

000 001 002 003 004 005 IMBALANCE: INFERENCE-TIME LATENT SEARCH 006 AGAINST DEGREE IMBALANCE IN LINK PREDICTION 007 008 009

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Paper under double-blind review

ABSTRACT

Knowledge Graph Embedding models have been extensively used to learn representations of entities and relations in Knowledge Graphs for predicting missing links. However, the quality of the learned representations varies a lot across different areas of the same Knowledge Graph. If previous research efforts have loosely linked the problem to relation types or degree bias, we show that it is much more widespread, and it more precisely lies in the degree imbalance of the entities in test triples. In particular, we show that the prediction of a target entity that has a degree much smaller than the degree of the anchor entity is extremely problematic. This is critical in use cases like *drug target discovery*, where these triples are predominant, or *recommender systems*, where they represent important corner cases. To address this issue, we propose an inference-time latent search optimization method capable of significantly improving model predictions on the most imbalanced triples. Built on top of a pre-trained model, it explores the embedding space at evaluation time, blending known and out-of-band information to mitigate the degree imbalance bias. We show the value of our approach on imbalanced triples from common benchmark datasets, where we outperform conventional methods, opening the door to the successful adoption of Knowledge Graph Embedding models on these critical corner cases.

1 INTRODUCTION

Knowledge Graphs (KGs) are flexible and scalable data structures that model factual knowledge by linking concepts through relationships (Hogan et al., 2021). The possibility of representing and integrating virtually any type of knowledge has made them scale up to include millions or billions of facts. However, such a size is incompatible with manual curation and leaves the door open for incompleteness (Dong et al., 2014). In this scenario, the design of a machine-based approach to infer missing links has attracted a lot of attention from the scientific community. This task is known as *link prediction*, where the goal is to predict whether a relation p exists between two entities s and o in the graph. In this context, a *query* takes the form of an incomplete triple, such as $(s, p, ?)$ or $(?, p, o)$ and the model must infer the missing entity. The entity that is already given in the query (e.g., the subject in $(s, p, ?)$) is referred to as the *anchor node*, since predictions are conditioned on it. The most successful attempts to solve this task have been carried out through Knowledge Graph Embedding (KGE) models, a family of scalable methods that learn low-dimensional representations for entities and relations.

Despite their success, KGEs suffer from limitations, as the quality of the learned representation significantly varies across the graph, strongly limiting the model performance under certain conditions. Previous work has attributed such behavior to relation types (Bordes et al., 2013; Wang et al., 2014; Lin et al., 2015; Ji et al., 2015; He et al., 2015) and degree bias (Mohamed et al., 2020; Shomer et al., 2023). In this work, we provide a more precise analysis, identifying the root cause of the problem in the *degree imbalance* of the head and tail entities in test triples. We show how the bigger the degree difference between the two, the poorer the prediction of the lower degree entity and the better the prediction of the higher degree entity. We further tie the problem to a two-fold learning issue that affects low-degree entities, highlighting how strong overfitting and failed convergence lie behind unsuccessful predictions.

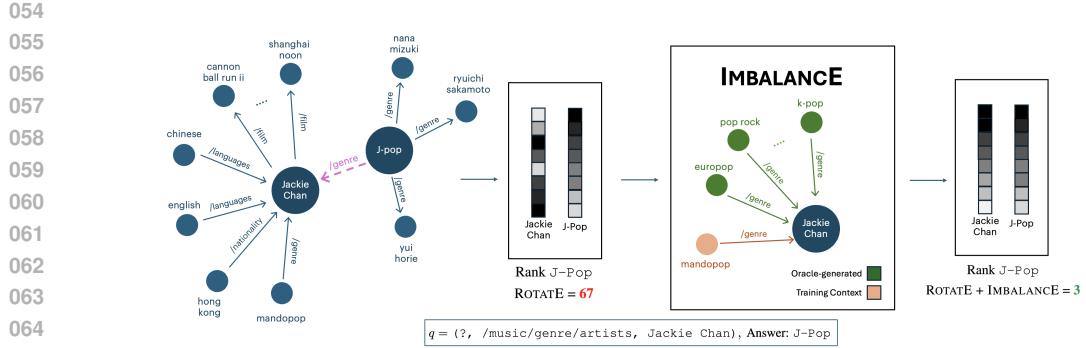


Figure 1: We consider a query q with high degree imbalance. In this example taken from FB15k-237, Jackie Chan has degree 49 and J-Pop has degree 8 (limited in the figure for clarity of visualization). IMBALANCE takes the pre-trained embeddings of the KGE model for the anchor entity and performs an inference-time latent search optimization on a subset of the original neighborhood enhanced with the triples generated by the oracle. It then outputs a representation for the anchor node that better aligns with the selected query, improving the ranking of the target answer from 67 down to 3.

For example, in drug discovery a key task is identifying proteins associated with a disease of interest. When framed as a link prediction task, where KG nodes are biological entities (diseases, proteins, genes...), top-scored proteins returned as solutions to $q = (?protein, associatedWith, disease)$ represent promising associations. These target triples are often imbalanced, as we typically have much more information (i.e., a much higher degree) for conditions as opposed to proteins (biologists have only identified a relatively small fraction of proteins with sufficient detail). Degree imbalance compromises our ability to successfully predict meaningful solutions, undermining the reliability of link prediction in this critical application.

Another example is *recommender systems*, where link prediction has been deployed frequently (Zhang et al., 2016). On a music streaming platform, we want to complement the information about artists as much as possible. In this way, the recommendations to accommodate user tastes are going to be more accurate. Consider (J-pop, /music/genre/artists, Jackie Chan), a test triple from FB15k-237 (Toutanova & Chen, 2015). The degree of J-pop is low, while Jackie Chan has a much higher number of connections. We want to answer the query $q = (?/music/genre/artists, Jackie Chan)$, so to tag Jackie Chan with an additional, related genre – in this case, J-pop – to broaden up its audience. However, when using embeddings from a pre-trained KGE model, the predicted ranking for J-pop is poor. This is because the large degree difference between Jackie Chan and J-pop causes the KGE model to place J-pop far down in the ranking (see Figure 1).

The problem affects a broad range of triples and various Graph Machine Learning (GraphML) models: from shallow architectures like TransE, DistMult, ComplEx and RotatE, to GNN-like ones, including the state-of-the-art NBFNet.

To address this issue, we propose IMBALANCE, an inference-time latent search method. Given the pre-trained embeddings of a KGE model and an imbalanced test triple $t = (s, p, o)$, in order to improve the prediction of the low-degree entity, IMBALANCE fine-tunes the embeddings of the anchor node and of few selected entities optimizing a dual-term objective function. These two terms extend the generalization of the embedding of the high-degree node and improve the quality of the embedding of the low-degree nodes. The former goal is achieved by an exploration of the embedding space guided by a selection of training facts: this second pass ring-fences the representation of the high-degree node in a region of the space suitable for the prediction. The latter goal, on the other hand, is achieved through an oracle that extends the little knowledge available in the KG for long-tail entities. Refer to Figure 1 for an example and an overview of the method.

We validate the value of our approach on the most imbalanced triples of common benchmark datasets. We show how IMBALANCE leads to significant improvements across multiple learning-to-rank metrics, enabling the prediction of low degree entities in critical applications, where these corner cases are predominant.

108 In summary, our work makes the following contributions:
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110 1 We analyze the degree imbalance affecting a wide variety of KGE models, and we further
 111 tie it to a two-fold learning issue that compromises the embedding quality of low-degree
 112 entities.
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114 2 We propose IMBALANCE, an inference-time latent search method that mitigates the above
 115 problem. We assess its impact on heavily imbalanced triples in popular *link prediction*
 116 benchmarks across multiple KGE models and show that it works as an effective plug-in
 117 to enhance KGE predictions. Moreover, we show how it can provide a lightweight and
 118 efficient alternative to much more involved and computationally expensive models.
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120 Our paper is structured as follows: we present the related work in Section 2, then we move on
 121 to introduce the degree imbalance problem and its implications for KGE models (Section 3). In
 122 Section 4 we describe IMBALANCE, from the underlying intuition to the details of the method,
 123 while in Section 5 we present the experimental results. We finally draw conclusions and highlight
 124 limitations and future directions in Sections 6 and 7.
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126 2 RELATED WORK 127

128 **Knowledge Graph Embedding Models** KGE models learn continuous representations of entities
 129 and relations in the KG from the graph topology and its soft regularities. A long list of methods
 130 (Cao et al., 2024) has followed seminal work in the space (Nickel et al., 2012). In this work, we
 131 limit our analysis to four traditional KGE models: TransE (Bordes et al., 2013), DistMult (Yang
 132 et al., 2015), ComplEx-N3 (Trouillon et al., 2016; Lacroix et al., 2018) and RotatE (Balažević et al.,
 133 2019). Despite others have claimed better performance Balažević et al. (2019), the performance gain
 134 is often marginal, and the small differences in modeling the representation of triples are not relevant
 135 to the core contribution of our work.
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137 A notable exception is represented by NBFNet (Zhu et al., 2021), a GNN-like architecture that has
 138 achieved substantial improvements on the aggregate metrics. We will show it is nonetheless affected
 139 by the degree imbalance, but we will not include it in our main experiments. Indeed, in NBFNet the
 140 representations of different entities are heavily entangled within the message-passing architecture,
 141 which makes the direct application of our approach to this architecture not scalable and prone to
 142 overfitting.
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144 **Topology-related Issues on KGs** The topology of KGs introduces significant challenges in training
 145 KGE models Sardina et al. (2024). Previous work (Mohamed et al., 2020; Rossi et al., 2021)
 146 has highlighted how link prediction models have a bias for high-degree nodes and have recognized
 147 how aggregate metrics can offer a distorted view on the actual performance of models. However,
 148 they loosely defined the issue and did not propose approaches to concretely tackle these challenges.
 149 Shomer et al. (2023) looked past the degree of single nodes into frequency among entity-relation
 150 pairs and proposed the synthetic generation of additional embeddings to compensate for long-tail
 151 entities distribution. Other works (Bordes et al., 2013; Ji et al., 2015; Lin et al., 2015; He et al.,
 152 2015), on the other hand, have identified the heterogeneity of relation types (and in particular, 1-
 153 to-many, many-to-1) as one of the main obstacles. They have tried to address the issue altering the
 154 representation of relations in a more expressive way, but were not able to solve the issue. Our work
 155 surpasses all these previous efforts as it defines the problem in a more systematic way, tracing it
 156 back to a two-fold learning issue of the training and extending the scope to a much broader set of
 157 triples.
 158

159 **Latent Search Optimization** The application of an inference-time latent search optimization has
 160 been carried out before in a completely different domain by Bonnet & Macfarlane (2024). To the
 161 best of our knowledge, our work is the first that leverage such a latent search optimization for KGE
 162 models. We refer to the work above for an overview of other related domains of application that are
 163 not pertinent to this work.
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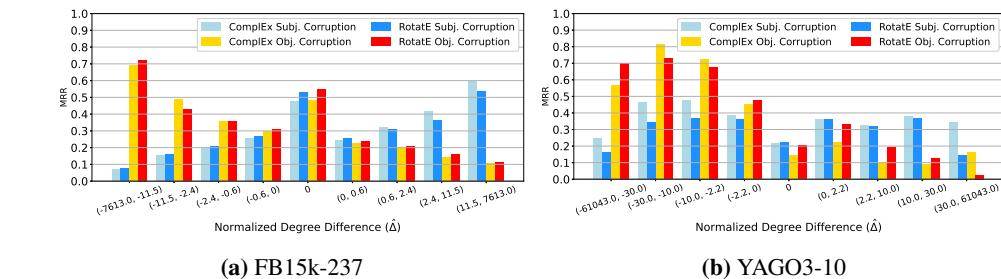


Figure 2: Performance of conventional KGE models on test triples binned by normalized degree difference. Bins are obtained using quartiles of $|\hat{\Delta}(t)|$ for $t \in \mathcal{G}$. On the left hand side we have triples with high-degree object and low-degree subject, on the right hand side, high-degree subjects and low-degree objects. We isolate subject and object corruption.

3 WHAT AGGREGATE METRICS DO NOT SHOW

KGE models have proven to be successful in reconstructing missing links in KGs. However, what the aggregated metrics often hide is that the quality of these predictions varies hugely in different groups of triples. Previous work has loosely associated the problem with the epistemic uncertainty of low-degree entities and has shown the challenge of predicting head (and tail) of 1-to-N (and N-to-1) relations. We extend the scope of the issue, proving that the fundamental challenge in predicting the head or tail of a triple lies in the degree difference (or *degree imbalance*) between the subject and the object: this makes the reconstruction of the higher degree entity much simpler, while leaving the lower degree entity poorly reconstructed.

In Figure 2, we plot the MRR of different ComplEx and RotatE, isolating the performance on subject and object corruptions and splitting the test triples of FB15k-237 (Toutanova & Chen, 2015) and Yago3-10 (Mahdisoltani et al., 2015) based on the normalized degree difference, that we define as:

$$\hat{\Delta}(t = (s, p, o)) = \frac{\delta(s) - \delta(p)}{\min \{\delta(s), \delta(o)\}},$$

where $\delta : \mathcal{E} \rightarrow \mathbb{R}$ is the degree function, that counts the incoming and outgoing edges for $e \in \mathcal{E}$. We can immediately observe the stark difference in the quality of the predictions for highly imbalanced triples and how this difference systematically decreases as the normalized degree difference $\hat{\Delta}$ approaches zero. Despite the aggregate metrics reported in Table 1 portray models with robust predictive power, a different point of view changes things entirely: indeed, such performance is inflated by easier predictions and these models could hardly be trusted in critical use cases where the prediction of low-degree entities is the predominant goal (see Appendix A).

Table 1: Aggregate performance of KGE models on FB15k-237 and YAGO3-10

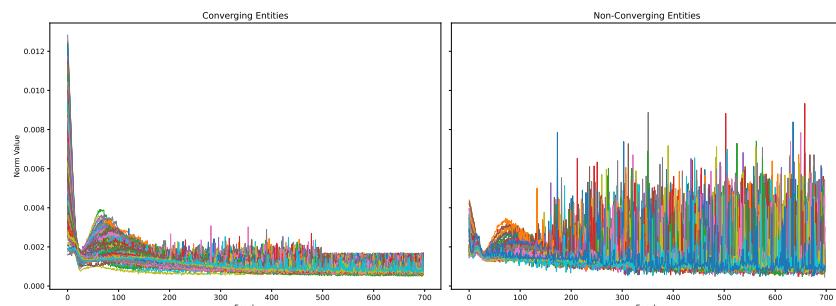
	FB15k-237		YAGO3-10	
	MRR	H@10	MRR	H@10
ComplEx	0.31	0.49	0.36	0.56
RotatE	0.31	0.51	0.37	0.57

To explain the degree imbalance issue, we take a step back and look into the training process. We carry out an analysis on FB15k-237 and report the results obtained for ComplEx.

In conventional KGE models, entities and relations are assigned a low-dimensional, continuous representation to capture relational patterns in the graph structure. During every training epoch, these representations are optimized to maximize the score assigned to the ground truth triples in the KG (*positives*), and minimize the score for false statements (*negatives*). In particular, the embedding of an entity is moved around the latent space every time the model processes a positive or a negative involving that entity.

By plotting the (euclidean) distance between the embeddings of the same entity across successive epochs, we can get an idea of the magnitude of the updates and, as it decreases, of the rate of

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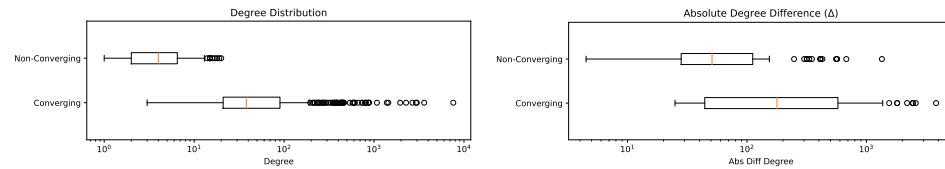
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(a) On the left, entities converge and the update reduces to approach zero. On the right, embeddings of the entities keep getting updated, proving there is an issue with the learning process.



(b) Degree distribution

(c) Absolute degree

Figure 3: (Top) Euclidean Norm of the difference between embeddings of converging and non-converging entities while training RotatE on FB15k-237. (Bottom) Comparison of degree statistics for converging vs. non-converging entities.

convergence of the learning process. Surprisingly, we observed that if there is one group of entities for which the magnitude of the updates stabilizes to very low values, indicating convergence, there is also a second group, which keeps receiving significant updates, with the repeating trend of random peaks indicating a failed convergence, rather than an insufficient amount of training (see Figure 3a).

Digging deeper into the characteristics of the two groups, we found a significant difference in the degree distributions, with the convergent group having a much higher degree (Figure 3b). Nonetheless, it would still contain entities with degree close to those of the non-converging group and so, to reduce the confounding factors of the analysis, we focused on this subset.

Comparing it to non-converging entities, we found that a higher degree imbalance in the *training* triples favors convergence (Figure 3c). Paired with the observed difficulty of predicting degree imbalanced triples at test-time, this tells us that the convergence of low-degree nodes is just a proxy for a strong overfitting. This is the case for J-Pop in our example above: it converges as only one relation type characterizes its neighborhood, but that also leads to overfitting the embedding. On the contrary, low-degree nodes connected to lower degree entities keep moving around the space, posing an equally hard challenge for the model at test-time in light of their instability.

Therefore, the challenge underlying the most imbalanced triples in the test set is two-folded: a strong overfitting on the one hand and an extremely poor learning on the other. We set to address this issue by proposing IMBALANCE, an inference-time latent search method that tries to improve model prediction on the most imbalanced triples involving low-degree entities.

4 INFERENCE TIME LATENT SEARCH FOR DEGREE IMBALANCE

4.1 PRELIMINARY AND NOTATION

A Knowledge Graph $\mathcal{G} = \{(s, p, o)\} \subseteq \mathcal{E} \times \mathcal{R} \times \mathcal{E}$ is a set of triples $t = (s, p, o)$, each including a subject (*head*) $s \in \mathcal{E}$, a predicate $p \in \mathcal{R}$, and an object (*tail*) $o \in \mathcal{E}$, where \mathcal{E} and \mathcal{R} are the sets of all entities and relation types, respectively. We refer to the task of predicting unseen triples in a KG as *Link Prediction*. It is formalized in literature as a learning-to-rank problem, where the objective

270 is learning a scoring function $f : \mathcal{E} \times \mathcal{R} \times \mathcal{E} \rightarrow \mathbb{R}$ that, given an input triple $t = (s, p, o)$, assigns a
 271 score $f(t) = f((s, p, o)) \in \mathbb{R}$ proportional to the likelihood that the fact t is true.
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273 **4.2 INTUITION**
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275 At the end of the training of a conventional KGE model, every entity and relation has a low-
 276 dimensional, continuous representation which is the result of updates based on all triples in the
 277 KG. As such, it is the representation of the entity that best fits *all* training triples. Given a query
 278 q , however, a one-size-fits-all representation of the anchor might not be precise enough to find the
 279 correct answer, especially if the anchor has high degree, which could thus introduce a lot of noise
 280 into the representation. Consider again the example above and the node Jackie Chan. All the
 281 relations in its neighborhood are different from /music/genre/artists except for one. As a
 282 result, the embedding will have little pertinence with the query. This is the reason why we refine its
 283 embedding with the training triples sharing the anchor and the predicate of q , i.e., (Mandopop,
 284 /music/genre/artists, Jackie Chan). In this way, the new representation is the one
 285 that best suits q and not *all* triples in the KG. We call this set the *training context* of q and denote it
 286 $\mathcal{C}_q = \{(s, p, o_i) | o_i \in \mathcal{E}\} \subset \mathcal{G}$ if $q = (s, p, ?)$ and, analogously, $\mathcal{C}_q = \{(s_i, p, o) | s_i \in \mathcal{E}\} \subset \mathcal{G}$ if
 287 $q = (?, p, o)$.
 288

289 The second issue of the optimization process we described above is related to low-degree entities.
 290 As we saw in the previous section, their embeddings are either heavily dependent on the few entities
 291 they are connected to and thus very prone to overfitting, or they suffer from an incomplete learning.
 292 This makes their reconstruction from the anchor node s very unlikely, as their representation share
 293 too little with s . The node J-Pop, for example, is only connected to musicians, while Jackie Chan
 294 is best known for his acting. Similarly, other possible genres that are plausible answers for the
 295 query could be far off from Jackie Chan for the same reason. Therefore, it is essential to bias their
 296 embeddings in favor of the query. To do so, we leverage an external oracle that has the advantage of
 297 being degree agnostic, to limit the risk of neglecting low-degree entities. This oracle suggests triples
 298 similar to those in the training context of the query. In this way, we will bring the embeddings of
 299 Jackie Chan and musical genres similar to Mandopop closer together, making a correct answer to
 300 the query more likely.
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302 **4.3 IMBALANCE**
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304 Inference-time latent search has been successfully applied to improve generalization and model pre-
 305 dictions in a completely different domain by Bonnet & Macfarlane (2024). As the degree imbalance
 306 could benefit from these properties, we design an inference-time latent search that can be selectively
 307 applied to target queries (see Algorithm 1). Built on top of a pre-trained KGE model with scoring
 308 function f , it takes a query $q = (s, p, ?)$ (analogous for $(?, p, o)$) and refines the representation of
 309 relevant entities optimizing for n epochs a two-term objective function:
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$$\mathcal{L}(q = (s, p, ?)) = \sum_{t^+ \in \mathcal{C}_q} f(t^+) + \sum_{t \in \Omega_q} f(t).$$

311 Crucially, our approach does not require any negatives, thus circumventing the problem of their
 312 synthetic generation that has attracted a lot of attention for its multiple criticalities (Kamigaito &
 313 Hayashi, 2022; Madushanka & Ichise, 2024). This aspect also limits the computational overhead of
 314 this method, that is extremely scalable. Moreover, it is extremely flexible, as it can be selectively
 315 applied to single queries, thus allowing to refine the prediction on single triples of interest.
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317 **Training Context Term** The first term of the loss function sums the scores assigned to the train-
 318 ing context of the query. These are ground truth triples that were already processed during training,
 319 and that the model should have already learned. However, as we observed above, the final embed-
 320 ding assigned to the anchor node is a one-size-fits-all representation that could underperform on a
 321 specific query. Therefore, during our latent search, we present these relevant triples again, to better
 322 tailor the anchor representation for the final prediction. Importantly, in this term we only optimize
 323 the embedding of the anchor node of the query, while keeping the others frozen. If we were to op-
 324 timize the embeddings of head, tail and predicate, we would overfit learned triples even more, thus
 325 worsening the already poor generalization of the model at test-time. Limiting the optimization to the
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324 **Algorithm 1** Inference-Time Latent Search for Degree Imbalance

325 **Require:** Pretrained KGE model with scoring function f and entity embedding matrix \mathbf{E} , a query $q = (s, p, ?)$,

326 training context \mathcal{C}_q and oracle triples Ω_q , number of latent search iterations T

327 **Ensure:** Ranked list of entities

328 1: $\mathbf{E}_0 \leftarrow \mathbf{E}$

329 2: Set the embeddings of s to $\mathbf{s}^0 \leftarrow \mathbf{E}_0[s]$

330 3: Set the embeddings of $o_j \in \{o | (s, p, o) \in \Omega_q\}$ to $\mathbf{o}_j^0 \leftarrow \mathbf{E}_0[o_j], \forall j = 1, \dots, |\Omega_q|$

331 4: **for** iteration = 0, ..., T **do**

332 5: Compute $\mathcal{L}(q)$ and gradients w.r.t. \mathbf{s}^i and $\mathbf{o}_j^i, j = 1, \dots, |\Omega_q|$

333 6: Update $\mathbf{s}^{i+1} \leftarrow \text{GRADIENTUPDATE}(\mathbf{s}^i)$

334 7: Update $\mathbf{o}_j^{i+1} \leftarrow \text{GRADIENTUPDATE}(\mathbf{o}_j^i), \forall j = 1, \dots, |\Omega_q|$

335 8: Update $\mathbf{E}_{i+1}[s] \leftarrow \mathbf{s}^{i+1}$ and $\mathbf{E}_{i+1}[o_j] \leftarrow \mathbf{o}_j^{i+1}, \forall j = 1, \dots, |\Omega_q|$

336 9: **end for**

337 10: Evaluate q with the updated embeddings and extract ranked list of candidates

338 11: **return** Ranked candidates based on final scores

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 340 anchor node, on the contrary, we refine its representation to best suit the training context, improving
 341 its generalization and ring-fencing it in an area of the embedding space suitable for the query.

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 343 **Oracle-Enhanced Term** If the first term improves the representation of the anchor, we might still
 344 get bad predictions if the embeddings of the nodes answering the query are inaccurate. This is
 345 particularly likely for low degree entities that are dependent on very few facts that could well be
 346 irrelevant for the target query q , or, as shown before, prone to overfitting or susceptible to a failed
 347 learning. Therefore, refining the anchor node only using the training context is not enough. We
 348 need to adjust the target embeddings, biasing them toward the query. Doing so is not trivial, as, at
 349 test-time, we have no sense of what the correct answers to the query are. Moreover, being our focus
 350 on low-degree entities, little information about them is available within the KG. For this reason, we
 351 enhance our latent search with a set of additional triples $\Omega_q = \{(s, p, o_i) | o_i \in \mathcal{E}\}$ generated by an
 352 oracle that considers them likely answers to q . This time around, we optimize both the anchor and
 353 the target entity embeddings, so that, following the oracle leads, also the landscape of the answer
 354 representations changes in favor of the query.

355 As we said, the KG provides limited information about the target entity. Therefore, our oracle can
 356 be any out-of-band source of information well aligned with the task at hand. For our experiments
 357 on benchmark datasets we have used a Large Language Model (LLM) defining similarity on the
 358 encoded textual descriptions of the entities (see Appendix C for the technical details). In a different
 359 scenario, like the gene-disease association example, the oracle can be formalized as the preference
 360 of biologists for a subset of genes involved in a biological pathway relevant for the disease of interest
 361 or any other source of expertise. A final observation is needed here: the oracle triples may include
 362 false positives, that could surface up in the evaluation ranking. However, this risk is mitigated by
 363 the regularization effect it has on the representation of the anchor and by biasing the representation
 364 of possible targets toward the query.

365 5 EXPERIMENTS

366 5.1 EXPERIMENTAL SETTINGS

367 **Datasets and Test Splits.** We evaluate IMBALANCE on three encyclopedic benchmark KGs,
 368 widely used in literature: FB15k-237 (Toutanova & Chen, 2015), WN18RR (Dettmers et al., 2018)
 369 and YAGO3-10 (Mahdisoltani et al., 2015). To prove its efficacy on highly imbalanced triples involv-
 370 ing low-degree entities, we identify *low-degree* nodes as those with degree below the first quartile
 371 of the degree distribution, and *high-degree* nodes as those with degree greater than the third quar-
 372 tile. The statistics of the resulting splits are reported in Table 8, while additional statistics about the
 373 dataset are reported in Appendix B.

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 375 **Evaluation Protocol and Metrics.** We evaluate IMBALANCE by corrupting the low-degree entity
 376 of the test triples with all entities in the KG and we consider the *filtered* setting (Bordes et al., 2013),
 377 i.e., we filter from the corruptions all facts in the training, validation or test sets. We then rank test

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431**Table 2:** Statistics of the datasets.

Dataset	#H Nodes (Min degree)	#L Nodes (Max degree)	#Valid H-L	#Valid L-H	#Test H-L	#Test L-H
FB15k-237	3536 (41)	4015 (11)	338	775	396	942
WN18RR	2296 (5)	32 697 (10)	295	689	277	753
Yago3-10	28 913 (5)	31 487 (16)	25	298	35	270

Table 3: Results on the three benchmark datasets on the High-Low and Low-High triples splits. The best value for each metric on each dataset is reported in **bold** except in case of a tie, when they are underlined.

Dataset	Model	High-Low				Low-High			
		MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10
FB15k-237	ComplEx-N3	0.03	0.01	0.01	0.06	0.03	0.01	0.02	0.08
	+IMBALANCE	<u>0.13</u>	0.05	0.14	0.28	<u>0.08</u>	<u>0.04</u>	0.08	<u>0.17</u>
	RotatE	0.02	0.01	0.01	0.05	0.04	0.01	0.04	0.09
	+IMBALANCE	<u>0.13</u>	0.06	0.11	0.36	<u>0.08</u>	<u>0.04</u>	0.09	<u>0.17</u>
Yago3-10	ComplEx-N3	0.06	0.00	0.03	0.23	0.07	<u>0.02</u>	0.07	0.17
	+IMBALANCE	0.06	0.00	0.03	0.23	0.09	<u>0.02</u>	0.11	0.21
	RotatE	0.18	<u>0.14</u>	<u>0.20</u>	0.26	0.03	0.01	0.03	0.05
	+IMBALANCE	0.19	<u>0.14</u>	<u>0.20</u>	0.29	0.03	0.01	0.03	0.08
WN18RR	ComplEx-N3	0.46	0.40	0.47	0.55	0.28	0.23	0.28	0.38
	+IMBALANCE	0.48	0.40	0.51	0.61	0.31	0.25	0.32	0.46
	RotatE	0.49	0.45	0.50	0.57	0.32	<u>0.27</u>	0.33	0.42
	+IMBALANCE	0.51	0.46	0.55	0.60	0.33	<u>0.27</u>	0.35	0.44

triples against all corruptions. The metrics used are the usual for link prediction: Mean Reciprocal Rank (MRR) and Hits at N (Hits@N).

Hyperparameter-Search Given a pre-trained KGE model, IMBALANCE has only three hyperparameters: the learning rate λ of an Adam (Kingma & Ba, 2015) optimizer, the number of latent search iterations and the number m of oracle-generated triples included in Ω_q . We selected optimal values based on the validation performance using MRR as the reference metric.

5.2 RESULTS

IMBALANCE Impact The results of our experiments are reported in Table 3. The impact of the latent search on FB15k-237 triples is striking, with metrics that improved at least by a factor of 2 on the Low-High split and over a factor of 6 on the High-Low one. This shows how IMBALANCE compensates the flaws of the pre-trained models. Also on WN18RR there is a noticeable improvement across all metrics, but it is smaller than on FB15k-237. The reason for it is that in WN18RR the degree distribution is concentrated around values much smaller than those of FB15k-237 (see Table 8). This reduced polarization results in a less pressing degree imbalance issue, making the improvement of IMBALANCE less striking. Finally, on Yago3-10, we only see an improvement on the Hits@10. This behavior can be justified by the fact that the oracle generated the triples for this dataset using only the labels of the entities of the KG, while it leveraged labels and additional descriptions for FB15k-237 and WN18RR. This impacted negatively the similarity of triples extracted by the oracle, introducing less relevant triples in the latent search that prevented the correct answers to reach the top positions of the ranking.

Baseline Comparison We compare the impact of our method with two different approaches in Table 4. KG-Mixup (Shomer et al., 2023) was designed to counter a vague notion of degree bias and to enhance performance on low-degree entities. However, we show how it provides little benefit on imbalanced triples compared to IMBALANCE. CSProm-KG (Chen et al., 2023), on the other hand, is a method that integrates an LLM to enhance link prediction and is way superior in aggregate metrics to ComplEx and RotatE ($MRR = 0.36$, $H@10 = 0.54$ on the entire FB15k-237 test set). On imbalanced triples, however, IMBALANCE outperforms it or falls behind by a slight margin. Therefore, our approach makes simpler models perform on the same level as more involved ones

432 **Table 4:** Results on FB15k-237 for IMBALANCE and two baselines. The best value for each metric on
 433 each dataset is reported in **bold** except in case of a tie, when they are underlined.

Dataset	Model	High-Low				Low-High			
		MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10
FB15k-237	ComplEx-N3 + IMBALANCE	0.13	0.05	0.14	0.28	0.08	0.04	0.08	<u>0.17</u>
	RotatE + IMBALANCE	<u>0.13</u>	0.06	0.11	0.36	0.08	0.04	<u>0.09</u>	<u>0.17</u>
	TuckER balazevic2019tucker +KG-Mixup shomer2023degree _{bias}	0.08	0.03	0.08	0.21	0.08	0.04	0.07	0.16
	CSProm-KG chen2023cspromkg	0.09	0.03	0.08	0.24	0.08	0.03	0.08	<u>0.17</u>

442 **Table 5:** Results on FB15k-237 isolating the contribution of the two terms of the loss function.

Dataset	Model	High-Low				Low-High			
		MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10
FB15k-237	ComplEx-N3	0.03	0.01	0.01	0.06	0.03	0.01	0.02	0.08
	+Context Only	0.05	0.01	0.04	0.14	0.05	0.02	0.04	0.12
	+Oracle Only	0.10	0.05	0.09	0.22	0.08	0.03	0.07	<u>0.17</u>
	+IMBALANCE	<u>0.13</u>	0.05	0.14	0.28	<u>0.08</u>	<u>0.04</u>	0.08	<u>0.17</u>
	RotatE	0.02	0.01	0.01	0.05	0.04	0.01	0.04	0.09
	+Context Only	0.06	0.02	0.06	0.16	0.06	0.02	0.06	0.14
	+Oracle Only	0.12	0.05	0.10	0.34	<u>0.08</u>	<u>0.04</u>	<u>0.09</u>	<u>0.17</u>
	+IMBALANCE	<u>0.13</u>	0.06	0.11	0.36	<u>0.08</u>	<u>0.04</u>	<u>0.09</u>	<u>0.17</u>

453 and does so preserving the computational efficiency that CSProm-KG and other LLM-enhanced
 454 KGE models cannot offer.

455 **Loss function terms contribution** To gauge the contribution of the training context term and
 456 of the oracle term, we run separate experiments where we switch off alternately one or the other.
 457 The results for FB15k-237 are reported in Table 5, while those for WN18RR and Yago3-10 are in
 458 Appendix D. Numbers clearly show that both terms provide a positive contribution on their own.
 459 This supports both our claims on how grounding the representation of the anchor node in a query-
 460 friendly way is essential and on the importance of adjusting the embeddings of other entities. We
 461 explain the bigger impact of the oracle term as it also contributes directly to the optimization of the
 462 anchor embedding. Finally, the joint contribution of the two outperforms the single terms, supporting
 463 their complementarity.

464 **Time-Complexity and Latency** The time-complexity needed to run one epoch of IMBALANCE is
 465 $\mathcal{O}(k \cdot (|\mathcal{C}_q| + |\Omega_q|))$. In fact, it applies the KGE scoring function and computes the gradients for the
 466 embeddings of the training-context and oracle-generated triples. The already low time-complexity
 467 to train a traditional KGE training for one epoch amounts to $\mathcal{O}(|\mathcal{G}|(\eta + 1)k)$, where \mathcal{G} is the training
 468 graph and η the number of negatives. Instead of re-traning such model, applying IMBALANCE to
 469 a test set \mathcal{T} is more efficient, as the size of \mathcal{T} is typically orders of magnitude smaller than \mathcal{G} .
 470 Moreover, \mathcal{C}_q is a limited subset of \mathcal{G} and the number of oracle triples are only a few dozens at most
 471 (in our experiments always ≤ 50), yielding $|\mathcal{T}| \cdot (|\mathcal{C}| + |\Omega|) \ll N$. Finally, IMBALANCE requires fewer
 472 epochs to reach convergence, thus confirming its efficiency even compared to the strong baseline.
 473 The run time latency of the system is reported in Table 6.

474 6 LIMITATIONS AND FUTURE DIRECTIONS

475 If this work sheds a light on a key issue affecting KGE models, it still has limitations that we will
 476 address in future work. First, IMBALANCE can only be applied on queries for which the training
 477 context is non-empty. Overcoming this limitation would mean refining the selection of training
 478 triples *tightly related* to the query. This would be of great interest, as it would explain which triples
 479 have the highest impact on a prediction. Second, we proved its applicability to High-Low and
 480 Low-High triples, but it remains a challenge to understand how it could benefit triples where the
 481 imbalance is more limited. The issue is limited as IMBALANCE can be applied selectively on single

486 **Table 6:** Latency of IMBALANCE compare to re-training the KGE model from scratch. The runtime
 487 is reported in seconds over 30 epochs of latent search for IMBALANCE, though we typically observe
 488 convergence within the first 10 epochs.

	Model	FB15k-237	Yago3-10
Single query	IMBALANCE	0.91	1.39
Full test set	IMBALANCE	440.7	400.1
Pre-training	RotatE	935.9	24,131.3

495 triples, but an extension that benefits all triples could stretch the applicability to wider use cases.
 496 Finally, the quality of the triples generated by the oracles affects the quality of the results, which
 497 made our results dependent on the LLM. We leave for future work an in-depth study of different
 498 oracles, that go beyond LLMs and potentially include human feedback.

500 7 CONCLUSIONS

502 This work has introduced the degree imbalance problem, an issue that heavily affects a wide variety
 503 of KGE models and hinders their predictive power. We provided deep insights on the learning issue
 504 from which the problem stems, proving how low-degree entities are impacted by strong overfitting
 505 or by a failed convergence. To mitigate the issue, we proposed IMBALANCE, the first inference-time
 506 latent search method applied in the realm of KGE embeddings. We validated its efficacy by obtaining
 507 reliable predictions on highly imbalanced triples, preserving model efficiency, and opening the door
 508 to the application of this method in use cases where degree imbalance is a consistent concern.

510 REFERENCES

512 Ivana Balažević, Carl Allen, and Timothy M Hospedales. Tucker: Tensor factorization for knowl-
 513 edge graph completion. In *Empirical Methods in Natural Language Processing*, 2019.

515 Clément Bonnet and Matthew V Macfarlane. Searching latent program spaces, 2024. URL <https://arxiv.org/abs/2411.08706>.

518 Antoine Bordes, Nicolas Usunier, Alberto Garcia-Duran, Jason Weston, and Oksana
 519 Yakhnenko. Translating embeddings for modeling multi-relational data. In C.J.
 520 Burges, L. Bottou, M. Welling, Z. Ghahramani, and K.Q. Weinberger (eds.), *Ad-
 521 vances in Neural Information Processing Systems*, volume 26. Curran Associates, Inc.,
 522 2013. URL https://proceedings.neurips.cc/paper_files/paper/2013/file/1cecc7a77928ca8133fa24680a88d2f9-Paper.pdf.

524 Jiahang Cao, Jinyuan Fang, Zaiqiao Meng, and Shangsong Liang. Knowledge graph embedding:
 525 A survey from the perspective of representation spaces. *ACM Comput. Surv.*, 56(6), mar 2024.
 526 ISSN 0360-0300. doi: 10.1145/3643806. URL <https://doi.org/10.1145/3643806>.

528 Chen Chen, Yufei Wang, Aixin Sun, Bing Li, and Kwok-Yan Lam. Dipping plms sauce: Bridging
 529 structure and text for effective knowledge graph completion via conditional soft prompting. In
 530 Anna Rogers, Jordan L. Boyd-Graber, and Naoaki Okazaki (eds.), *Findings of the Association
 531 for Computational Linguistics: ACL 2023, Toronto, Canada, July 9-14, 2023*, pp. 11489–11503.
 532 Association for Computational Linguistics, 2023. doi: 10.18653/V1/2023.FINDINGS-ACL.729.
 533 URL <https://doi.org/10.18653/v1/2023.findings-acl.729>.

534 Tim Dettmers, Pasquale Minervini, Pontus Stenetorp, and Sebastian Riedel. Convolutional 2d
 535 knowledge graph embeddings. In Sheila A. McIlraith and Kilian Q. Weinberger (eds.), *Pro-
 536 ceedings of the Thirty-Second AAAI Conference on Artificial Intelligence, (AAAI-18), the 30th
 537 innovative Applications of Artificial Intelligence (IAAI-18), and the 8th AAAI Symposium on Ed-
 538 ucational Advances in Artificial Intelligence (EAAI-18), New Orleans, Louisiana, USA, Febru-
 539 ary 2-7, 2018*, pp. 1811–1818. AAAI Press, 2018. doi: 10.1609/AAAI.V32I1.11573. URL
<https://doi.org/10.1609/aaai.v32i1.11573>.

540 Xin Dong, Evgeniy Gabrilovich, Jeremy Heitz, Wilko Horn, Ni Lao, Kevin Murphy, Thomas
 541 Strohmaier, Shaohua Sun, and Wei Zhang. Knowledge vault: a web-scale approach to proba-
 542 bility knowledge fusion. In *Proceedings of the 20th ACM SIGKDD International Conference*
 543 *on Knowledge Discovery and Data Mining*, KDD '14, pp. 601–610, New York, NY, USA, 2014.
 544 Association for Computing Machinery. ISBN 9781450329569. doi: 10.1145/2623330.2623623.
 545 URL <https://doi.org/10.1145/2623330.2623623>.

546 Shizhu He, Kang Liu, Guoliang Ji, and Jun Zhao. Learning to represent knowledge graphs with
 547 gaussian embedding. In *Proceedings of the 24th ACM International Conference on Informa-
 548 tion and Knowledge Management*, CIKM '15, pp. 623–632, New York, NY, USA, 2015. Associa-
 549 tion for Computing Machinery. ISBN 9781450337946. doi: 10.1145/2806416.2806502. URL
 550 <https://doi.org/10.1145/2806416.2806502>.

551 Aidan Hogan, Eva Blomqvist, Michael Cochez, Claudia D'amato, Gerard De Melo, Claudio Gutier-
 552 rez, Sabrina Kirrane, José Emilio Labra Gayo, Roberto Navigli, Sebastian Neumaier, Axel-
 553 Cyrille Ngonga Ngomo, Axel Polleres, Sabbir M. Rashid, Anisa Rula, Lukas Schmelzeisen, Juan
 554 Sequeda, Steffen Staab, and Antoine Zimmermann. Knowledge graphs. 54(4), jul 2021. ISSN
 555 0360-0300. doi: 10.1145/3447772. URL <https://doi.org/10.1145/3447772>.

556 Guoliang Ji, Shizhu He, Liheng Xu, Kang Liu, and Jun Zhao. Knowledge graph embedding via
 557 dynamic mapping matrix. In Chengqing Zong and Michael Strube (eds.), *Proceedings of the 53rd*
 558 *Annual Meeting of the Association for Computational Linguistics and the 7th International Joint*
 559 *Conference on Natural Language Processing (Volume 1: Long Papers)*, pp. 687–696, Beijing,
 560 China, July 2015. Association for Computational Linguistics. doi: 10.3115/v1/P15-1067. URL
 561 <https://aclanthology.org/P15-1067/>.

562 Hidetaka Kamigaito and Katsuhiko Hayashi. Comprehensive analysis of negative sampling in
 563 knowledge graph representation learning. In Kamalika Chaudhuri, Stefanie Jegelka, Le Song,
 564 Csaba Szepesvári, Gang Niu, and Sivan Sabato (eds.), *International Conference on Machine*
 565 *Learning, ICML 2022, 17-23 July 2022, Baltimore, Maryland, USA*, volume 162 of *Pro-
 566 ceedings of Machine Learning Research*, pp. 10661–10675. PMLR, 2022. URL <https://proceedings.mlr.press/v162/kamigaito22a.html>.

567 Diederik P. Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In Yoshua
 568 Bengio and Yann LeCun (eds.), *3rd International Conference on Learning Representations, ICLR*
 569 *2015, San Diego, CA, USA, May 7-9, 2015, Conference Track Proceedings*, 2015. URL <http://arxiv.org/abs/1412.6980>.

570 Timothee Lacroix, Nicolas Usunier, and Guillaume Obozinski. Canonical tensor decomposition for
 571 knowledge base completion. In *International Conference on Machine Learning*, pp. 2869–2878,
 572 2018.

573 Yankai Lin, Zhiyuan Liu, Maosong Sun, Yang Liu, and Xuan Zhu. Learning entity and relation
 574 embeddings for knowledge graph completion. *AAAI'15*, pp. 2181–2187. AAAI Press, 2015.
 575 ISBN 0262511290.

576 Tirosh Madushanka and Ryutaro Ichise. Negative sampling in knowledge graph representation
 577 learning: A review. *CoRR*, abs/2402.19195, 2024. doi: 10.48550/ARXIV.2402.19195. URL
 578 <https://doi.org/10.48550/arXiv.2402.19195>.

579 Farzaneh Mahdisoltani, Joanna Biega, and Fabian M. Suchanek. YAGO3: A knowledge base from
 580 multilingual wikipedias. In *Seventh Biennial Conference on Innovative Data Systems Research,
 581 CIDR 2015, Asilomar, CA, USA, January 4-7, 2015, Online Proceedings*. www.cidrdb.org, 2015.
 582 URL http://cidrdb.org/cidr2015/Papers/CIDR15_Paper1.pdf.

583 Aisha Mohamed, Shameem Parambath, Zoi Kaoudi, and Ashraf Aboulnaga. Popularity agnostic
 584 evaluation of knowledge graph embeddings. In Jonas Peters and David Sontag (eds.), *Proceedings*
 585 *of the 36th Conference on Uncertainty in Artificial Intelligence (UAI)*, volume 124 of *Proceedings*
 586 *of Machine Learning Research*, pp. 1059–1068. PMLR, 03–06 Aug 2020. URL <https://proceedings.mlr.press/v124/mohamed20a.html>.

594 Maximilian Nickel, Volker Tresp, and Hans-Peter Kriegel. Factorizing YAGO: scalable machine
 595 learning for linked data. In Alain Mille, Fabien Gandon, Jacques Misselis, Michael Rabinovich,
 596 and Steffen Staab (eds.), *Proceedings of the 21st World Wide Web Conference 2012, WWW 2012,*
 597 *Lyon, France, April 16-20, 2012*, pp. 271–280. ACM, 2012. doi: 10.1145/2187836.2187874.
 598 URL <https://doi.org/10.1145/2187836.2187874>.

599 Andrea Rossi, Denilson Barbosa, Donatella Firmani, Antonio Matinata, and Paolo Merialdo.
 600 Knowledge graph embedding for link prediction: A comparative analysis. *ACM Trans. Knowl.
 601 Discov. Data*, 15(2), January 2021. ISSN 1556-4681. doi: 10.1145/3424672. URL <https://doi.org/10.1145/3424672>.
 602
 603

604 Jeffrey Sardina, John D. Kelleher, and Declan O’Sullivan. A survey on knowledge graph structure
 605 and knowledge graph embeddings, 2024. URL <https://arxiv.org/abs/2412.10092>.
 606

607 Harry Shomer, Wei Jin, Wentao Wang, and Jiliang Tang. Toward degree bias in embedding-
 608 based knowledge graph completion. In *Proceedings of the ACM Web Conference 2023, WWW
 609 '23*, pp. 705–715, New York, NY, USA, 2023. Association for Computing Machinery. ISBN
 610 9781450394161. doi: 10.1145/3543507.3583544. URL <https://doi.org/10.1145/3543507.3583544>.
 611

612 Kristina Toutanova and Danqi Chen. Observed versus latent features for knowledge base and text
 613 inference. In Alexandre Allauzen, Edward Grefenstette, Karl Moritz Hermann, Hugo Larochelle,
 614 and Scott Wen-tau Yih (eds.), *Proceedings of the 3rd Workshop on Continuous Vector Space
 615 Models and their Compositionality, CVSC 2015, Beijing, China, July 26-31, 2015*, pp. 57–66.
 616 Association for Computational Linguistics, 2015. doi: 10.18653/v1/W15-4007. URL <https://doi.org/10.18653/v1/W15-4007>.
 617

618 Théo Trouillon, Johannes Welbl, Sebastian Riedel, Éric Gaussier, and Guillaume Bouchard. Complex
 619 embeddings for simple link prediction. In *ICML*, pp. 2071–2080, 2016.

620 Zhen Wang, Jianwen Zhang, Jianlin Feng, and Zheng Chen. Knowledge graph embedding by translating
 621 on hyperplanes. In *Proceedings of the Twenty-Eighth AAAI Conference on Artificial Intelligence,
 622 AAAI'14*, pp. 1112–1119. AAAI Press, 2014.

623 Bishan Yang, Wen-tau Yih, Xiaodong He, Jianfeng Gao, and Li Deng. Embedding entities and
 624 relations for learning and inference in knowledge bases. In Yoshua Bengio and Yann LeCun
 625 (eds.), *3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA,
 626 USA, May 7-9, 2015, Conference Track Proceedings*, 2015. URL <http://arxiv.org/abs/1412.6575>.
 627

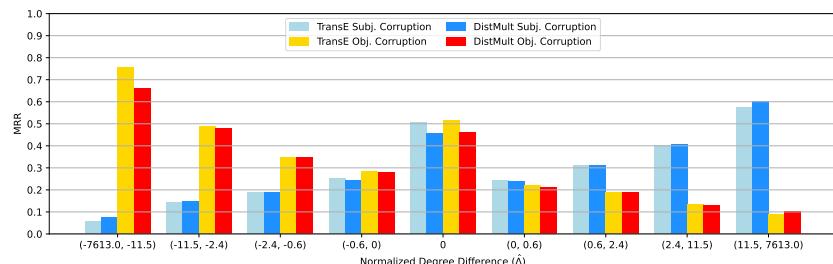
628 Fuzheng Zhang, Nicholas Jing Yuan, Defu Lian, Xing Xie, and Wei-Ying Ma. Collaborative knowl-
 629 edge base embedding for recommender systems. In *Proceedings of the 22nd ACM SIGKDD
 630 International Conference on Knowledge Discovery and Data Mining, KDD '16*, pp. 353–362,
 631 New York, NY, USA, 2016. Association for Computing Machinery. ISBN 9781450342322. doi:
 632 10.1145/2939672.2939673. URL <https://doi.org/10.1145/2939672.2939673>.
 633

634 Zhaocheng Zhu, Zuobai Zhang, Louis-Pascal Xhonneux, and Jian Tang. Neural bellman-ford net-
 635 works: A general graph neural network framework for link prediction. *Advances in Neural Infor-
 636 mation Processing Systems*, 34, 2021.

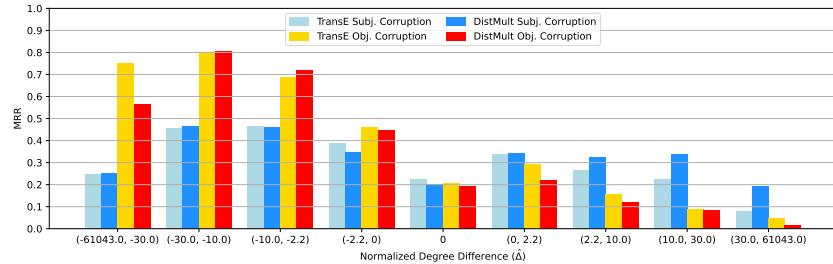
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648 **A DEGREE IMBALANCE FOR ADDITIONAL MODELS**
649650 In Table 7 we report the aggregate metrics for TransE and DistMult.
651652 In Figure 4 we report the degree imbalance for TransE and DistMult. As we can see, the behaviour
653 resembles that of ComplEx and DistMult.654 **Table 7:** Aggregate performance of KGE models on FB15k-237 and YAGO3-10
655

	FB15k-237		YAGO3-10	
	MRR	H@10	MRR	H@10
TransE	0.31	0.49	0.35	0.55
DistMult	0.30	0.48	0.34	0.53



(a) FB15k-237



(b) YAGO3-10

683 **Figure 4:** Performance of conventional KGE models on test triples binned by normalized degree difference.
684 Bins are obtained using quartiles of $|\hat{\Delta}(t)|$ for $t \in \mathcal{G}$. On the left hand side we have triples with
685 high-degree object and low-degree subject, on the right hand side, high-degree subjects and low-degree
686 objects. We isolate subject and object corruption.
687688 As we can see from the Figure 5, NBFNet (Zhu et al., 2021), despite its superior performance,
689 suffers from the same degree imbalance issue as conventional KGE models. Compared to the results
690 in Figure 2, the individual performance on most buckets is slightly better.
691692 **B DATASET STATISTICS**
693694 Statistics of the three datasets are reported in Table 8.
695696 **Table 8:** Statistics of the datasets.
697

Dataset	#Entities	#Relations	#Train	#Valid	#Test
FB15k-237	15 541	237	272 115	17 535	20 466
WN18RR	40 943	18	86 835	3034	3134
Yago3-10	123 182	37	1 079 040	5000	5000

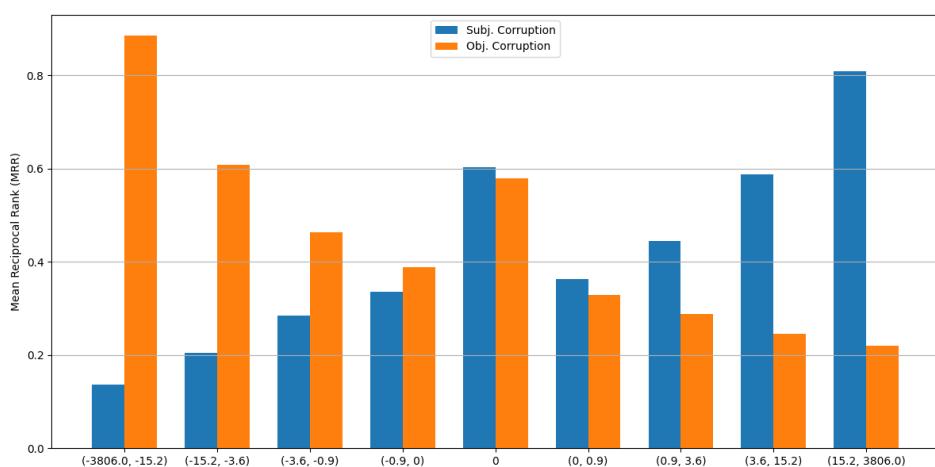


Figure 5: Performance of NBFNet on test triples binned by normalized degree difference. Bins are obtained using quartiles of $|\hat{\Delta}(t)|$ for $t \in \mathcal{G}$. On the left hand side we have triples with high-degree object and low-degree subject, on the right hand side, high-degree subjects and low-degree objects. We isolate subject and object corruption.

C ORACLE TECHNICAL DETAILS

For all our experiments, we have used NovaSearch/stella_en_400M_v5¹ with an embedding dimension of 1,024 to encode the labels and descriptions of the entities in our datasets (with the exception of Yago3-10, for which we used only the labels). Given $q = (s, p, ?)$, to generate the m triples in Ω_q , we have extracted the top- m entities closest to the barycenter of the set $\{e_j \in \mathcal{G} | (s, p, e_j) \in \mathcal{C}_q\}$ in the embedding space of the LLM. The distance between entities was computed using the cosine similarity.

As querying an LLM can introduce a significant overhead, we have extracted the LLM-triples as a preprocessing step ahead of running the experiments. In this way, the method remains lightweight.

D ADDITIONAL EXPERIMENTS

Loss Function Terms Contribution See results in Table 9. As the improvement in Table 3 showed a smaller increase on these datasets compared to FB15k-237, it is harder to appreciate significant differences in the contribution of the separate terms of the loss. However, the aggregation of the two yields the best results.

Sankey Plots To further explore how IMBALANCE impacts the ranks assigned to test triples, we leverage Sankey plots that show how ranks flow from the values assigned by the pre-trained model (on the left), to the ranks assigned using IMBALANCE (on the right). From Figures 8-9-10 we can observe how IMBALANCE improves ranks of hundreds of positions. Despite the metrics observed in Table 3 do not necessarily show it, bringing up ranks that are > 500 at the top positions of the ranking is a huge achievement, testifying the value of the approach.

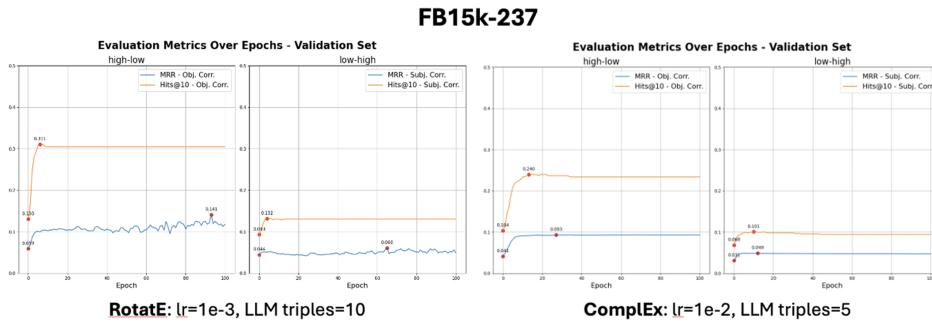
Degenerate Behaviour Loss Function The loss function is purely maximization over a set of known positive facts (\mathcal{C}_q) and oracle-suggested positive facts (Ω_q). This might lead to questioning the stability of the optimization process, with the risk of collapse or severe overfitting. We did not observe such behaviour, as the design of ImbalancE prevents it for two reasons:

- The first term of the loss function is used to optimize only the embedding of the anchor node, while the embeddings of the target entities of the training context remain fixed, thus preventing a collapse of the embeddings.

¹https://huggingface.co/NovaSearch/stella_en_400M_v5

756 **Table 9:** Results on WN18RR and YAGO3-10 isolating the contribution of the two terms of the loss
 757 function.

759 Dataset	Model	High-Low				Low-High			
		MRR	H@1	H@3	H@10	MRR	H@1	H@3	H@10
760 Yago3-10	ComplEx-N3	0.06	0.00	0.03	0.23	0.07	<u>0.02</u>	0.07	0.17
	+Context Only	0.06	0.00	0.03	0.23	0.06	<u>0.02</u>	0.07	0.17
	+Oracle Only	0.06	0.00	0.03	0.23	0.07	0.00	0.10	0.20
	+IMBALANCE	0.06	0.00	0.03	0.23	0.08	<u>0.02</u>	0.11	0.20
	RotatE	<u>0.18</u>	<u>0.14</u>	<u>0.20</u>	0.26	0.03	0.01	0.03	0.05
	+Context Only	0.18	0.14	0.20	0.26	0.04	0.02	0.03	0.08
	+Oracle Only	0.19	0.14	0.20	0.29	0.03	0.01	0.03	0.06
	+IMBALANCE	0.19	<u>0.14</u>	<u>0.20</u>	0.29	0.03	0.01	0.03	0.08
767 WN18RR	ComplEx-N3	0.46	0.40	0.47	0.55	0.28	0.23	0.28	0.38
	+Context Only	0.45	0.40	0.47	0.55	0.28	0.22	0.28	0.38
	+Oracle Only	0.48	0.40	0.51	<u>0.61</u>	0.31	0.25	0.33	0.45
	+IMBALANCE	0.48	0.40	0.51	<u>0.61</u>	0.31	0.25	0.32	0.46
	RotatE	0.49	0.45	0.50	0.57	0.32	<u>0.27</u>	0.33	0.42
	+Context Only	0.49	0.45	0.5	0.58	0.32	<u>0.27</u>	0.33	0.42
	+Oracle Only	0.49	0.43	0.53	0.6	0.32	<u>0.27</u>	0.34	0.42
	+IMBALANCE	0.51	0.46	0.55	0.60	0.33	<u>0.27</u>	0.35	0.44



785 **Figure 6:** Validation metrics for FB15k-237 extending the latent search to 100 epochs.

788 • Similarly, in both terms of the loss function, the embedding of the relation is kept constant,
 789 providing a similar constraint that prevents an excess of overfitting.

791 To further validate such a claim, we ran Imbalance on top of our models for 100 epochs for FB15k-
 792 237. As evident from Figure 6, MRR and H@10 on the validation set plateau after the first few
 793 epochs and then remain pretty much constant, showing non-degenerate behavior. Moreover, we
 794 remark once more how Imbalance is meant to be used at inference time: as such, we expect to run
 795 it for a limited number of iterations, as plots below show is recommendable.

796 **Training Context Size v Performance** We try to understand whether there is any correlation
 797 between the size of the training context and the improvement that IMBALANCE achieves on imbal-
 798 anced triples. However, Figure 7 shows how there is no such correlation, showing that the method
 799 can benefit imbalanced triples independently on the amount of information available in the training
 800 data, proving once more that the true issue lies in the degree imbalance.

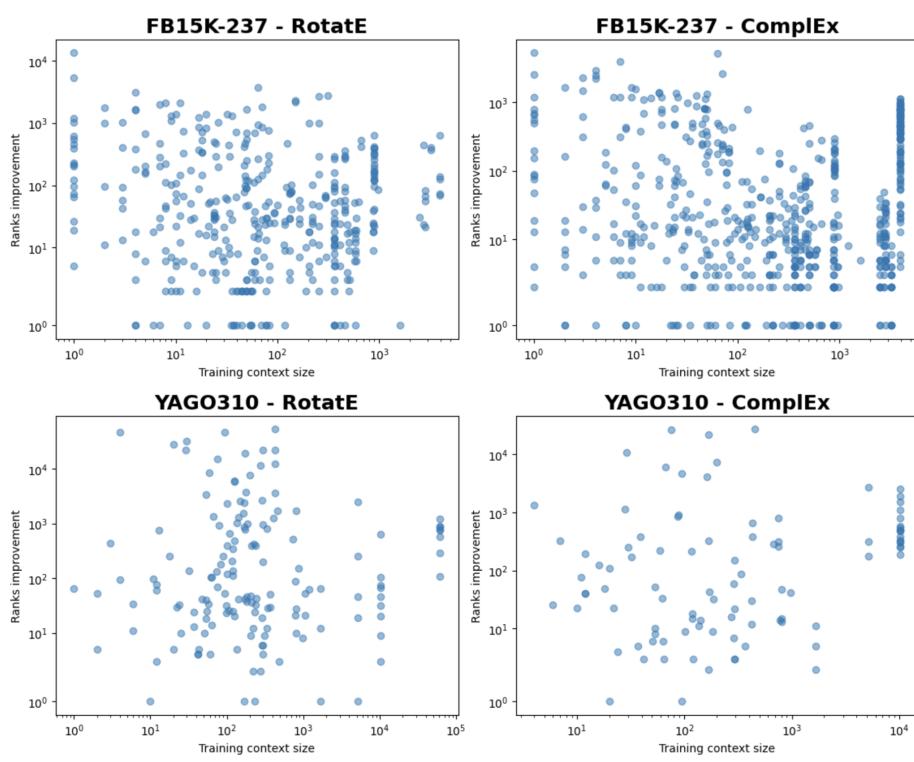


Figure 7: Correlation between the size of the training context and the rank improvement in FB15k-237 and Yago3-10.

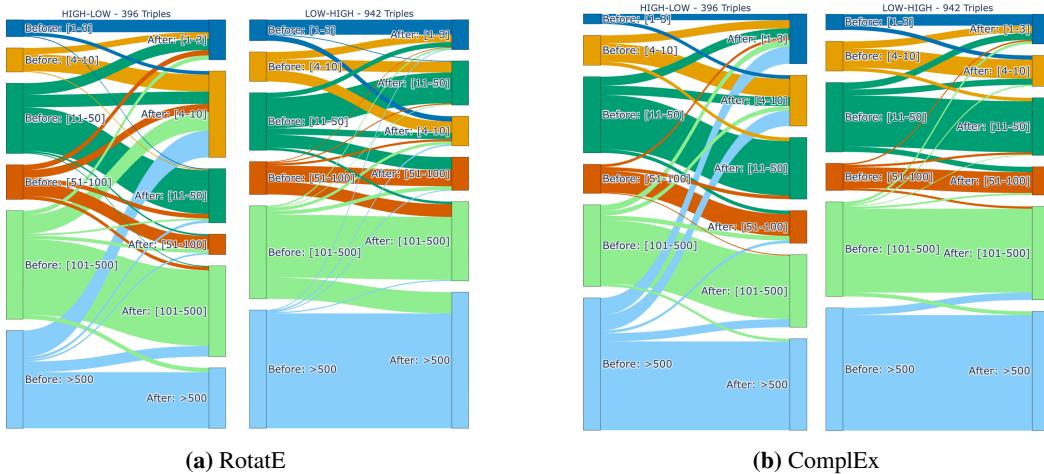


Figure 8: Sankey plots detailing how IMBALANCE altered RotatE and ComplEx ranks on FB15k-237.

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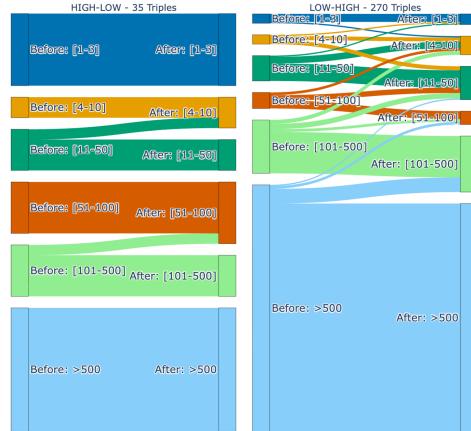
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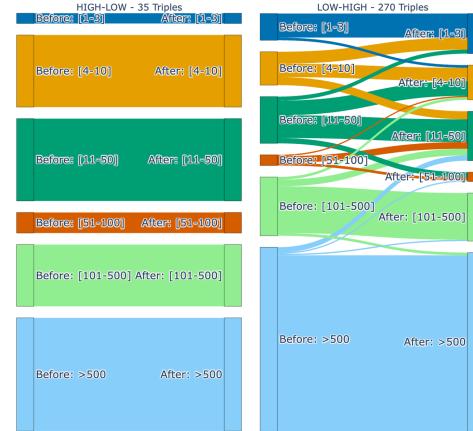
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(a) RotateE



(b) ComplEx

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Figure 9: Sankey plots detailing how IMBALANCE altered RotatE and ComplEx ranks on Yago3-10.

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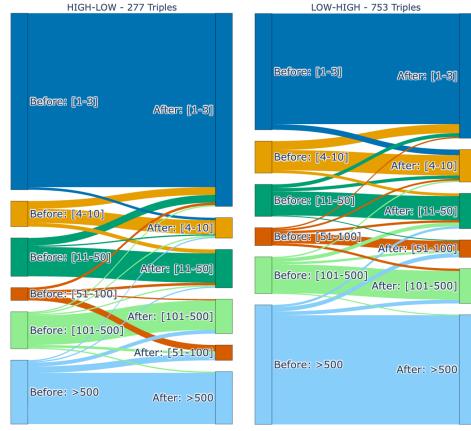
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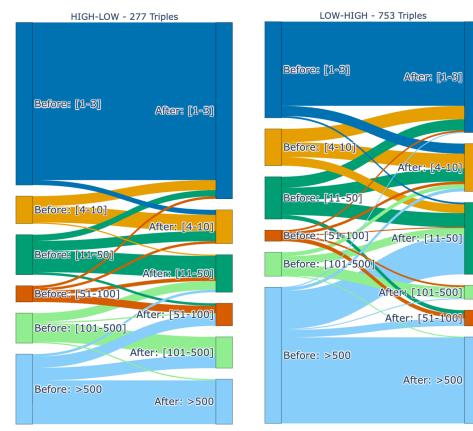
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(a) RotateE



(b) ComplEx

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Figure 10: Sankey plots detailing how IMBALANCE altered RotatE and ComplEx ranks on WN18RR.

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