

000 001 002 003 004 005 IMLP: AN ENERGY-EFFICIENT CONTINUAL LEARNING 006 METHOD FOR TABULAR DATA STREAMS 007 008 009

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ABSTRACT

Tabular data streams are rapidly emerging as a dominant modality for real-time decision-making in healthcare, finance, and the Internet of Things (IoT). These applications commonly run on edge and mobile devices, where energy budgets, memory, and compute are strictly limited. Continual learning (CL) addresses such dynamics by training models sequentially on task streams while preserving prior knowledge and consolidating new knowledge. While recent CL work has advanced in mitigating catastrophic forgetting and improving knowledge transfer, the practical requirements of energy and memory efficiency for tabular data streams remain underexplored. In particular, existing CL solutions mostly depend on replay mechanisms whose buffers grow over time and exacerbate resource costs.

We propose a *context-aware incremental Multi-Layer Perceptron (IMLP)*, a compact continual learner for tabular data streams. IMLP incorporates a windowed scaled dot-product attention over a sliding latent feature buffer, enabling constant-size memory and avoiding storing raw data. The attended context is concatenated with current features and processed by shared feed-forward layers, yielding lightweight per-segment updates. We evaluate IMLP against state-of-the-art (SOTA) tabular models on real-world concept drift benchmark tabular datasets designed to assess models under temporal distribution shifts. Compared to TabPFNv2 under the *incremental* concept drift, IMLP has 22.7% total energy reduction while only a 0.05 final balanced accuracy drop. The results show that the proposed attention-based feature memory design can effectively guide the energy consumption while achieving the highest final accuracy in the abrupt concept drifts among all network baselines.

1 INTRODUCTION

Tabular data, structured as a collection of features and instances, is one of the most common and practical data types in practical machine learning applications, [for example, in both high-stakes domains and lower-stakes domains \(Amrollahi et al., 2022; Ramjattan et al., 2024; Li et al., 2025b\)](#). As such domains increasingly rely on streaming data sources, tabular data streams are gaining significant attention due to their ability to capture continuous, real-time updates rather than static snapshots (Borisov et al., 2022). In particular, most such scenarios often occur on edge devices, IoT systems, and mobile platforms, where energy budgets, battery life, and computational resources are severely constrained Chang et al. (2021).

To tackle those real-world dynamics, Continual Learning (CL) (Wang et al., 2024a), also referred to as lifelong learning (Lee & Lee, 2020), enables models to incrementally acquire, update, accumulate, and exploit knowledge over time. While significant progress has been made on overcoming catastrophic forgetting (Kemker et al., 2018; Li et al., 2019; Bhat et al., 2022) and knowledge transfer (Ke et al., 2021; Li et al., 2024; Shi et al., 2024a), much less is known about their computational analysis and energy efficiency (Li et al., 2023; Trinci et al., 2024).

Energy-efficient continual learning (EECL) has become a practical necessity for real-world applications that [require real-time adaptation](#) on resource-constrained platforms (Chavan et al., 2023; Shi et al., 2024b; Trinci et al., 2024; Xiao et al., 2024). Meanwhile, most CL progress to date targets image (Trinci et al., 2024; Chavan et al., 2023; Shi et al., 2024b) and language tasks (Li et al., 2025a; Wang et al., 2024b). In contrast, tabular data streams remain underexplored. Tabular models that

054 excel on static datasets do not transfer directly to non-stationary streams with tight memory, compute,
 055 and energy budgets. Existing CL methods rarely target these constraints. In particular, replay-based
 056 strategies rely on buffers that grow over time, increasing storage and compute, and hindering on-
 057 device deployment. This gap motivates methods for tabular streaming CL that sustain accuracy under
 058 distribution shift while operating at low energy cost, with fixed memory, and without storing raw
 059 examples. **Moreover, trade-offs between energy consumption and predictive performance matter in**
 060 **lower-stakes domains, especially when the cost of electricity is taken into account.** Achieving this
 061 under strict resource budgets while mitigating catastrophic forgetting remains a central challenge for
 062 Green AI (Henderson et al., 2020; Bouza et al., 2023; Trinci et al., 2024; Rózycki et al., 2025).

063 This paper introduces *Incremental Multi-Layer Perceptron (IMLP)*, a novel method for energy-
 064 efficient continual learning, particularly focusing on tabular data streams. IMLP augments a simple
 065 MLP with self-attention capabilities, while maintaining efficiency in compute, memory, and energy
 066 usage. To be specific: 1) IMLP employs a windowed scaled dot-product attention with a sliding
 067 feature buffer, enabling the model to adaptively attend to the most relevant parts of the stream while
 068 storing only latent features without needing to revisit raw historical data. 2) The resulting attended
 069 representation is concatenated and passed through two shared feed-forward layers followed by a
 070 classifier head, serving as the MLP learner for classification tasks. This design avoids the unbounded
 071 **memory** growth inherent to replay baselines (Rebuffi et al., 2017; Li & Hoiem, 2017; Lopez-Paz &
 072 Ranzato, 2017), while remaining computationally lightweight on resource-constrained devices. **To**
 073 **evaluate hardware-grounded energy-accuracy trade-offs in CL on tabular data streams, we provide**
 074 **quantitative Pareto AUC and global efficiency analysis.**

075 2 RELATED WORK

076 Traditional tabular data models can be roughly categorized into three main groups: Gradient-Boosted
 077 Decision Trees (GBDTs) (Friedman, 2001), Neural Networks (NNs) (Goodfellow et al., 2016), and
 078 classic models (e.g., SVMs (Cortes & Vapnik, 1995), k-NN (Cover & Hart, 1967), linear model (Cox,
 079 1958), and simple decision trees (Loh, 2011)).

080 **GBDTs and their variants for CL.** Traditional GBDTs such as XGBoost (Chen & Guestrin, 2016),
 081 LightGBM (Ke et al., 2017), and CatBoost (Prokhorenkova et al., 2019) remain strong baselines
 082 for tabular classification due to their efficiency and robustness, especially on large or irregular static
 083 datasets. However, they are not naturally suited for **CL**: (1) new data typically requires retraining
 084 from scratch, since tree splits and boosting weights depend on the full dataset (Chen & Guestrin,
 085 2016; Ke et al., 2017; Prokhorenkova et al., 2019); (2) without access to past data, models trained
 086 only on new samples overwrite previous knowledge, causing catastrophic forgetting (Wang et al.,
 087 2024a); and (3) unlike **NNs**, GBDTs lack mechanisms for knowledge transfer across tasks (Ke et al.,
 088 2021; Parisi et al., 2019; De Lange et al., 2021). Extensions such as online bagging and boosting (Oza
 089 & Russell, 2001) or warm-starting (Pedregosa et al., 2011), and adaptive XGBoost (Montiel et al.,
 090 2020), partially mitigate these issues, but remain limited in long-term knowledge retention due to the
 091 lack of representation reuse, especially when compared to neural **CL** methods.

092 **Classic models in CL.** Both standard SVMs (Cortes & Vapnik, 1995) and decision trees (Loh, 2011)
 093 are batch learners, requiring retraining on the full dataset when new tasks arrive. SVMs can be
 094 extended to **CL** through incremental or online variants such as incremental SVM (Cauwenberghs &
 095 Poggio, 2000), LASVM (Bordes et al., 2005), and NORMA (Kivinen et al., 2004), which handle
 096 streaming updates but still face challenges with scalability, memory growth, and forgetting. k-
 097 NNs (Cover & Hart, 1967) trivially avoid forgetting if all data is stored, but this violates the constraint
 098 of no access to past raw inputs and is impractical under resource limits. Linear models (Cox, 1958)
 099 are efficient but prone to forgetting under distribution shifts, as updates overwrite prior knowledge.
 100 Incremental decision trees, such as Hoeffding Trees (Domingos & Hulten, 2000), and streaming
 101 ensembles (Bifet et al., 2010; Gomes et al., 2017) can adapt to data streams without full retraining.
 102 **Still**, their accuracy degrades under severe drift, **since** they lack strong representation learning, and
 103 ensemble methods can be computationally expensive.

104 **Neural models in CL.** Recent studies demonstrate that advanced NNs (Zabërgja et al., 2024; Arik
 105 & Pfister, 2021; Kadra et al., 2021; Gorishniy et al., 2023a; Hollmann et al., 2025b; Ye et al., 2024;
 106 Gorishniy et al., 2024) can surpass GBDTs on static tabular data in certain regimes, e.g., with well-
 107 regularized MLPs (Kadra et al., 2021), attention-based models such as SAINT (Somepalli et al., 2021),

108 or meta-learned foundation models like TabPFN and its variants (Hollmann et al., 2025b). While
 109 their training is typically computationally intensive than that of GBDTs unless carefully tuned (Kadra
 110 et al., 2021), NNs are generally better suited for streaming data, owing to their rich representations,
 111 incremental updates via stochastic gradient descent, and flexible architectures. However, vanilla NNs
 112 still suffer from catastrophic forgetting in the absence of CL strategies (Wang et al., 2024a).

113 **CL strategies with neural models.** In NNs, CL strategies are commonly categorized into
 114 regularization-based approaches (Kirkpatrick et al., 2017; Zenke et al., 2017), replay-based strate-
 115 gies (Rebuffi et al., 2017; Shin et al., 2017), attention-based retrieval mechanisms (Chaudhry et al.,
 116 2019; Aljundi et al., 2017), and architectural methods (Rusu et al., 2016). Regularization-based
 117 methods, such as EWC (Kirkpatrick et al., 2017), SI (Zenke et al., 2017), MAS (Aljundi et al., 2017),
 118 and LwF (Li & Hoiem, 2016), mitigate forgetting by constraining updates to parameters deemed
 119 important for previously learned tasks. Replay-based strategies, including iCaRL (Rebuffi et al.,
 120 2017) and generative replay (Shin et al., 2017), maintain past knowledge by rehearsing stored samples
 121 or synthetic data. Attention-based retrieval mechanisms, such as A-GEM with attention (Chaudhry
 122 et al., 2019) and attentive experience replay (Aljundi et al., 2017), employ attention to prioritize and
 123 retrieve relevant past experiences. Architectural methods, exemplified by PNNs (Rusu et al., 2016),
 124 expand model capacity by freezing previously trained components and introducing new modules for
 125 incoming tasks.

126 Despite recent progress, energy-efficient CL for tabular data streams remains largely unexplored (Chav-
 127 an et al., 2023; Trinci et al., 2024). Real-world tables frequently undergo domain drift (e.g., quarterly
 128 finance transactions, evolving sensor logs, healthcare data) without changes to the label space. **Yet, no**
 129 **standardized domain-incremental learning benchmark that considers energy-performance trade-offs**
 130 currently exists for tabular streams. Moreover, pre-trained transformers for tabular data (Gorishniy
 131 et al., 2023b; Hollmann et al., 2025b) and feature-level or attention-based CL strategies (Pelle-
 132 grini et al., 2020; Vaswani et al., 2017a; Jha et al., 2023) show promise for low-storage, privacy-
 133 preserving CL, but their effectiveness under domain drift has not been systematically evaluated.
 134 Here, we bridge this gap by introducing our method, establishing fair comparisons, and quantifying
 135 energy–performance trade-offs.

3 IMLP: AN INCREMENTAL MLP FOR TABULAR DATA STREAMS

139 Owing to the general difficulty and diversity of challenges in CL, we focus on a simplified task
 140 incremental learning setting (Parisi et al., 2019; De Lange et al., 2021). In this setting, a model is
 141 trained on a sequence of tasks $\{\mathcal{T}_t\}_{t=1}^T$, where the data for each task arrives incrementally at time t .

3.1 PROBLEM STATEMENT

145 **Problem Setup.** We consider a sequence of tasks $\{\mathcal{T}_t\}_{t=0}^T$, where the training data for each task
 146 arrives incrementally at time t . Each task \mathcal{T}_t is associated with data $(\mathcal{X}_t, \mathcal{Y}_t)$ randomly drawn from
 147 distribution \mathcal{D}_t , where \mathcal{X}_t denotes the set of data samples and \mathcal{Y}_t is the corresponding ground truth
 148 labels. Our goal is to design an incremental learner f_θ that updates online and minimizes the expected
 149 risk $\hat{L}_t(\theta)$ across all observed tasks, with limited or no access to the data from earlier tasks $t < T$,
 150

$$151 \hat{L}_t(\theta) := \sum_{t=0}^T \mathbb{E}_{(\mathcal{X}_t, \mathcal{Y}_t) \sim \mathcal{D}_t} [\ell_t(\theta)], \quad (1)$$

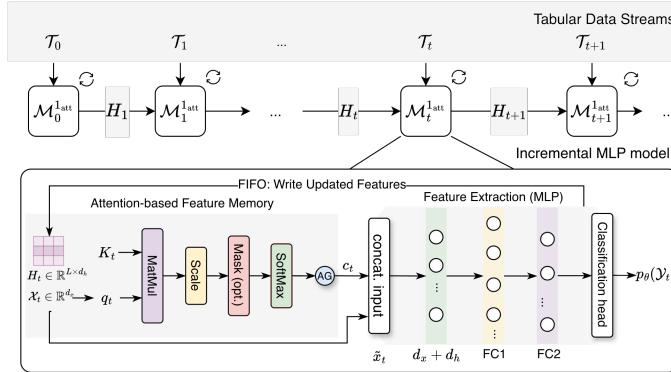
154 where $\ell_t(\theta)$ represents the loss function of the model $f_\theta(\mathcal{X}_t, H_t)$ with input \mathcal{X}_t , parameter θ , and the
 155 historical features H_t at time t . Additionally, we aim to achieve energy efficiency.

157 **Assumptions.** We formalize this with standard non-convex optimization assumptions for NNs.

158 **(A1)** *There exists $R_{\mathcal{X}} > 0$ such that $\|\mathcal{X}_t\|_2 \leq R_{\mathcal{X}}$ for all samples in the arrived stream \mathcal{T}_t .*

159 **(A2)** *The precomputed latent features are ℓ_2 -normalized, i.e., $\|h_{t,j}\|_2 \leq 1$ for all t, j .*

160 **(A3)** *Training is performed with weight decay and early stopping, so that for some $R_\theta > 0$,
 161 $\|\theta\|_2 \leq R_\theta$ throughout optimization.*

162 3.2 ARCHITECTURE OVERVIEW
163164 For efficient learning from the current task \mathcal{T}_t while maintaining performance on previously learned
165 tasks, we propose an incremental multi-layer perceptron (IMLP) architecture, as shown in Figure 1.
166 We employ two strategies: (1) processing each task with an augmented MLP learner module $\mathcal{M}^{1_{\text{att}}}$
167180 Figure 1: IMLP architecture. IMLP sequentially takes \mathcal{T}_t as raw input and outputs predictive
181 performance $p_\theta(\mathcal{Y}_t)$.182 that incorporates limited historical context H_t in a window size W through a variant of scaled
183 dot-product attention (Vaswani et al., 2017b); and (2) maintaining an FIFO feature buffer with fixed
184 memory over time to handle the concept drifts (Hoens et al., 2012), which facilitates representation
185 reuse while keeping memory and computation cost constrained as new data evolves.186 Given the current input $\mathcal{X}_t \in \mathbb{R}^{d_x}$, a hidden dimension d_h , and a FIFO memory of the past L features.
187 We denote by $H_t = [h_{t,1}, \dots, h_{t,L}]^\top \in \mathbb{R}^{L \times d_h}$ the matrix that stacks the latent features in the
188 window associated with time t , with learnable maps $W_q \in \mathbb{R}^{d_h \times d_x}$ and $W_k \in \mathbb{R}^{d_h \times d_h}$, IMLP forms
189

190
$$q_t = W_q \mathcal{X}_t + b_q \in \mathbb{R}^{d_h}, \quad (2)$$

191
$$K_t = H_t W_k^\top + \mathbf{1}_L b_k^\top \in \mathbb{R}^{L \times d_h}, \quad (3)$$

192
$$s_t = K_t q_t \in \mathbb{R}^L, \quad (4)$$

193
$$\tilde{s}_t = \frac{1}{\sqrt{d_h}} s_t, \quad (5)$$

194
$$\alpha_t = \text{softmax}(\tilde{s}_t) \in \mathbb{R}^L, \quad (6)$$

195 where

196
$$\alpha_{t,j} = \frac{\exp(\tilde{s}_{t,j})}{\sum_{j=1}^L \exp(\tilde{s}_{t,j})} \quad \text{for } j = 1, \dots, L (L \leq W), \quad (7)$$

197 and $\mathbf{1}_L \in \mathbb{R}^L$ is the all-ones vector used to broadcast the bias b_k to all L rows. $h_{t,j} \in \mathbb{R}^{d_h}$ is the
198 feature vector of the j -th most recent sample before \mathcal{X}_t (with $j = 1$ being the most recent). The j -th
199 row $K_{t,j} \in \mathbb{R}^{d_h}$ is the key for the j -th past feature in the window.200 The attention-based feature memory is the weighted sum of the keys $c_t = \alpha_t^\top K_t$. Finally, IMLP
201 concatenates the context with the current input $\tilde{x}_t = [\mathcal{X}_t, c_t]^\top \in \mathbb{R}^{d_x + d_h}$ and then feeds it to the
202 feature extractor $f_{t,\theta} = \phi(\tilde{x}_t) \in \mathbb{R}^{d_h}$, where $\phi(\cdot)$ is a two-layer MLP, and the classifier outputs
203 $\hat{y}_t = W_{cls} \cdot f_{t,\theta} + b_{cls}$ where $W_{cls} \in \mathbb{R}^{C \times d_h}$ and b_{cls} denotes the weight matrix and bias, respectively.
204 This corresponds to the model's performance $p_\theta(\mathcal{Y}_t)$ at the time t .205 In the following, we detail the properties of the proposed attention-based feature memory design to
206 achieve EECL over tabular data streams.213 3.3 CONVERGENCE ANALYSIS OF IMLP
214215 Let $\mathcal{A}(\mathcal{X}_t, H_t; \theta_{\text{att}}) := c_t$ denote the attention-based context, where $\theta_{\text{att}} = (W_q, b_q, W_k, b_k)$ and H_t
216 collects the latent features in the window at the time t .

216 **Lemma 3.1** (Bounded context vector). *Under (A2), there exists $B_c > 0$ (depending only on W_k and
217 b_k) such that $\|c_t\|_2 \leq B_c$ for all i .*

218 **Lemma 3.2** (Smooth attention map). *Under (A1)–(A3), the map $(\mathcal{X}_t, H_t, \theta_{\text{att}}) \mapsto \mathcal{A}(\mathcal{X}_t, H_t; \theta_{\text{att}})$
219 is continuously differentiable, and its Jacobian with respect to θ_{att} is bounded on the compact set
220*

$$221 \quad \mathcal{K} := \{(\mathcal{X}_t, H_t, \theta_{\text{att}}) : \|\mathcal{X}_t\|_2 \leq R_{\mathcal{X}}, \|h_{t,j}\|_2 \leq 1, \|\theta_{\text{att}}\|_2 \leq R_{\theta}\}. \quad (8)$$

222 *In particular, there exists $L_{\text{att}} > 0$ such that*

$$224 \quad \|\mathcal{A}(\mathcal{X}_t, H_t; \theta_{\text{att}}^{(1)}) - \mathcal{A}(\mathcal{X}_t, H_t; \theta_{\text{att}}^{(2)})\|_2 \leq L_{\text{att}} \|\theta_{\text{att}}^{(1)} - \theta_{\text{att}}^{(2)}\|_2, \quad (9)$$

225 *for all $\theta_{\text{att}}^{(1)}, \theta_{\text{att}}^{(2)} \in \mathcal{K}$.*

227 Correspondingly, the full network can be written as

$$229 \quad f_{\theta}(\mathcal{X}_t, H_t) := W_{\text{cls}} \phi_{\theta}([\mathcal{X}_t; \mathcal{A}(\mathcal{X}_t, H_t; \theta_{\text{att}})]) + b_{\text{cls}}, \quad (10)$$

230 where ϕ_{θ} is the two-layer ReLU feature extractor, and the per-sample loss is

$$231 \quad \ell_t(\theta) = \text{CE}(\text{softmax}(f_{\theta}(\mathcal{X}_t, H_t)), \mathcal{Y}_t). \quad (11)$$

234 **Lemma 3.3** (Smooth network and loss). *Under (A1)–(A3), $f_{\theta}(\mathcal{X}_t, H_t)$ is continuously differentiable
235 with bounded Jacobian on $\{\theta : \|\theta\|_2 \leq R_{\theta}\}$, and $\ell_t(\theta)$ has Lipschitz-continuous gradient on the
236 same set.*

237 For a fixed segment (task) \mathcal{T}_t with data $(\mathcal{X}_t, H_t, \mathcal{Y}_t)$, where $\{(\mathcal{X}_{t,i}, H_{t,i}, \mathcal{Y}_{t,i})\}_{i=1}^{n_t}$ denotes the samples
238 in this segment, we define $\hat{\ell}_t(\theta) := \frac{1}{n_t} \sum_{i=1}^{n_t} \ell_{t,i}(\theta)$.

240 **Theorem 3.4** (Segment-wise smooth empirical loss). *Under (A1)–(A3), the empirical loss $\hat{\ell}_t(\theta)$
241 is (i) bounded below; (ii) continuously differentiable on $\{\theta : \|\theta\|_2 \leq R_{\theta}\}$; and (iii) has Lipschitz-
242 continuous gradient on this compact set.*

243 **Corollary 3.5** (Per-segment convergence of IMLP). *Consider optimizing $\hat{\ell}_t(\theta)$ with a stochastic
244 first-order method (e.g., SGD or Adam) under standard step-size conditions and weight decay, as in
245 our training loop. Then the iterates on segment \mathcal{T}_t converge to a first-order stationary point in the
246 sense that*

$$247 \quad \|\nabla \hat{\ell}_t(\theta_k)\|_2 \rightarrow 0 \quad \text{as } k \rightarrow \infty, \quad (12)$$

249 *or, in the practical finite-epoch setting, reach a parameter θ^* with small gradient norm ε . In
250 particular, the attention-based feature memory acts as a bounded, smooth transformation of a finite
251 latent window, so IMLP behaves like a standard MLP with an augmented input \tilde{s}_t and inherits the
252 usual segment-wise convergence guarantees of non-convex deep networks.*

253 **Remark on non-stationary streams.** The analysis above is *segment-wise*. Under standard
254 assumptions, the attention-based feature memory yields a bounded, smooth network with
255 Lipschitz-continuous gradients, so first-order optimizers converge to a stationary point of the
256 empirical loss on each fixed segment \mathcal{T}_t . However, we do not claim convergence to any global
257 limit when the data-generating process is non-stationary across t . Instead, the theory guarantees
258 that, conditional on the data observed in each segment, the optimization problem remains
259 well-behaved despite using attention over a finite feature memory.

261 *Proofs of Lemmas 3.1–3.3 and Theorem 3.4 are given in the appendix A.*

265 3.4 FIFO ATTENTION-BASED FEATURE MEMORY AND TIME COMPLEXITY

267 Unlike replay buffers that grow with the number of seen samples, our FIFO memory has constant
268 memory complexity in time. The attention module adds the query layer $W_q \in \mathbb{R}^{d_h \times d_x}, b_q \in$
269 \mathbb{R}^{d_h} and key layer $W_k \in \mathbb{R}^{d_h \times d_h}, b_k \in \mathbb{R}^{d_h}$, hence, the parameter memory for attention denotes
 $\mathcal{O}(d_x \cdot d_h + d_h^2)$, which is constant with respect to the stream length and number of tasks.

270 At time t , the FIFO buffer stores W latent feature vectors $H_t \in \mathbb{R}^{W \times d_h}$, it costs $\mathcal{O}(Wd_h)$ memory
 271 per stream, independent of how long the stream has run. In the batched implementation, each FIFO
 272 buffer holds W feature tensors, each of shape $[B, d_h]$; hence, the runtime memory overhead is
 273 $\mathcal{O}(BWd_h)$.

274 For a single forward step with batch size B , the total computational efficiency of IMLP is given
 275 by query computation $\mathcal{O}(Bd_x d_h)$, key computation $\mathcal{O}(BWd_h^2)$, attention scores and weights
 276 $\mathcal{O}(Bd_x d_h + BWd_h^2 + BWd_h)$, as well as the rest of the network $\mathcal{O}(B(d_x + d_h) \cdot 512)$, where 512
 277 is the feature dimension of the FC1, FC2 layer in the feature extraction module using MLP. Therefore,
 278 the incremental cost scales linearly in B and W , its per-step computational cost scales as
 279

$$280 \mathcal{O}(Bd_h(d_x + Wd_h)) = \underbrace{\mathcal{O}(Bd_h(d_x + Wd_h))}_{\text{query and key}} + \underbrace{\mathcal{O}(Bd_h(d_x + Wd_h + W))}_{\text{attention scores and weights}} + \underbrace{\mathcal{O}(B(d_x + d_h)512)}_{\text{feature MLP}}, \quad (13)$$

$$281$$

282 where for a fixed B and d_h , the incremental overhead of IMLP over a vanilla MLP is controlled and
 283 linear in the window size W .

284 Therefore, FIFO attention-based feature memory adds $\mathcal{O}(d_x d_h + d_h^2)$ parameters and $\mathcal{O}(BWd_h)$
 285 runtime memory, while its per-step computational cost scales as $\mathcal{O}(Bd_h(d_x + Wd_h))$, yielding
 286 constant memory in time with respect to the length of the data stream.

288 3.5 ENERGY EFFICIENCY ANALYSIS OF IMLP

290 **Energy model and assumptions.** We assume that for a fixed device and implementation, energy
 291 is approximately linear in the number of floating-point operations (FLOPs), up to device-specific
 292 constants and small overhead. Let F_{train} denote the number of FLOPs required to perform one
 293 forward-and-backward pass of IMLP on a single sample. We adopt a standard abstract energy model
 294 with the following assumptions:

295 **(A4)** *On a fixed hardware platform (GPU/CPU), there exist constants $0 < \eta_{\min} \leq \eta_{\max}$ such that
 296 the energy consumed per FLOP lies in $[\eta_{\min}, \eta_{\max}]$.*

297 **(A5)** *The additional system overhead per training step (e.g., kernel launches, bookkeeping) is
 298 bounded by a constant E_0 independent of the sample index and segment.*

299 **Lemma 3.6** (FLOP complexity per sample). *Let C be the number of classes and d_{in} be the input
 300 dimension. For a single sample $(\mathcal{X}_{t,i}, H_{t,i}, \mathcal{Y}_{t,i})$, the FLOP count of a forward-and-backward step of
 301 IMLP satisfies*

$$302 F_{\text{train}} \leq K_{\text{arch}}(d_{\text{in}} d_h + Wd_h^2 + d_h^2 + d_h C), \quad (14)$$

$$303$$

304 for an architecture-dependent constant $K_{\text{arch}} > 0$ that does not depend on n_t or t .

306 **Theorem 3.7** (Per-segment energy complexity bound). *Consider a segment \mathcal{T}_t with n_t training
 307 samples. Under (A4)-(A5) and Lemma 3.6, the total training energy consumed by IMLP on this
 308 segment satisfies*

$$310 E_t^{\text{train}} \leq C_{\text{train}} E_{\max} n_t (d_{\text{in}} d_h + Wd_h^2 + d_h^2 + d_h C) + C_0, \quad (15)$$

$$311$$

312 for some hardware- and implementation-dependent constants $C_{\text{train}} > 0$ and $C_0 \geq 0$. Similarly, the
 313 inference energy on the test set of size n^{test} admits

$$314 E_t^{\text{infer}} \leq C_{\text{infer}} n^{\text{test}} (d_{\text{in}} d_h + Wd_h^2 + d_h^2 + d_h C) + C'_0, \quad (16)$$

$$315$$

316 with another constant $C_{\text{infer}} > 0$ and overhead $C'_0 \geq 0$.

317 Proofs of Lemma 3.6 and Theorem 3.7 are given in the appendix B.

318 **Corollary 3.8** (Energy complexity over the full non-stationary stream). *Let the data stream be
 319 partitioned into T segments $\{\mathcal{T}_t\}_{t=0}^T$ with sizes $\{n_t\}_{t=1}^T$. Under the same assumptions as Theorem 3.7,
 320 the total training energy over the entire stream satisfies*

$$322 E_{\text{total}}^{\text{train}} = \sum_{t=1}^T E_t^{\text{train}} \leq C_{\text{train}} E_{\max} \left(\sum_{t=1}^T n_t \right) (d_{\text{in}} d_h + Wd_h^2 + d_h^2 + d_h C) + T C_0, \quad (17)$$

$$323$$

324 and the total inference energy satisfies
 325

$$326 E_{\text{total}}^{\text{infer}} \leq C_{\text{infer}} n_{\text{total}}^{\text{test}} (d_{\text{in}} d_h + W d_h^2 + d_h^2 + d_h C) + T C'_0. \quad (18)$$

327 Theoretically, for a fixed d_h , W , and E_{max} , both training and inference energy grow at most linearly
 328 in the total number of processed examples $\sum_t n_t$ and in the effective model size. The attention-based
 329 feature memory only adds the bounded term $W d_h^2$ and does not change this linear energy scaling.
 330

331 **Remark on the energy complexity bounds.** The bounds above explain two aspects of our
 332 empirical observations: (1) on a fixed device, IMLP has a predictable energy profile, scaling
 333 linearly with the number of samples and epochs; and (2) The attention-based feature memory
 334 contributes a controlled overhead proportional to $W d_h^2$, which remains small in our IMLP
 335 because W and d_h are fixed. Our measured Joule values are therefore consistent with an energy
 336 complexity that is linear in the stream size, and the theoretical bounds clarify that this behavior
 337 is not specific to a particular dataset, but a structural property of the IMLP architecture and
 338 training procedure.

342 4 ENERGY-ACCURACY TRADE-OFFS

344 In many optimization problems, objectives are inherently conflicting; for instance, improving the
 345 accuracy of a NN increases energy consumption or latency. A classical way to study such trade-offs
 346 is through Pareto front analysis (Giakiozis & Fleming, 2014).

347 Our convergence and energy bounds naturally lead to a bi-objective viewpoint, where we jointly
 348 consider predictive performance and energy consumption. For a fixed segment \mathcal{T}_t , an IMLP configu-
 349 ration is determined by its architecture (d_h, W) and optimization budget (e.g., number of iterations,
 350 learning-rate schedule). Each such configuration yields a pair $(E_t(\theta), P_t(\theta))$, where $E_t(\theta)$ denotes
 351 the total energy consumed on \mathcal{T}_t and $P_t(\theta) = p_\theta(\mathcal{Y}_t)$ denotes the resulting segment-wise model
 352 performance (e.g., balanced accuracy). We say that a configuration $\theta^{(1)}$ *Pareto-dominates* $\theta^{(2)}$ if
 353

$$354 E_t(\theta^{(1)}) \leq E_t(\theta^{(2)}), \quad P_t(\theta^{(1)}) \geq P_t(\theta^{(2)}), \quad (19)$$

355 with at least one strict inequality. The *Pareto set*

$$357 \mathcal{P}_t := \{\theta : \nexists \theta' \text{ s.t. } E_t(\theta') \leq E_t(\theta), P_t(\theta') \geq P_t(\theta) \text{ and one inequality is strict}\} \quad (20)$$

358 collects all Pareto-efficient configurations, and its image in the energy–accuracy plane forms the
 359 *Pareto frontier*.

361 Intuitively, on each segment \mathcal{T}_t , our smoothness and Lipschitz-gradient assumptions imply that
 362 stochastic first-order methods require on the order of $1/\varepsilon^2$ iterations to reach an ε -stationary point
 363 of the empirical loss $\hat{L}_t(\theta)$, i.e., $\|\nabla \hat{L}_t(\theta_k)\|_2 \leq \varepsilon$. Each iteration has a bounded computational cost
 364 proportional to $d_{\text{in}} d_h + W d_h^2 + d_h^2 + d_h C$ (Lemma 3.6), and our energy model (Theorem 3.7) shows
 365 that energy is proportional to this cost up to device-dependent constants. Combining these results
 366 yields the scaling

$$367 E_t^{\text{train}}(\varepsilon) = \mathcal{O}\left(\frac{d_{\text{in}} d_h + W d_h^2 + d_h^2 + d_h C}{\varepsilon^2}\right). \quad (21)$$

369 Thus, reducing the optimization tolerance ε leads to a more than linear increase in training energy,
 370 with the rate governed by the architectural parameters (d_h, W) . For any fixed architecture, attainable
 371 pairs $(E_t(\theta), P_t(\theta))$ therefore lie on or above a decreasing curve in the energy–accuracy plane:
 372 improving performance inevitably requires a disproportionately large increase in energy.

374 5 EXPERIMENTS

375 **Setup and Configuration.** All experiments were conducted on a single workstation equipped with an
 376 Intel® Core™ i5-8600K Processor, a NVIDIA GeForce RTX 2080 Ti GPU, 16GB DDR4 RAM, and

an NVMe SSD for data and model checkpoints. To obtain ground-truth measurements, we instrument our CL pipeline with an ElmorLabs PMD-USB power meter (ElmorLabs, 2023) and corresponding PCI-E slot adapter (ElmorLabs, 2025) for real-life energy consumption measurement.

Datasets and Baselines. We evaluate our method on real-world data streams using the River’s INSECTS datasets¹, which are specifically chosen to represent challenging concept drift scenarios. The datasets include tasks that exhibit both *abrupt* and *incremental* concept drift as the underlying data distribution changes over time (Souza et al., 2020). We compare our IMLP model against a comprehensive set of seven recent SOTA methods for tabular classification, covering three distinct model categories: 1) foundation models: *TabPFNv2*; 2) deep NN baselines: *RealMLP*, *ModernNCA*, and *MLP*; and 3) GBDTs: *CatBoost*, *XGBoost*, and *LightGBM*. More details are in Appendix C.2.

Evaluation Protocol. A crucial consideration for this study is that our selected baselines were primarily developed for static, independent and identically distributed data. While an ideal comparison in our incremental environment would involve creating a dedicated CL variant of each GBDT and NN baseline, e.g., equipped with specialized components for memory and catastrophic forgetting mitigation, such an undertaking is outside the scope of this work. To establish a methodologically sound comparison, we standardize the data flow for all models by applying our FIFO buffer mechanism. This enforces a *segmental training mode* with a limited memory window, thereby comparing IMLP against the SOTA under the same challenging, resource-constrained sequential protocol.

Statistical Analysis and Metrics. For a fair evaluation, all datasets undergo the same preprocessing pipeline, with an 85%-15% stratified split used for training and validation/testing within the segmental mode. To assess the statistical significance of performance differences across the evaluated scenarios, we first conduct the Friedman test (Friedman, 1937). If the null hypothesis is rejected, we perform post-hoc analyses using the Wilcoxon signed-rank test (Wilcoxon, 1945) with Holm correction (Holm, 1979), along with critical difference analysis (Nemenyi, 1963). All models are evaluated based on six key metrics: balanced accuracy, log-loss, energy consumption, execution time, and the composite Pareto (AUC and global efficiency) metrics that capture the crucial energy-accuracy trade-offs.

5.1 ABLATION STUDY: IMPACT OF ATTENTION, d_h , W , AND BUFFER STRATEGY CHOICES

Attention and Buffer Strategy Choices. We first ablated the core components, i.e., the attention module and buffer strategy, on the *Insects-abrupt-drift* dataset to evaluate the impact of the attention and buffer strategy choices on model performance and energy consumption. We compared our default FIFO strategy against a *similarity-based* strategy, which replaces the most similar feature in the buffer to maximize diversity. The results are presented in Figure 2.

Figure 2a compares models with and without attention, while Figure 2b reports results for different buffer replacement strategies. Enabling attention substantially improves the predictive performance (the median balanced accuracy increases from 0.376 to 0.568) but also raises total energy consumption by approximately 47.11%. The FIFO strategy outperforms the diversity-maximizing similarity strategy, which highlights the importance of recency in drifting data streams, temporal locality (p. diversity. Meanwhile, the similarity consumption compared to the FIFO tends to trade additional energy for control the energy footprint at a given

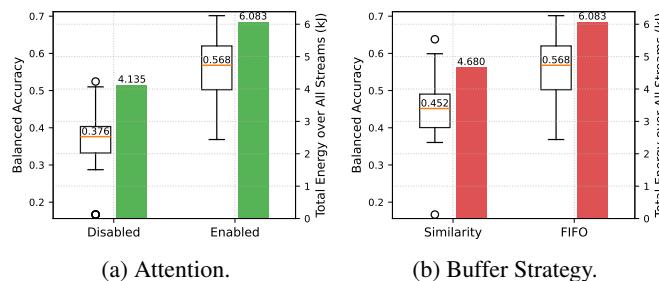
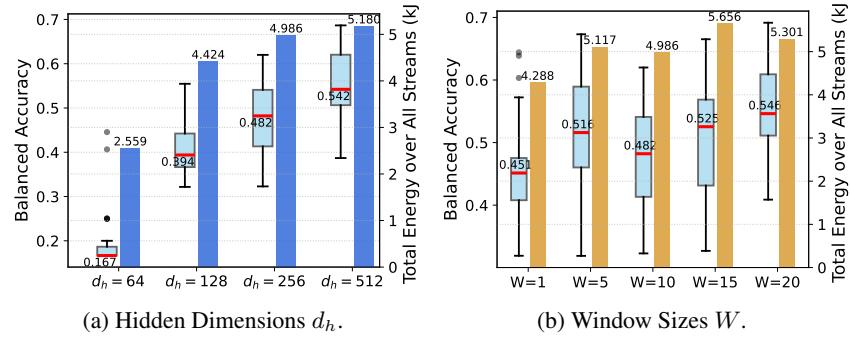


Figure 2: Ablation study. (a) Attention. (b) Buffer strategy. teams. The results indicate that for handling concept drift in serving the most recent samples) is more critical than feature based buffer strategy has a smaller influence on the total energy strategy. Overall, these ablations indicate that the buffer strategy improved accuracy, while attention acts as the primary lever to

¹<https://riverml.xyz/dev/api/datasets/Insects/>

432
 433 **Impact of d_h and W .** We also ablated the window size W and hidden dimensions d_h on the model
 434 performance and total energy cost under the non-stationary distributions, as depicted in Figure 3.



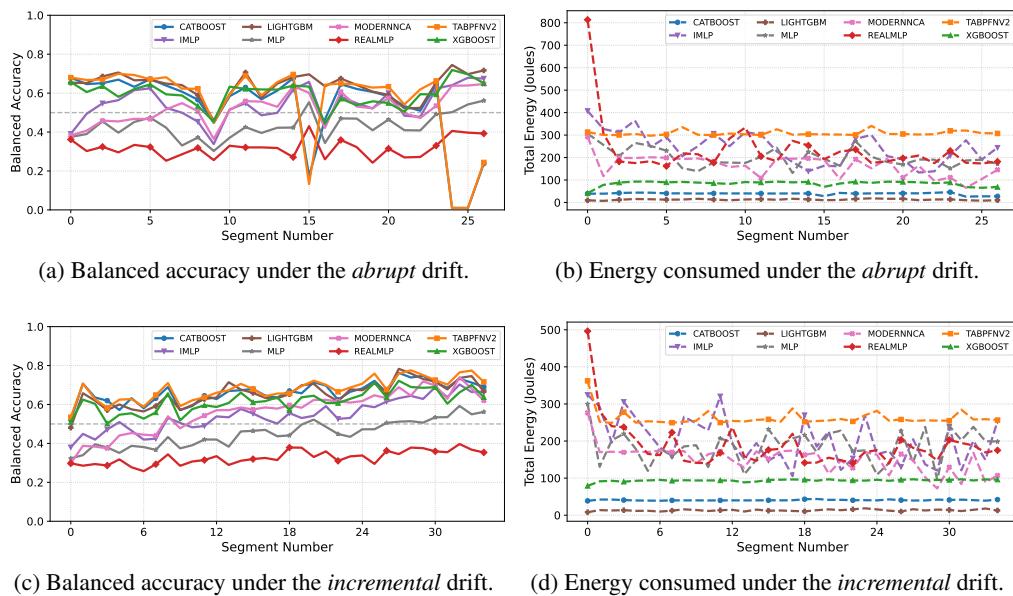
445
 446 Figure 3: Overview of the impact of d_h and W under the *Insects-abrupt-drift* dataset.
 447

448 Figure 3a compares with different hidden dimension values ($d_h = 64, 128, 256, 512$) with a fixed
 449 window size ($W = 10$), in which the median balanced accuracy improves significantly as the d_h
 450 increases from 64 to 512. Correspondingly, the total energy consumption increases from 64 to 512.
 451 Figure 3b presents the results for different window sizes ($W = 1, 5, 10, 15, 20$) with a fixed $d_h = 256$.
 452 The median balanced accuracy slightly improves as the window size increases, plateauing around
 453 $W = 10$ to $W = 20$. However, its upward trend is not linearly dependent on the window size.

454 Therefore, the *attention-based FIFO feature buffer* module, including the hidden dimension setting,
 455 significantly impacts the predictive performance improvement and energy consumption reduction
 456 under the non-stationary distributions.

459 5.2 EVALUATION UNDER ABRUPT AND INCREMENTAL DRIFTS

460 Figure 4 compares the dynamic performance and energy consumed when data arrives in sequence on
 461 the *abrupt*- and *incremental-balanced* drift scenarios.



483 Figure 4: Overview of the model performance and energy consumption under different concept drifts.
 484

485 **Abrupt drifts.** Figure 4a shows that IMLP reacts strongly to abrupt drifts with immediate accuracy
 486 degradation, while it also demonstrates a robust ability to recover in the following segment using

486 the new training data. For example, segments 9 and 16 show significant drops in balanced accuracy
 487 (0.338 and 0.436); however, the accuracy recovers to 0.548 at segment 11 and 0.582 at segment 17,
 488 respectively, and the final accuracy ranked second (lower than LightGBM equipped with FIFO buffer)
 489 among the methods evaluated. The training energy required for such adaptation and recovery tends to
 490 be significantly higher than that of the average non-drift segment, as IMLP is effectively retrained
 491 from the previous segment’s state, which necessitates gradient updates across all layers. Additionally,
 492 it must utilize the attention mechanism to analyze features from the memory buffer. Notably, the
 493 results show that the FIFO buffer keeps the LightGBM stable and prevents forgetting, while it ensures
 494 that this stability is maintained with the lowest computational cost and fastest adaptation speed among
 495 the compared models, as shown in Figure 4b.

496 **Incremental drifts.** Figure 4c depicts that TabPFNv2 (0.716) outperforms under the incremental
 497 concept drifts, followed by CatBoost (0.691), LightGBM (0.666), and IMLP (0.666), while it
 498 consumes the highest total energy as shown in Figure 4d. Still, GBDTs keep the lowest energy
 499 consumption. The IMLP’s energy profile is volatile because it is a gradient-based model operating
 500 with an aggressive adaptation policy controlled by a performance-based early stopping mechanism.
 501 This design means its energy consumption becomes a direct, fluctuating measure of the difficulty
 502 of adapting to the new segment’s concept. Consequently, most NN baselines (MLP, RealMLP,
 503 ModernNCA), when equipped with the FIFO buffer, exhibit similar energy volatility.

504 **Energy-accuracy trade-offs.** Table 1 presents the trade-off analysis based on final balanced accuracy,
 505 total energy consumed, quantitative Pareto AUC, and global Pareto efficiency.

506 In *abrupt* concept drift, IMLP achieves the
 507 highest final balanced accuracy (0.675),
 508 while costing 49.7% energy more than that
 509 of ModernNCA. Compared to TabPFNv2
 510 under the *incremental* concept drift, IMLP
 511 has 22.7% total energy reduction while
 512 only a 0.05 final balanced accuracy drop.
 513 Both IMLP and ModernNCA remain a
 514 global Pareto efficiency of 1.0 in both con-
 515 cept drifts, indicating that they are most
 516 often on the neural-global Pareto frontier.

517 **Summary of IMLP’s strengths.** IMLP
 518 offers several notable advantages over related tabular methods: (1) it is simple and inherently suitable
 519 for streaming tabular learning without replaying past raw inputs; and (2) it is lightweight **and tunable**
 520 in both computation and memory, with costs independent of the length of the data stream, yielding an
 521 **energy-efficient** solution.

522 6 CONCLUSION

523 This paper addresses the critical gap of EECL on tabular data streams by introducing IMLP, a
 524 novel incremental MLP model. IMLP employs a **novel** attention-based feature replay with context
 525 retrieval and sliding buffer updates, integrated into a minibatch training loop for streaming tabular
 526 learning. Experiments show that IMLP matches the accuracy of neural baselines under no replay
 527 while substantially reducing runtime and energy costs. IMLP achieves up to **22.7% energy reduction**
 528 compared to TabPFNv2, while maintaining competitive average accuracy. Positioned optimally on
 529 the neural Pareto frontier, IMLP consistently delivers efficiency gains across **abrupt and incremental**
 530 **concept drift datasets**.

531 **Limitations and Future Work.** Despite these exciting findings, IMLP currently treats baselines on
 532 **River’s Insets** benchmarks in an experimental setting. A promising next step is to compare the method
 533 with up-to-date models on real-life lifelong settings, thereby enriching the benchmarks. Beyond that,
 534 **building a comprehensive evaluation framework** would shed light on the influence of alternative CL
 535 strategies **for SOTA baselines**. Ultimately, an important future direction **for EECL** is to extend IMLP
 536 toward jointly optimizing the trade-offs between energy efficiency and predictive performance with
 537 tunable parameters, ideally supported by theoretical guarantees or unified analytical frameworks **for**
 538 **different CL strategies on different models under non-stationary distributions**.

Table 1: Trade-off analysis.

Data	Method	FinalAcc (\uparrow)	TotalEnergy (\downarrow)	AUC (\uparrow)	Efficiency (\uparrow)
abrupt	TabPFNv2	0.244	8316.162	0	0
	RealMLP	0.393	6325.562	0.177	0
	MLP	0.562	5408.748	0.551	0
	ModernNCA	0.647	4424.620	0.935	1.0
	IMLP	0.675	6622.577	0.435	1.0
incremental	TabPFNv2	0.716	9159.547	0	1.0
	RealMLP	0.354	6568.508	0	0
	MLP	0.562	<u>6500.125</u>	0.396	0
	ModernNCA	0.620	5291.275	0.737	1.0
	IMLP	<u>0.666</u>	7082.398	<u>0.463</u>	1.0

540
541 **Ethics statement.** This work contributes to an energy-efficient alternative to full retraining for
542 tabular data streams. By a windowed scaled dot-product attention over a sliding latent feature buffer,
543 it enables lightweight computation and avoids unbounded memory growth in continual learning,
544 while achieving efficient energy consumption [for deep networks](#). This method will be beneficial for
545 Green AI, especially in resource-constrained tabular data learning. All experiments are conducted
546 on publicly available benchmark datasets and baselines. Regarding the large language model use,
547 ChatGPTs, [Gemini](#), and Grammarly were used to assist us with writing and editing, retrieving related
548 work, coding improvement, but all the ideas, designs, plots, and analyses are our own.
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864 A FORMAL PROPERTIES OF THE ATTENTION-BASED FEATURE MEMORY

866 We recall that the IncrementalMLP (IMLP) augments the input $x_i \in \mathbb{R}^{d_{\text{in}}}$ with a context vector $c_i \in \mathbb{R}^{256}$
 867 obtained from a finite window of past latent features $H_i = \{h_{i,1}, \dots, h_{i,W}\}$, and then feeds the concatenated
 868 vector $[x_i; c_i]$ into a two-layer ReLU network followed by a linear classifier.

869 **Attention-based feature memory.** Given x_i and a set of past latent features H_i , IMLP computes

$$871 \quad q_i = W_q x_i + b_q \in \mathbb{R}^{256}, \quad (22)$$

$$872 \quad k_{i,j} = W_k h_{i,j} + b_k \in \mathbb{R}^{256}, \quad (23)$$

$$874 \quad e_{i,j} = \frac{1}{\sqrt{256}} k_{i,j}^\top q_i, \quad (24)$$

$$876 \quad \alpha_{i,j} = \frac{\exp(e_{i,j})}{\sum_{\ell=1}^W \exp(e_{i,\ell})}, \quad (25)$$

$$878 \quad c_i = \sum_{j=1}^W \alpha_{i,j} k_{i,j}. \quad (26)$$

881 In practice, the latent features $h_{i,j}$ are ℓ_2 -normalized after precomputation.

882 The feature extractor and classifier then read

$$883 \quad z_i^{(1)} = \text{ReLU}(W_1[x_i; c_i] + b_1), \quad (27)$$

$$884 \quad z_i^{(2)} = \text{ReLU}(W_2 z_i^{(1)} + b_2), \quad (28)$$

$$885 \quad o_i = W_{\text{cls}} z_i^{(2)} + b_{\text{cls}}, \quad (29)$$

$$886 \quad p_i = \text{softmax}(o_i), \quad (30)$$

$$887 \quad \ell_i(\theta) = \text{CE}(p_i, y_i), \quad (31)$$

889 where θ collects all network parameters, and CE denotes cross-entropy loss.

891 **Assumptions.** We make the following mild assumptions, which are standard in non-convex optimization for
 892 neural networks:

894 (A1) (*Bounded inputs*) There exists $R_x > 0$ such that $\|x_i\|_2 \leq R_x$ for all samples in the segment.

895 (A2) (*Bounded latent features*) The precomputed latent features are ℓ_2 -normalized, i.e., $\|h_{i,j}\|_2 \leq 1$ for all
 896 i, j .

897 (A3) (*Bounded parameters*) Training is performed with weight decay and early stopping, so that for some
 898 $R_\theta > 0$, $\|\theta\|_2 \leq R_\theta$ throughout optimization.

900 These assumptions hold in our implementation due to explicit normalization of $h_{i,j}$ and the use of weight decay
 901 and patience-based early stopping.

902 **Lemma A.1** (Bounded context vector). *Suppose (A2) holds, and let $\|W_k\|_{2 \rightarrow 2}$ denote the operator norm of
 903 W_k . Then there exists a constant $B_c > 0$ depending only on W_k and b_k such that*

$$904 \quad \|c_i\|_2 \leq B_c \quad \text{for all } i.$$

905 In particular, one can take $B_c = \|W_k\|_{2 \rightarrow 2} + \|b_k\|_2$.

907 *Proof.* For each j , we have

$$909 \quad \|k_{i,j}\|_2 = \|W_k h_{i,j} + b_k\|_2 \leq \|W_k\|_{2 \rightarrow 2} \|h_{i,j}\|_2 + \|b_k\|_2 \leq \|W_k\|_{2 \rightarrow 2} + \|b_k\|_2. \quad (32)$$

910 Define $B_c := \|W_k\|_{2 \rightarrow 2} + \|b_k\|_2$. Since $(\alpha_{i,1}, \dots, \alpha_{i,W})$ is a probability vector, the context vector c_i is a
 911 convex combination of the keys:

$$912 \quad c_i = \sum_{j=1}^W \alpha_{i,j} k_{i,j}.$$

914 Thus

$$916 \quad \|c_i\|_2 \leq \sum_{j=1}^W \alpha_{i,j} \|k_{i,j}\|_2 \leq \sum_{j=1}^W \alpha_{i,j} B_c = B_c.$$

917 \square

918 **Lemma A.2** (Smoothness and Lipschitzness of the attention map). *Let $\mathcal{A}(x_i, H_i; \theta_{\text{att}}) := c_i$ denote the
919 attention-based feature memory, where θ_{att} collects (W_q, b_q, W_k, b_k) . Under assumptions (A1)–(A3), the map
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$$921 \quad (x_i, H_i, \theta_{\text{att}}) \mapsto \mathcal{A}(x_i, H_i; \theta_{\text{att}})$$

922 *is continuously differentiable, and its Jacobian with respect to θ_{att} is bounded on the compact set*

$$923 \quad \mathcal{K} := \{(x_i, H_i, \theta_{\text{att}}) : \|x_i\|_2 \leq R_x, \|h_{i,j}\|_2 \leq 1, \|\theta_{\text{att}}\|_2 \leq R_\theta\}.$$

925 *Consequently, there exists $L_{\text{att}} > 0$ such that for all (x_i, H_i) and all $\theta_{\text{att}}^{(1)}, \theta_{\text{att}}^{(2)}$ in this set,*

$$927 \quad \|\mathcal{A}(x_i, H_i; \theta_{\text{att}}^{(1)}) - \mathcal{A}(x_i, H_i; \theta_{\text{att}}^{(2)})\|_2 \leq L_{\text{att}} \|\theta_{\text{att}}^{(1)} - \theta_{\text{att}}^{(2)}\|_2.$$

929 *Proof.* The attention map \mathcal{A} is a composition of: (i) linear maps $(x, h) \mapsto (W_q x + b_q, W_k h + b_k)$, (ii) bilinear
930 inner products and scaling $(k, q) \mapsto k^\top q / \sqrt{256}$, (iii) the softmax function on \mathbb{R}^W , and (iv) a weighted sum
931 $c = \sum_j \alpha_j k_j$. Each of these operations is smooth. Therefore, their composition is continuously differentiable
932 in $(x_i, H_i, \theta_{\text{att}})$.

933 On the compact set \mathcal{K} , all partial derivatives are bounded, hence the Jacobian $\nabla_{\theta_{\text{att}}} \mathcal{A}$ is bounded in operator
934 norm. This implies global Lipschitzness in θ_{att} on \mathcal{K} with some constant $L_{\text{att}} > 0$. \square

935 We now consider the full network mapping

$$937 \quad f_\theta(x_i, H_i) := W_{\text{cls}} \phi_\theta(x_i, H_i) + b_{\text{cls}},$$

938 where ϕ_θ denotes the two-layer ReLU feature extractor applied to $[x_i; \mathcal{A}(x_i, H_i; \theta_{\text{att}})]$, and θ collects both the
939 attention parameters and the MLP parameters.

940 **Lemma A.3** (Smoothness of the network and loss). *Under assumptions (A1)–(A3), the mapping*

$$942 \quad \theta \mapsto f_\theta(x_i, H_i)$$

943 *is continuously differentiable with bounded Jacobian on $\{\theta : \|\theta\|_2 \leq R_\theta\}$. Consequently, the per-sample loss*

$$945 \quad \ell_i(\theta) = \text{CE}(\text{softmax}(f_\theta(x_i, H_i)), y_i)$$

946 *is continuously differentiable with Lipschitz-continuous gradient on this compact set.*

948 *Proof.* By Lemma A.2, the attention map is smooth with bounded derivatives on bounded inputs. The feature
949 extractor is a composition of affine maps and ReLU activations:

$$950 \quad \phi_\theta = \text{ReLU} \circ (W_2 \cdot + b_2) \circ \text{ReLU} \circ (W_1 \cdot + b_1),$$

952 which is piecewise linear and globally Lipschitz, and smooth almost everywhere with respect to θ on any
953 compact subset of parameter space. Composition with the final linear classifier preserves these properties for f_θ .

954 The softmax function and cross-entropy loss are smooth with bounded derivatives when their inputs are bounded,
955 which follows from Lemma A.1 and (A1)–(A3). Hence $\ell_i(\theta)$ is continuously differentiable with Lipschitz-
956 continuous gradient on $\{\|\theta\| \leq R_\theta\}$. \square

957 We now move from individual samples to the empirical loss over a fixed segment (task) \mathcal{T}_t .

959 **Theorem A.4** (Segment-wise smooth empirical loss). *For a fixed segment \mathcal{T}_t with data $\{(x_i, H_i, y_i)\}_{i=1}^{n_t}$, define
960 the empirical loss*

$$961 \quad \hat{L}_t(\theta) := \frac{1}{n_t} \sum_{i=1}^{n_t} \ell_i(\theta).$$

963 *Under assumptions (A1)–(A3), $\hat{L}_t(\theta)$ is:*

- 965 (i) *bounded below, since $\ell_i(\theta) \geq 0$ for all i ;*
- 966 (ii) *continuously differentiable on $\{\theta : \|\theta\| \leq R_\theta\}$; and*
- 967 (iii) *has Lipschitz-continuous gradient on $\{\theta : \|\theta\| \leq R_\theta\}$.*

970 *Proof.* Each per-sample loss $\ell_i(\theta)$ is non-negative and continuously differentiable with Lipschitz gradient on
971 the compact parameter set by Lemma A.3. A finite average of such functions preserves these properties. Thus
972 \hat{L}_t is bounded below, continuously differentiable, and has Lipschitz-continuous gradient on $\{\|\theta\| \leq R_\theta\}$. \square

972 **Corollary A.5** (Per-segment convergence of IMLP training). *Consider optimizing $\hat{L}_t(\theta)$ by a stochastic first-
973 order method (e.g., SGD or Adam) with standard hyperparameters and weight decay, as implemented in our
974 training loop. Under Theorem A.4 and the usual step-size conditions from non-convex optimization theory, the
975 iterates $\{\theta_k\}$ produced by the optimizer on segment \mathcal{T}_t converge to a first-order stationary point of \hat{L}_t , in the
976 sense that*

$$977 \quad \lim_{k \rightarrow \infty} \|\nabla \hat{L}_t(\theta_k)\|_2 = 0,$$

978 *or, in the practical finite-epoch setting, reach a parameter θ^* with small gradient norm $\|\nabla \hat{L}_t(\theta^*)\|_2$. In
979 particular, the attention-based feature memory, being a bounded and smooth transformation of a finite latent
980 feature window, does not alter the fundamental optimization character of the model: IMLP behaves like
981 a conventional MLP with an augmented input $[x_i; c_i]$ and inherits the standard per-segment convergence
982 guarantees of non-convex deep networks.*

983 **Remark on non-stationary streams.** The analysis above is *segment-wise*: for each fixed segment \mathcal{T}_t , we
984 assume a finite dataset and study the optimization of the empirical risk $\hat{L}_t(\theta)$. This does not imply convergence
985 of the model to any limiting distribution when the underlying data-generating process is non-stationary across
986 t . Instead, our result shows that, *conditional on the observed stream in each segment*, the attention-based
987 feature memory yields a well-behaved optimization problem (smooth, with Lipschitz gradients), so that standard
988 optimizers can reliably minimize the empirical loss on that segment even in the presence of non-stationarity
989 across segments.

991 B ENERGY COMPLEXITY OF IMLP IN OUR EXPERIMENTAL SETTING

993 We now provide a simple energy-complexity characterization of IMLP in the experimental setup of Section 6.
994 The goal is not to predict the exact Joule values measured by our energy monitor, but to show that, under mild
995 hardware assumptions, the total energy consumed by IMLP is bounded and scales in a controlled way with the
996 model size and the number of training examples.

997 **Setup.** Recall that IMLP uses:

- 999 • input dimension d_{in} ,
- 1000 • fixed hidden size $H = 256$,
- 1001 • number of classes C ,
- 1002 • a finite feature-memory window of size $W \leq W_{\text{max}}$,
- 1003 • at most E_{max} training epochs per segment, enforced by early stopping (default $E_{\text{max}} = 100$),
- 1004 • mini-batch training with batch size B and Adam/AdamW optimization.

1006 For a given segment \mathcal{T}_t with n_t training samples, our code performs at most E_{max} full passes over the segment
1007 before stopping.

1009 **Hardware and energy model.** Let F_{train} denote the number of floating-point operations (FLOPs) required
1010 to perform *one* forward-and-backward pass of IMLP on a single sample (including the attention-based feature
1011 memory). We adopt a standard abstract energy model:

- 1012 (H1) On a fixed hardware platform (GPU/CPU), there exist constants $0 < \eta_{\text{min}} \leq \eta_{\text{max}}$ such that the
1013 energy consumed per FLOP lies in $[\eta_{\text{min}}, \eta_{\text{max}}]$.
- 1015 (H2) The additional system overhead per training step (e.g., kernel launches, bookkeeping) is bounded by a
1016 constant E_0 independent of the sample index and segment.

1017 These assumptions reflect that, for a fixed device and implementation, energy is approximately linear in the
1018 number of FLOPs, up to device-specific constants and small overhead.

1019 **Lemma B.1** (FLOP complexity per sample). *For a single sample (x_i, H_i, y_i) , the FLOP count of a forward-
1020 and-backward step of IMLP satisfies*

$$1021 \quad F_{\text{train}} \leq K_{\text{arch}}(d_{\text{in}}H + WH^2 + H^2 + HC),$$

1023 *for some architecture-dependent constant $K_{\text{arch}} > 0$ that does not depend on n_t or t . In particular, since $H=256$
1024 and $W \leq W_{\text{max}}$ are fixed in our experiments, F_{train} grows at most linearly in d_{in} and C .*

1025 *Proof.* We count FLOPs layer by layer:

1026 • **Attention block.**

1027 – Query: $x_i \mapsto q_i = W_q x_i + b_q$ costs $O(d_{\text{in}} H)$ FLOPs.

1028 – Keys: each $h_{i,j}$ is mapped to $k_{i,j} = W_k h_{i,j} + b_k$ with cost $O(H^2)$; for W keys this is $O(WH^2)$.

1029 – Attention scores and softmax: computing $e_{i,j} = k_{i,j}^\top q_i / \sqrt{H}$ costs $O(WH)$, softmax costs $O(W)$, and forming $c_i = \sum_j \alpha_{i,j} k_{i,j}$ costs $O(WH)$. Altogether $O(WH)$ FLOPs.

1030 Thus the attention block has FLOP complexity $O(d_{\text{in}} H + WH^2)$.

1031 • **Feature extractor.** The two ReLU layers operate on dimensions $(d_{\text{in}} + H) \rightarrow 512 \rightarrow H$, which costs

$$1032 \quad O((d_{\text{in}} + H) \cdot 512) + O(512 \cdot H) = O(d_{\text{in}} H + H^2).$$

1033 • **Classifier.** The final linear layer $H \rightarrow C$ costs $O(HC)$ FLOPs.

1034 • **Backward pass.** The backward pass through these linear and ReLU layers multiplies the forward FLOP count by a constant factor (depending only on the layer type), which we absorb into K_{arch} .

1043 Summing these contributions gives

$$1044 \quad F_{\text{train}} \leq K_{\text{arch}} (d_{\text{in}} H + WH^2 + H^2 + HC)$$

1045 for some constant $K_{\text{arch}} > 0$. □

1046 **Theorem B.2** (Per-segment energy complexity bound). *Consider a segment \mathcal{T}_t with n_t training samples. Under assumptions (H1)–(H2) and Lemma B.1, the total training energy consumed by IMLP on this segment satisfies*

$$1047 \quad E_t^{\text{train}} \leq C_{\text{train}} E_{\max} n_t (d_{\text{in}} H + WH^2 + H^2 + HC) + C_0,$$

1048 for some hardware- and implementation-dependent constants $C_{\text{train}} > 0$ and $C_0 \geq 0$. Similarly, the inference energy on the test set of size n^{test} admits

$$1049 \quad E_t^{\text{infer}} \leq C_{\text{infer}} n^{\text{test}} (d_{\text{in}} H + WH^2 + H^2 + HC) + C'_0,$$

1050 with another constant $C_{\text{infer}} > 0$ and overhead $C'_0 \geq 0$.

1051 *Proof.* For each epoch, the optimizer processes all n_t samples once (up to mini-batch granularity). Thus, the total FLOP count per segment is at most

$$1052 \quad F_t^{\text{seg}} \leq E_{\max} n_t F_{\text{train}},$$

1053 where F_{train} is bounded as in Lemma B.1. By (H1), energy per FLOP lies in $[\eta_{\min}, \eta_{\max}]$, so there exists \tilde{C}_{train} such that

$$1054 \quad E_t^{\text{train}} \leq \eta_{\max} F_t^{\text{seg}} + (\text{overhead}) \leq \tilde{C}_{\text{train}} E_{\max} n_t (d_{\text{in}} H + WH^2 + H^2 + HC) + C_0.$$

1055 We rename \tilde{C}_{train} as C_{train} for simplicity. The inference bound follows analogously, using only a forward pass per sample (no backward pass) and absorbing the constant factor into C_{infer} . □

1056 **Corollary B.3** (Energy complexity over the full non-stationary stream). *Let the data stream be partitioned into T segments $\{\mathcal{T}_t\}_{t=1}^T$ with sizes $\{n_t\}_{t=1}^T$. Under the same assumptions as Theorem B.2, the total training energy over the entire stream satisfies*

$$1057 \quad E_{\text{total}}^{\text{train}} = \sum_{t=1}^T E_t^{\text{train}} \leq C_{\text{train}} E_{\max} \left(\sum_{t=1}^T n_t \right) (d_{\text{in}} H + WH^2 + H^2 + HC) + T C_0,$$

1058 and the total inference energy satisfies

$$1059 \quad E_{\text{total}}^{\text{infer}} \leq C_{\text{infer}} n_{\text{total}}^{\text{test}} (d_{\text{in}} H + WH^2 + H^2 + HC) + T C'_0.$$

1060 In particular, for our experimental setting where $H=256$, $W \leq W_{\max}$, and E_{\max} are fixed constants, both training and inference energy grow at most linearly in the total number of processed examples $\sum_t n_t$ and in the effective model size. The attention-based feature memory only adds the bounded term WH^2 and does not change this linear energy scaling.

1080
 1081 **Discussion.** The bounds above explain two aspects of our empirical observations: (i) on a fixed device,
 1082 IMLP has a *predictable* energy profile, scaling linearly with the number of samples and epochs; and (ii) the
 1083 attention-based feature memory contributes a controlled overhead proportional to WH^2 , which remains small in
 1084 our experiments because W and H are fixed ($W \leq 10$, $H = 256$). Our measured Joule values are therefore
 1085 consistent with an energy complexity that is linear in the stream size, and the theoretical bounds clarify that this
 1086 behavior is not specific to a particular dataset, but a structural property of the IMLP architecture and training
 1087 procedure.
 1088

1088 C EXTENDED EXPERIMENTS

1090 C.1 DATASETS AND STREAM SEGMENTATION

1092 We evaluate IMLP on 36 classification tasks from the TabZilla benchmark (McElfresh & Talwalkar, 2023),
 1093 selected from OpenML based on three criteria: (1) sufficient data size to create meaningful segments, (2)
 1094 balanced representation of binary and multi-class problems, and (3) diverse feature dimensionalities and class
 1095 distributions. To simulate the data stream in incremental learning scenarios, Table 2 lists every OpenML task in
 1096 our benchmark together with basic statistics and the fixed stream segmentation applied *in original row order*
 1097 (rows 1 … k form Segment 0, rows $k+1$ … $2k$ form Segment 1, etc.).

1098 \dagger Class counts show *label ID : instances* after preprocessing. Binary tasks list two numbers; multi-class tasks
 1099 list one count per class. For tasks with many classes, we show representative counts or use compact notation
 1100 (e.g., “25 × 300” for 25 classes with 300 instances each).

1101 C.1.1 STREAM SEGMENTATION ALGORITHM

1103 Our segmentation follows a principled approach to create balanced segments while minimizing data waste:

1105 Algorithm 1 Optimal Segment Size Selection

1106 **Require:** Dataset with N training instances, bounds $k_{\min} = 500$, $k_{\max} = 1000$

1107 **Ensure:** Segment size k^* that minimizes remainder

```
1108 1: best_remainder  $\leftarrow N$ 
1109 2:  $k^* \leftarrow k_{\min}$ 
1110 3: for  $k = k_{\min}$  to  $\min(k_{\max}, N)$  do
1111 4:   num_segments  $\leftarrow \lfloor N/k \rfloor$ 
1112 5:   remainder  $\leftarrow N \bmod k$ 
1113 6:   if remainder = 0 then
1114 7:     return  $k$  ▷ Perfect division found
1115 8:   if remainder < best_remainder then
1116 9:     best_remainder  $\leftarrow$  remainder
1117 10:     $k^* \leftarrow k$ 
1118 11: return  $k^*$ 
```

1119
 1120 The choice of segment size bounds (500–1000 instances) balances three considerations: (1) *statistical power*,
 1121 each segment must contain sufficient samples for reliable learning, (2) *IMLP coherence*, segments should be
 1122 large enough for the attention mechanism to learn meaningful feature relationships within each temporal chunk,
 1123 and (3) *computational efficiency*, larger segments would increase memory requirements and training time per
 1124 segment without proportional benefits.

1125 When the optimal segment size k^* leaves a remainder $r = N \bmod k^*$, we apply *round-robin redistribution*: the
 1126 first r segments each receive one additional instance, ensuring segment sizes differ by at most 1. This maintains
 1127 temporal ordering while achieving optimal balance.

1128 C.2 DATA RETRIEVAL AND PREPROCESSING PROTOCOL

1129 C.2.1 DATASET ACQUISITION

1132 All datasets are retrieved via the OpenML Python API (v0.15.2) with local caching enabled. We use the default
 1133 target attribute specified in each OpenML task definition. Raw data is downloaded in DataFrame format to
 1134 preserve both feature names and categorical indicators.

1134 Table 2: Statistics of datasets. OpenML classification tasks and stream-segmentation parameters used
 1135 in this study. # Inst. stands for the number of instances, # Feat. stands for the number of features.
 1136 Seg. size stands for the segment size bound. # Segs stands for the number of segments. Numbers are
 1137 produced by the data-processing pipeline and reproduced by the helper script in §C.3.

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1140	ID	Name	# Inst.	#Feat.	Class balance [†]	Seg. size	#Segs
1141	146820	wilt	4,839	5	4,578; 261	514	8
1142	14964	artificial-characters	10,218	7	1,196; 600; 1,192; 1,416; 808; 1,008; ...	579	15
1143	14969	GesturePhaseSegmentation	9,873	32	2,741; 998; 2,097; 1,087; 2,950	839	10
1144	14951	eeg-eye-state	14,980	14	8,257; 6,723	749	17
1145	146206	magic	19,020	10	12,332; 6,688	951	17
1146	167211	Satellite	5,100	36	75; 5,025	867	5
1147	167141	churn	5,000	29	4,293; 707	850	5
1148	168910	fabert	8,237	800	933; 1,433; 1,927; 1,515; 979; 948; 502	500	14
1149	168912	sylvine	5,124	20	2,562; 2,562	871	5
1150	190410	philippine	5,832	308	2,916; 2,916	708	7
1151	2074	satimage	6,430	36	1,531; 703; 1,356; 625; 707; 1,508	683	8
1152	28	optdigits	5,620	64	554; 571; 557; 572; 568; 558; ...	597	8
1153	32	pendigits	10,992	16	1,143; 1,143; 1,144; 1,055; 1,144;	519	18
1154					...		
1155	146607	SpeedDating	8,378	442	6,998; 1,380	712	10
1156	168908	christine	5,418	1,611	2,709; 2,709	921	5
1157	14952	PhishingWebsites	11,055	38	4,898; 6,157	522	18
1158	3510	JapaneseVowels	9,961	14	1,096; 991; 1,614; 1,473; 782; ...	529	16
1159	3735	pollen	3,848	5	1,924; 1,924	545	6
1160	3711	elevators	16,599	18	5,130; 11,469	641	22
1161	3896	ada_agnostic	4,562	48	3,430; 1,132	646	6
1162	14970	har	10,299	561	1,722; 1,544; 1,406; 1,777; 1,906; 1,944	547	16
1163	3686	house_16H	22,784	16	6,744; 16,040	842	23
1164	3897	eye_movements	10,936	27	3,804; 4,262; 2,870	715	13
1165	3904	jm1	10,885	21	8,779; 2,106	514	18
1166	43	spambase	4,601	57	2,788; 1,813	782	5
1167	3954	MagicTelescope	19,020	10	12,332; 6,688	951	17
1168	9952	phoneme	5,404	5	3,818; 1,586	574	8
1169	3950	musk	6,598	267	5,581; 1,017	701	8
1170	9960	wall-robot-navigation	5,456	24	2,205; 2,097; 328; 826	515	9
1171	3889	sylva_agnostic	14,395	216	13,509; 886	941	13
1172	9985	first-order-theorem-proving	6,118	51	1,089; 486; 748; 617; 624; 2,554	520	10
1173	3481	isolet	7,797	617	25 \times 300 (class 0...24)	552	12
1174	45	splice	3,190	227	767; 768; 1,655	542	5
1175	9986	gas-drift	13,910	128	2,565; 2,926; 1,641; 1,936; 3,009; 1,833	563	21
1176	9987	gas-drift-different-conc.	13,910	129	2,565; 2,926; 1,641; 1,936; 3,009; 1,833	563	21
1177	168909	dilbert	10,000	2,000	1,988; 2,049; 1,913; 2,046; 2,004	500	17
1178	99901	Insects Abrupt	52,847	33	Balanced (6 classes)	1,957	27
1179	99902	Insects Incremental	57,017	33	Balanced (6 classes)	1,629	35

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C.2.2 FEATURE PREPROCESSING PIPELINE

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Our preprocessing pipeline follows scikit-learn best practices with separate transformers for numerical and categorical features:

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1189**Numerical Features:**1190
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1. **Imputation:** Missing values filled with column medians
2. **Standardization:** Zero mean, unit variance scaling via StandardScaler

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1195**Categorical Features:**1196
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1. **Imputation:** Missing values filled with constant ‘missing’
2. **Encoding:** One-hot encoding with `drop='first'` to avoid multicollinearity
3. **Unknown handling:** `handle_unknown='ignore'` for robust inference

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The ColumnTransformer ensures preprocessing consistency across all data splits. After transformation, all features are converted to `float32` for memory efficiency.

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1202**C.2.3 TARGET PROCESSING AND TASK TYPE DETECTION**1203
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Target variables are processed based on OpenML task type:

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- **Binary classification:** 2 unique labels \rightarrow LabelEncoder $\rightarrow \{0, 1\}$
- **Multi-class classification:** $C > 2$ unique labels \rightarrow LabelEncoder $\rightarrow \{0, \dots, C-1\}$
- **Regression:** Direct conversion to `float32` (not used in this study)

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1211**C.2.4 DATA SPLITTING STRATEGY**1212
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Our splitting protocol ensures a realistic evaluation:

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1. **Test Set Isolation:** A stratified 15% test split is carved out *before* any stream processing, using `random_seed=42` for reproducibility.
2. **Training Stream Creation:** The remaining 85% forms the chronologically ordered training stream, preserving the original row order from OpenML.
3. **Per-Segment Validation:** Each segment (or cumulative data) is further split with stratified 15% validation, using `random_seed=42+segment_idx` to ensure different splits per segment while maintaining reproducibility.

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This approach simulates realistic continual learning where: 1) The test set represents future unseen data, 2) Each segment represents a temporal chunk of arriving data, 3) Validation splits enable early stopping without future data leakage, and 4) All models use consistent 15% validation splits for hyperparameter selection and early stopping criteria.

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1226**C.2.5 MODEL TRAINING PROTOCOLS**

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Our experimental design follows two distinct training protocols based on model type:

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Cumulative Training: Traditional baselines (XGBoost, LightGBM, CatBoost, kNN, SVM, Decision Trees, Random Forest, and neural baselines like TabNet, SAINT) are retrained from scratch at each segment using all available data up to that point. For the segment, these models train on the union $\bigcup_{t=0}^T \mathcal{T}_t$ where \mathcal{T}_t denotes the t -th data segment. This protocol maximizes baseline performance by leveraging all historical data, representing the standard approach in tabular learning.

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Incremental Training: Our proposed IMLP trains only on the current segment S_t while accessing previous feature representations through the attention mechanism. This protocol tests true incremental learning capabilities without replay of raw historical data.

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Both protocols use identical validation procedures: each model’s hyperparameters are selected via early stopping on the 15% validation split, ensuring fair comparison despite different training paradigms.

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1239**C.2.6 REPRODUCIBILITY MEASURES**1240
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All steps are deterministic with fixed random seeds, including 1) Global seed: `random_seed = 42`, 2) Per-segment validation: `random_seed = 42 + segment_idx`, and 3) Preprocessing: Deterministic transformers with fixed parameters.

1242 C.3 DATASET SUMMARY REGENERATION SCRIPT

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1244 For full reproducibility, we provide a helper script that regenerates Table 2 from the processed data:

```

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12461 # dataset_summary.py (runs in < 2 seconds)
12472 import json, csv, gzip, numpy as np, pathlib
12483
12494 def regenerate_dataset_summary():
12505     """Regenerate the dataset summary CSV from processed metadata."""
12516     META = pathlib.Path("processed_datasets_summary.json")
12527     ROOT = pathlib.Path("full_datasets")
12538     OUT = pathlib.Path("dataset_summary.csv")
12549
125510    # Load processing metadata
125611    with META.open() as f:
125712        meta = json.load(f)
125813
125914    rows = []
126015    for tid, m in meta.items():
126116        # Load target labels to compute class balance
126217        y = np.load(gzip.open(ROOT/m["dataset_name"]/"y_full.npy.gz"))
126318        counts = np.bincount(y.astype(int))
126419
126520        rows.append({
126621            "task_id": int(tid),
126722            "name": m["original_name"],
126823            "instances": int(m["num_instances"]),
126924            "features": int(m["num_features"]),
127025            "class_balance": ";" .join(map(str, counts)),
127126            "segment_size": int(m["segment_size"]),
127227            "num_segments": int(m["num_segments"])
127328        })
127429
127530    # Write CSV output
127631    with OUT.open("w", newline="") as f:
127732        writer = csv.DictWriter(f, fieldnames=rows[0].keys())
127833        writer.writeheader()
127934        writer.writerows(rows)
128035
128136        print(f"Wrote {OUT} with {len(rows)} tasks")
128237
128338    if __name__ == "__main__":
128439        regenerate_dataset_summary()

```

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Running this script in the project root recreates the CSV that backs Table 2. The script requires the preprocessed datasets, but no pipeline re-execution.

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C.4 BASELINES

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We implement most of the baseline methods according to the publicly available codebases and integrate them into the same backbone for benchmarking.

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- XGBoost (Chen & Guestrin, 2016). <https://github.com/dmlc/xgboost>
- LightGBM (Ke et al., 2017). <https://github.com/microsoft/LightGBM>
- CatBoost (Prokhorenkova et al., 2019). <https://github.com/catboost/catboost>
- TabPFN v2 (Hollmann et al., 2025a). <https://github.com/automl/TabPFN>
- TabM (Gorishniy et al., 2024). <https://github.com/yandex-research/tabm>
- Real-MLP (Holzmüller et al., 2024). https://github.com/dholzmueller/realmpl-td-s_standalone
- TabR (Gorishniy et al., 2023a). <https://github.com/yandex-research/tabular-dl-tabr>
- ModernNCA (Ye et al., 2024). <https://github.com/YyzHarry/ModernNCA>
- MLP (Taud & Mas, 2017). https://scikit-learn.org/stable/modules/neural_networks_supervised.html
- TabNet (Arik & Pfister, 2021). <https://github.com/dreamquark-ai/tabnet>
- DANet (Chen et al., 2022). <https://github.com/QwQ2000/DANets>
- ResNet (Gorishniy et al., 2021). <https://github.com/yandex-research/tabular-dl-revisiting-models>
- STG (Jana et al., 2023). <https://github.com/runopti/stg>
- VIME (Yoon et al., 2020). <https://github.com/jsyoon0823/VIME>
- k-NN (Guo et al., 2003). <https://scikit-learn.org/stable/modules/neighbors.html>
- SVM (Jakkula, 2006). <https://scikit-learn.org/stable/modules/svm.html>
- Linear Model (Kiebel & Holmes, 2007). https://scikit-learn.org/stable/modules/linear_model.html
- Random Forest (Rigatti, 2017). <https://scikit-learn.org/stable/modules/ensemble.html#random-forests>
- Decision Tree (Rokach & Maimon, 2005). <https://scikit-learn.org/stable/modules/tree.html>

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In revisions, we changed the evaluation protocol and only include the recent SOTA works.

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D IMLP IMPLEMENTATION DETAILS

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D.1 ARCHITECTURE OVERVIEW AND DESIGN RATIONALE

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IMLP extends the standard MLP architecture with an attention-based memory mechanism designed specifically for tabular continual learning. The key innovation lies in storing and retrieving *feature representations* rather than raw data, enabling privacy-preserving incremental learning with constant memory requirements.

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D.1.1 COMPARISON WITH STANDARD MLP

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Table 4 contrasts IMLP with a standard MLP of equivalent capacity:

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D.2 LAYER-WISE ARCHITECTURE SPECIFICATION

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Design Choices:

- **Hidden size 256:** Balances expressiveness with computational efficiency across all datasets
- **No dropout/normalization:** Empirically found to hurt performance in continual learning setting
- **ReLU activations:** Simple, stable gradients for incremental training
- **Fixed architecture:** Same capacity across all 36 datasets for fair comparison

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Table 4: Architectural comparison between standard MLP and IMLP.

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Component	MLP	IMLP	IMLP Notes
Input processing	$d_{\text{in}} \rightarrow 512$	$d_{\text{in}} \rightarrow 256$	Query projection
Memory mechanism	None	Attention	Key-value retrieval
Feature extraction	$512 \rightarrow 256$	$(d_{\text{in}} + 256) \rightarrow 512 \rightarrow 256$	Context-augmented
Memory complexity	$\mathcal{O}(1)$	$\mathcal{O}(W)$	W = window size
Time complexity	$\mathcal{O}(1)$	$\mathcal{O}(W \cdot d)$	d = hidden dim
Privacy	Requires raw data	Feature-only	No raw data storage

Table 5: Detailed layer-wise specification of IMLP architecture.

Component	Output dim.	Activation	Notes
Input feature vector	d_{in}	–	Raw tabular features after preprocessing
<i>Attention Module</i>			
Query projection Q	256	–	$\text{Linear}(d_{\text{in}}, 256)$
Key projection K	256	–	$\text{Linear}(256, 256)$ applied to each stored feature
Context computation	256	–	Scaled dot-product attention over window
<i>Feature Extraction</i>			
Concatenated input (x, c)	$d_{\text{in}} + 256$	–	Only if attention enabled; c = context vector
FC 1	512	ReLU	$\text{Linear}(d_{\text{in}} + 256, 512)$
FC 2	256	ReLU	$\text{Linear}(512, 256)$
<i>Classification Head</i>			
Classifier	C	–	$\text{Linear}(256, C)$ where C = number of classes

D.3 ATTENTION MECHANISM DESIGN

D.3.1 SCALED DOT-PRODUCT ATTENTION

IMLP uses a simplified attention mechanism to retrieve relevant historical features. For a batch of size B :

$$Q = W_q \cdot x \in \mathbb{R}^{B \times 1 \times 256} \quad (\text{query from current input}) \quad (33)$$

$$K = W_t \cdot H_{\text{stacked}} \in \mathbb{R}^{B \times W \times 256} \quad (\text{keys from previous features}) \quad (34)$$

$$\text{Scores} = \text{bmm}(K, Q^T) \in \mathbb{R}^{B \times W \times 1} \quad (35)$$

$$\alpha = \text{softmax}(\text{Scores}. \text{squeeze}()) \in \mathbb{R}^{B \times W} \quad (36)$$

$$\text{Context} = \text{bmm}(\alpha. \text{unsqueeze}(1), K) \in \mathbb{R}^{B \times 1 \times 256} \quad (37)$$

where:

- $H_{\text{stacked}} = \text{stack}(\{h_{t-W}, \dots, h_{t-1}\}) \in \mathbb{R}^{B \times W \times 256}$
- bmm denotes batch matrix multiplication
- No scaling factor is applied (unlike standard scaled dot-product attention)
- Values equal keys: $V = K$

D.3.2 WINDOW MANAGEMENT STRATEGY

The sliding window maintains a FIFO queue of the most recent W feature vectors:

1404 **Algorithm 2** Sliding Window Update
 1405 **Require:** Current input x , previous features H_{prev} , window size W
 1406 **Ensure:** Updated window H_{new}

- 1: $h_{\text{current}} \leftarrow \text{FeatureExtractor}(x, \text{Context}(x))$
- 2: $H_{\text{new}} \leftarrow H_{\text{prev}} \cup \{h_{\text{current}}\}$
- 3: **if** $|H_{\text{new}}| > W$ **then**
- 4: $H_{\text{new}} \leftarrow H_{\text{new}}[1 :]$ ▷ Remove oldest feature
- 5: **return** H_{new}

D.3.3 FEATURE NORMALIZATION

To improve attention stability, stored features are L2-normalized during precomputation:

$$\tilde{h}_i = \frac{h_i}{\|h_i\|_2 + \epsilon} \quad (38)$$

where $\epsilon = 10^{-8}$ prevents division by zero. This normalization ensures attention weights focus on feature directions rather than magnitudes and is applied in the `precompute` method during segmental training.

```

1458 D.4 COMPLETE IMPLEMENTATION
1459
1460 1 import torch
1461 2 import torch.nn as nn
1462 3 import torch.nn.functional as F
1463 4
1464 5 class IncrementalMLP(nn.Module):
1465 6     """
1466 7         Incremental MLP with attention-based feature replay for continual
1467 8             ↳ learning.
1468 9
1469 10     Args:
1470 11         input_size (int): Number of input features
1471 12         num_classes (int): Number of output classes
1472 13         use_attention (bool): Whether to use attention mechanism
1473 14         window_size (int): Size of sliding memory window
1474 15     """
1475 16     def __init__(self, input_size, num_classes, use_attention=True,
1476 17         ↳ window_size=10):
1477 18         super().__init__()
1478 19         self.window_size = window_size
1479 20         self.use_attention = use_attention
1480 21         self.hidden_size = 256
1481 22
1482 23     # Attention projections
1483 24     self.query = nn.Linear(input_size, 256)
1484 25     self.key = nn.Linear(256, 256)
1485 26
1486 27     # Feature extraction pathway
1487 28     total_input_size = input_size + (256 if use_attention else 0)
1488 29     self.feature_extractor = nn.Sequential(
1489 30         nn.Linear(total_input_size, 512),
1490 31         nn.ReLU(),
1491 32         nn.Linear(512, self.hidden_size),
1492 33         nn.ReLU()
1493 34     )
1494 35
1495 36     # Classification head
1496 37     self.classifier = nn.Linear(self.hidden_size, num_classes)
1497 38
1498 39     def compute_context(self, x, prev_features):
1499 40         """
1500 41             Compute attention-weighted context from previous features.
1501 42
1502 43             Args:
1503 44                 x (Tensor): Current input batch [B, D]
1504 45                 prev_features (List[Tensor]): Previous feature vectors [W x
1505 46                     ↳ [256]]
1506 47
1507 48             Returns:
1508 49                 Tensor: Context vector [B, 256]
1509 50
1510 51             if not prev_features or self.window_size == 0:
1511 52                 return torch.zeros(x.size(0), 256, device=x.device)
1512 53
1513 54             # Stack previous features: [B, W, 256]
1514 55             stacked_prev = torch.stack(prev_features, dim=1)
1515 56
1516 57             # Compute keys and queries
1517 58             keys = self.key(stacked_prev) # [B, W, 256]
1518 59             query = self.query(x).unsqueeze(1) # [B, 1, 256]
1519 60
1520 61             # Scaled dot-product attention
1521 62             scores = torch.bmm(keys, query.transpose(1, 2)).squeeze(-1) # [B,
1522 63             ↳ W]

```

```

1512      attention_weights = F.softmax(scores, dim=1)  # [B, W]
1513
1514      # Compute weighted context
1515      context = torch.bmm(attention_weights.unsqueeze(1),
1516      → keys).squeeze(1)  # [B, 256]
1517
1518      return context
1519
1520  def forward(self, x, prev_features=None):
1521      """
1522          Forward pass with optional attention over previous features.
1523
1524          Args:
1525              x (Tensor): Input features [B, D]
1526              prev_features (List[Tensor]): Previous features for attention
1527
1528          Returns:
1529              Tuple[Tensor, Tensor]: (logits, current_features)
1530
1531          # Compute attention context
1532          context = torch.zeros(x.size(0), 256, device=x.device)
1533          if self.use_attention and prev_features:
1534              context = self.compute_context(x, prev_features)
1535
1536          # Concatenate input with context
1537          if self.use_attention:
1538              augmented_input = torch.cat([x, context], dim=1)
1539          else:
1540              augmented_input = x
1541
1542          # Extract features and classify
1543          features = self.feature_extractor(augmented_input)
1544          logits = self.classifier(features)
1545
1546          return logits, features
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```

D.5 COMPUTATIONAL COMPLEXITY ANALYSIS

D.5.1 TIME COMPLEXITY

For each forward pass with batch size B , input dimension d_{in} , hidden dimension $d_h = 256$, and window size W :

$$\text{Query projection: } \mathcal{O}(B \cdot d_{\text{in}} \cdot d_h) \quad (39)$$

$$\text{Key projection: } \mathcal{O}(B \cdot W \cdot d_h^2) \quad (40)$$

$$\text{Attention scores: } \mathcal{O}(B \cdot W \cdot d_h) \quad (41)$$

$$\text{Context aggregation: } \mathcal{O}(B \cdot W \cdot d_h) \quad (42)$$

$$\text{Feature extraction: } \mathcal{O}(B \cdot (d_{\text{in}} + d_h) \cdot 512) \quad (43)$$

$$\text{Total: } \mathcal{O}(B \cdot (d_{\text{in}} \cdot d_h + W \cdot d_h^2)) \quad (44)$$

For typical values ($W = 10$, $d_h = 256$, $d_{\text{in}} \lesssim 2000$), the attention overhead is $\mathcal{O}(W \cdot d_h^2) = \mathcal{O}(655,360)$ operations per sample.

D.5.2 MEMORY COMPLEXITY

IMLP maintains constant memory usage per segment:

- **Model parameters:** $\approx 1.2\text{M}$ parameters (fixed)
- **Feature buffer:** $W \times 256 \times 4$ bytes = 10,240 bytes for $W = 10$
- **Attention matrices:** $B \times W \times 256 \times 4$ bytes during computation

Unlike replay-based methods, memory usage does not grow with the number of segments, enabling indefinite continual learning.

1566 D.5.3 COMPARISON WITH REPLAY METHODS
15671568 Table 6 compares IMLP with alternative continual learning approaches:
15691570 Table 6: Complexity comparison of continual learning approaches.
1571

Method	Memory	Time per step	Privacy
Naive retraining	$\mathcal{O}(T \cdot N)$	$\mathcal{O}(T \cdot N)$	Requires raw data
Experience replay	$\mathcal{O}(M)$	$\mathcal{O}(N + M)$	Requires raw data
Generative replay	$\mathcal{O}(1)$	$\mathcal{O}(N + G)$	Private
IMLP (ours)	$\mathcal{O}(W)$	$\mathcal{O}(N + W \cdot d^2)$	Private

1578 where T = number of tasks, N = samples per task, M = replay buffer size, G = generative model cost, W =
1579 window size, d = feature dimension.
15801581 D.6 HYPERPARAMETER CONFIGURATION
15821583 IMLP uses the following default hyperparameters across all experiments:
15841585 Table 7: IMLP hyperparameter configuration.
1586

Parameter	Value	Description
Window size (W)	10	Number of previous feature vectors stored
Hidden dimension	256	Feature representation size
Learning rate	10^{-3}	Adam optimizer learning rate
Batch size	128	Training batch size
Weight decay	10^{-5}	L2 regularization strength
Early stopping patience	10	Epochs without improvement before stopping
Max epochs	100	Maximum training epochs per segment
Normalization ϵ	10^{-8}	Small constant for L2 normalization

1596 The window size $W = 10$ was chosen to balance memory efficiency with sufficient historical context. The
1597 hidden dimension of 256 provides adequate representational capacity while maintaining computational efficiency
1598 across diverse tabular datasets.
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