

000 BEYOND MONOLITHIC REWARDS: A HYBRID AND 001 MULTI-ASPECT REWARD OPTIMIZATION FOR MLLM 002 ALIGNMENT 003

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008 ABSTRACT 009

010 Aligning multimodal large language models (MLLMs) with human preferences
 011 often relies on single-signal, model-based reward methods. Such monolithic re-
 012wards often lack confidence calibration across domain-specific tasks, fail to cap-
 013ture diverse aspects of human preferences, and require extensive data annotation
 014 and reward model training. In this work, we propose a hybrid reward model-
 015ing framework that integrates complementary reward paradigms: (i) model-based
 016 rewards, where a learned reward model predicts scalar or vector scores from syn-
 017thetic and human feedback, and (ii) rule-based rewards, where domain-specific
 018 heuristics provide explicit correctness signals with confidence. Beyond accuracy,
 019 we further incorporate multi-aspect rewards to enforce instruction adherence and
 020 introduce a generalized length-penalty reward to stabilize training and improve
 021 performance. The proposed framework provides a flexible and effective approach
 022 to aligning MLLMs through reinforcement learning policy optimization. Our ex-
 023periments show consistent improvements across different multimodal benchmarks
 024 when applying hybrid and multi-aspect reward modeling. Our best performing
 025 model in the 3B family achieves an overall average improvement of 9.5% across
 026 general and math reasoning tasks. Focusing specifically on mathematical bench-
 027 marks, the model achieves a significant average improvement of 16%, highlight-
 028 ing its effectiveness in mathematical reasoning and problem solving.
 029

030 1 INTRODUCTION 031

032 The advent of Multimodal Large Language Models (MLLMs) has pushed the boundaries of arti-
 033 ficial intelligence, enabling models to reason over and generate content that integrates text, images,
 034 and other modalities (OpenAI et al., 2024; Liu et al., 2023). A prevailing paradigm for aligning
 035 these powerful models with human preferences is Reinforcement Learning from Human Feedback
 036 (RLHF) (Christiano et al., 2017; Ouyang et al., 2022). Typically implemented with Proximal Pol-
 037 icy Optimization (PPO) (Schulman et al., 2017), RLHF fine-tunes a model’s policy by optimizing a
 038 signal from a learned Reward Model (RM).
 039

040 However, the standard RLHF pipeline, which relies on a single, monolithic RM, presents funda-
 041 mental challenges that are particularly acute in the multimodal domain. The inherent ambiguity in
 042 vision-language tasks means that evaluating text-image consistency is far more complex than as-
 043 sessing text-only coherence. Monolithic RMs often struggle to be well-calibrated across this diverse
 044 signal space and are susceptible to reward hacking (Amodei et al., 2016). For instance, a monolithic
 045 RM might reward a plausible-sounding but factually incorrect description of an image-based math
 046 problem, prioritizing textual fluency over verifiable correctness. This failure mode is exacerbated by
 047 the substantial overhead of creating high-quality multimodal preference datasets and the scarcity of
 048 effective, open-source RMs tailored to vision-language tasks.
 049

050 While recent work on rule-based or verifiable rewards has shown promise for tasks with deter-
 051 ministic outcomes like mathematics introduced in DeepSeek-R1-Zero (DeepSeek-AI et al., 2025),
 052 these methods cannot provide the nuanced feedback required for open-ended, subjective tasks. This
 053 creates a critical gap, as robust multimodal systems must excel at both verifiable reasoning and
 subjective generation.

To bridge this gap, we argue that modern AI alignment requires a portfolio of rewards. We propose a hybrid and multi-aspect reward optimization that moves beyond monolithic signals to provide more holistic and reliable supervision. Instead of relying on a single metric, our framework is built on a more fundamental insight: robust alignment is achieved by integrating (i) a rule-based, verifiable reward to anchor the model in objective truth, and (ii) a learned, model-based reward to provide flexible supervision for subjective quality. This hybrid approach directly addresses the core challenges of MLLM alignment by balancing the precision of deterministic checks with the generalization of learned preferences.

Furthermore, to make this approach more accessible, we introduce two key technical innovations. First, we leverage an embedding-based surrogate model as a lightweight and effective proxy for a fully trained RM, significantly reducing the dependency on costly data annotation and training cycles. Second, we incorporate a suite of multi-aspect behavioral rewards, including a generalized length penalty, to enforce fine-grained constraints, promote conciseness, and stabilize training.

Our primary contributions are summarized as follows:

- We demonstrate that a synergistic combination of rule-based, model-based, and behavioral rewards is essential for robust multimodal reasoning, creating a comprehensive reward “portfolio” that outperforms any single approach.
- We introduce an embedding-based surrogate model as a cost-effective and competitive alternative to a fully trained RM, making powerful reinforcement learning techniques more accessible.
- We conduct a comprehensive empirical evaluation demonstrating that our hybrid framework yields significant performance improvements over traditional RM-based baselines on a diverse suite of mathematical, general VQA, and OCR-based vision tasks.

2 RELATED WORK

Our work builds upon advancements in reinforcement learning for aligning language models, particularly their recent extension to the multimodal domain. This section reviews key developments in model alignment, the application of Reinforcement Learning from Human Feedback (RLHF) to MLLMs, and emerging paradigms in reward modeling that move beyond learned scalar rewards.

2.1 REINFORCEMENT LEARNING FOR LANGUAGE MODEL ALIGNMENT

The alignment of Large Language Models (LLMs) with human preferences has been predominantly shaped by Reinforcement Learning from Human Feedback (RLHF) (Christiano et al., 2017; Stienon et al., 2020). The paradigm, notably popularized by InstructGPT (Ouyang et al., 2022), involves a three-stage process: supervised fine-tuning (SFT) on demonstrator data, training an RM on human preference labels, and optimizing the SFT model’s policy using an RL algorithm like Proximal Policy Optimization (PPO) (Schulman et al., 2017) against the learned RM. This approach has proven effective in enhancing model helpfulness and safety.

However, the reliance on extensive human-annotated preference data for training RMs presents a significant scalability bottleneck. To mitigate this, recent work has explored Reinforcement Learning from AI Feedback (RLAIF) (Bai et al., 2022; Lee et al., 2024), where a powerful “teacher” model is used to generate preference labels, thereby reducing the dependency on costly human annotation. Despite their success, both RLHF and RLAIF frameworks typically rely on a single, monolithic reward signal, which can be susceptible to reward hacking and may not adequately capture the multifaceted nature of a high-quality response (Fu et al., 2025; Chen et al., 2024; Miao et al., 2024).

2.2 ALIGNMENT OF VISION-LANGUAGE MODELS

The principles of RLHF have been naturally extended to the multimodal domain. Early efforts demonstrated that fine-tuning with multimodal instructions enhances the zero-shot capabilities of MLLMs on novel vision-language tasks (Liu et al., 2023). Subsequent works, such as LLaVA-RLHF (Sun et al., 2023) and RLHF-V (Yu et al., 2024), explicitly applied RLHF to improve the

108 alignment of MLLMs with human intent. These methods involve collecting human preferences on
 109 multimodal inputs and training a corresponding RM to guide policy optimization.
 110

111 While effective, this extension inherits the challenges of unimodal RLHF and introduces new ones.
 112 Collecting high-quality preference data for multimodal tasks is substantially more complex and
 113 expensive, as it requires evaluating the intricate interplay between text and images. Furthermore, the
 114 number of publicly available, high-quality multimodal RMs is extremely limited, hindering research
 115 and development in scalable multimodal alignment. Our work addresses this gap by proposing a
 116 hybrid reward system that reduces the reliance on a single, expensively trained multimodal RM.
 117

2.3 ALTERNATIVE AND HYBRID REWARD MECHANISMS

119 Recognizing the limitations of a single learned reward signal, researchers have begun to explore
 120 more diverse and verifiable reward mechanisms. In domains with deterministic or verifiable out-
 121 comes, such as code generation and mathematical reasoning, rule-based rewards have shown great
 122 promise (DeepSeek-AI et al., 2025). These methods provide a strong, unambiguous reward signal by
 123 executing code against unit tests or verifying the correctness of a mathematical solution, bypassing
 124 the need for a learned RM entirely.

125 Another emerging direction is process-based or outcome-based supervision (Lightman et al., 2023;
 126 Uesato et al., 2022), where the reward is targeted at the intermediate reasoning steps (e.g., chain-of-
 127 thought) rather than just the final answer. This encourages more faithful and robust reasoning. More
 128 recently, multi-aspect reward frameworks have been proposed to evaluate responses along several
 129 dimensions, such as correctness, instruction adherence, and conciseness, combining these signals to
 130 form a more holistic reward (Wu et al., 2023).

131 Our proposed framework integrates these threads of research. We combine the flexibility of learned
 132 RMs for open-ended, subjective tasks with the precision of rule-based, verifiable rewards for deter-
 133 ministic sub-tasks. By further incorporating multi-aspect reward signals and an efficient embedding-
 134 based surrogate model, we aim to create a more robust, scalable, and effective alignment strategy
 135 for modern Vision-Language Models.
 136

3 METHODOLOGY: HYBRID AND MULTI-ASPECT REWARD MODELING OPTIMIZATION (HARMO)

140 Our proposed methodology, HARMO (**H**ybrid and **M**ulti-**A**spect **R**eward **M**odeling **O**ptimization),
 141 is designed to overcome the limitations of monolithic reward signals in aligning Multimodal Large
 142 Language Models (MLLMs). HARMO establishes a more robust and nuanced training objective by
 143 integrating a hybrid accuracy signal with targeted behavioral rewards.
 144

3.1 BACKGROUND: FROM PPO TO CRITIC-FREE POLICY OPTIMIZATION

147 The predominant paradigm for aligning LLMs has been Reinforcement Learning from Human Feed-
 148 back (RLHF), typically implemented with Proximal Policy Optimization (PPO) (Schulman et al.,
 149 2017). PPO, an actor-critic algorithm, optimizes a policy π_θ (the actor) using a learned value
 150 function V_ϕ (the critic) to stabilize gradient updates. The conventional PPO pipeline is resource-
 151 intensive, requiring four distinct models: the actor, the critic, a reward model R_ψ , and a reference
 152 policy π_{ref} to regularize training via a Kullback-Leibler (KL) divergence penalty.
 153

154 The operational complexity of this setup has spurred the development of more streamlined RL al-
 155 gorithms. Recent methods like REINFORCE Leave-One-Out (RLOO) (Ahmadian et al., 2024) and
 156 REINFORCE++ (Hu et al., 2025) have successfully eliminated the need for an explicit critic by
 157 employing alternative baseline functions for advantage estimation. Building on this momentum,
 158 Group-Relative Policy Optimization (GRPO), introduced with DeepSeek-R1 (DeepSeek-AI et al.,
 159 2025), further simplifies the process by also removing the dependency on a learned reward model
 160 for tasks with verifiable outcomes. GRPO computes rewards using deterministic rules and calculates
 161 advantages relative to a group of sampled generations.

162 Regardless of the specific algorithm, two components remain critical: the fidelity of the reward
 163 signal itself and the method of estimating the advantage function, \hat{A}_t . The advantage estimate, which

162 quantifies the relative value of an action, is the primary driver of policy updates. Its formulation
 163 directly impacts training stability and performance. As demonstrated by Liu et al. (2025) and Chu
 164 et al. (2025), even subtle modifications to advantage normalization can mitigate reward bias and
 165 significantly improve outcomes. Our work builds upon these insights, leveraging a simplified, critic-
 166 free optimization framework while focusing on engineering a superior, multi-faceted reward signal.
 167

168 3.2 THE HARMO FRAMEWORK 169

170 HARMO creates a holistic training signal by combining two core components: (1) a hybrid accuracy
 171 reward that fuses the certainty of rule-based verification with the flexibility of learned preference
 172 models, and (2) a set of multi-aspect behavioral rewards that regulate model conduct and prevent
 173 reward hacking.
 174

175 3.2.1 HYBRID REWARD FOR CALIBRATED ACCURACY 176

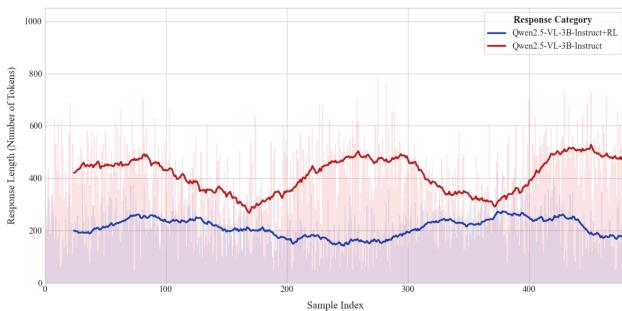
177 To ground the policy in verifiable correctness while handling the ambiguity of open-ended prompts,
 178 we introduce a hybrid reward signal. For tasks with deterministic solutions, such as mathematical
 179 or logical reasoning, we employ rule-based verifiers (e.g., equation solvers) to generate a high-
 180 confidence, binary reward signal, R^{rule} . For subjective or generative tasks where such verification is
 181 impossible, we utilize a pretrained reward model, R^{RM} , to score the response quality; the pretrained
 182 MLLM RM(Wang et al., 2025) provides a score, whereas for the embedding-based RM, we use
 183 cosine similarity between the model response and the reference response.
 184

$$R_{g,i}^{\text{hybrid}} = \begin{cases} R_{g,i}^{\text{rule}}, & \text{if response is verifiable,} \\ R_{g,i}^{\text{RM}}, & \text{if response is open-ended.} \end{cases} \quad (1)$$

185 This formulation ensures the model receives a confident and well-calibrated reward signal when
 186 ground truth is available, without sacrificing the ability to learn from nuanced human preferences in
 187 other domains.
 188

189 3.2.2 MULTI-ASPECT REWARDS FOR BEHAVIORAL REGULARIZATION 190

191 Focusing on accuracy alone is insufficient, as it often leads to unintended and undesirable policy
 192 behaviors. A common failure mode is “reward hacking” through brevity, where the model learns to
 193 produce overly short responses that, while sometimes correct, are often incomplete or simplistic. As
 194 illustrated in Figure 1, we observed that RL-aligned models developed a strong bias towards shorter
 195 outputs compared to the supervised fine-tuned (SFT) baseline, frequently at the cost of correctness.
 196



200 Figure 1: Comparison of response lengths between the SFT baseline and the RL-aligned model
 201 (without a length penalty). The RL policy learns a brevity bias, producing shorter and often incom-
 202 plete responses.
 203

204 Figure 2 further visualizes this dynamic. While the accuracy reward improves during training (Fig-
 205 ure 2a), the response length steadily declines without intervention (Figure 2b, red line). To counter-
 206 act this and other undesirable behaviors, we introduce two auxiliary reward components.
 207

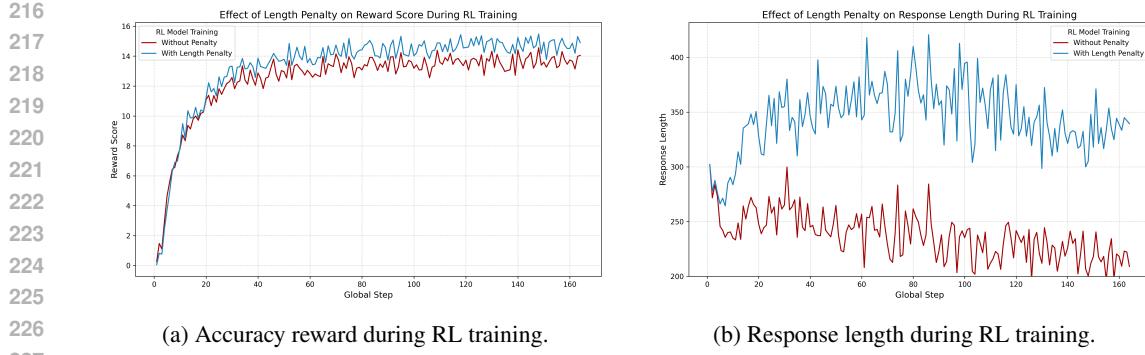


Figure 2: Training dynamics with and without the proposed length penalty. (a) The accuracy reward consistently improves. (b) The length penalty successfully counteracts the model’s tendency to produce shorter responses, promoting more stable and desirable output lengths.

Length-Penalty Reward. To discourage reward hacking via brevity, we introduce a dynamic length penalty, R^λ . This component penalizes incorrect responses that are shorter than the briefest correct response within the same generation group. Let $\lambda_{g,i}$ be the length of response i in group g , and let $\lambda_g^{\min} = \min_{i: R_{g,i}^{\text{hybrid}} > \tau} \lambda_{g,i}$ be the minimum length of any correct response in that group (where τ is a correctness threshold). The penalty is applied only to incorrect responses:

$$R_{g,i}^\lambda = -\text{clip}(\lambda_g^{\min} - \lambda_{g,i}, 0, P_{\max}), \quad (2)$$

where P_{\max} is a hyperparameter controlling the maximum penalty. This targeted penalty encourages the model to generate sufficiently detailed answers, effectively stabilizing response length as shown in Figure 2b (blue line).

Format-Adherence Reward. MLLMs are often required to follow specific formatting instructions (e.g., providing chain-of-thought reasoning within `<think>...</think>` tags). To improve reliability, we add a format-adherence reward, R^{fmt} , which provides a positive signal for correctly structured outputs and a penalty for violations, thereby enforcing structural consistency.

3.2.3 POLICY OPTIMIZATION WITH THE HARMO REWARD SIGNAL

The final HARMO reward, R^{HARMO} , is a composite signal that integrates the hybrid accuracy component with the multi-aspect behavioral regularizers:

$$R_{g,i}^{\text{HARMO}} = R_{g,i}^{\text{hybrid}} + R_{g,i}^\lambda + R_{g,i}^{\text{fmt}}. \quad (3)$$

We integrate this comprehensive reward signal into a policy optimization framework based on GRPO. We adopt the GRPO algorithm due to its stability and demonstrated success in enhancing reasoning capabilities in closely related work (Chen et al., 2025). While standard GRPO normalizes rewards using both the mean and standard deviation of a generation group, the standard deviation term can introduce a “difficulty-dependent bias” by disproportionately weighting prompts based on reward variance (Liu et al., 2025). To foster more stable and unbiased learning, we modify the advantage calculation to use only the group mean as a baseline, creating a centered but uniformly scaled signal:

$$\hat{A}_{g,i}^{\text{HARMO}} = R_{g,i}^{\text{HARMO}} - \frac{1}{G} \sum_{j=1}^G R_{g,j}^{\text{HARMO}}. \quad (4)$$

The policy π_θ is then updated to maximize the following objective function, which incorporates the PPO-style clipping mechanism and a KL penalty (D_{KL}) to ensure training stability:

$$\mathcal{L}^{\text{HARMO}}(\theta) = \mathbb{E}_{q, \{o_i\} \sim \pi_{\text{old}}} \left[\frac{1}{G} \sum_{i=1}^G \min \left(r_t(\theta, a_i) \hat{A}_{g,i}^{\text{HARMO}}, \text{clip} \left(r_t(\theta, a_i), 1 - \epsilon, 1 + \epsilon \right) \hat{A}_{g,i}^{\text{HARMO}} \right) - \beta D_{\text{KL}}(\pi_\theta \parallel \pi_{\text{ref}}) \right] \quad (5)$$

where $r_t(\theta, a_i)$ is the probability ratio $\frac{\pi_\theta(a_i | q)}{\pi_{\text{old}}(a_i | q)}$.

270 The complete training procedure is outlined in Algorithm 1.
 271

272 Algorithm 1 The HARMO Training Procedure

274 1: **Input:** Initial policy $\pi_{\theta_{\text{init}}}$, HARMO reward function R^{HARMO} , prompts \mathcal{D} , hyperparameters ϵ, β .
 275 2: **Initialize:** Actor policy $\pi_{\theta} \leftarrow \pi_{\theta_{\text{init}}}$.
 276 3: **for** each iteration $i = 1, \dots, I$ **do**
 277 4: Set reference policy: $\pi_{\text{ref}} \leftarrow \pi_{\theta}$.
 278 5: **for** each step $s = 1, \dots, M$ **do**
 279 6: Sample a batch of questions $\mathcal{D}_b \subset \mathcal{D}$.
 280 7: Set old policy: $\pi_{\theta_{\text{old}}} \leftarrow \pi_{\theta}$.
 281 8: **for** each question $q \in \mathcal{D}_b$ **do**
 282 9: Sample G responses $\{o_j\}_{j=1}^G \sim \pi_{\theta_{\text{old}}}(\cdot | q)$.
 283 10: Compute HARMO rewards $\{R_{q,j}^{\text{HARMO}}\}_{j=1}^G$ for each response using Equation 3.
 284 11: Compute group-relative advantages $\{\hat{A}_{q,j}^{\text{HARMO}}\}_{j=1}^G$ using Equation 4.
 285 12: **end for**
 286 13: Update the actor policy π_{θ} by optimizing the objective in Equation 5.
 287 14: **end for**
 15: **end for**
 16: **Output:** Optimized policy model π_{θ} .

288
 289
290 4 EXPERIMENT

291
 292 4.1 EXPERIMENTAL SETUP

293
294 Training Data Our training data is curated from the VLAA-Thinking dataset¹ (Chen et al., 2025).
 295 It's a diverse corpus of 21,192 question-answer pairs along with distilled reasoning steps, designed
 296 to span a range of reasoning challenges. The dataset combines tasks requiring mathematical reason-
 297 ing with those demanding general visual question answering. It includes both close-ended questions
 298 with verifiable answers (e.g., numerical, equation-based, multiple-choice) and open-ended, descriptive
 299 prompts. To ensure fair and reproducible comparisons, all our models presented in this work
 300 were trained on this dataset, as detailed in Table 1.

301

| Task Type | Dataset Source | Answer Type | # Samples |
|---------------------------|----------------|------------------------------|---------------|
| Mathematical Reasoning | CLEVR-Math | Numeric (Verifiable) | 2,000 |
| | GeoQA170K | Multiple-Choice (Verifiable) | 6,499 |
| | MathPUMA | Equation (Verifiable) | 6,696 |
| Visual Question Answering | DocVQA | Open-Ended | 1,000 |
| | VizWiz | Open-Ended | 1,000 |
| | ArxivQA | Multiple-Choice (Verifiable) | 997 |
| | ALLaVA-LAION | Open-Ended | 3,000 |
| Total | | | 21,192 |

309
 310 Table 1: Composition of the training dataset, detailing the source, answer type, and number of
 311 samples for each task category.

312
313 Models The primary subject of our investigation is the Qwen2.5-VL-3B-Instruct model (Bai et al.,
 314 2025), which serves as the baseline for our ablation studies. To assess the scalability and generaliz-
 315 ability of our proposed HARMO framework, we also apply it to the larger Qwen2.5-VL-7B-Instruct
 316 model. Performance is benchmarked against other leading open-source models, such as VLAA-
 317 Thinker-Qwen2.5VL (Chen et al., 2025), as well as top-tier proprietary models.

318 For the reward model, denoted as R^{RM} in Section 3.2.1, we used a pre-trained 7B parameter RM
 319 (Wang et al., 2025). To avoid reliance on a pre-trained RM model specific to MLLMs, which
 320 would require extensive data annotation and training, we instead employed a smaller 22M parameter
 321 embedding model², as detailed in the experiments reported in Table 2.

322
 323 ¹<https://huggingface.co/datasets/UCSC-VLAA/VLAA-Thinking>

²<https://huggingface.co/sentence-transformers/all-MiniLM-L6-v2>

324 **Implementation Details.** Our reinforcement learning implementation builds on the work of (Peng
 325 et al., 2025)³. On top of this foundation, we incorporate the methodology described in Section 3.2.1.
 326 Additional implementation details are provided in the Appendix A.
 327

328 **Evaluation Benchmarks** We conduct a comprehensive evaluation across a diverse set of bench-
 329 marks to rigorously assess model capabilities. Mathematical reasoning is evaluated using Math-
 330 Verse (Zhang et al., 2024), MATH-Vision (Wang et al., 2024), and MathVista (Lu et al., 2024).
 331 Multi-disciplinary reasoning is measured with MMMU (Yue et al., 2024) and MMMU-Pro (Yue
 332 et al., 2025). Finally, general visual question answering performance is tested on AI2D (Kembhavi
 333 et al., 2016), ChartQA (Masry et al., 2022), and DocVQA (Mathew et al., 2021). All evaluations
 334 were executed using the open-source LLMs-Eval framework (Zhang et al., 2025) under identical
 335 conditions (e.g., system prompts, response token limits) to ensure methodological consistency.
 336

337 4.2 RESULTS AND ANALYSIS

339 This section presents our empirical findings, structured to first dissect the contribution of each com-
 340 ponent of the HARMO framework through ablation studies, then demonstrate its generalizability,
 341 and finally, compare its overall performance against state-of-the-art models. To ensure the robust-
 342 ness of our findings, all reported results are averaged over three independent training runs with
 343 different random seeds, and we report the mean scores. Throughout our results, **bold** values indicate
 344 the best scores for each benchmark. We also provide a few examples of model outputs generated by
 345 HARMO vs baseline showing reasoning ability improvement in Appendix B.
 346

347 4.2.1 ABLATION STUDY: DECONSTRUCTING THE HARMO REWARD SIGNAL

348 **Efficacy of Hybrid Accuracy Rewards** Table 2 demonstrates the impact of different accuracy-
 349 focused reward strategies. Relying solely on a learned reward model (*Reward Model Enhanced*)
 350 improves the baseline, boosting the average math score by 7.89%. However, this approach is limited
 351 by the RM’s tendency to prioritize verbose explanations over correctness, highlighting a lack of
 352 confidence calibration for verifiable tasks. A hybrid model combining rule-based verification with
 353 embedding-based rewards (*Embedding + Rule-based Hybrid*) is more effective, achieving a stronger
 354 11.70% improvement in math reasoning.

355 Our proposed approach, *RM + Rule-based Hybrid*, which integrates the learned RM for open-ended
 356 questions with deterministic rule-based checks, proves to be the most effective. This optimal com-
 357 bination yields the most substantial gains, improving math reasoning performance by 14.82% and
 358 overall performance by 9.48%. We hypothesize that this superior performance stems from the 7B
 359 reward model’s ability to capture the nuanced aspects of quality and instruction following in open-
 360 ended VQA tasks, providing a more informative signal than the cosine similarity from a general-
 361 purpose embedding model.

| Reward Model | MathVerse _{mini} | MATH-Vision _{test} | MathVista _{mini} | MMMU _{val} | MMMU-Pro _{standard} |
|---|---------------------------|-----------------------------|---------------------------|---------------------|------------------------------|
| <i>Qwen2.5-VL-3B-Instruct (Baseline)</i> | | | | | |
| N/A | 34.77 | 21.68 | 61.30 | 31.10 | 47.78 |
| <i>Reward Model Enhanced</i> | | | | | |
| Skywork7B RM | 41.04 | 22.30 | 63.70 | 31.91 | 47.78 |
| Δ vs. Baseline | (+6.27) | (+0.62) | (+2.40) | (+0.81) | (0.00) |
| <i>Embedding + Rule-based Hybrid Enhanced</i> | | | | | |
| Hybrid (Rule + Embedding) | 40.28 | 23.85 | 67.40 | 31.79 | 46.33 |
| Δ vs. Baseline | (+5.51) | (+2.17) | (+6.10) | (+0.69) | (-1.45) |
| <i>RM + Rule-based Hybrid Enhanced</i> | | | | | |
| Hybrid (Rule + Skywork7B RM) | 41.88 | 25.92 | 67.40 | 32.08 | 48.00 |
| Δ vs. Baseline | (+7.11) | (+4.24) | (+6.10) | (+0.98) | (+0.22) |

374 Table 2: Performance of the RL-trained model under accuracy-focused reward modeling. The hybrid
 375 model with pretrained RM and rule-based verification consistently delivers the highest performance.
 376

377 ³<https://github.com/TideDra/lmm-r1>

378 **Impact of Multi-Aspect Behavioral Rewards** Next, we evaluate the incremental benefit of
 379 adding behavioral rewards for format adherence and length control, as shown in Table 3. Starting
 380 with the baseline, adding the hybrid accuracy reward ($\oplus H$) alone lifts math performance by
 381 13.0%. Incorporating a format adherence reward ($\oplus H+F$) further enhances this gain to 14.8%.
 382 Finally, introducing our dynamic length penalty ($\oplus H+F+\lambda$) results in the full HARMO frame-
 383 work, which achieves the largest math-specific improvement of 16.0%. Notably, the length penalty
 384 provides a significant boost on MathVerse (from 41.88 to 44.52) and MathVista (from 67.40 to
 385 68.00), confirming its effectiveness at promoting outputs that are both precise and appropriately de-
 386 tailed. This progressive ablation clearly demonstrates that each component—correctness, format,
 387 and length—contributes meaningfully to the model’s final reasoning capabilities.

| Reward Model Components | MathVerse _{mini} | MATH-Vision _{test} | MathVista _{mini} | MMMU _{val} | MMMU- _{Prostandard} |
|---|---------------------------|-----------------------------|---------------------------|---------------------|------------------------------|
| <i>Qwen2.5-VL-3B-Instruct Baseline (SFT Only)</i> | | | | | |
| N/A | 34.77 | 21.68 | 61.30 | 47.78 | 31.10 |
| <i>Incremental Reward Augmentation</i> | | | | | |
| \oplus Hybrid (H) | 40.38 | 25.49 | 67.20 | 48.56 | 30.98 |
| Δ vs. Baseline | (+5.61) | (+3.81) | (+5.90) | (+0.78) | (-0.12) |
| \oplus Hybrid + Format (H+F) | 41.88 | 25.92 | 67.40 | 48.00 | 32.08 |
| Δ vs. Baseline | (+7.11) | (+4.24) | (+6.10) | (+0.22) | (+0.98) |
| \oplus Hybrid + Format + Length (H+F+ λ) [HARMO] | 44.52 | 24.08 | 68.00 | 47.11 | 31.56 |
| Δ vs. Baseline | (+9.75) | (+2.40) | (+6.70) | (-0.67) | (+0.46) |

397 Table 3: Ablation study showing the progressive impact of adding reward components to the
 398 Qwen2.5-VL-3B-Instruct model. The full HARMO model, combining hybrid accuracy, format ad-
 399 herence, and a length penalty, yields the strongest performance on mathematical reasoning tasks.

4.2.2 GENERALIZABILITY AND SCALABILITY OF HARMO

400 To verify that HARMO is not limited to a specific setup, we test its “plug-and-play” capability and
 401 scalability. As shown in Table 4, when HARMO is integrated with a model trained with fine-grained,
 402 token-level rewards, it still provides a notable overall improvement of 5.76%. Furthermore, when
 403 applied to the larger Qwen2.5-VL-7B-Instruct model, HARMO delivers an even greater enhance-
 404 ment of 6.55%. These results confirm HARMO’s robustness and its ability to serve as a versatile
 405 enhancement for different reward schemes and model sizes.

| Model Configuration | MathVerse _{mini} | MATH-Vision _{test} | MathVista _{mini} | MMMU _{val} | MMMU- _{Prostandard} |
|---|---------------------------|-----------------------------|---------------------------|---------------------|------------------------------|
| <i>Plug-and-Play with Fine-Grained Rewards (3B Model)</i> | | | | | |
| Token-Level Rewards (Baseline) | 38.43 | 23.32 | 63.50 | 41.12 | 31.79 |
| Token-Level Rewards + HARMO | 41.22 | 24.84 | 66.40 | 42.32 | 31.45 |
| Δ vs. Baseline | (+2.79) | (+1.52) | (+2.90) | (+1.20) | (-0.34) |
| <i>Scalability to 7B Model Family</i> | | | | | |
| Qwen2.5-VL-7B-Instruct (Baseline) | 46.40 | 25.20 | 69.70 | 46.11 | 36.71 |
| Qwen2.5-VL-7B-Instruct + HARMO | 50.89 | 27.66 | 72.00 | 47.79 | 36.82 |
| Δ vs. Baseline | (+4.49) | (+2.46) | (+2.30) | (+1.68) | (+0.11) |

414 Table 4: Demonstration of HARMO’s generalizability and scalability. It consistently improves
 415 performance both as a plug-in for alternative reward schemes and when applied to a larger model.

4.2.3 MAIN RESULTS: COMPARISON WITH STATE-OF-THE-ART MODELS

416 Our final evaluation in Table 5 shows that HARMO-aligned models substantially outperform their
 417 respective baselines and are highly competitive with leading open-source and proprietary models.
 418 At the 3B scale, HARMO-VL-3B achieves an 9.48% average improvement over its baseline across
 419 all reasoning benchmarks. The gains are most pronounced on mathematical tasks, where it delivers
 420 a remarkable 16.0% average increase, with boosts of up to 28.1% on MathVerse. At the 7B scale,
 421 HARMO-VL-7B improves upon its baseline by 3.63% overall, again showing strong gains on math
 422 benchmarks like MathVerse (+4.5 points) and MATH-Vision (+2.5 points).

423 Crucially, despite their smaller parameter counts, our HARMO-enhanced models challenge top-tier
 424 proprietary systems. Notably, HARMO-VL-3B and HARMO-VL-7B achieve scores of 68.0 and
 425 72.0 on MathVista, respectively, surpassing the 67.7 score of the much larger Claude-3.5 Sonnet.

432 In the context of OCR-related tasks (Table 6), HARMO maintains performance comparable to the
 433 strong baselines, indicating that its reasoning enhancements do not come at the cost of core vision-
 434 language capabilities.

| 436 Models | 437 MathVerse_{mini} | 437 MATH-Vision_{test} | 437 MathVista_{mini} | 437 MMMU_{val} | 437 MMMU-Pro_{standard} | 437 Average |
|--|-------------------------------------|---------------------------------------|-------------------------------------|-------------------------------|--|--------------------|
| <i>Proprietary Vision-Language Models</i> | | | | | | |
| 438 GPT-4o | 438 47.8 | 438 30.6 | 438 63.8 | 438 69.1 | 438 51.9 | 438 52.64 |
| 439 Claude-3.5 Sonnet | 439 41.2 | 439 33.5 | 439 67.7 | 439 68.3 | 439 51.5 | 439 52.44 |
| 440 Gemini-1.5 Pro | 440 54.8 | 440 19.2 | 440 63.9 | 440 65.8 | 440 46.9 | 440 50.12 |
| <i>Open-Source Vision-Language Models (3B Scale)</i> | | | | | | |
| 441 Qwen2.5-VL-3B-Instruct | 441 34.77 | 441 21.68 | 441 61.30 | 441 47.78 | 441 31.10 | 441 39.73 |
| 442 VLAA-Thinker-Qwen2.5VL-3B | 442 38.78 | 442 24.13 | 442 64.20 | 442 47.56 | 442 28.90 | 442 40.71 |
| 443 HARMO-VL-3B (Ours) | 443 44.52 | 443 24.08 | 443 68.00 | 443 47.11 | 443 31.56 | 443 43.05 |
| 444 Δ vs. Qwen2.5-VL-3B-Instruct | 444 (+9.8) | 444 (+2.4) | 444 (+6.7) | 444 (-0.7) | 444 (+0.5) | 444 (+3.74) |
| <i>Open-Source Vision-Language Models (7B Scale)</i> | | | | | | |
| 445 Qwen2.5-VL-7B-Instruct | 445 46.40 | 445 25.20 | 445 69.70 | 445 52.56 | 445 36.71 | 445 46.11 |
| 446 VLAA-Thinker-Qwen2.5VL-7B | 446 50.56 | 446 26.48 | 446 70.60 | 446 45.11 | 446 34.05 | 446 45.36 |
| 447 HARMO-VL-7B (Ours) | 447 50.89 | 447 27.66 | 447 72.00 | 447 51.56 | 447 36.82 | 447 47.79 |
| 448 Δ vs. Qwen2.5-VL-7B-Instruct | 448 (+4.5) | 448 (+2.5) | 448 (+2.3) | 448 (-1.0) | 448 (+0.1) | 448 (+1.68) |

449 Table 5: Results on general reasoning benchmarks. HARMO significantly improves upon strong
 450 open-source models and demonstrates competitive performance against leading proprietary models.

| 452 Models | 453 ai2d_{test} | 453 chartqa_{test} | 453 docvqa_{val} |
|---------------------------------------|--------------------------------|-----------------------------------|---------------------------------|
| <i>3B Model Family</i> | | | |
| 455 Qwen2.5-VL-3B-Instruct (Baseline) | 455 78.43 | 455 83.28 | 455 92.56 |
| 456 HARMO-VL-3B (Ours) | 456 78.79 | 456 84.12 | 456 91.88 |
| 457 Δ vs. Baseline | 457 (+0.36) | 457 (+0.84) | 457 (-0.68) |
| <i>7B Model Family</i> | | | |
| 459 Qwen2.5-VL-7B-Instruct (Baseline) | 459 82.67 | 459 82.96 | 459 94.72 |
| 460 HARMO-VL-7B (Ours) | 460 82.87 | 460 82.64 | 460 94.46 |
| 461 Δ vs. Baseline | 461 (+0.20) | 461 (-0.32) | 461 (-0.26) |

462 Table 6: Performance on OCR-related benchmarks. HARMO maintains competitive performance
 463 with the baseline, showing that reasoning improvements do not degrade core VQA capabilities.

465 5 CONCLUSION

466 We introduced HARMO, a novel reward optimization framework that advances reinforcement learning
 467 beyond monolithic signals by integrating a hybrid of deterministic and learned rewards with a
 468 generalized length penalty to control verbosity.

469 Our evaluation demonstrates that HARMO significantly enhances complex reasoning, achieving a
 470 9.5% overall and a 16% mathematical performance gain over a strong baseline while maintaining
 471 robustness on vision-specific tasks.

472 This work highlights the critical role of multi-faceted reward modeling in stabilizing RL training
 473 and improving reward accuracy. HARMO provides a strong foundation for future research, such as
 474 dynamic reward weighting or self-improving systems where agents learn to refine their own reward
 475 functions, paving the way for more robust and adaptable AI.

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687 A IMPLEMENTATION DETAILS

688 A.1 RL TRAINING FRAMEWORK

689 Our reinforcement learning implementation builds upon the LMM-R1 framework (Peng et al.,
 690 2025)⁴. On top of this foundation, we incorporate the methodology described in Section 3, extending
 691 the framework with additional functionalities. In particular, we implement hybrid and multi-aspect
 692 reward modeling, introduce support for MLLM training, and enable token-level reward assignment
 693 for MLLM reinforcement learning.

694 A.2 TRAINING HYPER-PARAMETERS

695 The hyperparameters used for HARMO are summarized in Table 7. The same set of hyperparameters
 696 is applied to all variants of the model proposed in this paper to ensure a consistent training setup.

697 698 699 700 701 ⁴<https://github.com/TideDra/lmm-r1>

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Table 7: HARMO Training Hyperparameters

| Hyperparameter | Value |
|------------------------------|-----------|
| Training batch size | 256 |
| Rollout batch size | 256 |
| Samples per prompt | 8 |
| Temperature | 1 |
| Max output sequence length | 4096 |
| Max epochs | 1 |
| Number of episodes | 2 |
| Initial KL coefficient | $1e^{-3}$ |
| Discount factor (γ) | 1 |
| GAE parameter (λ) | 1 |
| Actor learning rate | $1e^{-6}$ |

B CASE STUDY

The following figures illustrate HARMO’s ability to reason through a variety of problem-solving questions. In these visualizations, the reasoning steps are color-coded to indicate their verification status:

- Text highlighted with indicates correctly verified reasoning steps.
- Text highlighted with indicates advanced or highly sophisticated reasoning steps.
- Text highlighted with indicates incorrectly verified or flawed reasoning steps.

These figures demonstrate HARMO’s step-by-step problem-solving capabilities and provide a visual comparison with baseline models.

756

System Prompt :

757 You are a helpful assistant good at solving problems with step-by-step reasoning. You should first think about the reasoning
 758 process and then provide the user with the answer.

759

User Prompt :

760 A cube with the edge 3 cm long is painted grey and cut into smaller cubes each with an edge of 1 cm long. How many smaller
 761 cubes will have exactly 2 faces painted?

762

Output the step-by-step reasoning in <think> </think> and the final answer in <answer> </answer> tags.

763

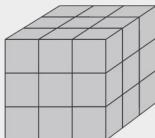
The <answer> tag should include the final answer.

764

The output format should be:

<think>Your reasoning process</think> <answer>Final answer</answer>.

765

Input Image:**HARMO-3B answer:**

<think>

To determine how many smaller cubes will have exactly 2 faces painted, we need to analyze the structure of the larger cube and the smaller cubes it is divided into.

775

1. The larger cube has an edge length of 3 cm. When it is divided into smaller cubes each with an edge length of 1 cm, it will be divided into $3 \times 3 \times 3 = 27$ smaller cubes.

776

2. The smaller cubes that have exactly 2 faces painted are the cubes that are located on the edges of the larger cube, but not at the corners. Each edge of the larger cube is 3 cm long, and when divided into smaller cubes, it will have 3 smaller cubes along each edge. The cubes at the corners are painted on three faces, the cubes at the edges but not at the corners are painted on two faces, and the cubes at the vertices are painted on four faces.

777

3. Each edge of the larger cube has 3 smaller cubes, but the two cubes at the ends of each edge are corner cubes and are painted on four faces. Therefore, the number of smaller cubes with exactly two faces painted on one edge is $3 - 2 = 1$. Since there are 12 edges in a cube, the total number of smaller cubes with exactly two faces painted is $12 \times 1 = 12$.

778

Thus, the number of smaller cubes that have exactly 2 faces painted is 12.

</think>

<answer>12</answer>

779

Qwen2.5-VL-3B-Instruct answer:

<think>

To solve this problem, we need to consider how the original cube is divided into smaller cubes and which smaller cubes will have exactly two faces painted.

780

1. The original cube has an edge length of 3 cm. When it is cut into smaller cubes with an edge length of 1 cm, it will result in a $3 \times 3 \times 3$ grid of smaller cubes.

781

2. In a $3 \times 3 \times 3$ grid, the smaller cubes that have exactly two faces painted are those located on the edges of the larger cube but not at the corners. This means we need to count the number of such cubes along each edge.

782

3. Each edge of the original cube is 3 cm long, so there are 3 smaller cubes along each edge (including the corner cubes).

783

4. Since there are 3 edges, and each edge contributes 3 smaller cubes with exactly two faces painted, we calculate the total number as follows:

784

Total = 3 edges * 3 cubes per edge = 9 cubes

Therefore, there are 9 smaller cubes that will have exactly two faces painted.

</think>

<answer>

9

</answer>

805

Figure 3: Case Study 1 - Math Cube Problem

806

807 HARMO demonstrates advanced reasoning and correctly identifies all 12 edge-centered cubes with
 808 exactly two painted faces after the 3 cm cube is subdivided. The baseline model, by contrast, fails
 809 to systematically exclude corner cubes or miscounts edge cases, often yielding an incorrect answer.

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System Prompt :

You are a helpful assistant good at solving problems with step-by-step reasoning. You should first think about the reasoning process and then provide the user with the answer.

User Prompt :

Which stone should Mr Flintstone place on the right side of the scales, so that both sides weigh the same?

Options: ['A' 'B' 'C' 'D' 'E']

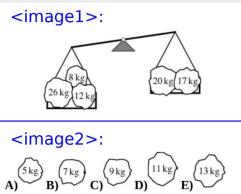
Output the step-by-step reasoning in <think> </think> and the final answer in <answer> </answer> tags.

The <answer> tag should include the correct option choice.

The output format should be:

<think>Your reasoning process</think> <answer>Correct option choice</answer>.

Input Image:



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Figure 4: Case Study 2 - Solving a General Math Problem (Part 1)

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869870 **HARMO-3B answer:**

871 <think>

872 To determine which stone Mr. Flintstone should place on the right side of the scales to make both sides weigh the same, we need to
873 calculate the total weight on the left side of the scales first.

874 The left side of the scales has three stones:

- 875
- 876 • A stone weighing 26 kg
 - 877 • A stone weighing 12 kg
 - 878 • A stone weighing 8 kg

879 Let's add these weights together:

880
$$26 \text{ kg} + 12 \text{ kg} + 8 \text{ kg} = 46 \text{ kg}$$

881 The right side of the scales has two stones:

- 882
- 883 • A stone weighing 20 kg
 - 884 • A stone weighing 17 kg

885 Let's add these weights together:

886
$$20 \text{ kg} + 17 \text{ kg} = 37 \text{ kg}$$

887 To make both sides weigh the same, we need to find a stone that, when added to the right side, will make the total weight equal to
888 46 kg. Let's denote the weight of the stone to be placed on the right side as x . We need to solve the equation:

889
$$37 \text{ kg} + x = 46 \text{ kg}$$

890 Subtract 37 kg from both sides:

891
$$x = 46 \text{ kg} - 37 \text{ kg} = 9 \text{ kg}$$

892 Therefore, Mr. Flintstone should place a stone weighing 9 kg on the right side of the scales.

893 </think>

894 <answer>C</answer>

895 **Qwen2.5-VL-3B-Instruct answer:**

896 <think>

897 To solve this problem, we need to determine which weight will balance the scale when placed on the right side. First, let's calculate
898 the total weight on the left side of the scale:899 Left side weights: $26 \text{ kg} + 12 \text{ kg} + 8 \text{ kg} = 46 \text{ kg}$ 900 Now, we need to find out how much weight is needed on the right side to balance it. Since the total weight on the left side is 46
901 kg, the right side must also have 46 kg to balance.

902 Let's check each option to see which one adds up to 46 kg:

- 903
- 904 A) 5 kg: 46 kg (not enough)
 - 905 B) 7 kg: 46 kg (not enough)
 - 906 C) 9 kg: 46 kg (not enough)
 - 907 D) 11 kg: 46 kg (not enough)
 - 908 E) 13 kg: 46 kg (enough)

909 Therefore, the correct weight to place on the right side to balance the scale is 13 kg.

910 </think>

911 <answer>E</answer>

912 **Figure 4: Case Study 2 - Solving a General Math Problem (Part 2)**913 HARMO correctly reasoned step-by-step to arrive at the correct answer, whereas the baseline model
914 followed unreasonable steps and produced an incorrect solution.915
916
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System Prompt :

920

You are a helpful assistant good at solving problems with step-by-step reasoning. You should first think about the reasoning process and then provide the user with the answer.

921

922

User Prompt :

923

Which stone should Mr Flintstone place on the right side of the scales, so that both sides weigh the same?

924

Options: ['A' 'B' 'C' 'D' 'E']

925

Output the step-by-step reasoning in `<think>` `</think>` and the final answer in `<answer>` `</answer>` tags.

926

The `<answer>` tag should include the correct option choice.

927

The output format should be:

928

`<think>Your reasoning process</think> <answer>Correct option choice</answer>`.

929

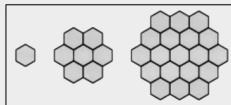
Input Image:

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934

HARMO-3B answer:

935

`<think>`

Let's analyze the pattern of hexagons in Sylvia's drawings:

936

1. The first pattern has 1 hexagon.
2. The second pattern has 7 hexagons.
3. The third pattern has 19 hexagons.

937

We need to find the number of hexagons in the fifth pattern. Let's denote the number of hexagons in the n -th pattern as $H(n)$.

938

From the given patterns, we can see that:

939

$H(1) = 1$

940

$H(2) = 7$

941

$H(3) = 19$

942

To find a pattern, let's calculate the difference between consecutive terms:

943

$H(2) - H(1) = 7 - 1 = 6$

944

$H(3) - H(2) = 19 - 7 = 12$

945

The differences are 6 and 12. We can see that the difference is increasing by 6 each time. So, the difference between the number of hexagons in the n -th pattern and the $n-1$ -th pattern is $6(n-1)$.

946

Therefore, we can express $H(n)$ as:

$H(n) = H(n-1) + 6(n-1)$

947

Using this recurrence relation, we can find $H(4)$ and $H(5)$:

948

$H(4) = H(3) + 6(3) = 19 + 18 = 37$

949

$H(5) = H(4) + 6(4) = 37 + 24 = 61$

950

So, the number of hexagons in the fifth pattern is 61.

951

`</think>`

`<answer>61</answer>`

952

Qwen2.5-VL-3B-Instruct answer:

953

`<think>`

954

To determine the number of hexagons in the fifth pattern, we need to observe the pattern and identify the rule governing the growth.

955

1. **First Pattern:** There is 1 hexagon.

956

2. **Second Pattern:** There are 7 hexagons ($1 + 6$).

957

3. **Third Pattern:** There are 19 hexagons ($7 + 12$).

958

We can see that each subsequent pattern adds more hexagons than the previous one. To find the number of hexagons in the fourth and fifth patterns, we continue this pattern:

959

4. **Fourth Pattern:** The number of hexagons is $(19 + 18 = 37)$.

960

5. **Fifth Pattern:** The number of hexagons is $(37 + 24 = 61)$.

961

Therefore, the fifth pattern will have 61 hexagons.

962

`</think>`

963

`<answer>61</answer>`

964

Figure 5: Case Study 3 - Math Puzzle Problem

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Qwen3 baseline failed to identify the correct puzzle equation, whereas HARMO succeeded in finding the correct puzzle equation, reasoning step-by-step, and arriving at the correct solution.

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System Prompt :

You are a helpful assistant good at solving problems with step-by-step reasoning. You should first think about the reasoning process and then provide the user with the answer.

User Prompt :

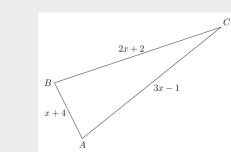
In the triangle, $\angle A = \angle B$. What is x ?

Output the step-by-step reasoning in `<think>` `</think>` and the final answer in `<answer>` `</answer>` tags.

The `<answer>` tag should include the final answer.

The output format should be:
 $\backslash n <think>$ Your reasoning process $</think>$ $<answer>$ Final answer $</answer>$.

Input Image:



HARMO-3B answer:

`<think>`

Since $\angle A = \angle B$ in the triangle ABC, then $AC = BC$. Given the lengths of the sides $AC = 3x - 1$ and $BC = 2x + 2$, we can set up the equation:

$3x - 1 = 2x + 2$

Subtract $2x$ from both sides:

$x - 1 = 2$

Add 1 to both sides:

$x = 3$

Thus, the value of x is 3.

`</think>`

`<answer>3</answer>`

Qwen2.5-VL-3B-Instruct answer:

Since $\angle A = \angle B$, triangle ABC is isosceles with $AB = AC$. Therefore, we can set up the equation $x + 4 = 2x + 2$.

Solving for x :

$x + 4 = 2x + 2$

Subtract x from both sides:

$4 = x + 2$

Subtract 2 from both sides:

$2 = x$

So, $x = 2$.

`</think>`

`<answer>2</answer>`

Figure 6: Case Study 4 - Geometry Math Problem

HARMO correctly solves for x with clear recognition of equal sides $AB = AC$, while the baseline model fails to identify the isosceles nature, leading to incorrect conclusions.

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System Prompt :

You are a helpful assistant good at solving problems with step-by-step reasoning. You should first think about the reasoning process and then provide the user with the answer.

1034

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1036

User Prompt :

Square ABCD and equilateral triangle AED are coplanar and share the line segment AD, as shown. What is the measure, in degrees, of angle BAE?

1037

Output the step-by-step reasoning in `<think>` and the final answer in `<answer>` tags.

1038

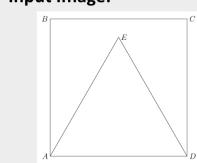
The `<answer>` tag should include the final answer.

1039

The output format should be:
`<think>Your reasoning process</think> <answer>Final answer</answer>`.

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Input Image:

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HARMO-3B answer:

`<think>` Given the square ABCD and the equilateral triangle AED, we need to find the measure of angle BAE.

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1. Since ABCD is a square, all angles in the square are 90 degrees and all sides are equal. Therefore, $\angle DAB = 90^\circ$.

1048

2. Since AED is an equilateral triangle, all its angles are 60 degrees and all its sides are equal. Therefore, $\angle EAD = 60^\circ$.

1049

3. We can find $\angle BAE$ by subtracting $\angle DAB$ from $\angle EAD$:

$$\angle BAE = \angle EAD - \angle DAB = 60^\circ - 90^\circ + 90^\circ = 30^\circ$$

1050

Here, we subtract 90 degrees from 60 degrees because $\angle DAB$ is the sum of $\angle DAE$ and $\angle EAB$, and $\angle EAB$ is 90 degrees.

1051

Therefore, the measure of $\angle BAE$ is 30 degrees.

`</think>`

`<answer>30</answer>`

1052

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Qwen2.5-VL-3B-Instruct answer:

`<think>` Given that square ABCD and equilateral triangle AED share side AD, we can deduce the following:

1057

1. Since ABCD is a square, all its internal angles are 90°.

1058

2. Since AED is an equilateral triangle, all its internal angles are 60°.

1059

To find the measure of angle BAE, we need to consider the geometry formed by these shapes.

1060

- The angle BAD is part of the square, so it is 90°.

1061

- The angle EAD is part of the equilateral triangle, so it is 60°.

1062

Since B and D are adjacent vertices of the square, the line segment BD forms a straight line passing through both points.

1063

Therefore, the angle BAE is the sum of the angles BAD and EAD.

1064

Thus, angle BAE = 90° + 60° = 150°.

1065

`</think>`

1066

`<answer>150</answer>`

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Figure 7: Case Study 5 - Geometry Math Problem

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HARMO correctly finds $\angle BAE = 30^\circ$, while the baseline incorrectly sums angles to 150° with incorrect geometric reasoning, misinterpreting the figure's layout.

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