Certifiable Factor Graph Optimization

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***Abstract*—We show that the *factor graph* and *certifiable estimation* paradigms can be naturally synthesized into a unified framework for *certifiable factor graph optimization*. The key insight is that the core constructions underpinning certifiable estimation (Shor's relaxation and Burer-Monteiro factorization) inherit a factor graph structure from the original problem: the resulting *lifted* problem has *identical* factor graph connectivity, with variables and factors obtained via one-to-one algebraic transformations (*lifts*) of those in the original factor graph. This structural preservation enables the Riemannian Staircase methodology to be implemented within existing factor graph libraries by simply substituting lifted variable and factor types. Experiments on pose graph optimization and range-aided SLAM benchmarks confirm functional equivalence with state-of-the-art hand-designed certifiable estimators, while reducing the required implementation effort from months to hours.**

***Index Terms*—Factor graphs, certifiable estimation, semidefinite optimization, Burer-Monteiro factorization, Riemannian Staircase, SLAM**

I. Introduction

State estimation is the problem of inferring the values of a set of unknown parameters from noisy sensor data, and it lies at the heart of mobile robotics [1], [2]. Canonical examples include SLAM [3], [4] and structure from motion [5], typically formalized as maximum likelihood estimation (MLE) [6].

Factor graphs [6] provide the dominant paradigm for these problems: practitioners compose standard variable types and measurement factors to specify estimation tasks, and deploy efficient local optimization via libraries such as GTSAM [7] and g2o [8]. However, *local* optimization offers no guarantee of global optimality, and can return egregiously wrong estimates *without warning* [4].

Certifiable estimation methods [9], [10] address this limitation via convex (semidefinite) relaxations that can recover *verifiably globally optimal* solutions. *Burer-Monteiro (BM) factorization* [11] and the *Riemannian Staircase* [12], [13] form the basis of essentially all state-of-the-art certifiable estimators for *large-scale* spatial estimation problems (SLAM and 3D visual reconstruction) [9], [14]–[16], but deploying them requires problem-specific SDP relaxations, custom Riemannian optimization algorithms, and substantial specialized expertise; this process demands months of effort per problem.

In this paper, we show that these two paradigms can be synthesized into a unified framework for *certifiable factor graph optimization*. We prove that the BM-factored Shor relaxation of a QCQP with factor graph structure admits a *lifted factor graph* with *identical connectivity*, whose variables and factors are one-to-one algebraic transformations of those in the original graph, enabling the Riemannian Staircase to be implemented within existing factor graph libraries with no custom solver development. We validate on PGO and range-aided SLAM benchmarks.

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II. Background

In this section, we briefly review the use of factor graphs and certifiable estimation, which form the basis of our approach.

A. Factor graph-based estimation

Let $X \triangleq (X_1, \dots, X_N)$ denote N unknown parameters to be inferred, and $\tilde{z} \triangleq \{\tilde{z}_k\}_{k=1}^K$ be a set of K noisy sensor measurements. Each measurement $\tilde{z}_k$ is sampled conditionally independently:

$$\tilde{z}_k \sim p_k(\cdot \mid X_{S_k}), \quad \forall k \in [K], \tag{1}$$

where $S_k \subseteq [N]$ specifies the subset X_{S_k} of parameters upon which the kth observation depends. Conditional independence implies $p(\tilde{z} \mid X) = \prod_{k=1}^K p_k(\tilde{z}_k \mid X_{S_k})$.

The *factor graph* $\mathcal{G} = (\mathcal{V}, \mathcal{F}, \mathcal{E})$ associated to this factorization is the bipartite graph with variable nodes $\mathcal{V} \triangleq \{X_1, \dots, X_N\}$, factor nodes $\mathcal{F} \triangleq \{p_1, \dots, p_K\}$, and edges $\mathcal{E} \triangleq \{(X_n, p_k) \mid X_n \in X_{S_k}\}$ [6]. Inference is performed via *maximum likelihood estimation* (MLE):

$$\hat{X}_{\text{MLE}}(\tilde{z}) \triangleq \operatorname*{argmin}_{X_n \in \mathcal{X}_n} \sum_{k=1}^K \ell_k(X_{S_k}; \tilde{z}_k), \tag{2}$$

where $\ell_k \triangleq -\log p_k(\tilde{z}_k \mid X_{S_k})$ and each X_n takes values in a set $\mathcal{X}_n$. Local optimization methods on manifolds [17] can efficiently recover critical points of (2), but cannot guarantee convergence to the *global* minimizer.

B. Certifiable estimation

Many estimation problems have MLEs (2) that take the form of *quadratically-constrained quadratic programs* (QCQPs):

$$\begin{aligned} f^\star_{\text{QCQP}} = \min_{X \in \mathbb{R}^{r \times d}} \; & \langle Q, XX^\mathsf{T} \rangle \\ \text{s.t.} \; & \langle A_m, XX^\mathsf{T} \rangle = b_m, \quad m \in [M], \end{aligned} \tag{3}$$

with $Q, A_m \in \mathbb{S}^r$ and $b \in \mathbb{R}^M$.

1) Shor's relaxation: Shor's relaxation [18] replaces XX^T in (3) with a generic PSD matrix $Z \in \mathbb{S}^r_+$, producing the convex SDP:

$$\begin{aligned} f^\star_{\text{SDP}} = \min_{Z \in \mathbb{S}^r_+} \; & \langle Q, Z \rangle \\ \text{s.t.} \; & \langle A_m, Z \rangle = b_m, \quad m \in [M]. \end{aligned} \tag{4}$$

This relaxation satisfies $f^\star_{\text{SDP}} \leq f^\star_{\text{QCQP}}$, and is typically *exact* when (3) arises from an MLE with sufficiently low noise [19].

2) Burer-Monteiro factorization: The $\mathcal{O}(r^2)$ cost of storing Z makes general-purpose SDP solvers impractical for large-scale problems [20]. The BM factorization [11] exploits low-rank solutions by replacing the original decision variable Z with an assumed low-rank factorization of the form YY^T, where $Y \in \mathbb{R}^{r\times p}$:

$$\begin{aligned} \min_{Y\in\mathbb{R}^{r\times p}} \quad & \langle Q, YY^\mathsf{T}\rangle \\ \text{s.t.} \quad & \langle A_m, YY^\mathsf{T}\rangle = b_m, \quad m \in [M], \end{aligned} \tag{5}$$

reducing memory to $\mathcal{O}(rp)$, while $YY^\mathsf{T} \succeq 0$ is automatically satisfied.

3) Verification and the Riemannian Staircase: Unlike the original SDP (4), the BM factorization (5) is *nonconvex*; consequently, local optimization methods applied to (5) may converge to stationary points that are *not* global minimizers. It is therefore necessary to verify whether a stationary point $Y^\star$ recovered from (5) is a low-rank factor for a *global* minimizer $Z^\star = Y^\star Y^{\star\mathsf{T}}$ of the SDP (4). Given a KKT point $Y^\star$ with multiplier $\lambda^\star$, the *certificate matrix* $S \triangleq Q + \mathcal{A}^\star(\lambda^\star)$ (where $\mathcal{A}^\star(\lambda) = \sum_m \lambda_m A_m$) certifies global optimality if $S \succeq 0$; otherwise, the minimum eigenvector of S furnishes a second-order descent direction into a higher-rank BM problem [13]. The *Riemannian Staircase* [12], [21] iterates this process until certification succeeds.

III. Certifiable Factor Graph Optimization

We now show that factor graph structure is *preserved* under Shor's relaxation and BM factorization.

A. Factor graph structure in QCQPs

Consider a QCQP (3) whose decision variable $X \in \mathbb{R}^{r\times d}$ is partitioned into N block rows:

$$X = \begin{bmatrix} X_1^\mathsf{T} & \cdots & X_N^\mathsf{T} \end{bmatrix}^\mathsf{T}, \tag{6}$$

with $X_n \in \mathbb{R}^{r_n\times d}$ and $r = \sum_{n=1}^N r_n$, and suppose that the resulting MLE admits a factor graph model $\mathcal{G} = (\mathcal{V}, \mathcal{F}, \mathcal{E})$:

$$\mathcal{V} \triangleq \{X_1, \ldots, X_N\}, \tag{7a}$$
$$\mathcal{F} \triangleq \{\ell_1, \ldots, \ell_K\}, \tag{7b}$$
$$\mathcal{E} \triangleq \{(X_n, \ell_k) \in \mathcal{V}\times\mathcal{F} \mid n \in S_k\}, \tag{7c}$$

where S_k indexes the variables of ℓ_k.

1) Block sparsity of objective data matrices: Under the stated hypotheses, each factor ℓ_k in (7b) is a quadratic function $\ell_k(X) = \langle Q_k, XX^\mathsf{T}\rangle$ for some $Q_k \in \mathbb{S}^r$. Partitioning Q_k conformally with (6):

$$Q_k = \begin{bmatrix} (Q_k)_{1,1} & \cdots & (Q_k)_{1,N} \\ \vdots & \ddots & \vdots \\ (Q_k)_{N,1} & \cdots & (Q_k)_{N,N} \end{bmatrix}, \tag{8}$$

substitution yields $\ell_k(X) = \sum_{i,j=1}^N \langle (Q_k)_{i,j}, X_i X_j^\mathsf{T}\rangle$, and the factor graph structure implies:

$$n \notin S_k \Longrightarrow (Q_k)_{n,i} = 0 \quad \forall i \in [N]; \tag{9}$$

that is, $\mathcal{G}$'s edge set $\mathcal{E}$ (7c) determines the block sparsity pattern of each Q_k.

2) Block-diagonal structure of constraint matrices: The feasible set of the factor graph MLE (2) is a *Cartesian product* $\mathcal{X} = \mathcal{X}_1 \times \cdots \times \mathcal{X}_N$, implying that the constraints in (3) are *separable* over $X_1, \ldots, X_N$. Partitioning the constraint indices into subsets $L_1, \ldots, L_N \subseteq [M]$ (where L_n indexes the constraints on X_n), each A_m has a *single nonzero block* on the diagonal:

$$m \in L_n \Longrightarrow A_m = \begin{bmatrix} 0 & & & & \\ & \ddots & & & \\ & & (A_m)_{n,n} & & \\ & & & \ddots & \\ & & & & 0 \end{bmatrix}. \tag{10}$$

The domain of X_n is thus the algebraic variety:

$$\mathcal{X}_n = \left\{ X_n \in \mathbb{R}^{r_n\times d} \mid \langle (A_m)_{n,n}, X_n X_n^\mathsf{T}\rangle = b_m, \ \forall m \in L_n \right\}. \tag{11}$$

3) Block decomposition of the QCQP: Taken together, these observations show that the structure of the factor graph $\mathcal{G}$ defined in (7) implies that the QCQP (3) must decompose into the following block form:

$$\begin{aligned} \min_{X_n\in\mathbb{R}^{r_n\times d}} \ & \sum_{k=1}^K \overbrace{\sum_{(i,j)\in S_k\times S_k} \langle (Q_k)_{i,j}, X_i X_j^\mathsf{T}\rangle}^{\ell_k(X_{S_k})} \\ \text{s.t.} \ & \langle (A_m)_{n,n}, X_n X_n^\mathsf{T}\rangle = b_m, \ \forall m \in L_n, \ n \in [N]. \end{aligned} \tag{12}$$

Conversely, any QCQP admitting such a sparse block decomposition defines a valid factor graph $\mathcal{G} = (\mathcal{V}, \mathcal{F}, \mathcal{E})$ via (7).

B. Shor relaxation and BM factorization inherit factor graph structure

The key observation is that *the same data matrices Q and A_m parameterize both the original QCQP* (3) *and its BM-factored Shor relaxation* (5), so any block sparsity and separability structure implied by $\mathcal{G}$ is *automatically* inherited. Partitioning the rows of Y in (5) conformally with (6):

$$Y = \begin{bmatrix} Y_1^\mathsf{T} & \cdots & Y_N^\mathsf{T} \end{bmatrix}^\mathsf{T}, \tag{13}$$

where $Y_n \in \mathbb{R}^{r_n\times p}$, the BM-factored Shor relaxation admits the analogous sparse block decomposition:

$$\begin{aligned} \min_{Y_n\in\mathbb{R}^{r_n\times p}} \ & \sum_{k=1}^K \overbrace{\sum_{(i,j)\in S_k\times S_k} \langle (Q_k)_{i,j}, Y_i Y_j^\mathsf{T}\rangle}^{\triangleq \bar{\ell}_k(Y_{S_k})} \\ \text{s.t.} \ & \langle (A_m)_{n,n}, Y_n Y_n^\mathsf{T}\rangle = b_m, \ \forall m \in L_n, \ n \in [N]. \end{aligned} \tag{14}$$

Since (14) is itself a QCQP, we may construct an associated *lifted* (or *certifiable*) *factor graph* $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{F}}, \bar{\mathcal{E}})$:

$$\bar{\mathcal{V}} \triangleq \{Y_1, \ldots, Y_N\}, \tag{15a}$$
$$\bar{\mathcal{F}} \triangleq \{\bar{\ell}_1, \ldots, \bar{\ell}_K\}, \tag{15b}$$
$$\bar{\mathcal{E}} \triangleq \{(Y_n, \bar{\ell}_k) \in \bar{\mathcal{V}}\times\bar{\mathcal{F}} \mid n \in S_k\}, \tag{15c}$$

TABLE I
Summary of common lifted variable and factor types

Type	Original	Lifted (rank-p)
Rotation	$R \in O(d)$	$Y \in \mathrm{St}(d,p)$
Unit sphere	$v \in S^{d-1}$	$u \in S^{p-1}$
Translation	$t \in \mathbb{R}^d$	$u \in \mathbb{R}^p$
Rel. rotation	$\kappa_{ij}\|R_j - R_i\tilde{R}_{ij}\|_F^2$	$\kappa_{ij}\|Y_j - Y_i\tilde{R}_{ij}\|_F^2$
Rel. translation	$\tau_{ij}\|t_j - t_i - R_i\tilde{t}_{ij}\|_2^2$	$\tau_{ij}\|u_j - u_i - Y_i\tilde{t}_{ij}\|_2^2$
Range	$\sigma_{ij}^{-2}\|t_j - t_i - \tilde{r}b\|_2^2$	$\sigma_{ij}^{-2}\|u_j - u_i - \tilde{r}s\|_2^2$

with variable domains:

$$\mathcal{Y}_n = \left\{ Y_n \in \mathbb{R}^{r_n \times p} \mid \langle (A_m)_{n,n}, Y_n Y_n^\mathsf{T} \rangle = b_m, \ \forall m \in L_n \right\}. \tag{16}$$

By construction, the variable and factor sets for $\bar{\mathcal{G}}$ and $\mathcal{G}$ are in *one-to-one correspondence*, and their *connectivities coincide*: each domain $\mathcal{X}_n$ is replaced by $\mathcal{Y}_n$, and each factor ℓ_k by $\bar{\ell}_k$. One thus obtains $\bar{\mathcal{G}}$ from $\mathcal{G}$ simply by *replacing the individual variables and factors with their lifted counterparts*.

C. Block separability enables efficient certificate computation

The block-diagonal structure (10) also enables efficient parallel construction of the certificate matrix S. The adjoint operator $\mathcal{A}^\star(\lambda) = \sum_m \lambda_m A_m$ decomposes as:

$$\mathcal{A}^\star(\lambda) = \begin{bmatrix} \sum_{m \in L_1} \lambda_m (A_m)_{1,1} & & \\ & \ddots & \\ & & \sum_{m \in L_N} \lambda_m (A_m)_{N,N} \end{bmatrix}; \tag{17}$$

that is, $\mathcal{A}^\star(\lambda)$ is block-diagonal, with the nth block constructed using *only* the constraint matrices and multipliers associated with Y_n. Moreover, the least-squares Lagrange multiplier computation [13] is *separable* over the N variables, yielding N *independent* low-dimensional problems:

$$\lambda_{L_n}^\star = \underset{\lambda_{L_n} \in \mathbb{R}^{|L_n|}}{\operatorname{argmin}} \left\| (QY)_n + \sum_{m \in L_n} \lambda_m (A_m)_{n,n} Y_n \right\|^2. \tag{18}$$

D. The Riemannian Staircase over factor graphs

Sections III-A–III-C enable the Riemannian Staircase to be implemented using existing factor graph libraries; Algorithm 1 summarizes this instantiation.

IV. Common Lifted Variables and Factors

Table I summarizes how common robotic estimation variables lift under our framework, including rotations, unit vectors, and translations.

V. Experiments

We evaluate our framework on pose graph optimization (PGO) and range-aided SLAM (RA-SLAM). For each, we present the original MLE and its lifted formulation (cf. Section III-B and Table I). Our implementation uses GTSAM [7] with Levenberg-Marquardt (LM) for the local factor graph optimizations in Algorithm 1. We will release the complete implementation soon.

Algorithm 1 Riemannian Staircase over Factor Graphs

Input: Initial feasible point $Y = (Y_1, \ldots, Y_N) \in \mathbb{R}^{r \times p}$ for p-dimensional BM-factored Shor relaxation (14).
Output: Factor $Y^\star$ for minimizer $Z^\star = Y^\star Y^{\star\mathsf{T}}$ of SDP, optimal value f^*_{SDP}.

function RIEMANNIANSTAIRCASE(Y)
  loop
    // Build lifted factor graph & find KKT point.
    Construct $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{F}}, \bar{\mathcal{E}})$ for rank-p BM factorization via (14)–(16).
    $Y^\star \leftarrow$ FACTORGRAPHOPTIMIZATION($\bar{\mathcal{G}}, Y$).
    Compute $\lambda^\star \in \mathbb{R}^M$ via (18).
    Construct certificate S from $\lambda^\star$ via (17).
    $(\lambda_{\min}, v_{\min}) \leftarrow$ MINIMUMEIGENPAIR(S).
    // Test optimality of $Z^\star = Y^\star Y^{\star\mathsf{T}}$ for SDP.
    if $\lambda_{\min} \geq 0$ **then**
      $f^*_{\mathrm{SDP}} \leftarrow \langle Q, Y^\star Y^{\star\mathsf{T}} \rangle$.
      return $\{Y^\star, f^*_{\mathrm{SDP}}\}$
    end if
    // Saddle escape and rank increase.
    $Y_+ \leftarrow (Y^\star \ 0), \quad \dot{Y}_+ \leftarrow (0 \ v_{\min})$
    $Y \leftarrow$ LINESEARCH($Y_+, \dot{Y}_+$)
  end loop
end function

A. Problem formulations

1) Pose graph optimization: In PGO, the goal is to jointly estimate n unknown poses $x_i = (t_i, R_i) \in \mathrm{SE}(d)$ given noisy pairwise relative measurements $\tilde{x}_{ij} = (\tilde{t}_{ij}, \tilde{R}_{ij}) \approx x_i^{-1} x_j$, represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ [22], where $\kappa_{ij} > 0$ and $\tau_{ij} > 0$ denote the rotational and translational measurement precisions, respectively. The MLE is [9]:

Problem 1 (MLE formulation of PGO)**.**

$$\min_{\substack{t_i \in \mathbb{R}^d \\ R_i \in \mathrm{SO}(d)}} \sum_{(i,j) \in \mathcal{E}} \kappa_{ij} \left\| R_j - R_i \tilde{R}_{ij} \right\|_F^2 + \tau_{ij} \left\| t_j - t_i - R_i \tilde{t}_{ij} \right\|_2^2. \tag{19}$$

The corresponding *lifted* formulation is:

Problem 2 (Lifted formulation of PGO)**.**

$$\min_{\substack{t_i \in \mathbb{R}^p \\ Y_i \in \mathrm{St}(d,p)}} \sum_{(i,j) \in \mathcal{E}} \kappa_{ij} \left\| Y_j - Y_i \tilde{R}_{ij} \right\|_F^2 + \tau_{ij} \left\| t_j - t_i - Y_i \tilde{t}_{ij} \right\|_2^2 \tag{20}$$

2) Range-aided SLAM: RA-SLAM generalizes PGO to the joint estimation of poses and m landmark positions $l_j \in \mathbb{R}^d$, given noisy scalar *range* measurements $\tilde{r}_{ij}$ between pose i and landmark j, where $\sigma_{ij} > 0$ denotes the standard deviation of the ijth ranging measurement and $\mathcal{R} \subseteq [n] \times [m]$ indexes the available range measurements [15]:

Problem 3 (MLE formulation of RA-SLAM).

$$\min_{\substack{t_i \in \mathbb{R}^d,\, R_i \in \mathrm{SO}(d) \\ l_j \in \mathbb{R}^d}} \left\{ \begin{aligned} & \sum_{(i,j)\in\mathcal{E}} \kappa_{ij} \left\| R_j - R_i \tilde{R}_{ij} \right\|_F^2 + \tau_{ij} \left\| t_j - t_i - R_i \tilde{t}_{ij} \right\|_2^2 \\ & + \sum_{(i,j)\in\mathcal{R}} \frac{1}{\sigma_{ij}^2} \left(\|l_j - t_i\|_2 - \tilde{r}_{ij} \right)^2 \end{aligned} \right\} \tag{21}$$

The lifted formulation additionally introduces auxiliary bearing variables $s_{ij} \in S^{p-1}$ associated with each range measurement [15], [23]:

Problem 4 (Lifted formulation of RA-SLAM).

$$\min_{\substack{t_i \in \mathbb{R}^p,\, Y_i \in \mathrm{St}(d,p) \\ l_j \in \mathbb{R}^p,\, s_{ij} \in S^{p-1}}} \left\{ \begin{aligned} & \sum_{(i,j)\in\mathcal{E}} \kappa_{ij} \left\| Y_j - Y_i \tilde{R}_{ij} \right\|_F^2 + \tau_{ij} \left\| t_j - t_i - Y_i \tilde{t}_{ij} \right\|_2^2 \\ & + \sum_{(i,j)\in\mathcal{R}} \frac{1}{\sigma_{ij}^2} \left\| l_j - t_i - \tilde{r}_{ij}\, s_{ij} \right\|_2^2 \end{aligned} \right\} \tag{22}$$

Problems 2 and 4 are the local optimizations instantiated and solved (via local factor graph optimization) at each level of the Riemannian Staircase (Algorithm 1).

B. Results

Table II compares our approach against state-of-the-art *custom* certifiable estimators: SE-Sync [9] for PGO and CORA [15] for RA-SLAM, with random initialization at $p_0 = d$. Our method recovers matching objective values across all instances, confirming functional equivalence. Runtime differences are primarily attributable to our use of GTSAM's general-purpose LM solver, whereas these specialized estimators employ custom Riemannian trust-region methods that exploit the quadratic structure of the objective to avoid explicit relinearization [9]; this is a difference of *implementation*, not framework; incorporating structure-exploiting Riemannian optimization techniques within the factor graph framework is a natural direction for future work. The elevated runtimes on RA-SLAM instances are likewise attributable to the use of a general-purpose LM solver on these inherently ill-conditioned problems [15].

Table III compares against GTSAM's LM solver, with odometric initialization for the pose variables and random initialization for the landmark variables. When LM recovers the global optimum (Sphere2500), our method terminates with comparable optimization time; on instances where LM converges to suboptimal local minima (Rim, Goats 16, Plaza 2), our method recovers the same certifiably optimal (or near-optimal) objective values as the specialized solvers in Table II, while LM returns solutions that are significantly worse.

These results confirm that our certifiable factor graph framework provides the same strong performance guarantees as state-of-the-art custom certifiable solvers, while substantially reducing the required implementation effort via a general-purpose factor graph optimization framework.

VI. Conclusions

In this paper, we propose *certifiable factor graph optimization*, a unified framework that synthesizes factor graph optimization and certifiable estimation. This enables certifiable estimators to be designed and implemented using the same factor graph-based modeling abstractions and software libraries already employed throughout robotics and computer vision, eliminating the need for custom semidefinite programming formulations and hand-designed Riemannian optimization algorithms, and substantially reducing the implementation effort and domain expertise required in practice [6].

TABLE II
COMPARISON WITH SPECIALIZED CERTIFIABLE ESTIMATORS

Random initialization. Opt. time: cumulative local optimization time (s). Lvl: terminal Riemannian Staircase level. For RA-SLAM: SDP and refined objective values.

	Specialized Solver			**Ours**		
PGO	Obj.	Opt. time	Lvl	Obj.	Opt. time	Lvl
Sphere2500	1.687×10^3	7.89	5	1.687×10^3	7.34	5
Rim	5.461×10^3	155.65	5	5.461×10^3	84.93	5
RA-SLAM	SDP / Ref.	Opt. time	Lvl	SDP / Ref.	Opt. time	Lvl
Goats 16	$3.69/3.89\times10^3$	0.40	4	$3.72/3.89\times10^3$	1.59	4
Plaza2	$7.24/7.34\times10^2$	1.83	3	$7.30/7.34\times10^2$	39.80	4

TABLE III
COMPARISON WITH LOCAL OPTIMIZATION (GTSAM)

Odometric initialization for poses, random initialization for landmarks. Opt. time: optimization time (s).

	Ours		**LM (GTSAM)**	
PGO	Obj.	Opt. time	Obj.	Opt. time
Sphere2500	1.687×10^3	0.18	1.687×10^3	0.23
Rim	5.461×10^3	77.90	1.510×10^4	7.25

Range-Aided SLAM: SDP and refined objectives for our method.

	Ours			**LM (GTSAM)**	
	SDP val.	Ref. val.	Opt. time	Obj.	Opt. time
Goats 16	3.720×10^3	3.894×10^3	0.77	2.713×10^4	0.03
Plaza2	7.302×10^2	7.343×10^2	90.50	2.182×10^4	2.76

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