THE POWER OF MINIMALISM IN LONG-TERM TIME SERIES FORECASTING

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Paper under double-blind review

ABSTRACT

Recently, transformer-based models have been widely applied to time series forecasting tasks due to their remarkable capability to capture complex interactions within sequential data. However, as the sequence length expands, Transformer-based models suffer from increased memory consumption, overfitting, and performance deterioration in capturing long-range dependencies. Recently, several studies have shown that MLP-based models can outperform advanced Transformer-based models for long-term time series forecasting (LTSF) tasks. Unfortunately, linear mappings often struggle to capture intricate dependencies when handling multivariate time series. Although modeling each channel independently can alleviate this issue, it will significantly increase the computational cost. To this end, we introduce a set of simple yet effective depthwise convolution models named `LTSF-Conv` to perform LTSF. Specifically, we apply unique filters to each channel to achieve channel independence, which plays a pivotal role in enhancing overall forecasting performance. Experimental results show that `LTSF-Conv` models outperform the state-of-the-art Transformer-based and MLP-based models across seven real-world LTSF benchmarks. Surprisingly, a two-layer non-stacked network can outperform the state-of-the-art Transformer model in 91% of cases with a significant reduction of computing resources. In particular, `LTSF-Conv` models substantially decrease the average number of trainable parameters (by $\sim 12\times$), maximum memory consumption (by $\sim 86\times$), running time (by $\sim 18\times$), and inference time (by $\sim 2\times$) on the Electricity benchmark. We hope this simple network unit opens up new research directions for the LTSF tasks.

1 INTRODUCTION

Time series forecasting is a prevalent problem across a series of domains, including energy management for efficient resource allocation (Shao et al., 2022b), meteorology for weather forecasting (Wang et al., 2019), transportation for optimizing traffic flow (Shao et al., 2022a), and finance for informed decision-making (Cheng et al., 2022). These applications emphasize the far-reaching impact of accurate predictions in the real world. In the era of data-driven decision-making, extending the forecasting horizon can enable organizations and policymakers to plan long-term goals and strategies more accurately. In this context, long-term time series forecasting (LTSF) emerges as an area of paramount significance. However, extending the length of time series input-output presents several challenges in modeling techniques for LTSF tasks, triggering significant interest among researchers (Zhou et al., 2021; Wu et al., 2021; Zhou et al., 2022; Zeng et al., 2022).

In recent years, the rapid development of deep learning technology has led to significant progress in time series forecasting tasks. Transformer-based models have attracted considerable attention among various approaches. The self-attention mechanism in Transformers enables it to effectively capture long-term dependencies and dynamically focus on different parts of the input sequence. Although Transformers have demonstrated exceptional performance in tasks like natural language processing (NLP) (Devlin et al., 2018; Brown et al., 2020), transferring their success to the time series domain presents challenges. The permutation-invariant self-attention mechanism in Transformers, although powerful, can result in some loss of temporal information. Some improved positional embeddings are added to convey temporal information, but they might not fully compensate for the inherent sequence ordering present in time series data, inadvertently leading to a loss of fine-grained temporal information that is vital for accurate forecasting. Moreover, the substantial size of Transformer models leads to longer training times and increased computational demands, especially during the training process where the risk of overfitting becomes more high. This hypothesis has received empirical validation in a study referenced as (Zeng et al., 2022), wherein a very simple linear model, named DLinear, managed to outperform the majority of the Transformer-based forecasting models mentioned earlier. This emphasizes the significance of considering model complexity, as even a seemingly basic model can yield competitive results in comparison to more intricate architectures. Built on this research, a series of subsequent approaches have arisen, focusing on the utilization of Multi-Layer Perceptron (MLP) based techniques. Although these stacked linear layer approaches slightly improve the predictive performance, MLP-based models often possess constrained representational capacity, especially when dealing with multi-channel datasets. A potential solution involves the adoption of channel-independent (CI) modeling, as proposed in reference (Nie et al., 2023). However, it concurrently introduces a substantial increase in computational demands. Among various deep learning models developed for long-term time series forecasting, it's difficult to strike a balance between performance and efficiency when using Transformer-based models or MLP-based models.

Convolutional neural networks possess an underestimated potential in the domain of long-term time series prediction, providing a promising path for addressing current issues. To answer this question, we engage in an investigation involving intensive experiments and analyses to delve into the intricate operational mechanisms of recent LTSF models. We then introduce a series of models named `LTSF-Conv`, which integrate an exceedingly simple basic network. Our models utilize a single depthwise convolution layer as the temporal feature extractor and employ a one-layer linear projection for the final prediction results. We perform comprehensive experiments on seven public real-world datasets across several domains: weather, traffic, electricity, etc. Surprisingly, our results reveal that `LTSF-Conv` models consistently outperform existing complex Transformer-based models for averaged performance across all scenarios in 91% cases. Furthermore, our models also surpass the state-of-the-art stacked MLP-based models for averaged performance in 86% cases. We find that, in contrast to the existing transformer-based and MLP-based architectures, `LTSF-Conv` models exhibit remarkable performance while maintaining a high level of efficiency. The main contributions of this work include:

- We have introduced a series of two-layer convolution models, called `LTSF-Conv`. It's important to emphasize that these models maintain an unstacked structure, representing a simple yet competitive basic unit with substantial potential for future research exploration in LTSF tasks.
- Surprisingly, experimental evaluations on seven popular public datasets show that our simple foundational component significantly outperforms the strongest Transformer-based and MLP-based models in LTSF tasks while achieving notable reductions in compute resources. For example, `Conv` model attains the best results in 100% of cases for average metrics across various horizons, while achieving a 95% reduction in the number of training parameters, a 99% reduction in GPU memory usage, a 90% reduction in running time, and a 54% reduction in inference time compared to the strongest Transformer model on the Weather benchmark.
- We conduct comprehensive empirical investigations across some dimensions of popular Transformer-based and MLP-based solutions, including the sensitivity to different input window sizes, the impact of encoder-decoder structure, and the limitations in dealing with time series with multiple periods among channels, etc. These findings can offer valuable insights for future research within the realm of LTSF.

2 Backgrounds

2.1 Transformer-based LTSF models

Over the past few years, researchers have dedicated significant effort to propose novel Transformer models for improving long sequence prediction performance. However, dealing with long sequences can drastically increase the computational complexity of the Transformer-based models, resulting in a greater demand for computational resources and processing time. To address the challenge from the quadratic complexity inherent in self-attention mechanisms, the LogTrans (Li et al., 2019)

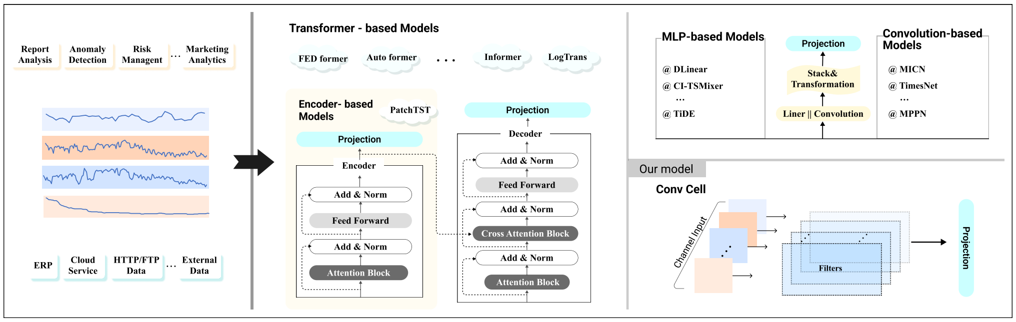


Figure 1: The pipeline of existing LTSF solutions.

model introduces an innovative attention mechanism called LogSparse attention, which effectively mitigates computational complexity, reducing it to $\mathcal{O}(L(logL)^2)$. Similarly, the Informer model (Zhou et al., 2021) incorporates the distilling operation and the ProbSparse attention mechanism based on the Kullback-Leibler divergence. These strategies further significantly reduce the computational complexity to $\mathcal{O}(L(logL))$. Autoformer (Wu et al., 2021) introduces an auto-correlation mechanism to enhance computational efficiency and the utilization of sequence information. Furthermore, it also incorporates a decomposition block that can progressively extract the long-term stationary trend. Building upon this scheme, FEDFormer (Zhou et al., 2022) improves decomposed blocks by integrating frequency domain representations of time series into attention computation. The forecasting accuracy is significantly influenced by the size of the look-back window. To capture more contextual information from a longer historical window, the PatchTST model (Nie et al., 2023) presents a patching component that segments the time series into subseries-level patches. PatchTST first applies channel-independence technology to Transformer-based models and significantly improves prediction accuracy.

2.2 Convolution and MLP-based LTSF models

Although Transformer-based models exhibit potential in time series forecasting, challenges arise from the disparities between NLP and time series data. Recently, DLinear (Zeng et al., 2022) achieved remarkable success by relying on a single linear layer, surpassing the performance of certain complex Transformer-based architectures by a significant margin. On top of this scheme, subsequent MLP-based approaches exhibit better performance in LTSF tasks. TSMixer (Vijay et al., 2023) presents a hybrid architecture of augmenting various reconciliation heads that enhances the learning capacity by integrating gated attention mechanisms into the channel-independent backbone. TiDE (Das et al., 2023) is a simple encoder-decoder model that encodes historical time-series data and covariates using dense MLPs, followed by decoding the time-series and future covariates using dense MLPs again. Many of the extracted patterns might encompass unpredictable noise and lack clear interpretability. Besides MLP-based solutions, other types of convolution networks are also widely explored. SCINet (Liu et al., 2022) employs a recursive downsample-convolve-interact architecture that iteratively downsamples the sequence into two sub-sequences to effectively extract temporal features. (Wu et al., 2022) proposes a task-general foundational model named TimesNet, which transforms 1D time series into a set of 2D tensors to more effectively capture intra-period and inter-period variations in the 2D space. MICN (Shao et al., 2022b) captures local correlations and global correlations by incorporating a multi-scale branch structure with down-sampled convolution. MPPN (Wang et al., 2023) constructs multi-resolution contextual-aware semantic units within time series and introduces a multi-period pattern mining mechanism to explicitly capture crucial time series patterns, which has better interpretability.

3 Theoretical Study on the Convolution Mapping

3.1 Notation

Considering a series of observed time series signals $X = [x_0, \ldots, x_t, \ldots, x_{L-1}] \in \mathbb{R}^{C \times L}$, where L represents the length of time steps and C denotes the recorded variates. Then, given a historical time series $X_{in} = [x_{t-sl}, \ldots, x_{t-1}]$, our goal is to learn a mapping function and forecast the subsequent T time steps $\hat{Y} = [x_t, \ldots, x_{t+T-1}] \in \mathbb{R}^{C \times T}$.

3.2 Theoretical Insights

In this paper, we consistently employ the following variable names: $sl \leq L$: the look-back window length, $kl \leq sl$: the convolutional kernel length, kn: the number of convolution kernels. The definition of a convolution operation is as follows

$$Z = K \otimes X_{in}, \tag{1}$$

where $Z \in \mathbb{R}^{C \times sl}$. The h-th element of the output sequence can be expressed as follows

$$z[h] = \sum_{i=0}^{kl-1} (w(i) * x(h+i)) + b, \tag{2}$$

where w represents the weights associated with the convolutional window, i is the relative position of the convolution window within the input sequence, and b is the bias term.

For time series forecasting tasks, periodicity and trend with tolerable noise are fundamental characteristics that play a crucial role in the success of time series forecasting (Holt, 2004) (Zhang & Qi, 2005). Periodicity refers to the data exhibiting regular and predictable fluctuations over a specific interval. By capturing the periodic behavior, forecasting models can identify repetitive patterns. The trend represents the underlying upward or downward movement observed in the series. Incorporating the trend component is essential in forecasting as it helps capture the overall trajectory and anticipate future changes in the data. We first consider the widely acknowledged assumption that the time series exhibits periodicity.

Theorem 1. *Given a seasonal time series that satisfies $x(t) = x(t-p)$, where $p \leq kl$ is the period. A solution exists for $valid$ convolution models to predict future values, which can be expressed as follows*

$$w(i) = \begin{cases} 1, & \text{if } i = (sl - \alpha \cdot p) \bmod p \\ 0, & \text{otherwise} \end{cases}, b_i = 0, \tag{3}$$

where $1 \leq \alpha \in \mathbb{Z} \leq \lfloor sl/p \rfloor$. Equation 3 implies that when the length of the input historical sequence and the convolutional kernel are equal to or greater than the period, convolutional maps have the ability to predict periodic signals. More proof is in Appendix A.

4 Convolution-based LTSF Solutions

`LTSF-Conv` is a set of convolution-based models. For clearness, we name these two types of temporal variations as `Conv` and `DConv` respectively.

4.1 Basic Backbone

We denote the i-th univariate series with a length of L, starting at time index 0, as $x^i_{0:L-1} = (x^i_0, x^i_1, ..., x^i_{L-1})$, where i ranges from 1 to C. The input is divided into C separate univariate series, where each series is independently fed into a separate kernel backbone. In LTSF tasks, maintaining channel independence has been observed to improve prediction performance as compared to channel mixing. Concretely, the input time series firstly segment undergoes reversible instance normalization, denoted as RevIN (Nie et al., 2023). RevIN is applied to standardize the data distribution by removing the mean and dividing by the standard deviation, a process designed to mitigate data shifts within the time series.

Conv uses a one-layer depthwise convolution operation, and each convolutional kernel is responsible for processing one input channel independently. Each channel is convolved with a separate kernel, and this process generates a feature map with the same number of channels as the input. We utilizes group-wise convolution to cleverly achieve channel independence. Then, we apply a one-layer linear projection to the feature map for the final prediction results.

Autoformer (Wu et al., 2021) first introduces a new methodology by integrating seasonal-trend decomposition ahead of each neural block, leveraging a widely employed technique in time series analysis (Cleveland et al., 1990; Hamilton, 2020) for enhancing the predictability of raw data. Inspired by this scheme, DConv combines a decomposition scheme with convolutional layers to create a powerful and lightweight model architecture. DConv initiates the modeling process by decomposing the raw input data into two main components: the trend component and the seasonal component. Then, we use the separate one-layer linear projection to each component for the final prediction results, as follows

$$X_t = Z^\intercal, \tag{4}$$

$$X_s = X - X_t, \tag{5}$$

$$\hat{Y} = (W_t(X_t) + W_s(X_s))^\intercal, \tag{6}$$

where Z can be obtained by Equation 1, and W_t and $W_s \in \mathbb{R}^{T\times sl}$ represent the one-layer linear model. This process allows the model to model the unique characteristics and patterns within each component individually. Subsequently, the features obtained from both components are summed up to obtain the final prediction. By incorporating these two linear layers and aggregating their outputs, DConv effectively combines the information from both the trend and remainder components to make a comprehensive prediction.

4.1.1 Loss Function

We choose to use the mean squared error (MSE) loss as a measure of the discrepancy between the predictions and the ground truth in each channel. The MSE loss in each channel, denoted as $||\hat{y}^i_{sl:sl+T-1} - y^i_{sl:sl+T-1}||^2_2$, is computed individually for each time series. Then, the losses from C time series are gathered and averaged to obtain the overall objective loss, as follows

$$\mathcal{L} = \mathbb{E}_X \frac{1}{C}\sum_{i=1}^{C} ||\hat{y}^i_{sl:sl+T-1} - y^i_{sl:sl+T-1}||^2_2 \tag{7}$$

5 Experimental Evaluation

5.1 Dataset

In this section, we assess the performance of the LTSF-Conv models on seven widely used multivariate time series datasets, including Weather, Traffic, Electricity, and four ETT datasets. Table 6 of the Appendix provides a brief overview of these datasets. For the Weather, Electricity, and Traffic datasets, adopted a standard split ratio of 7:1:2 for the training set, validation set, and test set, respectively. For the ETT dataset, adopted a standard split ratio of 6:2:2 for the training set, validation set, and test set. Since our primary interest lies in long-term forecasting results, we have excluded the ILI dataset with shorter horizons.

5.2 Baselines and Configuration

We choose recent popular Transformer models in the domain of LTSF for comparison, including PatchTST (Nie et al., 2023), Preformer (Du et al., 2023), FEDformer (Zhou et al., 2022), and Autoformer (Wu et al., 2021). Additionally, we also compare the latest non-Transformer models: MPPN (Wang et al., 2023), TimesNet (Wu et al., 2022), MICN-regre (Shao et al., 2022b), TiDE (Das et al., 2023), and DLinear (Zeng et al., 2022). By default, all methods adhere to a uniform experimental setup with prediction lengths of 96, 192, 336, 720. We utilize Mean Squared Error (MSE) and Mean Absolute Error (MAE) as the standard error metrics for evaluation. To ensure fairness in comparison, in Table 1 and Table 4, we perform experiments with a fixed look-back window size of 512.

Table 1: Multivariate long-term time series forecasting results in terms of MSE and MAE, the lower the better. The best numbers in each row are highlighted in bold and the second best numbers are highlighted with an underline. Among them, * denotes re-implementation with a fixed look-back window of 512. Other results collect from PatchTST (Nie et al., 2023).

Methods		Conv* (Ours)		DConv* (Ours)		PatchTST 2023		TimesNet* 2022		MICN-regre* 2022		FEDformer 2022		Autoformer 2022		DLinear 2022		Informer 2021	
Metric		MSE	MAE	MAE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Weather	96	**0.140**	**0.190**	0.169	0.223	0.149	0.198	0.159	0.215	0.174	0.236	0.238	0.314	0.249	0.329	0.176	0.237	0.354	0.405
	192	**0.183**	**0.230**	0.214	0.260	0.194	0.241	0.220	0.267	0.221	0.282	0.275	0.329	0.325	0.37	0.220	0.282	0.419	0.434
	336	**0.234**	**0.271**	0.259	0.294	0.245	0.282	0.274	0.306	0.257	0.316	0.339	0.377	0.351	0.391	0.265	0.319	0.583	0.543
	720	**0.306**	**0.325**	0.324	0.342	0.314	0.334	0.347	0.356	0.319	0.359	0.389	0.409	0.415	0.426	0.323	0.362	0.916	0.705
	Avg	**0.216**	**0.254**	0.242	0.280	0.225	0.263	0.250	0.286	0.243	0.298	0.310	0.357	0.335	0.379	0.246	0.300	0.568	0.521
Electricity	96	0.132	0.227	0.135	0.231	**0.129**	**0.222**	0.181	0.286	0.155	0.265	0.186	0.302	0.196	0.313	0.140	0.237	0.304	0.393
	192	**0.145**	0.241	0.146	0.243	0.147	**0.240**	0.192	0.294	0.185	0.293	0.197	0.311	0.211	0.324	0.153	0.249	0.327	0.417
	336	**0.161**	**0.257**	0.163	0.259	0.163	0.259	0.196	0.299	0.180	0.292	0.213	0.328	0.214	0.327	0.169	0.267	0.333	0.422
	720	0.201	**0.289**	0.203	0.292	**0.197**	0.290	0.226	0.323	0.207	0.316	0.233	0.344	0.236	0.342	0.203	0.301	0.351	0.427
	Avg	0.160	0.254	0.162	0.256	**0.159**	**0.252**	0.199	0.301	0.182	0.292	0.207	0.321	0.214	0.326	0.166	0.263	0.328	0.414
ETTm2	96	**0.161**	**0.249**	0.163	0.252	0.166	0.256	0.191	0.280	0.176	0.275	0.180	0.271	0.205	0.293	0.167	0.260	0.355	0.462
	192	**0.216**	**0.288**	0.218	0.289	0.223	0.296	0.257	0.324	0.254	0.334	0.252	0.318	0.278	0.336	0.224	0.303	0.595	0.586
	336	**0.271**	0.327	**0.271**	**0.325**	0.274	0.329	0.295	0.349	0.288	0.351	0.324	0.364	0.343	0.379	0.281	0.342	1.270	0.871
	720	0.361	0.387	**0.357**	**0.380**	0.362	0.385	0.452	0.432	0.417	0.440	0.410	0.420	0.414	0.419	0.397	0.421	3.001	1.267
	Avg	**0.252**	0.313	**0.252**	**0.312**	0.256	0.316	0.299	0.346	0.284	0.350	0.291	0.343	0.310	0.356	0.267	0.331	1.305	0.796
ETTm1	96	**0.292**	**0.338**	0.304	0.346	0.293	0.346	0.335	0.376	0.311	0.364	0.326	0.390	0.510	0.492	0.299	0.343	0.626	0.560
	192	**0.332**	**0.361**	0.335	0.365	0.333	0.370	0.396	0.410	0.356	0.388	0.365	0.415	0.514	0.495	0.335	0.365	0.725	0.619
	336	**0.364**	**0.380**	0.366	0.383	0.369	0.392	0.441	0.433	0.407	0.422	0.392	0.425	0.510	0.492	0.369	0.386	1.005	0.741
	720	0.418	**0.411**	0.419	0.412	**0.416**	0.420	0.551	0.495	0.464	0.462	0.446	0.458	0.527	0.493	0.425	0.421	1.133	0.845
	Avg	**0.352**	**0.373**	0.356	0.377	**0.352**	0.382	0.431	0.429	0.408	0.425	0.382	0.422	0.515	0.493	0.357	0.378	0.872	0.691
ETTh1	96	**0.365**	0.393	0.366	**0.392**	0.37	0.400	0.443	0.458	0.389	0.424	0.376	0.415	0.435	0.446	0.375	0.399	0.941	0.769
	192	0.401	0.416	**0.400**	**0.412**	0.413	0.429	0.492	0.491	0.481	0.495	0.423	0.446	0.456	0.457	0.405	0.416	1.007	0.786
	336	**0.419**	0.437	0.420	**0.424**	0.422	0.440	0.483	0.487	0.516	0.524	0.444	0.462	0.486	0.487	0.439	0.443	1.038	0.784
	720	0.464	0.472	**0.435**	**0.451**	0.447	0.468	0.545	0.520	0.743	0.664	0.469	0.492	0.515	0.517	0.472	0.490	1.144	0.857
	Avg	0.412	0.430	**0.405**	**0.420**	0.413	0.434	0.491	0.489	0.532	0.527	0.428	0.453	0.473	0.476	0.422	0.437	1.032	0.799
ETTh2	96	**0.269**	0.339	**0.269**	**0.334**	0.274	0.337	0.383	0.420	0.313	0.396	0.332	0.374	0.332	0.368	0.289	0.353	1.549	0.952
	192	**0.329**	0.383	0.330	**0.376**	0.341	0.382	0.405	0.437	0.498	0.519	0.407	0.446	0.426	0.434	0.383	0.418	3.792	1.542
	336	0.335	0.394	**0.327**	0.387	0.329	**0.384**	0.390	0.437	0.929	0.705	0.400	0.447	0.477	0.479	0.448	0.465	4.215	1.642
	720	**0.379**	0.424	0.381	0.432	**0.379**	**0.422**	0.460	0.471	1.256	0.817	0.412	0.469	0.453	0.490	0.605	0.551	3.656	1.619
	Avg	0.328	0.385	**0.327**	0.382	0.330	**0.381**	0.410	0.441	0.749	0.609	0.387	0.434	0.422	0.442	0.431	0.446	3.303	1.438
Traffic	96	0.396	0.275	0.391	0.272	**0.360**	**0.249**	0.602	0.321	0.473	0.306	0.576	0.359	0.597	0.371	0.410	0.282	0.733	0.410
	192	0.407	0.279	0.396	0.273	**0.379**	**0.256**	0.615	0.331	0.475	0.298	0.610	0.380	0.607	0.382	0.423	0.287	0.777	0.435
	336	0.417	0.285	0.408	0.281	**0.392**	**0.264**	0.614	0.333	0.493	0.307	0.608	0.375	0.623	0.387	0.436	0.296	0.776	0.434
	720	0.453	0.304	0.451	0.301	**0.432**	**0.286**	0.648	0.348	0.531	0.325	0.621	0.375	0.639	0.395	0.466	0.315	0.827	0.466
	Avg	0.426	0.286	0.412	0.282	**0.390**	**0.263**	0.620	0.333	0.493	0.309	0.603	0.372	0.616	0.383	0.433	0.295	0.778	0.436

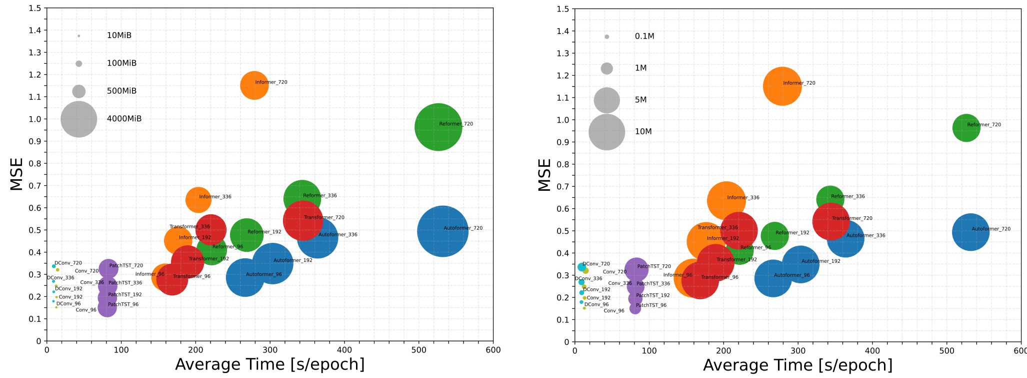
Table 2: Multivariate long-term time series forecasting results. *Best* denotes re-implementation with a longer look-back window of $\{336, 512, 720, 1600\}$, consistently choosing the most optimal results. Other results collect from Tide (Nie et al., 2023) and MPPN (Wang et al., 2023).

Methods	Metric	Weather					Electricity					ETTm2					ETTm1				
		96	192	336	720	Avg	96	192	336	720	Avg	96	192	336	720	Avg	96	192	336	720	Avg
Conv-Best	MSE	**0.140**	**0.182**	**0.234**	**0.294**	**0.213**	**0.129**	**0.143**	**0.159**	**0.195**	**0.157**	**0.161**	0.217	**0.257**	**0.325**	0.240	**0.287**	**0.328**	0.364	0.394	**0.343**
	MAE	**0.188**	**0.230**	**0.271**	**0.324**	**0.253**	0.225	**0.238**	**0.255**	**0.286**	**0.251**	**0.249**	**0.287**	0.329	0.379	0.311	**0.334**	**0.358**	**0.380**	0.408	**0.370**
Dconv-Best	MSE	0.166	0.209	0.253	0.306	0.234	0.130	0.144	0.160	0.199	0.158	**0.161**	**0.213**	0.258	**0.325**	**0.239**	0.300	0.335	**0.356**	**0.393**	0.346
	MAE	0.220	0.259	0.293	0.335	0.277	0.225	**0.238**	0.256	0.288	0.252	0.253	0.292	**0.325**	**0.369**	**0.310**	0.342	0.363	0.384	**0.405**	0.374
PatchTST-Best	MSE	0.149	0.194	0.245	0.306	0.224	**0.129**	0.147	0.163	0.197	0.159	0.165	0.220	0.274	0.350	0.252	0.290	0.332	0.366	0.407	0.349
	MAE	0.198	0.241	0.282	0.334	0.263	**0.222**	0.240	0.259	0.290	0.252	0.255	0.292	0.329	0.380	0.314	0.342	0.369	0.392	0.421	0.381
Preformer-Best	MSE	0.227	0.275	0.324	0.394	0.305	0.180	0.189	0.201	0.232	0.201	0.213	0.269	0.324	0.418	0.306	0.516	0.556	0.572	0.598	0.561
	MAE	0.292	0.322	0.352	0.393	0.340	0.297	0.302	0.319	0.342	0.315	0.295	0.329	0.363	0.416	0.351	0.482	0.491	0.509	0.533	0.504
TiDE	MSE	0.166	0.209	0.254	0.313	0.235	0.132	0.147	0.161	0.196	0.159	**0.161**	0.215	0.267	0.352	0.248	0.306	0.335	0.364	0.413	0.354
	MAE	0.222	0.263	0.301	0.340	0.281	0.229	0.243	0.261	0.294	0.256	0.251	0.289	0.326	0.383	0.312	0.349	0.366	0.384	0.413	0.378
MPPN	MSE	0.144	0.189	0.240	0.310	0.220	0.131	0.145	0.162	0.200	0.159	0.162	0.217	0.273	0.368	0.255	**0.287**	0.330	0.369	0.426	0.353
	MAE	0.196	0.240	0.281	0.333	0.262	0.226	0.239	0.256	0.289	0.252	0.250	0.288	**0.325**	0.383	0.311	0.335	0.360	0.382	0.414	0.372

In Table 2, Table 3, and Table 5, we conducted experiments within a broader window size range of $\{336, 512, 720, 1600\}$ to explore better results. Further implementation details can be found in the Appendix B.

5.3 RESULTS

Multivariate Results In multivariate long-term TS forecasting tasks, we compare `Conv` and `DConv` to the latest competitive Transformer-based and non-Transformer baselines on seven popular benchmarks. For fixed look-back window size of 512, Table 1 shows that our method can match or outperform the strongest model PatchTST on popular LTSF benchmarks. However, our model can further enhance predictive capabilities by utilizing a longer look-back window. In Table 2 and Table 5, from an overall perspective, our models achieve the best results in 86% cases in MSE and 86% cases in MAE for all compared models. Specifically, our models outperform the strongest Transformer baseline PatchTST in 86% cases in MSE and 75% cases in MAE. Additionally, our models perform best in 86% cases in MSE and 86% cases in MAE compared to the state-of-the-art MLP-based models (TiDE). Please note that our models have not performed effectively on the largest Traffic dataset. This limitation is due to the simplicity of our model, which comprises only a single convolution layer. We recognize the potential of stacking our Conv-based unit. By designing



(a): GPU Memory (MiB). (b): Trainable Parameter (M).

Figure 2: The ball chart illustrates the efficiency of various models with look-back window size 336 and forecasting lengths of {96, 192, 336, 720}, evaluated on the Weather dataset. Each ball's size is indicative of the model's complexity.

Table 3: Univariate long-term time series forecasting results. *Best* denotes re-implementation with a longer look-back window of {336, 512, 720, 1600}, consistently choosing the most optimal results.

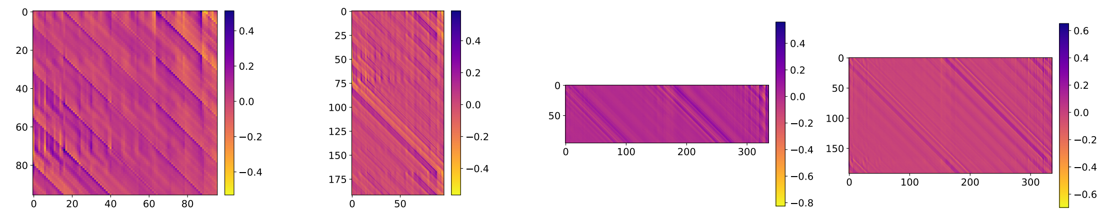
Methods	Metric	ETTm1					ETTm2					ETTh1					ETTh2				
		96	192	336	720	Avg	96	192	336	720	Avg	96	192	336	720	Avg	96	192	336	720	Avg
Conv-Best	MSE	**0.025**	0.038	0.050	**0.065**	**0.044**	**0.062**	**0.089**	**0.116**	0.163	**0.107**	**0.052**	**0.064**	**0.075**	**0.078**	**0.067**	0.129	**0.167**	**0.170**	0.186	**0.163**
	MAE	**0.120**	**0.147**	**0.169**	**0.196**	**0.158**	**0.181**	**0.223**	**0.257**	0.316	**0.244**	**0.174**	**0.197**	**0.217**	0.228	**0.204**	0.278	0.325	0.338	0.351	**0.323**
Dconv-Best	MSE	**0.025**	**0.037**	**0.049**	0.066	**0.044**	**0.062**	**0.089**	**0.116**	**0.162**	**0.107**	0.053	0.069	0.077	0.079	0.069	**0.128**	0.168	0.175	0.199	0.167
	MAE	**0.120**	**0.147**	0.170	0.199	0.159	0.182	**0.223**	**0.257**	**0.315**	**0.244**	0.177	0.204	0.220	**0.225**	0.206	0.277	**0.322**	0.337	0.360	0.324
PatchTST-Best	MSE	0.026	0.039	0.053	0.073	0.048	0.065	0.093	0.120	0.171	0.112	0.055	0.071	0.076	0.087	0.072	0.129	0.168	0.171	0.223	0.173
	MAE	0.121	0.150	0.173	0.206	0.163	0.186	0.231	0.265	0.322	0.251	0.179	0.205	0.220	0.232	0.209	0.282	0.328	**0.336**	0.380	0.332
TimesNet-Best	MSE	0.029	0.047	0.059	0.077	0.053	0.066	0.113	0.133	0.183	0.124	0.056	0.072	0.081	0.082	0.073	0.136	0.186	0.197	**0.172**	0.173
	MAE	0.127	0.163	0.188	0.211	0.172	0.187	0.250	0.277	0.335	0.262	0.182	0.209	0.225	0.228	0.211	0.286	0.340	0.360	**0.344**	0.333
MICN-Best	MSE	0.033	0.050	0.065	0.089	0.059	0.065	0.105	0.132	0.166	0.117	0.063	0.075	0.092	0.129	0.090	0.125	0.181	0.203	0.272	0.195
	MAE	0.134	0.166	0.190	0.221	0.178	0.187	0.240	0.275	0.317	0.255	0.186	0.208	0.239	0.288	0.230	**0.267**	0.327	0.362	0.424	0.345

advanced architectures to enhance the model's capability to handle complex patterns and improve its performance on the larger Traffic dataset.

Univariate Results We provide details about the univariate results of four typical ETT datasets. In Table 4, we compare some baselines under a look-back window size of 512. In Table 3, with an increase in input length, `LTSF-Conv` achieves the best results in 100% of cases for both average MSE and MAE metrics across various horizons. In particular, compared with the advanced Transformer-based solution PatchTST, our methods match or outperform it in all of the settings. Also, we observe that `LTSF-Conv` outperforms TimesNet by an avg. of 7.8%, and outperforms MICN by an avg. of 13.3%. For more exhaustive results, refer to the Appendix Table 4.

Effciency Comparison In this section, we aim to illustrate that `LTSF-Conv` models exhibit significantly improved efficiency compared to Transformer-based models in terms of trainable param, GPU memory, running time, training time, and inference times. The ball charts presented in Figures 2(a) and 2(b) illustrate the MSE metric on the Weather dataset relative to the GPU memory and trainable parameter of the different models, where the size of each ball corresponds to the complexity of the model. Overall, It can be intuitively observed that Transformer-based models require approximately $13 \sim 67\times$ the number of parameters and $162 \sim 407\times$ the GPU memory compared to `LTSF-Conv` models, yet they exhibit higher mean squared error. More detailed results about the comparison of different datasets can be found in Table 13-15 of the Appendix. In particular, under a look-back length of 336 on the Electricity dataset, `Conv` achieves a 94% reduction (2.21$\rightarrow$0.13) in the number of training parameters required, a 99% reduction (16032.05$\rightarrow$165.96) in GPU memory usage, a 95% reduction (265.65$\rightarrow$13.98) in running time, and a 66% reduction (56.74$\rightarrow$19.11) in inference time compared to the advanced patchTST model.

Interpretability `LTSF-Conv` is a set of convolution-based models, and the weights visualization within the projection layer can provide valuable insights into prediction. Here, let's take the `Conv` model as an example. We visualize the weights with two fixed input lengths and two different forecasting horizons. Specifically, Figure 3 illustrates the weight distribution of the `Conv` model



(a) In-96-Out-96. (b) In-96-Out-192. (c) In-336-Out-96. (d) In-336-Out-192.

Figure 3: Visualization of the weights of Conv on Electricity benchmark. The X-axis represents the look-back window size, and the Y-axis represents the prediction length.

on the Electricity dataset. We can observe that the model assigns considerable weight to time steps with a 24-hour interval within the look-back window, indicating their higher relevance. Please note that the data is recorded at hourly intervals from 2012 to 2014, with each day comprising 24 time steps. This demonstrates that our model has the capability to capture a daily periodicity. Then we further expand the look-back window size to 336 which encompasses weekly patterns. Within these forecasting time steps, some time steps with a 168-hour interval within the look-back window are also assigned greater weight. This suggests that the `Conv` model is capable of capturing a weekly periodic pattern. More visualization results of `DConv` can be found in Figure 7 of the Appendix.

6 More Analyses on LTSF-Transformer

In time series forecasting tasks, extending the forecasting horizon presents a practical but challenging problem. Although Transformer has achieved breakthrough success in some domains, applying it directly to long-term time series forecasting tasks remains a significant set of problems. Firstly, transformer-based models heavily rely on self-attention mechanisms to capture long-range dependencies and interactions within the data. However, the increased input and output sizes lead to a higher risk of overfitting, particularly in comparison to short-term forecasting scenarios. Secondly, the complex $encoder$-$decoder$ structure of Transformers magnifies the memory bottleneck. In this section, we will explore the Transformer-based methods' suitability for long sequence prediction from two perspectives.

6.1 Do we need an Encoder-Decoder structure to model

Over the past few decades, there has been significant progress in artificial neural networks, driven by the belief that increasing network complexity can improve performance. These sophisticated networks are constructed with multiple layers comprising a large number of neurons or transformer blocks (Vaswani et al., 2017; Liu et al., 2021). Successful architectural design patterns for one task are often applied to address related tasks. Given the significant success in natural language processing, it is natural to use an $encoder$-$decoder$ structure in LTSF tasks would be advantageous and imperative. However, as these models grow in complexity, they face challenges in optimization, and intricate operations, such as transformer models, call for a shift toward a simple structure. A recent paper has already questioned whether the $encoder$-$decoder$ structure is effective for time series forecasting from a theoretical perspective (Liu et al., 2023). We performed experiments on nine real-world benchmarks to rigorously evaluate the performance and generalizability of the proposed conclusion. For a fair comparison, we chose three representative models with $encoder$-$decoder$ structure and adopted the experimental settings used in the studies by (Wu et al., 2021). We maintained a consistent look-back length of 96 across all experiments. In the Ori structure, both the encoder and decoder are kept intact, representing the original and complete architecture of the LTSF-Transformer models. In contrast, the $Half$-Ex structure preserves the decoder as is, thereby reducing its complexity and capacity. By evaluating these two structures, our primary goal is to understand the influence of the structure reduction on the overall capabilities of the model. Table 16 of the Appendix presents a comparison of the performance between the complete $Encoder$-$Decoder$ structure and a reduced version comprising only half of the original components. As shown in the table, for the average performance across four different output horizons, the FEDformer-based reduced structure achieves the best results in 89% cases in terms of MSE and MAE metrics. The reduced structure based on Autoformer achieves the best results in 83% of the cases for both MSE and

MAE metrics. Similarly, the LogTrans-based reduced structure also exhibits strong performance, securing the best results in 94% of the cases for MSE and MAE metrics. Overall, by analyzing its performance across various metrics, we can gain valuable insights into the impact of reducing the $encoder$-$decoder$ structure's complexity for the model's forecasting capabilities. More analysis of Transformers can be found in the Appendix C.1.

7 More Analyses on LTSF-Linear

Recently, a series of LTSF-MLP models have emerged in the time series forecasting field. These models are known for their lightweight and fast nature, while still achieving performance comparable to or even better than Transformer-based methods. Although linear mapping can effectively learn periodic patterns in time series data, there are still some limitations compared to convolutional models for LTSF tasks. For the following subsection, we use the variable names: Linear-S: every channel shares the same linear layer, and Linear-CI: a linear layer for each channel individually (Channel Independent).

7.1 Limitations of dealing with multiple periods among channels

In the context of LTSF tasks, linear mapping proves effective in capturing and modeling periodicity in univariate data (Li et al., 2023). However, when dealing with tasks involving multiple periods across channels, where each channel represents a distinct aspect or feature of the data, the limitations of the Linear-S mapping become apparent. It lacks the capacity to capture intricate interactions between channels, making it challenging to model complex patterns and interactions accurately. Furthermore, LTSF tasks require capturing dependencies over extended periods. With multiple channels, the complexity of capturing long-term dependencies increases further. As a result, the forecasting accuracy of the model may be limited. Figure 6 of the Appendix shows the forecasting results of different Linear-S models applied to the real-world dataset consisting of distinct periodic channels. The results indicate that the proposed `Conv` basic unit is capable of capturing the underlying curves, whereas Linear-S struggles to fit the multiple channels data more effectively. Specifically, our model demonstrates accuracy in predicting the peaks and troughs at the early stages, while also achieving impressive precision in forecasting the trend in the long-term future. However, the Linear-S model exhibits an over-smoothing problem, causing the predicted trends to deviate further from the actual data trends. As the prediction length increases, this trend deviation becomes even more pronounced, highlighting the limitations of the model's ability to capture the true dynamics of the data.

To address this challenge, a potential solution is to use Linear-CI modeling. However, this approach does come with a trade-off, as it noticeably increases the computational overhead required for the training process. Table 17 of the Appendix measures the computational cost of each Linear-CI model on the Electricity benchmark. It can be observed that the utilization of Linear-CI models leads to a substantial increase in the number of parameters, memory usage (Max), and running time (Avg) compared to Linear-S models. Specifically, RLinear-CI leads to a substantial increase in the average number of trainable parameters (by $\sim 320\times$), maximum GPU memory consumption (by $\sim 2\times$), and epoch time (by $\sim 7\times$). Similarly, DLinear-CI results in a significant rise in the average number of trainable parameters (by $\sim 320\times$), maximum GPU memory usage (by $\sim 3\times$), and running time (by $\sim 5\times$). This increase is primarily due to the number of channels. Notably, when the look-back window size is increased, some datasets with a large number of channels, such as Traffic (862 channels), encounter an unacceptable training time.

8 Conclusion

This work systematically investigates the limitations of Transformer-based and MLP-based solutions in long-term time series forecasting tasks. We use an embarrassingly simple convolution unit as a temporal feature extractor to verify our claims. By embracing the depthwise convolution mechanism, the `LTSF-Conv` models exhibit their adeptness in achieving a balance between computational efficiency and model performance. This progress is particularly valuable in situations involving the processing of long sequences, as it adeptly tackles the computational limitations inherent in conventional Transformers and MLPs. Note that LTSF-Conv models serve as straightforward yet competitive basic units, exhibiting promising potential for further expansion.

REFERENCES

Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. Language models are few-shot learners. *Advances in neural information processing systems*, 33:1877–1901, 2020.

Dawei Cheng, Fangzhou Yang, Sheng Xiang, and Jin Liu. Financial time series forecasting with multi-modality graph neural network. *Pattern Recognition*, 121:108218, 2022.

Robert B Cleveland, William S Cleveland, Jean E McRae, and Irma Terpenning. Stl: A seasonal-trend decomposition. *J. Off. Stat*, 6(1):3–73, 1990.

Abhimanyu Das, Weihao Kong, Andrew Leach, Rajat Sen, and Rose Yu. Long-term forecasting with tide: Time-series dense encoder. *arXiv preprint arXiv:2304.08424*, 2023.

Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. Bert: Pre-training of deep bidirectional transformers for language understanding. *arXiv preprint arXiv:1810.04805*, 2018.

Dazhao Du, Bing Su, and Zhewei Wei. Preformer: predictive transformer with multi-scale segment-wise correlations for long-term time series forecasting. In *ICASSP 2023-2023 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pp. 1–5. IEEE, 2023.

James D Hamilton. *Time series analysis*. Princeton university press, 2020.

Charles C Holt. Forecasting seasonals and trends by exponentially weighted moving averages. *International journal of forecasting*, 20(1):5–10, 2004.

Shiyang Li, Xiaoyong Jin, Yao Xuan, Xiyou Zhou, Wenhu Chen, Yu-Xiang Wang, and Xifeng Yan. Enhancing the locality and breaking the memory bottleneck of transformer on time series forecasting. *Advances in neural information processing systems*, 32, 2019.

Zhe Li, Shiyi Qi, Yiduo Li, and Zenglin Xu. Revisiting long-term time series forecasting: An investigation on linear mapping. *arXiv preprint arXiv:2305.10721*, 2023.

Bing Liu, Wei Luo, Gang Li, Jing Huang, and Bo Yang. Do we need an encoder-decoder to model dynamical systems on networks? *arXiv preprint arXiv:2305.12185*, 2023.

Minhao Liu, Ailing Zeng, Muxi Chen, Zhijian Xu, Qiuxia Lai, Lingna Ma, and Qiang Xu. Scinet: Time series modeling and forecasting with sample convolution and interaction. *Advances in Neural Information Processing Systems*, 35:5816–5828, 2022.

Ze Liu, Yutong Lin, Yue Cao, Han Hu, Yixuan Wei, Zheng Zhang, Stephen Lin, and Baining Guo. Swin transformer: Hierarchical vision transformer using shifted windows. In *Proceedings of the IEEE/CVF international conference on computer vision*, pp. 10012–10022, 2021.

Yuqi Nie, Nam H Nguyen, Phanwadee Sinthong, and Jayant Kalagnanam. A time series is worth 64 words: Long-term forecasting with transformers. *arXiv preprint arXiv:2211.14730*, 2023.

Zezhi Shao, Zhao Zhang, Fei Wang, Wei Wei, and Yongjun Xu. Spatial-temporal identity: A simple yet effective baseline for multivariate time series forecasting. In *Proceedings of the 31st ACM International Conference on Information & Knowledge Management*, pp. 4454–4458, 2022a.

Zezhi Shao, Zhao Zhang, Fei Wang, and Yongjun Xu. Pre-training enhanced spatial-temporal graph neural network for multivariate time series forecasting. In *Proceedings of the 28th ACM SIGKDD Conference on Knowledge Discovery and Data Mining*, pp. 1567–1577, 2022b.

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. *Advances in neural information processing systems*, 30, 2017.

E Vijay, Arindam Jati, Nam Nguyen, Gift Sinthong, and Jayant Kalagnanam. Tsmixer: Lightweight mlp-mixer model for multivariate time series forecasting. In *ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, 2023.

Bin Wang, Jie Lu, Zheng Yan, Huaishao Luo, Tianrui Li, Yu Zheng, and Guangquan Zhang. Deep uncertainty quantification: A machine learning approach for weather forecasting. In *Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, pp. 2087–2095, 2019.

Xing Wang, Zhendong Wang, Kexin Yang, Junlan Feng, Zhiyan Song, Chao Deng, et al. Mppn: Multi-resolution periodic pattern network for long-term time series forecasting. *arXiv preprint arXiv:2306.06895*, 2023.

Haixu Wu, Jiehui Xu, Jianmin Wang, and Mingsheng Long. Autoformer: Decomposition transformers with auto-correlation for long-term series forecasting. *Advances in Neural Information Processing Systems*, 34:22419–22430, 2021.

Haixu Wu, Tengge Hu, Yong Liu, Hang Zhou, Jianmin Wang, and Mingsheng Long. Timesnet: Temporal 2d-variation modeling for general time series analysis. In *The Eleventh International Conference on Learning Representations*, 2022.

Ailing Zeng, Muxi Chen, Lei Zhang, and Qiang Xu. Are transformers effective for time series forecasting? In *Proceedings of the AAAI conference on artificial intelligence*, volume 37, pp. 11121–11128, 2022.

G Peter Zhang and Min Qi. Neural network forecasting for seasonal and trend time series. *European journal of operational research*, 160(2):501–514, 2005.

Haoyi Zhou, Shanghang Zhang, Jieqi Peng, Shuai Zhang, Jianxin Li, Hui Xiong, and Wancai Zhang. Informer: Beyond efficient transformer for long sequence time-series forecasting. In *Proceedings of the AAAI conference on artificial intelligence*, volume 35, pp. 11106–11115, 2021.

Tian Zhou, Ziqing Ma, Qingsong Wen, Xue Wang, Liang Sun, and Rong Jin. Fedformer: Frequency enhanced decomposed transformer for long-term series forecasting. In *International Conference on Machine Learning*, pp. 27268–27286. PMLR, 2022.

Table 4: Univariate long-term time series forecasting results on ETT full benchmark, with a fixed input length sl=512.

Methods		Conv (Ours)		DConv (Ours)		PatchTST 2023		TimesNet 2023		MICN-regre 2023		FEDformer 2022		Autoformer 2022		DLinear 2022		Informer 2021		LogTrans 2019	
Metric		MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
ETTm1	96	**0.026**	**0.122**	**0.026**	**0.122**	**0.026**	0.123	0.031	0.135	0.036	0.142	0.033	0.140	0.056	0.183	0.028	0.123	0.109	0.277	0.049	0.171
	192	**0.039**	**0.150**	**0.039**	**0.150**	0.040	0.151	0.048	0.169	0.052	0.171	0.058	0.186	0.081	0.216	0.045	0.156	0.151	0.310	0.157	0.317
	336	**0.052**	**0.173**	**0.052**	**0.173**	0.053	0.174	0.059	0.188	0.070	0.202	0.084	0.231	0.076	0.218	0.061	0.182	0.427	0.591	0.289	0.459
	720	0.072	**0.204**	**0.070**	**0.204**	0.073	0.206	0.077	0.211	0.089	0.221	0.102	0.250	0.110	0.267	0.080	0.210	0.438	0.586	0.430	0.579
	Avg	**0.047**	**0.162**	**0.047**	**0.162**	0.048	0.163	0.054	0.176	0.062	0.184	0.069	0.201	0.080	0.221	0.053	0.167	0.281	0.441	0.231	0.381
ETTm2	96	**0.063**	**0.182**	**0.063**	0.183	0.065	0.187	0.078	0.212	0.077	0.208	0.067	0.198	0.065	0.189	**0.063**	0.183	0.088	0.225	0.075	0.208
	192	0.091	**0.225**	**0.090**	**0.225**	0.093	0.231	0.108	0.252	0.093	0.229	0.102	0.245	0.118	0.256	0.092	0.227	0.132	0.283	0.129	0.275
	336	0.119	**0.261**	**0.118**	**0.261**	0.121	0.266	0.138	0.289	0.132	0.278	0.130	0.279	0.154	0.305	0.119	**0.261**	0.180	0.336	0.154	0.302
	720	0.172	0.321	0.171	0.320	0.172	0.322	0.183	0.335	0.166	**0.317**	0.178	0.325	0.182	0.335	0.175	0.320	0.300	0.435	**0.160**	0.321
	Avg	**0.111**	**0.247**	**0.111**	**0.247**	0.112	0.251	0.127	0.272	0.117	0.258	0.119	0.261	0.129	0.271	0.112	0.247	0.175	0.319	0.129	0.276
ETTh1	96	**0.053**	**0.177**	0.056	0.183	0.059	0.189	0.063	0.196	0.066	0.198	0.079	0.215	0.071	0.206	0.056	0.180	0.193	0.377	0.283	0.468
	192	**0.065**	**0.198**	0.072	0.212	0.074	0.215	0.079	0.224	0.100	0.248	0.104	0.245	0.114	0.262	0.071	0.204	0.217	0.395	0.234	0.409
	336	**0.075**	**0.217**	0.077	0.221	0.076	0.220	0.081	0.225	0.115	0.269	0.119	0.270	0.107	0.258	0.098	0.244	0.202	0.381	0.386	0.546
	720	**0.082**	**0.227**	**0.082**	0.229	0.087	0.236	**0.082**	0.228	0.258	0.424	0.142	0.299	0.126	0.283	0.189	0.359	0.183	0.355	0.475	0.629
	Avg	**0.069**	**0.205**	0.071	0.211	0.074	0.215	0.076	0.218	0.135	0.285	0.111	0.257	0.104	0.252	0.103	0.246	0.198	0.377	0.344	0.513
ETTh2	96	0.135	0.285	0.135	0.285	0.131	0.284	0.146	0.300	0.146	0.299	**0.128**	**0.271**	0.153	0.306	0.131	0.279	0.213	0.373	0.217	0.379
	192	0.173	**0.328**	0.173	**0.328**	**0.171**	0.329	0.195	0.351	0.184	0.338	0.185	0.330	0.204	0.351	0.176	0.329	0.227	0.387	0.281	0.429
	336	0.179	0.342	0.177	0.340	**0.171**	**0.336**	0.197	0.360	0.203	0.362	0.231	0.378	0.246	0.389	0.209	0.367	0.242	0.401	0.293	0.437
	720	0.219	0.376	0.219	0.375	0.223	0.380	**0.172**	**0.344**	0.410	0.524	0.278	0.420	0.268	0.409	0.276	0.426	0.291	0.439	0.218	0.387
	Avg	0.177	0.333	0.176	**0.332**	**0.174**	**0.332**	0.178	0.339	0.236	0.381	0.205	0.349	0.217	0.363	0.198	0.350	0.243	0.400	0.252	0.408

Table 5: Multivariate long-term time series forecasting best results.

Methods	Metric	ETTh1					ETTh2					Traffic				
		96	192	336	720	Avg	96	192	336	720	Avg	96	192	336	720	Avg
Conv-Best	MSE	**0.365**	0.401	0.424	0.450	0.411	**0.268**	0.327	0.329	**0.379**	0.326	0.383	0.397	0.411	0.450	0.410
	MAE	0.393	0.416	0.428	0.460	0.425	0.339	0.382	0.390	0.424	0.384	0.271	0.275	0.282	0.302	0.283
Dconv-Best	MSE	0.366	**0.400**	**0.421**	**0.429**	**0.404**	0.269	**0.326**	**0.321**	0.382	**0.325**	0.378	0.390	0.404	0.442	0.404
	MAE	**0.382**	**0.412**	**0.422**	**0.446**	**0.416**	**0.334**	**0.375**	0.386	0.428	0.381	0.264	0.269	0.275	0.294	0.276
PatchTST-Best	MSE	0.370	0.414	0.422	0.447	0.413	0.274	0.339	0.329	**0.379**	0.330	0.360	0.379	0.392	0.432	0.390
	MAE	0.400	0.421	0.440	0.468	0.432	0.336	0.379	**0.384**	**0.422**	**0.380**	**0.249**	**0.256**	0.264	0.286	0.263
Preformer-Best	MSE	0.438	0.457	0.464	0.503	0.466	0.339	0.385	0.394	0.466	0.396	0.560	0.565	0.577	0.597	0.575
	MAE	0.455	0.474	0.481	0.512	0.481	0.384	0.435	0.441	0.479	0.435	0.349	0.349	0.351	0.358	0.352
TiDE	MSE	0.375	0.412	0.435	0.454	0.419	0.270	0.332	0.360	0.419	0.345	**0.336**	**0.346**	**0.355**	**0.386**	**0.355**
	MAE	0.398	0.422	0.433	0.465	0.429	0.336	0.380	0.407	0.451	0.393	0.253	0.257	**0.260**	**0.273**	**0.260**
MPPN	MSE	0.371	0.405	0.426	0.436	0.409	0.278	0.344	0.362	0.393	0.344	0.387	0.396	0.410	0.449	0.410
	MAE	0.393	0.413	0.425	0.452	0.420	0.335	0.380	0.400	0.434	0.387	0.271	0.273	0.279	0.301	0.281

A PROOFS

Corollary 1. *When considering transformed periodic sequences $x(t) = a \cdot x(t-p) + c$, the convolutional model still has an explicit solution to Equation 2 as*

$$w(i) = \begin{cases} a, & \text{if } i = (sl - \alpha \cdot p) \bmod p \\ 0, & \text{otherwise} \end{cases}, b_i = c. \tag{8}$$

We have discovered that a single convolutional network can successfully capture periodicity in a time series. However, a general time series can be decomposed into two components: a periodic sequence and a sequence with a smooth trend (Wu et al., 2021) (Zhou et al., 2022).

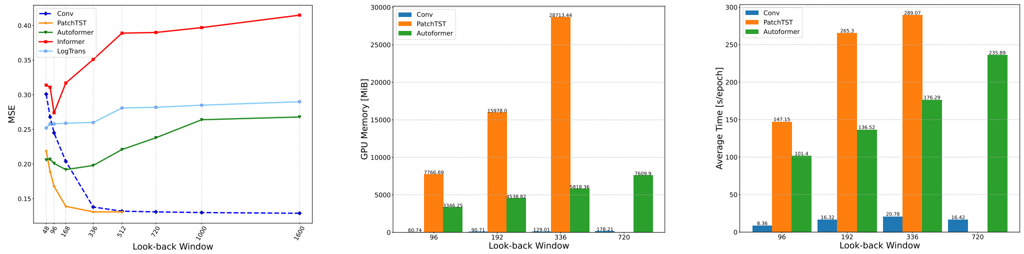
Theorem 2. *Let $x(t) = s(t) + g(t)$, where $s(t)$ represents the seasonal signal with period p and $g(t)$ satisfies V-Lipschitz smooth (i.e. $|g(a) - g(b)| \leq V|a - b|$). Then there exists a convolutional model with look-back window length $sl = p + \epsilon, \epsilon \geq 0$, such that $|x[sl+h] - \hat{x}[sl+h]| \leq V \cdot (p + \epsilon), h = 0, \ldots, H-1$.*

Proof. We denote the length of historical data as sl. In terms of the ground truth of the future time series, it can be described as follows:

$$x[sl+h] = s(p + \epsilon + h) + g(p + \epsilon + h) \tag{9}$$

Assuming that the convolutional network is only capable of capturing periodic patterns, we can use Equation 3 directly as an approximate solution. Then the h-th true value will be predicted:

$$\begin{aligned} \hat{x}[sl+h] &= x(h + (p + \epsilon - \alpha \cdot p) \bmod p) \\ &= s(h + (p + \epsilon) \bmod p) + g(h + (p + \epsilon - \alpha \cdot p) \bmod p) \\ &= s(p + \epsilon + h) + g((p + \epsilon - \alpha \cdot p) \bmod p + h) \end{aligned} \tag{10}$$



(a) (b) (c)

Figure 4: The MSE, GPU memory, and average running time (Y-axis) of models under different look-back window sizes (X-axis) on the Electricity benchmark (T=96).

So we can get

$$\begin{aligned} |x[sl+h] - \hat{x}[sl+h]| &= g(p+\epsilon+h) - g((p+\epsilon-\alpha\cdot p) \bmod p + h) \\ &\leq V\cdot(p+\epsilon-(p+\epsilon-\alpha\cdot p) \bmod p) \\ &\leq V\cdot(p+\epsilon) \end{aligned} \tag{11}$$

This analysis highlights the associated error is bounded given an appropriate look-back window size.

B Implementation Details

All experiments are conducted on the NVIDIA GeForce RTX 2080 Ti GPU, implemented in PyTorch. The training process is early stopped within five epochs. We use the ADAM optimizer with a decay learning rate schedule, which gradually decreases the learning rate over time in a smooth and controlled manner. The initial learning rate is set to 5·1e-3. and the hyper-parameter convolution kernel size is fine-tuned within the range {20, 35, 55, 128, 196}. Under the configuration with a look-back window size of 512, the specific hyper-parameters chosen of the Conv model for each dataset are summarized in Table 7 and Table 8. For long-term time series forecasting, all best experimental results are derived from three runs with different random seeds, and then calculating the mean. The related standard deviation and mean values results are summarized in Table 9.

C More Experimental Results

Table 6: Summary of seven benchmarks.

Datasets	Weather	Electricity	Traffic	ETTm1	ETTm2	ETTh1	ETTh2
Features	21	321	862	7	7	7	7
Timesteps	52,696	26,304	17,544	69,680	69,680	17,420	17,420
Granularity	10 Minutes	1 Hour	1 Hour	15 Minutes	15 Minutes	1 Hour	1 Hour
Date	2020/1-2021/1	2016/7-2019/7	2016/7-2018/7	2016/7-2018/6	2016/7-2018/6	2016/7-2018/6	2016/7-2018/6

C.1 Can existing LTSF Transformers benefit from longer look-back windows

For the LTSF tasks, a longer look-back window expands the receptive field, which can potentially improve the predictive accuracy of forecasting models. Here, we compare `Conv` and Transformer-based solutions across various look-back window sizes on the Electricity benchmark. We conducted experiments using a range of look-back window sizes {48, 72, 96, 168, 336, 512, 720, 1000, 1600} for long-term forecasting, where T=96. In Figure 4 (a), it is evident that the performance of existing Transformer-based models either degrades or remains stable as the look-back window size increases. These models struggle to efficiently extract more valuable temporal information when extending the input horizon. This finding contradicts our initial hypothesis. However, as the look-back window size increases, `Conv` demonstrates significant improvements in performance, highlighting its capacity to effectively learn from extended historical data.

Table 7: Hyperparameters of multivariate long-term time series forecasting tasks. The look-back window size is 512.

Dataset	Pre_len	Batch_Size	Learning_Rate	Kernel_Size	Individual
ETTh1	96	16	0.005	55	0
	192	16	0.005	55	0
	336	128	0.005	78	1
	720	16	0.005	55	0
ETTh2	96	128	0.005	55	0
	192	128	0.005	78	0
	336	128	0.0005	78	0
	720	128	0.0001	78	0
ETTm1	96	16	0.005	78	1
	192	16	0.005	35	0
	336	16	0.005	35	0
	720	16	0.005	35	0
ETTm2	96	16	0.005	24	1
	192	16	0.005	55	1
	336	16	0.005	35	0
	720	64	0.005	24	0
Weather	96	16	0.005	55	0
	192	16	0.005	78	1
	336	16	0.005	24	1
	720	128	0.005	78	0
Electricity	96	16	0.005	55	0
	192	128	0.005	55	1
	336	64	0.005	55	0
	720	16	0.005	55	0
Traffic	96	16	0.005	35	0
	192	16	0.005	35	0
	336	16	0.005	24	0
	720	16	0.005	24	0

Table 8: Hyperparameters of univariate long-term time series forecasting tasks. The look-back window size is 512.

Dataset	Pre_Len	Batch_size	Learning_rate	Kernel_Size	Individual
ETTh1	96	16	0.005	24	0
	192	16	0.005	55	0
	336	64	0.005	55	0
	720	128	0.005	78	0
ETTh2	96	128	0.005	24	0
	192	16	0.005	35	0
	336	128	0.005	24	0
	720	128	0.005	24	0
ETTm1	96	64	0.005	55	0
	192	64	0.005	24	0
	336	16	0.005	35	0
	720	16	0.005	35	0
ETTm2	96	16	0.005	55	0
	192	16	0.005	24	0
	336	64	0.005	24	0
	720	16	0.005	24	0
Weather	96	64	0.005	78	0
	192	16	0.005	24	0
	336	16	0.005	78	0
	720	128	0.005	55	0
Electricity	96	16	0.005	35	0
	192	64	0.005	35	0
	336	64	0.005	35	0
	720	16	0.005	35	0
Traffic	96	16	0.005	24	0
	192	64	0.005	35	0
	336	16	0.005	35	0
	720	16	0.005	24	0

In Figure 4 (b) and (c), we illustrate the GPU memory utilization and average running time of Transformer-based models and `Conv` model as the look-back window size is varied from 48 to 1000. Notably, PatchTST and Autoformer exhibit heightened sensitivity to changes in window size. Moreover, PatchTST runs out of memory when the look-back window size is greater than or equal to 720. In contrast, our model maintains relatively stable memory utilization while the lookback window increases. `Conv` achieves superior accuracy while concurrently sustaining significantly improved memory efficiency and reduced average running time.

Table 9: The error bars of Conv-Best with 3 runs, output length $H \in \{96, 192, 336, 720\}$ on Traffic, Electricity, and ETTm2.

Dataset		Traffic					Electricity					ETTm2				
Metric		Seed1	Seed2	Seed3	Mean	Std.	Seed1	Seed2	Seed3	Mean	Std.	Seed1	Seed2	Seed3	Mean	Std.
96	MSE	0.3837	0.3838	0.3833	0.3836	0.00021	0.1294	0.1294	0.1295	0.1295	0.00005	0.1615	0.1621	0.1618	0.1618	0.00023
	MAE	0.2714	0.2715	0.2708	0.2712	0.00029	0.2249	0.2249	0.2250	0.2250	0.00005	0.2492	0.2494	0.2495	0.2494	0.00012
192	MSE	0.3977	0.3970	0.3970	0.3972	0.00035	0.1438	0.1441	0.1437	0.1439	0.00015	0.2174	0.2171	0.2171	0.2172	0.00014
	MAE	0.2755	0.2751	0.2752	0.2753	0.00018	0.2385	0.2390	0.2385	0.2387	0.00024	0.2879	0.2878	0.2878	0.2878	0.00006
336	MSE	0.4114	0.4110	0.4110	0.4111	0.00022	0.1594	0.1594	0.1598	0.1595	0.00019	0.2576	0.2577	0.2575	0.2576	0.00008
	MAE	0.2820	0.2819	0.2820	0.2820	0.00005	0.2549	0.2553	0.2553	0.2551	0.00018	0.3292	0.3294	0.3292	0.3293	0.00007
720	MSE	0.4509	0.4499	0.4507	0.4505	0.00041	0.1955	0.1949	0.1950	0.1951	0.00023	0.3230	0.3268	0.3243	0.3247	0.00157
	MAE	0.3027	0.3017	0.3019	0.3021	0.00042	0.2863	0.2861	0.2863	0.2863	0.00008	0.3787	0.3794	0.3796	0.3793	0.00038

C.2 Ablation Studies

In this section, we conduct additional ablation experiments to verify the validity of the group-wise convolution (the number of channels is equal to the variable dimension and the number of filters) and cross-channel convolution in the `Conv` model. Tables 10 and 11 display quantitative results with fixed input lengths of 336 and 512 and four different forecasting horizons. We use the following variable names: ① `Conv` uses depthwise convolution (channel independence), ② $\texttt{Conv}_{CD}$ uses so-called channel-dependent convolution. It proves that the group-wise convolution can better mine the long sequence information, at least for existing benchmarks.

Table 10: Ablations on depthwise convolution with a look-back window size of 336 on six benchmarks.

Benchmark		Conv				Conv$_{CD}$			
		96	192	336	720	96	192	336	720
ETTh1	MSE	0.370	0.407	0.422	0.451	0.394	0.475	0.464	0.535
	MAE	0.394	0.417	0.428	0.458	0.418	0.461	0.465	0.509
ETTh2	MSE	0.277	0.341	0.330	0.379	0.340	0.398	0.373	0.452
	MAE	0.343	0.383	0.387	0.420	0.394	0.433	0.421	0.466
ETTm1	MSE	0.287	0.328	0.366	0.422	0.305	0.346	0.378	0.430
	MAE	0.334	0.358	0.380	0.413	0.352	0.375	0.390	0.421
ETTm2	MSE	0.163	0.217	0.271	0.368	0.175	0.247	0.330	0.437
	MAE	0.250	0.288	0.324	0.385	0.264	0.310	0.368	0.426
Weather	MSE	0.143	0.185	0.237	0.313	0.145	0.192	0.247	0.321
	MAE	0.189	0.230	0.271	0.326	0.199	0.243	0.284	0.333
Electricity	MSE	0.136	0.150	0.167	0.204	0.250	0.279	0.274	0.310
	MAE	0.230	0.243	0.259	0.292	0.356	0.371	0.372	0.396

Table 11: Ablations on depthwise convolution with a look-back window size of 512 on six benchmarks.

Benchmark		Conv				Conv$_{CD}$			
		96	192	336	720	96	192	336	720
ETTh1	MSE	0.365	0.401	0.419	0.464	0.450	0.441	0.512	0.605
	MAE	0.393	0.416	0.437	0.472	0.455	0.460	0.501	0.541
ETTh2	MSE	0.269	0.329	0.335	0.379	0.336	0.371	0.369	0.470
	MAE	0.339	0.383	0.394	0.424	0.385	0.411	0.419	0.481
ETTm1	MSE	0.292	0.332	0.364	0.418	0.315	0.358	0.390	0.445
	MAE	0.338	0.361	0.380	0.411	0.359	0.385	0.405	0.441
ETTm2	MSE	0.161	0.216	0.271	0.361	0.179	0.248	0.311	0.430
	MAE	0.249	0.288	0.327	0.387	0.272	0.318	0.363	0.429
Weather	MSE	0.140	0.183	0.234	0.306	0.150	0.199	0.246	0.315
	MAE	0.190	0.230	0.271	0.325	0.203	0.251	0.283	0.334
Electricity	MSE	0.132	0.145	0.161	0.201	0.252	0.246	0.263	0.282
	MAE	0.227	0.241	0.257	0.289	0.360	0.351	0.367	0.378

C.3 Robustness Experiments

In order to assess the potential influence of different random initializations for long-term forecasting, we conduct experiments on Weather, Electricity, Traffic, ETTm2, ETTh1, and ETTh2 datasets. Each experimental configuration was repeated with five random seeds. The robust experimental results are provided in Figure 5. In general, we observe that `DConv` exhibits reliability and stability towards different initialization. However, on the traffic dataset, there is a certain degree of fluctuation in the results. We think the reason for the poor performance of `DConv` on traffic datasets is excessively high variable dimensionality ($C = 862$). A simple two-layer network is insufficient for modeling high-dimensional and large-scale datasets due to the limited parameter capacity of the model.

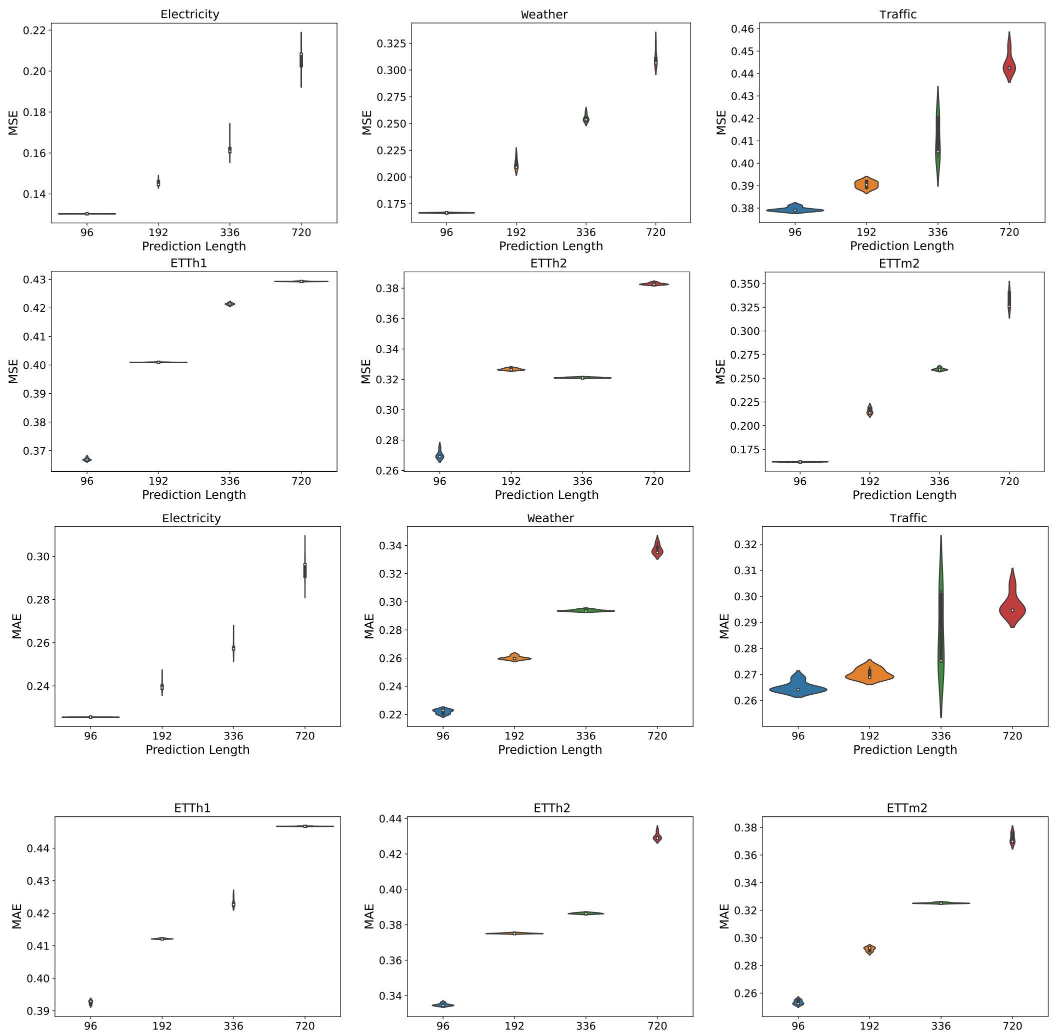


Figure 5: Robust experiments of DConv with different random seeds. The X-axis corresponds to the prediction length, and the Y-axis represents the MSE/MAE metrics.

C.4 More Experimental Datasets

In this section, we add additional experiments to further verify the validity of LTSF-Conv model in different LTSF tasks. Solar power prediction is a crucial aspect within the realm of renewable energy, wielding significant influence across diverse domains. We add two datasets from real-world applications, namely the Solar-Jinta and Solar-Alabama benchmarks. Solar-Jinta records seven key

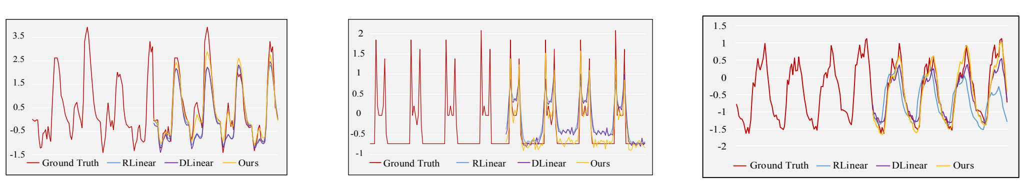
Table 12: Multivariate long-term time series forecasting results on Solar benchmarks.

Benchmarks	Methods	Metric									
		MSE					MAE				
		96	192	336	720	Avg	96	192	336	720	Avg
Solar-Energy	DConv	**0.173**	**0.189**	**0.201**	**0.206**	**0.192**	**0.229**	**0.237**	**0.244**	**0.245**	**0.239**
	PatchTST	0.178	0.192	0.203	0.218	0.197	0.248	0.265	0.267	0.274	0.263
	Flowformer	0.190	0.267	0.279	0.243	0.245	0.229	0.263	0.268	0.252	0.253
	Reformer	0.195	0.223	0.253	0.281	0.238	0.234	0.252	0.286	0.286	0.265
	Informer	0.195	0.220	0.259	0.246	0.230	0.241	0.241	0.274	0.266	0.256
	LogTrans	0.219	0.217	0.224	0.241	0.225	0.240	0.244	0.258	0.269	0.252
	Dlinear	0.221	0.231	0.247	0.255	0.239	0.294	0.301	0.317	0.314	0.307
Solar- Jinta	DConv	**0.480**	**0.521**	**0.539**	**0.648**	**0.547**	0.481	**0.496**	**0.522**	**0.573**	**0.518**
	PatchTST	0.491	0.602	0.617	0.710	0.605	**0.464**	0.523	0.533	0.601	0.530
	Flowformer	0.646	0.792	0.748	1.065	0.812	0.565	0.624	0.611	0.783	0.645
	Reformer	0.954	0.934	0.993	0.995	0.969	0.773	0.757	0.742	0.730	0.751
	Informer	0.718	0.821	0.847	1.102	0.872	0.616	0.656	0.737	0.855	0.716
	LogTrans	0.704	0.817	0.754	1.012	0.821	0.594	0.651	0.642	0.762	0.662
	Dlinear	0.523	0.588	0.638	0.734	0.620	0.496	0.531	0.562	0.616	0.551

meteorological factors of solar radiation, collected by the hour. Solar-Alabama [1] contains the solar power production of 137 PV plants in the USA, with a data granularity of 10 minutes. The solar power of different PV plants is influenced by varying geographical and weather conditions. Table 12 provides a summary of prediction results from several popular baselines on the Solar-Energy datasets. It can be observed that DConv outperforms the other baselines for most horizons by a large margin. Moreover, compared to other datasets, the Solar-Jinta benchmark has a smaller size. The experimental results also indicate that our model performs equally well on small datasets, ensuring its generalizability and robustness.

D Limitations and Future Work

In this work, we reconsider the significance of model complexity on prediction results, even seemingly straightforward models can yield competitive results. However, for high-dimensional and large-scale datasets, we need to further stack our proposed basic units to increase the model's capacity, enabling it to better adapt to intricate tasks, enhancing representational power, depth, feature extraction, and generalization performance. In practical scenarios, a substantial amount of time series data frequently incorporates spatial information. In future work, we intend to construct additional spatial modules built upon the LTSF-Conv basic unit to strengthen the adaptability of the model. Furthermore, we plan to extend LTSF-Conv to various other downstream tasks, such as classification and anomaly detection.



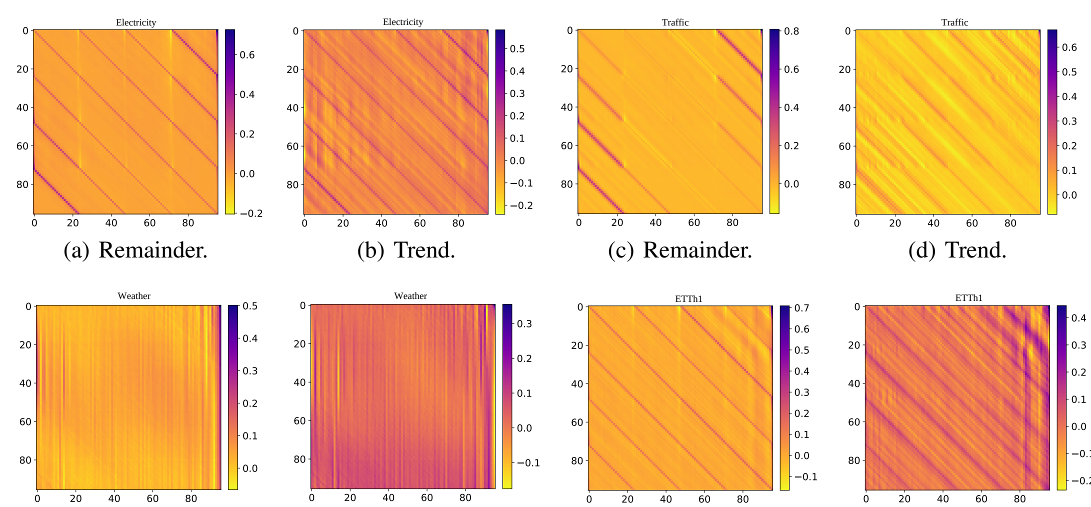
(a): Channel 1. (b): Channel 2. (c): Channel 3.

Figure 6: Forecasting results on Electricity with three random channels of different periods under the predict-96 setting.

[1] https://github.com/laiguokun/multivariate-time-series-data/

Table 13: Model efficiency comparison under $sl = 336$ on Electricity.

Models	Series Length	Trainable Parameter (M)	GPU Memory (MiB)	Running Time (s / iter)	Iters (nums)	Training Time (s)	Inference Time (s)	GPUs (nums)
Conv	96	0.0446	90.71	10.56	20	211.27	10.75	1
	192	0.0769	117.45	11.89	22	261.59	14.40	1
	336	0.1254	170.00	14.04	19	266.75	21.58	1
	720	0.2548	285.68	19.43	30	582.79	29.69	1
	Avg	0.1254	165.96	13.98	22.75	330.60	19.11	1
DConv	96	0.0763	119.99	11.68	26	303.65	15.40	1
	192	0.1410	146.42	13.08	29	379.41	24.64	1
	336	0.2380	207.08	15.08	19	286.46	34.72	1
	720	0.4968	349.91	20.06	20	401.24	59.55	1
	Avg	0.2380	205.85	14.98	23.5	342.69	33.58	1
PatchTST	96	0.9212	15978.00	265.30	50	13264.98	40.55	2
	192	1.4374	15999.82	265.87	19	5051.50	48.58	2
	336	2.2117	16034.15	266.07	49	13037.56	57.42	2
	720	4.2764	16116.24	265.34	22	5837.58	80.39	2
	Avg	2.2116	16032.05	265.65	35	9297.90	56.74	2
Autoformer	96	12.1439	4538.82	136.52	6	819.10	30.14	1
	192	12.1439	5223.64	156.88	7	1098.13	37.52	1
	336	12.1439	5513.15	185.88	6	1115.30	51.26	1
	720	12.1439	8529.48	273.23	7	1912.64	79.31	1
	Avg	12.1439	5951.27	188.18	6.5	1236.29	49.56	1
Informer	96	12.4537	2315.98	86.87	8	694.94	19.08	1
	192	12.4537	2444.42	96.50	15	1447.57	25.46	1
	336	12.4537	2086.85	113.19	8	905.48	33.97	1
	720	12.4537	2590.08	149.48	18	2690.69	56.90	1
	Avg	12.4537	2359.33	111.51	12.25	1434.67	33.85	1
Reformer	96	6.4382	2858.95	115.18	10	1151.81	22.61	1
	192	6.4382	3467.09	137.28	7	960.94	29.53	1
	336	6.4382	4378.95	170.68	10	1706.78	41.98	1
	720	6.4382	7049.06	263.51	13	3425.62	67.07	1
	Avg	6.4382	4438.52	171.66	10	1811.29	40.30	1
LogTrans	96	11.6657	3087.83	88.51	8	708.07	17.79	1
	192	11.6657	3375.00	98.27	14	1375.80	23.89	1
	336	11.6657	2996.17	113.14	7	791.99	34.02	1
	720	11.6657	5033.59	175.74	28	4920.77	56.55	1
	Avg	11.6657	3623.15	118.92	14.25	1949.16	33.07	1



(a) Remainder. (b) Trend. (c) Remainder. (d) Trend.

(e) Remainder. (f) Trend. (g) Remainder. (h) Trend.

Figure 7: Visualization of the weights of DConv on different benchmarks. The X-axis represents the look-back window size, and the Y-axis represents the prediction length.

Table 14: Model efficiency comparison under $sl = 336$ on Weather.

Models	Series Length	Trainable Parameter (M)	GPU Memory (MiB)	Running Time (s / iter)	Iters (nums)	Training Time (s)	Inference Time (s)	GPUs (nums)
Conv	96	0.0332	6.19	7.58	11	83.42	5.91	1
	192	0.0655	8.40	7.54	20	150.73	6.51	1
	336	0.1140	12.09	7.97	15	119.48	7.58	1
	720	0.2434	21.94	8.62	21	181.04	8.82	1
	Avg	0.1140	12.15	7.93	16.75	133.67	7.20	1
DConv	96	0.0654	8.49	8.85	8	70.78	7.68	1
	192	0.1302	11.04	9.24	10	92.38	8.55	1
	336	0.2272	15.43	8.94	25	223.56	9.47	1
	720	0.4860	28.12	9.36	10	93.57	13.47	1
	Avg	0.2272	15.77	9.10	13.25	120.08	9.79	1
PatchTST	96	0.9212	1065.29	81.14	23	1866.22	13.29	1
	192	1.4374	1073.08	81.33	13	1057.25	14.44	1
	336	2.2117	1086.64	81.73	11	898.98	15.63	1
	720	4.2764	1122.29	82.86	12	994.39	19.41	1
	Avg	2.2116	1086.82	81.76	14.75	1204.20	15.69	1
Autoformer	96	10.6076	4465.52	266.57	6	1599.39	42.04	1
	192	10.6076	5142.19	303.50	6	1820.98	48.28	1
	336	10.6076	5125.08	363.88	6	2183.29	69.68	1
	720	10.6076	8016.37	532.23	6	3193.39	96.93	1
	Avg	10.6076	5687.29	366.54	6	2199.26	64.23	1
Informer	96	11.3782	2251.79	158.98	7	1112.86	19.923	1
	192	11.3782	2369.26	176.47	7	1235.31	22.22	1
	336	11.3782	1997.09	203.78	6	1222.69	32.52	1
	720	11.3782	2435.73	278.99	11	3068.91	45.62	1
	Avg	11.3782	2263.47	204.56	7.75	1659.94	30.07	1
Reformer	96	5.8235	2784.63	221.54	11	2436.91	27.35	1
	192	5.8235	3371.91	268.70	8	2149.58	32.99	1
	336	5.8235	4256.18	343.30	7	2403.12	46.07	1
	720	5.8235	6892.63	526.34	10	5263.38	72.81	1
	Avg	5.8235	4326.33	339.97	9	3063.25	44.80	1
LogTrans	96	10.5902	3018.15	168.47	6	1010.84	19.03	1
	192	10.5902	3297.23	189.21	6	1135.27	25.35	1
	336	10.5902	2892.28	220.57	6	1323.41	30.14	1
	720	10.5902	4877.56	344.36	6	2066.19	43.56	1
	Avg	10.5902	3521.30	230.65	6	1383.92	29.52	1

Table 15: Model efficiency comparison under $sl = 336$ on ETTm2.

Models	Series Length	Trainable Parameter (M)	GPU Memory (MiB)	Running Time (s / iter)	Iters (nums)	Training Time (s)	Inference Time (s)	GPUs (nums)
Conv	96	0.0326	2.40	6.11	26	158.79	5.84	1
	192	0.0649	3.46	6.18	18	111.20	5.92	1
	336	0.1135	5.19	6.27	27	169.33	6.32	1
	720	0.2429	9.79	6.80	16	108.74	7.15	1
	Avg	0.1134	5.21	6.34	21.75	137.02	6.31	1
DConv	96	0.0650	3.55	10.90	22	239.78	8.36	1
	192	0.1297	4.88	12.19	15	182.87	10.29	1
	336	0.2267	7.05	12.16	16	194.59	8.73	1
	720	0.4855	13.18	10.36	16	165.82	10.13	1
	Avg	0.2267	7.17	11.40	17.25	195.76	9.38	1
PatchTST	96	0.9212	365.60	29.28	11	322.07	8.27	1
	192	1.4374	376.49	30.79	11	338.71	8.91	1
	336	2.2117	387.91	30.04	10	300.44	9.56	1
	720	4.2764	421.48	30.20	9	271.81	10.37	1
	Avg	2.2116	387.87	30.08	10.25	308.26	9.28	1
Autoformer	96	10.5370	3046.10	191.56	9	1724.04	37.30	1
	192	10.5370	3378.39	215.69	7	1509.83	41.80	1
	336	10.5370	2909.60	246.71	10	2467.10	53.38	1
	720	10.5370	4449.39	346.27	6	2077.63	72.89	1
	Avg	10.5370	3445.87	250.06	8	1944.65	51.35	1
Informer	96	11.3290	2258.83	161.44	6	968.66	26.85	1
	192	11.3290	2381.14	180.83	6	1084.99	30.28	1
	336	11.3290	2005.51	204.47	8	1635.76	36.36	1
	720	11.3290	3128.20	282.11	15	4231.67	50.62	1
	Avg	11.3290	2443.42	207.21	8.75	1980.27	36.03	1
Reformer	96	5.7953	2780.49	208.82	6	1252.94	31.75	1
	192	5.7953	3368.69	253.63	6	1521.75	37.71	1
	336	5.7953	4249.20	317.01	8	2536.10	47.34	1
	720	5.7953	6884.70	489.97	7	3429.76	73.77	1
	Avg	5.7953	4320.77	317.36	6.75	2185.14	47.64	1
LogTrans	96	10.5411	3015.32	159.83	9	1438.46	22.07	1
	192	10.5411	3293.75	178.98	7	1252.85	25.12	1
	336	10.5411	2888.15	207.05	7	1449.32	28.89	1
	720	10.5411	4870.88	322.14	7	2254.95	42.60	1
	Avg	10.5411	3517.03	217.00	7.5	1598.89	29.67	1

Table 16: The performance comparison of two structures. Ori represents the original $encoder$-$decoder$ structure, and a modified version is denoted as $Half$-Ex, which retains half of the original design, only the decoder part.

Methods		FEDformer				Autoformer				LogTrans			
		MSE		MAE		MSE		MAE		MSE		MAE	
Predict Length		Ori	$Half$-Ex	Ori	$Half$-Ex	Ori	$Half$-Ex	Ori	$Half$-Ex	Ori	$Half$-Ex	Ori	$Half$-Ex
ETTh1	96	0.376	0.374	0.419	0.414	0.449	0.387	0.459	0.423	0.878	0.839	0.74	0.708
	192	0.42	0.422	0.448	0.446	0.5	0.437	0.482	0.447	1.037	1.056	0.824	0.823
	336	0.459	0.447	0.465	0.463	0.521	0.502	0.496	0.491	1.238	1.005	0.932	0.811
	720	0.506	0.487	0.507	0.492	0.514	0.493	0.512	0.501	1.135	1.212	0.852	0.914
	Avg	0.44	**0.432**	0.459	**0.453**	0.496	**0.454**	0.487	**0.465**	1.072	**1.028**	0.837	**0.814**
ETTh2	96	0.346	0.34	0.388	0.383	0.358	0.405	0.397	0.422	2.116	1.772	1.197	1.074
	192	0.429	0.431	0.439	0.438	0.456	0.426	0.452	0.433	4.315	3.986	1.635	1.601
	336	0.496	0.481	0.487	0.479	0.482	0.452	0.486	0.463	4.511	3.722	1.758	1.588
	720	0.463	0.466	0.474	0.478	0.515	0.467	0.511	0.482	3.188	3.161	1.54	1.473
	Avg	0.433	**0.429**	0.447	**0.444**	0.452	**0.437**	0.461	**0.45**	3.532	**3.16**	1.532	**1.434**
ETTm1	96	0.379	0.361	0.419	0.408	0.505	0.517	0.475	0.484	0.6	0.548	0.546	0.529
	192	0.426	0.404	0.441	0.432	0.553	0.557	0.496	0.504	0.837	0.659	0.7	0.602
	336	0.445	0.454	0.459	0.461	0.621	0.541	0.537	0.496	1.124	1.011	0.832	0.77
	720	0.543	0.508	0.49	0.488	0.671	0.559	0.561	0.507	1.153	1.079	0.82	0.806
	Avg	0.448	**0.431**	0.452	**0.447**	0.587	**0.543**	0.517	**0.497**	0.928	**0.824**	0.724	**0.676**
ETTm2	96	0.203	0.188	0.287	0.281	0.255	0.214	0.339	0.296	0.768	0.555	0.642	0.547
	192	0.269	0.255	0.328	0.322	0.281	0.273	0.34	0.33	0.989	0.854	0.757	0.713
	336	0.325	0.318	0.366	0.363	0.339	0.328	0.372	0.363	1.334	1.153	0.872	0.794
	720	0.421	0.427	0.415	0.422	0.422	0.428	0.419	0.421	3.048	3.048	1.328	1.31
	Avg	0.304	**0.297**	0.349	**0.347**	0.324	**0.31**	0.367	**0.352**	1.534	**1.402**	0.899	**0.841**
Traffic	96	0.587	0.569	0.366	0.352	0.613	0.615	0.388	0.396	0.684	0.676	0.384	0.369
	192	0.604	0.599	0.373	0.373	0.616	0.626	0.382	0.394	0.685	0.674	0.39	0.367
	336	0.621	0.614	0.383	0.376	0.622	0.619	0.337	0.385	0.734	0.658	0.408	0.358
	720	0.626	0.624	0.382	0.38	0.66	0.635	0.408	0.395	0.717	0.679	0.396	0.363
	Avg	0.609	**0.601**	0.376	**0.37**	0.627	**0.623**	**0.378**	0.392	0.705	**0.671**	0.394	**0.364**
Exchange	96	0.148	0.135	0.278	0.263	0.197	0.153	0.323	0.285	0.968	0.59	0.812	0.594
	192	0.271	0.278	0.38	0.384	0.3	0.268	0.369	0.378	1.04	1.076	0.851	0.783
	336	0.46	0.448	0.5	0.492	0.509	0.45	0.524	0.499	1.659	1.28	1.081	0.923
	720	1.195	1.163	0.841	0.824	1.447	1.141	0.941	0.827	1.941	2.545	1.127	1.366
	Avg	0.518	**0.506**	0.499	**0.49**	0.613	**0.503**	0.539	**0.497**	1.402	**1.372**	0.967	**0.916**
ILI	24	3.228	3.171	1.26	1.235	3.483	3.277	1.287	1.255	4.48	4.226	1.444	1.325
	36	2.679	2.598	1.08	1.056	3.103	2.772	1.148	1.118	4.799	4.598	1.467	1.405
	48	2.622	2.48	1.078	1.045	2.669	2.697	1.085	1.114	4.8	4.967	1.468	1.462
	60	2.857	2.721	1.157	1.118	2.77	2.737	1.125	1.122	5.278	4.772	1.56	1.461
	Avg	2.846	**2.742**	1.143	**1.113**	3	**2.87**	1.161	**1.152**	4.839	**4.64**	1.484	**1.413**
Electricity	96	0.193	0.184	0.308	0.299	0.201	0.195	0.317	0.307	0.258	0.271	0.357	0.365
	192	0.201	0.196	0.315	0.31	0.222	0.225	0.334	0.335	0.266	0.278	0.368	0.37
	336	0.214	0.212	0.329	0.326	0.231	0.251	0.338	0.359	0.28	0.283	0.38	0.373
	720	0.246	0.239	0.355	0.348	0.254	0.28	0.361	0.383	0.283	0.289	0.376	0.375
	Avg	0.213	**0.207**	0.326	**0.32**	**0.227**	0.237	**0.337**	0.346	**0.271**	0.28	**0.37**	**0.37**
Weather	96	0.217	0.264	0.239	0.345	0.266	0.225	0.336	0.3	0.458	0.435	0.49	0.462
	192	0.276	0.254	0.336	0.316	0.307	0.298	0.367	0.353	0.658	0.459	0.589	0.474
	336	0.339	0.359	0.38	0.395	0.359	0.345	0.395	0.382	0.797	0.501	0.652	0.504
	720	0.403	0.398	0.428	0.411	0.419	0.411	0.428	0.42	0.675	0.668	1.13	0.587
	Avg	**0.308**	0.318	**0.345**	0.366	0.337	**0.319**	0.381	**0.363**	0.647	**0.515**	0.715	**0.506**

Table 17: Comparison of practical efficiency of LTSF-Linear and LTSF-Conv under $sl = 336$ on the Electricity. All results are the average test of three runs.

Models		RLinear		DLinear		Conv
		CI	S	CI	S	-
Trainable Parameter (M)	96	10.39	0.03	31.15	0.1	0.04
	192	20.77	0.07	62.31	0.19	0.08
	336	36.35	0.11	109.04	0.34	0.13
	720	77.89	0.24	233.66	0.73	0.25
GPU Memory (MiB)	96	536.93	307.47	716.18	398.01	336.79
	192	737.83	400.058	1190.77	501.78	452.89
	336	1148.48	581.36	1905.77	661.24	634.2
	720	2263.83	1064.83	3974.65	1148.93	1117.67
Running Time (s / epoch)	96	68.47	7.83	50.81	8.48	7.18
	192	73.13	9.095	53.46	9.96	8.47
	336	78.92	11.19	64.29	13.96	12.99
	720	94.57	16.25	93.61	20.4	16.54