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Spectral fluctuations and crossovers in multilayer network

Keywords: Spectral graph theory, graph adjacency matrix, multilayer network, random matrix theory, and protein crystal structures

Abstract

In the past few decades, network theory has established itself as a powerful framework for analyzing a wide range of real-world systems, including social, biological, economic, transportation, and information networks [1, 2]. Many of these systems are inherently multilayered, as they encompass multiple types of interactions, relations, or dynamical processes occurring simultaneously [3]. Motivated by this, we employ the framework of random matrix theory (RMT) to investigate spectral fluctuations in multilayer networks, with the aim of distinguishing universal and non-universal spectral features[4, 5].

In this context, we consider general multilayer architectures—including purely intra-layer, purely inter-layer, combined, and multiplex structures—by constructing ensembles of random multilayer networks with Erdős-Rényi (ER) random graph connectivity and systematically varying intra- and inter-layer connection probabilities to examine the dependence of spectral statistics on connectivity patterns [5]. The adjacency matrix of a multilayer network naturally exhibits a block structure, with diagonal blocks representing intra-layer connectivity and off-diagonal blocks capturing inter-layer couplings. In this work, fluctuations are characterized using higher-order spacing ratios distibution(SRD), which are sensitive to spectral correlations [6]. To ensure comparability of spectral fluctuations across layers, we introduce scaling factors for diagonal and off-diagonal blocks that normalize variances irrespective of block size or connection probability.

In this work further, we are introducing a crossover model for bilayer networks, and capture the smooth transition of spectral properties from block-diagonal (two independent GOEs) to single-layer (one GOE) statistics as the relative strength of inter-layer to intra-layer connection varies. Furthermore, we apply the framework to empirical multilayer networks derived from protein crystal structures (1EWT, 1EWK, and 1UW6), modeling residues as nodes and spatial proximity as edges. By varying distance thresholds, we probe how changes in intra- and inter-layer connectivity drive transitions between universality classes, using both SRD and cumulative SRD (CSRD) to quantify the behavior.

Our findings show that, under appropriate scaling, multilayer networks exhibit the universal spectral fluctuations predicted by RMT, despite heterogeneity in layer structure and topology. This demonstrates that spectral universality is a robust feature of layered architectures and that the interplay between intra- and inter-layer couplings can be rigorously quantified within the RMT framework. Beyond protein systems, these results establish a general methodology for probing universality and coupling effects in complex multilayered systems, with broad relevance to communication, transportation, and biological networks.

References

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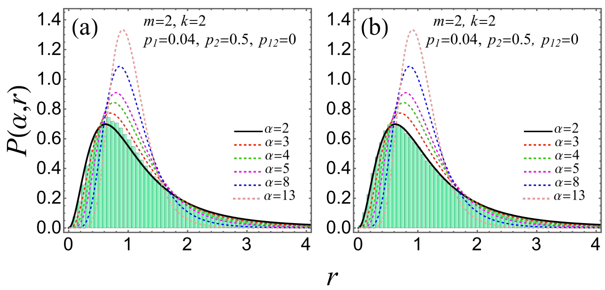

Figure 1: **Effect of scaling.** Higher-order SRD histograms for a bilayer network ($m = 2$) without inter-layer connections. The diagonal blocks of the adjacency matrix have dimensions 350×350 and 450×450, respectively. The histograms are obtained from numerical simulations over an ensemble of 250 adjacency matrices. The order of the spacing distribution and the edge probabilities used for the simulations are $(k, p_1, p_2) = (2, 0.04, 0.5)$ in both cases. (a) Without scaling the two blocks, the resulting distributions deviate from RMT predictions. (b) With appropriate scaling of the blocks, the histograms closely follow the analytical RMT expression corresponding to $\alpha = 2$.