Score-Based Likelihood Characterization for Inverse Problems in the Presence of Non-Gaussian Noise

Ronan Legin[1,2,3] **Alexandre Adam**[1,2,3]
Yashar Hezaveh[1,2,3,4,5,6] **Laurence Perreault-Levasseur**[1,2,3,4,5,6]
[1]Université de Montréal [2]Ciela Institute [3]Mila
[4]CCA, Flatiron Institute [5]Trottier Space Institute [6]Perimeter Institute
{ronan.legin,alexandre.adam,yashar.hezaveh,
laurence.perreault.levasseur}@umontreal.ca

Abstract

Likelihood analysis is typically limited to normally distributed noise due to the difficulty of determining the probability density function of complex, high-dimensional, non-Gaussian, and anisotropic noise. This work presents Score-based LIkelihood Characterization (SLIC), a framework that resolves this issue by building a data-driven noise model using a set of noise realizations from observations. We show that the approach produces unbiased and precise likelihoods even in the presence of highly non-Gaussian correlated and spatially varying noise. We use diffusion generative models to estimate the gradient of the probability density of noise with respect to data elements. In combination with the Jacobian of the physical model of the signal, we use Langevin sampling to produce independent samples from the unbiased likelihood. We demonstrate the effectiveness of the method using real data from the Hubble Space Telescope and James Webb Space Telescope.

1 Introduction

The presence of measurement errors, or noise, in data leads to uncertainty in the inference of variables of interest. Modeling this uncertainty requires the likelihood function, which depends on the statistics of the measurement errors [1]. These errors can be caused by various physical processes, resulting in different forms of noise [e.g., 2–8].

Although the assumption of Gaussian noise models for the likelihood is prevalent in many fields [e.g., 9–11], in practice, however, the statistics of noise often deviate significantly from a simple normal distribution and could be highly correlated, anisotropic, and non-Gaussian [e.g., 12–19]. Traditionally, the approach to deal with these scenarios has been to adopt an approximate normal noise model for the likelihood (and accept the potential biases that result), model and remove specific forms of noise (e.g., cosmic rays removal from CCD images [20–22]), masking a subset of data with significant non-Gaussian noise, or avoiding using likelihood analysis altogether. These sub-optimal approaches are driven by the fact that the probability density of complex, high-dimensional, non-Gaussian noise is not known and is intractable with closed-form expressions.

In this work, we build a noise model using a set of observed noise realizations in high dimensions (e.g., pixel space) and show that this can produce unbiased likelihoods even for noise with highly non-Gaussian statistics. We focus on the ubiquitous case of additive noise and assume that the noise realizations used for learning the noise model are either simulated or real data in the absence of the signal of interest (e.g., regions in a CCD image that do not contain the signal of interest).

Our framework, Score-based Likelihood Characterization (SLIC), employs score-based diffusion generative models [23–25] to estimate the gradient of the probability density of noise relative to

37th Conference on Neural Information Processing Systems (NeurIPS 2023).

image pixels. We demonstrate that, when combined with the Jacobian of the forward model of the signal, this approach enables unbiased inference of model parameters for inverse problems where the data contains non-Gaussian noise.

2 Methods

2.1 Learning the noise model

We assume that we have a dataset of n images of noise, where each image has m elements (i.e. the number of pixels), $D_{\mathrm{Training}} = \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n\}$. We construct this training set with i.i.d. realizations of noise from an underlying true noise probability density distribution, $Q_{\mathrm{True}}(\mathbf{x})$. Here, $\mathbf{x}$ refers to any m-dimensional point (i.e. an image) in the data space. When evaluated on a particular data, e.g., $\mathbf{x}_0$, this function informs us of how likely it is that $\mathbf{x}_0$ is a realization of noise.

In the context of score-based models, instead of learning Q directly, we learn the gradient of the logarithm of Q with respect to its variable $\mathbf{x}$, written as $\mathbf{s}(\mathbf{x}_0) = \partial \log Q(\mathbf{x}_0)/\partial \mathbf{x}$. We refer the reader to [25] for a comprehensive description of how to learn $\mathbf{s}(\mathbf{x})$. In summary, the method involves training a neural network model (e.g., a UNet [26]) using the score matching technique [27–29] to estimate the score of a distribution.

2.2 Sampling the parameters of the forward model

Our goal is to generate samples of the parameters of the forward model, $\boldsymbol{\eta} \in \mathbb{R}^l$, based on the likelihood function. The data-generating process can be written as

$$\mathbf{x} = \mathbf{M}(\boldsymbol{\eta}) + \mathbf{N}, \tag{1}$$

where $\mathbf{M}(\boldsymbol{\eta})$ encodes the signal (the forward model) as a function of the parameters of interest $\boldsymbol{\eta}$ and $\mathbf{N}$ is a vector of additive noise. A particular observation could be written as $\mathbf{x}_0 = \mathbf{M}(\boldsymbol{\eta}_{\mathrm{True}}) + \mathbf{N}_0$, where $\mathbf{N}_0$ is the true (but unknown) realization of noise.

Since here we are only considering non-stochastic forward models, the likelihood $P(\mathbf{x}_0|\boldsymbol{\eta})$ can be written as

$$P(\mathbf{x}_0|\boldsymbol{\eta}) = Q(\mathbf{x}_0 - \mathbf{M}(\boldsymbol{\eta})), \tag{2}$$

where Q is the probability density of noise.

If we can compute the score of the likelihood with respect to the model parameters $\nabla_{\boldsymbol{\eta}} \log Q$, we can sample from the underlying likelihood function, $Q(\mathbf{x}_0 - \mathbf{M}(\boldsymbol{\eta}))$, using gradient-based sampling methods [23, 25, 30, 31]. To get the score of the likelihood in the parameters of the model, we use the chain rule to write:

$$\nabla_{\boldsymbol{\eta}} \log Q = \nabla_{\mathbf{x}} \log Q \cdot \nabla_{\boldsymbol{\eta}} \mathbf{x}. \tag{3}$$

Here, $\nabla_{\mathbf{x}} \log Q$ is the score of the noise model with respect to the pixel values, which is the variable learned by the neural network score function and $\nabla_{\boldsymbol{\eta}} \mathbf{x}$ is the gradient of the pixel values with respect to the variables of interest $\boldsymbol{\eta}$. Since the pixel values only change when the model $\mathbf{M}(\boldsymbol{\eta})$ changes, this is equivalent to $\partial \mathbf{M}/\partial \boldsymbol{\eta}$, which is the Jacobian of the forward model.

Afterwards, we can generate samples from the likelihood function using a chosen sampling method. In this work, we use the unadjusted Langevin sampling algorithm

$$\boldsymbol{\eta}_{t+1} = \boldsymbol{\eta}_t + \tau \nabla_{\boldsymbol{\eta}} \log Q(\mathbf{x}_0 - \mathbf{M}(\boldsymbol{\eta})) + \sqrt{2\tau}\boldsymbol{\xi}_t, \tag{4}$$

where τ is the step size and $\boldsymbol{\xi}_t \sim \mathcal{N}(0, \mathbb{1})$ is a random variable drawn from an l-dimensional multivariate normal distribution.

To summarize, the procedure to perform inference with SLIC involves:

1. Learning a score model for the distribution of noise in the data space, $\mathbf{s}(\mathbf{x}) = \nabla_{\mathbf{x}} \log Q(\mathbf{x})$.
2. Calculating the Jacobian of the forward model, $\partial \mathbf{M}/\partial \boldsymbol{\eta}$.
3. Use a gradient-based sampling algorithm, such as the unadjusted langevin sampling algorithm, to perform inference using equation 3 to construct the likelihood function.

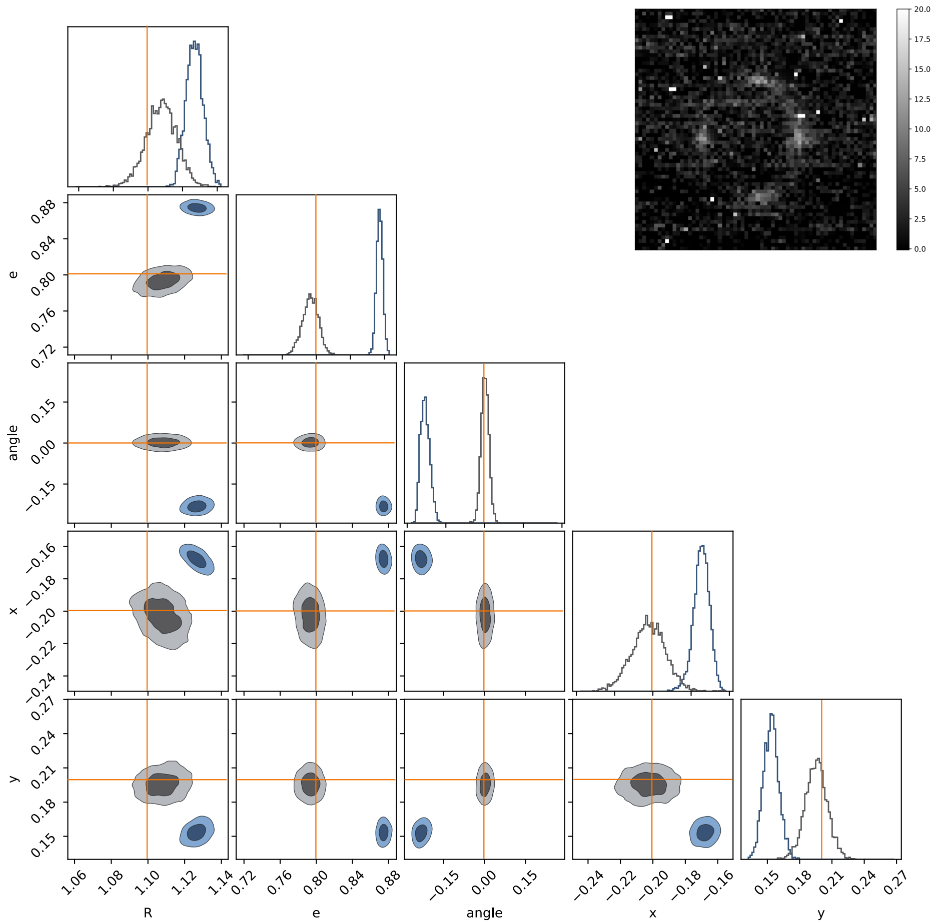

Figure 1: The analysis of a gravitational lensing signal with noise taken from *JWST* data. The data is shown on the top panel (in linear intensity units). The gray contours show samples generated with SLIC and the dark blue contours show biased results when a Gaussian likelihood is sampled.

3 Experiments and results

In this section, we present the results of our experiments to model and sample likelihood functions in the presence of non-Gaussian noise using SLIC. We use *Hubble Space Telescope* (*HST*) and *James Webb Space Telescope* (*JWST*) imaging data, publicly available at `https://archive.stsci.edu/` and `https://hla.stsci.edu/`, to test the proposed framework. We also compare the samples from our likelihood to those generated from a Gaussian likelihood with an MCMC sampler.

For *JWST*, we use five 2048×2048 stage-2 calibrated exposure-based images to produce empty noise cutouts. In total, we produce a set of approximately 800 128×128 *JWST* images to train the score model. For *HST*, we use around 50 different snapshots mainly consisting of background noise and cosmic rays, and produce a second training set of approximately 150000 64×64 *HST* cutouts.

To produce simulated observations with real non-Gaussian *HST* and *JWST* noise, we inject the signal of a strongly lensed galaxy into real noise samples. The signal is produced by distorting the image of a Sérsic background source by the gravity of a Singular Isothermal Ellipsoid (SIE) foreground structure. Our gravitational lensing model is implemented in `JAX` and is fully differentiable, allowing us to easily calculate the Jacobian of the model. Our goal is to infer the parameters of the foreground model (mass, ellipticity, orientation angle, position x and y).

We use Langevin dynamics to produce 2000 samples from the SLIC likelihood functions using equation 4, with a step size of $\tau = 10^{-5}$, and compare our results for both the *HST* and *JWST* examples using a Gaussian likelihood in Figure 1 and 4.

In Figure 2, we show results of a coverage probability test calculated using 200 different realizations of *HST* noise. In every experiment, we use the same ground truth parameters but add different realizations of noise to the signal and obtain the corresponding SLIC likelihood. The coverage probabilities are calculated using the method proposed by Lemos et al. [32].

We also visually assess the quality of the score models performance by generating new realizations of noise using the method described in Song et al. [25]. Figure 3 of section A shows examples of real *HST* and *JWST* noise along with noise realizations generated from the score model.

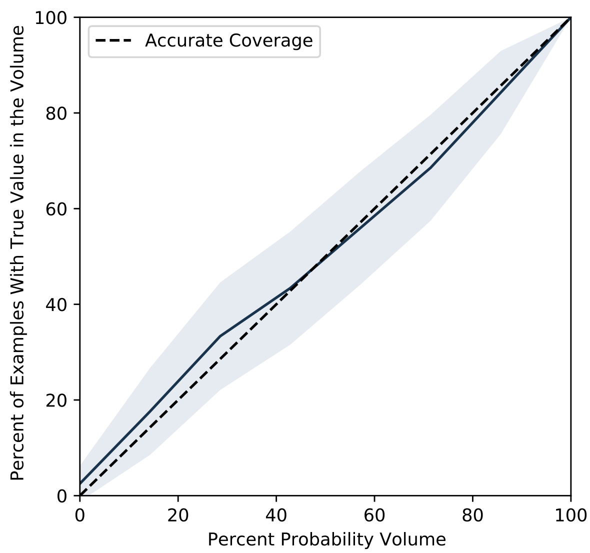

Figure 2: The coverage probability calculated by estimating the SLIC likelihoods for 200 different realizations of *HST* noise for a signal similar to that of figure 4 from section A. The 3σ error region (shaded) is calculated using jacknifing. The trend of following the dashed diagonal line shows that the likelihood is accurate (unbiased).

4 Discussion and conclusion

Our experiments show that by modeling the empirical distribution of noise and having access to differentiable forward models, it is possible to build accurate likelihood models for highly non-Gaussian noise, resulting in accurate inference without the need for major approximations.

As shown in Figure 1 and 4, the procedure provides likelihoods with high accuracy while maintaining the precision and the information content of the data. Note that the framework is not only limited to instrumental noise and can also be used to treat any other sources of additive errors. Additionally, while we demonstrated the method using gravitational lensing as an example, the framework is general and can be used for inference with any differentiable forward model.

The proposed framework also offers the advantage of utilizing the learned noise models for generating new noise realizations, as shown in Figure 3. These simulations can be used to train other inference models (e.g., in the context of simulation-based inference).

While in this work we used an unadjusted Langevin method to produce samples from the score of the likelihood, it is also possible to use Hamiltonian MCMC to sample the non-Gaussian likelihoods using only the score of the distribution [e.g., see 31].

We conclude by summarizing that given a set of possibly highly non-Gaussian noise realizations, SLIC provides a framework to use a score-based generative model to characterize the likelihood, allowing accurate analysis of highly non-Gaussian likelihoods without approximations that result in biases or loss of precision.

Acknowledgements

This work is partially supported by Schmidt Futures, a philanthropic initiative founded by Eric and Wendy Schmidt as part of the Virtual Institute for Astrophysics (VIA). The work is in part supported by computational resources provided by Calcul Quebec and the Digital Research Alliance of Canada. Y.H. and L.P. acknowledge support from the Natural Sciences and Engineering Research Council of Canada (NSERC) grant RGPIN-2020-05073 and 05102, the Fonds de recherche du Québec grant 2022-NC-301305 and 300397, and the Canada Research Chairs Program. The work of A.A. and R.L was partially funded by an NSERC CGS D scholarship. R.L. acknowledges support from the Centre for Research in Astrophysics of Quebec and the hospitality of the Flatiron Institute.

References

[1] David J. C. Mackay. *Information Theory, Inference, and Learning Algorithms*. Cambridge University Press, September 2003. ISBN 9780521642989.

[2] K. Somiya, Y. Chen, S. Kawamura, and N. Mio. Erratum: Frequency noise and intensity noise of next-generation gravitational-wave detectors with RF/DC readout schemes [Phys. Rev. D 73, 122005 (2006)]. , 75(4):049905, February 2007. doi: 10.1103/PhysRevD.75.049905.

[3] Ronald L. Gilliland, William J. Chaplin, Edward W. Dunham, Vic S. Argabright, William J. Borucki, Gibor Basri, Stephen T. Bryson, Derek L. Buzasi, Douglas A. Caldwell, Yvonne P. Elsworth, Jon M. Jenkins, David G. Koch, Jeffrey Kolodziejczak, Andrea Miglio, Jeffrey van Cleve, Lucianne M. Walkowicz, and William F. Welsh. Kepler Mission Stellar and Instrument Noise Properties. , 197(1):6, November 2011. doi: 10.1088/0067-0049/197/1/6.

[4] Mikhail Konnik and James Welsh. High-level numerical simulations of noise in CCD and CMOS photosensors: review and tutorial. *arXiv e-prints*, art. arXiv:1412.4031, December 2014. doi: 10.48550/arXiv.1412.4031.

[5] S E Harper, C Dickinson, R A Battye, S Roychowdhury, I W A Browne, Y-Z Ma, L C Olivari, and T Chen. Impact of simulated 1/f noise for HI intensity mapping experiments. *Monthly Notices of the Royal Astronomical Society*, 478(2):2416–2437, 05 2018. ISSN 0035-8711. doi: 10.1093/mnras/sty1238. URL `https://doi.org/10.1093/mnras/sty1238`.

[6] T. Chen, R. A. Battye, A. A. Costa, C. Dickinson, and S. E. Harper. Impact of 1/f noise on cosmological parameter constraints for SKA intensity mapping. , 491(3):4254–4266, January 2020. doi: 10.1093/mnras/stz3307.

[7] B. P. Abbott, R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, V. B. Adya, C. Affeldt, M. Agathos, K. Agatsuma, N. Aggarwal, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, T. Alford, G. Allen, A. Allocca, M. A. Aloy, P. A. Altin, A. Amato, A. Ananyeva, S. B. Anderson, W. G. Anderson, S. V. Angelova, S. Antier, S. Appert, K. Arai, M. C. Araya, J. S. Areeda, M. Arène, N. Arnaud, K. G. Arun, S. Ascenzi, G. Ashton, S. M. Aston, P. Astone, F. Aubin, P. Aufmuth, K. AultONeal, C. Austin, V. Avendano, A. Avila-Alvarez, S. Babak, P. Bacon, F. Badaracco, M. K. M. Bader, S. Bae, P. T. Baker, F. Baldaccini, G. Ballardin, S. W. Ballmer, S. Banagiri, J. C. Barayoga, S. E. Barclay, B. C. Barish, D. Barker, K. Barkett, S. Barnum, F. Barone, B. Barr, L. Barsotti, M. Barsuglia, D. Barta, J. Bartlett, I. Bartos, R. Bassiri, A. Basti, M. Bawaj, J. C. Bayley, M. Bazzan, B. Bécsy, M. Bejger, I. Belahcene, A. S. Bell, D. Beniwal, B. K. Berger, G. Bergmann, S. Bernuzzi, J. J. Bero, C. P. L. Berry, D. Bersanetti, A. Bertolini, J. Betzwieser, R. Bhandare, J. Bidler, I. A. Bilenko, S. A. Bilgili, G. Billingsley, J. Birch, R. Birney, O. Birnholtz, S. Biscans, S. Biscoveanu, A. Bisht, M. Bitossi, M. A. Bizouard, J. K. Blackburn, C. D. Blair, D. G. Blair, R. M. Blair, S. Bloemen, N. Bode, M. Boer, Y. Boetzel, G. Bogaert, F. Bondu, E. Bonilla, R. Bonnand, P. Booker, B. A. Boom, C. D. Booth, R. Bork, V. Boschi, S. Bose, K. Bossie, V. Bossilkov, J. Bosveld, Y. Bouffanais, A. Bozzi, C. Bradaschia, P. R. Brady, A. Bramley, M. Branchesi, J. E. Brau, T. Briant, J. H. Briggs, F. Brighenti, A. Brillet, M. Brinkmann, V. Brisson, P. Brockill, A. F. Brooks, D. D.

Brown, S. Brunett, A. Buikema, T. Bulik, H. J. Bulten, A. Buonanno, D. Buskulic, C. Buy, R. L. Byer, M. Cabero, L. Cadonati, G. Cagnoli, C. Cahillane, J. Calderón Bustillo, T. A. Callister, E. Calloni, J. B. Camp, W. A. Campbell, M. Canepa, K. C. Cannon, H. Cao, J. Cao, E. Capocasa, F. Carbognani, S. Caride, M. F. Carney, G. Carullo, J. Casanueva Diaz, C. Casentini, S. Caudill, M. Cavaglià, F. Cavalier, R. Cavalieri, G. Cella, P. Cerdá-Durán, G. Cerretani, E. Cesarini, O. Chaibi, K. Chakravarti, S. J. Chamberlin, M. Chan, S. Chao, P. Charlton, E. A. Chase, E. Chassande-Mottin, D. Chatterjee, M. Chaturvedi, K. Chatziioannou, B. D. Cheeseboro, H. Y. Chen, X. Chen, Y. Chen, H. P. Cheng, C. K. Cheong, H. Y. Chia, A. Chincarini, A. Chiummo, G. Cho, H. S. Cho, M. Cho, N. Christensen, Q. Chu, S. Chua, K. W. Chung, S. Chung, G. Ciani, A. A. Ciobanu, R. Ciolfi, F. Cipriano, A. Cirone, F. Clara, J. A. Clark, P. Clearwater, F. Cleva, C. Cocchieri, E. Coccia, P. F. Cohadon, D. Cohen, R. Colgan, M. Colleoni, C. G. Collette, C. Collins, L. R. Cominsky, Jr. Constancio, M., L. Conti, S. J. Cooper, P. Corban, T. R. Corbitt, I. Cordero-Carrión, K. R. Corley, N. Cornish, A. Corsi, S. Cortese, C. A. Costa, R. Cotesta, M. W. Coughlin, S. B. Coughlin, J. P. Coulon, S. T. Countryman, P. Couvares, P. B. Covas, E. E. Cowan, D. M. Coward, M. J. Cowart, D. C. Coyne, R. Coyne, J. D. E. Creighton, T. D. Creighton, J. Cripe, M. Croquette, S. G. Crowder, T. J. Cullen, A. Cumming, L. Cunningham, E. Cuoco, T. Dal Canton, G. Dálya, S. L. Danilishin, S. D'Antonio, K. Danzmann, A. Dasgupta, C. F. Da Silva Costa, L. E. H. Datrier, V. Dattilo, I. Dave, M. Davier, D. Davis, E. J. Daw, D. DeBra, M. Deenadayalan, J. Degallaix, M. De Laurentis, S. Deléglise, W. Del Pozzo, L. M. DeMarchi, N. Demos, T. Dent, R. De Pietri, J. Derby, R. De Rosa, C. De Rossi, R. DeSalvo, O. de Varona, S. Dhurandhar, M. C. Díaz, T. Dietrich, L. Di Fiore, M. Di Giovanni, T. Di Girolamo, A. Di Lieto, B. Ding, S. Di Pace, I. Di Palma, F. Di Renzo, A. Dmitriev, Z. Doctor, F. Donovan, K. L. Dooley, S. Doravari, I. Dorrington, T. P. Downes, M. Drago, J. C. Driggers, Z. Du, J. G. Ducoin, P. Dupej, S. E. Dwyer, P. J. Easter, T. B. Edo, M. C. Edwards, A. Effler, P. Ehrens, J. Eichholz, S. S. Eikenberry, M. Eisenmann, R. A. Eisenstein, R. C. Essick, H. Estelles, D. Estevez, Z. B. Etienne, T. Etzel, M. Evans, T. M. Evans, V. Fafone, H. Fair, S. Fairhurst, X. Fan, S. Farinon, B. Farr, W. M. Farr, E. J. Fauchon-Jones, M. Favata, M. Fays, M. Fazio, C. Fee, J. Feicht, M. M. Fejer, F. Feng, A. Fernandez-Galiana, I. Ferrante, E. C. Ferreira, T. A. Ferreira, F. Ferrini, F. Fidecaro, I. Fiori, D. Fiorucci, M. Fishbach, R. P. Fisher, J. M. Fishner, M. Fitz-Axen, R. Flaminio, M. Fletcher, E. Flynn, H. Fong, J. A. Font, P. W. F. Forsyth, J. D. Fournier, S. Frasca, F. Frasconi, Z. Frei, A. Freise, R. Frey, V. Frey, P. Fritschel, V. V. Frolov, P. Fulda, M. Fyffe, H. A. Gabbard, B. U. Gadre, S. M. Gaebel, J. R. Gair, L. Gammaitoni, M. R. Ganija, S. G. Gaonkar, A. Garcia, C. García-Quirós, F. Garufi, B. Gateley, S. Gaudio, G. Gaur, V. Gayathri, G. Gemme, E. Genin, A. Gennai, D. George, J. George, L. Gergely, V. Germain, S. Ghonge, Abhirup Ghosh, Archisman Ghosh, S. Ghosh, B. Giacomazzo, J. A. Giaime, K. D. Giardina, A. Giazotto, K. Gill, G. Giordano, L. Glover, P. Godwin, E. Goetz, R. Goetz, B. Goncharov, G. González, J. M. Gonzalez Castro, A. Gopakumar, M. L. Gorodetsky, S. E. Gossan, M. Gosselin, R. Gouaty, A. Grado, C. Graef, M. Granata, A. Grant, S. Gras, P. Grassia, C. Gray, R. Gray, G. Greco, A. C. Green, R. Green, E. M. Gretarsson, P. Groot, H. Grote, S. Grunewald, P. Gruning, G. M. Guidi, H. K. Gulati, Y. Guo, A. Gupta, M. K. Gupta, E. K. Gustafson, R. Gustafson, L. Haegel, O. Halim, B. R. Hall, E. D. Hall, E. Z. Hamilton, G. Hammond, M. Haney, M. M. Hanke, J. Hanks, C. Hanna, M. D. Hannam, O. A. Hannuksela, J. Hanson, T. Hardwick, K. Haris, J. Harms, G. M. Harry, I. W. Harry, C. J. Haster, K. Haughian, F. J. Hayes, J. Healy, A. Heidmann, M. C. Heintze, H. Heitmann, P. Hello, G. Hemming, M. Hendry, I. S. Heng, J. Hennig, A. W. Heptonstall, Francisco Hernandez Vivanco, M. Heurs, S. Hild, T. Hinderer, D. Hoak, S. Hochheim, D. Hofman, A. M. Holgado, N. A. Holland, K. Holt, D. E. Holz, P. Hopkins, C. Horst, J. Hough, E. J. Howell, C. G. Hoy, A. Hreibi, E. A. Huerta, D. Huet, B. Hughey, M. Hulko, S. Husa, S. H. Huttner, T. Huynh-Dinh, B. Idzkowski, A. Iess, C. Ingram, R. Inta, G. Intini, B. Irwin, H. N. Isa, J. M. Isac, M. Isi, B. R. Iyer, K. Izumi, T. Jacqmin, S. J. Jadhav, K. Jani, N. N. Janthalur, P. Jaranowski, A. C. Jenkins, J. Jiang, D. S. Johnson, A. W. Jones, D. I. Jones, R. Jones, R. J. G. Jonker, L. Ju, J. Junker, C. V. Kalaghatgi, V. Kalogera, B. Kamai, S. Kandhasamy, G. Kang, J. B. Kanner, S. J. Kapadia, S. Karki, K. S. Karvinen, R. Kashyap, M. Kasprzack, S. Katsanevas, E. Katsavounidis, W. Katzman, S. Kaufer, K. Kawabe, N. V. Keerthana, F. Kéfélian, D. Keitel, R. Kennedy, J. S. Key, F. Y. Khalili, H. Khan, I. Khan, S. Khan, Z. Khan, E. A. Khazanov, M. Khursheed, N. Kijbunchoo, A. X. Kim, Chunglee Kim, J. C. Kim, K. Kim, W. Kim, W. S. Kim, Y. M. Kim, C. Kimball, E. J. King, P. J. King, M. Kinley-Hanlon, R. Kirchhoff, J. S. Kissel, L. Kleybolte, J. H. Klika, S. Klimenko, T. D. Knowles, P. Koch, S. M. Koehlenbeck, G. Koekoek, S. Koley, V. Kondrashov, A. Kontos, N. Koper, M. Korobko, W. Z. Korth, I. Kowalska, D. B. Kozak, V. Kringel, N. Krishnendu,

A. Królak, G. Kuehn, A. Kumar, P. Kumar, R. Kumar, S. Kumar, L. Kuo, A. Kutynia, S. Kwang, B. D. Lackey, K. H. Lai, T. L. Lam, M. Landry, B. B. Lane, R. N. Lang, J. Lange, B. Lantz, R. K. Lanza, S. Larson, A. Lartaux-Vollard, P. D. Lasky, M. Laxen, A. Lazzarini, C. Lazzaro, P. Leaci, S. Leavey, Y. K. Lecoeuche, C. H. Lee, H. K. Lee, H. M. Lee, H. W. Lee, J. Lee, K. Lee, J. Lehmann, A. Lenon, N. Leroy, N. Letendre, Y. Levin, J. Li, K. J. L. Li, T. G. F. Li, X. Li, F. Lin, F. Linde, S. D. Linker, T. B. Littenberg, J. Liu, X. Liu, R. K. L. Lo, N. A. Lockerbie, L. T. London, A. Longo, M. Lorenzini, V. Loriette, M. Lormand, G. Losurdo, J. D. Lough, C. O. Lousto, G. Lovelace, M. E. Lower, H. Lück, D. Lumaca, A. P. Lundgren, R. Lynch, Y. Ma, R. Macas, S. Macfoy, M. MacInnis, D. M. Macleod, A. Macquet, F. Magaña-Sandoval, L. Magaña Zertuche, R. M. Magee, E. Majorana, I. Maksimovic, A. Malik, N. Man, V. Mandic, V. Mangano, G. L. Mansell, M. Manske, M. Mantovani, F. Marchesoni, F. Marion, S. Márka, Z. Márka, C. Markakis, A. S. Markosyan, A. Markowitz, E. Maros, A. Marquina, S. Marsat, F. Martelli, I. W. Martin, R. M. Martin, D. V. Martynov, K. Mason, E. Massera, A. Masserot, T. J. Massinger, M. Masso-Reid, S. Mastrogiovanni, A. Matas, F. Matichard, L. Matone, N. Mavalvala, N. Mazumder, J. J. McCann, R. McCarthy, D. E. McClelland, S. McCormick, L. McCuller, S. C. McGuire, J. McIver, D. J. McManus, T. McRae, S. T. McWilliams, D. Meacher, G. D. Meadors, M. Mehmet, A. K. Mehta, J. Meidam, A. Melatos, G. Mendell, R. A. Mercer, L. Mereni, E. L. Merilh, M. Merzougui, S. Meshkov, C. Messenger, C. Messick, R. Metzdorff, P. M. Meyers, H. Miao, C. Michel, H. Middleton, E. E. Mikhailov, L. Milano, A. L. Miller, A. Miller, M. Millhouse, J. C. Mills, M. C. Milovich-Goff, O. Minazzoli, Y. Minenkov, A. Mishkin, C. Mishra, T. Mistry, S. Mitra, V. P. Mitrofanov, G. Mitselmakher, R. Mittleman, G. Mo, D. Moffa, K. Mogushi, S. R. P. Mohapatra, M. Montani, C. J. Moore, D. Moraru, G. Moreno, S. Morisaki, B. Mours, C. M. Mow-Lowry, Arunava Mukherjee, D. Mukherjee, S. Mukherjee, N. Mukund, A. Mullavey, J. Munch, E. A. Muñiz, M. Muratore, P. G. Murray, A. Nagar, I. Nardecchia, L. Naticchioni, R. K. Nayak, J. Neilson, G. Nelemans, T. J. N. Nelson, M. Nery, A. Neunzert, K. Y. Ng, S. Ng, P. Nguyen, D. Nichols, S. Nissanke, F. Nocera, C. North, L. K. Nuttall, M. Obergaulinger, J. Oberling, B. D. O'Brien, G. D. O'Dea, G. H. Ogin, J. J. Oh, S. H. Oh, F. Ohme, H. Ohta, M. A. Okada, M. Oliver, P. Oppermann, Richard J. Oram, B. O'Reilly, R. G. Ormiston, L. F. Ortega, R. O'Shaughnessy, S. Ossokine, D. J. Ottaway, H. Overmier, B. J. Owen, A. E. Pace, G. Pagano, M. A. Page, A. Pai, S. A. Pai, J. R. Palamos, O. Palashov, C. Palomba, A. Pal-Singh, Huang-Wei Pan, B. Pang, P. T. H. Pang, C. Pankow, F. Pannarale, B. C. Pant, F. Paoletti, A. Paoli, A. Parida, W. Parker, D. Pascucci, A. Pasqualetti, R. Passaquieti, D. Passuello, M. Patil, B. Patricelli, B. L. Pearlstone, C. Pedersen, M. Pedraza, R. Pedurand, A. Pele, S. Penn, C. J. Perez, A. Perreca, H. P. Pfeiffer, M. Phelps, K. S. Phukon, O. J. Piccinni, M. Pichot, F. Piergiovanni, G. Pillant, L. Pinard, M. Pirello, M. Pitkin, R. Poggiani, D. Y. T. Pong, S. Ponrathnam, P. Popolizio, E. K. Porter, J. Powell, A. K. Prajapati, J. Prasad, K. Prasai, R. Prasanna, G. Pratten, T. Prestegard, S. Privitera, G. A. Prodi, L. G. Prokhorov, O. Puncken, M. Punturo, P. Puppo, M. Pürrer, H. Qi, V. Quetschke, P. J. Quinonez, E. A. Quintero, R. Quitzow-James, F. J. Raab, H. Radkins, N. Radulescu, P. Raffai, S. Raja, C. Rajan, B. Rajbhandari, M. Rakhmanov, K. E. Ramirez, A. Ramos-Buades, Javed Rana, K. Rao, P. Rapagnani, V. Raymond, M. Razzano, J. Read, T. Regimbau, L. Rei, S. Reid, D. H. Reitze, W. Ren, F. Ricci, C. J. Richardson, J. W. Richardson, P. M. Ricker, K. Riles, M. Rizzo, N. A. Robertson, R. Robie, F. Robinet, A. Rocchi, L. Rolland, J. G. Rollins, V. J. Roma, M. Romanelli, R. Romano, C. L. Romel, J. H. Romie, K. Rose, D. Rosińska, S. G. Rosofsky, M. P. Ross, S. Rowan, A. Rüdiger, P. Ruggi, G. Rutins, K. Ryan, S. Sachdev, T. Sadecki, M. Sakellariadou, L. Salconi, M. Saleem, A. Samajdar, L. Sammut, E. J. Sanchez, L. E. Sanchez, N. Sanchis-Gual, V. Sandberg, J. R. Sanders, K. A. Santiago, N. Sarin, B. Sassolas, B. S. Sathyaprakash, P. R. Saulson, O. Sauter, R. L. Savage, P. Schale, M. Scheel, J. Scheuer, P. Schmidt, R. Schnabel, R. M. S. Schofield, A. Schönbeck, E. Schreiber, B. W. Schulte, B. F. Schutz, S. G. Schwalbe, J. Scott, S. M. Scott, E. Seidel, D. Sellers, A. S. Sengupta, N. Sennett, D. Sentenac, V. Sequino, A. Sergeev, Y. Setyawati, D. A. Shaddock, T. Shaffer, M. S. Shahriar, M. B. Shaner, L. Shao, P. Sharma, P. Shawhan, H. Shen, R. Shink, D. H. Shoemaker, D. M. Shoemaker, S. ShyamSundar, K. Siellez, M. Sieniawska, D. Sigg, A. D. Silva, L. P. Singer, N. Singh, A. Singhal, A. M. Sintes, S. Sitmukhambetov, V. Skliris, B. J. J. Slagmolen, T. J. Slaven-Blair, J. R. Smith, R. J. E. Smith, S. Somala, E. J. Son, B. Sorazu, F. Sorrentino, T. Souradeep, E. Sowell, A. P. Spencer, A. K. Srivastava, V. Srivastava, K. Staats, C. Stachie, M. Standke, D. A. Steer, M. Steinke, J. Steinlechner, S. Steinlechner, D. Steinmeyer, S. P. Stevenson, D. Stocks, R. Stone, D. J. Stops, K. A. Strain, G. Stratta, S. E. Strigin, A. Strunk, R. Sturani, A. L. Stuver, V. Sudhir, T. Z. Summerscales, L. Sun,

S. Sunil, J. Suresh, P. J. Sutton, B. L. Swinkels, M. J. Szczepańczyk, M. Tacca, S. C. Tait, C. Talbot, D. Talukder, D. B. Tanner, M. Tápai, A. Taracchini, J. D. Tasson, R. Taylor, F. Thies, M. Thomas, P. Thomas, S. R. Thondapu, K. A. Thorne, E. Thrane, Shubhanshu Tiwari, Srishti Tiwari, V. Tiwari, K. Toland, M. Tonelli, Z. Tornasi, A. Torres-Forné, C. I. Torrie, D. Töyrä, F. Travasso, G. Traylor, M. C. Tringali, A. Trovato, L. Trozzo, R. Trudeau, K. W. Tsang, M. Tse, R. Tso, L. Tsukada, D. Tsuna, D. Tuyenbayev, K. Ueno, D. Ugolini, C. S. Unnikrishnan, A. L. Urban, S. A. Usman, H. Vahlbruch, G. Vajente, G. Valdes, N. van Bakel, M. van Beuzekom, J. F. J. van den Brand, C. Van Den Broeck, D. C. Vander-Hyde, J. V. van Heijningen, L. van der Schaaf, A. A. van Veggel, M. Vardaro, V. Varma, S. Vass, M. Vasúth, A. Vecchio, G. Vedovato, J. Veitch, P. J. Veitch, K. Venkateswara, G. Venugopalan, D. Verkindt, F. Vetrano, A. Viceré, A. D. Viets, D. J. Vine, J. Y. Vinet, S. Vitale, T. Vo, H. Vocca, C. Vorvick, S. P. Vyatchanin, A. R. Wade, L. E. Wade, M. Wade, R. Walet, M. Walker, L. Wallace, S. Walsh, G. Wang, H. Wang, J. Z. Wang, W. H. Wang, Y. F. Wang, R. L. Ward, Z. A. Warden, J. Warner, M. Was, J. Watchi, B. Weaver, L. W. Wei, M. Weinert, A. J. Weinstein, R. Weiss, F. Wellmann, L. Wen, E. K. Wessel, P. Weßels, J. W. Westhouse, K. Wette, J. T. Whelan, B. F. Whiting, C. Whittle, D. M. Wilken, D. Williams, A. R. Williamson, J. L. Willis, B. Willke, M. H. Wimmer, W. Winkler, C. C. Wipf, H. Wittel, G. Woan, J. Woehler, J. K. Wofford, J. Worden, J. L. Wright, D. S. Wu, D. M. Wysocki, L. Xiao, H. Yamamoto, C. C. Yancey, L. Yang, M. J. Yap, M. Yazback, D. W. Yeeles, Hang Yu, Haocun Yu, S. H. R. Yuen, M. Yvert, A. K. Zadrożny, M. Zanolin, T. Zelenova, J. P. Zendri, M. Zevin, J. Zhang, L. Zhang, T. Zhang, C. Zhao, M. Zhou, Z. Zhou, X. J. Zhu, M. E. Zucker, J. Zweizig, LIGO Scientific Collaboration, and Virgo Collaboration. A guide to LIGO-Virgo detector noise and extraction of transient gravitational-wave signals. *Classical and Quantum Gravity*, 37(5):055002, March 2020. doi: 10.1088/1361-6382/ab685e.

[8] Everett Schlawin, Jarron Leisenring, Karl Misselt, Thomas P. Greene, Michael W. McElwain, Thomas Beatty, and Marcia Rieke. Jwst noise floor. i. random error sources in jwst nircam time series. *The Astronomical Journal*, 160(5):231, oct 2020. doi: 10.3847/1538-3881/abb811. URL `https://dx.doi.org/10.3847/1538-3881/abb811`.

[9] Carron, J. On the assumption of gaussianity for cosmological two-point statistics and parameter dependent covariance matrices. *A&A*, 551:A88, 2013. doi: 10.1051/0004-6361/201220538. URL `https://doi.org/10.1051/0004-6361/201220538`.

[10] Sylvain Fichet. Taming systematic uncertainties at the lhc with the central limit theorem. *Nuclear Physics B*, 911:623–637, 2016. ISSN 0550-3213. doi: https://doi.org/10.1016/j.nuclphysb.2016.08.029. URL `https://www.sciencedirect.com/science/article/pii/S0550321316302590`.

[11] Mike Shengbo Wang, Will J. Percival, Santiago Avila, Robert Crittenden, and Davide Bianchi. Cosmological inference from galaxy-clustering power spectrum: Gaussianization and covariance decomposition. , 486(1):951–965, June 2019. doi: 10.1093/mnras/stz829.

[12] Guillaume Payeur, Étienne Artigau, Laurence Perreault Levasseur, and René Doyon. Correlated Read Noise Reduction in Infrared Arrays Using Deep Learning. , 163(6):292, June 2022. doi: 10.3847/1538-3881/ac69d2.

[13] Giovanna Giardino, Stephan Birkmann, Massimo Robberto, Pierre Ferruit, Bernard J. Rauscher, Marco Sirianni, Catarina Alves de Oliveira, Torsten Boeker, Nora Luetzgendorf, Maurice te Plate, Elena Puga, and Tim Rawle. The Impact of Cosmic Rays on the Sensitivity of JWST/NIRSpec. , 131(1003):094503, September 2019. doi: 10.1088/1538-3873/ab2fd6.

[14] Nathan D. Miles, Susana E. Deustua, Gonzalo Tancredi, Germán Schnyder, Sergio Nesmachnow, and Geoffrey Cromwell. Using Cosmic Rays Detected by HST as Geophysical Markers. I. Detection and Characterization of Cosmic Rays. , 918(2):86, September 2021. doi: 10.3847/1538-4357/abfa9b.

[15] Mathew S. Madhavacheril, Kendrick M. Smith, Blake D. Sherwin, and Sigurd Naess. CMB lensing power spectrum estimation without instrument noise bias. *arXiv e-prints*, art. arXiv:2011.02475, November 2020. doi: 10.48550/arXiv.2011.02475.

[16] Sukanta Bose, Bernard Hall, Nairwita Mazumder, Sanjeev Dhurandhar, Anuradha Gupta, and Andrew Lundgren. Tackling excess noise from bilinear and nonlinear couplings in gravitational-wave interferometers. *Journal of Physics: Conference Series*, 716(1):012007, may 2016. doi: 10.1088/1742-6596/716/1/012007. URL `https://dx.doi.org/10.1088/1742-6596/716/1/012007`.

[17] Jade Powell, Sarah E. Gossan, Joshua Logue, and Ik Siong Heng. Inferring the core-collapse supernova explosion mechanism with gravitational waves. , 94(12):123012, December 2016. doi: 10.1103/PhysRevD.94.123012.

[18] Barak Zackay, Tejaswi Venumadhav, Javier Roulet, Liang Dai, and Matias Zaldarriaga. Detecting gravitational waves in data with non-stationary and non-Gaussian noise. , 104(6):063034, September 2021. doi: 10.1103/PhysRevD.104.063034.

[19] Hang Yu and Rana X. Adhikari. Nonlinear noise cleaning in gravitational-wave detectors with convolutional neural networks. *Frontiers in Artificial Intelligence*, 5, 2022. ISSN 2624-8212. doi: 10.3389/frai.2022.811563. URL `https://www.frontiersin.org/articles/10.3389/frai.2022.811563`.

[20] Rogier A. Windhorst, Barbara E. Franklin, and Lyman W. Neuschaefer. Removing cosmic-ray hits from multi-orbit hst wide field camera images. *Publications of the Astronomical Society of the Pacific*, 106(701):798, jul 1994. doi: 10.1086/133443. URL `https://dx.doi.org/10.1086/133443`.

[21] Wojtek Pych. A Fast Algorithm for Cosmic-Ray Removal from Single Images. , 116(816): 148–153, February 2004. doi: 10.1086/381786.

[22] Keming Zhang and Joshua S. Bloom. deepCR: Cosmic Ray Rejection with Deep Learning. , 889(1):24, January 2020. doi: 10.3847/1538-4357/ab3fa6.

[23] Yang Song and Stefano Ermon. Generative Modeling by Estimating Gradients of the Data Distribution. *arXiv e-prints*, art. arXiv:1907.05600, July 2019. doi: 10.48550/arXiv.1907.05600.

[24] Jonathan Ho, Ajay Jain, and Pieter Abbeel. Denoising Diffusion Probabilistic Models. *arXiv e-prints*, art. arXiv:2006.11239, June 2020. doi: 10.48550/arXiv.2006.11239.

[25] Yang Song, Jascha Sohl-Dickstein, Diederik P. Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. Score-Based Generative Modeling through Stochastic Differential Equations. *arXiv e-prints*, art. arXiv:2011.13456, November 2020. doi: 10.48550/arXiv.2011.13456.

[26] Olaf Ronneberger, Philipp Fischer, and Thomas Brox. U-Net: Convolutional Networks for Biomedical Image Segmentation. *arXiv e-prints*, art. arXiv:1505.04597, May 2015. doi: 10.48550/arXiv.1505.04597.

[27] Aapo Hyvärinen. Estimation of non-normalized statistical models by score matching. *J. Mach. Learn. Res.*, 6:695–709, dec 2005. ISSN 1532-4435.

[28] Pascal Vincent. A connection between score matching and denoising autoencoders. *Neural Comput.*, 23(7):1661–1674, 2011. doi: 10.1162/NECO_a_00142. URL `https://doi.org/10.1162/NECO_a_00142`.

[29] Guillaume Alain and Yoshua Bengio. What regularized auto-encoders learn from the data-generating distribution. *J. Mach. Learn. Res.*, 15(1):3563–3593, jan 2014. ISSN 1532-4435.

[30] Radford Neal. MCMC Using Hamiltonian Dynamics. In *Handbook of Markov Chain Monte Carlo*, pages 113–162. 2011. doi: 10.1201/b10905.

[31] Benjamin Remy, Francois Lanusse, Niall Jeffrey, Jia Liu, Jean-Luc Starck, Ken Osato, and Tim Schrabback. Probabilistic Mass Mapping with Neural Score Estimation. *arXiv e-prints*, art. arXiv:2201.05561, January 2022. doi: 10.48550/arXiv.2201.05561.

[32] Pablo Lemos, Adam Coogan, Yashar Hezaveh, and Laurence Perreault-Levasseur. Sampling-Based Accuracy Testing of Posterior Estimators for General Inference. *submitted to ICML*, January 2023.

A Supplementary Material

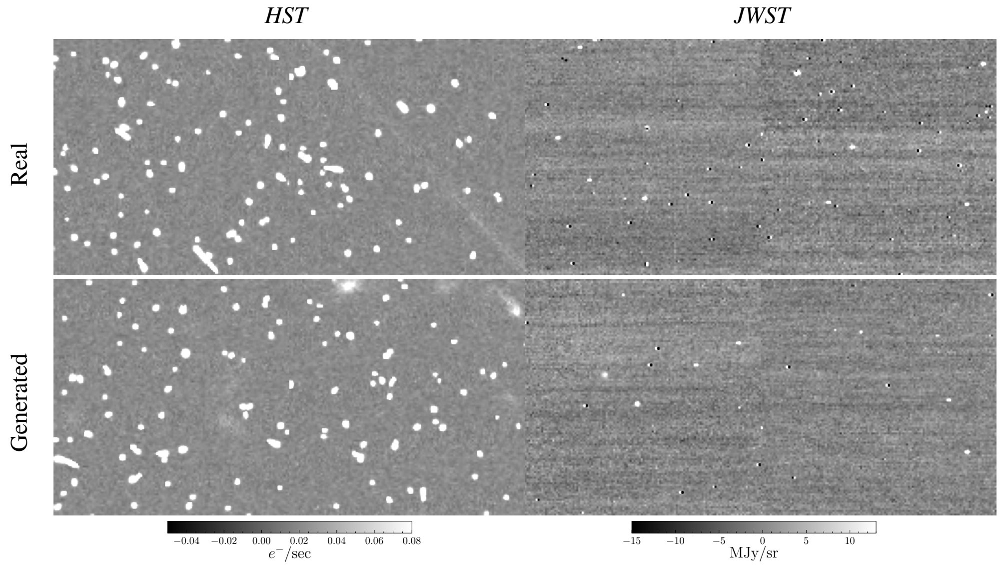

Figure 3: *Top row*: Examples of noise patches from the datasets. *Bottom row*: Examples of noise patches generated using the score models used in this work. These examples are generated by solving a reverse-time variance-exploding stochastic differential (VESDE, [25]) equation with 1000 steps of the discretized Euler-Maruyama solver.

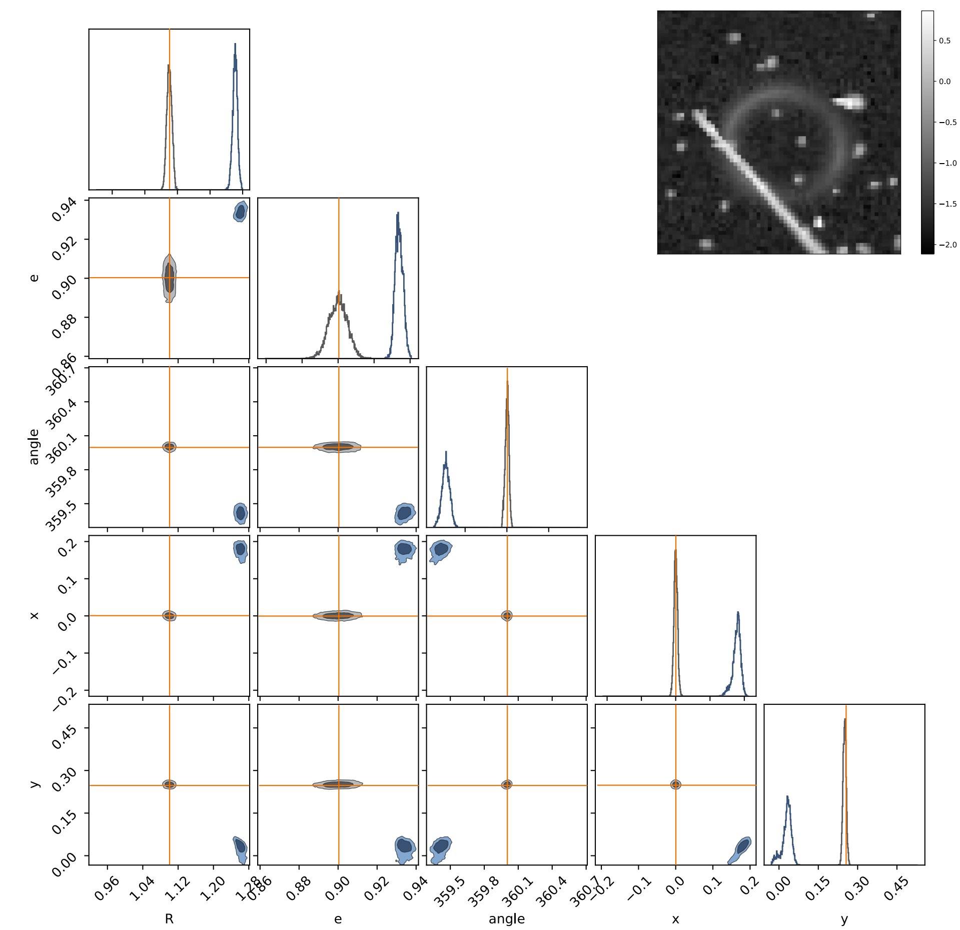

Figure 4: Same as Figure 1 but performed on *HST* data (image intensity is shown in $\log_{10}$).