-ahead uncertainty for querying a silver label from channel k is

$$\hat{s}_{t+1}^{(k)}(X_i) = \sum_{z\in\{0,1\}} \hat{\mathbb{P}}\Big(Z_{i,t+1}^{(k)} = z \mid X_i, \mathbf{Z}_{i,t}\Big)\, f\Big(p_{t+1}^{(k,z)}(X_i)\Big),$$

where

$$\hat{\mathbb{P}}\Big(Z_{i,t+1}^{(k)} = 1 \mid X_i, \mathbf{Z}_{i,t}\Big) = e_k(X_i) + (1 - 2e_k(X_i))\, p_t(X_i).$$

The sampling score becomes

$$u_{i,t,\textsc{Silver}(k)} = \frac{s_t(X_i) - \hat{s}_{t+1}^{(k)}(X_i)}{c_s^{(k)}}.$$

The labeling action space extends to

$$\{\textsc{Gold},\ \textsc{Silver}(1), \ldots, \textsc{Silver}(K), \textsc{None}\},$$

and the softmax preference in Section 3.3 is applied over these actions. This extension enables CAL to automatically determine *which silver channel to query* based on its reliability and cost, without any modification to the core algorithm or theory.

A.2. Multiclass Extension

The CAL framework also extends naturally to multiclass labeling problems where $Y \in \{1, \ldots, C\}$. The G-model now outputs a probability vector

$$p(X;\theta) = \big(p^{(1)}(X), \ldots, p^{(C)}(X)\big), \qquad p^{(c)}(X) = \mathbb{P}(Y = c \mid X).$$

For silver labels, we model channel-specific confusion through

$$e_{c\to c'}^{(k)}(X) = \mathbb{P}\Big(Z^{(k)} = c' \mid Y = c, X\Big),$$

which generalizes the binary error rate to a multiclass transition matrix. We assume the diagonal dominates, i.e., $e_{c\to c}^{(k)}(X) > e_{c\to c'}^{(k)}(X)$ for $c' \neq c$.

If instance i has received silver labels from various channels, the posterior update becomes

$$p_t^{(c)}(X_i) \propto p^{(c)}(X_i;\hat{\theta}) \prod_{k} \prod_{j=1}^{m_{i,t}^{(k)}} e_{c\to Z_{i,j}^{(k)}}^{(k)}(X_i;\hat{\phi}_k).$$

Uncertainty is computed from the updated distribution, for example using entropy

$$s_t(X_i) = -\sum_{c=1}^{C} p_t^{(c)}(X_i) \log p_t^{(c)}(X_i).$$

The one-stage-ahead uncertainty and sampling score for each action (Gold or Silver from any channel) are defined in the same way as in the binary case, replacing scalar probabilities by the multiclass posterior.

This extension preserves the structure of CAL while allowing silver labels to provide class-dependent diagnostic information through the estimated confusion matrices.

A.3. Extension to General Empirical Risk Minimization

In the main text, we use the population mean $\mu = \mathbb{E}[Y]$ as a motivating example. The same construction applies directly to general empirical risk objectives.

Let $\ell(Y, X)$ be a bounded loss function and define the target risk

$$R := \mathbb{E}_{X \sim P_X^*}\big[\ell(Y, X)\big].$$

Using the adjusted weight D_i defined in Section 4.2, the adjusted IPW estimator becomes

$$\hat{R}_{\texttt{IPW,CAL}} = \frac{1}{n} \sum_{i=N+1}^{N+n} D_i \, \ell(Y_i, X_i).$$

The unbiasedness, consistency, and asymptotic normality follow from the same arguments as for $\hat{\mu}_{\texttt{IPW,CAL}}$, since the outcome Y is replaced by the loss $\ell(Y, X)$.

For the augmented version, let

$$m_{t-1}(X) = \hat{\mathbb{E}}[\ell(Y, X) \mid X]$$

denote the loss prediction from the silver-guided G-model at stage $t-1$. Define

$$\psi_i^{(\ell)} = D_i \, \ell(Y_i, X_i) + \frac{1}{T} \sum_{t \le \min\{\tau_i, T\}} \left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right) m_{t-1}(X_i).$$

The adjusted AIPW estimator and its variance estimator are

$$\hat{R}_{\texttt{AIPW,CAL}} = \frac{1}{n} \sum_{i=N+1}^{N+n} \psi_i^{(\ell)}, \qquad \hat{\mathbb{V}}_{\texttt{AIPW,CAL}} = \frac{1}{n} \sum_{i=N+1}^{N+n} \big(\psi_i^{(\ell)} - \bar{\psi}^{(\ell)}\big)^2.$$

All theoretical guarantees established for $\hat{\mu}_{\texttt{AIPW,CAL}}$ extend directly to this setting by replacing Y with $\ell(Y, X)$ in the proofs.

B. Collaborative Adaptive Labeling Pseudocode

Algorithm 1.

C. Proof

C.1. Proof of Lemma 5.1

Proof. For any score U, define the quantile threshold

$$t_b(U) := \inf\{t : \ \mathbb{P}(U \ge t) \le b\}.$$

Then $S_b(U) = \{U \ge t_b(U)\}$ and

$$G_b(U) = \mathbb{P}(W = 1 \mid U \ge t_b(U)) = \frac{\mathbb{P}(U \ge t_b(U) \mid W = 1)\, \mathbb{P}(W = 1)}{\mathbb{P}(U \ge t_b(U))}.$$

Since $\mathbb{P}(U \ge t_b(U)) = b$, we obtain

$$G_b(U) = \frac{\mathbb{P}(W = 1)}{b}\, \mathbb{P}(U \ge t_b(U) \mid W = 1).$$

Algorithm 1 Collaborative Adaptive Labeling (CAL)

Input: historical data $\mathcal{D}$, unlabeled pool $\mathcal{D}^*$, budgets $\{B_t\}_{t=1}^T$, costs c_g, c_s, temperature τ
Train G-model $p(X;\hat{\theta}_0)$ and D-model $e(X;\hat{\phi}_0)$ on $\mathcal{D}$
for $t = 1$ **to** T **do**
  for each instance $i \in \mathcal{D}^*$ **do**
    Compute silver-guided prediction $p_t(X_i)$
    Compute uncertainty score $s_t(X_i)$
    Compute one-step-ahead uncertainty $\hat{s}_{t+1}(X_i)$
    $u_{i,t,G} \leftarrow s_t(X_i)/c_g$
    $u_{i,t,S} \leftarrow \big(s_t(X_i) - \hat{s}_{t+1}(X_i)\big)/c_s$
    $q_{i,t,G} \leftarrow \dfrac{\exp(u_{i,t,G}/\tau)}{\exp(u_{i,t,G}/\tau) + \exp(u_{i,t,S}/\tau)}$
    $\tilde{u}_{i,t} \leftarrow q_{i,t,G}u_{i,t,G} + (1 - q_{i,t,G})u_{i,t,S}$
    $\tilde{c}_{i,t} \leftarrow q_{i,t,G}c_g + (1 - q_{i,t,G})c_s$
  end for
  Compute weights $\tilde{w}_{i,t} = \frac{\exp(\tilde{u}_{i,t}/\tau)}{\sum_j \exp(\tilde{u}_{j,t}/\tau)}$
  $\alpha_t \leftarrow \dfrac{B_t}{\sum_i \tilde{w}_{i,t}\tilde{c}_{i,t}}$
  for each instance $i \in \mathcal{D}^*$ **do**
    Sample i with probability $\alpha_t\tilde{w}_{i,t}$
    if i is selected **then**
      Sample action: GOLD with prob. $q_{i,t,G}$, SILVER otherwise
      if GOLD **then**
        Obtain gold label Y_i
        Remove i from $\mathcal{D}^*$
      else
        Obtain silver label $Z_{i,t}$
        Update silver counts for i
      end if
    end if
  end for
  Update G-model and D-model using all collected data: $\hat{\theta}_{t-1} \to \hat{\theta}_t$, $\hat{\phi}_{t-1} \to \hat{\phi}_t$.
end for
Output: collected labels and sampling probabilities

Hence

$$\int_0^1 G_b(U)\,db = \mathbb{P}(W=1)\int_0^1 \frac{\mathbb{P}(U \geq t_b(U) \mid W=1)}{b}\,db.$$

Let $F_{U|W=1}$ denote the CDF of U conditional on $W = 1$, and write $Q_U(b)$ for the $(1-b)$-quantile of U, so that $t_b(U) = Q_U(b)$. A change of variables yields

$$\int_0^1 G_b(U)\,db = \mathbb{P}(W=1)\int_{\mathbb{R}} \mathbb{P}(U \geq u \mid W=1)\,dF_U(u).$$

An analogous identity holds for any score S.

Observe that

$$\mathrm{AUC}(U,W) = \mathbb{P}(U^+ > U^-) + \frac{1}{2}\mathbb{P}(U^+ = U^-) = \int_{\mathbb{R}} \mathbb{P}(U \geq u \mid W=1)\,dF_{U|W=0}(u).$$

Thus, a larger AUC means that, in an average sense over thresholds u drawn from the distribution of $U \mid W = 0$, the tail probability $\mathbb{P}(U \geq u \mid W = 1)$ is larger.

Since F_U is a mixture of $F_{U|W=1}$ and $F_{U|W=0}$, a larger AUC implies that the integral

$$\int_{\mathbb{R}} \mathbb{P}(U \geq u \mid W = 1)\, dF_U(u)$$

is larger. Therefore,

$$\int_0^1 G_b(S)\, db > \int_0^1 G_b(U)\, db.$$

□

C.2. Proof of Proposition 5.4

Proof. Recall that

$$\mathrm{AUC}(Z, W) = \mathbb{P}(Z^+ > Z^-) + \tfrac{1}{2}\mathbb{P}(Z^+ = Z^-),$$

where $Z^+ \sim Z \mid (W = 1)$ and $Z^- \sim Z \mid (W = 0)$ are independent. Let (U^+, M^+) and (U^-, M^-) be independent draws from the conditional distributions given $W = 1$ and $W = 0$, respectively. Define

$$D := U^+ - U^-, \qquad \Gamma := \Delta^+ - \Delta^-,$$

so that

$$T^{\mathrm{sil},+} - T^{\mathrm{sil},-} = D + \Gamma.$$

Under Assumptions 5.2–5.3, conditioning on (Y, X) gives

$$\mathbb{P}(M = 0 \mid W = 1) = 1 - e(X), \qquad \mathbb{P}(M = 0 \mid W = 0) = e(X),$$

and since $e(X) \leq \bar{e} < 1/2$, we have

$$\mathbb{P}(M = 0 \mid W = 1) > \mathbb{P}(M = 0 \mid W = 0).$$

Because disagreement yields a positive increment δ^+ while agreement yields a negative decrement $-\delta^-$ with $\delta^+ + \delta^- > 0$, it follows that

$$\mathbb{E}[\Gamma] > 0.$$

Moreover, Γ takes both positive and negative values with positive probability. Hence, for any fixed value d,

$$\mathbb{P}(d + \Gamma > 0) \;\geq\; \mathbf{1}\{d > 0\},$$

with strict inequality whenever $d \leq 0$. Integrating over the distribution of D yields

$$\mathbb{P}(D + \Gamma > 0) > \mathbb{P}(D > 0)$$

whenever $\mathbb{P}(D \leq 0) > 0$, i.e., whenever $\mathrm{AUC}(U, W) < 1$.

The same argument applies to equality events, which gives

$$\mathrm{AUC}(T^{\mathrm{sil}}, W) > \mathrm{AUC}(U, W),$$

unless $\mathrm{AUC}(U, W) = 1$. □

C.3. Proof of Proposition 5.5

C.3.1. UNBIASEDNESS

Proof. We first note that the stopping time τ_i is measurable with respect to the filtration $\{\mathcal{F}_t\}$ since

$$\{\tau_i = t\} = \{\delta_{i,1} = 0, \ldots, \delta_{i,t-1} = 0, \delta_{i,t} = 1\}$$

is determined entirely by the adaptive sampling rule based on past information.

Next, observe the identity

$$(T - \tau_i)_+ = T - \min(\tau_i, T),$$

so that

$$\begin{aligned} D_i &= \frac{1}{T}\left(\sum_{t \le \min(\tau_i, T)} \frac{\delta_{i,t}}{\pi_{i,t}} + (T - \tau_i)_+\right) \\ &= 1 + \frac{1}{T}\left(\sum_{t \le \min(\tau_i, T)} \frac{\delta_{i,t}}{\pi_{i,t}} - \min(\tau_i, T)\right). \end{aligned}$$

We now compute the expectation of the summation term. Using iterated expectation and the fact that

$$\mathbb{E}[\delta_{i,t} \mid \mathcal{F}_{t-1}] = \pi_{i,t},$$

we have

$$\begin{aligned} \mathbb{E}\left[\sum_{t \le \min(\tau_i, T)} \frac{\delta_{i,t}}{\pi_{i,t}}\right] &= \sum_{t=1}^{T} \mathbb{E}\left[\mathbf{1}\{t \le \tau_i\} \frac{\delta_{i,t}}{\pi_{i,t}}\right] \\ &= \sum_{t=1}^{T} \mathbb{E}\left[\mathbf{1}\{t \le \tau_i\} \frac{\mathbb{E}[\delta_{i,t} \mid \mathcal{F}_{t-1}]}{\pi_{i,t}}\right] \\ &= \sum_{t=1}^{T} \mathbb{E}[\mathbf{1}\{t \le \tau_i\}] = \mathbb{E}[\min(\tau_i, T)]. \end{aligned}$$

Hence,

$$\mathbb{E}[D_i] = 1.$$

We now prove unbiasedness using the IPW randomization identity. For any stage t,

$$\mathbb{E}\left[\frac{\delta_{i,t}}{\pi_{i,t}} \;\middle|\; \mathcal{F}_{t-1},\, Y_i\right] = 1,$$

since $\pi_{i,t} = \mathbb{P}(\delta_{i,t} = 1 \mid \mathcal{F}_{t-1})$ and the sampling decision does not depend on the unobserved gold label Y_i.

Therefore,

$$\begin{aligned} \mathbb{E}[D_i Y_i] &= \frac{1}{T}\mathbb{E}\left[Y_i \left(\sum_{t \le \min(\tau_i, T)} \frac{\delta_{i,t}}{\pi_{i,t}} + (T - \tau_i)_+\right)\right] \\ &= \frac{1}{T}\mathbb{E}[Y_i \left(\min(\tau_i, T) + (T - \tau_i)_+\right)] \\ &= \mathbb{E}[Y_i] = \mu. \end{aligned}$$

Averaging over i yields

$$\mathbb{E}[\hat{\mu}_{\texttt{IPW,CAL}}] = \mu.$$

□

C.3.2. CONSISTENCY

Proof. From the martingale decomposition, write

$$M_{i,t} := \delta_{i,t} - \pi_{i,t}, \qquad \mathbb{E}[M_{i,t} \mid \mathcal{F}_{t-1}] = 0.$$

Using the identity

$$\sum_{t\le\min(\tau_i,T)}\frac{\delta_{i,t}}{\pi_{i,t}}=\min(\tau_i,T)+\sum_{t\le\min(\tau_i,T)}\frac{M_{i,t}}{\pi_{i,t}},$$

and the fact that $\min(\tau_i,T)+(T-\tau_i)_+=T$, we obtain

$$D_i=1+\frac{1}{T}\sum_{t\le\min(\tau_i,T)}\frac{M_{i,t}}{\pi_{i,t}}.$$

Hence

$$D_iY_i-\mu=\frac{Y_i}{T}\sum_{t\le\min(\tau_i,T)}\frac{M_{i,t}}{\pi_{i,t}}.$$

Since instances are independent across i, we have

$$\mathrm{Var}(\hat{\mu}_{\texttt{IPW,CAL}})=\frac{1}{n}\,\mathrm{Var}(D_iY_i)\,.$$

We now bound $\mathrm{Var}(D_iY_i)$. Using iterated expectation and the martingale property,

$$\begin{aligned}\mathrm{Var}(D_iY_i)&=\mathbb{E}\left[\left(\frac{Y_i}{T}\sum_{t\le\min(\tau_i,T)}\frac{M_{i,t}}{\pi_{i,t}}\right)^2\right]\\&\le\frac{Y_i^2}{T^2}\mathbb{E}\left[\sum_{t=1}^{T}\frac{M_{i,t}^2}{\pi_{i,t}^2}\mathbf{1}\{\tau_i\ge t\}\right],\end{aligned}$$

where cross terms vanish because $\{M_{i,t}\}$ is a martingale difference sequence.

Since $\delta_{i,t}\in\{0,1\}$,

$$\mathbb{E}[M_{i,t}^2\mid\mathcal{F}_{t-1}]=\mathrm{Var}(\delta_{i,t}\mid\mathcal{F}_{t-1})\le\pi_{i,t},$$

hence

$$\mathbb{E}\left[\frac{M_{i,t}^2}{\pi_{i,t}^2}\right]\le\mathbb{E}\left[\frac{1}{\pi_{i,t}}\right].$$

Under the positivity condition $\pi_{i,t}\ge\underline{\pi}>0$, we obtain

$$\mathrm{Var}(D_iY_i)\le C$$

for some finite constant C independent of n.

Therefore,

$$\mathrm{Var}(\hat{\mu}_{\texttt{IPW,CAL}})\le\frac{C}{n}.$$

Applying Chebyshev's inequality, for any $\varepsilon>0$,

$$\mathbb{P}\big(\big|\hat{\mu}_{\texttt{IPW,CAL}}-\mu\big|>\varepsilon\big)\le\frac{C}{n\varepsilon^2}\to0.$$

Hence,

$$\hat{\mu}_{\texttt{IPW,CAL}}\xrightarrow{P}\mu.$$

□

C.3.3. ASYMPTOTIC NORMALITY

Proof. Recall the decomposition

$$D_i = 1 + \frac{1}{T} \sum_{t \le \min(\tau_i, T)} \frac{M_{i,t}}{\pi_{i,t}}, \qquad M_{i,t} = \delta_{i,t} - \pi_{i,t},$$

with $\mathbb{E}[M_{i,t} \mid \mathcal{F}_{t-1}] = 0$. Hence

$$D_i Y_i - \mu = \frac{Y_i}{T} \sum_{t \le \min(\tau_i, T)} \frac{M_{i,t}}{\pi_{i,t}}.$$

For each fixed i, the sequence

$$\left\{ Y_i \frac{M_{i,t}}{\pi_{i,t}} \mathbf{1}\{\tau_i \ge t\} \right\}_{t=1}^{T}$$

forms a martingale difference sequence with respect to $\{\mathcal{F}_t\}$. Therefore $D_i Y_i$ can be written as a finite sum of martingale increments.

Since instances are independent across i, the random variables

$$Z_i := D_i Y_i$$

are i.i.d. with $\mathbb{E}[Z_i] = \mu$ and finite variance (shown in the consistency proof). Hence, by the CLT,

$$\sqrt{n}\big(\hat{\mu}_{\texttt{IPW,CAL}} - \mu\big) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (Z_i - \mu) \Rightarrow \mathcal{N}(0, \sigma^2),$$

where $\sigma^2 = \mathrm{Var}(Z_i)$.

It remains to show consistency of the variance estimator

$$\hat{\mathbb{V}}_{\texttt{IPW,CAL}} = \frac{1}{n} \sum_{i=1}^{n} \left(Z_i - \hat{\mu}_{\texttt{IPW,CAL}}\right)^2.$$

Since $\{Z_i\}$ are i.i.d. with finite second moment, the law of large numbers implies

$$\hat{\mathbb{V}}_{\texttt{IPW,CAL}} \xrightarrow{P} \sigma^2.$$

Finally, Slutsky's theorem yields

$$\frac{\sqrt{n}\big(\hat{\mu}_{\texttt{IPW,CAL}} - \mu\big)}{\sqrt{\hat{\mathbb{V}}_{\texttt{IPW,CAL}}}} \Rightarrow \mathcal{N}(0, 1).$$

□

C.4. Proof of Proposition 5.7

C.4.1. UNBIASEDNESS

Proof. We first prove unbiasedness. Recall

$$\psi_i = D_i Y_i + \frac{1}{T} \sum_{t \le \min\{\tau_i, T\}} \left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right) p_{t-1}(X_i).$$

From the proof of Proposition 5.5, we already have

$$\mathbb{E}[D_i Y_i] = \mu.$$

Hence it suffices to show that the augmentation term has mean zero. For each $t \le T$, since $\pi_{i,t} = \mathbb{P}(\delta_{i,t} = 1 \mid \mathcal{F}_{t-1})$ and $p_{t-1}(X_i)$ is $\mathcal{F}_{t-1}$-measurable,

$$\begin{aligned}\mathbb{E}\left[\left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right) p_{t-1}(X_i) \;\middle|\; \mathcal{F}_{t-1},\, \tau_i \ge t\right] &= p_{t-1}(X_i)\left(1 - \frac{\mathbb{E}[\delta_{i,t} \mid \mathcal{F}_{t-1}]}{\pi_{i,t}}\right) \\ &= 0.\end{aligned}$$

Taking expectation over the stopping time τ_i yields

$$\mathbb{E}\left[\frac{1}{T}\sum_{t \le \min\{\tau_i, T\}} \left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right) p_{t-1}(X_i)\right] = 0.$$

Therefore,

$$\mathbb{E}[\psi_i] = \mu, \qquad \mathbb{E}[\hat{\mu}_{\texttt{AIPW,CAL}}] = \mu.$$

□

C.4.2. Consistency and Asymptotic Normality

Proof. We prove consistency and asymptotic normality by comparing ψ_i with an idealized version that uses the probability limits of the prediction models.

Step 1: Decompose the estimator. Write

$$\psi_i = D_i Y_i + \frac{1}{T}\sum_{t \le \min\{\tau_i, T\}} \left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right) p_{t-1}(X_i),$$

and define the deterministic limit version

$$\psi_i^* = D_i Y_i + \frac{1}{T}\sum_{t \le \min\{\tau_i, T\}} \left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right) p^*(X_i),$$

where $p^*(X)$ is the probability limit implied by Assumption 5.6.

Then

$$\hat{\mu}_{\texttt{AIPW,CAL}} - \frac{1}{n}\sum_{i=1}^n \psi_i^* = \frac{1}{nT}\sum_{i=1}^n \sum_{t \le \min\{\tau_i, T\}} \left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right)\Big(p_{t-1}(X_i) - p^*(X_i)\Big).$$

Denote this difference by R_n.

Step 2: Show $R_n = o_p(n^{-1/2})$. For each fixed i and t, the term

$$\left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right)\Big(p_{t-1}(X_i) - p^*(X_i)\Big)\mathbf{1}\{\tau_i \ge t\}$$

is a martingale difference with respect to $\{\mathcal{F}_t\}$, since $p_{t-1}(X_i)$ is $\mathcal{F}_{t-1}$-measurable and $\mathbb{E}[\delta_{i,t} \mid \mathcal{F}_{t-1}] = \pi_{i,t}$.

Using independence across i and the martingale property (cross terms vanish),

$$\begin{aligned}\mathrm{Var}(R_n) &= \mathrm{Var}\left(\frac{1}{nT}\sum_{i=1}^n \sum_{t \le \min\{\tau_i, T\}} \left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right)\big(p_{t-1}(X_i) - p^*(X_i)\big)\right) \\ &\le \frac{1}{n^2T^2}\sum_{i=1}^n \mathrm{Var}\left(\sum_{t \le \min\{\tau_i, T\}} \left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right)\big(p_{t-1}(X_i) - p^*(X_i)\big)\right) \\ &= \frac{1}{n^2T^2}\sum_{i=1}^n \sum_{t=1}^T \mathbb{E}\left[\left(1 - \frac{\delta_{i,t}}{\pi_{i,t}}\right)^2 \big(p_{t-1}(X_i) - p^*(X_i)\big)^2 \mathbf{1}\{\tau_i \ge t\}\right] \\ &\le \frac{C}{n}\,\mathbb{E}\Big[\big(p_{t-1}(X) - p^*(X)\big)^2\Big],\end{aligned}$$

for some constant C depending only on T and the positivity bound on $\pi_{i,t}$.

By Assumption 5.6, $p_{t-1}(X) \xrightarrow{p} p^*(X)$ and the second moment is bounded, hence

$$\mathbb{E}\Big[\big(p_{t-1}(X) - p^*(X)\big)^2\Big] \to 0.$$

Therefore,

$$\mathrm{Var}(R_n) = o\left(\frac{1}{n}\right), \qquad R_n = o_p(n^{-1/2}).$$

Step 3: Analyze the idealized estimator. For ψ_i^*, the prediction term is now deterministic given X_i. As in the proof of Proposition 5.5, ψ_i^* is a finite sum of martingale differences and therefore has finite variance. Moreover,

$$\mathbb{E}[\psi_i^*] = \mu.$$

Since instances are independent across i, the CLT gives

$$\sqrt{n}\left(\frac{1}{n}\sum_{i=1}^{n}\psi_i^* - \mu\right) \Rightarrow \mathcal{N}(0, \sigma^2), \quad \sigma^2 = \mathrm{Var}(\psi_i^*).$$

Step 4: Transfer the result back to $\hat{\mu}_{\texttt{AIPW,CAL}}$. Because $R_n = o_p(n^{-1/2})$, Slutsky's theorem yields

$$\sqrt{n}\big(\hat{\mu}_{\texttt{AIPW,CAL}} - \mu\big) \Rightarrow \mathcal{N}(0, \sigma^2).$$

Consistency follows immediately from the law of large numbers applied to ψ_i^* and the fact that $R_n \to 0$ in probability.

Finally, the sample variance $\hat{\mathbb{V}}_{\texttt{AIPW,CAL}}$ is consistent for σ^2 by the law of large numbers, and another application of Slutsky's theorem gives

$$\frac{\sqrt{n}\big(\hat{\mu}_{\texttt{AIPW,CAL}} - \mu\big)}{\sqrt{\hat{\mathbb{V}}_{\texttt{AIPW,CAL}}}} \Rightarrow \mathcal{N}(0, 1).$$

□

D. Synthetic Data Details and Additional Results

This section provides details of the synthetic experiments used to illustrate the diagnostic role of silver labels in Section 5.1. All experiments measure the alignment between the uncertainty score $s(p) = -p\log p - (1-p)\log(1-p)$ and the true inaccuracy indicator $I = \mathbf{1}\{\hat{Y} \neq Y\}$ through

$$\mathrm{AUC}\big(s(p), I\big).$$

At each step, we simulate the effect of repeatedly querying silver labels and updating the posterior prediction using the update rule in Section 3.3.2.

Data generation

We generate n instances by

$$U \sim \mathcal{N}(0,1), \qquad p_0 = \frac{1}{1+e^{-U}}, \qquad Y \sim \mathrm{Bernoulli}(p_0).$$

The initial model prediction is p_0, and uncertainty is measured by entropy $s(p_0)$.

Given a silver label S, the posterior is updated by

$$p \leftarrow \frac{p\big((1-e)\mathbf{1}\{S=1\} + e\mathbf{1}\{S=0\}\big)}{p\big((1-e)\mathbf{1}\{S=1\} + e\mathbf{1}\{S=0\}\big) + (1-p)\big((1-e)\mathbf{1}\{S=0\} + e\mathbf{1}\{S=1\}\big)}.$$

This process is repeated for $k = 0, 1, \ldots, K$ silver labels, and the AUC is recorded at each step.

Experiment 1: Heterogeneous error rates

We draw instance-wise error rates

$$e(X) \sim \text{Uniform}(a, b),$$

with increasing heterogeneity: low $(0.1, 0.6)$, mid $(0.1, 0.7)$, high $(0.1, 0.8)$. Figure 6 shows that larger heterogeneity in $e(X)$ leads to slower improvement in the alignment between uncertainty and true inaccuracy, but the improvement is still robust.

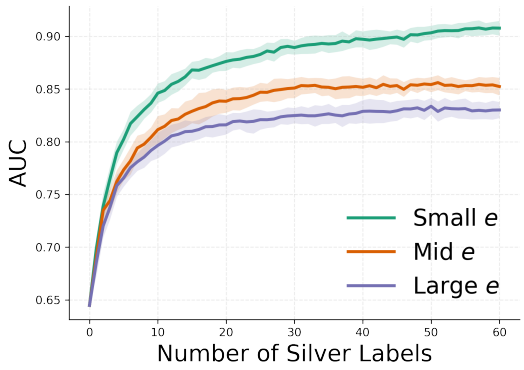

Figure 6. Effect of heterogeneous silver error rates on AUC(uncertainty, inaccuracy).

Experiment 2: Misspecified error rate

We examine robustness when the error rate used in the posterior update is misspecified. Specifically, the error rate used for updating is

$$e_{\text{used}} = \text{clip}\big(e + \sigma\epsilon,\, 0.01, 0.49\big), \qquad \epsilon \sim \mathcal{N}(0, 1),$$

where σ controls the noise level. We set $\sigma = 0.05$ to represent mild misspecification and $\sigma = 0.35$ to represent severe misspecification.

Experiment 3: Correlated silver labels

To simulate dependence among repeated silver labels, we introduce a shared latent bias term and generate silver labels according to

$$\mathbb{P}(S = Y) = (1 - e)(1 - \rho) + \rho\,\sigma(b),$$

where $\sigma(\cdot)$ is the sigmoid function and b is a randomly generated instance-specific latent variable. The parameter $\rho \in [0, 1]$ controls the strength of correlation across silver labels for the same instance.