
Simultaneously aligning cells and features of single-cell multi-omic datasets with co-optimal transport

Anonymous Author(s)

Affiliation

Address

email

Abstract

1 Availability of different single-cell multi-omic datasets provide an opportunity to
2 study various aspects of the genome at the single-cell resolution. Jointly studying
3 multiple genomic features can help us understand gene regulatory mechanisms.
4 Although there are experimental challenges to jointly profile multiple genomic
5 features on the same single-cell, computational methods have been develop to align
6 unpaired single-cell multi-omic datasets. Despite the success of these alingment
7 methods, studying how genomic features interact in gene regulation requires the
8 alignment of features, too. However, most single-cell multi-omic alignment tools
9 can only align cells across different measurements. Here, we introduce SCOOTR,
10 which aligns both cells and features of the single-cell multi-omic datasets. Our
11 preliminary results show that SCOOTR provides quality alignments for datasets
12 with sparse correspondences, and for datasets with more complex relationships,
13 supervision on one level (e.g. cells) improves alignment performance on the other
14 level (e.g. features).

15 1 Introduction

16 Recent experimental developments have enabled us to measure various aspects of the genome, such as
17 gene expression, chromatin confirmation, chromatin accessibility and methylation, at the single-cell
18 resolution [1–4]. Studying the multiple views of the genome together can allow biologists to learn
19 how they interact to regulate cellular processes. Although we can experimentally combine some
20 measurements on the same single-cell using co-assays, for most measurement combinations, there
21 are no co-assays available [4]. Moreover, simultaneous profiling of multiple features can yield more
22 noisy data than single-omic experiments [5]. As a result, various computational methods [6–12],
23 including the ones based on optimal transport theory [9–11], have been developed to successfully
24 align single-cell measurements from non-co-assay (i.e. unpaired) experiments.

25 Despite the success of these methods, studying cross-modality feature relationships also requires the
26 alignment of features. Due to the number of features and the complexity of their relationship, we need
27 new computational approaches to infer these alignments. Unfortunately, most single-cell alignment
28 methods can only yield alignments on the cell level, with the exception of bindSC [12]. Although
29 bindSC performs both cell and feature alignments, it requires prior knowledge of feature relationships
30 using a gene activity matrix. This gene activity matrix is computed between gene expression features
31 and the chromatin accessibility or methylation signals in the neighborhood of these genes. Therefore,
32 it can only work with a few measurements (like chromatin accessibility and methylation) that have
33 known relationships with gene expression and ignores most intergenic regions.

34 We introduce SCOOTR, which simultaneously aligns both the cells and the features of unpaired
 35 single-cell multi-omic datasets in a modality-agnostic manner and without systematically ignoring
 36 intergenic regions, using co-optimal transport. Our results demonstrate that SCOOTR can yield
 37 quality alignments for both cells and features between datasets with sparse correspondences. For
 38 datasets with more complex relationships, supervision on one level (e.g. cell-type alignments)
 39 improves alignment performance on the other level (e.g. features, or vice-versa).

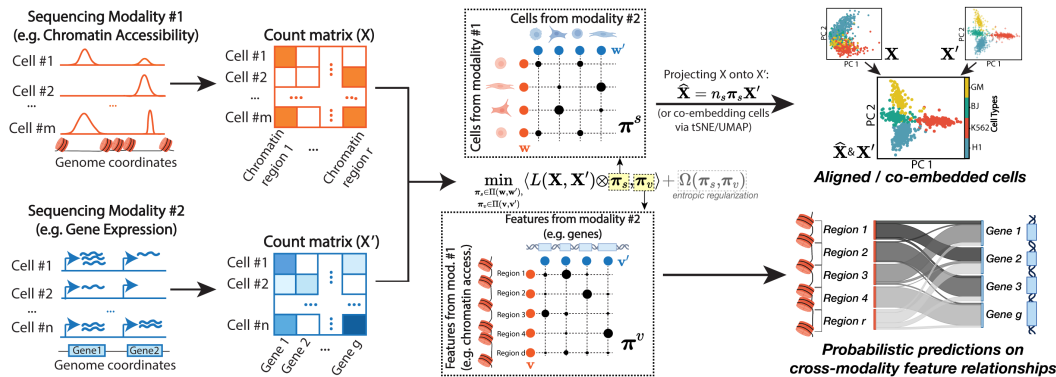


Figure 1: **Overview of the SCOOTR method on SNARE-seq dataset** Given two unpaired single-cell datasets with different genomic measurements (e.g. chromatin accessibility and gene expression), SCOOTR simultaneously solves for two probabilistic correspondence matrices: one between features, and one between cells across the two datasets.

40 2 Method

41 We first give a brief introduction to optimal transport and explain how the existing optimal transport-
 42 based single-cell multi-omic alignment methods discard features during alignment. We then introduce
 43 the SCOOTR framework.

44 2.1 Background on Optimal Transport

45 Optimal transport is a mathematical framework for relating probability distributions or discrete
 46 measures to one another. Here, we focus on discrete measures because we work with sequencing
 47 datasets that contain empirical measurements on a finite set of samples. Consider two datasets
 48 with n and n' data points in each, represented by matrices $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_n]^T \in \mathbb{R}^{n \times d}$ and
 49 $\mathbf{X}' = [\mathbf{x}'_1, \dots, \mathbf{x}'_{n'}]^T \in \mathbb{R}^{n' \times d}$. We let $\mu = \sum_{i=1}^n w_i \delta_{\mathbf{x}_i}$ and $\mu' = \sum_{j=1}^{n'} w'_j \delta_{\mathbf{x}'_j}$ be two empirical
 50 distributions related to their samples. Here $\delta_{\mathbf{x}_i}$ is the Dirac measure, the probabilities placed on
 51 data points are non-negative, $w_i \geq 0, w'_j \geq 0$, and sum up to one for each dataset, $\sum_{i=1}^n w_i = 1 =$
 52 $\sum_{j=1}^{n'} w'_j$. We refer in the following to $\mathbf{w} = [w_1, \dots, w_n]^T \in \Delta_n$ and $\mathbf{w}' = [w'_1, \dots, w'_{n'}]^T \in \Delta_{n'}$
 53 as sample weights vectors that both lie in the simplex.

54 Given a cost function $L(\mathbf{x}_i, \mathbf{x}'_j)$ that describes how “expensive” it is to match one data point (\mathbf{x}_i)
 55 with another (\mathbf{x}'_j) across the two datasets, Kantorovich formulation of optimal transport sets out to
 56 find an optimal coupling π that attains:

$$\min_{\pi \in \Pi(\mathbf{w}, \mathbf{w}')} \sum_{i,j} L(\mathbf{x}_i, \mathbf{x}'_j) \pi_{ij} \quad (1)$$

$$\text{such that } \Pi(\mathbf{w}, \mathbf{w}') = \{\pi | \pi \geq \mathbf{0}, \pi \mathbf{1}_{n'} = \mathbf{w}, \pi^T \mathbf{1}_n = \mathbf{w}'\}. \quad (2)$$

57 Here, the coupling π holds the alignment probabilities between each pair of data points across the two
 58 datasets to optimally transform one into the other and Equation 2 defines the set of linear transport
 59 constraints. Most of the practical applications of optimal transport includes an entropic regularization
 60 over the coupling matrix to split the alignment probabilities across multiple samples and also to
 61 make the optimization computationally more efficient. For more detailed background on optimal
 62 transport, we refer readers to Villani, 2008 [13] (for theory), and Peyré and Cuturi (2019) [14] (for
 63 computational aspects).

64 **2.2 Previous Optimal Transport-Based Single-cell Alignment Methods**

65 Previously, three single-cell multi-omic alignment algorithms have been proposed based on optimal
 66 transport: SCOT [9], Pamona [10], and SCOTv2 [11]. In a single-cell multi-omic alignment task,
 67 the datasets to be aligned contain measurements from different modalities (with potentially different
 68 number of features d and d'). Performing alignment using the formulation in Equation 1 would
 69 require defining a cost function over samples of different metric spaces, which is not possible. To get
 70 around this, SCOT [9] used Gromov-Wasserstein optimal transport, which extends the formulation
 71 in 1 with a cost function defined over intra-domain sample distances, making it amenable to use for
 72 multi-omic datasets:

$$GW(\mathbf{w}, \mathbf{w}') = \min_{\pi \in \Pi(\mathbf{w}, \mathbf{w}')} \sum_{ik}^n \sum_{jl}^{n'} L(D_{ik}^X, D_{jl}^{X'}) \pi_{ij} \pi_{kl} \quad (3)$$

$$\text{such that } \Pi(\mathbf{w}, \mathbf{w}') = \{\pi | \pi \geq \mathbf{0}, \pi \mathbf{1}_{n'} = \mathbf{w}, \pi^\top \mathbf{1}_n = \mathbf{w}'\}. \quad (4)$$

73 With this formulation, Gromov-Wasserstein optimal transport considers aligning a pair of samples
 74 $\mathbf{x}_i, \mathbf{x}_k$ in one dataset (\mathbf{X}) with a pair of samples $\mathbf{x}'_j, \mathbf{x}'_l$ in the other dataset (\mathbf{X}') by comparing the
 75 distances between sample pairs in each domain D_{ik}^X and $D_{jl}^{X'}$. Similarly to 2, the linear constraints
 76 placed on the coupling matrix requires that the probability distributions in its column marginals
 77 and row marginals match the empirical probabilities defined over the datasets. Observing that this
 78 constraint in practice can lead to undesirable alignments for datasets with different representations
 79 of cell-type proportions, Pamona [10] and SCOTv2 [11] proposed variations on SCOT to relax this
 80 constraint in different ways. Despite these variations, all three methods construct nearest neighbor
 81 graphs on the input datasets and compute pairwise distances on these graphs to extract intra-domain
 82 sample distances. This procedure discards the features, which are not considered in the alignment.

83 **2.3 Single-cell Co-Optimal Transport (SCOOTR)**

84 Unlike the previous optimal transport-based single-cell alignment methods, SCOOTR does not
 85 discard dataset features and optimizes over two coupling matrices, one over the samples (π^s) and
 86 one over the features (π^f) to attain:

$$\min_{\pi^s \in \Pi(\mathbf{w}, \mathbf{w}'), \pi^f \in \Pi(\mathbf{v}, \mathbf{v}')} \sum_{i,j,k,l} L(X_{i,k}, X'_{j,l}) \pi^s_{ij} \pi^f_{kl} + \Omega(\pi^s, \pi^f) \quad (5)$$

87 where $\Omega(\pi^s, \pi^f)$ is the entropic regularization term with $\Omega(\pi^s, \pi^f) = \varepsilon_1 H(\pi^s | \mathbf{w}\mathbf{w}'^T) +$
 88 $\varepsilon_2 H(\pi^f | \mathbf{v}\mathbf{v}'^T)$ and $H(\pi^s | \mathbf{w}\mathbf{w}'^T) = \sum_{i,j} \log(\frac{\pi^s_{ij}}{w_i w'_j}) \pi^s_{ij}$ being the relative entropy. Here, $\mathbf{w} \in \Delta_n$
 89 and $\mathbf{w}' \in \Delta_{n'}$ represent the empirical measures defined over samples, as described in Section 2.1,
 90 and similarly, $\mathbf{v} \in \Delta_d$ and $\mathbf{v}' \in \Delta_{d'}$ are uniform measures defined over the features. This time,
 91 while the scripts i and j still refer to sample indices, k and l refer to feature indices in the datasets
 92 \mathbf{X} and \mathbf{X}' , respectively. Intuitively, $L(X_{i,k}, X'_{j,l}) = (X_{i,k} - X'_{j,l})^2$ compares each feature in each
 93 pair of cells across the two modalities after both the cells and the features of one modality have
 94 been transformed with respect to the two coupling matrices. Since the feature space is also being
 95 transformed by π^f , this formulation allows us to compare multi-omic datasets without discarding
 96 features. The hyperparameters ε_1 and ε_2 control the extent of entropic regularization over the two
 97 coupling matrices, which controls their sparsity.

98 This optimization formulation is based on Co-Optimal Transport, introduced by Redko *et al* [15],
 99 which uses an alternating block coordinate descent procedure to solve for both π^s and π^f . We
 100 describe the details of the optimization procedure in Supplementary Algorithm 1.

101 One of the advantages of aligning both the samples and the features is the opportunity to provide
 102 supervision on one of them (e.g. cell-type alignments) to improve alignments on the other (e.g.
 103 features, or vice-versa). To do this, we optionally provide a ‘‘penalization matrix’’ to scale the cost of
 104 certain alignments, as detailed in Supplementary Materials.

105 **3 Results**

106 We apply SCOOTR to three real-world datasets with some ground-truth information to benchmark its
 107 alignment performance: (1) a CITE-seq dataset, with gene expression measurements and antibody

108 abundance profiles for ten antibodies from human peripheral blood mononuclear cells [2], (2) a
 109 SNARE-seq dataset, with chromatin accessibility and gene expression profiles from a mixture of four
 110 cell lines: BJ, H1, K562, and GM12878 [1], and finally (3) a multi-species scRNA-seq dataset with
 111 gene expression measurements from mouse prefrontal cortex and bearded lizard pallium [16]. For
 112 CITE-seq, SCOOTR yields quality alignments for both cells and features in its unsupervised setting.
 113 Figure 2(A) visualizes the feature alignment matrix (π^f) recovered by SCOOTR, where the rows
 114 are antibodies and the columns are the ten genes that encode them, in their respective order. We
 115 observe that SCOOTR recovers some correspondence for all antibodies and their encoding gene
 116 (along the diagonal), and $\sim 72\%$ of the antibodies were assigned the highest coupling probability
 117 with their encoding gene. Figure S1(B) visualizes the cell alignments by projecting cells from the
 118 one domain (gene expression) onto the cells of the other domain (antibody abundance) by taking a
 119 weighted average of cells in the latter domain according to their coupling probabilities the cells in
 120 the former domain, as recovered in (π^s) (a.k.a. “barycentric projection”, also used by the previous
 121 optimal transport-based alignment methods [9–11]). We visualize that the cells are correctly aligned
 122 with their corresponding cell-types and yield a low alignment error of 0.141 (compared to 0.154 by
 123 bindSC), as measured by the commonly used “average fraction of sample closer than true match”
 124 metric (FOSCTTM) [7, 9, 11, 12].

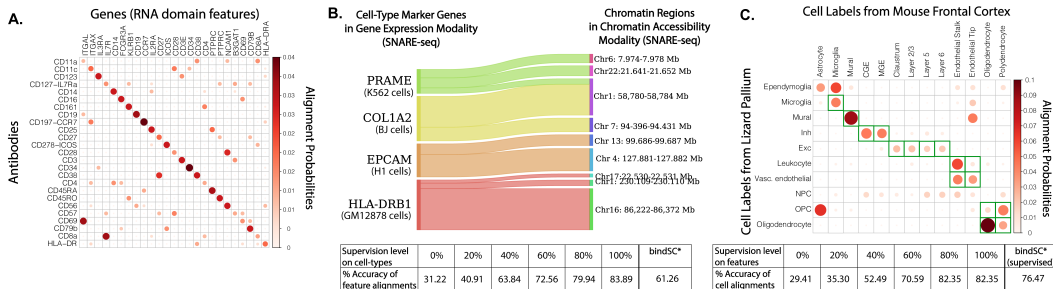


Figure 2: SCOOTR feature alignment results **A.** Feature coupling matrix between the ten antibodies and their encoding genes of the CITE-seq dataset. Larger and darker circles correspond to higher alignment probabilities. **B.** Sankey plot visualizing example feature alignments for the four cell-type marker genes in this dataset, with validations from the literature described in Supplementary Materials. The table below this figure shows the increase in feature alignment accuracy with supervision, as benchmarked against the regulatory relationships predicted by CellOracle software [17], which constructs gene regulatory networks based on gene expression and chromatin accessibility data. Similarly, we observe that supervision on the feature level improves cell-type alignments for the cross-species gene expression dataset. In Figure 2(C), we visualize the cell-type alignments between the gene expression datasets for the two species, after averaging cell alignment probabilities based on cell-type annotations. Here, we provide full supervision on the feature-level alignments by only penalizing the alignment of non-paralogous gene pairs. As the table below this figure indicates, cell-type alignment improves with increased percentage of the paralogous genes used for supervision. We compare our fully supervised alignment accuracy (82.35%) with the bindSC (76.47%), which is also in its fully supervised setting. Since bindSC requires a prior on feature matchings and this dataset involves the alignment of the same modality (gene expression), we construct this prior matrix (“gene activity matrix”) based on paralogous gene matches. Despite this, we find that the cell-type alignments by SCOOTR are more accurate than the alignments by bindSC.

125 We observe that, when aligning datasets with more complex relationships, such as chromatin accessi-
 126 bility and gene expression alignments for the SNARE-seq dataset, where the underlying correspon-
 127 dences are not expected to be 1 – 1, supervision on cell-type annotations improves feature alignment
 128 performance. Figure 2(B) visualizes an example of feature alignments recovered by SCOOTR for the
 129 four cell-type marker genes in this dataset, with validations from the literature described in Supple-
 130 mentary Materials. The table below this figure shows the increase in feature alignment accuracy with
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144 **References**

- 145 [1] Song Chen, Blue B. Lake, and Kun Zhang. High-throughput sequencing of the transcriptome and chromatin
146 accessibility in the same cell. *Nature Biotechnology*, 37(12):1452–1457, 2019.
- 147 [2] Marlon Stoeckius, Christoph Hafemeister, William Stephenson, Brian Houck-Loomis, Pratip K Chat-
148 topadhyay, Harold Swerdlow, Rahul Satija, and Peter Smibert. Simultaneous epitope and transcriptome
149 measurement in single cells. *Nature Methods*, 14(9):865–868, 2017.
- 150 [3] Stephen J. Clark, Ricard Argelaguet, Chantriolnt-Andreas Kapourani, Thomas M. Stubbs, Heather J. Lee,
151 Celia Alda-Catalinas, Felix Krueger, Guido Sanguinetti, Gavin Kelsey, John C. Marioni, Oliver Stegle, and
152 Wolf Reik. scnm-seq enables joint profiling of chromatin accessibility dna methylation and transcription
153 in single cells. *Nature Communications*, 9(1):781, 2018.
- 154 [4] Anjun Ma, Adam McDermaid, Jennifer Xu, Yuzhou Chang, and Qin Ma. Integrative methods and practical
155 challenges for single-cell multi-omics. *Trends in Biotechnology*, 38(9):1007–1022, 2020.
- 156 [5] Michael Eisenstein. The secret life of cells. *Nature Methods*, 17(1):7–10, 2020.
- 157 [6] Jie Liu, Yuanhao Huang, Ritambhara Singh, Jean-Philippe Vert, and William Stafford Noble. Jointly
158 Embedding Multiple Single-Cell Omics Measurements. In *19th International Workshop on Algorithms in*
159 *Bioinformatics (WABI 2019)*, volume 143, pages 10:1–10:13, 2019.
- 160 [7] Ritambhara Singh, Pinar Demetci, Giancarlo Bonora, Vijay Ramani, Choli Lee, He Fang, Zhijun Duan,
161 Xinxian Deng, Jay Shendure, Christine Disteche, and William Stafford Noble. Unsupervised manifold
162 alignment for single-cell multi-omics data. ACM-BCB, 2020.
- 163 [8] Kai Cao, Xiangqi Bai, Yiguang Hong, and Lin Wan. Unsupervised topological alignment for single-cell
164 multi-omics integration. *Bioinformatics*, 36(Supplement_1):i48–i56, 2020.
- 165 [9] Pinar Demetci, Rebecca Santorella, Björn Sandstede, William Stafford Noble, and Ritambhara Singh.
166 Gromov-wasserstein optimal transport to align single-cell multi-omics data. *bioRxiv*, 2020.
- 167 [10] Kai Cao, Yiguang Hong, and Lin Wan. Manifold alignment for heterogeneous single-cell multi-omics data
168 integration using Pamona. *Bioinformatics*, 08 2021. btab594.
- 169 [11] Pinar Demetci, Rebecca Santorella, Björn Sandstede, and Ritambhara Singh. Unsupervised Integration of
170 Single-Cell Multi-omics Datasets with Disproportionate Cell-Type Representation. In *26th International*
171 *Conference on Research in Computational Molecular Biology (RECOMB 2022)*, pages 3–19. Springer
172 International Publishing, 2022.
- 173 [12] Jinzhuang Dou, Shaoheng Liang, Vakul Mohanty, Qi Miao, Yuefan Huang, Qingnan Liang, Xuesen Cheng,
174 Sangbae Kim, Jongsu Choi, Yumei Li, Li Li, May Daher, Rafet Basar, Katayoun Rezvani, Rui Chen, and
175 Ken Chen. Bi-order multimodal integration of single-cell data. *Genome Biology*, 23(1):112, 2022.
- 176 [13] Cédric Villani. *Optimal Transport: Old and New*. Grundlehren der mathematischen Wissenschaften.
177 Springer, 2009 edition, September 2008.
- 178 [14] Gabriel Peyré and Marco Cuturi. Computational optimal transport. *Foundations and Trends® in Machine*
179 *Learning*, 11:355–607, 2019.
- 180 [15] Ievgen Redko, Titouan Vayer, Rémi Flamary, and Nicolas Courty. Co-optimal transport, 2020.
- 181 [16] April R. Kriebel and Joshua D. Welch. Uinmf performs mosaic integration of single-cell multi-omic
182 datasets using nonnegative matrix factorization. *Nature Communications*, 13(1):780, 2022.
- 183 [17] Kenji Kamimoto, Christy M. Hoffmann, and Samantha A. Morris. Celloracle: Dissecting cell identity via
184 network inference and in silico gene perturbation. *bioRxiv*, 2020.