

# 000 001 002 003 004 005 006 007 008 009 PROTEIN AUTOREGRESSIVE MODELING VIA MULTI- SCALE STRUCTURE GENERATION

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Paper under double-blind review

## ABSTRACT

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We present *protein autoregressive modeling* (PAR), the *first* multi-scale autoregressive framework for protein backbone generation via coarse-to-fine next-scale prediction. Using the hierarchical nature of proteins, PAR generates structures that mimic sculpting a statue, forming a coarse topology and refining structural details over scales. To achieve this, PAR consists of three key components: *(i)* multi-scale downsampling operations that represent protein structures across multiple scales during training; *(ii)* an autoregressive transformer that encodes multi-scale information and produces conditional embeddings to guide structure generation; *(iii)* a flow-based backbone decoder that generates backbone atoms conditioned on these embeddings. Moreover, autoregressive models suffer from exposure bias, caused by the training and the generation procedure mismatch, and substantially degrades structure generation quality. We effectively alleviate this issue by adopting noisy context learning and scheduled sampling, enabling robust backbone generation. Notably, PAR exhibits strong zero-shot generalization, supporting flexible human-prompted conditional generation and motif scaffolding *without* requiring fine-tuning. On the unconditional generation benchmark, PAR effectively learns protein distributions and produces backbones of high design quality, and exhibits favorable scaling behavior. Together, these properties establish PAR as a promising framework for protein structure generation.

## 028 029 1 INTRODUCTION

030  
031 Deep generative modeling of proteins has emerged as a way to design and model novel structures  
032 with desired functions and properties, with broad applications in biomedicine and nanotechnology  
033 (Huang et al., 2016; Kuhlman & Bradley, 2019). A widely adopted approach is to directly model  
034 the distribution of three-dimensional protein structures, which govern protein function. Typically,  
035 structure generative models produce protein backbones without sequences or side chains. Prior  
036 work in this area could be broadly categorized into methods that predict the SE(3) backbone frame  
037 representations (Yim et al., 2023a; Watson et al., 2023) and those that directly model atoms, *e.g.*,  $\text{C}\alpha$   
038 coordinates for simplicity and scalability (Geffner et al., 2025; Lin & AlQuraishi, 2023). However,  
039 all these works are based on diffusion models and their variations (*e.g.*, flow matching).

040  
041 On the other hand, autoregressive (AR) modeling has emerged as a powerful paradigm for large  
042 language models (Achiam et al., 2023; Touvron et al., 2023). AR models employ *next-token prediction*  
043 to model the probability of each token based on prior ones, showing striking empirical behaviors such  
044 as scalability (Kaplan et al., 2020) and *zero-shot generalization* to unseen tasks (Brown et al., 2020).

045  
046 Despite its success in other domains, AR modeling has received little attention in backbone modeling.  
047 We identify two main reasons. **(i)** Extending AR models to continuous data, *e.g.* atomic positions in  
048 3D, often relies on data discretization (Esser et al., 2021b;a), which can reduce structural fidelity and  
049 fine-grained details for proteins, limiting generative performance (Hsieh et al., 2025). **(ii)** Protein  
050 residues exhibit strong *bidirectional* dependencies: residues distant in sequence may be spatially  
051 close and form hydrogen bonds or hydrophobic contacts. This mutual dependency conflicts with the  
052 *unidirectional* assumption of standard AR models, and thus limits the quality of previous attempts on  
053 autoregressive structure generation (Gaujac et al., 2024). A natural question therefore arises: *can we*  
054 *apply AR modeling to protein backbone design?*

055  
056 In this paper, we answer the above question affirmatively, and propose **PAR**, a protein **autoregressive**  
057 framework, to unlock the power of AR models for protein backbone generation. We take initiative  
058 from the hierarchical nature of proteins: their structures span multiple *scales* of granularity, from  
059 coarse 3D topology and tertiary fold arrangements, local secondary structures, to the finest atomic

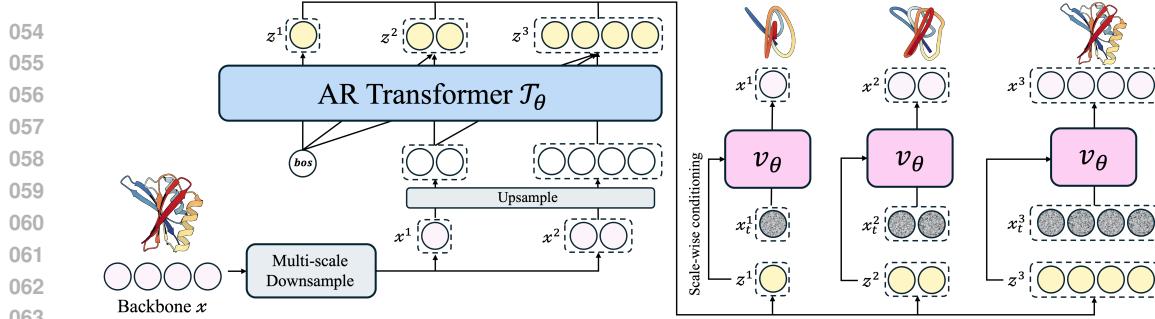


Figure 1: **Overview of PAR.** PAR comprises the autoregressive (AR) transformer  $\mathcal{T}_\theta$  (left) and the flow-based backbone decoder  $\mathbf{v}_\theta$  (right). During training, we downsample a backbone  $\mathbf{x} \in \mathbb{R}^{L \times 3}$  into multi-scale representations  $\{\mathbf{x}^1, \dots, \mathbf{x}^n\}$ . AR transformer performs next-scale prediction, producing conditional embeddings  $(\mathbf{z}^1, \dots, \mathbf{z}^n)$  from  $(bos, \dots, \mathbf{x}^{n-1})$ . The shared flow-based decoder learns to denoise backbones  $\mathbf{x}^i$  at each scale conditioned on  $\mathbf{z}^i$ . At inference, PAR autoregressively generates  $\mathbf{x}^i$  until the final structure  $\mathbf{x}$  is constructed.

coordinates. PAR thus adopts a multi-scale autoregressive framework via next-scale prediction, predicting each scale conditioned on prior coarser scales. This strategy, inspired by advances in image generation, enabled AR models to surpass strong diffusion models in image synthesis for the first time (Tian et al., 2024), and further allows multimodal LLMs to achieve unified text and image generation framework (Li et al., 2025).

Building on this multi-scale framework, PAR includes three key components (Fig. 1). The *multi-scale downsampling* creates coarse-to-fine structural representations to serve as structural context and targets during training. *AR transformer*, a stack of non-equivariant attention layers (Vaswani et al., 2017), encodes all preceding scales to produce a scale-wise conditional embedding following Li et al. (2024). The *flow-based backbone decoder* is conditioned on this embedding to model  $C\alpha$  backbone atoms directly. As a result, PAR avoids both discretization of protein structures and residue-wise unidirectional autoregressive ordering, thereby overcoming the two aforementioned limitations that compromise structural fidelity and generative quality. Moreover, training on ground-truth structural context, AR models suffer from *exposure bias* (Arora et al., 2022), which is a key challenge substantially reducing structure generation quality in our preliminary study. We effectively mitigate such issue via noisy context learning and scheduled sampling, allowing the model to learn from corrupted, or partially inaccurate context.

This multi-scale approach introduces several notable model behaviors. PAR generates backbones by establishing a global topology and performing refinements, analogous to progressively sculpting a statue into a masterpiece. For unconditional generation, PAR exhibits favorable scaling behavior, yielding competitive results on distributional metrics like Fréchet Protein Structure Distance (FPSD). Unlike diffusion models, which operate at a single scale, PAR flexibly handles inputs at various granularities, and hence shows zero-shot generalization in tasks like prompt-based generation and motif scaffolding. Finally, PAR provides a more general framework, incorporating flow-based models as a special case when restricted to a single scale, and thus remains compatible with techniques from flow-based models like self-conditioning (Chen et al., 2022).

**Main contributions:** (i) We present PAR, the *first* multi-scale AR model for protein backbone generation that addresses key limitations of existing AR methods. (ii) PAR comprises multi-scale downsampling, AR transformer, and a flow-based decoder, to directly model  $C\alpha$  atom, avoiding fidelity loss from discretization. (iii) We alleviate exposure bias through noisy context learning and scheduled sampling, effectively improving structure generation. (iv) Our model shows an interpretable generation process that forms coarse backbone topology and refines it progressively. (v) Benchmarking results show that PAR effectively captures protein data distributions, achieving FPSD score of 231.5 against PDB dataset that further scale with training compute. (vi) PAR exhibits zero-shot generalization potential, reflecting the versatility of AR large language models.

## 2 BACKGROUND AND RELATED WORK

**Flow and diffusion-based structure generative models.** Flow-based and diffusion methods (Lipman et al., 2022; Ho et al., 2020) operate by transforming samples from a prior distribution to the target data distribution, and have been widely applied to protein backbone generation. These methods either predict per-residue rotations and translations using a frame-based Riemannian manifold

representation (Yim et al., 2023b; Bose et al., 2023; Yim et al., 2023a; Watson et al., 2023; Ingraham et al., 2023) or directly model atom coordinates, such as  $C\alpha$  positions (Lin & AlQuraishi, 2023; Lin et al., 2024; Geffner et al., 2025), with some approaches generating fully atomistic proteins including side chains (Qu et al., 2024; Chu et al., 2024). Discrete diffusion methods (Hayes et al., 2025; Wang et al., 2024), trained on structure tokens, often reduce structural fidelity and limit generation quality (Hsieh et al., 2025). Unlike most diffusion approaches, which are single-scale, PAR models protein structures across multiple scales using a parameterized upsampling autoregressive process from short to long, allowing flexible handling of different structural granularities and zero-shot generalization to tasks like prompt-based generation. In addition, PAR provides a more general framework, as it naturally reduces to a flow-based model when restricted to a single scale.

**Autoregressive modeling.** Autoregressive (AR) modeling has been driving natural language processing and computer vision due to its strong scalability and zero-shot generalization (Tian et al., 2024; Touvron et al., 2023; Achiam et al., 2023). The approach relies on *next-token prediction* that predicts the distribution of the next token based on prior ones in a *unidirectional* sequence. However, adapting autoregressive models to continuous domains, like image generation, often involves tokenizers such as VQVAE (Esser et al., 2021b;a), which discretizes the data for transformer training and may discard fine-grained details. Recently, Li et al. (2024) used the AR model that produces conditioning for a diffusion network (*e.g.*, a small MLP) to model image latents, unlocking the operations of AR models in a continuous-valued space. In addition, defining appropriate autoregressive orders that preserve data properties is crucial. Since next-token prediction inherently discards spatial locality by flattening the 2D image feature map into a 1D sequence, VAR (Tian et al., 2024) introduced *next-scale prediction*. Leveraging a multi-scale VQVAE, the image feature map is quantized into  $n$  multi-scale token maps that preserve the spatial and *bidirectional* correlations. To our knowledge, autoregressive modeling has not been widely applied to protein structure generation despite their success in other domains. The only exception is Gaujac et al. (2024), which models structure tokens with a causal transformer. In contrast, we design a multi-scale autoregressive framework that operates directly in continuous backbone space using a flow-based backbone decoder, thereby addressing the limitations of discrete token maps while respecting the bidirectional biophysical characteristics of protein structures.

### 3 PROTEIN AUTOREGRESSIVE MODELING

In this section, we introduce PAR, a multi-scale autoregressive (AR) framework for protein backbone generation. Formally, we want to model a protein backbone  $C\alpha$  structure with  $L$  residues  $\mathbf{x} \in \mathbb{R}^{L \times 3}$  in an autoregressive manner as follows:

$$p_{\theta}(\mathbf{x}) = \mathbb{E}_{X \sim q_{\text{decompose}}(\cdot | \mathbf{x})} p_{\theta}(X = \{\mathbf{x}^1, \dots, \mathbf{x}^n\}) = \mathbb{E}_{X \sim q_{\text{decompose}}(\cdot | \mathbf{x})} \prod_{i=1}^n p_{\theta}(\mathbf{x}^i | X^{<i}), \quad (1)$$

where  $q_{\text{decompose}}(\cdot | \mathbf{x})$  defines a decomposition of autoregressive order for protein structure  $\mathbf{x}$  into  $n$  scales  $X = \{\mathbf{x}^1, \dots, \mathbf{x}^n\}$  with  $\mathbf{x}^n = \mathbf{x}$ , while  $p_{\theta}(\mathbf{x}^i | X^{<i})$  is the desired PAR model learning to generate  $\mathbf{x}$  via a scale-wise autoregression.

The design space of  $q_{\text{decompose}}$  and  $p_{\theta}$  under this formulation (Eqn. 1) can be flexible. Recall that our goal is to enable AR modeling to preserve spatial dependencies and avoid discretization, as discussed in §1. To this end, in §3.1, we devise a non-parametric and deterministic  $q_{\text{decompose}}$  by *multi-scale protein downsampling* (Fig. 1, left) that represents protein backbones at multiple scales via hierarchical down-sampling (Eqn. 2), providing structural context and training targets. In §3.2, we parameterize PAR  $p_{\theta}$  as a backbone autoregressive upsampling process via next-scale prediction and achieve direct  $C\alpha$  modeling in the continuous space (Eqn. 3). This comprises two key components: (*i*) an autoregressive transformer (Fig. 1, left) that produces scale-wise conditional embeddings informed by preceding scales to guide generation (Eqn. 4); and (*ii*) a flow-based backbone decoder (Fig. 1, right) which samples  $C\alpha$  backbone coordinates conditioned on the learned embeddings (Eqn. 5).

Finally, in §3.3, we dedicated strategies to mitigate *exposure bias* (Arora et al., 2022), a mismatch between training on ground-truth data and inference on model predictions that leads to error accumulations and degrading generation quality in AR models. Together, these components enable PAR to robustly generate protein backbones in a coarse-to-fine manner.

#### 3.1 MULTI-SCALE PROTEIN DOWNSAMPLING

We construct the multi-scale representations of protein structures via hierarchical downsampling to serve as training context and targets for PAR (Fig. 1, left). Given a protein structure  $\mathbf{x} \in \mathbb{R}^{L \times 3}$ , it

162 produces a hierarchy of coarse-to-fine scales by progressively downsampling  $\mathbf{x}$  into  $n$  scales:  
 163

$$\begin{aligned} q_{\text{decompose}} : \mathbf{x} \rightarrow X &= \{\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^n\} \\ 164 \quad &= \{\text{Down}(\mathbf{x}, \text{size}(1)), \text{Down}(\mathbf{x}, \text{size}(2)), \dots, \mathbf{x}\}, \end{aligned} \quad (2)$$

166 where  $\text{Down}(\mathbf{x}, \text{size}(i)) \in \mathbb{R}^{\text{size}(i) \times 3}$  denotes a downsampling operation that interpolates  $\mathbf{x}$  along  
 167 the sequence dimension, leading to  $\text{size}(i)$  3D centroids that provide a coarse structural layout.  
 168 Since  $q_{\text{decompose}}$  is designed as a deterministic mapping for every  $\mathbf{x}$ , the likelihood of Eqn. 1 can be  
 169 simplified without marginalization:  $p_{\theta}(\mathbf{x}) = \prod_{i=1}^n p_{\theta}(\mathbf{x}^i | X^{<i})$ .  
 170

**Scale configurations**  $\mathcal{S} = \{\text{size}(1), \dots, \text{size}(n)\}$  could be defined in two ways. When defined  
 171 by *length*, scales are chosen as hyperparameters, e.g.,  $\mathcal{S} = \{64, 128, 256\}$ . In this case, if  $L$  lies in  
 172  $(\text{size}(i), \text{size}(i+1)]$ , the protein could be generated with only  $i+1$  autoregressive steps. When  
 173 defined by *ratio*, scales are adaptively determined based on protein length, e.g.,  $\mathcal{S} = \{L/4, L/2, L\}$ .  
 174 Empirically, defining scales by length yields slightly better results in modeling data distributions.  
 175 We adopt this as the default configuration. This design enables training PAR with flexible scale  
 176 configurations. In the following sections, we describe how this hierarchy of representations are  
 177 modeled using the autoregressive transformer and backbone decoder.  
 178

### 3.2 COARSE-TO-FINE BACKBONE AUTOREGRESSIVE MODELING

179 Preserving the inherent dependencies in data when defining the autoregressive order is crucial and  
 180 affects generation performance (Tian et al., 2024). Standard AR models assume *unidirectional*  
 181 dependency, which conflicts with the strong *bidirectional* interactions in protein sequences, e.g.,  
 182 spatially close residues can form hydrophobic contacts or hydrogen bonds even if distant in sequence.  
 183 PAR addresses this with a multi-scale AR framework via next-scale prediction, capturing mutual  
 184 structural dependency over each scale. Motivated by Li et al. (2024), we propose to use an AR  
 185 Transformer with diffusion/flow-based regression loss to enable modeling of  $\text{C}\alpha$  atoms directly in  
 186 continuous space. That is, we could rewrite the likelihood as:  
 187

$$p_{\theta}(X = \{\mathbf{x}^1, \dots, \mathbf{x}^n\}) = \prod_{i=1}^n p_{\theta}(\mathbf{x}^i | X^{<i}) = \prod_{i=1}^n p_{\theta}(\mathbf{x}^i | \mathbf{z}^i = \mathcal{T}_{\theta}(X^{<i})), \quad (3)$$

190 where  $\mathcal{T}_{\theta}$  is an AR Transformer that produces scale-wise conditioning  $\mathbf{z}^i$  while  $p_{\theta}(\mathbf{x}^i | \mathbf{z}^i)$  is optimized  
 191 with a flow-based atomic decoder  $\mathbf{v}_{\theta}$  with flow matching. This avoids discretizing protein structures  
 192 into tokens, preserving structural details and generation fidelity. We describe each component below.  
 193

**Autoregressive transformer for scale-wise conditioning.** To formulate the autoregressive order, we  
 194 leverage the hierarchical nature of proteins, where a protein structure could span various levels of  
 195 representations from coarse tertiary topology to the finest atomic coordinates. We adopt the next-scale  
 196 prediction to model per-scale distribution based on prior coarser scales, which further ensures that  
 197 the *bidirectional* dependencies of residues are modeled over each scale. We train our autoregressive  
 198 model (Fig. 1, left), a non-equivariant transformer  $\mathcal{T}_{\theta}$ , to produce scale-wise conditioning embedding  
 199  $\mathbf{z}^i$  for scale  $i$  depending on prior scales  $X^{<i} = \mathbf{x}^1, \dots, \mathbf{x}^{i-1}$ :  
 200

$$\mathbf{z}^i = \mathcal{T}_{\theta}(X^{<i}) = \mathcal{T}_{\theta}\left([\text{bos}, \text{Up}(\mathbf{x}^1, \text{size}(2)), \dots, \text{Up}(\mathbf{x}^{i-1}, \text{size}(i))]\right), \quad (4)$$

201 where  $\text{bos} \in \mathbb{R}^{\text{size}(1) \times 3}$  is a learnable embedding, and  $\text{Up}(\mathbf{x}^{i-1}, \text{size}(i))$  interpolates  $\mathbf{x}^{i-1}$  to  
 202  $\text{size}(i)$  3D points. All inputs are concatenated along the sequence dimension before being fed into  
 203  $\mathcal{T}_{\theta}$ . The embedding  $\mathbf{z}^i$  is then used to condition the flow matching decoder to predict the backbone  
 204 coordinates  $\mathbf{x}^i$ , detailed as follows.  
 205

**Flow-based atomic decoder.** We enable PAR to directly model  $\text{C}\alpha$  positions  $\mathbf{x}$ , wherein  $p_{\theta}(\mathbf{x} | \mathbf{z}^i)$   
 206 is parameterized by an atomic decoder  $\mathbf{v}_{\theta}$  with flow matching (FM, Lipman et al., 2022), which  
 207 maps standard normal distribution to the target data distribution. We condition the  $\mathbf{v}_{\theta}$  with scale-wise  
 208 conditioning  $\mathbf{z}^i$  predicted by the AR Transformer  $\mathcal{T}_{\theta}$  at each scale  $i$  (Fig. 1, right). During training,  
 209 we sample the noise  $\epsilon^i \sim \mathcal{N}(0, I)$  and a time variable  $t^i \in [0, 1]$ , and compute the interpolated  
 210 sample as  $\mathbf{x}_{t^i}^i = t^i \cdot \mathbf{x}^i + (1 - t^i) \cdot \epsilon^i$ . As such, we can jointly train  $\mathbf{v}_{\theta}$  and  $\mathcal{T}_{\theta}$  with an FM objective:  
 211

$$\mathcal{L}(\theta) = \mathbb{E}_{\mathbf{x} \sim p_{\mathcal{D}}} \left[ \frac{1}{n} \sum_{i=1}^n \frac{1}{\text{size}(i)} \mathbb{E}_{t^i \sim p(t^i), \epsilon^i \sim \mathcal{N}(0, I)} \left\| \mathbf{v}_{\theta}(\mathbf{x}_{t^i}^i, t^i, \mathbf{z}^i) - (\mathbf{x}^i - \epsilon^i) \right\|^2 \right], \quad (5)$$

212 where  $p_{\mathcal{D}}(\mathbf{x})$  denotes the training data distribution and  $p(t)$  denotes the t-sampling distribution in  
 213 Geffner et al. (2025). The conditioning embedding  $\mathbf{z}^i$  is injected into the atomic decoder network  
 214  $\mathbf{v}_{\theta}$  through adaptive layer norms (Peebles & Xie, 2023). We further concatenate a learnable scale  
 215

embedding alongside  $\mathbf{z}^i$  to help the model identify different scales and incorporate self-conditioning input as an additional condition (Chen et al., 2022), though we omit them in the equation for simplicity. To formulate the indices for positional encoding  $p^i$  at scale  $i$ , we uniformly sample  $\text{size}(i)$  numbers from the interval  $[1, L]$ , i.e.,  $p^i = \text{linspace}(1, L, \text{size}(i))$ . At coarse scales, the wide spacing between adjacent indices encourages the model to capture global structural layout, while at finer scales the dense indices allow the model to focus on local details. For more details, please refer to §A.1.

Leveraging the learned flow network  $\mathbf{v}_\theta$ , sampling could be performed at each scale through ordinary differential equation (ODE)  $d\mathbf{x}_t = \mathbf{v}_\theta(\mathbf{x}_t, t) dt$ , with the scale superscript  $i$  omitted for simplicity. Moreover, we could define the stochastic differential equation (SDE) for sampling:

$$d\mathbf{x}_t = \mathbf{v}_\theta(\mathbf{x}_t, t) dt + g(t) \mathbf{s}_\theta(\mathbf{x}_t, t) dt + \sqrt{2g(t)\gamma} d\mathcal{W}_t, \quad (6)$$

where  $g(t)$  is a time-dependent scaling function for the score function  $\mathbf{s}_\theta(\mathbf{x}_t, t)$  (Albergo et al., 2023; Ma et al., 2024) and the noise term,  $\gamma$  is a noise scaling parameter, and  $\mathcal{W}_t$  is a standard Wiener process. The score function, defined as the gradient of the log-probability of the noisy data distribution at time  $t$ , could be computed as  $\mathbf{s}_\theta(\mathbf{x}_t, t) = \frac{t \mathbf{v}_\theta(\mathbf{x}_t, t) - \mathbf{x}_t}{1-t}$ .

**Multi-scale structure generation.** At inference, the autoregressive transformer first produces  $\mathbf{z}^1$  at the coarsest scale, which conditions the flow matching decoder to generate  $\mathbf{x}^1$  either via ODE or SDE sampling in Eqn. 6. We upsample  $\mathbf{x}^1$  using  $\text{Up}(\mathbf{x}^1, \text{size}(2))$  and send it back into the autoregressive transformer to predict the next scale embedding  $\mathbf{z}^2$ . This coarse-to-fine process iterates  $n$  times until the flow-matching model generates the full-resolution backbone  $\mathbf{x}$ . KV cache is applied throughout the autoregressive process for efficiency.

### 3.3 MITIGATING EXPOSURE BIAS

Training AR models typically uses teacher forcing (Williams & Zipser, 1989), where ground-truth data are fed as context to stabilize learning. However, during inference the model is conditioned on its own predictions, creating a training-inference mismatch known as *exposure bias* (Arora et al., 2022; He et al., 2019). Errors can then accumulate across autoregressive steps, degrading output quality. Our preliminary study shows that teacher forcing greatly reduces the designability of generated structures. To mitigate this, we adapt Noisy Context Learning (NCL) and Scheduled Sampling (SS), techniques from language and image AR modeling (Ren et al., 2025; Bengio et al., 2015), for PAR.

**Noisy context learning.** We train PAR with noisy context, adding noise to the ground-truth prior-scale input during training. This encourages the model to learn the per-scale distribution without relying on perfectly accurate context, improving robustness. We randomly sample  $n$  noise weights  $\{w_{\text{ncl}}^1, \dots, w_{\text{ncl}}^n\} \in [0, 1]$ , and draw  $n$  noise samples  $\{\epsilon_{\text{ncl}}^1, \dots, \epsilon_{\text{ncl}}^n\} \in \mathcal{N}(0, I)$ . Each input context  $\mathbf{x}^i$  is corrupted as  $\mathbf{x}_{\text{ncl}}^i = w_{\text{ncl}}^i \cdot \mathbf{x}^i + (1 - w_{\text{ncl}}^i) \cdot \epsilon_{\text{ncl}}^i$ . This perturbation is applied to the input context *only* during training, which updates the autoregressive step in Eqn. 4 as  $\mathbf{z}^i = \mathcal{T}_\theta([\text{bos}, \text{Up}(\mathbf{x}_{\text{ncl}}^1, \text{size}(2)), \dots, \text{Up}(\mathbf{x}_{\text{ncl}}^{i-1}, \text{size}(i))])$ .

**Scheduled sampling.** During training, we use scheduled sampling (Bengio et al., 2015) by running the forward process iteratively across scales. At the  $i$ -th scale, the flow-based backbone decoder predicts the clean data  $\mathbf{x}_{\text{pred}}^i = \mathbf{x}_t^i + (1 - t^i) \mathbf{v}_\theta(\mathbf{x}_t^i, t^i, \mathbf{z}^i)$ . With a probability of 0.5, we replace the ground truth context  $\mathbf{x}^i$  with this prediction  $\mathbf{x}_{\text{pred}}^i$  at later scales. This exposes the model to its own output and reduces the train-test gap. Notably, we could combine noisy context learning with this technique by adding noises to the model predicted context  $\mathbf{x}_{\text{pred}}^i$ .

Leveraging these designs, PAR robustly models Cα backbone coordinates within a coarse-to-fine autoregressive framework, enabling it to effectively learn the underlying protein data distribution. Further, the multi-scale formulation leads to various notable model behaviors, *e.g.*, zero-shot generalization. We present these analyses in the following section.

## 4 EXPERIMENTS

We begin by evaluating PAR on unconditional backbone generation and compare it with existing structure generative methods in §4.1. Next, we examine its zero-shot generalization ability in §4.2. We then study scaling behavior and propose strategies to mitigate exposure bias as well as additional ablation studies in §4.3.

### 4.1 PROTEIN BACKBONE GENERATION

**Generation over scales.** We illustrate PAR’s backbone generation using a 5-scale model ( $\mathcal{S} = \{L/16, L/8, L/4, L/2, L\}$ ) in Fig. 2, showing generated structures with target lengths of

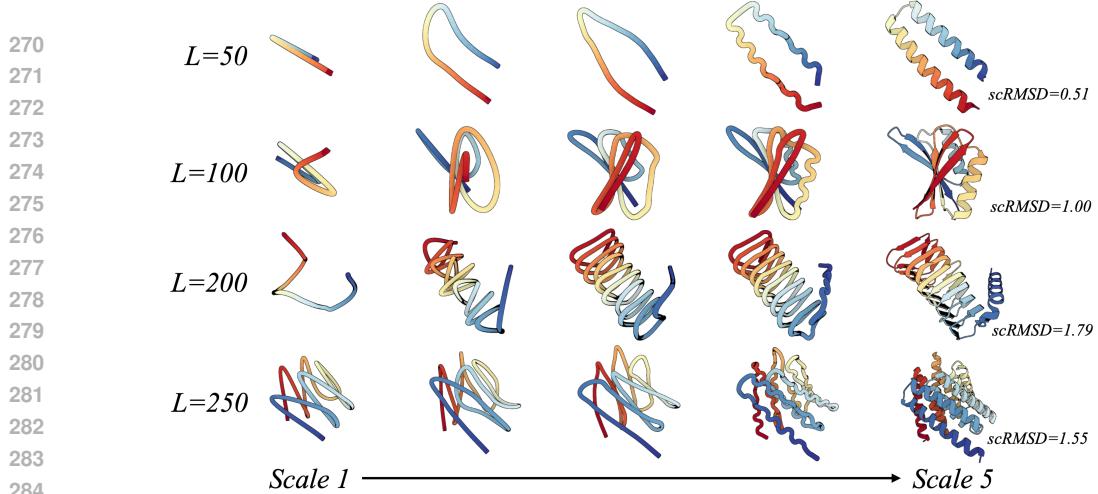


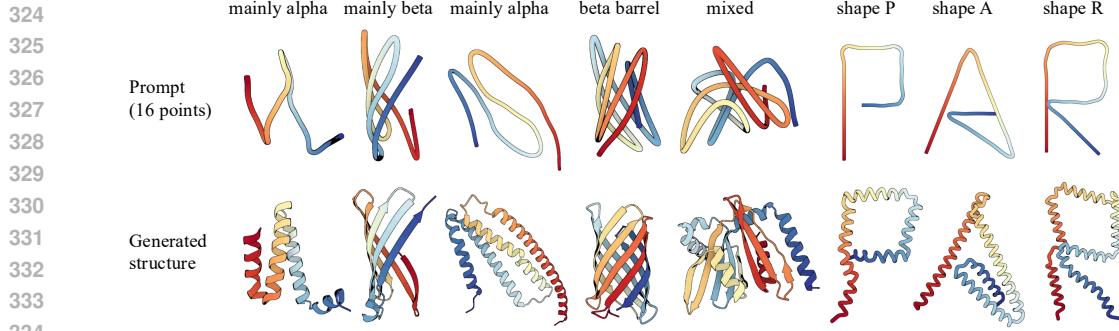
Figure 2: **Samples generated by PAR over scales.** We illustrate PAR’s generation process across five scales. Much like sculpting a statue, the model first formulates the global structural layout at coarse scales and progressively refines the details at later scales.

Table 1: **Unconditional backbone generation performance.** We follow Geffner et al. (2025) in adopting **FPSD** and **fS** to evaluate the model’s ability to capture the data distribution.  $\text{PAR}_{\text{pdb}}$  denotes the 400M model finetuned on the PDB subset.

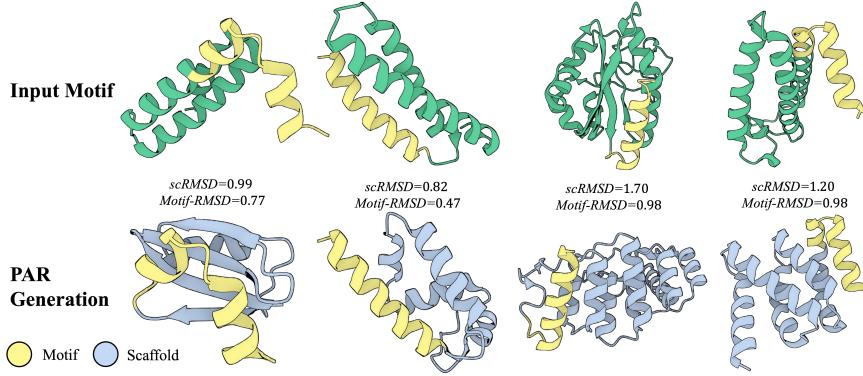
Method	Designability		Diversity TM-Sc.↓	FPSD vs.		fS (C / A / T)↑	Sec. Struct. % ( $\alpha / \beta$ )
	(%)↑	sc-RMSD↓		PDB↓	AFDB↓		
FrameDiff (17M)	65.4	-	0.40	194.2	258.1	2.46/5.78/23.35	64.9/11.2
RFDiffusion (60M)	94.4	-	0.42	253.7	252.4	2.25/5.06/19.83	64.3/17.2
ESM3 (1.4B)	22.0	-	0.42	933.9	855.4	3.19/6.71/17.73	64.5/8.5
Genie2 (16M)	95.2	-	0.38	350.0	313.8	1.55/3.66/11.65	72.7/4.8
Proteina (200M)	92.8	1.14	0.37	282.3	285.6	2.17/6.22/21.48	66.3/9.2
Proteina (400M)	92.6	1.09	0.37	271.3	272.6	2.13/6.14/21.18	65.1/9.5
$\text{Proteina}_{\text{pdb}}$	<b>94.8</b>	<b>1.02</b>	<b>0.36</b>	<b>181.5</b>	<b>257.3</b>	<b>2.64/6.48/30.10</b>	<b>46.9/17.6</b>
PAR (200M)	87.0	1.33	0.37	252.0	237.9	2.11/6.41/19.22	64.3/8.8
PAR (400M)	88.0	1.28	<b>0.36</b>	231.5	<b>211.8</b>	2.20/6.59/20.96	63.2/9.7
$\gamma=0.30$	<b>96.0</b>	<b>1.01</b>	<b>0.39</b>	<b>313.9</b>	<b>296.4</b>	<b>2.24/6.60/16.71</b>	<b>66.3/8.9</b>
$\text{PAR}_{\text{pdb}}$	<b>96.6</b>	1.04	0.43	<b>161.0</b>	<b>228.4</b>	<b>2.57/7.42/23.61</b>	50.2/16.7

$\{50, 100, 200, 250\}$  residues. Generation proceeds in a coarse-to-fine manner, which resonates with statue sculpting: the coarser scales establish a rough global layout, and finer scales progressively add local details. This multi-scale formulation yields a clear and interpretable generation process. We present the quantitative analysis on PAR’s backbone generation in the next paragraph.

**Unconditional generation benchmark.** We compare 3-scale PAR’s ( $S = \{64, 128, 256\}$ ) backbone generation performance with other baselines in Tab. 1, following the evaluation protocol in Geffner et al. (2025). The baselines span three categories: frame-based diffusion methods (Yim et al., 2023c; Watson et al., 2023), multimodal protein language models (Hayes et al., 2025), and diffusion/flow-based  $C\alpha$  generators (Lin et al., 2024; Geffner et al., 2025). We train both PAR and Proteina for 200k steps on our training data, and report results of other baselines from Geffner et al. (2025) for fair comparison. Evaluation metrics and baseline categories are detailed in §A.2§A.3. As shown in Tab. 1, PAR generates samples that closely match the reference data distribution and maintaining competitive designability. PAR achieves high FPSD and fold score (fS), which assess generation quality and diversity by comparing generated and reference distributions. When comparing to Proteina, which also models  $C\alpha$  with non-equivariant transformers, PAR more effectively models the data distribution, achieving a lower FPSD (231.5 vs. 271.3). **By reducing the noise scaling parameter  $\gamma$  from 0.45 to 0.3 in Equation 6 for the SDE sampling, we can reduce sampling stochasticity and improve sample quality, improving the designability from 88.0% to 96.00% without additional training.** Moreover, PAR demonstrates superior generation diversity among all methods. **Meanwhile, we note that Geffner et al. (2025) also report results using fine-tuned models on a PDB subset, which improves FPSD while maintaining strong designability.** Following their approach, we curated a PDB subset of 21K designable samples and fine-tuned PAR on this subset for 5K steps. After fine-tuning, PAR achieved



335 **Figure 3: Backbone generation with human prompt.** Given a small number of points (e.g., 16)  
 336 as prompt, PAR can generate protein backbones that adhere to the global arrangements specified by  
 337 these points, *without* any finetuning. For visualization, input points are interpolated to match the  
 338 length of the generated structure.



351 **Figure 4: Zero-shot motif scaffolding.** Given a motif structure, PAR can generate diverse, plausible  
 352 scaffold structures that accurately preserve the motif via teacher-forcing the motif coordinates at each  
 353 scale, without additional conditioning or fine-tuning.

355 **96.6% designability and 161.0 FPSD against the PDB, outperforming all diffusion-based baselines  
 356 and even surpassing Proteina fine-tuned on the same subset.**

#### 4.2 ZERO-SHOT TASK GENERALIZATION

358 **Guiding backbone generation with human-specified prompt.** Proteins possess hierarchical and  
 359 complex structures, which makes it challenging to directly specify a target shape and design proteins  
 360 accordingly. By leveraging PAR’s coarse-to-fine generation, a simple prompt (e.g., 16 points) can  
 361 specify a protein’s coarse layout, from which the model generates the complete structure as shown in  
 362 Fig. 3. In particular, we first obtain a 16-point input prompt either by downsampling a real protein  
 363 structure from the test set, or by specifying the points manually (the top row in Fig. 3). Using a 5-scale  
 364 PAR ( $\mathcal{S} = \{16, 32, 64, 128, 256\}$ ), we initialize the first-scale prediction with the 16-point prompt  
 365 and autoregressively upsample until the full protein structure is generated, as illustrated in the bottom  
 366 row of Fig. 3. Following this process, PAR can generate a new structure that preserves the coarse  
 367 structural layout (first five examples), and explore entirely novel structures (last three examples). If  
 368 desired, longer prompts (e.g., 32 points) could be specified to achieve more finer-grained control over  
 369 backbone generation. As later shown in Tab. 5, we quantitatively evaluate the structural consistency  
 370 (TM-score) between the prompted layout and the final generation.

371 **Motif scaffolding.** Besides the point-based layout, PAR can preserve finer-grained prompts like  
 372 atomic coordinates. Fig. 4 highlights the zero-shot motif scaffolding capabilities of PAR. Using a  
 373 5-scale PAR, we downsample a raw protein structure into five scales and teacher-force the ground-  
 374 truth motif coordinates at each scale before propagating into the next scale. To avoid clashes or  
 375 discontinuities, we superimpose the ground-truth motif residues and the generated motif segments  
 376 before replacement. With no fine-tuning and no conditioning, PAR generates plausible scaffolds  
 377 that preserve motif structures with high fidelity. This stands in contrast to diffusion or flow-based  
 378 frameworks, which typically require fine-tuning on additional conditions such as masks or motif  
 379 coordinates, or rely on decomposition strategies (Geffner et al., 2025; Watson et al., 2023; Wang

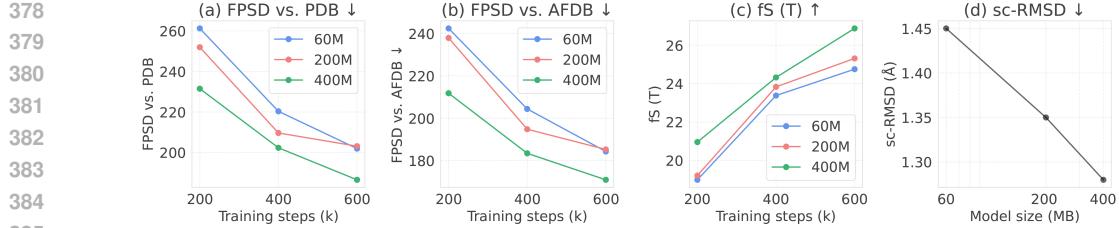


Figure 5: **Scaling effects of PAR.** Performance of four metrics over varying training steps and model sizes, (a) FPSD vs. PDB, (b) FPSD vs. AFDB, (c) fS(T), (d) sc-RMSD.

et al., 2022). Moreover, the generated scaffolds differ substantially from the input structure, showing that PAR generates structurally diverse scaffolds rather than merely copying. For example, the leftmost example in Fig. 4 preserves the yellow motif helix while introducing new secondary structure elements like  $\beta$ -sheet and loops, in contrast to the original helices.

### 4.3 EMPIRICAL ANALYSIS OF MULTISCALE PAR

**Scaling effects of PAR.** We examine the model’s behaviors by varying the backbone decoder’s size and number of training steps in Fig. 5. We train PAR with 3 scales over three different model sizes with 60,200,400 million parameters and three training durations over 200, 400, 600K steps. PAR demonstrates favorable behavior when scaling both model size and training duration, effectively improving its ability to capture the protein data distribution with FPSD scores of 187 against PDB and 170 against AFDB (first two columns in Fig. 5). Further, the fS scores, which reflect quality and diversity, increase with larger model sizes and greater computational budgets. While extending training duration alone offers negligible gains, increasing model size substantially enhances designability, leading to lower sc-RMSD values. Meanwhile, we empirically observe that scaling the autoregressive transformer has minimal impacts on the evaluation results. This is consistent with the module’s role to generate scale-wise conditioning to guide the backbone generation, which does not require large model capacity. Similar trends have been observed in image generation (Chen et al., 2025), where outputs from one scale are directly passed to the next without relying on additional encoding modules. This allows us to prioritize increasing the backbone decoder’s model capacity that effectively improves metrics.

**Efficient sampling with multi-scale orchestration of SDE/ODE.** While Tab. 1 reports results using a uniform number of sampling steps across scales, the multi-scale formulation of PAR actually offers advantages in sampling efficiency. More specifically, (1) sampling at the coarser scale (e.g., first scale) is more efficient than sampling at finer scales (e.g., 2nd scale) due to shorter sequence length; (2) we can use less number of sampling steps at finer scales than coarser scales. As shown in Tab. 2, by using SDE sampling only at the first scale,

Table 2: **Performance of different sampling methods and steps.** Combining SDE and ODE sampling across scales yields a  $2.5\times$  inference speedup compared to the single-scale 400-step baseline, shown in the first and the last row. We generate 100 samples at each length.

Sampling	Steps	Length 150		Length 200	
		Time (s)	Design. (%)	Time (s)	Design. (%)
Proteina (SDE)	0/0/400	131	97%	170	92%
Proteina (SDE)	0/0/200	67	89%	86	80%
All SDE	400/400/400	312	97%	351	94%
All SDE	400/400/2	184	0%	-	-
All ODE	400/400/400	312	28%	-	-
S/S/O	400/400/400	312	98%	-	-
S/S/O	400/400/2	184	99%	186	91%
S/O/O	400/400/400	312	96%	-	-
S/O/O	400/2/2	67	97%	68	94%

and switching to ODE sampling for the remaining scales, PAR could dramatically reduce the diffusion steps from 400 to 2 steps at the last two scales without harming designability (97%), yielding a 4.7x inference speedup. This is possible because a high-quality coarse topology places the model near high-density regions, enabling efficient refinement with ODE sampling. Naively reducing the SDE sampling steps significantly harms designability, dropping to 22% when reducing steps to 50, as shown in Fig. 8. This is consistent with the observation of single-scale models like Proteina, where designability degrades to 89% when reducing SDE sampling steps to 200 in Tab. 2. Crucially, SDE sampling at the first scale is necessary for establishing a reliable global topology, given that ODE-only sampling exhibits poor designability. Compared to the single-scale 400-step baseline, PAR achieves 1.96x and 2.5x sampling speedup at length 150 and 200, respectively. This improvement is driven by speeding up the final scales, where the longer sequence lengths cause computational costs to grow

432 quadratically in transformer architectures. Moreover, the computational costs remain constant at the  
 433 first scale because it has a fixed size 64, even when generating longer sequences.

434 **Mitigating exposure bias.** To  
 435 mitigate exposure bias, we  
 436 adopted noisy context learning  
 437 (NCL) and scheduled sampling  
 438 (SS) as defined in §3.3. Noisy  
 439 context learning encourages the  
 440 model to infer structural guid-  
 441 ance from corrupted context and  
 442 boosts the structure generation  
 443 quality. Tab. 3 shows that noisy  
 444 context learning effectively improves the sc-RMSD of the generated structure from 2.20 to 1.58,  
 445 and reduces FPSD against AFDB to 23.69 when using ODE sampling. The designability further  
 446 improved to 1.48 along with scheduled sampling, which makes the training process more aligned  
 447 with the inference scenario. Results are obtained with 60M PAR trained for 100K steps.

448 **Interpreting multi-scale PAR.**  
 449 We visualize the attention maps  
 450 of the autoregressive transformer  
 451 at each scale (Fig. 6). **We aver-**  
 452 **age the attention scores within**  
 453 **each scale, normalize them such**  
 454 **that the scores across scales sum**  
 455 **to 1, and average them over 50**  
 456 **test samples to obtain the scale-**  
 457 **level attention distribution dur-**  
 458 **ing inference.** We summarize  
 459 three key observations: *(i)* Most  
 460 scales barely attend to the first  
 461 scale, since the input to this scale,  
 462 a `bos` token, carries little structural signal. *(ii)* Each scale primarily  
 463 attends to the previous scale, which typically contains richer contextual and structural information.  
 464 *(iii)* Despite focusing most heavily on the current scale, the model still retains non-negligible attention  
 465 to earlier scales. This indicates that PAR effectively integrates information across multiple scales  
 466 and maintains structural consistency during generation. This aligns with results in Tab. 5 where the  
 467 autoregressive Transformer effectively improves consistency with the given prompt.

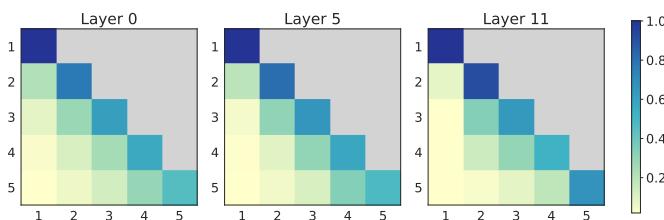


Figure 6: Visualization of the average attention scores in PAR autoregressive transformer over 5 scales, obtained from samples with lengths in  $(128, 256]$ . We provide attention map visualization for shorter proteins in §C.3

468 **Multi-scale formulation.** We  
 469 ablate the effect of defining scale  
 470 by *length* versus *ratio*, as shown  
 471 in §3.1. Tab. 4 shows that under  
 472 comparable levels of upsample-  
 473 ring ratio ( $\{64, 128, 256\}$  and  
 474  $\{L/4, L/2, L\}$ ), the *by-length*  
 475 strategy outperforms *by-ratio*.  
 476 Meanwhile, PAR obtains better  
 477 designability and FPSD when increasing from two scales to three scales. Beyond this point, increasing  
 478 the scale configurations to four and five scales results in degraded designability, potentially due to  
 479 error accumulation and exposure bias. These results support our choice of adopting the 3-scale PAR  
 480 as the default. All results are obtained using the 60M model.

481 **AR Transformer improves**  
 482 **structural consistency.** We  
 483 conduct an ablation study to  
 484 evaluate the effectiveness of  
 485 the autoregressive transformer  
 486 in Tab. 5. We compare two  
 487 different strategies for encoding  
 488 prior-scale structural context,  
 489 including *(i)* direct input, where the multi-scale structures are directly fed into the backbone decoder  
 490 without any intermediate encoding; and *(ii)* transformer encoder, where all scales are processed

Table 4: **Multi-scale formulation.** We ablate different strategies for scale configuration in downsampling.

Define scale	Designability		FPSD vs.		fs (C / A / T)↑
	(%)↑	(sc-RMSD)↓	PDB↓	AFDB↓	
$\{64, 256\}$	83.0	1.38	282.85	274.32	2.14/6.58/20.66
$\{64, 128, 256\}$	85.0	1.39	279.63	267.35	2.15/6.52/20.35
$\{64, 128, 192, 256\}$	77.8	1.55	296.70	282.69	2.05/6.04/18.69
$\{64, 96, 128, 192, 256\}$	81.0	1.51	276.00	263.58	2.17/6.31/20.65
$\{L/4, L/2, L\}$	86.4	1.49	310.64	298.30	2.00/5.87/18.91

Table 5: **Structural consistency for prompted generation.** Using a transformer to encode prior-scale structural conditions shows better prompt-following than direct input.

Length	RMSD vs. Reference ↓			TM-score vs. Prompt ↑		
	(32.64]	(64,128]	(64,128]	(32.64]	(64,128]	(64,128]
Reference	-	-	-	0.60	0.61	0.59
Direct Input	2.13	3.38	6.51	0.58	0.61	0.59
Trans. Encode	1.45	2.72	5.75	0.60	0.64	0.61

486 autoregressively by a Transformer encoder, and the resulting encoded representation is then passed to  
 487 the backbone decoder. We train two 60M models and evaluate both models by downsampling 588  
 488 testing structures as prompts and re-upsamples them with PAR. As shown in Tab. 5, the transformer  
 489 encoder demonstrates better structural consistency, indicating that autoregressive encoding across  
 490 scales produces coherent structural guidance over scales, consistent with attention maps in Fig. 6.

491 **Ablation with self-conditioning.** Multi-scale  
 492 autoregressive modeling and self-conditioning  
 493 similarly guide the generation with a coarse estimate  
 494 of the structure. To evaluate the role of self-  
 495 conditioning in our multiscale framework, we  
 496 conducted an ablation study (Fig. 7), where the  
 497 results are from the same 60M model in the previous  
 498 ablation study. Across all length ranges, the  
 499 model with self-conditioning consistently  
 500 generates higher-quality protein structures, in  
 501 terms of sc-RMSD. Although self-conditioning  
 502 also supplies an intermediate structural estimate  
 503 during generation, it is complementary to the  
 504 multi-scale formulation and yields further im-  
 505 provements in structural quality.

## 5 DISCUSSION

506 PAR is the first multi-scale autoregressive model for protein backbone generation, offering a general  
 507 framework that includes flow-based methods as a special case. PAR addressed limitations of standard  
 508 autoregressive models, such as unidirectional dependency, discretization, and exposure bias. Our  
 509 method robustly models structures over multiple granularities and in turn enables strong zero-shot  
 510 generalization. This capability includes coarse-prompted conditional generation using points (e.g.,  
 511 16 points) as structural layout and finer-grained controls such as atomic-coordinate-based motif  
 512 scaffolding. For unconditional backbone generation, PAR exhibits powerful distributional fidelity  
 513 and generation quality. The analysis of scale-level attention map provides additional insights into  
 514 how the multi-scale formulation operates.

515 We hope that PAR unlocks the potential of autoregressive modeling for protein design. Some promising  
 516 open directions include: (1) *Conformational dynamics modeling*. PAR can, in principle, perform  
 517 zero-shot modeling of conformational distributions: we downsample a structure and upsample it with  
 518 PAR to mimic local molecular dynamics. We leave this exciting application for future research. (2)  
 519 *All-atom modeling*. This work focuses on backbone C $\alpha$  atoms to prioritize autoregressive design, but  
 520 it's natural to extend to full-atom representations (Qu et al., 2024). The multi-scale framework offers  
 521 an advantage for flexible zero-shot prompt-based all-atom designs.

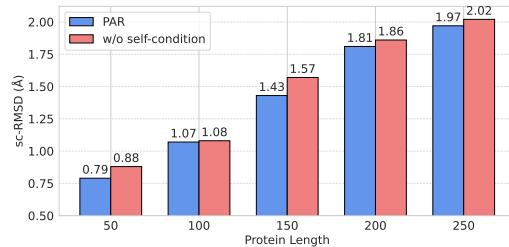
## 523 ETHICS STATEMENT

524 Protein design holds significant potential in drug development, vaccine and antibody discovery,  
 525 industrial biotechnology, and sustainable chemistry. Generative models provide new opportunities to  
 526 accelerate discovery and deepen our understanding of protein structures, which may bring positive  
 527 impact to medicine, materials science, and manufacturing.

528 However, we also acknowledge the potential risks of generative models. To mitigate such risks,  
 529 this study is conducted solely on publicly available datasets and strictly adheres to relevant ethical  
 530 guidelines. We advocate for the responsible research and application of protein generative models to  
 531 ensure that their development truly benefits society.

## 533 REPRODUCIBILITY STATEMENT

534 We ensure that the training data, training and inference procedures, and result evaluations are all  
 535 reproducible. The appendix provides all necessary details and offers a comprehensive explanation of  
 536 each component of this work. The datasets used are publicly available, and the model implementation  
 537 is based on the open-source Proteina (Geffner et al., 2025) codebase. The code and models used  
 538 for evaluation are also publicly accessible and cited in the appendix. Furthermore, we describe the  
 539 training and inference hyperparameters in detail in the appendix, thereby ensuring that the entire  
 experimental process is fully reproducible.



500 **Figure 7: Ablation with self-conditioning.** Self-  
 501 conditioning consistently improves backbone gen-  
 502 eration performance of PAR across varying protein  
 503 lengths, showing that both methods are compati-  
 504 ble.

## 540 REFERENCES

541  
 542 Josh Achiam, Steven Adler, Sandhini Agarwal, Lama Ahmad, Ilge Akkaya, Florencia Leoni Aleman,  
 543 Diogo Almeida, Janko Altenschmidt, Sam Altman, Shyamal Anadkat, et al. Gpt-4 technical report.  
 544 *arXiv preprint arXiv:2303.08774*, 2023.

545 Michael S Albergo, Nicholas M Boffi, and Eric Vanden-Eijnden. Stochastic interpolants: A unifying  
 546 framework for flows and diffusions. *arXiv preprint arXiv:2303.08797*, 2023.

547  
 548 Kushal Arora, Layla El Asri, Hareesh Bahuleyan, and Jackie Chi Kit Cheung. Why exposure bias  
 549 matters: An imitation learning perspective of error accumulation in language generation. *arXiv  
 550 preprint arXiv:2204.01171*, 2022.

551 Samy Bengio, Oriol Vinyals, Navdeep Jaitly, and Noam Shazeer. Scheduled sampling for sequence  
 552 prediction with recurrent neural networks. *Advances in neural information processing systems*, 28,  
 553 2015.

554 Avishek Joey Bose, Tara Akhoud-Sadegh, Guillaume Huguet, Kilian Fatras, Jarrid Rector-Brooks,  
 555 Cheng-Hao Liu, Andrei Cristian Nica, Maksym Korablyov, Michael Bronstein, and Alexander Tong.  
 556 Se (3)-stochastic flow matching for protein backbone generation. *arXiv preprint arXiv:2310.02391*,  
 557 2023.

558 Tom Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared D Kaplan, Prafulla Dhariwal,  
 559 Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, et al. Language models are  
 560 few-shot learners. *Advances in neural information processing systems*, 33:1877–1901, 2020.

561 Andrew Campbell, Jason Yim, Regina Barzilay, Tom Rainforth, and Tommi Jaakkola. Generative  
 562 flows on discrete state-spaces: Enabling multimodal flows with applications to protein co-design.  
 563 *arXiv preprint arXiv:2402.04997*, 2024.

564 Shoufa Chen, Chongjian Ge, Shilong Zhang, Peize Sun, and Ping Luo. Pixelflow: Pixel-space  
 565 generative models with flow. *arXiv preprint arXiv:2504.07963*, 2025.

566 Ting Chen, Ruixiang Zhang, and Geoffrey Hinton. Analog bits: Generating discrete data using  
 567 diffusion models with self-conditioning. *arXiv preprint arXiv:2208.04202*, 2022.

568 Alexander E Chu, Jinho Kim, Lucy Cheng, Gina El Nesr, Minkai Xu, Richard W Shuai, and Po-Ssu  
 569 Huang. An all-atom protein generative model. *Proceedings of the National Academy of Sciences*,  
 570 121(27):e2311500121, 2024.

571 Justas Dauparas, Ivan Anishchenko, Nathaniel Bennett, Hua Bai, Robert J Ragotte, Lukas F Milles,  
 572 Basile IM Wicky, Alexis Courbet, Rob J de Haas, Neville Bethel, et al. Robust deep learning-based  
 573 protein sequence design using proteinmpnn. *Science*, 378(6615):49–56, 2022.

574 Patrick Esser, Robin Rombach, and Bjorn Ommer. Taming transformers for high-resolution image  
 575 synthesis. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*,  
 576 pp. 12873–12883, 2021a.

577 Patrick Esser, Robin Rombach, and Bjorn Ommer. Taming transformers for high-resolution image  
 578 synthesis. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*,  
 579 pp. 12873–12883, 2021b.

580 Benoit Gaujac, Jérémie Donà, Liviu Copoiu, Timothy Atkinson, Thomas Pierrot, and Thomas D  
 581 Barrett. Learning the language of protein structure. *arXiv preprint arXiv:2405.15840*, 2024.

582 Tomas Geffner, Kieran Didi, Zuobai Zhang, Danny Reidenbach, Zhonglin Cao, Jason Yim, Mario  
 583 Geiger, Christian Dallago, Emine Kucukbenli, Arash Vahdat, et al. Proteina: Scaling flow-based  
 584 protein structure generative models. *arXiv preprint arXiv:2503.00710*, 2025.

585 Thomas Hayes, Roshan Rao, Halil Akin, Nicholas J Sofroniew, Deniz Oktay, Zeming Lin, Robert  
 586 Verkuil, Vincent Q Tran, Jonathan Deaton, Marius Wigert, et al. Simulating 500 million years of  
 587 evolution with a language model. *Science*, 387(6736):850–858, 2025.

594 Tianxing He, Jingzhao Zhang, Zhiming Zhou, and James Glass. Exposure bias versus self-  
 595 recovery: Are distortions really incremental for autoregressive text generation? *arXiv preprint*  
 596 *arXiv:1905.10617*, 2019.

597 Martin Heusel, Hubert Ramsauer, Thomas Unterthiner, Bernhard Nessler, and Sepp Hochreiter. Gans  
 598 trained by a two time-scale update rule converge to a local nash equilibrium. *Advances in neural*  
 599 *information processing systems*, 30, 2017.

600 Jonathan Ho, Ajay Jain, and Pieter Abbeel. Denoising diffusion probabilistic models. *Advances in*  
 601 *neural information processing systems*, 33:6840–6851, 2020.

602 Cheng-Yen Hsieh, Xinyou Wang, Daiheng Zhang, Dongyu Xue, Fei Ye, Shujian Huang, Zaixiang  
 603 Zheng, and Quanquan Gu. Elucidating the design space of multimodal protein language models.  
 604 *arXiv preprint arXiv:2504.11454*, 2025.

605 Chin-Wei Huang, Milad Aghajohari, Joey Bose, Prakash Panangaden, and Aaron C Courville.  
 606 Riemannian diffusion models. *Advances in Neural Information Processing Systems*, 35:2750–2761,  
 607 2022.

608 Po-Ssu Huang, Scott E Boyken, and David Baker. The coming of age of de novo protein design.  
 609 *Nature*, 537(7620):320–327, 2016.

610 John B Ingraham, Max Baranov, Zak Costello, Karl W Barber, Wujie Wang, Ahmed Ismail, Vincent  
 611 Frappier, Dana M Lord, Christopher Ng-Thow-Hing, Erik R Van Vlack, et al. Illuminating protein  
 612 space with a programmable generative model. *Nature*, 623(7989):1070–1078, 2023.

613 John Jumper, Richard Evans, Alexander Pritzel, Tim Green, Michael Figurnov, Olaf Ronneberger,  
 614 Kathryn Tunyasuvunakool, Russ Bates, Augustin Žídek, Anna Potapenko, et al. Highly accurate  
 615 protein structure prediction with alphafold. *nature*, 596(7873):583–589, 2021.

616 Jared Kaplan, Sam McCandlish, Tom Henighan, Tom B Brown, Benjamin Chess, Rewon Child, Scott  
 617 Gray, Alec Radford, Jeffrey Wu, and Dario Amodei. Scaling laws for neural language models.  
 618 *arXiv preprint arXiv:2001.08361*, 2020.

619 Brian Kuhlman and Philip Bradley. Advances in protein structure prediction and design. *Nature*  
 620 *reviews molecular cell biology*, 20(11):681–697, 2019.

621 Patrick Kunzmann and Kay Hamacher. Biotite: a unifying open source computational biology  
 622 framework in python. *BMC bioinformatics*, 19(1):346, 2018.

623 Gilles Labesse, N Colloc'h, Joël Pothier, and J-P Mornon. P-sea: a new efficient assignment of  
 624 secondary structure from  $\text{c}\alpha$  trace of proteins. *Bioinformatics*, 13(3):291–295, 1997.

625 Han Li, Xinyu Peng, Yaoming Wang, Zelin Peng, Xin Chen, Rongxiang Weng, Jingang Wang,  
 626 Xunliang Cai, Wenrui Dai, and Hongkai Xiong. Onecat: Decoder-only auto-regressive model for  
 627 unified understanding and generation. *arXiv preprint arXiv:2509.03498*, 2025.

628 Tianhong Li and Kaiming He. Back to basics: Let denoising generative models denoise. *arXiv*  
 629 *preprint arXiv:2511.13720*, 2025.

630 Tianhong Li, Yonglong Tian, He Li, Mingyang Deng, and Kaiming He. Autoregressive image  
 631 generation without vector quantization. *Advances in Neural Information Processing Systems*, 37:  
 632 56424–56445, 2024.

633 Yeqing Lin and Mohammed AlQuraishi. Generating novel, designable, and diverse protein structures  
 634 by equivariantly diffusing oriented residue clouds. *arXiv preprint arXiv:2301.12485*, 2023.

635 Yeqing Lin, Minji Lee, Zhao Zhang, and Mohammed AlQuraishi. Out of many, one: Designing  
 636 and scaffolding proteins at the scale of the structural universe with genie 2. *arXiv preprint*  
 637 *arXiv:2405.15489*, 2024.

638 Zeming Lin, Halil Akin, Roshan Rao, Brian Hie, Zhongkai Zhu, Wenting Lu, Nikita Smetanin,  
 639 Robert Verkuil, Ori Kabeli, Yaniv Shmueli, et al. Evolutionary-scale prediction of atomic-level  
 640 protein structure with a language model. *Science*, 379(6637):1123–1130, 2023.

648 Yaron Lipman, Ricky TQ Chen, Heli Ben-Hamu, Maximilian Nickel, and Matt Le. Flow matching  
 649 for generative modeling. *arXiv preprint arXiv:2210.02747*, 2022.  
 650

651 Nanye Ma, Mark Goldstein, Michael S Albergo, Nicholas M Boffi, Eric Vanden-Eijnden, and  
 652 Saining Xie. Sit: Exploring flow and diffusion-based generative models with scalable interpolant  
 653 transformers. In *European Conference on Computer Vision*, pp. 23–40. Springer, 2024.

654 William Peebles and Saining Xie. Scalable diffusion models with transformers. In *Proceedings of*  
 655 *the IEEE/CVF international conference on computer vision*, pp. 4195–4205, 2023.  
 656

657 Wei Qu, Jiawei Guan, Rui Ma, Ke Zhai, Weikun Wu, and Haobo Wang. P (all-atom) is unlocking  
 658 new path for protein design. *bioRxiv*, pp. 2024–08, 2024.  
 659

660 Sucheng Ren, Qihang Yu, Ju He, Xiaohui Shen, Alan Yuille, and Liang-Chieh Chen. Beyond next-  
 661 token: Next-x prediction for autoregressive visual generation. *arXiv preprint arXiv:2502.20388*,  
 662 2025.

663 Tim Salimans, Ian Goodfellow, Wojciech Zaremba, Vicki Cheung, Alec Radford, and Xi Chen.  
 664 Improved techniques for training gans. *Advances in neural information processing systems*, 29,  
 665 2016.  
 666

667 Keyu Tian, Yi Jiang, Zehuan Yuan, Bingyue Peng, and Liwei Wang. Visual autoregressive modeling:  
 668 Scalable image generation via next-scale prediction. *Advances in neural information processing*  
 669 *systems*, 37:84839–84865, 2024.

670 Hugo Touvron, Thibaut Lavril, Gautier Izacard, Xavier Martinet, Marie-Anne Lachaux, Timothée  
 671 Lacroix, Baptiste Rozière, Naman Goyal, Eric Hambro, Faisal Azhar, et al. Llama: Open and  
 672 efficient foundation language models. *arXiv preprint arXiv:2302.13971*, 2023.  
 673

674 Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz  
 675 Kaiser, and Illia Polosukhin. Attention is all you need. *Advances in neural information processing*  
 676 *systems*, 30, 2017.  
 677

678 Xinyou Wang, Zaixiang Zheng, Fei Ye, Dongyu Xue, Shujian Huang, and Quanquan Gu. Dplm-2: A  
 679 multimodal diffusion protein language model. *arXiv preprint arXiv:2410.13782*, 2024.

680 Yinhuai Wang, Jiwen Yu, and Jian Zhang. Zero-shot image restoration using denoising diffusion  
 681 null-space model. *arXiv preprint arXiv:2212.00490*, 2022.  
 682

683 Joseph L Watson, David Juergens, Nathaniel R Bennett, Brian L Trippe, Jason Yim, Helen E Eisenach,  
 684 Woody Ahern, Andrew J Borst, Robert J Ragotte, Lukas F Milles, et al. De novo design of protein  
 685 structure and function with rfdiffusion. *Nature*, 620(7976):1089–1100, 2023.  
 686

687 Ronald J Williams and David Zipser. A learning algorithm for continually running fully recurrent  
 688 neural networks. *Neural computation*, 1(2):270–280, 1989.

689 Jason Yim, Andrew Campbell, Andrew YK Foong, Michael Gastegger, José Jiménez-Luna, Sarah  
 690 Lewis, Victor Garcia Satorras, Bastiaan S Veeling, Regina Barzilay, Tommi Jaakkola, et al. Fast  
 691 protein backbone generation with se (3) flow matching. *arXiv preprint arXiv:2310.05297*, 2023a.  
 692

693 Jason Yim, Brian L Trippe, Valentin De Bortoli, Emile Mathieu, Arnaud Doucet, Regina Barzilay,  
 694 and Tommi Jaakkola. Se (3) diffusion model with application to protein backbone generation.  
 695 *arXiv preprint arXiv:2302.02277*, 2023b.  
 696

697 Jason Yim, Brian L Trippe, Valentin De Bortoli, Emile Mathieu, Arnaud Doucet, Regina Barzilay,  
 698 and Tommi Jaakkola. Se (3) diffusion model with application to protein backbone generation. In  
 699 *International Conference on Machine Learning*, pp. 40001–40039. PMLR, 2023c.

700 Boyang Zheng, Nanye Ma, Shengbang Tong, and Saining Xie. Diffusion transformers with represen-  
 701 tation autoencoders. *arXiv preprint arXiv:2510.11690*, 2025.

## 702 A IMPLEMENTATION AND EVALUATION DETAILS

704 We follow the implementation of Proteina (Geffner et al., 2025) for training PAR, using the same  
 705 architecture and hyperparameter setup. Training is conducted on 8 H100 GPUs, with a batch size of  
 706 15 per GPU, for a total of 200k steps. We train the flow-based backbone decoder with 60 M, 200 M,  
 707 and 400 M parameters, using the same non-equivariant transformer architecture as Proteina. For the  
 708 autoregressive module, we adopt Proteina’s smallest configuration (60 M parameters), as we find that  
 709 a small AR module is enough to yield competitive generation quality, discussed in §4.3. For a fair  
 710 comparison, we trained Proteina from scratch under the same setting and achieved similar or even  
 711 better performance than results reported in the original paper. For other baselines, we directly obtain  
 712 the results from Geffner et al. (2025). Model and training configurations can be found in Tab. 6. Note  
 713 that we remove pair representations and triangle update for memory and training efficiency, and the  
 714 additional trainable parameters come from the 60M autoregressive transformer encoder.

715 Table 6: Hyperparameters for PAR models.

716 PAR Architecture	717 $T_\theta$		718 $v_\theta$	
	719 60M	720 60M	721 200M	722 400M
723 initialization	random	random	random	random
sequence repr dim	512	512	768	1024
sequence cond dim	128	128	512	512
$t$ sinusoidal enc dim	196	196	256	256
interpolated position enc dim	196	196	128	128
# attention heads	12	12	12	16
# transformer layers	12	12	15	18
# trainable parameters	60M	60M	200M	400M

### 724 A.1 IMPLEMENTATION DETAILS

726 In §3.2 we briefly introduce two novel techniques for our autoregressive modeling: scale embedding  
 727 and interpolated position embedding.

728 **Scale Embedding.** Since we use a shared decoder to train across all scales, we introduce a scale  
 729 embedding to distinguish data distributions at different scales. Each scale is assigned a unique scale  
 730 id, which is incorporated into the model to help disambiguate the varying statistical characteristics  
 731 associated with different scales.

732 **Interpolated Position Embedding.** Interpolated position embedding is a natural extension to the  
 733 standard position embedding for sequence representation. In the raw structure, each residue is  
 734 associated with a 3D coordinate and a position ID ranging from 1 to  $L$ , where  $L$  is the protein  
 735 length. Our downsampled structure and interpolated position embeddings are derived from the  
 736 raw structure and position IDs via interpolation, following the sequential order of residues. Each  
 737 interpolated residue is computed by interpolating the coordinates of neighboring real residues, while  
 738 each interpolated position ID is obtained by interpolating over the corresponding relative positions.  
 739 This approach has the advantage that, across inputs of different lengths (i.e., different scales), the  
 740 interpolated positions still reflect the relative location of each interpolated residue within the original  
 741 structure, providing a coarse-grained view of the real protein.

### 742 A.2 EVALUATION METRICS

743 We evaluate the model from multiple perspectives, including quality and diversity, following evalua-  
 744 tion protocols established in prior literature by Yim et al. (2023c); Bose et al. (2023). Specifically, we  
 745 sample 100 structures for each of the five sequence lengths: 50, 100, 150, 200, and 250, resulting in a  
 746 total of 500 structures for evaluation.

747 **Designability.** Following the procedure from Yim et al. (2023c), we generate 8 candidate sequences  
 748 for each structure using ProteinMPNN Dauparas et al. (2022) with a temperature of 0.1. Each  
 749 sequence is folded into a predicted structure using ESMFold Lin et al. (2023). We compute the root-  
 750 mean-square deviation (RMSD) between each predicted structure and the original generated structure,  
 751 and record the minimum RMSD across the 8 predictions. A structure is considered designable if its  
 752 minimum RMSD is less than 2 Å. We report the proportion of designable structures and the average  
 753 minimum RMSD across all samples.

754 **Diversity.** Following Bose et al. (2023), we compute the average pairwise TM-score among all  
 755 designable structures for each sequence length. The final diversity score is obtained by averaging  
 756 these values across all five lengths.

**Secondary Structure** To analyze secondary structure characteristics, we annotate all designable structures using the P-SEA algorithm Labesse et al. (1997) as implemented in Biotite Kunzmann & Hamacher (2018). For each structure, we compute the proportion of alpha helices and beta sheets, and report the average proportions across all samples.

To better assess the model’s overall structural fidelity at the distributional level, we adopt two metrics introduced in Geffner et al. (2025). We randomly sample 125 structures at each sequence length from 60 to 255 (with a step size of 5), resulting in 5,000 structures in total. Importantly, no designability filtering is applied during this stage; all samples are used for evaluation.

**Fréchet Protein Structure Distance (FPSD).** Analogous to the Fréchet Inception Distance (FID) Heusel et al. (2017), FPSD measures the Wasserstein distance between the distributions of generated and reference structures. Structures are embedded into a feature space defined by a fold class predictor, and the distance is computed based on the resulting Gaussian approximations.

**Protein Fold Score (fS).** Inspired by the Inception Score (IS) Salimans et al. (2016), the fS metric encourages both diversity and sample-level quality. High-quality generations lead to confident fold class predictions, while diversity is captured by the entropy across the predicted fold distribution.

### A.3 UNCONDITIONAL BACKBONE GENERATION

We train 200M and 400M models for Proteina and PAR for 200k steps, using Adam optimizer with learning rate 1e-4, no warmup applied. For evaluation, we sample from Proteina and PAR with the same techniques below. We follow the optimal configuration and sample 400 steps for Proteina. For PAR, we find 1k steps show better results.

**Self conditioning.** Self-conditioning has been widely employed in protein design. During sampling, the model’s own previous predictions

$$\hat{\mathbf{x}}(\mathbf{x}_t) = \mathbf{x}_t + (1-t)\mathbf{v}_t^\theta(\mathbf{x}_t) \quad (7)$$

are fed back as conditions to guide subsequent generation. During training, the model is conditioned on its own predictions with a probability of 50%. Sampling can be performed either with or without self-conditioning.

**Low temperature sampling.** In Eqn. 6, the parameter  $\gamma$  is injected to control the scale of noise. When  $\gamma = 1$ , this SDE yields the same marginals as the ODE defined by flow model. In practice, it is common to use a lower  $\gamma < 1$  which empirically improves designability at the cost of diversity. In this paper, we use  $\gamma = 0.45$  for Proteina and PAR.

**Category of unconditional backbone generation baselines.** We categorize each baseline based on their modeling types and frameworks in the table below.

Table 7: Category of unconditional backbone generation baselines.

Method	Type	Framework
FrameDiff	Frame	Diffusion
RFDiffusion	Frame	Diffusion
ESM3	Token	PLM
Genie2	Ca	Diffusion
Proteina	Ca	FM
PAR	Ca	PAR

## B DATASETS

The training data is derived from the curated AFDB representative dataset (denoted as  $D_{FS}$ , containing 0.6M structures), as processed by Proteina. This dataset ensures both high quality (pLDDT > 80) and structural diversity, with sequence lengths ranging from 32 to 256 residues. We follow Geffner et al. (2025) and split it by 98:19:1 for training, validation and testing.

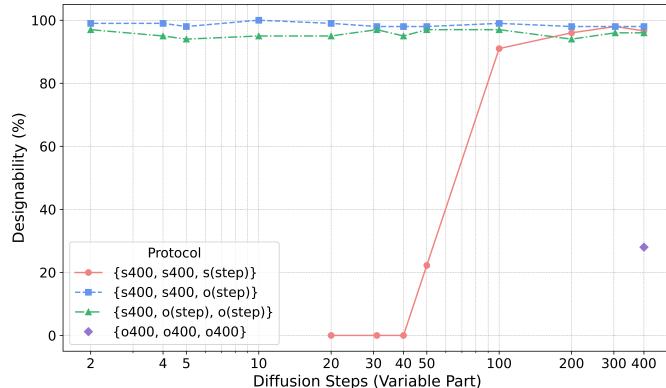
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C MORE EMPIRICAL ANALYSIS

Figure 8: **Designability analysis of multi-scale SDE/ODE sampling methods.** Naively reducing the SDE sampling steps substantially degrades the designability (red). Using ODE alone exhibits limited designability (purple). Orchestrating SDE and ODE sampling enables reduced sampling steps while retaining designability (blue and green).

## C.1 EFFICIENT SAMPLING WITH SDE/ODE ORCHESTRATION

We report the designability over varying sampling steps in Fig. 8. Leveraging SDE sampling at the first scale and ODE for the remaining scales, PAR could effectively reduce diffusion steps without harming designability, highlighting the unique advantage of multi-scale design to orchestrate SDE and ODE sampling at different scales. In addition, aggressively reducing SDE steps or replacing SDE with ODE across all scales yields much worse designability, highlighting the necessity of combining both sampling methods.

## C.2 LONG PROTEIN GENERATION

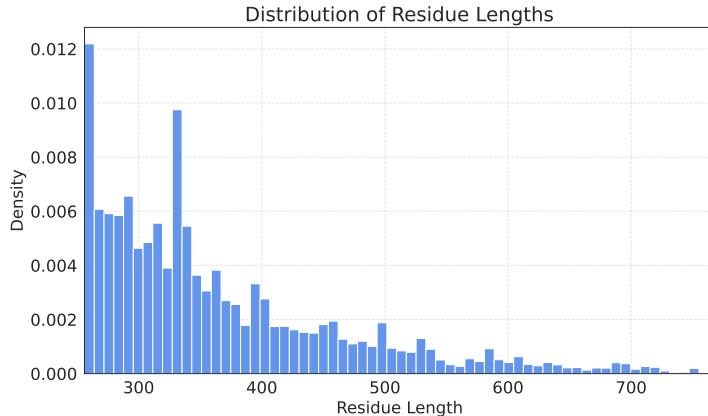


Figure 9: Protein length distribution for long protein finetuning.

**Finetuning on longer protein chains.** We follow Proteina to finetune our models on datasets with longer proteins. Since Proteina has not released its long-protein dataset, we cannot fully reproduce their experiment setups. Instead, we follow the filtering procedure described in their appendix on PDB structures to curate a long-protein dataset. We filter PDB structures to lengths between 256 and 768 residues and keep only designable samples, resulting in 26k high-quality proteins. The length-distribution of this dataset (Fig. 9) exhibits a long-tail shape with peaks around 300-400 residues. We then finetune the 400M PAR and Proteina models in Tab. 1 on this dataset for 10k steps.

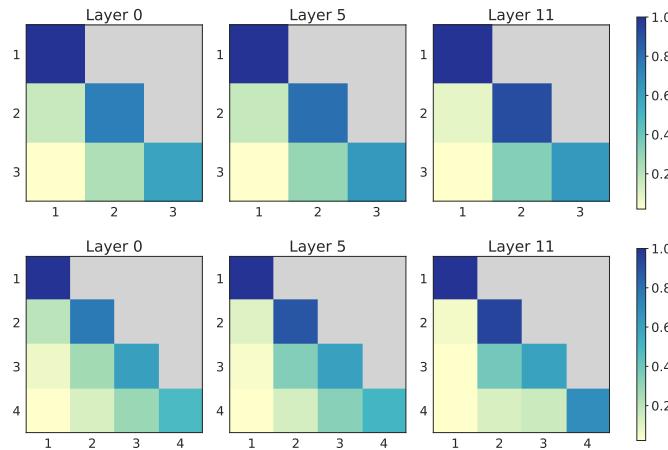
**Long-protein generation.** We generate 100 proteins for each length in  $\{300, 400, 500, 600, 700\}$ . PAR exhibits higher designability at lengths  $\{300, 400\}$ , consistent with the higher density of training samples in this range. At lengths between 500 to 700, both Proteina and PAR show degraded designability, while PAR demonstrating slightly better results. We attribute this to the long-tail nature of the training set, which includes far fewer samples in the length range between 500 and 700. The

864 limited size of the training set (26K) also potentially hinders the model from reaching its full potential.  
 865 We leave scaling up long-protein data as a promising direction for future work.

866 Table 8: **Long protein generation.** scR: sc-RMSD (Å)  $\uparrow$ . DesA: Designability (%)  $\downarrow$ .

	300		400		500		600		700	
	scR	DesA	scR	DesA	scR	DesA	scR	DesA	scR	DesA
Proteina	1.91	85	2.70	61	4.09	49	7.90	21	13.32	4
PAR	<b>1.28</b>	<b>93</b>	<b>1.65</b>	<b>72</b>	<b>3.19</b>	<b>52</b>	<b>6.80</b>	<b>29</b>	<b>11.29</b>	<b>10</b>

## 872 C.3 VISUALIZATION OF ATTENTION SCORES.

873 We provide attention score visualization for shorter proteins in Fig. 10. The pattern generally aligns  
 874 with Fig. 6, where each scale primarily attends to its previous scale.  
 875890 Figure 10: **Visualization of the average attention scores in PAR autoregressive transformer over 3/4**  
 891 **scales. Top Length  $\in (32, 64]$ . Bottom Length  $\in (64, 128]$ .**

## 892 C.4 SCALE-AGNOSTIC INFERENCE

893 In our original setup, we included a learnable scale embedding vector as part of the AR module’s con-  
 894 ditioning. This embedding allows the model to identify the current scale and adjust its behavior (e.g.,  
 895 generating coarse vs. fine structures). However, since the dimensionality of this learnable embedding  
 896 is fixed to the number of scales, the model cannot be applied to a different scale configuration at  
 897 inference.898 To explore flexible scale configurations, we finetune an alternative model that simply discards the  
 899 learnable embedding on the PDB designable subset for 5k steps. This formulation cancels the  
 900 embedding from a fixed number of scales and enables inference across arbitrary scale settings.  
 901 As shown in the Tab. 9, when inferring with five scales using this 3-scale model, FPSD remains  
 902 stable, suggesting that the model still captures the underlying data distribution under altered scale  
 903 configurations. However, the designability substantially drops, indicating that sampling with an  
 904 unseen scale configuration fails to preserve structural detail, ultimately leading to lower-quality  
 905 results.906 Table 9: **Inference with flexible scale configuration.**

	Designability (%) $\uparrow$	sc-RMSD $\downarrow$	FPSD $\downarrow$ vs. PDB	FPSD $\downarrow$ vs. AFDB	fS $\uparrow$ (C/A/T)
PAR (3 scale)	96.6	1.04	160.99	228.44	2.57/7.42/23.61
w/o scale emb	92.8	1.16	175.09	246.34	2.54/7.66/26.68
5 scale inference	72.6	1.74	177.01	246.76	2.56/7.53/26.78

## 913 C.5 ABLATING AR AND DECODER SIZE

914 We introduced an ablation study examining the AR encoder size, and discussed crucial design choices  
 915 for both the AR encoder and flow-based decoder. We summarize key findings below.916 **Per-token vs per-scale decoder.** In our preliminary study, we implemented the model with a 200M-  
 917 parameter AR module and, following MAR (Li et al., 2024), used a 3-layer MLP ( 20M) as the  
 918 diffusion head. However, this setup failed to generate reasonable structures, yielding an average

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Table 10: **Effect of AR module and decoder size.** Both AR and decoder utilize transformer-based architectures.

AR	Decoder	sc-RMSD	Designability (%)
400M	60M	1.26	87.80
60M	400M	1.01	96.00
60M	60M	1.19	92.60

sc-RMSD of 16. This likely occurs because a per-token decoder is not expressive enough to capture the global correlations between atoms that is required to produce a reliable coarse structure at the first scale, which is crucial for the subsequent coarse-to-fine refinement. These observations motivated our shift to a per-scale transformer-based decoder.

**Large vs. small decoder.** As shown in Tab. 10 and our scaling experiments in §4.3, using a large decoder brings effective improvements to generation quality.

**Large AR vs small AR.** With the decoder size fixed, increasing the AR transformer size from 60M to 400M does not offer improvements. We believe this is due to exposure bias: the AR module overfits to ground truth context to stabilize training, resulting in a mismatch with inference, where the model relies on its predictions as context. This issue becomes more severe under several conditions:

- (1) Larger AR models tend to overfit the context more strongly, making exposure bias more severe.
- (2) Limited data increases overfitting risks: our 588K training structures (32–256 residues each) provide far less coverage than datasets like ImageNet (1.28M 256x256 images).
- (3) High precision tasks like protein modeling are sensitive to small errors, making exposure bias more serious than in image generation, where the compressed VAE latents lie in a smoother Gaussian space that is robust to small errors at the cost of some visual details (Zheng et al., 2025; Li & He, 2025).

Our noisy context learning and scheduled sampling mitigate this issue for the 60M PAR, but scaling the AR transformer appears to intensify this issue. Exploring more training data is a potential solution and we leave this for future work.

#### C.6 SEQUENCE-BASED DOWNSAMPLING PRESERVES PAIRWISE SPATIAL RELATIONSHIPS

Size(i)	16	32	64	128
RMSE	0.362	0.275	0.217	0.170
LLDT	1	1	1	1

Table 11: RMSE and LDDT across different downsample sizes.

We discuss whether 1D downsampling properly preserves pairwise spatial relationships. To study this, we attempt to investigate the difference between pairwise distances computed after downsampling the 1D coordinate sequence and those obtained by downsampling the full-resolution 2D distance map. We discuss details below.

**Spatial relationships in downsampled 1D sequence.** We follow the process below to quantify the spatial relationships:

1. Downsample the coordinate sequence from  $\mathbb{R}^{L \times 3}$  to  $\mathbb{R}^{size(i) \times 3}$  for each scale  $i$ .
2. We compute pairwise distance maps using the downsampled sequence, leading to a  $size(i) \times size(i)$  map.

**Spatial relationships in 3D space after downsampling.** We quantify this using the pairwise distance map calculated from the full-resolution structure:

1. Calculate the pairwise distance map of the structure, producing a  $L \times L$  map.
2. We downsample pairwise map this using the `F.interpolate(mode='bicubic')` operation, resulting in a  $size(i) \times size(i)$  map.

**Does sequence-based downsampling preserve spatial relationships?** We select all samples from the testing set, and calculate the RMSE and lddt between the aforementioned two  $size(i) \times size(i)$  pairwise maps for each sample. As expected, rmse slightly increases as  $size(i)$  decreases, reflecting the loss of fine-grained details at coarser scales. However, lddt remains consistently at 1 and the rmse values remain low across all scales. Together, these results indicate that, despite small

972 information loss at the coarse scales, 1D sequence downsampling preserves the essential pairwise  
 973 spatial correlations captured by the downsampled 2D distance map.  
 974

### 975 C.7 FOLDSEEK CLUSTER DIVERSITY 976

977 **Table 12: Foldseek cluster diversity.**

$\gamma$	Designable Clusters
0.35	118
0.40	125
0.45	141
0.50	139
0.60	163
0.70	159
0.80	145

986 We investigated the foldseek cluster diversity of PAR-generated samples. A larger  $\gamma$  increases  
 987 sampling stochasticity and improves the diversity, reaching its peak value at  $\gamma=0.6$ . We generate 500  
 988 structures, with 100 samples for each length in  $\{50, 100, 150, 200, 250\}$ . We use the same foldseek  
 989 command following Geffner et al. (2025) with a tmscore threshold of 0.5. The command is

990 `foldseek easy-cluster <path_samples> <path_tmp>/res <path_tmp>`  
 991 `--alignment-type 1 --cov-mode 0 --min-seq-id 0`  
 992 `--tmscore-threshold 0.5`

### 993 C.8 ZERO-SHOT MOTIF SCAFFOLD BENCHMARK 994

995 **Table 13: Zero-shot motif scaffold benchmark.** PAR\* indicates our zero-shot model, producing 100  
 996 samples, while other baselines *require finetuning*. Baseline results are taken directly from Geffner  
 997 et al. (2025), which reports results using 1000 samples. SR: success rate.

		Unique Solutions (%)				
		PAR*	Proteina	Genie2	RFDiffusion	FrameFlow
1000	1PRW	0	0.3	0.2	0.1	0.3
1001	1BCF	0	0.1	0.1	0.1	0.1
1002	5TPN	0	0.4	0.8	0.5	0.6
1003	5IUS	0	0.1	0.1	0.1	0
1004	3LXT	9.0	0.8	1.4	0.3	0.8
1005	5YUI	0	0.5	0.3	0.1	0.1
1006	1QJG	3.0	0.3	0.5	0.1	1.8
1007	1YCR	4.0	24.9	13.4	0.7	14.9
1008	2KL8	4.0	0.1	0.1	0.1	0.1
1009	7MRX.60	0	0.2	0.5	0.1	0.1
1010	7MRX.85	1.0	3.1	2.3	1.3	2.2
1011	7MRX.128	1.0	5.1	2.7	6.6	3.5
1012	4JHW	0	0	0	0	0
1013	4ZYP	0	1.1	0.3	0.6	0.4
1014	5WN9	0	0.2	0.1	0	0.3
1015	5TRV_short	0	0.1	0.3	0.1	0.1
1016	5TRV_med	0	2.2	2.3	1.0	2.1
1017	5TRV_long	0	17.9	9.7	2.3	7.7
1018	6E6R_short	8.0	5.6	2.6	2.3	2.5
1019	6E6R_med	2.0	41.7	27.2	15.1	9.9
1020	6E6R_long	3.0	71.3	41.5	38.1	11.0
1021	6EXZ_short	2.0	0.3	0.2	0.1	0.3
1022	6EXZ_med	9.0	4.3	5.4	2.5	11.0
1023	6EXZ_long	12.0	29.0	32.6	16.7	40.3
1024	# tasks (SR $\geq 1\%$ )	11	11	11	9	11

1025 We quantify the **zero-shot** motif scaffolding performance of PAR in Tab. 13. For other training-  
 1026 based methods, we directly quote the results reported in Proteina (Geffner et al., 2025). We use  
 1027 PAR to generate 100 backbone structures for each benchmark problem in Watson et al. (2023).  
 1028 Following Proteina’s evaluation protocol, we produce 8 ProteinMPNN sequences with the motif  
 1029 residues fixed, and feed each sequence to ESMFold. Using the predicted structure, we calculate  
 1030 ca-RMSD and MotifRMSD. A design is considered a success if any sequence achieves scRMSD  $\leq$   
 1031 2 $\text{\AA}$ , a motifRMSD  $\leq$  1 $\text{\AA}$ , pLDDT  $\geq$  70, and pAE  $\leq$  5. Note that our method is the only one evaluated  
 1032 in a *zero-shot* setting, whereas all other baselines rely on training or finetuning with additional motif  
 1033 conditioning.

1026 **D OTHER RELATED WORK**  
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1028 **Flow and diffusion-based structure generative models.** Flow-based and diffusion methods have  
1029 been widely applied to protein backbone generation, with examples including RFDiffusion (Watson  
1030 et al., 2023) and Chroma (Ingraham et al., 2023). Subsequently, various protein representations have  
1031 been proposed for protein structure generation. FrameDiff, FoldFlow and FrameFlow (Yim et al.,  
1032 2023b; Bose et al., 2023; Yim et al., 2023a) model protein structures through per-residue rotation  
1033 and translation predictions, employing a frame-based Riemannian manifold representation (Jumper  
1034 et al., 2021; Huang et al., 2022). Building upon FrameFlow, Multiflow (Campbell et al., 2024)  
1035 jointly models sequence and structures. In contrast, Genie and Genie2 (Lin & AlQuraishi, 2023; Lin  
1036 et al., 2024) generate protein backbones by diffusing the  $C\alpha$  coordinates. Pallatom and Protopardelle  
1037 (Qu et al., 2024; Chu et al., 2024) further generate fully atomistic proteins that include side-chains.  
1038 Meanwhile, Proteina (Geffner et al., 2025) leverages a non-equivariant transformer architecture to  
1039 model the  $C\alpha$  backbone coordinates, exhibiting scalability and simplicity. In addition to continuous  
1040 diffusion and flow-matching based approaches, discrete diffusion methods like ESM3 (Hayes et al.,  
1041 2025) and DPLM-2 (Wang et al., 2024) have been trained on structure tokens, which often reduce  
1042 structural fidelity and thus limit structure generation quality (Hsieh et al., 2025).

1043 **E THE USE OF LARGE LANGUAGE MODELS**  
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1045 We employ large language models exclusively for language-editing, which is limited to polishing  
1046 text to improve readability. No language models contributed to the development of research ideas,  
1047 analysis, model, or interpretation of results.

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