

000 MERGE BEFORE FORGET: A SINGLE LORA CONTIN- 001 002 UAL LEARNING VIA CONTINUAL MERGING 003 004

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007 008 ABSTRACT 009

010 Parameter-efficient continual learning has emerged as a promising approach for
011 large language models (LLMs) to mitigate catastrophic forgetting while enabling
012 adaptation to new tasks. Current Low-Rank Adaptation (LoRA) continual learning
013 techniques often retain and freeze previously learned LoRAs or generate data repre-
014 sentations to overcome forgetting, typically utilizing these to support new LoRAs
015 learn new tasks. However, these methods not only ignore growing computational
016 memory with tasks and limited storage space but also suffer from potential task
017 interference due to the lack of effective LoRA merging mechanisms. In this paper,
018 we propose a novel continual learning method that orthogonally initializes and
019 sequentially merges LoRAs updates into a single unified LoRA. Our method lever-
020 ages orthogonal basis extraction from previously learned LoRA to initialize the
021 learning of new tasks, further exploits the intrinsic asymmetry property of LoRA
022 components by using a time-aware scaling mechanism to balance new and old
023 knowledge during continual merging. Our approach maintains constant memory
024 complexity with respect to the number of tasks, minimizes interference between
025 past and new tasks via orthogonal basis initialization, and improves performance
026 over asymmetric LoRA merging via adaptive scaling. We provide theoretical
027 analysis to justify our design and conduct extensive experiments across diverse
028 continual learning benchmarks using various Llama models, demonstrating the
029 effectiveness and efficiency of our method.

030 1 INTRODUCTION

031 Large Language Models (LLMs) (Raffel et al., 2020; Achiam et al., 2023; Touvron et al., 2023) have
032 been growing as the cornerstone of modern machine learning, achieving remarkable performance
033 across a wide range of downstream tasks. However, despite their impressive capabilities, LLMs
034 still suffer from catastrophic forgetting (McCloskey & Cohen, 1989; Zhu et al., 2024a; Yang et al.,
035 2024) when fine-tuning sequential tasks, and their huge model capacity makes full fine-tuning
036 computationally expensive and memory-intensive (Zhao et al., 2024a). These challenges have led
037 to increasing attention in parameter-efficient continual learning, particularly via techniques such as
038 LoRA (Low-Rank Adaptation) (Hu et al., 2022), that injects trainable low-rank matrices A and B into
039 pre-trained models, enabling task adaptation with minimal additional parameters. Existing methods
040 have shown progress in mitigating forgetting in LLMs through LoRA-based continual learning.
041 For example, O-LoRA (Wang et al., 2023) freezes previously learned LoRAs and incrementally
042 learns new tasks in their orthogonal subspace; InfLoRA (Liang & Li, 2024) preserves prior LoRAs
043 and uses task-dependent input matrices to define orthogonal subspaces for initializing new ones;
044 SAPT-LoRA (Zhao et al., 2024b) retains earlier LoRAs and leverages generated previous tasks'
045 data to align new LoRA learning with shared modules; SD-LoRA (Wu et al., 2025) incrementally
046 decouples the learning of magnitude and direction in LoRA components while preserving directions
047 learned from previous tasks. However, these methods either keep and freeze previously learned
048 LoRAs, resulting in parameter growth of the form $[B_1 A_1, \dots, B_t A_t]$, or generate and maintain
049 task-specific data representations, leading to (i) *linear growth* in memory usage with the number of
050 tasks, (ii) *limited scalability* due to constrained storage space, and (iii) *potential task interference* in
051 the absence of principled LoRA merging mechanisms. These limitations motivate the question:

052 *Can we enable continual learning only using a single shared LoRA, without learning or storing
053 task-specific LoRAs or data representations?*

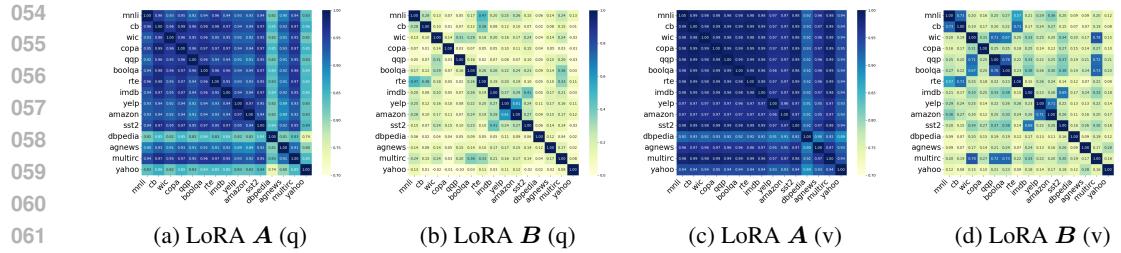


Figure 1: Cosine similarity between 15 tasks from the large number of tasks benchmark for fine-tuned q and v attention LoRA A and B in the last layer (32nd) of Llama-2-7B-chat.

To address this question, we take inspiration from model merging (Garipov et al., 2018; Draxler et al., 2018; Wortsman et al., 2022), an emerging paradigm that aims to combine multiple task-specific models into a single unified model without retraining (Stoica et al., 2024; Ilharco et al., 2023; Yadav et al., 2023; Ortiz-Jimenez et al., 2023). By extending this idea, we frame continual learning as a sequential model merging problem, where the objective shifts from keeping all task-specific LoRAs or data to continually integrating their updates into a single shared LoRA as new tasks arrive. Recently, model merging has successfully been extended to the LoRA regime: KnOTS (Stoica et al., 2025) leverages singular value decomposition to project LoRA updates into a shared latent space, where existing merging methods can be applied; LoRA-LEGO (Zhao et al., 2025) decomposes LoRAs into minimal semantic units via grouping and clustering, enabling a reconstruction of multiple LoRAs into one. However, these LoRA merging methods generally assume *concurrent access* to all task-specific LoRAs fine-tuned from the same pre-trained model, which limits their applicability to the continual merging scenarios (Dziadzio et al., 2025), where tasks arrive sequentially. In such settings, the order of merging becomes critical and may degrade the performance of the final model. Moreover, continual LoRA merging remains underexplored in existing literature. While in the full-model setting, continual merging has received more attention, e.g., OPCM (Tang et al., 2025) sequentially projects new model updates onto subspaces orthogonal to the previously merged model and uses adaptive scaling to mitigate interference. However, these methods are not designed for LoRA, and the objective of merging differs from that of continual learning (Ortiz-Jimenez et al., 2023). These challenges lead to the following question we aim to answer:

How can we enable continual learning through LoRA-based continual merging?

We answer this question by maintaining a single pair of low-rank matrices $\{\mathbf{A}, \mathbf{B}\}$, shared across tasks. Achieving this necessitates addressing key challenges, including how to initialize and continually update the shared LoRA to effectively balance the trade-off between forgetting and generalization. Moreover, in contrast to full-model continual merging, \mathbf{A} and \mathbf{B} play different roles in continual merging with LoRA. For instance, prior works (Zhu et al., 2024b; Sun et al., 2024; Zhang et al., 2023b; Kopiczko et al., 2024) have shown that in LoRA fine-tuning, training \mathbf{B} (initialized to zero) is critical for the performance, even randomly initialized \mathbf{A} often suffices, but reversing the roles of \mathbf{A} and \mathbf{B} substantially decreases performance. To further investigate the asymmetry of LoRA components, we separately fine-tune 15 tasks from a standard large number of tasks benchmark (Wang et al., 2023) in continual learning using 15 independent LoRAs on Llama-2-7B-chat (Touvron et al., 2023), and compute cosine similarity of \mathbf{A} and \mathbf{B} across 15 tasks using their last layer LoRA. Figure 1 shows that \mathbf{A} exhibits significantly higher similarity across tasks compared to \mathbf{B} , suggesting that LoRA components follow inherently different learning dynamics. This motivates us to treat \mathbf{A} and \mathbf{B} differently in continual merging.

To address the above questions, we propose a novel parameter-efficient continual learning method via continual merging into a single LoRA, which initializes new task learning in an orthogonal subspace and sequentially merges LoRA updates. We name this method **SLAO** (Single LoRA continual learning with Orthogonal initialization via continual merging). Specifically, SLAO initializes each new task learning LoRA using orthogonal basis extracted from previously learned LoRA components, and exploits the asymmetric roles of \mathbf{A} and \mathbf{B} by applying a time-aware scaling mechanism that balances knowledge retention and plasticity during continual merging. As shown in Figure 2, our approach ensures constant memory overhead regardless of the number of tasks. Additionally, it reduces interference between past and new tasks via orthogonal basis initialization and enhances performance through adaptive continual merging that considers LoRA asymmetry.

108 **Summary of contributions.** This paper makes the following key contributions: (1) A novel parameter-efficient continual learning method for LLMs that continually merges new task LoRAs into a single LoRA via orthogonal basis initialization and a time-aware scaling mechanism, reducing catastrophic forgetting and improving generalization. (2) A theoretical analysis of how our design mitigates forgetting and improves intransigence. (3) Comprehensive experiments on various continual learning benchmarks using Llama models and [Qwen models](#) of varying sizes, demonstrating effectiveness and efficiency of our proposed method.

2 BACKGROUND AND MOTIVATION

123 **Problem setup.** Let $f_{\mathbf{W}_0} : \mathcal{X} \rightarrow \mathcal{Y}$ denote a pre-trained model parameterized by $\mathbf{W}_0 \in \mathbb{R}^{m \times n}$, which 124 remains frozen throughout the continual learning (CL) process. Here, \mathcal{X} and \mathcal{Y} represent the input 125 and output spaces, respectively. We consider a sequence of T tasks. For each task $t \in \{1, 2, \dots, T\}$, 126 the model is continually fine-tuned using the LoRA algorithm based on its associated training dataset 127 $\mathcal{D}_t = \{(\mathbf{X}_{t,i}, \mathbf{Y}_{t,i})\}_{i=1}^N$, and evaluated on a separate test dataset $\mathcal{D}'_t = \{(\mathbf{X}'_{t,i}, \mathbf{Y}'_{t,i})\}_{i=1}^{N'}$, where N 128 and N' denote the number of training and testing samples, respectively. The goal is to continually 129 learn a single set of LoRA parameters, specifically, matrices $\mathbf{B} \in \mathbb{R}^{m \times r}$ and $\mathbf{A} \in \mathbb{R}^{r \times n}$ with 130 $r \ll \min(m, n)$ such that the resulting merged LoRA model remains competitive with models 131 optimized for expected risk (multi-task objective):

$$\min_{\mathbf{B}, \mathbf{A}} \sum_{t=1}^T \sum_{(\mathbf{X}_t, \mathbf{Y}_t) \in \mathcal{D}'_t} \mathcal{L}_t(f_{\mathbf{W}_0 + \mathbf{B}\mathbf{A}}(\mathbf{X}_t), \mathbf{Y}_t), \quad (1)$$

134 where \mathcal{L}_t denotes the empirical risk (e.g., cross-entropy or mean squared error) for task t .

135 In the single shared LoRA setting for CL, we are restricted to maintaining only one pair of LoRA 136 parameters, denoted by $\mathbf{A}_{\text{merge}}^t$ and $\mathbf{B}_{\text{merge}}^t$, across all tasks. When task t arrives, we fine-tune on 137 its training data, possibly initialized with current merged models $\{\mathbf{A}_{\text{merge}}^{t-1}, \mathbf{B}_{\text{merge}}^{t-1}\}$, to obtain task- 138 specific LoRA parameters $\mathbf{B}_{\text{ft},t} \in \mathbb{R}^{m \times r}$ and $\mathbf{A}_{\text{ft},t} \in \mathbb{R}^{r \times n}$. The fine-tuned model for task t is 139 represented as $f_{\mathbf{W}_0 + \mathbf{B}_{\text{ft},t} \mathbf{A}_{\text{ft},t}}(\cdot)$. After fine-tuning, we merge the previously accumulated LoRA 140 parameters $\mathbf{B}_{\text{merge}}^{t-1} \mathbf{A}_{\text{merge}}^{t-1}$ with the new task-specific parameters $\mathbf{B}_{\text{ft},t} \mathbf{A}_{\text{ft},t}$, resulting in an updated 141 merged representation $\mathbf{B}_{\text{merge}}^t \mathbf{A}_{\text{merge}}^t$. Due to the inherent asymmetry in LoRA components, the 142 merging of the LoRA components is performed separately for \mathbf{B} and \mathbf{A} as formalized below: 143

$$\mathbf{B}_{\text{merge}}^t = \text{ContinualMerge.B}(\mathbf{B}_{\text{merge}}^{t-1}; \mathbf{B}_{\text{ft},t}), \quad \mathbf{A}_{\text{merge}}^t = \text{ContinualMerge.A}(\mathbf{A}_{\text{merge}}^{t-1}; \mathbf{A}_{\text{ft},t}), \quad t \geq 2$$

144 where $\mathbf{B}_{\text{ft},0} = \mathbf{0}$ and $\mathbf{A}_{\text{ft},0}$ is initialized using a Gaussian distribution, following the standard LoRA 145 initialization (Hu et al., 2022). $\mathbf{B}_{\text{merge}}^1 = \mathbf{B}_{\text{ft},1}$ and $\mathbf{A}_{\text{merge}}^1 = \mathbf{A}_{\text{ft},1}$ are initialized as the first task 146 fine-tuned LoRA, and $\mathbf{B}_{\text{merge}}^t \mathbf{A}_{\text{merge}}^t$ is to optimize Equation 1. 147

2.1 OPPORTUNITIES AND CHALLENGES IN CONTINUAL LEARNING VIA CONTINUAL MERGING

152 **Storage and memory efficiency** is one of the core advantages of continual merging for CL. Unlike 153 existing continual learning methods that benefit from freezing and retaining previously fine-tuned 154 LoRAs, using continual merging after fine-tuning task t only requires storing a fixed number of 155 LoRAs: (1) the current merged LoRA and (2) the fine-tuned LoRA to be merged. This strategy 156 results in a constant memory complexity of $\mathcal{O}(|\mathbf{B}| + |\mathbf{A}|) = \mathcal{O}((m+n)r)$, where $|\mathbf{B}| + |\mathbf{A}|$ denote 157 the parameter sizes of a single LoRA. Critically, this memory requirement remains independent 158 of the number of sequential tasks T . In contrast, as shown in Figure 2, existing freezing-based 159 continual learning methods require storing all LoRAs, incurring a linear memory complexity of 160 $\mathcal{O}(T(|\mathbf{B}| + |\mathbf{A}|))$, which is $\mathcal{O}(T(m+n)r)$, growing linearly with the number of tasks.

161 **Training efficiency** is an evident advantage of continual merging for CL. Prior works do not simply 162 keep previous tasks' LoRAs without any operations. Instead, when training new tasks, prior works

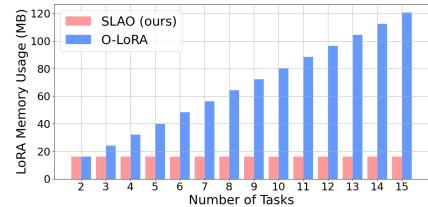


Figure 2: Comparison of SLAO and O-LoRA memory usage of large number of tasks benchmark via Llama-2-7B-chat.

use these multiple LoRAs during training through constraints, i.e., making new task LoRA parameters orthogonal to all previous LoRAs, heavily increasing computational cost during training. However, continual merging in CL would only use the parameters of a single previously fine-tuned LoRA to initialize new task LoRA parameters before training, avoiding extra computation during training.

Difference between continual learning and merging is mainly in the objective. In the context of multi-task model merging, the task arithmetic property, as defined by Ortiz-Jimenez et al. (2023), refers to the ability to add task-specific vectors without interfering with performance on other tasks. However, in CL, the objective extends beyond retention: the model must both preserve previously acquired knowledge and generalize effectively to unseen data. Hence, while merging can support CL, its underlying objectives are not entirely equivalent to those of CL, leading to fundamental differences in both theoretical analysis and algorithmic design.

2.2 ORTHOGONAL INITIALIZATION MOTIVATED BY LoRA NTK ANALYSIS

To inform our algorithmic design, we evaluate the performance of CL using LoRA by two key metrics, forgetting error (Lin et al., 2023) and intransigence error (Li et al., 2023), defined as below:

(1) **Forgetting error:** It measures how much knowledge of old tasks has been forgotten after learning the current task. Specifically, after learning task $t \in [2, T]$, the average forgetting over all old tasks $i \in [1, t - 1]$ is defined as:

$$\mathcal{F}_t = \sum_{i=1}^{t-1} (\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_t \mathbf{A}_t) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i)) \quad (2)$$

In Equation 2, $\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_t \mathbf{A}_t) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i)$ denotes the performance difference between $\mathbf{B}_i \mathbf{A}_i$ (result after training task i) and $\mathbf{B}_t \mathbf{A}_t$ (result after training task t) on test data of task i .

(2) **Intransigence error:** It evaluates the ability of the algorithm to adapt to a new task after having already adapted to a sequence of old tasks.

$$\mathcal{I}_t = \sum_{i=1}^t (\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i^* \mathbf{A}_i^*)) \quad (3)$$

In Equation 3, $\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i^* \mathbf{A}_i^*)$ denotes the performance difference between $\mathbf{B}_i^* \mathbf{A}_i^*$ (optimal result of training task i) and $\mathbf{B}_i \mathbf{A}_i$ (result after training task i) on test data of task i .

To examine these errors, we draw on the empirical observation (Malladi et al., 2023) that, when prompt-based fine-tuning is employed (Schick & Schütze, 2021; Gao et al., 2021), the fine-tuning of a pre-trained language model tends to remain within the Neural Tangent Kernel (NTK) regime. Specifically, under the NTK regime, assuming $\mathcal{D}_t = \{(\mathbf{X}_i, \mathbf{Y}_i)\}_{i \in \{1, \dots, N\}}$, the empirical risk for task t using LoRA can be approximated as (Jang et al., 2024)

$$\mathcal{L}_t = \frac{1}{N} \sum_{i=1}^N \ell_i(f_{\mathbf{W}_0}(\mathbf{X}_i) + \langle \nabla_{\mathbf{W}} f_{\mathbf{W}_0}(\mathbf{X}_i), \mathbf{B}_i \mathbf{A}_i \rangle, \mathbf{Y}_i) \quad (4)$$

As detailed in Appendix C (Lemma 1), by extending the analysis in Jang et al. (2024); Maurer (2016), we show that, under the NTK regime, the term in forgetting error can be bounded as follows:

$$\begin{aligned} \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_t \mathbf{A}_t) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i) &\leq G \|\langle \mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i, \nabla_{\mathbf{W}} f_{\mathbf{W}_0}(\mathbf{X}_i) \rangle\|_2 \\ &\leq G \sqrt{\sum_{j=1}^K \|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F^2 \|\nabla_{\mathbf{W}} f_{\mathbf{W}_0}^{(j)}(\mathbf{X}_i)\|_F^2} \leq G \sqrt{\sum_{j=1}^K \|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F^2 R^2} \end{aligned} \quad (5)$$

where K is output dimension and $\|\nabla_{\mathbf{W}} f_{\mathbf{W}_0}^{(j)}(\mathbf{X}_i)\|_F \leq R$. Thus, to minimize Equation 2, we should make $\|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F$ as small as possible. Similarly, to minimize Equation 3, $\|\mathbf{B}_i \mathbf{A}_i - \mathbf{B}_i^* \mathbf{A}_i^*\|_F$ should be minimized. Thus, the term in forgetting-intransigence decomposition can be written as:

$$\begin{aligned} &\|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F + \|\mathbf{B}_i \mathbf{A}_i - \mathbf{B}_i^* \mathbf{A}_i^*\|_F \\ &\leq \|\mathbf{B}_t(\mathbf{A}_t - \mathbf{A}_i)\|_F + \|(\mathbf{B}_t - \mathbf{B}_i)\mathbf{A}_i\|_F + \|\mathbf{B}_i(\mathbf{A}_i - \mathbf{A}_i^*)\|_F + \|(\mathbf{B}_i - \mathbf{B}_i^*)\mathbf{A}_i^*\|_F \end{aligned} \quad (6)$$

Algorithmic motivation. From above bound, we observe that forgetting-intransigence error in CL with LoRA depends asymmetrically on the choice of frozen and trainable components. For example, freezing \mathbf{A} and fine-tuning \mathbf{B} is at least as effective, if not better, than the reverse (Zhu et al., 2024b). However, if we apply freezing \mathbf{A} in CL, then $\|\mathbf{A}_t - \mathbf{A}_i\|_F = 0$ but $\|\mathbf{A}_i - \mathbf{A}_i^*\|_F$ may

be unintentionally increased due to random \mathbf{A}_i . Instead, if we propose to fine-tune \mathbf{A}_i and extract orthogonal basis \mathbf{Q}_{i-1} from \mathbf{A}_{i-1} , where $\mathbf{Q}_{i-1}\mathbf{Q}_{i-1}^\top = \mathbf{I}_r$, to initialize $\mathbf{A}_i^{(0)}$ via \mathbf{Q}_{i-1} , then we have $\mathbf{A}_i^{(0)}(\mathbf{A}_i^{(0)})^\top = \mathbf{I}_r$ where $i \in [1, \dots, t]$. This orthogonal structure not only keeps geometric consistency across tasks but also allows \mathbf{A}_j ($t \geq j > i$), to remain well-aligned with previous \mathbf{A}_i , i.e. $\mathbb{E}[\mathbf{A}_j \mathbf{A}_i^\top] \approx \mathbf{I}_r$, thereby minimizing both $\|\mathbf{A}_t - \mathbf{A}_i\|_F$ and $\|\mathbf{A}_i - \mathbf{A}_i^*\|_F$. This motivates our design of orthogonal initialization. The complete derivation is in Appendix C.1.

2.3 CONTINUAL MERGING MOTIVATED BY LoRA ASYMMETRY ANALYSIS

To provide the analysis of merging \mathbf{B} , we consider a scenario where a single LoRA is continually fine-tuned for sequential tasks, which means each task starts from the previous task's fine-tuned LoRA. We initialize task 1 as $\mathbf{B}_0 = 0$ and $\mathbf{A}_0 \sim \mathcal{N}(0, \sigma^2)$ (Hu et al., 2022). After fine-tuning with T steps, we obtain its parameters:

$$\mathbf{W}_0 + (\mathbf{B}_0 + \Delta\mathbf{B}_1)(\mathbf{A}_0 + \Delta\mathbf{A}_1) = \mathbf{W}_0 + \Delta\mathbf{B}_1\mathbf{A}_0 + \Delta\mathbf{B}_1\Delta\mathbf{A}_1 \quad (7)$$

Since $\mathbf{B}_0 = 0$, we have that

$$\|(\Delta\mathbf{B}_1)^\top \mathbf{B}_0\|_F = 0, \quad \|\mathbf{A}_0(\Delta\mathbf{A}_1)^\top\|_F \neq 0 \quad (8)$$

Based on theorem in Hao et al. (2024), we write task 1 fine-tuned LoRA:

$$\mathbf{B}_1 = \eta f_B(T)\mathbf{A}_0^\top, \quad \mathbf{A}_1 = \mathbf{A}_0 + \eta \mathbf{A}_0 f_A(T) \quad (9)$$

Using recursion, for task i where $i \geq 2$, the orthogonality measures become:

$$\|\Delta\mathbf{B}_i^\top \mathbf{B}_{i-1}\|_F \approx \|(\eta f_B(T)\mathbf{A}_{i-1}^\top)^\top (\eta f_B(T)\mathbf{A}_{i-2}^\top)\|_F = \eta^2 \|\mathbf{A}_{i-1} f_B(T)^\top f_B(T)\mathbf{A}_{i-2}^\top\|_F \quad (10)$$

$$\begin{aligned} \|\mathbf{A}_{i-1} \Delta\mathbf{A}_i^\top\|_F &\approx \|(\mathbf{A}_{i-2} + \eta \mathbf{A}_{i-2} f_A(T))(\eta \mathbf{A}_{i-1} f_A(T))^\top\|_F \\ &= \eta \|\mathbf{A}_{i-2} f_A(T)^\top \mathbf{A}_{i-1}^\top + \eta \cdot \mathbf{A}_{i-2} f_A(T) f_A(T)^\top \mathbf{A}_{i-1}^\top\|_F \end{aligned} \quad (11)$$

Our insight is that the second term in Equation 11 has a smaller magnitude when learning rate is not large, since when $\eta \ll 1/L$, $\lim_{t \rightarrow \infty} \eta \|f_A(t)\| \ll 1$ (Hao et al., 2024), thus the second term is significantly smaller than the first term, and $\|\Delta\mathbf{B}_i^\top \mathbf{B}_{i-1}\|_F < \|\mathbf{A}_{i-1} \Delta\mathbf{A}_i^\top\|_F$. Hence, the update of \mathbf{B} is more orthogonal to its initialization than the update of \mathbf{A} is to its initialization. Based on the findings in Wei et al. (2025), task vectors in model merging are inherently close orthogonal to minimize interference, which indicates that in our case, merging \mathbf{B} rather than merging \mathbf{A} provides better task isolation and reduced interference, motivating our choice to perform merging \mathbf{B} . Then, for the operation of merging \mathbf{B} , we build on parameter-efficient module linear arithmetic composition, including addition and negation. Linear connectivity implies that model parameters fine-tuned from the same pretrained checkpoint can be added to improve generalization (Wortsman et al., 2022), a property that extends to PEFT adapters, whose small updates likewise allow linear composition (Zhang et al., 2023a). Hence, we can write merging operations on \mathbf{B} using task vectors

$$\mathbf{B}_{\text{merge}} = \mathbf{B}_{\text{merge}} + \lambda \cdot (\mathbf{B}_{\text{new}} - \mathbf{B}_{\text{merge}}) \quad (12)$$

This makes the foundation of the operation of merging \mathbf{B} in continual merging.

3 METHODOLOGY

3.1 SLAO: SINGLE LoRA CONTINUAL LEARNING

Based on the above analysis, we propose our method, SLAO, to utilize continual merging into a single LoRA to minimize task interference and improve generalization for CL. SLAO is motivated by four key insights: (1) *Orthogonal Retention*: To minimize forgetting error and intransigence error, it is crucial to maintain orthogonality in the LoRA components across tasks; (2) *Continual Merging*: To reduce memory usage in LoRA-based CL, continually merging new task fine-tuned LoRA updates into a single merged LoRA is a highly efficient strategy; (3) *Asymmetry of LoRA*: Given the distinct learning roles of LoRA components \mathbf{A} and \mathbf{B} , they should be handled separately; and (4) *Time-aware Scaling*: To retain prior knowledge while adapting to new tasks, the merging process for new LoRA updates should be scaled in a time-aware manner that reflects its training trajectory.

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Algorithm 1 SLAO: Single LoRA Continual Learning

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1: Initialize  $\mathbf{B}_{\text{merge}}^1 = \mathbf{B}_{\text{ft},1}$ ,  $\mathbf{A}_{\text{merge}}^1 = \mathbf{A}_{\text{ft},1}$ , scaling factor  $\lambda(1) = 1$ , number of tasks  $T$ .
2: for  $i = 2$  to  $T$  do
3:    $\mathbf{Q}_i \mathbf{R}_i = QR((\mathbf{A}_{\text{ft},i-1})^\top)$ ,  $\mathbf{Q}_i = \mathbf{Q}_i \cdot \text{sign}(\text{diag}(\mathbf{R}_i))^\top$  // Extract orthogonal basis of  $\mathbf{A}_{\text{ft},i-1}$ 
4:    $\mathbf{A}_{\text{ft},i}^{(0)} = \mathbf{Q}_i^\top, \mathbf{B}_{\text{ft},i}^{(0)} = \mathbf{B}_{\text{ft},i-1}$  // Initialize  $\mathbf{A}_{\text{ft},i}$  and  $\mathbf{B}_{\text{ft},i}$  for task  $i$ 
5:    $\mathbf{B}_{\text{ft},i} \mathbf{A}_{\text{ft},i} \leftarrow \text{fine-tune}(\mathbf{W}_0, \mathbf{B}_{\text{ft},i}^{(0)} \mathbf{A}_{\text{ft},i}^{(0)})$  // Fine-tune  $\mathbf{A}_{\text{ft},i}$  and  $\mathbf{B}_{\text{ft},i}$  for task  $i$ 
6:    $\mathbf{A}_{\text{merge}}^i = \mathbf{A}_{\text{ft},i}$ 
7:    $\mathbf{B}_{\text{merge}}^i = \mathbf{B}_{\text{merge}}^{i-1} + \lambda(i)(\mathbf{B}_{\text{ft},i} - \mathbf{B}_{\text{merge}}^{i-1})$ 
8:   Use merged LoRA  $\mathbf{B}_{\text{merge}}^i \mathbf{A}_{\text{merge}}^i$  for inference until new task comes
9: end for
10: return  $\mathbf{B}_{\text{merge}}^T \mathbf{A}_{\text{merge}}^T$ 

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The SLAO consists of two main operations: (1) Initialize each new task learning LoRA by extracted orthogonal basis from the previous task’s fine-tuned LoRA; (2) After fine-tuning on the new task, we utilize the asymmetry of LoRA components to employ adaptive time-varying scaling for new LoRA updates to merge into the merged LoRA. The complete procedure is outlined in Algorithm 1 and illustrated in Figure 4. Starting with the first task’s fine-tuned LoRA $\mathbf{B}_{\text{ft},1} \mathbf{A}_{\text{ft},1}$ by standard fine-tuning, our method iteratively integrates LoRA updates of subsequent tasks in a continual fashion.

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Orthogonal basis extraction for initialization. For new task i , we first extract orthogonal basis of previous fine-tuned $\mathbf{A}_{\text{ft},i-1}$, use that to initialize $\mathbf{A}_{\text{ft},i}^{(0)}$, making $\mathbf{A}_{\text{ft},i}^{(0)} (\mathbf{A}_{\text{ft},i}^{(0)})^\top = \mathbf{I}_r$. We utilize QR decomposition to extract orthogonal matrix from $\mathbf{A}_{\text{ft},i-1}$, which is:

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$$\mathbf{Q}_i \mathbf{R}_i = QR((\mathbf{A}_{\text{ft},i-1})^\top) \rightarrow \mathbf{Q}_i = \mathbf{Q}_i \cdot \text{sign}(\text{diag}(\mathbf{R}_i))^\top \rightarrow \mathbf{A}_{\text{ft},i}^{(0)} = \mathbf{Q}_i^\top \quad (13)$$

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As a result, the initialization $\mathbf{A}_{\text{ft},i}^{(0)}$ has orthogonal rows. For \mathbf{B} , we directly initialize $\mathbf{B}_{\text{ft},i}^{(0)}$ by $\mathbf{B}_{\text{ft},i-1}$ which is the fine-tuned \mathbf{B} of previous task $i-1$.

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Asymmetrically merging LoRA via time-aware scaling. After fine-tuning task i , we merge its LoRA updates into $\{\mathbf{B}_{\text{merge}}^{i-1}, \mathbf{A}_{\text{merge}}^{i-1}\}$. Due to the intrinsic asymmetry of \mathbf{B} and \mathbf{A} in LoRA, we update $\mathbf{A}_{\text{merge}}^i = \mathbf{A}_{\text{ft},i}$, and we merge \mathbf{B} by time-aware coefficient $\lambda(i)$ for new task updates:

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$$\mathbf{B}_{\text{merge}}^i = \mathbf{B}_{\text{merge}}^{i-1} + \lambda(i) \cdot (\mathbf{B}_{\text{ft},i} - \mathbf{B}_{\text{merge}}^{i-1}) \quad (14)$$

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where $\lambda(i)$ is introduced to maintain a consistent magnitude of the merged \mathbf{B} ’s deviation from previous tasks throughout the merging process. In our method, the scaling factor can be set to $\lambda(i) = \frac{1}{\sqrt{i}}$, which follows the continual merging method proposed in Tang et al. (2025). The findings in Ilharco et al. (2023); Tang et al. (2024) indicate that task vectors from different tasks tend to be approximately orthogonal, and since \mathbf{B} s across tasks are approximately orthogonal to each other, as shown in Figure 1, which indicates that \mathbf{B} task vectors are approximately orthogonal. This orthogonality makes $\lambda(i) = \frac{1}{\sqrt{i}}$ a natural choice for the scaling factor, since it helps maintain the magnitude of parameter changes across merging steps Tang et al. (2025).

291

3.2 DYNAMICS OF SLAO

292

To better understand the effectiveness of SLAO, inspired by the analysis of Hao et al. (2024), in the following theorem we analyze the dynamics of task-specific parameters’ update in CL scenario.

293

Theorem 1. Let the parameters \mathbf{A} and \mathbf{B} be updated using SGD at each step s for task i as follows:

294

$$\mathbf{A}_i^{s+1} = \mathbf{A}_i^s - \eta (\mathbf{B}_i^s)^\top (\nabla_{\mathbf{W}} \mathcal{L}_i^s), \quad \mathbf{B}_i^{s+1} = \mathbf{B}_i^s - \eta (\nabla_{\mathbf{W}} \mathcal{L}_i^s) (\mathbf{A}_i^s)^\top \quad (15)$$

295

where η is the learning rate. We assume $\mathbf{A}_i^s = \mathbf{A}_i^{(0)} + \eta \mathbf{A}_i^{(0)} f_A(s)$ and $\mathbf{B}_i^s = \mathbf{B}_i^{(0)} + \eta f_B(s) (\mathbf{A}_i^{(0)})^\top$ holds with such functions f_A and f_B for $1, \dots, s$, and $\|\sum_{s=1}^S \nabla_{\mathbf{W}} \mathcal{L}_i^{(s)}\|_F \leq L$ for every S during training task i , which implies that the model stays within a finite Euclidean ball. If we assume $\mathbf{A}_i^{(0)} (\mathbf{A}_i^{(0)})^\top = \mathbf{I}_r$, in this case, the dynamics of \mathbf{A}_i satisfies $\|f_A(s)\|_2 \leq \frac{\eta L^2 (1 - (\eta^2 L^2)^s)}{1 - \eta^2 L^2}$, and the dynamics of \mathbf{B} satisfies $f_B(s) = -\sum_{j=0}^{s-1} (\nabla_{\mathbf{W}} \mathcal{L}_i^j) (\eta f_A^\top(j) + \mathbf{I})$. When η is small, we have

324 $f_B(s) \approx -\sum_{j=0}^{s-1} (\nabla_{\mathbf{W}} \mathcal{L}_i^j)$. Thus $B_i^S = \eta f_B(S)(\mathbf{A}_i^{(0)})^\top$, and total update for \mathbf{B}_i is $\Delta \mathbf{B}_i =$
 325 $-\eta (\sum_{s=0}^S (\nabla_{\mathbf{W}} \mathcal{L}_i^s))(\mathbf{A}_i^{(0)})^\top$.
 326

327 The proof is deferred to Appendix C. This analysis, under the orthogonal initialization of \mathbf{A} , suggests
 328 that \mathbf{B} may update across different initialization subspaces, effectively increasing the rank of \mathbf{B} and
 329 thereby aiding generalization. We note that the key difference between ours and Hao et al. (2024) lies
 330 in the initialization of LoRA: while they use standard initialization with $\mathbf{B}_i^{(0)} = \mathbf{0}$, we initialize \mathbf{B}
 331 using the previously fine-tuned LoRA parameters, resulting in $\mathbf{B}_i^{(0)} \neq \mathbf{0}$, complicating the analysis.
 332

334 4 EXPERIMENTS

336 4.1 EXPERIMENTAL SETUP

338 **Models and datasets.** We evaluate our approach across three Llama models: Llama-2-7B-chat,
 339 Llama-2-13B-chat, and Llama-3-2-3B, and two Qwen models: Qwen2.5-3B and Qwen2.5-7B. All
 340 experiments are conducted on NVIDIA A100 GPUs utilizing DeepSpeed repository. We consider
 341 three continual learning benchmarks: (1) **Standard CL benchmark**: AG News, Amazon, Reviews,
 342 Yelp Reviews, DBpedia, and Yahoo Answers. (2) **Large number of tasks**: five standard CL bench-
 343 mark tasks, four GLUE tasks (MNLI, QQP, RTE, SST-2), five SuperGLUE tasks (WiC, CB, COPA,
 344 MultiRC, BoolQ), and IMDB movie reviews. Following O-LoRA (Wang et al., 2023), each task
 345 uses 1000 randomly sampled training samples and 500 validation samples per class. (3) **SuperNI**
 346 **Benchmark** (Wang et al., 2022a): A diverse collection of NLP tasks with expert-written instructions,
 347 covering dialogue generation, information extraction, question answering, summarization, and senti-
 348 ment analysis. We follow task selection and ordering in SAPT (Zhao et al., 2024b), using 1,000
 349 training instances and 100 for validation/testing per task.

350 **Baselines.** We compare our method SLAO with the following baselines: (1) *Continual learning*
 351 *baselines*: SeqLoRA: sequentially fine-tunes a single LoRA on multiple tasks without con-
 352 straints; IncLoRA: incrementally adds a new LoRA per task while freezing previous LoRAs; O-
 353 LoRA (Wang et al., 2023); InfLoRA (Liang & Li, 2024); SAPT-LoRA (Zhao et al., 2024b); MTL:
 354 a single model is trained jointly on all tasks; LoRM (Salami et al., 2025); CorDA (knowledge-
 355 preserved adaptation) (Yang et al., 2024); Magmax (Marczak et al., 2024). (2) *LoRA merging*
 356 *baselines*: LoRA-LEGO (Zhao et al., 2025); KnOTS (Stoica et al., 2025). (3) *Continual merging*
 357 *baseline*: OPCM (Tang et al., 2025). To fairly evaluate existing merging methods in LoRA-based
 358 continual learning, we extend full-model merging methods to LoRA and equally treat components of
 359 LoRA, and all merging methods are achieved sequentially.

360 **Evaluation metrics.** To evaluate our proposed approach, we employ three key metrics: (1) av-
 361 erage accuracy (AA), calculated as the mean accuracy across all tasks after training on the last
 362 task: $\frac{1}{T} \sum_{i=1}^T a_{i,T}$, where $a_{i,T}$ is accuracy for classification tasks and Rouge-L for other tasks; (2)
 363 backward transfer (BWT) (Lin et al., 2022), defined as $\frac{1}{T-1} \sum_{i=1}^{T-1} (a_{i,T} - a_{i,i})$, and experimental
 364 results are shown in Appendix E.4; (3) maximum order-normalized performance disparity (MOPD)
 365 and average order-normalized performance disparity (AOPD) (Yoon et al., 2020), which evaluate
 366 order-robustness, and experimental results are shown in Appendix E.5.

367 4.2 OVERALL RESULTS

368 **Continual learning performance results analysis** As shown in Table 1, our method consistently
 369 outperforms all data-free baselines across three benchmarks using Llama-2-7B-chat. **LoRA-Based**
 370 **continual learning**: SeqLoRA performs worst among LoRA-based methods, as unconstrained
 371 continual fine-tuning on a single LoRA causes severe forgetting. IncLoRA improves by freezing prior
 372 learned LoRAs to isolate subspaces, though its subspace separation is simple. InfLoRA outperforms
 373 O-LoRA in standard CL benchmark due to orthogonal input-based subspaces, but drops on large
 374 number of tasks and SuperNI benchmark, due to sensitivity to manually tuned DualGPM threshold.
 375 SAPT-LoRA achieves the highest average performance among LoRA-based methods, but relies on
 376 generated previous task pseudo samples, unrealistic in many LLM scenarios, and is more order-
 377 sensitive than ours. LoRM-BA (begin with freezing \mathbf{B}) and LoRM-AB yield nearly identical results,
 378 suggesting that the freezing order of LoRA components in CL matters little. CorDA performs

Table 1: Testing performance (%) on three CL benchmarks using Llama-2-7B-chat across different task orders, where each result is run three random times, where O_i denotes i th task order.

	Standard CL Benchmark				Large Number of Tasks				SuperNI Benchmark		
Method	O1	O2	O3	avg	O4	O5	O6	avg	O1	O2	avg
SeqLoRA	73.3	76.2	78.4	76.0	69.1	66.0	71.1	68.7	18.4	26.8	22.6
IncLoRA	75.3	77.3	78.3	77.0	72.2	71.6	73.8	72.5	22.0	25.6	23.8
O-LoRA	76.1	76.3	79.2	77.2	74.0	72.0	74.6	73.5	23.3	28.4	25.9
InfLoRA	78.4	80.4	79.9	79.6	69.4	67.4	72.5	69.8	16.5	22.1	19.3
SPAT-LoRA	82.9	81.8	78.7	81.1	84.7	78.9	82.2	81.9	53.2	48.5	50.9
LoRM-BA	76.0	76.8	78.3	77.0	71.4	69.0	70.3	70.2	25.6	18.7	22.2
LoRM-AB	77.5	74.7	75.9	76.0	71.0	69.5	70.2	70.2	25.6	23.7	24.7
CorDA	78.4	79.3	80.0	79.2	73.4	72.7	74.0	73.4	20.9	16.0	18.5
MagMax	80.1	80.6	80.3	80.3	72.3	73.5	74.5	73.4	15.3	7.0	11.2
KnOTS	67.9	65.9	70.8	68.2	61.5	60.1	58.0	59.9	34.6	30.1	32.4
LoRA-LEGO	68.3	66.0	70.9	68.4	58.8	58.7	53.2	56.9	32.8	26.7	29.8
OPCM	61.9	62.0	56.7	60.2	51.9	52.8	46.9	50.5	11.6	12.3	12.0
SLAO(ours)	80.1	80.8	80.4	80.4	75.0	74.4	75.1	74.8	38.7	35.7	37.2
Multi-Task	80.9				78.1				45.2		

Table 2: Comparison of initialization strategies on testing performance across three standard CL benchmarks using Llama-2-7B-chat under different task orders, where O_i denotes i th task order.

	Standard CL Benchmark				Large Number of Tasks				SuperNI Benchmark		
Initialization	O1	O2	O3	avg	O4	O5	O6	avg	O1	O2	avg
Random (Zero)	66.4	62.4	68.4	65.7	61.4	60.3	57.2	59.6	33.3	28.9	31.1
Last-Merge	80.1	80.8	80.1	80.3	74.7	72.8	75.0	74.2	37.4	30.5	34.0
Last-FT (ours)	80.1	80.8	80.4	80.4	75.0	74.4	75.1	74.8	38.7	35.7	37.2

well on standard CL benchmark and large number of tasks, but drops significantly on SuperNI benchmark, likely due to relying on nullspace selection from pretrained models and lacking time-aware merging. **MagMax** performs comparably to ours on standard CL benchmark, slightly worse on large number of tasks, but underperforms on SuperNI benchmark, where task similarity is lower, thus only keeping weights which have the largest absolute value would cause forgetting. **LoRA merging baselines:** KnOTS and LoRA-LEGO perform similarly in the standard CL benchmark, but KnOTS outperforms in the large number of tasks and SuperNI. KnOTS may benefit from flexible SVD-merging mechanism so that we apply time-aware scaling on merging, while LoRA-LEGO treats tasks equally, lacks prioritization, and is ineffective in complex CL contexts. **Continual merging approaches:** since OPCM is designed for full model, directly applying it to LoRA by treating its two components identically leads to suboptimal performance. The results under Llama-2-13B-chat, Qwen2.5-3B, and Qwen2.5-7B are shown in Appendix E.

Impact of initialization strategies. We compare three different initialization strategies for learning new tasks: (1) random (zero) initialization, (2) initialization from last merging point, (3) initialization from last fine-tuning point (ours). As shown in Table 2, initializing from last fine-tuning point consistently outperforms other two strategies across all three benchmarks. Using last merging point performs slightly worse, while random (zero) initialization performs the worst. The performance gap is due to how initialization affects LoRA’s learning trajectory and merging way. Random initialization places A far away from optimal task-specific A^* , making intransigence worse. Initialization from last merging point fixes time coefficients after merging back to a single LoRA, limiting its flexibility, while initialization from last fine-tuning point allows the merged LoRA to implicitly reweight previous tasks’ updates when merging. This adaptive adjustment yields better CL performance.

432 Table 3: Comparison of merging strategies on testing performance on three standard CL benchmarks
 433 using Llama-2-7B-chat across different task orders, where O_i denotes i th task order.

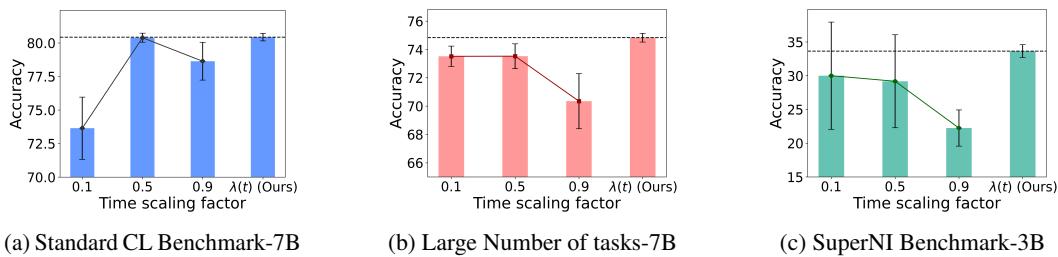
Merging	Standard CL Benchmark				Large Number of Tasks				SuperNI Benchmark		
	O1	O2	O3	avg	O4	O5	O6	avg	O1	O2	avg
FREB-MA	77.7	78.0	76.2	77.3	70.8	66.1	72.1	69.7	13.9	25.4	19.7
FREA-MB	78.7	79.3	78.0	78.7	72.3	73.0	73.5	72.9	23.7	29.2	26.5
FTBA-MA	76.2	79.0	79.9	78.4	71.6	69.5	74.1	71.7	21.1	30.7	25.9
FTBA-MBA	79.7	80.4	80.2	80.1	73.2	73.9	74.5	73.9	33.8	32.7	33.3
FTBA-MB	79.3	80.8	80.1	80.1	74.1	74.0	74.8	74.3	32.4	35.2	33.8
SLAO (ours)	80.1	80.8	80.4	80.4	75.0	74.4	75.1	74.8	38.7	35.7	37.2

445 Table 4: Comparison of model variants and sizes on testing performance across three standard CL
 446 benchmarks using Llama-2-7B-chat in different task orders, where O_i denotes i th task order.

Model	Standard CL Benchmark				Large Number of Tasks				SuperNI Benchmark		
	O1	O2	O3	avg	O4	O5	O6	avg	O1	O2	avg
Llama-3-2-3B	74.3	75.8	75.3	75.1	73.3	72.5	74.9	73.6	32.7	34.6	33.7
Llama-2-7B-chat	80.1	80.8	80.4	80.4	75.0	74.4	75.1	74.8	38.7	35.7	37.2
Llama-2-13B-chat	80.8	81.1	81.1	81.0	76.5	75.9	76.0	76.1	42.3	42.2	42.3

456 **Asymmetry in LoRA merging.** To investigate the asymmetry in LoRA merging, we compare
 457 different continual merging strategies for LoRA: (1) Freeze A Merge B (FREA-MB), (2) Freeze
 458 B Merge A (FREB-MA), (3) Fine-tune BA Merge A (FTBA-MA), (4) Fine-tune BA Merge BA
 459 (FTBA-MBA), (5) Fine-tune BA Merge B (FTBA-MB). As shown in Table 3, only FTBA-MB
 460 consistently outperforms other strategies, except ours. FTBA-MBA has comparable performance
 461 compared to FTBA-MB, but FTBA-MA yields the poorest performance among fine-tuning LoRA
 462 methods. When freezing one component of LORA, FREA-MB is better than FREB-MA, consistent
 463 with Zhu et al. (2024b) that freezing A and fine-tuning B is at least better than the reverse. It
 464 highlights asymmetry in LoRA components and the importance of asymmetric merging based on
 465 their fundamental roles in adaptation.

466 **Effect of model variants and sizes.** We evaluate our method on three LLM variants: Llama-2-7B-
 467 chat, Llama-2-13B-chat, Llama-3-3B. The results in Table 4 indicate that model variants and model
 468 size play crucial roles in average performance across different task orders and benchmarks. Llama-
 469 3-3B performs the worst, while in same generation of Llama, larger models consistently achieve
 470 better accuracy: Llama-2-13B-chat relatively has higher accuracy compared to Llama-2-7B-chat.
 471 This trend suggests that increased model capacity enhances both reducing catastrophic forgetting
 472 and improving generalization in continual learning. Moreover, we observe that larger models exhibit
 473 greater robustness to task order variations compared to smaller models.



482 Figure 3: Comparison of model performance across time coefficients.

483 **Time-varying coefficient analysis.** To evaluate impact of adaptive time-varying scaling in continual
 484 merging for CL, we compare it against fixed factors $\{0.1, 0.5, 0.9\}$ on three benchmarks via Llama-
 485 2-7B-chat and Llama-3-2-3B. As shown in Figure 3, adaptive strategy consistently achieves highest

486 average accuracy with lower variance across task orders and models. For simpler standard CL
 487 benchmark, larger fixed value 0.9 outperforms smaller one 0.1, while for more complex or long
 488 benchmarks, smaller values perform better; 0.5 is relatively stable but consistently suboptimal.
 489

490 **5 CONCLUSION**
 491

492 In this work, we proposed a novel parameter-efficient continual learning based on continual merging
 493 of LoRA, enabling no additional training or access to any data representations. Our approach
 494 leverages the orthogonal basis from previous fine-tuned LoRA to initialize for new task learning and
 495 constructs a single shared merged LoRA via time-aware scaling, thus ensuring constant memory usage
 496 regardless of task number. Through comprehensive experiments, we demonstrated the effectiveness
 497 and efficiency of our method across multiple benchmarks and model scales.
 498

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702 **A APPENDIX**
703704 The appendix is organized as follows:
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707
- 708 • Appendix C provides theoretical analysis.
709
- 710 • Appendix D provides the overview of SLAO.
711
- 712 • Appendix E provides more experimental setup details and additional experimental results.
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738
- 739 • Appendix F provides related works.
740

741 **B THE USE OF LARGE LANGUAGE MODELS (LLMs)**
742743 We utilize LLMs to polish the paper writing.
744745 **C THEORETICAL ANALYSIS**
746747 **C.1 THE FORGETTING-INTRANSIGENCE ERROR DECOMPOSITION UNDER NTK**
748

749 **Lemma 1.** (Jang *et al.*, 2024; Maurer, 2016) Assume \mathcal{D} is i.i.d N random samples sampled from
750 probability distribution \mathcal{P} . Let $\mathcal{A}_D = \{\mathbf{X}_i \rightarrow f_{\mathbf{W}_0}(\mathbf{X}_i) + \langle \nabla_{\mathbf{W}} f_{\mathbf{W}_0}(\mathbf{X}_i), \boldsymbol{\delta} \rangle \in \mathbb{R}^K : \|\boldsymbol{\delta}\|_* \leq D, \boldsymbol{\delta} \in \mathbb{R}^{m \times n}\}$ is class of affine predictors with bounded nuclear norm D . For $1 \leq j \leq K$, suppose
751 $\|\nabla_{\mathbf{W}} f_{\mathbf{W}_0}^{(j)}(\mathbf{X})\|_F \leq R$ almost surely with respect to the random data $\mathbf{X}_i \sim \mathcal{P}$. For $1 \leq i \leq N$,
752 suppose $\ell_i = \ell(\cdot, \mathbf{Y}_i)$ is G -Lipschitz continuous on \mathcal{A} on the first argument (with respect to the
753 Euclidean norm) for almost surely with respect to the random data $\mathbf{X}_i \subseteq \mathcal{D} \sim \mathcal{P}$. That is
754

$$755 |\ell_i(a(\mathbf{X}_1)) - \ell_i(a'(\mathbf{X}_2))| \leq G\|a(\mathbf{X}_1) - a'(\mathbf{X}_2)\|_2 \quad \text{for any } a, a' \in \mathcal{A}, \mathbf{X}_1, \mathbf{X}_2 \subseteq \mathcal{D} \sim \mathcal{P} \quad (16)$$

756 *Proof.* First, let $g : \mathcal{X} \rightarrow \mathbb{R}$ be a function satisfying the following property with $c > 0$:
757

$$758 |g(\mathbf{X}_1, \dots, \mathbf{X}_{i-1}, \mathbf{X}_i, \mathbf{X}_{i+1}, \dots, \mathbf{X}_N) - g(\mathbf{X}_1, \dots, \mathbf{X}_{i-1}, \mathbf{X}'_i, \mathbf{X}_{i+1}, \dots, \mathbf{X}_N)| \leq c \quad (17)$$

759 for all $\mathbf{X}_1, \dots, \mathbf{X}_N, \mathbf{X}'_i \in \mathcal{X}$. Then, for all $\epsilon > 0$,

$$760 \mathbb{P}(|g(\mathbf{X}_1, \dots, \mathbf{X}_N) - \mathbb{E}[g(\mathbf{X}_1, \dots, \mathbf{X}_N)]| \geq \epsilon) \leq \exp\left(-\frac{2\epsilon^2}{Nc^2}\right) \quad (18)$$

761 Take g to be $g = \sup_{\|\boldsymbol{\delta}\|_* \leq D} (\hat{\mathcal{L}}(\boldsymbol{\delta}_0) - \hat{\mathcal{L}}(\boldsymbol{\delta}) - \mathcal{L}(\boldsymbol{\delta}_0) + \mathcal{L}(\boldsymbol{\delta}))$, which is a function of $\mathbf{X}_1, \dots, \mathbf{X}_N$.
762 Since $\|\boldsymbol{\delta}\|_* \leq D$ implies $\|\boldsymbol{\delta}\|_F \leq D$ and by the Lipschitz continuity of $\ell(\cdot, \mathbf{Y}_i)$, we have the following
763 for any $(\mathbf{X}_i, \mathbf{Y}_i) \in \mathcal{D}$:

$$764 |\ell(f_{\mathbf{W}_0}(\mathbf{X}_i) + \langle \mathbf{G}(\mathbf{X}_i), \boldsymbol{\delta}_0 \rangle, \mathbf{Y}_i) - \ell(f_{\mathbf{W}_0}(\mathbf{X}_i) + \langle \mathbf{G}(\mathbf{X}_i), \boldsymbol{\delta} \rangle, \mathbf{Y}_i)| \leq G\|\langle \boldsymbol{\delta}_0 - \boldsymbol{\delta}, \mathbf{G}(\mathbf{X}_i) \rangle\|_2$$

$$\begin{aligned}
&\leq G \sqrt{\sum_{j=1}^K \|\delta_0 - \delta\|_F^2 \|\mathbf{G}^{(j)}(\mathbf{X}_i)\|_F^2} \\
&\leq G \sqrt{\sum_{j=1}^K \|\delta_0 - \delta\|_*^2 \|\mathbf{G}^{(j)}(\mathbf{X}_i)\|_F^2} \\
&\leq G \sqrt{\sum_{j=1}^K 4D^2 \cdot R^2} \\
&= 2GRD\sqrt{K}.
\end{aligned} \tag{19}$$

□

Thus, from this lemma, we apply it to LoRA-based continual learning and have

$$\begin{aligned}
\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_t \mathbf{A}_t) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i) &\leq G \|\langle \mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i, \nabla_{\mathbf{W}} f_{\mathbf{W}_0}(\mathbf{X}_i) \rangle\|_2 \\
&\leq G \sqrt{\sum_{j=1}^K \|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F^2 \|\nabla_{\mathbf{W}} f_{\mathbf{W}_0}^{(j)}(\mathbf{X}_i)\|_F^2} \\
&\leq G \sqrt{\sum_{j=1}^K \|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F^2 R^2}
\end{aligned} \tag{20}$$

where K is the output dimension. From this, we can see that to minimize forgetting error, we should make $\|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F$ as small as possible. Similarly, to minimize intransigence error, $\|\mathbf{B}_i \mathbf{A}_i - \mathbf{B}_i^* \mathbf{A}_i^*\|_F$ should be also small. Besides, it's evident that $\mathcal{L}_t(\mathbf{W}_0 + \mathbf{B}_t \mathbf{A}_t) - \mathcal{L}_t(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i) = 0$. Thus, if we makes $\|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F \leq D$ and $\|\mathbf{B}_i \mathbf{A}_i - \mathbf{B}_i^* \mathbf{A}_i^*\|_F \leq D$, then we have

$$\begin{aligned}
&\mathcal{F}_t + \mathcal{I}_t \\
&= \sum_{i=1}^{t-1} (\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_t \mathbf{A}_t) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i)) + \sum_{i=1}^t (\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i^* \mathbf{A}_i^*)) \\
&= \sum_{i=1}^t (\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_t \mathbf{A}_t) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i)) + \sum_{i=1}^t (\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i^* \mathbf{A}_i^*)) \\
&= \sum_{i=1}^t (\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_t \mathbf{A}_t) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i)) + (\mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i \mathbf{A}_i) - \mathcal{L}_i(\mathbf{W}_0 + \mathbf{B}_i^* \mathbf{A}_i^*)) \\
&\leq \sum_{i=1}^t G \sqrt{\sum_{j=1}^K \|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F^2 R^2} + G \sqrt{\sum_{j=1}^K \|\mathbf{B}_i \mathbf{A}_i - \mathbf{B}_i^* \mathbf{A}_i^*\|_F^2 R^2} \\
&= GR \sum_{i=1}^t \sqrt{\sum_{j=1}^K \|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F^2} + \sqrt{\sum_{j=1}^K \|\mathbf{B}_i \mathbf{A}_i - \mathbf{B}_i^* \mathbf{A}_i^*\|_F^2} \\
&\leq 4DGR \sum_{i=1}^t \sqrt{K}
\end{aligned} \tag{21}$$

To make both $\|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F$ and $\|\mathbf{B}_i \mathbf{A}_i - \mathbf{B}_i^* \mathbf{A}_i^*\|_F$ minimized, the forgetting-intransigence decomposition can be written as:

$$\begin{aligned}
&\|\mathbf{B}_t \mathbf{A}_t - \mathbf{B}_i \mathbf{A}_i\|_F + \|\mathbf{B}_i \mathbf{A}_i - \mathbf{B}_i^* \mathbf{A}_i^*\|_F \\
&\leq \|\mathbf{B}_t(\mathbf{A}_t - \mathbf{A}_i)\|_F + \|(\mathbf{B}_t - \mathbf{B}_i)\mathbf{A}_i\|_F + \|\mathbf{B}_i(\mathbf{A}_i - \mathbf{A}_i^*)\|_F + \|(\mathbf{B}_i - \mathbf{B}_i^*)\mathbf{A}_i^*\|_F
\end{aligned} \tag{22}$$

Algorithmic motivation. From the above bound, we observe that generalization error in CL with LoRA depends asymmetrically on the choice of frozen and trainable components. Interestingly, this insight contrasts with standard fine-tuning practices. For example, as concluded in (Zhu et al., 2024b), freezing \mathbf{A} and fine-tuning \mathbf{B} is at least as effective, if not better, than the reverse. However, if we apply freezing \mathbf{A} in CL, it implies $\|\mathbf{A}_t - \mathbf{A}_i\|_F = 0$, which may unintentionally increase $\|\mathbf{A}_i - \mathbf{A}_i^*\|_F$ due to limited task-specific expressiveness. In Hu et al. (2022), \mathbf{A} is initialized to a random Gaussian matrix satisfying $\mathbb{E}[\mathbf{A}^{(0)}(\mathbf{A}^{(0)})^\top] = \mathbf{I}_r$. Instead, if we propose to fine-tune \mathbf{A}_i

810 while extracting orthogonal basis \mathbf{Q}_{i-1} from \mathbf{A}_{i-1} , where $\mathbf{Q}_{i-1}\mathbf{Q}_{i-1}^\top = \mathbf{I}_r$, to initialize $\mathbf{A}_i^{(0)}$ via
 811 \mathbf{Q}_{i-1} , then we will have $\mathbf{A}_i^{(0)}(\mathbf{A}_i^{(0)})^\top = \mathbf{I}_r$ where $i \in [1, \dots, t]$. This orthonormal structure not
 812 only keeps geometric consistency across tasks but also allows \mathbf{A}_j ($t \geq j > i$), to remain well-aligned
 813 with previous \mathbf{A}_i , i.e. $\mathbb{E}[\mathbf{A}_j \mathbf{A}_i^\top] \approx \mathbf{I}_r$, thereby minimizing both $\|\mathbf{A}_t - \mathbf{A}_i\|_F$ and $\|\mathbf{A}_i - \mathbf{A}_i^*\|_F$.
 814 This motivates our design of continual merging with orthogonal initialization to reduce forgetting and
 815 maintain adaptability.
 816

817 C.2 DYNAMICS OF LOW-RANK ADAPTERS UPDATED BY SLAO

818 We analyze the dynamics of \mathbf{A}_i^s and \mathbf{B}_i^s in continual learning setting. This analysis, under the
 819 orthogonal initialization of \mathbf{A} , suggests that \mathbf{B} may update across different initialization subspaces,
 820 effectively increasing the rank of \mathbf{B} and thereby aiding generalization.
 821

822 **Theorem 2.** *Let the parameters \mathbf{A} and \mathbf{B} be updated using SGD at each step s for task i as follows:*

$$824 \quad \mathbf{A}_i^{s+1} = \mathbf{A}_i^s - \eta(\mathbf{B}_i^s)^\top(\nabla_{\mathbf{W}} \mathcal{L}_i^s), \quad \mathbf{B}_i^{s+1} = \mathbf{B}_i^s - \eta(\nabla_{\mathbf{W}} \mathcal{L}_i^s)(\mathbf{A}_i^s)^\top \quad (23)$$

825 where η is the learning rate. We assume $\mathbf{A}_i^s = \mathbf{A}_i^{(0)} + \eta \mathbf{A}_i^{(0)} f_A(s)$ and $\mathbf{B}_i^s = \mathbf{B}_i^{(0)} + \eta f_B(s)(\mathbf{A}_i^{(0)})^\top$
 826 holds with such functions f_A and f_B for $1, \dots, s$, and $\|\sum_{s=1}^S \nabla_{\mathbf{W}} \mathcal{L}_i^{(s)}\|_F \leq L$ for every S during
 827 training task i , which implies that the model stays within a finite Euclidean ball. If we assume
 828 $\mathbf{A}_i^{(0)}(\mathbf{A}_i^{(0)})^\top = \mathbf{I}_r$, in this case, the dynamics of \mathbf{A}_i satisfies $\|f_A(s)\|_2 \leq \frac{\eta L^2(1 - (\eta^2 L^2)^s)}{1 - \eta^2 L^2}$, and
 829 the dynamics of \mathbf{B} satisfies $f_B(s) = -\sum_{j=0}^{s-1}(\nabla_{\mathbf{W}} \mathcal{L}_i^j)(\eta f_A^\top(j) + \mathbf{I})$. When η is small, we have
 830 $f_B(s) \approx -\sum_{j=0}^{s-1}(\nabla_{\mathbf{W}} \mathcal{L}_i^j)$. Thus $\mathbf{B}_i^S = \eta f_B(S)(\mathbf{A}_i^{(0)})^\top$, and total update for \mathbf{B}_i is $\Delta \mathbf{B}_i =$
 831 $-\eta \left(\sum_{s=0}^S (\nabla_{\mathbf{W}} \mathcal{L}_i^s) \right) (\mathbf{A}_i^{(0)})^\top$.
 832
 833

834
 835 *Proof.* We start by noting the fact that for **Task 1**, when $s = 0$, $f_A(0) = f_B(0) = 0$. For $s > 0$,
 836 assume $\mathbf{A}_1^s = \mathbf{A}_0 + \eta \mathbf{A}_0 f_A(s)$ and $\mathbf{B}_1^s = \eta f_B(s) \mathbf{A}_0^\top$. Since the first task training is the same as the
 837 LoRA fine-tuning in Hao et al. (2024) for the dynamics of \mathbf{A} and \mathbf{B} we have:
 838

$$839 \quad \begin{aligned} \mathbf{A}_1^{s+1} &= \mathbf{A}_1^s - \eta(\mathbf{B}_1^s)^\top(\nabla_{\mathbf{W}_0} \mathcal{L}_1^s) \\ &= \mathbf{A}_0 + \eta \mathbf{A}_0 f_A(s) - \eta^2 \mathbf{A}_0 f_B^\top(s)(\nabla_{\mathbf{W}_0} \mathcal{L}_1^s) \\ &= \mathbf{A}_0 + \eta \mathbf{A}_0 f_A(s+1) \end{aligned} \quad (24)$$

842 and

$$844 \quad \begin{aligned} \mathbf{B}_1^{s+1} &= \mathbf{B}_1^s - \eta(\nabla_{\mathbf{W}_0} \mathcal{L}_1^s)(\mathbf{A}_1^s)^\top \\ &= \eta f_B(s+1) \mathbf{A}_0^\top \end{aligned} \quad (25)$$

847 Thus, by rearranging the terms, we have:

$$848 \quad f_A(s) = -\eta \sum_{j=0}^{s-1} f_B^\top(j)(\nabla_{\mathbf{W}_0} \mathcal{L}_1^j) \quad (26)$$

$$852 \quad f_B(s) = -\sum_{j=0}^{s-1} (\nabla_{\mathbf{W}_0} \mathcal{L}_1^j)(\eta f_A^\top(j) + \mathbf{I}) \quad (27)$$

855 Since $\mathbf{W}_0 + \mathbf{B}\mathbf{A} \approx \mathbf{W}_0 + \Delta \mathbf{B}\mathbf{A}_0$ when learning rate η is small, the change in \mathbf{B} dominates the
 856 final weight update. Thus, if freezing \mathbf{A} , we obtain

$$857 \quad \Delta \mathbf{B}_1 \approx -\eta \left(\sum_{s=1}^S \nabla_{\mathbf{W}_0} \mathcal{L}_1^s \right) \mathbf{A}_0^\top \quad (28)$$

860 and

$$862 \quad f_B(s) \approx -\sum_{j=0}^{s-1} \nabla_{\mathbf{W}_0} \mathcal{L}_1^j \quad (29)$$

864 Task i ($i > 1$): when $s = 0$, $f_A(0) = f_B(0) = 0$.

865 For $s > 0$: Assume $\mathbf{A}_i^s = \mathbf{A}_i^{(0)} + \eta \mathbf{A}_i^{(0)} f_A(s)$ and $\mathbf{B}_i^s = \mathbf{B}_i^{(0)} + \eta f_B(s) (\mathbf{A}_i^{(0)})^\top$ hold with such
866 functions f_A and f_B for $1, \dots, s$. Then, for $s + 1$, we have

$$\begin{aligned} 868 \quad \mathbf{A}_i^{s+1} &= \mathbf{A}_i^s - \eta (\mathbf{B}_i^s)^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) \\ 869 \quad &= \mathbf{A}_i^{(0)} + \eta \mathbf{A}_i^{(0)} f_A(s) - \eta (\mathbf{B}_i^{(0)} + \eta f_B(s) (\mathbf{A}_i^{(0)})^\top)^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) \\ 870 \quad &= \mathbf{A}_i^{(0)} + \eta \mathbf{A}_i^{(0)} f_A(s) - \eta ((\mathbf{B}_i^{(0)})^\top + \eta \mathbf{A}_i^{(0)} f_B^\top(s)) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) \\ 871 \quad &= \mathbf{A}_i^{(0)} + \eta \mathbf{A}_i^{(0)} (f_A(s) - \eta f_B^\top(s) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s)) - \eta (\mathbf{B}_i^{(0)})^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) \end{aligned} \quad (30)$$

872 We would like to express \mathbf{A}_i^{s+1} as
873

$$\mathbf{A}_i^{s+1} = \mathbf{A}_i^{(0)} + \eta \mathbf{A}_i^{(0)} f_A(s + 1) \quad (31)$$

874 So compare both sides:
875

$$\eta \mathbf{A}_i^{(0)} f_A(s + 1) = \eta \mathbf{A}_i^{(0)} (f_A(s) - \eta f_B^\top(s) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s)) - \eta (\mathbf{B}_i^{(0)})^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) \quad (32)$$

876 Divide both sides by η , rearrange:
877

$$\mathbf{A}_i^{(0)} f_A(s + 1) = \mathbf{A}_i^{(0)} (f_A(s) - \eta f_B^\top(s) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s)) - (\mathbf{B}_i^{(0)})^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) \quad (33)$$

878 Since by our initialization $\mathbf{A}_i^{(0)} (\mathbf{A}_i^{(0)})^\top = \mathbf{I}_r$, then we have
879

$$\mathbf{A}_i^{(0)} f_A(s + 1) = \mathbf{A}_i^{(0)} (f_A(s) - \eta f_B^\top(s) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s)) - \mathbf{I}_r (\mathbf{B}_i^{(0)})^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) \quad (34)$$

$$\mathbf{A}_i^{(0)} f_A(s + 1) = \mathbf{A}_i^{(0)} (f_A(s) - \eta f_B^\top(s) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) - (\mathbf{A}_i^{(0)}) (\mathbf{A}_i^{(0)})^\top (\mathbf{B}_i^{(0)})^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s)) \quad (35)$$

$$f_A(s + 1) = f_A(s) - \eta f_B^\top(s) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) - (\mathbf{A}_i^{(0)})^\top (\mathbf{B}_i^{(0)})^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) \quad (36)$$

$$f_A(s + 1) = -\eta \sum_{j=0}^s (f_B^\top(j) - (\mathbf{A}_i^{(0)})^\top (\mathbf{B}_i^{(0)})^\top) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j) \quad (37)$$

890 For \mathbf{B} , we have:
891

$$\begin{aligned} 892 \quad \mathbf{B}_i^{s+1} &= \mathbf{B}_i^s - \eta (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) (\mathbf{A}_i^s)^\top \\ 893 \quad &= \mathbf{B}_i^{(0)} + \eta f_B(s) (\mathbf{A}_i^{(0)})^\top - \eta (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) (\mathbf{A}_i^{(0)} + \eta \mathbf{A}_i^{(0)} f_A(s))^\top \\ 894 \quad &= \mathbf{B}_i^{(0)} + \eta (f_B(s) (\mathbf{A}_i^{(0)})^\top - (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) (\mathbf{A}_i^{(0)} + \eta \mathbf{A}_i^{(0)} f_A(s))^\top) \\ 895 \quad &= \mathbf{B}_i^{(0)} + \eta (f_B(s) (\mathbf{A}_i^{(0)})^\top - (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) (\mathbf{A}_i^{(0)})^\top - \eta (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) f_A^\top(s) (\mathbf{A}_i^{(0)})^\top) \\ 896 \quad &= \mathbf{B}_i^{(0)} + \eta (f_B(s) - (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) - \eta (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) f_A^\top(s)) (\mathbf{A}_i^{(0)})^\top \end{aligned} \quad (38)$$

897 Thus,
898

$$\begin{aligned} 899 \quad f_B(s + 1) &= f_B(s) - (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) - \eta (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) f_A^\top(s) \\ 900 \quad &= f_B(s) - (\nabla_{\mathbf{W}_0} \mathcal{L}_i^s) (\eta f_A^\top(s) + \mathbf{I}) \\ 901 \quad &= -\sum_{j=0}^s (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j) (\eta f_A^\top(j) + \mathbf{I}) \end{aligned} \quad (39)$$

902 Then, we have:
903

$$\begin{aligned} 904 \quad \|f_A(s)\|_F &= \left\| \eta \sum_{j=0}^{s-1} \left(\sum_{m=0}^{j-1} (\eta f_A(m) + \mathbf{I}) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^m)^\top + (\mathbf{A}_i^{(0)})^\top (\mathbf{B}_i^{(0)})^\top \right) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j) \right\|_F \\ 905 \quad &\leq \eta^2 \left\| \sum_{m=0}^{s-2} (f_A(m)) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^m)^\top \sum_{j=m+1}^{s-1} (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j) \right\|_F \end{aligned}$$

$$\begin{aligned}
& + \eta \left\| \sum_{j=0}^{s-1} \sum_{m=0}^{j-1} (\nabla_{\mathbf{W}_0} \mathcal{L}_i^m)^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j) \right\|_F \\
& + \eta \left\| \sum_{j=0}^{s-1} (\mathbf{A}_i^{(0)})^\top (\mathbf{B}_i^{(0)})^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j) \right\|_F \\
& \leq \eta^2 L \left\| \sum_{m=0}^{s-2} (f_A(j)) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^m)^\top \right\|_F + \eta L^2 + \eta^2 L^2 s \|(\mathbf{A}_i^{(0)})^\top (\mathbf{B}_i^{(0)})^\top\|_F
\end{aligned} \tag{40}$$

$$\|(\mathbf{A}_i^{(0)})^\top (\mathbf{B}_i^{(0)})^\top\|_F = \|(\mathbf{B}_i^{(0)} \mathbf{A}_i^{(0)})^\top\|_F = \sqrt{\sum_{p=1}^r \sigma_p^2(\mathbf{B}_i^{(0)} \mathbf{A}_i^{(0)})} \leq \sqrt{r} \tag{41}$$

If $\|f_A(s)\| \leq a_s = \frac{\eta L^2 (1 - (\eta^2 L^2)^s)}{1 - \eta^2 L^2}$, then

$$\begin{aligned}
\|f_A(s)\|_F & \leq \eta^2 L^2 a_{s-1} + \eta L^2 + \eta^2 L^2 s \sqrt{r} \\
& = \eta^2 L^2 \frac{\eta L^2 (1 - (\eta^2 L^2)^s)}{1 - \eta^2 L^2} + \eta L^2 + \eta^2 L^2 s \sqrt{r} + \eta^2 L^2 s \sqrt{r} \\
& = \frac{\eta^3 L^4 (1 - (\eta^2 L^2)^s) + \eta L^2 - \eta^3 L^4 + \eta^2 L^2 s \sqrt{r} - \eta^4 L^4 s \sqrt{r}}{1 - \eta^2 L^2} \\
& = \frac{\eta L^2 (1 - (\eta^2 L^2)^s) + \eta^2 L^2 s \sqrt{r} (1 - \eta^2 L^2)}{1 - \eta^2 L^2}
\end{aligned} \tag{42}$$

We have

$$\|f_A(s)\|_2 \leq \|f_A(s)\|_F \tag{43}$$

If $\eta \ll 1/L$,

$$\begin{aligned}
\eta \|f_A(s)\|_F & \leq \eta \frac{\eta L^2 (1 - (\eta^2 L^2)^s) + \eta^2 L^2 s \sqrt{r} (1 - \eta^2 L^2)}{1 - \eta^2 L^2} \\
& \leq \eta \frac{\eta L^2 (1 - (\eta^2 L^2)^s)}{1 - \eta^2 L^2} \\
& \leq \eta a_s
\end{aligned} \tag{44}$$

Thus, we have $\eta \|f_A(s)\| \leq \eta a_s$. The dynamics are:

$$f_A(s) = -\eta \sum_{j=0}^{s-1} f_B^\top(j) (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j), \quad f_B(s) = -\sum_{j=0}^{s-1} (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j) (\eta f_A^\top(j) + \mathbf{I}) \tag{45}$$

In our algorithm, we fine-tune $\mathbf{A}_i^{(0)}$ in our algorithm and we have $\eta \|f_A(s)\| \ll \mathbf{I}$, then

$$f_A(s) = -\eta \sum_{j=0}^{s-1} \left(-\sum_{m=0}^{j-1} (\nabla_{\mathbf{W}_0} \mathcal{L}_i^m)^\top \right)^\top (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j), \quad f_B(s) = -\sum_{j=0}^{s-1} (\nabla_{\mathbf{W}_0} \mathcal{L}_i^j) \tag{46}$$

Therefore, we have

$$\Delta \mathbf{B}_i \approx -\eta \left(\sum_{s=0}^S \nabla_{\mathbf{W}_0} \mathcal{L}_i^s \right) (\mathbf{A}_i^{(0)})^\top \tag{47}$$

□

D OVERVIEW OF SLAO

We show the detailed overview of SLAO in Figure 4. It presents a framework where fine-tuned LORA and merged LoRA are processed over time, and specializes key components: (1) orthogonal initialization, and (2) time-aware continual merging.

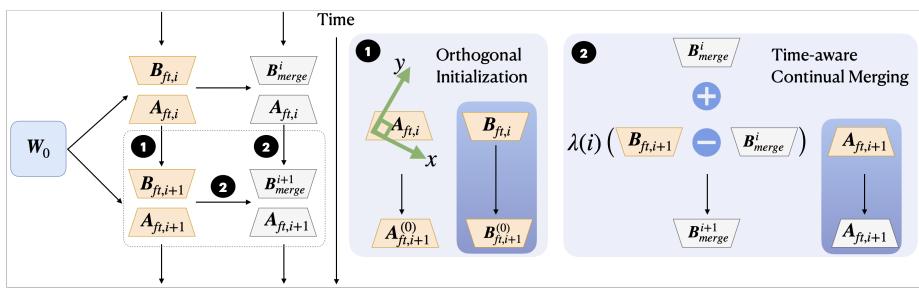


Figure 4: Overview of SLAO. Left area is a framework where fine-tuned LoRA (orange) and merged LoRA (gray) are processed over time. Right area (2 blue boxes) highlights key components: (1) Orthogonal initialization for new task $i + 1$ learning LoRA, where orthogonal basis is extracted from $A_{ft,i}$ to initialize $A_{ft,i+1}$ such that $A_{ft,i+1}^{(0)}(A_{ft,i+1}^{(0)})^\top = I_r$ and $B_{ft,i+1}$ is initialized by previous $B_{ft,i}$; (2) Time-aware continual merging for $B_{ft,i+1}$ and B_{merge}^i , and update A_{merge}^{i+1} via $A_{ft,i+1}$.

E EXPERIMENTS DETAILS

Our experiments are conducted on 4 NVIDIA A100 GPUs using the DeepSpeed repository. We evaluate three LLMs: Llama-2-7B-chat, Llama-2-13B-chat, and Llama-3-2-3B. Each individual experiment (e.g., running a single task order from the large number of tasks benchmark on Llama-2-13B-chat) can be executed on a single A100 GPU. We apply LoRA to the query and value projection matrices in the attention modules of each Llama model, with a fixed rank of 8. Each task order is evaluated over 3 random seeds.

For Standard CL benchmark and the large number of tasks benchmark on Llama-2-7B-chat and Llama-2-13B-chat, we follow the setting in Wang et al. (2023), and train the models with one epoch, a constant learning rate of 1e-4, the training batch size is 1, and the gradient accumulation step is 8. And for Standard CL benchmark and the large number of tasks benchmark on Llama-3-2-3B, we set the learning rate as 1e-4. For SuperNI benchmark using Llama-2-7B-chat and Llama-2-13B-chat, we follow the learning rate, training batch size, and gradient accumulation steps in Zhao et al. (2024b), and we use 5e-5 and train the models with five epochs with a training batch size of 2, and gradient accumulation steps of 4. And for SuperNI benchmark on Llama-3-2-3B, we set the learning rate as 5e-5.

E.1 OVERALL RESULTS ON LLAMA-2-13B-CHAT

Continual learning performance analysis. As shown in Table 5, our method consistently outperforms all data-free baselines across two benchmarks using Llama-2-13B-chat model.

LoRA-Based continual learning: IncLoRA improves upon SeqLoRA by freezing previously learned LoRAs to isolate subspaces, though its subspace separation is simplistic. SeqLoRA performs a little better than O-LoRA in the standard CL benchmark, since we use the hyperparameter $\lambda = 0.5$ in the orthogonal loss in O-LoRA that may be adjusted along with the different models and datasets, while O-LoRA outperforms SeqLoRA and IncLoRA in large number of tasks benchmark. SAPT-LoRA achieves the highest average performance among LoRA-based methods, but it relies on generated previous task pseudo samples, unrealistic in many LLM scenarios, and is more sensitive to task ordering than our method. LoRM-BA (from second task, begin with freezing B) and LoRM-AB (from second task, begin with freezing A) yield nearly identical results in large number of tasks benchmark, suggesting that the order of alternating LoRA components in learning sequential tasks does not significantly affect outcomes, but LoRM-BA outperforms LoRM-AB in standard CL benchmark. CorDA performs well in standard CL benchmark and large number of tasks, but both of them are lower than our method. MagMax performs comparably to our approach on standard CL benchmark, and slightly worse on large number of tasks, thus only keeping the weights which have the largest absolute value would cause catastrophic forgetting.

LoRA merging baselines: KnOTS and LoRA-LEGO perform similarly in the standard CL benchmark, but KnOTS outperforms in the large number of tasks. KnOTS may benefit from flexible SVD-merging mechanism so that we apply time-aware scaling on merging, while LoRA-LEGO treats tasks equally, lacks prioritization, and is ineffective in complex CL contexts.

1026 Table 5: Testing performance (%) on three CL benchmarks using Llama-2-13B-chat across different
 1027 task orders, where each result is run three random times, where O_i denotes i th task order.

1029 1030 1031 Method	1032 1033 1034 1035 Standard CL Benchmark				1036 1037 1038 1039 1040 1041 1042 1043 1044 1045 1046 Large Number of Tasks			
	O1	O2	O3	avg	O4	O5	O6	avg
SeqLoRA	77.1	77.4	78.6	77.7	74.1	74.2	74.5	74.3
IncLoRA	78.3	79.6	79.5	79.1	74.2	76.1	75.1	75.1
O-LoRA	76.1	77.1	78.5	77.2	75.5	75.5	74.8	75.3
SPAT-LoRA	83.2	82.4	80.1	81.9	83.9	80.2	82.3	82.1
LoRM-BA	78.4	80.2	80.4	79.7	74.9	71.9	70.5	72.4
LoRM-AB	76.2	74.1	75.3	75.2	74.3	70.1	72.3	72.2
CorDA	79.1	80.4	80.6	80.0	75.9	75.8	72.9	74.9
MagMax	80.9	80.6	80.7	80.7	73.7	73.4	76.0	74.4
KnOTS(zero init)	71.9	73.1	73.9	73.0	66.8	65.5	66.0	66.1
LoRA-LEGO	73.0	72.3	72.9	72.7	64.3	63.3	64.0	63.9
OPCM	68.8	63.6	61.6	64.7	57.6	60.2	58.8	58.9
SLAO (ours)	80.8	81.1	81.1	81.0	76.5	75.9	76.1	76.2
Multi-Task	81.4				79.2			

1047 Table 6: Comparison of merging strategies on testing performance on two CL benchmarks using
 1048 Llama-2-13B-chat across different task orders, where O_i denotes i th task order.

1050 1051 1052 Method	1053 1054 1055 1056 1057 1058 Standard CL Benchmark				1059 1060 1061 1062 1063 1064 1065 1066 1067 1068 1069 1070 1071 1072 1073 1074 1075 1076 1077 1078 1079 1080 1081 1082 1083 1084 1085 1086 1087 1088 1089 1090 1091 1092 1093 1094 1095 1096 1097 1098 1099 1100 1101 1102 1103 1104 1105 1106 1107 1108 1109 1110 1111 1112 1113 1114 1115 1116 1117 1118 1119 1120 1121 1122 1123 1124 1125 1126 1127 1128 1129 1130 1131 1132 1133 1134 1135 1136 1137 1138 1139 1140 1141 1142 1143 1144 1145 1146 1147 1148 1149 1150 1151 1152 1153 1154 1155 1156 1157 1158 1159 1160 1161 1162 1163 1164 1165 1166 1167 1168 1169 1170 1171 1172 1173 1174 1175 1176 1177 1178 1179 1180 1181 1182 1183 1184 1185 1186 1187 1188 1189 1190 1191 1192 1193 1194 1195 1196 1197 1198 1199 1200 1201 1202 1203 1204 1205 1206 1207 1208 1209 1210 1211 1212 1213 1214 1215 1216 1217 1218 1219 1220 1221 1222 1223 1224 1225 1226 1227 1228 1229 1230 1231 1232 1233 1234 1235 1236 1237 1238 1239 1240 1241 1242 1243 1244 1245 1246 1247 1248 1249 1250 1251 1252 1253 1254 1255 1256 1257 1258 1259 1260 1261 1262 1263 1264 1265 1266 1267 1268 1269 1270 1271 1272 1273 1274 1275 1276 1277 1278 1279 1280 1281 1282 1283 1284 1285 1286 1287 1288 1289 1290 1291 1292 1293 1294 1295 1296 1297 1298 1299 1299 1300 1301 1302 1303 1304 1305 1306 1307 1308 1309 1309 1310 1311 1312 1313 1314 1315 1316 1317 1318 1319 1319 1320 1321 1322 1323 1324 1325 1326 1327 1328 1329 1329 1330 1331 1332 1333 1334 1335 1336 1337 1338 1339 1339 1340 1341 1342 1343 1344 1345 1346 1347 1348 1349 1349 1350 1351 1352 1353 1354 1355 1356 1357 1358 1359 1359 1360 1361 1362 1363 1364 1365 1366 1367 1368 1369 1369 1370 1371 1372 1373 1374 1375 1376 1377 1378 1379 1379 1380 1381 1382 1383 1384 1385 1386 1387 1388 1389 1389 1390 1391 1392 1393 1394 1395 1396 1397 1398 1399 1399 1400 1401 1402 1403 1404 1405 1406 1407 1408 1409 1409 1410 1411 1412 1413 1414 1415 1416 1417 1418 1419 1419 1420 1421 1422 1423 1424 1425 1426 1427 1428 1429 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1611 1612 1613 1614 1615 1616 1617 1618 1619 1619 1620 1621 1622 1623 1624 1625 1626 1627 1628 1629 1629 1630 1631 1632 1633 1634 1635 1636 1637 1638 1639 1639 1640 1641 1642 1643 1644 1645 1646 1647 1648 1649 1649 1650 1651 1652 1653 1654 1655 1656 1657 1658 1659 1659 1660 1661 1662 1663 1664 1665 1666 1667 1668 1669 1669 1670 1671 1672 1673 1674 1675 1676 1677 1678 1679 1679 1680 1681 1682 1683 1684 1685 1686 1687 1688 1689 1689 1690 1691 1692 1693 1694 1695 1696 1697 1698 1699 1699 1700 1701 1702 1703 1704 1705 1706 1707 1708 1709 1709 1710 1711 1712 1713 1714 1715 1716 1717 1718 1719 1719 1720 1721 1722 1723 1724 1725 1726 1727 1728 1729 1729 1730 1731 1732 1733 1734 1735 1736 1737 1738 1739 1739 1740 1741 1742 1743 1744 1745 1746 1747 1748 1749 1749 1750 1751 1752 1753 1754 1755 1756 1757 1758 1759 1759 1760 1761 1762 1763 1764 1765 1766 1767 1768 1769 1769 1770 1771 1772 1773 1774 1775 1776 1777 1778 1779 1779 1780 1781 1782 1783 1784 1785 1786 1787 1788 1789 1789 1790 1791 1792 1793 1794 1795 1796 1797 1798 1799 1799 1800 1801 1802 1803 1804 1805 1806 1807 1808 1809 1809 1810 1811 1812 1813 1814 1815 1816 1817 1818 1819 1819 1820 1821 1822 1823 1824 1825 1826 1827 1828 1829 1829 1830 1831 1832 1833 1834 1835 1836 1837 1838 1839 1839 1840 1841 1842 1843 1844 1845 1846 1847 1848 1849 1849 1850 1851 1852 1853 1854 1855 1856 1857 1858 1859 1859 1860 1861 1862 1863 1864 1865 1866 1867 1868 1869 1869 1870 1871 1872 1873 1874 1875 1876 1877 1878 1879 1879 1880 1881 1882 1883 1884 1885 1886 1887 1888 1889 1889 1890 1891 1892 1893 1894 1895 1896 1897 1898 1899 1899 1900 1901 1902 1903 1904 1905 1906 1907 1908 1909 1909 1910 1911 1912 1913 1914 1915 1916 1917 1918 1919 1919 1920 1921 1922 1923 1924 1925 1926 1927 1928 1929 1929 1930 1931 1932 1933 1934 1935 1936 1937 1938 1939 1939 1940 1941 1942 1943 1944 1945 1946 1947 1948 1949 1950 1951 1952 1953 1954 1955 1956 1957 1958 1959 1960 1961 1962 1963 1964 1965 1966 1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1979 1980 1981 1982 1983 1984 1985 1986 1987 1988 1989 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2009 2010 2011 2012 2013 2014 2015 2016 2017 2018 2019 2019 2020 2021 2022 2023 2024 2025 2026 2027 2028 2029 2029 2030 2031 2032 2033 2034 2035 2036 2037 2038 2039 2039 2040 2041 2042 2043 2044 2045 2046 2047 2048 2049 2050 2051 2052 2053 2054 2055 2056 2057 2058 2059 2060 2061 2062 2063 2064 2065 2066 2067 2068 2069 2070 2071 2072 2073 2074 2075 2076 2077 2078 2079 2080 2081 2082 2083 2084 2085 2086 2087 2088 2089 2089 2090 2091 2092 2093 2094 2095 2096 2097 2098 2099 2099 2100 2101 2102 2103 2104 2105 2106 2107 2108 2109 2109 2110 2111 2112 2113 2114 2115 2116 2117 2118 2119 2119 2120 2121 2122 2123 2124 2125 2126 2127 2128 2129 2129 2130 2131 2132 2133 2134 2135 2136 2137 2138 2139 2139 2140 2141 2142 2143 2144 2145 2146 2147 2148 2149 2149 2150 2151 2152 2153 2154 2155 2156 2157 2158 2159 2159 2160 2161 2162 2163 2164 2165 2166 2167 2168 2169 2169 2170 2171 2172 2173 2174 2175 2176 2177 2178 2179 2179 2180 2181 2182 2183 2184 2185 2186 2187 2188 2189 2189 2190 2191 2192 2193 2194 2195 2196 2197 2198 2199 2199 2200 2201 2202 2203 2204 2205 2206 2207 2208 2209 2209 2210 2211 2212 2213 2214 2215 2216 2217 2218 2219 2219 2220 2221 2222 2223 2224 2225 2226 2227 2228 2229 2229 2230 2231 2232 2233 2234 2235 2236 2237 2238 2239 2239 2240 2241 2242 2243 2244 2245 2246 2247 2248 2249 2249 2250 2251 2252 2253 2254 2255 2256 2257 2258 2259 2259 2260 2261 2262 2263 2264 2265 2266 2267 2268 2269 2269 2270 2271 2272 2273 2274 2275 2276 2277 2278 2279 2279 2280 2281 2282 2283 2284 2285 2286 2287 2288 2289 2289 2290 2291 2292 2293 2294 2295 2296 2297 2298 2299 2299 2300 2301 2302 2303 2304 2305 2306 2307 2308 2309 2309 2310 2311 2312 2313 2314 2315 2316 2317 2318 2319 2319 2320 2321 2322 2323 2324 2325 2326 2327 2328 2329 2329 2330 2331 2332 2333 2334 2335 2336 2337 2338 2339 2339 2340 2341 2342 2343 2344 2345 2346 2347 2348 2349 2349 2350 2351 2352 2353 2354 2355 2356 2357 2358 2359 2359 2360 2361 2362 2363 2364 2365 2366 2367 2368 2369 2369 2370 2371 2372 2373 2374 2375 2376 2377 2378 2379 2379 2380 2381 2382 2383 2384 2385 2386 2387 2388 2389 2389 2390 2391 2392 2393 2394 2395 2396 2397 2398 2399 2399 2400 2401 2402 2403 2404 2405 2406 2407 2408 2409 2409 2410 2411 2412 2413 2414 2415 2416 2417 2418 2419 2419 2420 2421 2422 2423 2424 2425 2426 2427 2428 2429 2429 2430 2431 2432 2433 2434 2435 2436 2437 2438 2439 2439 2440 2441 2442 2443 2444 2445 2446 2447 2448 2449 2449 2450 2451 2452 2453 2454 2455 2456 2457 2458 2459 2459 2460 2461 2462 2463 2464 2465 2466 2467 2468 2469 2469 2470 2471 2472 2473 2474 2475 2476 2477 2478 2479 2479 2480 2481 2482 2483 2484 24			

1080 Table 7: Comparison of testing performance on SuperNI benchmark using Qwen2.5-3B and Qwen2.5-
1081 7B models across two different task orders.
1082

Method	Qwen2.5-3B			Qwen2.5-7B		
	O1	O2	avg	O1	O2	avg
O-LoRA	31.1	29.8	30.5	34.3	32.6	33.4
InfLoRA	35.6	25.6	30.6	43.5	31.0	37.3
SLAO	37.8	32.4	35.1	41.0	35.5	38.3

1089 Table 8: Comparison of initialization strategies for existing LoRA merging methods on testing
1090 performance on two CL benchmarks using Llama-2-7B-chat and Llama-2-13B-chat across different
1091 task orders, where O_i denotes i th task order.
1092

Init	Model	Method	Standard CL Benchmark				Large Number of Tasks			
			O1	O2	O3	avg	O4	O5	O6	avg
Random (Zero)	7B	KnOTS	67.9	65.9	70.8	68.2	61.5	60.1	58.0	59.9
Random (Zero)	7B	LoRA-LEGO	68.3	66.0	70.9	68.4	58.8	58.7	53.2	56.9
Last-FT	7B	KnOTS	79.7	80.7	79.9	80.1	74.0	72.5	75.0	73.8
Last-FT	7B	LoRA-LEGO	78.8	79.8	79.5	79.4	72.2	70.4	74.2	72.3
Random (Zero)	13B	KnOTS	71.9	73.1	73.9	73.0	66.8	65.5	66.0	66.1
Random (Zero)	13B	LoRA-LEGO	73.0	72.3	72.9	72.7	64.3	63.3	64.0	63.9
Last-FT	13B	KnOTS	80.1	79.6	80.3	80.0	75.4	74.9	75.8	75.4
Last-FT	13B	LoRA-LEGO	79.8	80.0	80.2	80.0	75.3	73.8	76.0	75.0

1107 To evaluate the performance of our SLAO using state-of-the-art LLMs, we use Qwen2.5-3B and
1108 Qwen2.5-7B to compare with the other two LoRA-based continual learning baselines on the SuperNI
1109 benchmark.1110 When evaluating SuperNI benchmarks, SLAO is consistently better than other two baselines and
1111 achieves the best average performance on both Qwen2.5-3B and Qwen2.5-7B models, demonstrating
1112 its robustness on Qwen2.5 model sizes. Also, the performance of SLAO on Qwen2.5-3B is better
1113 than that on Qwen2.5-7B.

1114 E.3 IMPACT OF INITIALIZATION STRATEGIES FOR EXISTING LORA MERGING METHODS

1116 We compare the testing performance of different initialization strategies for existing LoRA merging
1117 methods in Table 8, where we compare two strategies: random (zero) initialization and the last
1118 fine-tuning point of previous tasks. For zero initialization for new tasks, when using Llama-2-7B-chat
1119 and Llama-2-13B-chat, KnOTS and LoRA-LEGO perform similarly in the standard CL benchmark,
1120 but KnOTS outperforms in the large number of tasks. For last fine-tuning point initialization, KnOTS
1121 and LoRA-LEGO perform similarly in the standard CL benchmark and the large number of tasks,
1122 while KnOTS is slightly over LoRA-LEGO. All results in the last fine-tuning point initialization
1123 are significantly better than the zero initialization. These results show that our last fine-tuning point
1124 initialization has better performance than zero initialization in continual merging scenarios.

1125 E.4 CONTINUAL LEARNING PERFORMANCE ON BACKWARD TRANSFER

1127 We evaluate the performance of backward transfer (BWT) using Llama-2-7B-chat on the large
1128 number of tasks benchmark. As shown in Table 9, SLAO demonstrates strong backward transfer
1129 ability. Among all methods, SeqLoRA performs the worst due to its lack of mechanisms to prevent
1130 forgetting. KnOTS and LoRA-LEGO also underperform, as they are primarily designed for model
1131 merging rather than continual learning. InLoRA exhibits limited BWT performance, while O-
1132 LoRA achieves better results by enforcing orthogonality during learning. InfLoRA slightly trails
1133 O-LoRA, and CorDA performs worse, possibly due to its reliance on nullspace projection without
time-aware updates. SAPT-LoRA achieves the best BWT overall, though it benefits from synthetic

1134 Table 9: Testing performance (%) of the average of backward transfer (BWT) on large number of
 1135 tasks using Llama-2-7B-chat across different task orders.

Method	BWT	Method	BWT
SeqLoRA	-17.2	LoRM-BA	-6.7
IncLoRA	-9.6	LoRM-AB	-4.1
O-LoRA	-4.0	MagMax	-3.8
InfLoRA	-4.9	OPCM	-3.9
SAPT-LoRA	-2.9	KnOTS (zero init)	-14.1
CorDA	-4.5	LoRA-LEGO	-15.6
SLAO (Ours)	-3.5		

1146 Table 10: Comparison of MOPD and AOPD on testing performance across three standard CL
 1147 benchmarks using Llama-2-7B-chat in different task orders.

Method	Standard CL Benchmark		Large Number of Tasks		SuperNI Benchmark	
	MOPD	AOPD	MOPD	AOPD	MOPD	AOPD
O-LoRA	9.84%	5.79%	17.87%	8.53%	22.16%	11.63%
SAPT-LoRA	8.75%	5.69%	18.94%	9.65%	25.28%	12.38%
InfLoRA	8.23%	2.54%	19.58%	10.01%	23.42%	11.46%
SLAO(ours)	1.72%	1.30%	15.17%	7.16%	18.94%	10.76%

1157 data from previous tasks, which may not be feasible in realistic settings. Between the two variants of
 1158 LoRM, LoRM-BA slightly outperforms LoRM-AB. Finally, MagMax and OPCM show comparable
 1159 performance, both designed to balance update integration during continual merging.

1161 E.5 CONTINUAL LEARNING PERFORMANCE ON ORDER-NORMALIZED PERFORMANCE 1162 DISPARITY

1164 We evaluate the performance of Order-normalized Performance Disparity (Yoon et al., 2020) using
 1165 Llama-2-7B-chat on three benchmarks. Order-normalized Performance Disparity is used to evaluate
 1166 order-sensitivity for each task t , defined as the disparity between its performance on R random task
 1167 sequences:

$$OPD_t = \max(\bar{P}_t^1, \dots, \bar{P}_t^R) - \min(\bar{P}_t^1, \dots, \bar{P}_t^R) \quad (48)$$

1170 where \bar{P}_t^r denotes the performance of task t to the task sequence r . The Maximum OPD is de-
 1171 fined as $MOPD = \max(OPD_1, \dots, OPD_t)$ and the Average OPD is defined as $AOPD =$
 1172 $\frac{1}{T} \sum_{t=1}^T OPD_t$, to evaluate order-robustness on the whole task set. Lower scores of both metrics
 1173 indicate higher robustness.

1174 Table 10 shows the performance of MOPD and AOPD on three benchmarks with their task sequences.
 1175 Our SLAO shows the most stable performance across different task orders, indicating that it handles
 1176 order sensitivities better compared to other baselines. While SAPT-LoRA achieves higher scores in
 1177 Table 1, it heavily depends on past tasks’ data information, so that SAPT-LoRA suffers from greater
 1178 variation in different task orders, mostly due to its past pseudo-sample generation.

1180 E.6 CHOICE OF ORTHOGONAL DECOMPOSITION STRATEGY

1182 In our algorithm, SLAO, we use QR decomposition to extract the orthogonal basis from previous
 1183 LoRA A . To evaluate the effectiveness of QR decomposition, we compare it against orthogonal bases
 1184 derived from (1) singular value decomposition (SVD), where the product UV^\top forms an orthogonal
 1185 approximation, and (2) randomized SVD, where the product QU forms an orthogonal approximation.
 1186 As shown in Table 11, QR initialization performs similarly to the SVD approach on standard CL
 1187 benchmark and large number of tasks, but the performance of QR on SuperNI benchmark is better
 1188 than that of SVD. The performance of randomized SVD is not better than SVD and QR across these

Table 11: Comparison of orthogonal decomposition strategies on testing performance on two CL benchmarks using Llama-2-7B-chat across different task orders, where O_i denotes i th task order.

	Standard CL Benchmark			Large Number of Tasks			SuperNI Benchmark				
Method	O1	O2	O3	avg	O4	O5	O6	avg	O1	O2	avg
Randomized SVD	72.6	69.1	73.3	71.7	52.3	62.5	57.8	57.5	11.8	22.8	17.3
SVD	79.9	80.8	80.2	80.3	75.3	74.4	75.1	74.9	36.9	33.7	35.3
QR	80.1	80.8	80.4	80.4	75.0	74.4	75.1	74.8	38.7	35.7	37.2

three benchmarks. This suggests that SLAO is robust to the choice of orthogonal decomposition technique.

E.7 ASYMMETRY OF LoRA

We separately fine-tune 15 tasks from the SuperNI benchmark using 15 LoRAs on Llama-2-7B-chat (Touvron et al., 2023), and compute cosine similarity of \mathbf{A} and \mathbf{B} across 15 tasks using the last layer LoRA. Figure 5 shows that \mathbf{A} exhibits significantly higher similarity across tasks compared to \mathbf{B} , suggesting that LoRA components follow inherently different learning dynamics.

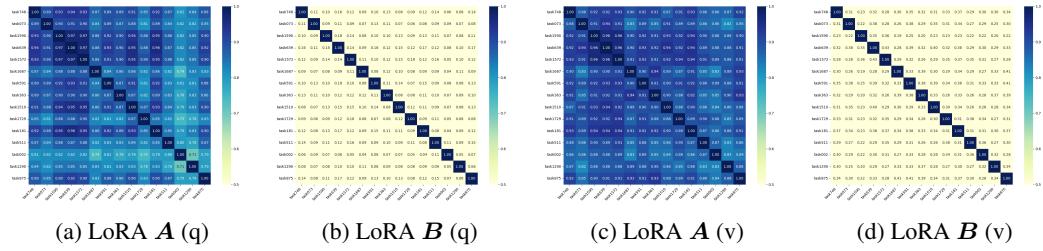


Figure 5: Cosine similarity between 15 tasks from SuperNI benchmark for fine-tuned q and v attention LoRA \mathbf{A} and \mathbf{B} in the last layer (32nd) of Llama-2-7B-chat.

E.8 IMPACT OF LEARNING RATE

To assess the effect of different learning rates, we evaluate SeqLoRA on the standard CL benchmark using Llama-2-7B-chat with learning rates of 1e-3, 1e-4, and 1e-5. As shown in Table 12, a learning rate of 1e-4 achieves the best performance, while 1e-3 performs the worst, significantly degrading the overall results. This highlights the importance of careful learning rate selection in LoRA-based continual learning.

Table 12: Impact of learning rate on testing performance of SeqLoRA using Llama-2-7B-chat across three task orders in Standard CJ Benchmark, where Q_i denotes i th task order.

learning rate	Standard CL Benchmark			
	O1	O2	O3	avg
$1e - 3$	6.0	0.0	19.7	8.6
$1e - 4$	73.3	76.2	78.4	76.0
$1e - 5$	73.6	71.8	76.0	73.8

E.9 IMPACT OF THE RANK OF LoRA

We assess the effect of different LoRA rank values in our algorithm by comparing three rank settings on both the standard CL benchmark and the large number of tasks benchmark using Llama-2-13B-chat. As shown in Table 13, a rank of 8 yields the best performance on the standard CL benchmark, while a rank of 4 performs best on the large number of tasks benchmark. Overall, the performance

1242 differences across the three ranks are relatively small, suggesting that our method is robust to the
 1243 choice of rank.
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1245 Table 13: Impact of the rank of LoRA on testing performance of SLAO using Llama-2-13B-chat
 1246 across Standard CL Benchmark and large number of tasks, where O_i denotes i th task order.
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rank	Standard CL Benchmark				Large Number of Tasks			
	O1	O2	O3	avg	O4	O5	O6	avg
$r = 4$	80.6	81.0	81.1	80.9	77.2	76.3	77.1	76.9
$r = 8$	80.8	81.1	81.1	81.0	76.5	75.9	76.1	76.2
$r = 16$	80.3	80.9	80.9	80.7	76.9	75.5	75.9	76.1

1255 E.10 COMPARISON OF TRAINING COST

1257 We compare the training cost among several baselines in Table 14. We use a single NVIDIA A100
 1258 GPU to fine-tune Llama-2-7B-chat. It shows that our SLAO is both memory usage efficient and
 1259 training efficient, since we only compute one-time QR matrix factorization at initialization, avoiding
 1260 additional computational cost during training. Besides, we observe that the walltime under orthogonal
 1261 initialization is often smaller than the walltime without orthogonal initialization. There is a similar
 1262 conclusion in Hu et al. (2020), which proves that drawing the initial weights from the orthogonal
 1263 group can speed up convergence. Therefore, SLAO provides an ideal balance between performance,
 1264 memory usage, and training speed.
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1265 E.11 COMPARISON OF ORTHOGONALITY AMONG LORA-BASED CL METHODS

1266 • **Initialization:**

- 1267 (a) O-LoRA: New task’s LoRA $\mathbf{B}_i \mathbf{A}_i$ is randomly initialized and is not orthogonal to
 1268 previous tasks’ LoRAs $\{\mathbf{B}_1 \mathbf{A}_1, \dots, \mathbf{B}_{i-1} \mathbf{A}_{i-1}\}$.
 1269
- 1270 (b) InfLoRA: Use all of new task i data to compute its input matrix \mathbf{H}_i on current full
 1271 model parameters, and use all previous $i - 1$ tasks’ gradient spaces which denote as
 1272 \mathbf{M}_i to make new $\mathbf{A}_i \in \mathbb{R}^{r \times d}$ lie in $\mathbf{N}_i \cap \mathbf{M}_i^\perp$ where \mathbf{N}_i is the subspace spanned by
 1273 the columns of \mathbf{H}_i . $\mathbf{B}_i \in \mathbb{R}^{d \times r}$ is initialized as zero.
 1274
- 1275 (c) SLAO: Extract orthogonal basis from previous fine-tuned $\mathbf{A}_{i-1} \in \mathbb{R}^{r \times d}$ as new task’s
 1276 \mathbf{A}_i , which makes $\mathbf{A}_i \mathbf{A}_i^\top = \mathbf{I}_r$. $\mathbf{B}_i \in \mathbb{R}^{d \times r}$ is initialized as previous fine-tuned \mathbf{B}_{i-1} .
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1278 • **Training:**

- 1279 (a) O-LoRA: Compute orthogonal loss to make new task’s $\mathbf{A}_i \in \mathbb{R}^{r \times d}$ orthogonal to all
 1280 previous tasks’ \mathbf{A} , and update \mathbf{A}_i and \mathbf{B}_i .
 1281
- 1282 (b) InfLoRA: Compute standard cross entropy loss and update $\mathbf{B}_i \in \mathbb{R}^{d \times r}$.
 1283
- 1284 (c) SLAO: Compute standard cross entropy loss and update $\mathbf{B}_i \in \mathbb{R}^{d \times r}$ and $\mathbf{A}_i \in \mathbb{R}^{r \times d}$.
 1285

1286 • **Post-Training:**

- 1287 (a) O-LoRA: Store all tasks’ LoRA $\{\mathbf{B}_1 \mathbf{A}_1, \dots, \mathbf{B}_i \mathbf{A}_i\}$.
 1288
- 1289 (b) InfLoRA: Use new task i data to compute new task input matrix \mathbf{R}_i on new learned
 1290 $\mathbf{B}_i \mathbf{A}_i$, then compute new gradient orthogonal bases memory \mathbf{M}_i through DualGPM,
 1291 where \mathbf{M}_i represents the gradient space of all i tasks. Then, integrate $\mathbf{B}_i \mathbf{A}_i$ to \mathbf{W}_{i-1}
 1292 and store the updated gradient space \mathbf{M}_i of all i tasks.
 1293
- 1294 (c) SLAO: Merge \mathbf{B}_i to previously merged \mathbf{B}_{i-1} and keep fine-tuned \mathbf{B}_i , merged \mathbf{B}_i , and
 1295 fine-tuned \mathbf{A}_i .
 1296

1297 Overall, InfLoRA and SLAO both focus on the orthogonality of initialization and post-training,
 1298 while O-LoRA focuses on the orthogonality of the updating process during training. Moreover, for
 1299 initialization, InfLoRA makes new \mathbf{A}_i lie at the intersection of input matrix and previous gradient
 1300 spaces \mathbf{M}_i , while SLAO extracts orthogonal basis from previous fine-tuned \mathbf{A}_{i-1} as new \mathbf{A}_i ; for
 1301 post-training, InfLoRA computes and stores all previous tasks’ orthogonal gradient spaces, while
 1302 SLAO uses the asymmetry of LoRA to obtain a merged \mathbf{B} .
 1303

1296 Table 14: Comparison of training cost across three standard CL benchmarks using Llama-2-7B-chat
 1297 (average cost across different task orders, FTBA-MBAOI: FTBA-MBA with orthogonal initialize A).
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1300 1301 1302 1303 1304 1305 1306 1307 1308 1309 1310 1311 1312 1313 1314 1315 1316 1317 1318 1319 1320 1321 1322 1323 1324 1325 1326 1327 1328 1329 1330 1331 1332 1333 1334 1335 1336 1337 1338 1339 1340 1341 1342 1343 1344 1345 1346 1347 1348 1349	Peak GPU Memory	GPU Walltime	Peak GPU Memory	GPU Walltime	Peak GPU Memory	GPU Walltime
O-LoRA	35.71GB	01:21:49	37.55GB	02:47:06	38.06GB	02:48:34
InfLoRA	45.21GB	01:47:48	57.66GB	04:57:22	58.99GB	05:01:01
FTBA-MBA	35.29GB	00:51:43	35.64GB	01:59:51	36.01GB	02:02:07
FTBA-MBAOI	35.43GB	00:51:27	36.23GB	01:59:12	37.59GB	02:01:35
FTBA-MB	35.17GB	00:51:36	35.47GB	01:59:26	35.91GB	02:01:49
SLAO (ours)	35.24GB	00:50:58	35.61GB	01:59:00	35.94GB	02:00:08

E.12 DESCRIPTIONS OF TASK SEQUENCE ORDERS

We report task descriptions and their metrics used for our CL experiments across Llama models in Table 15 and Table 16. And we show eight task orders in Table 17.

Table 15: Descriptions of 15 datasets in Large Number of Tasks benchmark and first 5 datasets from standard CL benchmark.

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|--|--|--|--|--|--|--|
| 1. Yelp | CL Benchmark | Sentiment analysis | Yelp reviews | Accuracy | | |
| 2. Amazon | CL Benchmark | Sentiment analysis | Amazon reviews | Accuracy | | |
| 3. DBpedia | CL Benchmark | Topic classification | Wikipedia | Accuracy | | |
| 4. Yahoo | CL Benchmark | Topic classification | Yahoo Q&A | Accuracy | | |
| 5. AG News | CL Benchmark | Topic classification | News | Accuracy | | |
| 6. MNLI | GLUE | Natural language inference | Various | Accuracy | | |
| 7. QQP | GLUE | Paragraph detection | Quora | Accuracy | | |
| 8. RTE | GLUE | Natural language inference | News, Wikipedia | Accuracy | | |
| 9. SST-2 | GLUE | Sentiment analysis | Movie reviews | Accuracy | | |
| 10. WiC | SuperGLUE | Word sense disambiguation | Lexical databases | Accuracy | | |
| 11. CB | SuperGLUE | Natural language inference | Various | Accuracy | | |
| 12. COPA | SuperGLUE | Question and answering | Blogs,encyclopedia | Accuracy | | |
| 13. BoolQA | SuperGLUE | Boolean question and answering | Wikipedia | Accuracy | | |
| 14. MultiRC | SuperGLUE | Question and answering | Various | Accuracy | | |
| 15. IMDB | SuperGLUE | Sentiment Analysis | Movie reviews | Accuracy | | |

Continual Learning. Continual learning aims to retain knowledge of previously learned tasks while adapting to new data. It faces two main challenges: (1) catastrophic forgetting (McCloskey & Cohen, 1989), where the performance of the model on earlier tasks significantly degrades as it learns new ones; and (2) knowledge transfer, where the model leverages previously acquired knowledge to improve learning on new tasks. Existing approaches are divided into three categories to address the issues:

(i) *Rehearsal-based methods* employ a memory buffer to store samples from previous tasks, enabling joint training with new tasks. Dark Experience Replay (Buzzeaga et al., 2020) encourages consistency with past knowledge by aligning the model’s current logits with those sampled earlier in the optimization trajectory. CLEAR (Rolnick et al., 2019), an experience replay method, effectively mitigates catastrophic forgetting in multi-task reinforcement learning. *Gradient episodic memory* (Lopez-Paz & Ranzato, 2017) stores task-specific gradients and projects new gradients to avoid interference with previous knowledge.

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Table 16: Descriptions of 15 datasets in SuperNI benchmark.

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Dataset number	Dataset name	Task	Metric
1. task639	multi-woz-user-utterance-generation	dialogue generation	Rouge-L
2. task1590	diplomacy-text-generation	dialogue generation	Rouge-L
3. task1729	personachat-generate-next	dialogue generation	Rouge-L
4. task181	outcome extraction	information extraction	Rouge-L
5. task748	glucose-reverse-cause-event-detection	information extraction	Rouge-L
6. task1510	evaluation-relation-extraction	information extraction	Rouge-L
7. task002	quoref-answer-generation	question answering	Rouge-L
8. task073	commonsenseqa-answer-generation	question answering	Rouge-L
9. task591	sciq-answer-generation	question answering	Rouge-L
10. task511	reddit-tifu-long-text-summarization	summarization	Rouge-L
11. task1290	xsum-summarization	summarization	Rouge-L
12. task1572	samsum-summary	summarization	Rouge-L
13. task363	sst2-polarity-classification	sentiment analysis	Accuracy
14. task875	emotion-classification	sentiment analysis	Accuracy
15. task1687	sentiment140-classification	sentiment analysis	Accuracy

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Table 17: Eight different task orders

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Order	Model	Task Sequence
1	Llama-2-7B-chat, Llama-2-13B-chat, Llama-3-2-3B	dbpedia → amazon → yahoo → ag
2	Llama-2-7B-chat, Llama-2-13B-chat, Llama-3-2-3B	dbpedia → amazon → ag → yahoo
3	Llama-2-7B-chat, Llama-2-13B-chat, Llama-3-2-3B	yahoo → amazon → ag → dbpedia
4	Llama-2-7B-chat, Llama-2-13B-chat, Llama-3-2-3B	mnli → cb → wic → copa → qqp → boolqa → rte → imdb → yelp → amazon → sst-2 → dbpedia → ag → multirc → yahoo
5	Llama-2-7B-chat, Llama-2-13B-chat, Llama-3-2-3B	multirc → boolqa → wic → mnli → cb → copa → qqp → rte → imdb → sst-2 → dbpedia → ag → yelp → amazon → yahoo
6	Llama-2-7B-chat, Llama-2-13B-chat, Llama-3-2-3B	yelp → amazon → mnli → cb → copa → qqp → rte → imdb → sst-2 → dbpedia → ag → yahoo → multirc → boolqa → wic
1 (SuperNI)	Llama-2-7B-chat, Llama-2-13B-chat, Llama-3-2-3B	task1572 → task363 → task1290 → task181 → task002 → task1510 → task639 → task1729 → task073 → task1590 → task748 → task511 → task591 → task1687 → task875
2 (SuperNI)	Llama-2-7B-chat, Llama-2-13B-chat, Llama-3-2-3B	task748 → task073 → task1590 → task639 → task1572 → task1687 → task591 → task363 → task1510 → task1729 → task181 → task511 → task002 → task1290 → task875

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(ii) *Regularization-based methods* utilize constraints on the parameters of the model to prevent model updates of new tasks from interfering with knowledge acquired on previous tasks. Elastic weight consolidation, *EWC* (Kirkpatrick et al., 2017), uses Fisher Information Matrix to identify and protect parameters critical for previous tasks. Orthogonal Gradient Descent, *OGD* (Farajtabar et al., 2020),

1404 projects gradients of new tasks onto a subspace that preserves model outputs on previous tasks, while
 1405 ensuring the direction remains effective for learning new tasks.

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 1407 (iii) *Architecture-based methods* dynamically adjust the structure of the model to isolate task-specific
 1408 weights or expand model capacity (Mallya & Lazebnik, 2018; Wang et al., 2022b). **Packnet** (Mallya
 1409 & Lazebnik, 2018) performs iterative pruning and network re-training. **Progressive Prompts** (Raz-
 1410 daibiedina et al., 2023) mitigate forgetting by maintaining a growing sequence of soft prompts, where
 1411 each new task contributes an additional prompt.

1412 **Parameter-efficient continual learning.** LoRA-based continual learning has emerged as a practical
 1413 and parameter-efficient technique for adapting LLMs to sequential tasks. **O-LoRA** (Wang et al., 2023)
 1414 freezes previously learned LoRAs and incrementally learns new tasks in their orthogonal subspace;
 1415 **InfLoRA** (Liang & Li, 2024) preserves prior LoRAs and uses task-dependent input matrices to define
 1416 orthogonal subspaces for initializing new ones; **SAPT-LoRA** (Zhao et al., 2024b) retains earlier
 1417 LoRAs and leverages generated previous tasks’ data to align new LoRA learning with shared modules;
 1418 **SD-LoRA** (Wu et al., 2025) incrementally decouples the learning of magnitude and direction in
 1419 LoRA components while preserving directions learned from previous tasks.

1420 **Merging and Continual Merging.** Model merging (Garipov et al., 2018; Draxler et al., 2018;
 1421 Wortsman et al., 2022) has emerged as an efficient paradigm that combines multiple task-specific
 1422 models into a single unified model without retraining (Stoica et al., 2024; Ilharco et al., 2023; Yadav
 1423 et al., 2023; Ortiz-Jimenez et al., 2023). This idea has recently extended to LoRA-based adaptation:
 1424 **KnOTS** (Stoica et al., 2025) leverages singular value decomposition to project LoRA updates into a
 1425 shared latent space, where existing merging methods can be applied; **LoRA-LEGO** (Zhao et al., 2025)
 1426 decomposes LoRAs into minimal semantic units via grouping and clustering, enabling a reconstruc-
 1427 tion of multiple LoRAs into one. However, both LoRA merging and full model merging generally
 1428 assume *simultaneous access to all task-specific LoRA or model* fine-tuned from the same initial
 1429 pre-trained model, which limits their applicability to the continual merging scenarios (Dziadzio et al.,
 1430 2025), where tasks arrive sequentially. Moreover, continual LoRA merging remains underexplored in
 1431 existing literature. While in full-model settings, continual merging has received more attention, i.e.,
 1432 **OPCM** (Tang et al., 2025) mitigates interference by sequentially projecting new model updates onto
 1433 subspaces orthogonal to the previously merged model, combined with adaptive scaling.

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