Data Sampling-driven Adaptive Modification of Bus Routes Under Time-Varying Road Conditions

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Abstract. In urban areas, fluctuating road speeds due to traffic congestion and accidents significantly impact bus operations and stop connectivity. Current approaches cannot maintain public transport (PT) network stability during adaptation to changing road conditions, undermining both operations and passenger experience. This paper proposes a data sampling-based adjustment strategy to adapt the time-varying road conditions. The innovation lies in utilising limited network modifications to enhance the existing static PT network instead of considering reconstruction from scratch or minor adjustments (such as stop-skipping), aiming to minimise both passenger travel time degradation and the operational duration of each transit line. Our proposed multi-objective optimization model leverages historical traffic data samples and integrates route variation quantification with penalty mechanisms to enable real-time adaptive routing decisions. The case studies utilising Mandl's network illustrate that our methodology can propose effective strategies for time-varying roads with any coefficient of variation. Experimental findings with high-variance samples indicate that our methodology decreases passenger travel time in roughly 80% of various scenarios compared to conventional static routes, providing a more efficient solution for public transport systems.

Keywords: Adaptive Network Optimization · Time-Varying Road Conditions · Data Sampling-driven

1 Introduction

Fluctuating road speeds in urban areas, caused by traffic congestion, accidents, and varying demand patterns, significantly impact bus operations within public transport (PT) network performance. These variations can disrupt bus schedules and connectivity between stops, presenting growing challenges as urban populations expand [1].

When road conditions alter, current bus routes—including both alignments and stops—must be modified to sustain service. This issue is termed the bus route adjustment problem, and the modified routes are identified as adaptive bus routes. Prior research typically suggests a singular adaptive pathway for each original route, whether through reconstructing PT networks from scratch or implementing minor adjustments (e.g., stop-skipping), that remains constant throughout the planning horizon [5,17]. However, the dynamic evolution of road conditions complicates practical implementation [30]. These approaches fail to minimize network modifications, thereby compromising operational manageability and potentially causing user disorientation.

In response to these limitations, this paper presents a novelty data sampling-based adjustment strategy designed to reduce extra passenger travel time resulting from fluctuating road conditions. Thus, initially, static bus routes can be dynamically adjusted as required, enhancing adaptability to constantly changing traffic conditions. When designing adaptive bus service networks, the main goal is usually to minimise the overall passenger cost, focusing solely on service-level factors while disregarding bus operating expenses. Nevertheless, designing adaptive networks without considering operational costs can be unfeasible or infrequently implemented [13]. Thus, the issue addressed here aims to minimise both overall passenger expenses and bus operational time. Furthermore, to comprehensively account for passenger expenses, our research considers passenger assignment as an endogenous variable by integrating in-vehicle travel costs, transfer costs, and waiting costs.

To the best of our knowledge, this paper is the first study to incorporate the minimisation of route modifications into the objective of adaptive bus network optimisation, thereby adapting the PT network to time-varying road conditions. Specifically, we propose a multi-objective optimisation model employing sampling methodology to reduce passenger travel time, minimise route alterations, and concurrently decrease the duration of public transport operations.

Based on extensive experiments with high-variance samples (coefficient of variation, $CV=1$), compared to conventional static PT routes, our proposed approach demonstrates passenger travel time reductions in approximately 80% of scenarios for each different test sample.

2 Related Work

2.1 Transit Network Design Problem

The Transit Network Design Problem (TNDP) is a critical strategic decision-making challenge within public transit. Traditional research generally delineates objective functions from two viewpoints: a user-centric perspective aimed at minimising the average travel time for passengers (comprising in-vehicle and transfer durations) and an operator-centric perspective concentrated on costs, typically represented by total route length. This fundamental version of the TNDP is often employed as a benchmark for evaluating various solutions. However, it has been established as NP-hard [19].

As research has advanced, the user dimension has been broadened in many ways. In addition to overall travel duration [16,14], recent studies also examine network coverage [4], route efficiency [22], and demand fulfillment [3]. Meanwhile, the operator dimension persists in prioritising the optimisation of overall route length and operational expenses [2,8]. Nonetheless, these traditional methodologies demonstrate limitations when confronted with dynamic urban settings and ever-evolving commuter requirements [30].

To balance diverse objectives, researchers have introduced multi-objective optimization models. Szeto and Jiang [23] established a bilevel transit network design framework, wherein the upper-level model aims to minimise passenger transfers, while the lower-level model focuses on transit assignment. Tian et al. [25] proposed an innovative trilevel programming model that not only considers congestion in public transit routes but also achieves more comprehensive network design through three levels of optimization: the upper-level minimizes both passenger and operating costs, the middle level formulates passenger routing strategies, and the lower-level describes the equilibrium state under congested public lines. Based on these approaches, Cervantes-Sanmiguel et al. [7] investigated the trade-off between cutting travel time and decreasing fares in further detail. Nonetheless, as noted by [24], existing route planning methodologies exhibit significant deficiencies, especially their incapacity to adjust to real-time variations in traffic patterns.

2.2 Bus Route Adjustment Problem

Various approaches can be employed in the bus route adjustment problem to generate adaptive bus routes, including rerouting, short-turning, skip-stopping, and branching [6,18,29]. Previous research on bus route localised adjustments has often aimed at two main objectives: accommodating demand fluctuations and enhancing integration with subway networks.

Guan et al. [11] focused on customizing bus routes for passengers with multiple trip requests. They proposed a bilevel planning model based on a spatiotemporal state network, aiming to optimize customized bus routes while considering trip characteristics, time windows, capacity constraints, and mixed loads. However, their approach creates entirely new routes for each demand pattern. Zhang et al. [31] investigated long-distance bus routes located near subway stations, seeking to adapt them to urban rail networks. To achieve this, the authors developed a bilevel planning model to optimize both bus route adjustments and vehicle headways and then solved it using a tailored genetic algorithm. While effective for subway integration, this method only addresses localized adjustments near stations rather than system-wide adaptation to varying road conditions. Under circumstances where passenger flow control is implemented in a subway system, Zhou et al. [33] adjusted existing bus routes to evacuate passengers stranded at regulated subway stations. Their study integrated passenger flow control with bus route adjustments in an optimization framework, aiming to relieve passenger congestion and reduce travel times. Wang et al. [28] proposed a two-stage method to mitigate disruptions to bus routes. First, they identified affected routes based on newly developed indicators and generated a set of candidate adaptive routes using skip-stop and detour strategies. Subsequently, an optimization model was formulated to determine the combination of these adaptive routes that maximizes the number of passengers served. While practical for local disruptions, their skip-stop and detour strategies are insufficient for handling system-wide performance degradation under highly variable conditions. Zheng et al. [32] proposed a methodology for generating adaptive bus routes that incorporate diversion, short-run, and cancellation strategies. Nonetheless, their localised adjustments depend on prior awareness of disruptions, rendering them less suitable for fluctuating road conditions where decisions must be made regarding the necessity of systematic network alterations to implement adaptive routes. Additionally, route cancellations can severely impact network connectivity.

Existing methods fall into two extremes: complete network redesign, which is computationally expensive and disrupts stability, or minor local adjustments (e.g., skip-stopping, detouring) with limited adaptability. More critically, current research rarely quantifies the operational costs of route modifications, yet frequent changes increase complexity and confuse passengers. Thus, balancing network stability with adaptation to time-varying conditions remains an unresolved challenge.

2.3 Our Contribution

As previously reviewed, to minimize travel time and the length of public transport's operating time, a large number of studies have delved into the transit network design and bus route adjusting problem by integrating route adjustment and passenger assignment optimization. *However*, to the best of our knowledge, to adapt to time-varying traffic conditions, no prior research has done such an integration while *minimizing route modifications*. Specifically, this study proposes a multi-objective optimization model that samples historical traffic data as inputs and incorporates route variation quantification and penalty mechanisms to make real-time decisions on adaptive routing, generating effective strategies that adapt to road fluctuations.

3 Methodology

In this section, we first present the graph model of the public transport (PT) network and describe our problem statement. We then formulate a multi-objective optimization problem to obtain the optimal PT routes that adapt to travel time variations while maintaining network stability.

3.1 Graph Model of PT Network

We model the original static PT network before travel time variation as graph $\mathcal{G}_{\mathrm{PT}}^{\mathrm{orig}} = (\mathcal{V}_{\mathrm{PT}}^{\mathrm{orig}}, \mathcal{E}_{\mathrm{PT}}^{\mathrm{orig}})$, where $\mathcal{V}_{\mathrm{PT}}^{\mathrm{orig}}$ represents the set of bus stops and $\mathcal{E}_{\mathrm{PT}}^{\mathrm{orig}}$ represents the set of links between consecutive bus stops. To clarify, in this paper, the term public transport exclusively denotes buses. $\mathcal{G}_{\mathrm{PT}}^{\mathrm{orig}}$ composed of multiple

PT lines. Each line ℓ has a headway h^ℓ, defined as the distance between two consecutive vehicles expressed in time [27]. The headway can be calculated using the formula $h^\ell = \frac{t^\ell}{N^\ell}$, where t^ℓ represents the round trip time along the entire line, and N^ℓ denotes the number of vehicles operating on that line (Eq. 4.4 of [27]). To avoid introducing nonlinearity into our model, we treat N^ℓ as a parameter rather than a decision variable in this work. A line consists of a sequence of stops connected by edges. For simplicity, we ignore the dwell time at stops. Assuming random passenger arrivals, the average waiting time for passengers' first boarding on line ℓ is $\frac{h^\ell}{2}$ [21]. We define t_{ij}, the time taken to travel between any two successive stops i and j. For interchange at stop i from line ℓ to line ℓ', we consider an average waiting time $\frac{h^{\ell'}}{2}$ at stop i.

3.2 Time-varying Road Network and Travel Time Sampling

Let $\mathcal{CV}$ be the set of coefficient of variation values. For a substrate network $\mathcal{G}_{\text{sub}} = (\mathcal{V}_{\text{sub}}, \mathcal{E}_{\text{sub}})$, representing the underlying physical road infrastructure upon which transit services operate. For simplicity, we consider $\mathcal{V}_{\text{sub}} = \mathcal{V}_{\text{PT}}^{\text{orig}}$, representing the set of bus stops, and $\mathcal{E}_{\text{sub}} = \mathcal{E}_{\text{PT}}^{\text{orig}}$, representing the road segments connecting these stops. We consider travel times with varying coefficients of variation $cv \in \mathcal{CV}$. For each cv, we generate a set of samples Ω_{cv}, where each sample $\omega \in \Omega_{cv}$ corresponds to a set of travel times $\{t_{i,j}^{cv,\omega} | \forall (i,j) \in \mathcal{E}_{\text{sub}}\}$ across the network. Given the periodic nature of traffic patterns [10], these samples can be viewed as historical observations that represent potential future travel times under the same CV. In this work, we use log-normal distribution to generate these samples, which is described in Section 4.2. For future work, we'll use actual historical data as Ω_{cv}.

3.3 Problem Statement

Our research addresses the challenge of optimizing conventional static PT networks under these varying road conditions. We aim to minimize both increased trip durations and total route lengths while maintaining network stability through limited line modifications. Fig. 1 illustrates this concept with a time-varying PT network comprising 16 bus stops. Based on the coefficient of variation $cv \in \mathcal{CV}$ for each time t, we sample a $\omega \in \Omega_{cv}$ to derive travel times $\{t_{i,j}^{cv,\omega} | \forall (i,j) \in \mathcal{E}_{sub}\}$ that reflect the prevailing road conditions, which informs our adaptation of bus routes.

Our optimization problem makes the following three decisions: (a) How to dynamically modify PT lines' topology to adapt to actual road conditions when they severely deviate from the expected one? (b) How to determine which routes should be made available for users to travel among OD pairs? and (c) How to optimally locate the terminals of each line?

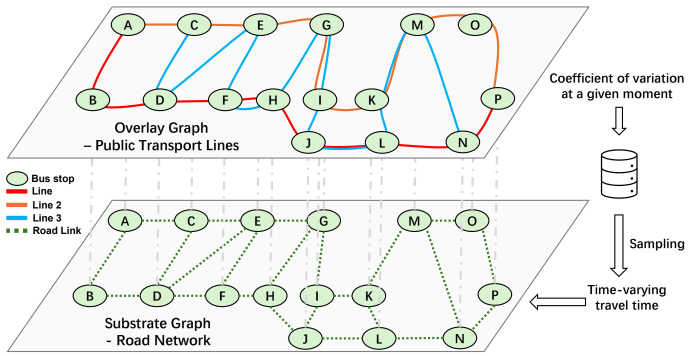


Fig. 1: Illustration of Bus Routes and Corresponding Road Network under Time-Varying Travel Times.

3.4 Multi-objective Optimization Model

For the origin-destination (OD) pair $k \in \mathcal{K}$, following sampling, we acquire a particular realisation of the travel times for each link $t_{i,j}$ within the road network $\mathcal{G}_{\text{sub}}$. Let t_k^o and t_k denote the travel times (comprising transfer, waiting, and in-vehicle time) attained with the original static PT network $\mathcal{G}_{\text{PT}}^{\text{orig}}$ and the adaptive PT network $\mathcal{G}_{\text{PT}}^{\text{adpt}}$, respectively. Our objective is to optimize the adaptive PT network to reduce both passenger travel time degradation and the operational duration of each transit line via constrained network alterations.

Objective Functions We formulate our public transport network adaptation problem as a three-objective optimization. The first objective Z_1 minimizes passenger travel time deterioration, where q_k is the demand for OD pair k, t_k and t_k^o are the travel times under adaptive and original networks respectively. Using $\max(t_k - t_k^o, 0)$ ensures only deteriorations are penalized. The second objective Z_2 minimizes operational costs through total vehicle-hours, where t_{ij} is the link travel time and x_{ij}^ℓ indicates whether link (i, j) is used by line ℓ. The third objective Z_3 minimizes network modifications, calculated as the sum of absolute differences $|x_{ij}^\ell - x_{ij}^{\ell o}|$ between adaptive and original network configurations, where $x_{ij}^{\ell o}$ denotes the original link usage. This preserves operational stability and reduces user disorientation. For a detailed explanation of the notation and variables used, please refer to Table 1.

$$\min_{\mathcal{G}_{\mathrm{PT}}^{\mathrm{adpt}}} \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} = \begin{pmatrix} \underbrace{\sum_{k \in \mathcal{K}} q_k \cdot \max(t_k - t_k^o, 0)}_{\text{passenger travel time deterioration}} \\ \underbrace{\sum_{\ell \in \mathcal{L}} \sum_{i,j \in \mathcal{N}} t_{ij} x_{ij}^{\ell}}_{\text{total operating time}} \\ \underbrace{\sum_{\ell \in \mathcal{L}} \sum_{i,j \in \mathcal{N}} |x_{ij}^{\ell} - x_{ij}^{\ell}{}^{o}|}_{\text{penalty for route modification}} \end{pmatrix} \tag{1}$$

Table 1: Notation for Adaptive PT Network Design Problem

Symbol	Description
Sets:	
$\mathcal{N}$	Set of nodes, indexed by $i, j \in \mathcal{N}$
$\mathcal{L}$	Set of transit lines, indexed by $\ell \in \mathcal{L}$
$\mathcal{K}$	Set of origin-destination pairs, indexed by $k \in \mathcal{K}$
$\mathcal{E}$	Set of feasible edges in the network, indexed by $(i, j) \in \mathcal{E}$
Parameters:	
$q_k, ; k \in \mathcal{K}$	Demands for OD pair k (passengers)
$t_{ij}{}^{o}, ; (i, j) \in \mathcal{E}$	Travel time on link (i, j) before changes (minutes)
$t_{ij}, ; (i, j) \in \mathcal{E}$	Travel time on link (i, j) (minutes)
$t_k{}^{o}, ; k \in \mathcal{K}$	Original travel time for OD pair k before changes (minutes)
α, β	Weights for objective functions
$N^{\ell}, ; \ell \in \mathcal{L}$	Number of vehicles on line ℓ (vehicles per hour)
M	Large constant
$o_k, d_k; k \in \mathcal{K}$	Origin and destination node of OD pair k
$x_{ij}^{\ell}{}^{o}, ; i, j \in \mathcal{N}, \ell \in \mathcal{L}$	Original network configuration (1 if link (i, j) is used by line ℓ)
Decision Variables:	
Binary Variables:	
$x_{ij}^{\ell}, ; i, j \in \mathcal{N}, \ell \in \mathcal{L}$	1 if link (i, j) is used by line ℓ in new network
$y_{ij}^{kl}, ; i, j \in \mathcal{N}, k \in \mathcal{K}, \ell \in \mathcal{L}$	1 if OD pair k uses link (i, j) on line ℓ
$a_i^{\ell}, b_i^{\ell}, ; i \in \mathcal{N}, \ell \in \mathcal{L}$	1 if node i is the start/end terminal of line ℓ
$tb_{ki}^{\ell}, ; k \in \mathcal{K}, i \in \mathcal{N}, \ell \in \mathcal{L}$	1 if OD pair k transfers by boarding line ℓ at node i
$dx_{ij}^{l+}, ; i, j \in \mathcal{N}, \ell \in \mathcal{L}$	1 if link (i, j) is added to line ℓ in new network
$dx_{ij}^{l-}, ; i, j \in \mathcal{N}, \ell \in \mathcal{L}$	1 if link (i, j) is removed from line ℓ in new network
Continuous Variables:	
$u_i^{\ell}, ; i \in \mathcal{N}, \ell \in \mathcal{L}$	Auxiliary variable for subtour elimination (MTZ)
$s_i^{k}, ; i \in \mathcal{N}, k \in \mathcal{K}$	Order of node i in the path of OD pair k (0 if not visited)
$tt_{ki}^{\ell}, ; k \in \mathcal{K}, i \in \mathcal{N}, \ell \in \mathcal{L}$	Transfer waiting time for OD pair k at node i for line ℓ
$ft_k^{\ell}, ; k \in \mathcal{K}, \ell \in \mathcal{L}$	Initial waiting time for OD pair k on line ℓ
$t^{\ell}, ; \ell \in \mathcal{L}$	Total travel time of line ℓ
$it_k, ; k \in \mathcal{K}$	Increased travel time for OD pair k

Subject to:

$$ft_k^\ell \geq \frac{t^\ell}{2N^\ell} - M(1 - \sum_{j \in \mathcal{N}} y_{o_k,j}^{k\ell}) \qquad \forall k \in \mathcal{K}, \ell \in \mathcal{L} \tag{2}$$

$$ft_k^\ell \leq M \cdot \sum_{j \in \mathcal{N}} y_{o_k,j}^{k\ell} \qquad \forall k \in \mathcal{K}, \ell \in \mathcal{L} \tag{3}$$

$$tt_{ki}^\ell \geq \frac{t^\ell}{2N^\ell} - M(1 - tb_{ki}^\ell) \qquad \forall k \in \mathcal{K}, i \in \mathcal{N}, \ell \in \mathcal{L} \tag{4}$$

$$tt_{ki}^\ell \leq M \cdot tb_{ki}^\ell \qquad \forall k \in \mathcal{K}, i \in \mathcal{N}, \ell \in \mathcal{L} \tag{5}$$

Line Constraints

$$\sum_{i \in \mathcal{N}} a_i^\ell M \geq \sum_{(i,j) \in \mathcal{E}} x_{ij}^\ell, \sum_{i \in \mathcal{N}} b_i^\ell M \geq \sum_{(i,j) \in \mathcal{E}} x_{ij}^\ell \qquad \forall \ell \in \mathcal{L} \tag{6}$$

$$\sum_{i \in \mathcal{N}} a_i^\ell \leq 1, \sum_{i \in \mathcal{N}} b_i^\ell \leq 1 \qquad \forall \ell \in \mathcal{L} \tag{7}$$

$$1 \leq u_i^\ell \leq |N| \qquad \forall i \in \mathcal{N}, \ell \in \mathcal{L} \tag{8}$$

$$u_i^\ell - u_j^\ell + 1 \leq (|N| - 1)(1 - x_{ij}^\ell) \qquad \forall (i,j) \in \mathcal{E}, \ell \in \mathcal{L} \tag{9}$$

$$\sum_{j:(j,i) \in \mathcal{E}} x_{ji}^\ell + a_i^\ell = \sum_{j:(i,j) \in \mathcal{E}} x_{ij}^\ell + b_i^\ell \qquad \forall i \in \mathcal{N}, \ell \in \mathcal{L} \tag{10}$$

$$\sum_{l \in \mathcal{L}} x_{ij}^\ell = \sum_{l \in \mathcal{L}} x_{ji}^\ell \qquad \forall (i,j) \in \mathcal{E} \tag{11}$$

Passenger Flow Constraints

$$y_{ij}^{k\ell} \leq x_{ij}^\ell \qquad \forall (i,j) \in \mathcal{E}, k \in \mathcal{K}, \ell \in \mathcal{L} \tag{12}$$

$$\sum_{j:(o_k,j) \in \mathcal{E}} \sum_{\ell \in \mathcal{L}} y_{o_k,j}^{k\ell} = 1 \qquad \forall k \in \mathcal{K} \tag{13}$$

$$\sum_{j:(j,d_k) \in \mathcal{E}} \sum_{\ell \in \mathcal{L}} y_{j,d_k}^{k\ell} = 1 \qquad \forall k \in \mathcal{K} \tag{14}$$

$$\sum_{j:(j,i) \in \mathcal{E}} \sum_{\ell \in \mathcal{L}} y_{ji}^{k\ell} = \sum_{j:(i,j) \in \mathcal{E}} \sum_{\ell \in \mathcal{L}} y_{ij}^{k\ell} \qquad \forall k \in \mathcal{K}, i \in \mathcal{N} \setminus \{o_k, d_k\} \tag{15}$$

$$s_{o_k}^k = 1 \qquad \forall k \in \mathcal{K} \tag{16}$$

$$s_j^k \geq s_i^k + 1 - M(1 - \sum_{\ell \in \mathcal{L}} y_{ij}^{k\ell}) \qquad \forall k \in \mathcal{K}, (i,j) \in \mathcal{E} \tag{17}$$

Transfer Constraints

$$tb_{ki}^{\ell} \leq \sum_{j:(j,i)\in\mathcal{E}} \sum_{\ell'\neq\ell} y_{ji}^{k\ell'} \qquad \forall k \in \mathcal{K}, i \in \mathcal{N}, \ell \in \mathcal{L} \tag{18}$$

$$tb_{ki}^{\ell} \leq \sum_{j:(i,j)\in\mathcal{E}} y_{ij}^{k\ell} \qquad \forall k \in \mathcal{K}, i \in \mathcal{N}, \ell \in \mathcal{L} \tag{19}$$

$$tb_{ki}^{\ell} \geq \sum_{j:(j,i)\in\mathcal{E}} \sum_{\ell'\neq\ell} y_{ji}^{k\ell'} + \sum_{j:(i,j)\in\mathcal{E}} y_{ij}^{k\ell} - 1 \qquad \forall k \in \mathcal{K}, i \in \mathcal{N}, \ell \in \mathcal{L} \tag{20}$$

$$tb_{ko_k}^{\ell} = 0, tb_{kd_k}^{\ell} = 0 \qquad \forall k \in \mathcal{K}, \ell \in \mathcal{L}, i = o_k \text{ or } d_k \tag{21}$$

$$\sum_{j:(i,j)\in\mathcal{E}} x_{ij}^{\ell} \geq tb_{ki}^{\ell} \qquad \forall k \in \mathcal{K}, i \in \mathcal{N}, \ell \in \mathcal{L} \tag{22}$$

$$\sum_{j:(j,i)\in\mathcal{E}} y_{ji}^{k\ell} = \sum_{j:(i,j)\in\mathcal{E}} y_{ij}^{k\ell} + \sum_{\ell'\neq\ell} tb_{ki}^{\ell'} \qquad \forall k \in \mathcal{K}, i \in \mathcal{N} \setminus \{o_k, d_k\}, \ell \in \mathcal{L} \tag{23}$$

Constraints (2)-(5) define passenger waiting time and transfer time at stops. Line constraints (6)-(11) ensure the feasibility of the transit network structure. Constraints (6)-(7) specify unique start and end points for each line, constraints (8)-(9) eliminate potential subtours, constraint (10) maintains line continuity, and constraint (11) ensures bidirectional connectivity at the network level by maintaining an equal number of transit lines between each node pair in both directions. Passenger flow constraints (12)-(17) govern passenger movement within the network. Constraint (12) restricts passenger flows to established transit lines, constraints (13)-(14) ensure all origin-destination demands are satisfied, and constraints (15)-(17) guarantee the validity of passenger paths by preventing cycles and ensuring logical progression from origin to destination. Transfer constraints (18)-(23) handle passenger transfers within the network. Constraints (18)-(20) define the conditions for transfers to occur, constraint (21) prohibits transfers at origin and destination nodes, constraint (22) ensures the presence of necessary lines at transfer points, and constraint (23) maintains flow balance considering transfers between lines.

3.5 Linearization

The objective function (1) contains nonlinear components, and in order to obtain an exact solution using the solver, we need to linearize it using the following constraints.

$$it_k \geq \sum_{(i,j)\in\mathcal{E}} \sum_{\ell\in\mathcal{L}} y_{ij}^{k\ell} t_{ij} + \sum_{i\in\mathcal{N}} \sum_{\ell\in\mathcal{L}} tt_{ki}^{\ell} + \sum_{\ell\in\mathcal{L}} ft_k^{\ell} - t_k^o \qquad \forall k \in \mathcal{K} \tag{24}$$

$$it_k \geq 0 \qquad \forall k \in \mathcal{K} \tag{25}$$

$$x_{ij}^{\ell} + dx_{ij}^{l-} = x_{ij}^{lo} + dx_{ij}^{l+} \qquad \forall (i,j) \in \mathcal{E}, \ell \in \mathcal{L} \tag{26}$$

$$dx_{ij}^{l-} + dx_{ij}^{l+} \leq 1 \qquad \forall (i,j) \in \mathcal{E}, \ell \in \mathcal{L} \tag{27}$$

Constraints (24)-(25) calculate and control changes in passenger travel time after network optimization. Constraints (26)-(27) manage line changes during network optimization.

After linearization, the optimization problems become as follows:

$$\text{Minimize} \quad Z = \beta \underbrace{\sum_{k\in\mathcal{K}} d_k \cdot it_k}_{\text{passenger inconvenience}} + (1-\beta) \underbrace{\sum_{\ell\in\mathcal{L}} \sum_{i,j\in\mathcal{N}} x_{ij}^{\ell} t_{ij}}_{\text{operating cost}} + \alpha \underbrace{\sum_{\ell\in\mathcal{L}} \sum_{i,j\in\mathcal{N}} (dx_{ij}^{l+} + dx_{ij}^{l-})}_{\text{network modification}} \tag{28}$$

s.t.
constraints (2)–(27)

4 Numerical Results

The performance of the proposed methodology is evaluated through numerical experiments. A case study on Mandl's Swiss network (see Section 4.1) is presented, along with a travel time variance sample pool generated using a log-normal distribution (see Section 4.2). The corresponding mixed-integer programming model is implemented in Python and solved using the Gurobi 11.0.3 solver on an Intel Core i5 PC operating at 4.6 GHz with 16.0 GB of RAM. A maximum CPU time of 1200 seconds is set for all computations. The code is available on this GitHub repository.

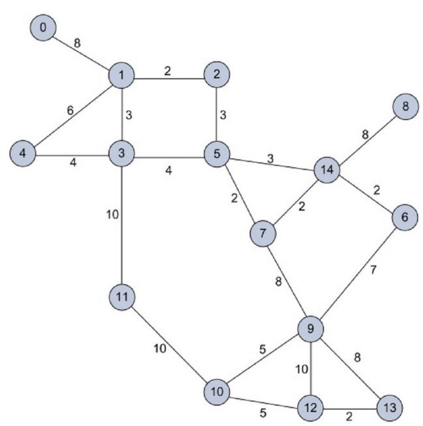


Fig. 2: Mandl's network [20].

4.1 Mandl's Swiss Network

Mandl's Swiss network (see Fig. 2) is a small network consisting of 15 nodes and 21 links, initially used by Mandl [20]. The network includes a total of 15,570 trips, and the demand is symmetric. This dataset is one of the few publicly accessible resources for the route planning problem and has emerged as the predominant benchmark instance.

The predominant route length restriction confines each route to a maximum of 8 nodes. In this paper, we minimise human bias by permitting an unlimited number of nodes per route, thereby enabling the model to determine the optimal routes autonomously. These "unlimited" routes remain constrained by the condition that each node may be visited no more than once per route.

We adopted the PT network reported by Vermeir et al. [26] that performed best in minimizing total passenger travel time. Under their assumption that passengers always choose the shortest path, we obtained the passenger travel times T_k^{ver} under their PT network. Nonetheless, as our goal encompasses not only the optimisation of passenger travel time but also the operational duration of lines, and considering the inherent complexity of the Transit Network Design Problem with differing assumptions across various studies, direct comparisons prove to be difficult [9]. We used these passenger travel times T_k^{ver} with the penalty parameter $\alpha = 0$ in our objective function (28) to generate our own transport network that optimizes both passenger travel times and operating times, which serves as our original existing static PT network $\mathcal{G}_{\mathrm{PT}}^{\mathrm{orig}}$.

4.2 Travel Time Variability Modeling

In this work, we account for the variability in travel times by modeling them with a log-normal distribution, which has been shown to capture the skewed nature of travel time data effectively [12]. For a log-normal distribution with parameters μ and σ, the mean and variance are given by $\mathrm{E}[X] = \exp\left(\mu + \frac{\sigma^2}{2}\right)$ and $\mathrm{Var}(X) = \left[\exp(\sigma^2) - 1\right] \exp\left(2\mu + \sigma^2\right)$, respectively. Therefore, the coefficient of variation (CV), defined as the ratio of the standard deviation to the mean, is

$$\mathrm{CV} = \frac{\sqrt{\mathrm{Var}(X)}}{\mathrm{E}[X]} = \frac{\sqrt{[\exp(\sigma^2) - 1] \exp\left(2\mu + \sigma^2\right)}}{\exp\left(\mu + \frac{\sigma^2}{2}\right)} = \sqrt{\exp(\sigma^2) - 1}.$$

Based on this property, for each link (i, j), we can compute the median as $\exp(\mu_{i,j})$ and the coefficient of variation as $CV_{i,j} = \sqrt{\exp(\sigma_{i,j}^2) - 1}$. Considering the Mandl's network (Fig. 2), we assume the nominal travel time on link (i, j) in the original dataset as the median, which we denote as $m_{i,j}$. Since $m_{i,j} = \exp(\mu_{i,j})$, we have $\mu_{i,j} = \ln(m_{i,j})$. Given a desired coefficient of variation $cv \in \mathcal{CV}$, we can derive $\sigma_{i,j} = \sqrt{\ln(cv^2 + 1)}$. The travel time $t_{i,j}^{cv,\omega}$ for each sample $\omega \in \Omega_{cv}$ is then generated as a realization of log-normal$(\mu_{i,j}, \sigma_{i,j}^2)$.

4.3 Performance Comparison under Different Scenarios

We now show the improvement in network performance, achieved via our method that adapts the PT network to current road conditions. To demonstrate our method's adaptability to travel time variations, for each $cv \in \mathcal{CV}$, we consider a subset of samples $\Omega'_{cv} = \{1, ..., 5\} \subset \Omega_{cv}$. For each sampled travel time realization $\{t_{i,j}^{cv,\omega} | \forall (i, j) \in \mathcal{E}\}$ where $\omega \in \Omega'_{cv}$. Let $z \in \mathcal{Z}$ denote the trips, each corresponding to a certain OD pair $k \in \mathcal{K}$. We denote by $T_z^{o,cv,\omega}$ and $T_z^{*,cv,\omega}$ the travel time of that trip when performed on the original PT network and the optimized one, respectively. We evaluate the network performance improvement by:

$$\frac{T_z^{o,cv,\omega} - T_z^{*,cv,\omega}}{T_z^{o,cv,\omega}} \times 100\% \tag{29}$$

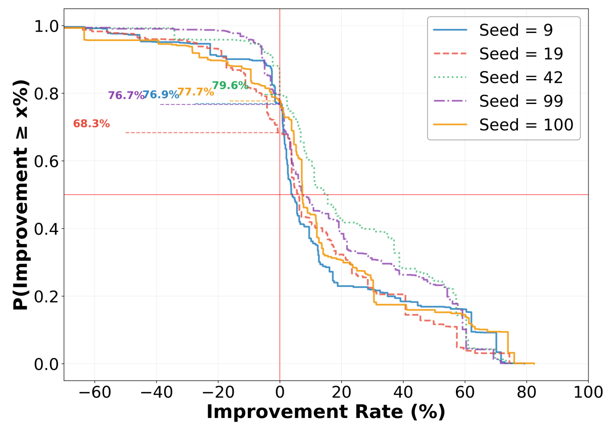
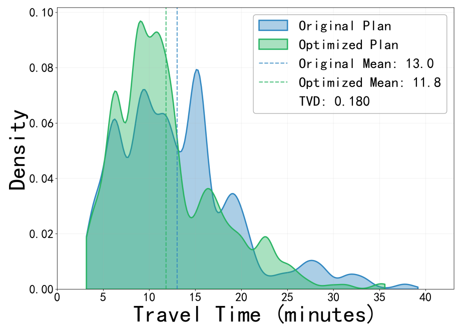


Fig. 3: Inverse Cumulative Distribution of Improvement Rates for Different Seeds (based on cv=1)

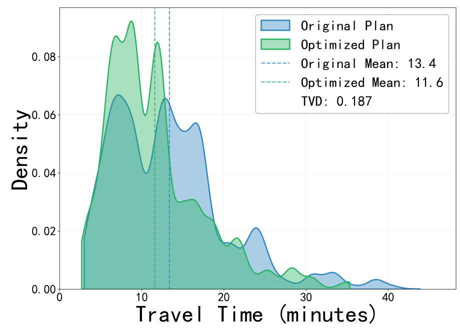
Figure 3 shows the inverse cumulative distribution of improvement rates for different seeds with $cv = 1$. For any point (x, y) on the curve, y represents the probability that a randomly selected trip will experience an improvement rate of at least $x\%$. The horizontal dashed lines highlight the probability of achieving a positive improvement for each seed value. The results reveal that our method yields positive enhancements in approximately 80% of trips across different instances. Furthermore, the consistency of the distribution curves across various seeds indicates that our method identifies the optimal value across all sample cases.

To ensure brevity, we perform the following analysis utilising a singular representative sample ($\omega = 1 \in \Omega'_{cv}$) for each $cv \in \mathcal{CV}$. The results presented herein are derived with $\alpha = 10$ and $\beta = 0.5$. In Figure 4, we represent, for different values of $cv \in \mathcal{CV}$, the distribution of travel times of the original PT network $\{T_z^{o,cv,\omega} | z \in \mathcal{Z}\}$ versus the ones on the optimized adaptive PT network $\{T_z^{*,cv,\omega} | z \in \mathcal{Z}\}$. We observe the improvement of travel times is consistent, and it is particularly strong when variability is high ($cv = 1$); indeed, in this case, adaptivity to actual road conditions is particularly beneficial. We also specify in the figure the Total Variation Distance (TVD) between the distribution of $\{T_z^{o,cv,\omega} | z \in \mathcal{Z}\}$ and $\{T_z^{*,cv,\omega} | z \in \mathcal{Z}\}$, which as expected increases with cv.

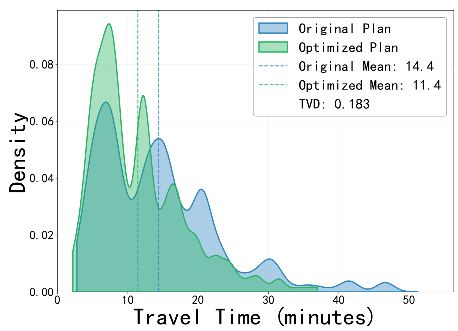
To quantify the similarity between the old and new routes. We adopt the Jaccard index [15], defined as $R = \frac{|A \cap B|}{|A \cup B|}$, where A and B denote the node sets of the original and new routes, respectively. Table 2 illustrates that our method adeptly reconciles route preservation with performance enhancement: it achieves reductions in operating time while ensuring substantial overlap with the original routes. *It's worth noting that* the most substantial time savings are realised in longer routes, as these routes present greater opportunities for optimisation through strategic adjustments. Line 3, the longest route in the network, exhibits a significant time reduction of 71.6 mins under $cv = 1$, while preserving a 71.4% overlap with its original trajectory.



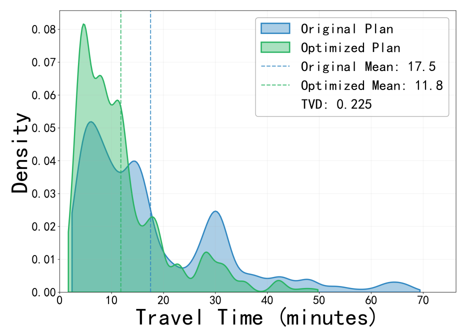
(a) Minimal Variation (**cv** = 0.05)



(b) Low Variation (**cv** = 0.25)



(c) Moderate Variation (**cv** = 0.50)



(d) High Variation (**cv** = 1.00)

Fig. 4: **Trip Time Distribution Analysis:** Comparison of Original Optimal Static and Adaptive Optimal PT Networks

5 Conclusion

This paper presents a method to modify the existing static bus lines to adapt to time-varying road conditions. Its novelty is that, instead of considering rebuilding from scratch or localized adjustments, we aim to optimize the original public transport (PT) network to reduce both passenger travel time degradation and the operational duration of each transit line via constrained network alterations.

We formulate the adaptive PT design problem as a mixed-integer program. The model samples historical traffic data as input parameters and incorporates route variation quantification and penalisation processes to develop resilient strategies that adjust to variations in road speed. The case studies utilising Mandl's network demonstrate that our methodology can suggest effective strategies for time-varying roads with any coefficient of variation.

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

Table 2: Comparison of Route Adjustments Under Different cv

Coefficient of variation (cv)	Line	Path Sequence	Overlap (%)	Time difference (mins)
Original	1	11→10→8→6→3→2→4→5	–	–
	2	2→4→6→15→9	–	–
	3	5→4→12→11→13→14→10→7→15→8→6→3→2→1	–	–
0.05	1	11→10→8→6→3→2→1	66.7	+1.7
	2	10→7→15→8→6→4→5	33.3	+3.9
	3	10→14→13→11→12→4→2→3→6→15→9	66.7	**-10.2**
0.25	1	1→2→3→6→8→15→9	36.4	+4.6
	2	11→13→14→10→8→6→4	20.0	+11.8
	3	5→2→4→12→11→10→7→15→6	64.3	**-24.0**
0.50	1	1→2→3→6→8→15→9	36.4	**-6.4**
	2	11→10→8→6→4→5	22.2	+8.6
	3	5→2→4→12→11→13→14→10→8→15→7	78.6	**-31.1**
1.00	1	9→15→6→3→2→4→12→11→13→14→10	46.2	+18.4
	2	11→10→8→6→3→2→5	20.0	+5.7
	3	1→2→4→6→8→15→7→10→13→14	71.4	**-71.6**

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