Cost and preferences in bipartite complex systems: inverting the (sub-)optimal transport problem

Keywords: random networks, mutualistic systems, optimal transport, ecology, economic complexity

Extended Abstract

The theory of Optimal Transport (OT) was originally devised to optimize the movement of materials between locations with different characteristics, such as the transportation of coal from mines to storage facilities. Its central objective is to minimize the total transportation cost, subject to constraints on resource availability at sources (r_x), storage capacity at destinations (c_y), and transportation costs between locations (ϕ_{xy}). Formally, the problem amounts to finding the transport matrix π_{xy}, representing the mass moved from location x to location y, that minimizes the global cost $C = \sum_{xy} \phi_{xy} \pi_{xy}$ while satisfying the conservation constraints $\sum_y \pi_{xy} = r_x$ and $\sum_x \pi_{xy} = c_y$.

Despite its apparent simplicity, OT is rarely studied in complex systems by simultaneously considering its two defining components: the constraint vectors (r_x, c_y) and the cost matrix ϕ_{xy}. Yet this combination has clear relevance to domains such as international trade, where it can capture the dual roles of resource availability and competitive costs in shaping global exchanges. Moreover, the development of fast algorithms, such as Sinkhorn's method, has made OT increasingly accessible through approximate solutions.

A well-known structural property of the OT solution is its network topology: the optimal transport plan forms a tree. However, perfectly tree-like networks are never observed in empirical data. To better understand deviations from this idealized structure, we study how sub-optimal solutions emerge when the system is driven away from strict optimality. Specifically, we introduce a random graph model in which global resources are conserved while the importance of the cost function is tuned by a parameter β[1]. In the limit $\beta \to \infty$, the model recovers the exact OT solution, while for $\beta = 0$ transport decisions are indifferent to costs, producing a regime of flat preferences. This framework allows us to interpolate between perfectly optimized and purely random transport structures.

A central challenge in classical OT is the 'inverse problem': estimating the underlying cost function ϕ_{xy} from the observed transport matrix π_{xy}. This is a central problem in this field, since usually one can only observe the transport plan and has no access to the cost function. Without a reliable way to estimate it, most of the potential applications of OT to networks and complex systems remain untapped. This task is notoriously difficult, since the transport plan depends on both costs and constraints in a highly nonlinear way, and strong assumptions on the functional form of ϕ_{xy} are typically required. In the classical OT, the information carried by the transport plan is very low, due the limited number of non-zero links of the solution. In contrast, a sub-optimal solution of the OT problem carries much more information and our sub-optimal transport model offers a natural resolution. Thanks to the geometric properties of the solution space at finite β, the inverse problem becomes tractable without imposing any prior structure on the cost matrix. This feature significantly broadens the applicability of our framework, as it enables inference of cost-related information directly from observed flows.

We apply this model to quantify the presence of OT-like processes in empirical systems, with particular focus on international trade and ecological bipartite networks such as plant–pollinator interactions. In these systems we frame the OT problem as a resources allocation problem with a gain function as described in the figure. By framing both systems within a unified theoretical setting, we aim to highlight common structural properties, identify domain-specific deviations, and provide new insights into how optimality and randomness jointly shape real-world transport processes.

[1] ”Maximum entropy modeling of Optimal Transport: the sub-optimality regime and the transition from dense to sparse networks.” arXiv preprint arXiv:2504.10444 (2025).

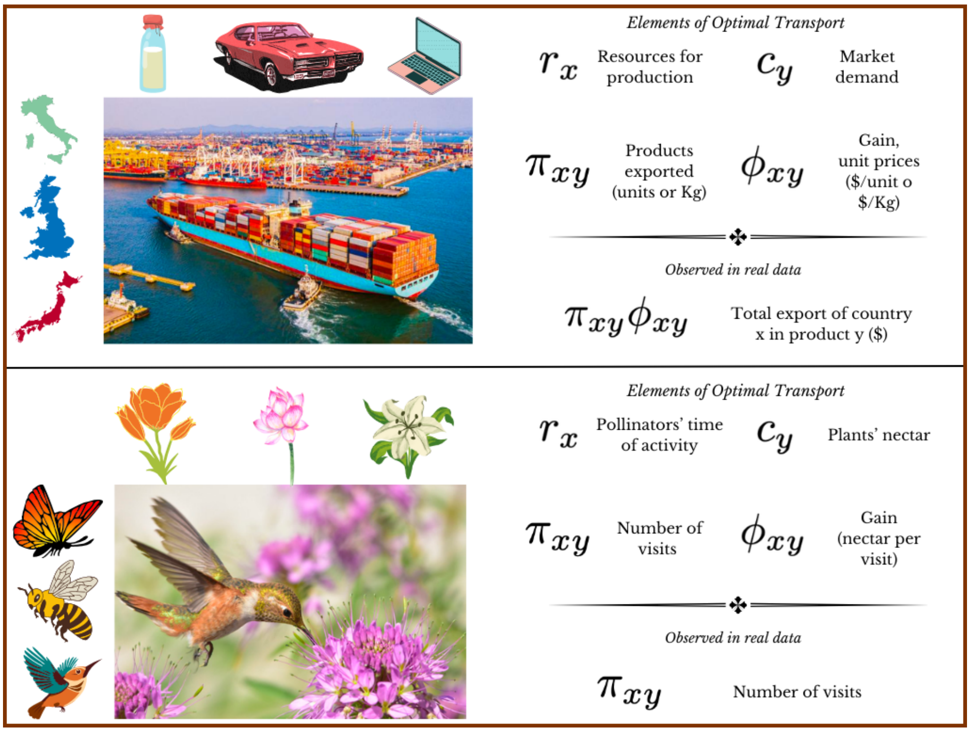

Figure 1: **The OT problem framed in plant-pollinator systems and international trade.** Resources allocation plan (π_{xy}) in both systems may be decided by optimizing the total gain and the resources availability. Upper panel) In the country-products bipartite network of international trade, the constraints are resources of countries and products demand in the market. Resource are allocated in the production of commodities trying to meet market demands and depending on the unitary prices of export. Usually the observed data are the monetary fluxes of trade. Lower panel) For plant-pollinator systems we have the constraints tightly related to the abundances of species. Pollinators decide which plant to visit based on their preferences and the abundance of the each specie of plant. The observed data are the frequencies of interactions (number of visits), assumed to be the resource allocation plan.

Preferred formats:
Best - oral presentation
Still ok - Lightining
Does not justify the very long travel - poster