RECURRENTLY CONTROLLING A RECURRENT NETWORK WITH RECURRENT NETWORKS CONTROLLED BY MORE RECURRENT NETWORKS

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ABSTRACT

This paper explores an intriguing idea of recursively parameterizing recurrent nets. Simply speaking, this refers to recurrently controlling a recurrent network with recurrent networks controlled by recurrent networks. The proposed architecture recursively parameterizes its gating functions whereby gating mechanisms of X-RNN are controlled by instances of itself, which are repeatedly called in a recursive fashion. We postulate that our proposed inductive bias provides modeling benefits pertaining to learning with inherently hierarchically-structured sequence data. To this end, we conduct extensive experiments on recursive logic tasks (sorting, tree traversal, logical inference), sequential pixel-by-pixel classification, semantic parsing, code generation, machine translation and polyphonic music modeling, demonstrating the widespread utility of the proposed approach, i.e., achieving optimistic and competitive results on all tasks.

1 INTRODUCTION

We explore the intriguing idea of recursively parameterizing recurrent networks. In our proposed approach, recurrent networks are controlled by recurrent networks which may or may not be controlled by more recurrent networks.

Why would we do something like this? In short, our proposed method, X-RNN (*Recurrently Controlling Recurrent Networks with Recurrent Networks Controlled by more Recurrent Networks*[1]), marries the benefits of recursive reasoning with recurrent models. Notably, we postulate that this formulation brings about benefits pertaining to modeling data that is intrinsically hierarchical (recursive) in nature, e.g., natural language, music and logic, an increasingly prosperous and emerging area of research (Shen et al., 2018; Wang et al., 2019; Choi et al., 2018).

In X-RNN, the gating functions of our model are now parameterized repeatedly by instances of itself which imbues our model with the ability to reason deeply and recursively about certain inputs. To achieve the latter, we propose a soft dynamic recursion mechanism, which softly learns the depth of recursive parameterization on a per-token basis. Notably, this is reminiscent of Adaptive Computation Time (ACT) (Graves, 2016), albeit operation at the parameter level[2]. Our formulation can be interpreted as a form of meta-gating since temporal compositionality is now being meta-controlled at various levels of abstractions.

Our Contributions Overall, the key contributions of this work are as follows:

- We propose a new sequence model - X-RNN. Our model is distinctly characterized by recursive parameterization of recurrent gates, i.e., compositional flow is controlled by instances of itself, á la repeatedly and recursively. We propose a soft dynamic recursion mechanism that dynamically and softly learns the recursive depth of the model at a token-level.

[1] We refer this to as X-RNN for short for the remainder of the paper.

[2] While seemingly similar, these methods are very different in the context of what the objective is. Our goal is to dynamically expand the parameters of the model, not dynamically decide how long to deliberate on input tokens.

- We evaluate our proposed method on a potpourri of sequence modeling tasks, i.e., logical recursive tasks (sorting, tree traversal, logical inference), pixel-wise sequential image classification, semantic parsing, neural machine translation and polyphonic music modeling. X-RNN achieves optimistic and competitive results.

2 Related Work

The study of effective inductive biases for sequential representation learning has been a prosperous research direction. This has spurred on research across multiple fronts, starting from gated recurrent models (Hochreiter & Schmidhuber, 1997; Cho et al., 2014), convolution (Bai et al., 2018a) to the recently popular self-attention based models (Vaswani et al., 2017).

The intrinsic hierarchical structure native to many forms of sequences has long fascinated and inspired many researchers (Socher et al., 2013; Bowman et al., 2014; 2016; Dyer et al., 2016). The study of recursive networks, popularized by (Socher et al., 2013) has provided a foundation for learning syntax-guided composition in language processing research. Along the same vein, (Tai et al., 2015) proposed Tree-LSTMs which guide LSTM composition with grammar. Recent attempts have been made to learn this process without guidance nor syntax-based supervision (Yogatama et al., 2016; Shen et al., 2017; Choi et al., 2018; Havrylov et al., 2019; Kim et al., 2019). Ordered Neuron LSTMs (Shen et al., 2018) proposed structured gating mechanisms, imbuing the recurrent unit with a tree-structured inductive bias. (Tran et al., 2018) shows that recurrence is important for modeling hierarchical structure. Notably, learning hierarchical representations across multiple time-scales (El Hihi & Bengio, 1996; Schmidhuber, 1992; Koutnik et al., 2014; Chung et al., 2016; Hafner et al., 2017) have also demonstrated reasonable success.

Learning an abstraction and controller over a base recurrent unit is also another compelling direction. First proposed by Fast Weights (Schmidhuber, 1992), several recent works explore this notion. HyperNetworks (Ha et al., 2016) learns to generate weights for another recurrent unit, i.e., a form of relaxed weight sharing. On the other hand, RCRN (Tay et al., 2018) explicitly parameterizes the gates of a RNN unit with other RNN units. Recent attempts to speed up the recurrent unit are also reminiscent of this particular notion (Bradbury et al., 2016; Lei et al., 2018).

The marriage of recursive and recurrent architectures is also notable. This direction is probably the closest relevance to our proposed method, although with vast differences. (Liu et al., 2014) proposed Recursive Recurrent Networks for machine translation which are concerned with the more traditional syntactic supervision concept of vanilla recursive nets. (Jacob et al., 2018) proposed RR-Net, which learns hierarchical structures on the fly. RR-Net proposes to learn to split or merge nodes at each time step, which makes it reminiscent of (Choi et al., 2018; Shen et al., 2018). (Alvarez-Melis & Jaakkola, 2016) proposed doubly recurrent decoders for tree-structured decoding. The core of their method is a depth and breath-wise recurrence which is similar to our model. However, X-RNN is concerned with learning gating controllers which is different from the objective of decoding trees.

Our work combines the idea of external meta-controllers (Schmidhuber, 1992; Ha et al., 2016; Tay et al., 2018) with recursive architectures. In particular, our recursive parameterization is also a form of dynamic memory which gives our model improved expressiveness in similar spirit to memory-augmented recurrent models (Santoro et al., 2018; Graves et al., 2014; Tran et al., 2016).

3 The Proposed Method

This section introduces our proposed model. X-RNN is fundamentally a recurrent model. The key difference is that the gating functions that control compositionality over time is parameterized by a recursion parameterization of itself.

3.1 X-RECURRENT UNIT

Our proposed model accepts a sequence of vectors $X \in \mathbb{R}^{\ell \times d}$ as input. The main unit of the X-RNN unit $h_t = \text{XRNN}_n(x_t, h_{t-1})$ is defined as follows:

$$f_t^n = \sigma_s(\alpha_t \, \text{XRNN}_{n+1}(x_t, h_{t-1}) + (1 - \alpha_t) \, F_n^F(x_t, h_{t-1})) \tag{1}$$

$$o_t^n = \sigma_s(\beta_t \, \text{XRNN}_{n+1}(x_t, h_{t-1}^n) + (1 - \beta_t) \, F_n^O(x_t, h_{t-1})) \tag{2}$$

$$z_t^n = \sigma_r(F_n^Z(x_t, h_t)) \text{ and } c_t^n = (1 - f_t) \odot h_{t-1} + (f_t) \odot z_t \tag{3}$$

$$h_t^n = o_t \odot c_t \text{ and } h_t^n = h_t + x_t \tag{4}$$

where σ_r is a nonlinear activation such as tanh. σ_s is the sigmoid activation function. In a nutshell, the X-RNN unit recursively calls itself until a max depth L is hit. When $n = L$, f_t and o_t are parameterized by:

$$f_t^L = \sigma_s(F_L^F(x_t, h_{t-1})) \text{ and } o_t^L = \sigma_s(F_L^O(x_t, h_{t-1})) \tag{5}$$

where f_t^L, o_t^L is the forget and output gate of X-RNN at time step t while at the maximum depth L. We also include an optional residual connection $h_t^n = h_t + x_t$ to facilitate gradient flow down the recursive parameterization of X-RNN.

Soft Dynamic Recursion We propose learning the depth of recursion in a data-driven fashion. To learn α_t, β_t, we use the following:

$$\alpha_t = F_\alpha(x_t) \text{ and } \beta_t = F_\beta(x_t)$$

where $F_*(xt) = Wx_t + b$ is a simple linear transformation layer applied to sequence X across the temporal dimension. Intuitively, α, β control the extent of recursion, enabling a *soft* depth pertaining to the hierarchical parameterization. Alternatively, we may also consider a static variation where:

$$\alpha_t = F_\alpha(\sum_{t=0}^{\ell} x_t) \text{ and } \beta_t = F_\beta(\sum_{t=0}^{\ell} x_t)$$

where the same value of α, β is computed based on global information from the entire sequence. Note that this strictly cannot be used for autoregressive decoding. Finally, we note that it is also possible to assign $\alpha \in \mathbb{R}, \beta \in \mathbb{R}$ to be trainable scalar parameters.

Depth-wise Parameterization Intuitively, $F_n^* \, \forall * \in F, O, Z$ are depth-wise parameters of X-RNN. We parameterize F_n with depth-wise RNN units.

$$F_n^*(x_t) = \text{RNN}_n^*(x_t, h_{t-1}^n)$$

Alternatively, we may also parameterize F_n with simple linear transformations.

$$F_n^*(x_t) = W_n^* x_t + b_n^*$$

Overall, X-RNN is agnostic to the choice of $F_n^*(x_t)$ and even the RNN unit. Note that for RNN, the hidden states are initialized with zero and each level uses a new initial hidden state.

Connections to other recurrent models When setting the max depth to $n = 1$, $\alpha = 0, \beta = 0$ and parameterizing all $F_n()$ with linear transforms, this reverts X-RNN to a simple recurrent model (SRU) Lei et al. (2018) with two gates (forget and output).

$$\begin{aligned}
f_t^n &= (1 - \alpha_t) \, F_n^F(x_t, h_{t-1})) \\
o_t^n &= (1 - \beta_t) \, F_n^O(x_t, h_{t-1})) \\
z_t^n &= \sigma_r(F_n^Z(x_t, h_t)) \\
c_t^n &= (1 - f_t) \odot h_{t-1} + (f_t) \odot z_t \\
h_t^n &= o_t \odot c_t
\end{aligned}$$

The key difference between the SRU and standard RNNs is that we do not include the previous hidden state h_{t-1} into the gate construction functions. When $F_n(x_t, h_t) = W([x_t, h_{t-1}]) + b$, this is similar fashion to the standard recurrent models like LSTMs or GRUs. Similarly, when $n = 1$ and $F_n(.)$ is a 1D convolution model, this becomes the Quasi RNN model Bradbury et al. (2016). When $F_n()$ is a LSTM/GRU unit, then this becomes the Recurrently Controlled Recurrent Network Tay et al. (2018) model.

Parallel Variation We postulate that X-RNN can also be useful as a non-autoregressive[3] parallel model.. This can be interpreted as a form of recursive feed-forward layer that is used in place of recurrent X-RNN for speed benefits. In early experiments, we find this a useful enhancement to state-of-the-art Transformer (Vaswani et al., 2017) models. The non-autoregressive variant of X-RNN is written as follows:

$$f_t^n = \sigma_s(\alpha_t \, \text{XRNN}_{n+1}(x_t) + (1 - \alpha_t) \, F_n^F(x_t)) \tag{6}$$

$$o_t^n = \sigma_s(\beta_t \, \text{XRNN}_{n+1}(x_t) + (1 - \beta_t) \, F_n^O(x_t)) \tag{7}$$

$$z_t^n = \sigma_r(F_n^Z(x_t)) \text{ and } h_t^n = (f_t^n \odot x_t) + (o_t \odot z_t^n) \text{ and } h_t^n = h_t + x_t \tag{8}$$

More concretely, we dispense with the reliance on the previous hidden state. This can be used in place of any position-wise feed-forward layer. In this case, note that $F_n^*(x_t)$ are typically position-wise functions as well.

4 EXPERIMENTS

We conduct experiments on a suite of diagnostic synthetic tasks and real world tasks.

4.1 LOGIC / RECURSIVE TASKS

We evaluate our model on three diagnostic logical tasks. The tasks are described as follows:

	TREE TRAVERSAL										SORT			
	$n=3$		$n=4$		$n=5$		$n=8$		$n=10$		$n=5$		$n=10$	
Model	EM	P	EM	P	EM	P	EM	P	EM	P	EM	P	EM	P
BiLSTM	100	1.0	96.9	2.4	60.3	2.4	5.6	30.6	2.2	132	79.9	1.2	78.9	1.2
S-BiLSTM	100	1.0	98.0	1.0	63.4	2.5	**5.9**	99.9	2.8	225	83.4	1.2	88.0	1.1
ON-LSTM	100	1.0	81.0	1.4	55.7	2.8	5.5	52.3	2.7	173	90.8	1.1	87.4	1.1
X-RNN	**100**	**1.0**	**98.4**	**1.0**	**63.4**	**1.8**	5.6	**20.4**	**2.8**	**119**	**92.2**	**1.1**	**90.6**	**1.1**

Table 1: Experimental Results on Tree Traversal and Sorting.

	# Operations					
Model	$n=7$	$n=8$	$n=9$	$n=10$	$n=11$	$n=12$
Tree-LSTM† (Tai et al., 2015)	93.0	90.0	87.0	89.0	86.0	87.0
LSTM (Bowman et al., 2014)	88.0	85.0	80.0	78.0	71.0	69.0
RRNet (Jacob et al., 2018)	84.0	81.0	78.0	74.0	72.0	71.0
Ordered Neurons (Shen et al., 2018)	91.0	87.0	86.0	81.0	78.0	76.0
X-RNN	**97.0**	**95.0**	**93.0**	**92.0**	**90.0**	**88.0**

Table 2: Experimental results on Logical Inference task. † denotes models with access to ground truth syntax. Results reported from (Shen et al., 2018). X-RNN achieves state-of-the-art performance.

Task 1 (SORT SEQUENCES) - The input to the model is a sequence of integers. The correct output is the sorted sequence of integers. Since mapping sorted inputs to outputs can be implemented in a recursive fashion, we evaluate our model's ability to better model recursively structured sequence data. Example input output pair would be $9, 1, 10, 5, 3 \rightarrow 1, 3, 5, 9, 10$.

Task 2 (TREE TRAVERSAL) - We construct a binary tree of maximum depth N. The goal is to generate the *postorder* tree traversal given the *inorder* **and** *preorder* traversal of the tree. Note that this is known to arrive at only one unique solution. The constructed trees have random sparsity where we assign a probability p of growing the tree up to maximum depth N. Hence, the trees can be of varying depths[4]. This requires inferring hierarchical structure and long-term reasoning across sequences. We concatenate the *postorder* and *inorder* sequences, delimited by a special token. We evaluate on $n \in \{3, 4, 5, 8, 10\}$. For $n = \{5, 8\}$, we ensure that each tree traversal has at least 10 tokens. For $n = 10$, we ensure that each path has at least 15 tokens. Example input output pair would be $13, 15, 4, 7, 5, X, 13, 4, 15, 5, 7 \rightarrow 7, 15, 13, 4, 5$.

[3]To clarify, decoding tasks (e.g., in NMT) are still done in a step-by-step autoregressive fashion.

[4]Note that all our models solve the problem entirely when the tree is fixed and full. Hence, random trees provide a necessary challenge.

Task 3 (LOGICAL INFERENCE) - We use the standard logical inference dataset[5] proposed in (Bowman et al., 2014). This is a classification task in which the goal is to determine the semantic equivalence of two statements expressed with logic operators such as *not*, *and*, and *or*. The language vocabulary is of six words and three logic operators. As per prior work (Shen et al., 2018), the model is trained on sequences with 6 or fewer operations and evaluated on sequences of 6 to 12 operations.

For Task 1 and Task 2, we frame these tasks as a Seq2Seq (Sutskever et al., 2014) task and evaluate models on exact match accuracy and perplexity (P) metrics. We use a standard encoder-decoder architecture with attention (Bahdanau et al., 2014). We vary the encoder module with BiLSTMs, Stacked BiLSTMs (3 layers) and Ordered Neuron LSTMs (Shen et al., 2018). For Task 3 (logical inference), we use the common setting in other published works.

Results on Sorting and Tree Traversal Table 1 reports our results on the Sorting and Tree Traversal task. All models solve the task with $n = 3$. However, the task gets increasingly harder with a greater maximum possible length and largely still remains a challenge for neural models today. The relative performance of X-RNN is on a whole better than any of the baselines, especially pertaining to perplexity. We also found that S-BiLSTMs are always better than LSTMs on this task and Ordered LSTMs are slightly worst than vanilla BiLSTMs. However, on sorting, ON-LSTMs are much better than standard BiLSTMs and S-BiLSTMs.

Results on Logical Inference Table 2 reports our results on logical inference task. We compare with mainly other published work. X-RNN is a strong and competitive model on this task, outperforming ON-LSTM by a wide margin ($+12\%$ on the longest number of operations). Performance of our model also exceeds Tree-LSTM, which has access to ground truth syntax. Our model achieves state-of-the-art performance on this dataset even when considering models with access to syntactic information.

4.2 PIXEL-WISE SEQUENTIAL IMAGE CLASSIFICATION

We evaluate our model on its ability to model and capture long-range dependencies. More specifically, the sequential pixel-wise image classification problem treats pixels in images as sequences. We use the well-established pixel-wise MNIST and CIFAR-10 datasets. We use 3 layered X-RNN of 128 hidden units each.

Model	$\|\theta\|$	MNIST	CIFAR
DilatedGRU (Chang et al., 2017)	-	99.00	-
IndRNN (Li et al., 2018a)	-	99.00	-
r-LSTM (Trinh et al., 2018)	-	98.52	72.20
Transformer (Trinh et al., 2018)	-	98.90	62.20
TrellisNet (Bai et al., 2018b)	8.0M	**99.20**	73.42
TrellisNet (Our run)	8.0M	97.59	55.83
X-RNN	0.9M	99.04	73.01
X-RNN$^\sharp$	0.9M	99.09	**73.95**

Table 3: Experimental results (accuracy) on Pixel-wise Sequential Image Classification. We also trained the recent R-Adam optimizer (Liu et al., 2019) which we found to have improved performance (results denoted with $\sharp$).

Results on Pixel-wise Image Classification Table 3 reports the results of X-RNN against other published works. Our method achieves state-of-the-art performance on the CIFAR-10 dataset, outperforming the recent Trellis Network (Bai et al., 2018b). On the other hand, results on MNIST are reasonable, outperforming a wide range of other published works. On top of that, our method has 8 times fewer parameters than Trellis network (Bai et al., 2018b) while achieving similar or better performance. This ascertains that X-RNN is a reasonably competitive long-range sequence encoder.

4.3 SEMANTIC PARSING / CODE GENERATION

We evaluate X-RNN on semantic parsing (GEO, ATIS, JOBS) and code generation (DJANGO), a task mainly concerned with learning to parse and generate structured data. We run our experiments on the publicly released source code[6] of (Yin & Neubig, 2018), replacing the recurrent decoder with our X-RNN decoder. We only replaced the recurrent decoder since early experiments showed that

[5] 3https://github.com/sleepinyourhat/vector-entailment.

[6] https://github.com/pcyin/tranX

varying the encoder did not yield any benefits in performance. Overall, our hyperparameter details followed the codebase of (Yin & Neubig, 2018) quite strictly, i.e., we ran all models from their codebase *as it is*.

Results on Semantic Parsing and Code Generation Table 4 reports our experimental results on Semantic Parsing (GEO, ATIS, JOBS) and Code Generation (DJANGO). We observe that TranX + X-RNN outperforms all competitor approaches, achieving state-of-the-art performance. On Django, we outperform TranX by $+1.6\%$ and $\approx +1\%$ on all semantic parsing tasks. More importantly, the performance gain over the base TranX method allows us to observe the ablative benefits of X-RNN which is achieved by only varying the recurrent decoder.

Model	GEO	ATIS	JOBS	DJANGO
(Dong & Lapata, 2016)	87.1	84.6	-	31.5
(Ling et al., 2016)	-	-	-	62.3
(Neubig, 2015)	-	-	-	45.1
(Yin & Neubig, 2017)	-	-	-	71.6
(Rabinovich et al., 2017)	87.1	85.9	-	-
(Yin & Neubig, 2018)	88.2	86.2	-	72.7
TranX (Code reported)	**88.6**	87.7	90.0	77.2
TranX (Our Run)	87.5	87.5	90.0	76.7
TranX + X-RNN	**88.6**	**88.4**	**90.7**	**78.3**

Table 4: Experimental results on Semantic Parsing and Code Generation.

4.4 NEURAL MACHINE TRANSLATION

This task is a sequence transduction task which is concerned with translating a source language to a target language. We conduct experiments on two IWSLT datasets which are collections derived from TED talks. Specifically, we compare on the IWSLT 2014 German-English and IWSLT 2015 English-Vietnamese datasets. We compare against a suite of published results and strong baselines. For our method, we replaced the multi-head aggregation layer in the Transformer networks (Vaswani et al., 2017) with a parallel version adaptation of X-RNN. The base models are all linear layers. For our experiments, we use the standard implementation and hyperparameters in Tensor2Tensor[7] (Vaswani et al., 2018), using the small (S) and base (B) setting for Transformers.

Model	BLEU
(Luong & Manning, 2015)	23.30
(Bahdanau et al., 2014)	26.10
(Huang et al., 2017)	28.07
Transformer B	28.43
Transformer B + X-RNN	**30.81**

Table 5: Experimental results on Neural Machine Translation on IWSLT 2015 En-Vi.

For evaluation, model averaging is used (5 recent checkpoints) and beam size of $8/4$ and length penalty of 0.6 is adopted for De-En and En-Vi respectively. For our model, max depth is tuned amongst $\{1, 2, 3\}$. We also ensure to compare, in an ablative fashion, our own reported runs of the base Transformer models.

Model	BLEU
(Ranzato et al., 2015)	21.83
(Bahdanau et al., 2016)	28.53
(Huang et al., 2017)	28.96
(Wang et al., 2018)	32.35
(He et al., 2018)	35.07
Transformer S (Our run)	34.68
Transformer B (Our run)	36.30
Transformer S + X-RNN	35.15
Transformer B + X-RNN	**37.09**

Table 6: Experimental results on Neural Machine Translation on IWSLT 2014 De-En.

Results on Neural Machine Translation Table 6 reports results on IWSLT 2014 de-en task. Our proposed model performs very competitively (37.09 BLEU), outperforming many well-established baselines. Our results also show that equipping Transformer models with X-RNN can also lead to improvements in performance. Notably there is a $+0.69$ BLEU improvement on Transformer Base and $+0.42$ BLEU improvement for Transformer Small. On the other hand, our method achieves 30.81 BLEU on the IWSLT 2015 En-Vi dataset, with $+2.38$ improvement in BLEU from the standard Transformer Base model.

4.5 POLYPHONIC MUSIC MODELING

We evaluate X-RNN on the polyphonic music modeling, i.e., generative modeling of musical sequences. We use three well-established datasets, namely Nottingham, JSB Chorales and Piano Midi (Boulanger-Lewandowski et al., 2012). The inputs to the model are 88-bit sequences, each

[7] https://github.com/tensorflow/tensor2tensor

corresponding to the 88 keys of the piano. The task is evaluated on the Negative Log-likelihood (NLL). We compare with a wide range of published works (Chung et al., 2014; Bai et al., 2018a; Song et al., 2019) on this task.

Model	Nott	JSB	Piano
GRU (Chung et al.)	3.13	8.54	8.82
Song et al.	3.25	8.61	7.99
Li et al.	3.21	8.67	8.18
Song et al.	3.16	8.30	7.55
Bai et al.	3.07	**8.10**	-
TCN (our run)	2.95	8.13	7.53
X-RNN	**2.88**	8.12	**7.49**

Table 7: Experimental Results (NLL) on Polyphonic Music Modeling.

Results on Music Modeling Table 7 reports our scores on this task. X-RNN achieves state-of-the-art performance on the Nottingham and Piano midi datasets. Our model also achieves competitive performance on the JSB dataset, only underperforming the state-of-the-art by 0.01 NLL. On a whole, our model also outperforms a wide range of competitive models such as Gumbel Gate LSTMs (Li et al., 2018b). Moreover, the performance gain over standard LSTM and GRUs are also relatively large.

4.6 Analysis and Discussion

This section reports some analysis and discussion regarding the proposed model. We hope to understand the behaviour of the model, at the activation level of different recursive depths.

Max N	Base Model	ATIS	DJANGO
2	Linear	**88.40**	77.56
3	Linear	88.21	77.62
4	Linear	87.80	76.84
2	LSTM	86.61	**78.33**
3	LSTM	85.93	77.39

Table 8: Ablation studies on Semantic Parsing and Code Generation.

Task	N	Base Unit
Tree Traversal	2	Recurrent
Sorting	2	Recurrent
Logical Inference	3	Recurrent
Pixel-wise Classification	2	Recurrent
Semantic Parsing	2	Linear
Code Generation	2	Recurrent
Machine Translation	3	Linear
Polyphonic Music	3	Linear

Table 9: Optimal Maximum Depth N and base unit for different tasks.

4.6.1 Effect of Maximum Depth and Base Unit

Table 8 reports some ablation studies on the semantic parsing and code generation tasks. We observe that the base unit and optimal maximum depth used is task dependent. For ATIS dataset, using the linear transform as the base unit performs the best. Conversely, the linear base unit performs worse than the recurrent base unit (LSTM) on the DJANGO dataset. On a whole, we also observed this across other tasks, i.e., the base unit and maximum depth of X-RNN is a critical choice for most tasks. Table 9 reports the optimal max depth N and best base unit for each task.

4.6.2 Analysis of Soft Dynamic Recursion

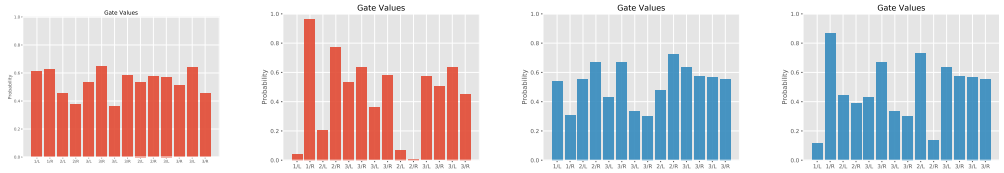

Figure 1: Initial (CIFAR) Figure 2: Epoch 10 (CIFAR) Figure 3: Initial (MNIST) Figure 4: Epoch 10 (MNIST)

Figure 5: Depth Gate Visualization on CIFAR and MNIST.

Figure 5 illustrates the depth gate values on CIFAR and MNIST datasets. These values reflect the α and β values in X-RNN, signifying how the parameter tree is being constructed during training. This is reflected as L and R in the figures representing left and right gates. Firstly, we observe that our

model indeed builds data-specific parameterization of the network. This is denoted by how X-RNN builds different[8] trees for CIFAR and MNIST.

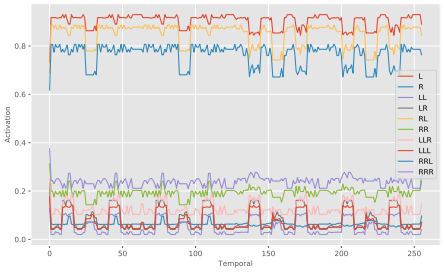

Figure 6: Visualisation of Dynamic Depth Recursion on Music.

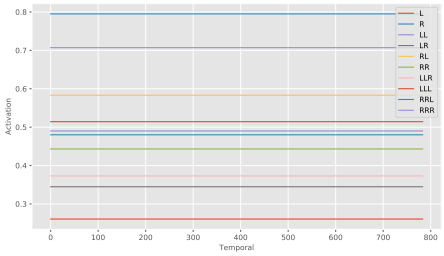

Figure 7: Visualisation of Dynamic Depth Recursion on MNIST.

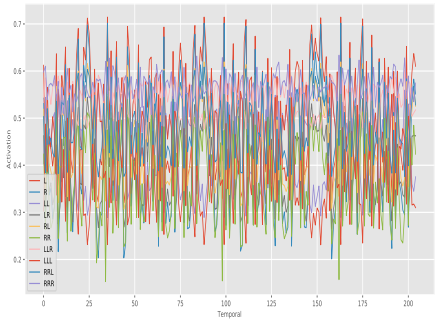

Figure 8: Visualisation of Dynamic Depth Recursion on CIFAR.

Secondly, we analyze the dynamic recursion depth with respect to time steps. The key observation that all datasets have very diverse construction of recursive parameters. The recursive gates fluctuate aggressively on CIFAR while remaining more stable on Music modeling. Moreover, we found that the recursive gates remain totally constant on MNIST. This demonstrates that our model has the ability to adjust the dynamic construction adaptively and can revert to static recursion over time if necessary. We find that compelling.

The adaptive recursive depth is made more intriguing by observing how the recursive parameterization alters on CIFAR and Music datasets. From Figure 6 we observe that the structure of the network changes in a rhythmic fashion, in line with our intuition of musical data. When dealing with pixel information, the tree structure changes adaptively according to the more complex information processed by the network.

5 Conclusion

We proposed X-RNN, a new sequence model characterized by recursive parameterization of gating functions. We explored the interesting idea of *recurrently controlling recurrent networks by recurrent networks controlled by more recurrent networks* and find that the proposed method achieves very promising and competitive results on a spectrum of benchmarks across multiple modalities (e.g., language, logic, music). Our proposed X-RNN outperforms the recent state-of-the-art Ordered Neurons on Logical Inference task along with other tree-structured analysis tasks. Additionally, we propose a parallel variation of X-RNN, which allows simple drop-in enhancement to state-of-the-art Transformers while retaining efficiency. We study and visualize our network as it learns a dynamic recursive parameterization, shedding light on the expressiveness and flexibility to learn dynamic parameter structures depending on the data.

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[8]Though not depicted, we also found that the probability of each node expanding to children has low variance across batches in the same dataset.

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