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Symbolic Higher-Order Analysis of Multivariate Time Series

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Extended Abstract

We propose a novel approach that allows to identify higher-order relations in multivariate time series based on mapping the original series into a symbolic sequence. Our method works in the case of M time series of binary signals $x_i(t) \in \{0, 1\}, \forall i = 1, ..., M$, with $t \in [0, T]$, where $x_i(t) = 1$ represents the occurrence of an event of type i at time t. In order to convert multivariate time series into a symbolic sequence, we first assign to each time series a different symbol from an alphabet of length M, Fig. 1(a-d). The M symbols are naturally represented as the M nodes of a network, where each k-tuple among the symbols defines a hyperedge of size k. Among all the observed tuples, our goal is to determine which ones are statistically significant—i.e., which represent actual motifs. To do so, we use a modified version of the method proposed by Sinatra et al. in Ref. [1] to compute $p_{\exp}(\alpha)$, the expected probability of observing a given tuple α of length l, given the observed occurrences of tuples of length $l - 1$ and $l - 2$. We then use a Bayesian approach to reject the null hypothesis that the tuples appear with probability $p_{\exp}$ [2, 3, 4]. The algorithm is information-theoretically optimal, achieving the minimum possible runtime and memory usage up to constant factors with respect to the sequence length and the number of distinct tuples. When benchmarked on artificial data, our methods show excellent performances even in presence of extreme noise level and outperforms standard measure of statistical significance, such as the z-score, both in terms of reliability and robustness.

We applied our method to mouse neuropixels spiking data (Allen Brain Institute [5]) to explore higher-order interactions in neuronal activity. In the case of neuropixel datasets containing the time stamps associated with the spikes of several neurons, a 1 in time series i represents the moment when the maximum potential of the spike event from neuron i was recorded, Fig. 1(a-b). In Fig. 1(e), we show a snapshot of the higher-order network reconstructed from brain data recorded from a mouse during a visual stimulus. The nodes represent individual neurons, with different node colors corresponding to different areas of the brain. Significant two-motifs are represented as pairwise edges connecting pairs of neurons, while the 3-hyperedges represent 3-motifs. Thanks to the flexibility of our method, we were able to analyze the data at two distinct spatial scales: one at the level of individual neurons, and the other aggregating neurons within the same functional brain area. While higher-order motifs were observed at both scales, we found that the occurrence of 3-motifs significantly increases when moving from the microscale of single neurons to the macroscale of functional brain areas. This suggests a non-negligible spatial effect that should be carefully considered in future studies.

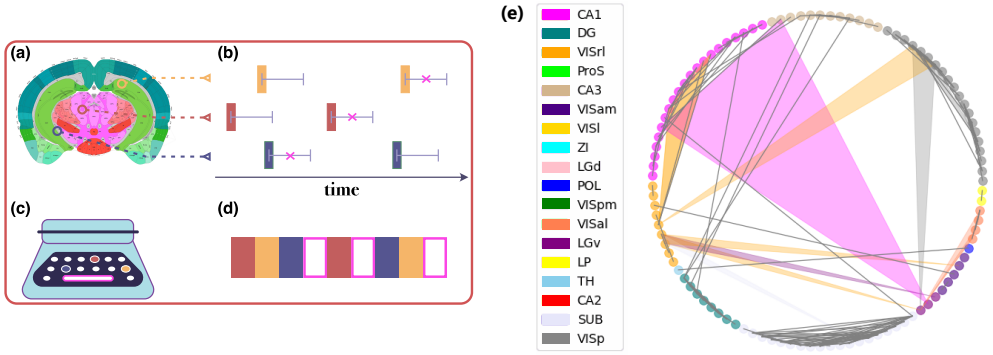

Figure 1: (a) Neural activity is recorded over time at a single-neuron resolution. (b) The temporal series displays spike events from three different neurons, where each neuron is assigned a unique color. This can be visualized as assigning distinct typewriter keys to each neuron, as shown in (c). (d) The original time series is converted into a symbolic sequence: when a spike from neuron i is followed by a spike from neuron j within a defined time interval Δt (indicated by grey lines in (b)), their corresponding symbols are placed adjacently in the symbolic sequence, maintaining the temporal order. If no spike from other neurons occurs within Δt (pink cross in (b)), a space is inserted in the sequence. (e) Higher-order brain network of a mouse showing pairwise and 3-body significant patterns of neural activity in a time window of 35s during the presentation of a visual stimulus. Different colors correspond to different brain areas, e.g., VIS=visual cortex, CA=hippocampus, TH=thalamus.

References

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