

EXCHANGE OF PERSPECTIVE PROMPTING ENHANCES REASONING IN LARGE LANGUAGE MODELS

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ABSTRACT

Large language models (LLMs) have made significant advancements in addressing diverse natural language processing (NLP) tasks. However, their performance is often limited by inherent comprehension of problems. To address this limitation, we propose Exchange-of-Perspective (EoP), a novel framework designed to exchange perspectives across different definitions of problem, so that it can break the fixed mindset from any particular formulation of the question. We conducted extensive and comprehensive experiments on 8 benchmarks. The results show that EoP can significantly improve performance. For instance, compared to the non-commutative baseline PHP, with GPT-3.5-Turbo and EoP, we observe a 3.6% improvement on AQuA (60.6% \rightarrow 64.2%), while GPT-4-powered EoP demonstrates a 7.7% enhancement on Math (53.9% \rightarrow 61.6%) and a 3.5% improvement on OlympiadBench Maths (43.5% \rightarrow 47.0%) when using Qwen-2.5-72b.

1 INTRODUCTION

LLMs have made significant progress in the field of NLP, but they often struggle to provide stable and accurate answers when faced with highly complex tasks. This issue cannot be resolved by simply scaling up the model size (Rae et al., 2021; Wang et al., 2023c).

To address this limitation, Wei et al. (2022) introduced the chain-of-thought (CoT) prompting method, which guides the model to generate a series of intermediate reasoning steps before arriving at the final answer. Subsequently, various self-correction strategies have emerged (Zheng et al., 2023; Welleck et al., 2023; Ganguli et al., 2023). They are designed to iteratively improve the quality of responses by using the model’s previous outputs.

Nevertheless, both CoT and self-correction techniques focus on reasoning process, and they rely heavily on the model’s own comprehension of the problem. Recent studies show that LLMs struggle to improve their responses without external feedback (Valmeekam et al., 2023; Stechly et al., 2023; Huang et al., 2024). This difficulty arises from their reliance on internal representations, making it challenging to overcome intrinsic capacity constraints (Yin et al., 2023).

To tackle the challenges outlined, we propose a different viewpoint. Instead of centering focus on the reasoning process, **let’s redirect our attention to the question**. We assert that it is preferable to thoroughly and deeply understand the question before formulating a solution rather than hastily

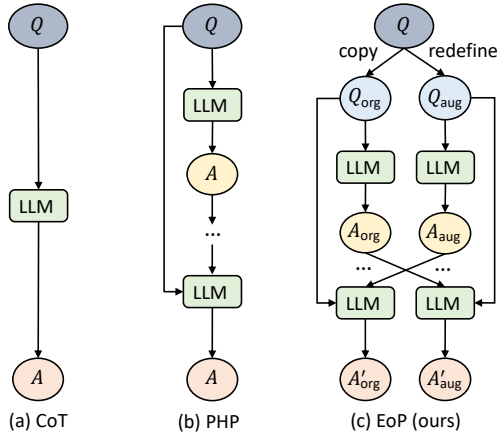


Figure 1: Comparison of CoT, PHP, and EoP. Both CoT and PHP rely on the model’s internal perspective to generate or refine output, lacking external insights. EoP enhances the model’s reasoning ability by incorporating external perspective through swapping answers for the same question presented with different definitions.

054 offering a solution and then trying to revise it repeatedly. One effective method of enhancing com-
 055 prehension is to view the question from different perspectives, we observe a phenomenon that when
 056 humans answer questions, different definitions of the same question can lead to varied responses.
 057 This indicates that diverse phrasing of question has potential to yield multiple perspectives. When
 058 the responses are consistent, the likelihood of the response being correct also increases.

059 Building on this insight, we introduce the Exchange-of-Perspective (EoP) framework, as shown
 060 in Figure 1. Unlike CoT and PHP (Zheng et al., 2023), EoP redefines the original question first,
 061 and then dynamically incorporates external perspectives by iteratively exchanging answers for the
 062 same question presented with different definitions. Figure 2 illustrates EoP further. It executes as
 063 outlined below: (1) For a given question, we redefine it into an augmented question with LLM,
 064 subsequently forming two branches: the original branch and the augmented branch. (2) Instruct the
 065 LLM to generate initial answers for both branches. (3) Swap the answers and combine them with
 066 the question from the other branch using the phrase "Hint: The answer is near to", which follows
 067 (Zheng et al., 2023), to derive follow-up answers. (4) Continue the process in step (3) until meeting
 068 termination condition.

069 We summarize our contributions as follows:

- 070 • We introduce EoP, a novel framework that integrates various perspectives on the question,
 071 **we are exploring a new direction for improving LLM performance by focusing on the**
 072 **input side of the question rather than the reasoning side.**
- 073 • We conduct extensive experiments across various complex reasoning tasks. Results show
 074 that our method significantly outperforms established strong baselines, highlighting the
 075 crucial role of external perspectives in enhancing the capabilities of LLMs.
- 076 • Our research offers a unique viewpoint that yields a fresh perspective by redefining the
 077 original problem. This method emphasizes the essential role of problem definition in influ-
 078 encing comprehension and solutions.

080 2 METHOD

081 2.1 ARCHITECTURE

082 To deepen the understanding of question and acquire external perspective to enhance reasoning
 083 capability, the original question q_{org} is passed through a redefinition function f , resulting in an
 084 augmented question, $q_{\text{aug}} = f(q_{\text{org}})$. The reasoning process then divides into two branches: the
 085 original branch and the augmented branch.

086 We denote the LLM with parameters θ as P_θ . In the first interaction, the LLM produces a rationale
 087 $r_{\text{org}}^{(1)}$ and an answer $a_{\text{org}}^{(1)}$ for the original question q_{org} . Similarly, it generates a rationale $r_{\text{aug}}^{(1)}$ and an
 088 answer $a_{\text{aug}}^{(1)}$ for the augmented question q_{aug} . The first iteration of the reasoning process for both the
 089 original and augmented questions can be represented probabilistically as:

$$090 (r_{\text{org}}^{(1)}, a_{\text{org}}^{(1)}) \sim P_\theta(r_{\text{org}}, a_{\text{org}} | q_{\text{org}}) \quad (1)$$

$$091 (r_{\text{aug}}^{(1)}, a_{\text{aug}}^{(1)}) \sim P_\theta(r_{\text{aug}}, a_{\text{aug}} | q_{\text{aug}}) \quad (2)$$

092 In subsequent iterations ($j > 1$), the model P_θ adapts its response strategy. It now generates a
 093 rationale $r_{\text{org}}^{(j)}$ and an answer $a_{\text{org}}^{(j)}$ for the original question q_{org} , taking into account the history of
 094 answers provided by the augmented question up to the $(j - 1)$ -th iteration, which is encapsulated in
 095 the set $\mathbb{A}_{\text{aug}}^{(j-1)} = \{a_{\text{aug}}^{(1)}, \dots, a_{\text{aug}}^{(j-1)}\}$. This process is modeled as:

$$096 (r_{\text{org}}^{(j)}, a_{\text{org}}^{(j)}) \sim P_\theta(r_{\text{org}}, a_{\text{org}} | q_{\text{org}}, \mathbb{A}_{\text{aug}}^{(j-1)}) \quad (3)$$

097 For augmented question q_{aug} , it uses $\mathbb{A}_{\text{org}}^{(j-1)} = \{a_{\text{org}}^{(1)}, \dots, a_{\text{org}}^{(j-1)}\}$ to generate $(r_{\text{aug}}^{(j)}, a_{\text{aug}}^{(j)})$:

$$098 (r_{\text{aug}}^{(j)}, a_{\text{aug}}^{(j)}) \sim P_\theta(r_{\text{aug}}, a_{\text{aug}} | q_{\text{aug}}, \mathbb{A}_{\text{org}}^{(j-1)}) \quad (4)$$

This cross-referencing between the original and augmented answers aims to acquire dynamic external perspective to break the inherent fixed mindset of LLM, and enhance the model’s ability to provide coherent responses, which can lead to more reliable and accurate results.

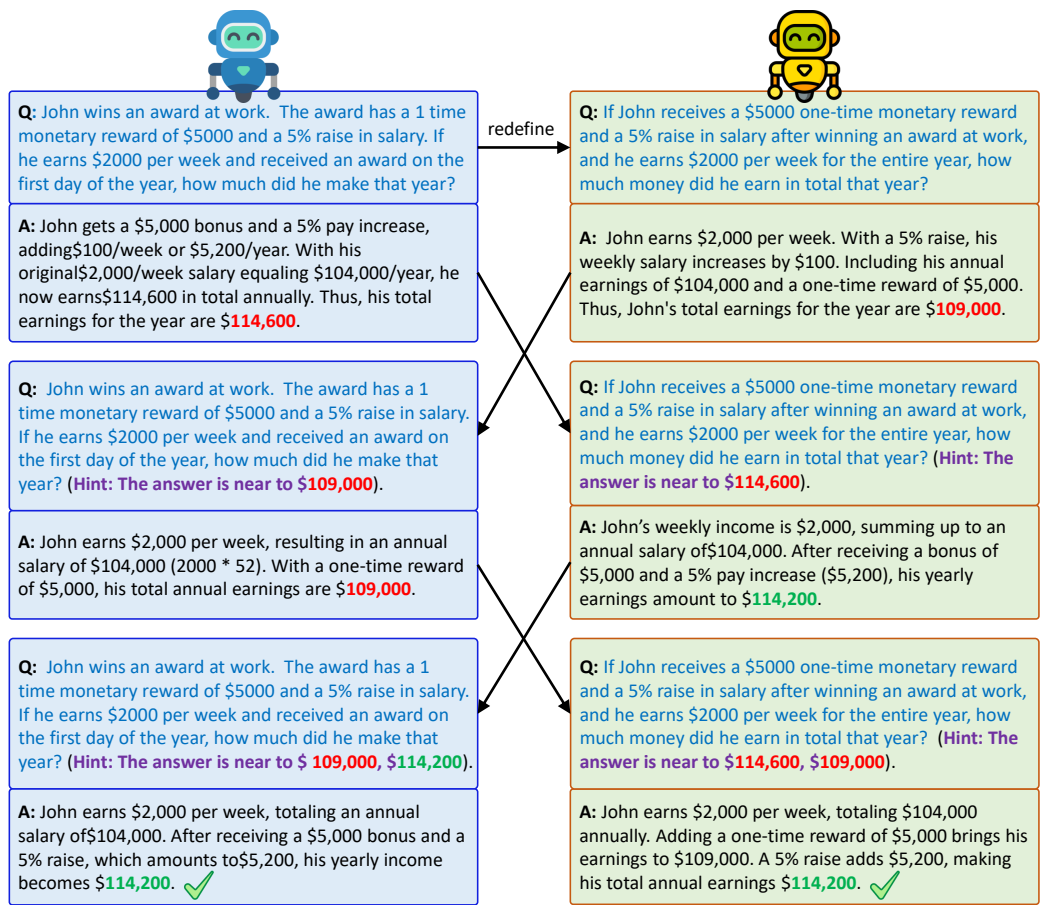


Figure 2: Our proposed EoP integrates the current question with answers from the alternative branch to facilitate perspective exchange. It consists of four stages: (1) We redefine the given question into an augmented version, subsequently forming two branches: **original branch** and **augmented branch**. (2) Instruct the LLM to produce initial answers for both the original and augmented branches by providing it with a combination of the question and a fundamental prompt, such as CoT or Complex CoT. (3) Swap the answers and combine them with the question from the other branch to generate subsequent answers. (4) We continue the process in step (3) until meeting termination condition.

2.2 REDEFINITION FUNCTION

The redefinition of question adheres to two principles: (1) preserving the original semantics and (2) not changing the final answer. In line with these, we introduce two redefinition strategies:

Premise Extraction and Concatenation (PEC) In Figure 3, we illustrate the PEC method. Given original question q_{org} , we pass it through LLM to extract key premises $[p_1, p_2, \dots, p_n]$ and the core question q_{core} . These elements are crucial for understanding the complex concepts presented in the question, and then they are concatenated to form an augmented question q_{aug} :

$$q_{aug} = [p_1, p_2, \dots, p_n] \oplus q_{core} \quad (5)$$

The symbol \oplus denotes concatenation. The main purpose of this method is to clarify the original question, making it clear and unambiguous.

Question Rephrasing (QR) Utilize a LLM to directly rephrase the original question. This procedure involves first grasping the original question and subsequently expressing the question in a manner that aligns with the LLM’s understanding.

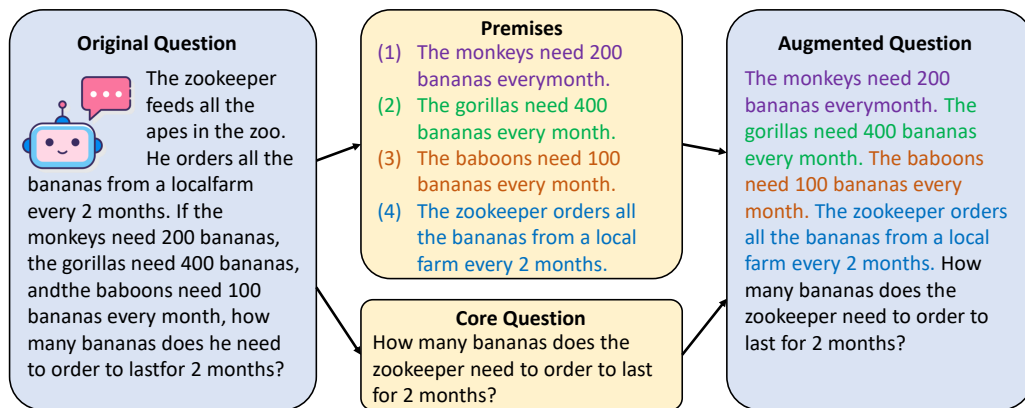


Figure 3: Illustration of PEC redefinition, which includes a two-stage process: (1) extract key premises and the core question from the original question; (2) combine the premises with the core question to form an augmented question.

2.3 TERMINATION CONDITIONS

The iteration process terminates upon meeting one of the following conditions:

Consensus Across Branches For the j -th iteration, if the output from the original branch matches the augmented branch, i.e., $a_{\text{org}}^{(j)} = a_{\text{aug}}^{(j)}$, it indicates that both branches have reached a consensus.

Stability Within Branch For a given branch, if the output in the j -th iteration is identical to that of the $(j - 1)$ -th iteration, either $a_{\text{org}}^{(j)} = a_{\text{org}}^{(j-1)}$ for the original branch or $a_{\text{aug}}^{(j)} = a_{\text{aug}}^{(j-1)}$ for the augmented branch, it indicates that the branch’s output is stable.

3 EXPERIMENT

3.1 EXPERIMENTAL SETTINGS

Datasets and Models We evaluate the performance of EoP on 8 datasets: AddSub (Hosseini et al., 2014), MultiArith (Roy & Roth, 2015), SingleEQ (Koncel-Kedziorski et al., 2015), SVAMP (Patel et al., 2021), GSM8K (Cobbe et al., 2021), AQUA (Ling et al., 2017) and Math (Hendrycks et al., 2021), OlympiadBench Maths¹ (He et al., 2024). These datasets were selected to focus on model’s mathematical reasoning capability. We utilized three types of prompts: Standard, Chain-of-Thought (CoT) (Wei et al., 2022), and Complex CoT (Fu et al., 2023). For more details, see Section A. To verify the effectiveness of our proposed method, we employ 4 models: GPT-3.5-Turbo, GPT-4 (Ouyang et al., 2022; OpenAI, 2023) and Qwen-2.5-7b, Qwen-2.5-72b (Yang et al., 2024).

Baselines We benchmark our proposed EoP against several strong baselines. For the Arithmetic dataset, the baselines include: (1) Chain-of-Thought prompting (CoT; Kojima et al., 2022), (2) Plan-and-Solve prompting (PS; Wang et al., 2023a), (3) Least-to-Most prompting (Zhou et al., 2023), (4) Contrastive Prompting (CP; CoT-CP; Yao, 2024), (5) Progressive-Hint Prompting (PHP; Zheng et al., 2023). Regarding the Math dataset, in addition to CoT and PHP, we also compared with following methods: (1) Program-Aided Language models (PAL; Gao et al., 2023b), (2) Tool-Integrated Reasoning Agent (ToRA; Gou et al., 2024b), (3) Skills-in-Context Prompting (SKiC; Chen et al., 2023a), (4) Cumulative Reasoning (CR; Zhang et al., 2023). Additionally, we compared EoP with

¹The data utilized adheres to: <https://github.com/QwenLM/Qwen2.5-Math>

self-consistency (SC; Wang et al., 2023c) on the Math and OlympiadBench datasets, and we set the temperature $T = 0.8$ for SC method, and $T = 0$ for other baselines during testing.

3.2 RESULTS

Performance on Arithmetic dataset Table 1 displays the results of existing baselines and our EoP approach on the Arithmetic dataset. EoP achieves the highest mean accuracy, with GPT-3.5-Turbo recording 85.3%, a 4.4% improvement over the CoT baseline and surpassing PHP by 1.1%. The AQuA dataset shows the most significant enhancement, with EoP scoring 64.2%, reflecting an 11.2% increase over CoT and a 3.6% rise over PHP. For GPT-4, all baselines show improved performance, with EoP maintaining the highest mean accuracy. These results demonstrate that EoP is a robust and efficient method across various LLMs and datasets, particularly excelling in complex reasoning tasks like AQuA, highlighting its suitability for deep understanding tasks.

Table 1: **Evaluation Results on Arithmetic Dataset:** When applied to various LLMs with complex CoT prompts and PEC redefinition, EoP outperforms baseline methods. **Avg.** indicates mean accuracy across all test datasets. Top results are in **bold**, and runner-up results are underlined. Performance improvements (Δ) are relative to each baseline method.

Method	Arithmetic Dataset						Avg.	Δ
	AddSub	MultiArith	SingleEQ	SVAMP	GSM8K	AQuA		
ChatGPT (GPT-3.5-Turbo)								
CoT	85.8	95.3	<u>93.5</u>	79.3	78.9	53.0	80.9	+4.4
PS	86.6	93.8	92.5	79.4	76.1	58.9	81.2	+4.1
Least-to-Most	91.3	95.5	<u>93.5</u>	80.9	77.5	57.4	82.6	+2.7
CP	<u>90.6</u>	95.2	91.7	83.2	73.2	40.2	80.8	+4.5
CoT-CP	88.6	96.2	92.3	85.9	73.5	60.6	82.9	+2.4
PHP	85.3	<u>98.0</u>	92.9	83.1	85.1	60.6	84.2	+1.1
EoP (ours)	87.3	98.2	93.6	<u>84.6</u>	<u>84.2</u>	64.2	85.3	
GPT-4								
CoT	92.4	97.8	95	90.4	94.6	72.8	90.6	+1.7
PS	<u>93.1</u>	<u>98.1</u>	95.3	<u>92.6</u>	94.3	75.5	<u>91.4</u>	+0.9
Least-to-Most	92.1	97.1	<u>95.0</u>	90.3	92.1	71.6	89.7	+2.6
CP	91.6	97.8	91.7	91.5	88.8	62.2	87.3	+5.0
CoT-CP	91.4	97.2	92.7	91.6	89.5	71.3	89.0	+3.3
PHP	89.6	<u>98.1</u>	93.1	91.9	<u>95.5</u>	79.9	<u>91.4</u>	+0.9
EoP (ours)	93.4	98.3	94.5	93.0	96.2	<u>78.4</u>	92.3	

3.3 ANALYSIS

Performance on Math and OlympiadBench Maths dataset Table 2 presents the results on the Math and OlympiadBench Maths datasets using the Qwen-2.5 series. EoP achieves the best performance on these challenging tasks. Table 3 further provides a breakdown of each question type on the Math dataset using GPT-4. This time we observe even more improvements for the EoP approach. It is worth noting that PHP can be seen as a special case of EoP, focusing solely on the original branch. The results reveal that the EoP approach gets a mean accuracy of 61.6%, surpassing PHP by 7.7%. Significantly, the EoP technique outperforms PAL and ToRA, which are code-based methods requiring the implementation of specific code.

Table 2: **Evaluation Results on Math and OlympiadBench Maths:** N represents the average interaction number required to obtain the final answer from the LLM in the reasoning phase.

LLM	Method	Math		Olympiad	
		N	Acc.	N	Acc.
Qwen2.5-7b	CoT	1.0	71.1	1.0	35.8
	PHP	2.4	<u>72.5</u>	2.5	<u>38.1</u>
	EoP (ours)	3.2	74.6	4.8	40.7
	SC	4.0	71.8	5.0	37.2
Qwen2.5-72b	CoT	1.0	78.5	1.0	42.1
	PHP	2.3	79.2	2.4	<u>43.5</u>
	EoP (ours)	2.9	81.7	4.2	47.0
	SC	4.0	<u>80.5</u>	5.0	43.1

The consistent and reliable results across various

mathematical problems highlight the method’s robustness and its potential for enhancing reasoning capabilities.

3.4 ABLATION STUDY

Why EoP can get significant performance improvement? Table 4 showcases the performance metrics for each branch. The Org Branch (with original question) yields 83.3% for CoT Prompt and 83.1% for Complex CoT Prompt, while the Aug Branch (with rephrased questions) results in 82.1% and 81.7% respectively. These results indicate that the rephrased questions perform even worse than the original questions. However, when integrating both branches, EoP framework outperforms individual branches with scores of 84.9% for CoT Prompt and 85.3% for Complex CoT Prompt. So the performance gain of EoP is not from rephrasing the question, while it comes from two main factors: (1) Error Correction, where insights from one branch can rectify misinterpretations from another, thereby improving problem analysis accuracy, and (2) Complementary Information, where merging branches provides more extensive and holistic insights to problem-solving. So EoP can effectively mitigating the potential inaccuracies introduced by rephrased question.

Table 3: **Evaluation Results on Math Dataset:** Significant improvements are observed for EoP technique on Math dataset. In addition to overall accuracy (%), we provide a breakdown of accuracy for various question types within the test set. Both PHP and EoP are applied with complex CoT prompt. The results are from GPT-4 with greedy decoding and PEC redefinition.

Method	Math Dataset							Avg.	Δ
	Algebra	Probability	Geometry	InterAlgebra	NumTheory	PreAlgebra	Precalculus		
CoT	70.8	53.1	36.5	23.4	49.6	71.6	26.7	50.3	+11.3
PAL	59.1	61.0	38	32.8	58.7	73.9	29.3	52.0	+9.6
ToRA	71.8	66.1	48.8	49.5	49.5	67.1	44.6	60.8	+0.8
SKiC	74.6	58.2	43.6	29.5	55.9	79.7	36.6	56.4	+5.2
CR	86.6	63.2	43.9	32	59.7	71.8	35.7	58.0	+3.6
PHP	74.3	56.3	41.9	26.3	55.7	73.8	29.8	53.9	+7.7
EoP (ours)	<u>80.1</u>	70.0	<u>47.6</u>	<u>35.2</u>	63.5	81.2	<u>36.8</u>	61.6	

Table 4: Ablation Study. We employ the prompt of CoT and Complex CoT. **Org**: original branch, **Aug**: augmented branch. According to the experiment results, we see that the performance of combined branch exceeds that of the individual branch across various tasks. The results are from GPT-3.5-Turbo with greedy decoding and PEC redefinition.

Prompt	Org	Aug	Arithmetic Dataset						Avg.
			AddSub	MultiArith	SingleEQ	SVAMP	GSM8K	AQuA	
CoT	✓	✗	89.4	97.3	93.9	79.7	78.6	60.6	83.3
	✗	✓	90.1	93.8	93.3	79.3	75.7	60.2	82.1
	✓	✓	90.4	97.7	94.5	82.1	80.4	64.2	84.9
Complex CoT	✓	✗	86.3	98.1	93.3	81.3	81.4	58.3	83.1
	✗	✓	86.3	94.2	92.3	80.7	79.5	57.5	81.7
	✓	✓	87.3	98.2	93.6	84.6	84.2	64.2	85.3

PEC redefinition method performs better Table 5 illustrates that the PEC method achieves a 1.3% enhancement over QR using Standard prompt. Additionally, PEC exceeds QR by 0.7% with CoT prompt, and demonstrates a 2.1% advantage with complex CoT prompt. These results indicate that PEC consistently outperforms QR across various prompt scenarios. The primary strength of the PEC approach lies in its capacity to clarify premises and the foundational question. This leads to more targeted and clear inquiries. Such precision is particularly beneficial for models tackling complex tasks, such as AddSub and AQuA, where a thorough understanding of the problem is

crucial for achieving superior accuracy. While QR employs LLM to rephrase questions directly, it may enhance the limited inherent comprehension of the question and finally diminish performance.

EoP outperforms PHP across various prompts, with more improvement for weaker prompt

Table 6 evaluates EoP’s performance against PHP across various prompts. The data reveals that EoP consistently outperforms PHP. For example, in the case of standard prompt, EoP’s accuracy exceeds PHP’s by 6.0%. As for the CoT prompt, EoP surpasses PHP by 1.6%. Additionally, for the standard prompt, EoP outperforms PHP by 1.1%. Furthermore, we can find that there is a pattern indicating more accuracy improvements with weaker prompts, where the complex CoT serves as the most effective prompt, whereas the standard prompt is the least effective, and it is more likely to produce errors, while EoP can assist in rectifying these issues by offering external perspective.

EoP performs better when addressing more difficult challenges

The data illustrated in Figure 4 indicates that EoP exhibits significant enhancements when tackling more complex challenges, surpassing both CoT and PHP. Initially, all methods demonstrate comparable performance. However, as the difficulty of the problems increases, EoP’s decline in performance becomes less evident, especially from Level 3 onwards. By Level 5, EoP achieves a score considerably higher than the other methods, boasting a 9.5% advantage over PHP and an 11.0% lead over CoT. This performance underscores EoP’s exceptional ability to handle intricate problems, primarily due to the collaborative and adaptive characteristics of the insight exchange process.

Table 5: Comparison of Question Redefinition Methods. **Blue**: The performance of PEC is better than that of QR. **Red**: The performance of PEC is worse than that of QR. According to the experiment results, we see that PEC redefinition method performs better in most cases. The results are from GPT-3.5-Turbo with greedy decoding.

Prompt	Redefinition Method	Arithmetic Dataset						Avg.
		AddSub	MultiArith	SingleEQ	SVAMP	GSM8K	AQuA	
Standard	QR	85.6	87.5	90.2	81.0	57.0	36.2	72.9
	PEC	90.9	85.2	90.7	80.2	57.2	40.9	74.2
	Δ Absolute gain	(+5.3)	(-2.3)	(+0.5)	(-0.8)	(+0.2)	(+4.7)	(+1.3)
CoT	QR	87.3	98.7	93.1	81.0	81.2	63.8	84.2
	PEC	90.4	97.7	94.5	82.1	80.4	64.2	84.9
	Δ Absolute gain	(+3.1)	(-1.0)	(+1.4)	(+1.1)	(-0.8)	(+0.4)	(+0.7)
Complex CoT	QR	85.3	98.0	93.3	82.9	81.9	57.1	83.2
	PEC	87.3	98.2	93.6	84.6	84.2	64.2	85.3
	Δ Absolute gain	(+2.0)	(+0.2)	(+0.3)	(+1.7)	(+2.3)	(+7.1)	(+2.1)

EoP does not result in a significant increase in inference cost

Based on our experiments, we have made the following three observations: (1) The EoP framework is efficient. Despite employing two branches, EoP does not lead to a significant increase in inference cost. As shown in Table 2, we compared the interaction number of EoP with PHP and SC methods. In the reasoning phase, for the math dataset, EoP requires approximately 3 interactions, compared to PHP’s 2.3, an increase of less than one interaction. For the OlympiadBench dataset, EoP requires about 4.5 interactions, while PHP requires 2.4, an increase of two interactions. (2) The complexity of the question correlates with the number of interactions required; the OlympiadBench dataset is the most challenging, resulting in the highest number of interactions, followed by the Math dataset, with the Arithmetic dataset requiring the fewest. Figure 5 further illustrates the interaction patterns within the Arithmetic dataset, showing that more difficult questions necessitate more clarifications and repeated responses to ensure a comprehensive understanding and accurate solutions. (3) The more powerful the model, the fewer interactions are needed. Stronger models are more likely to provide correct initial responses and subsequent answers, which ultimately reduces the number of interactions required to arrive at a stable final answer.

Table 6: Performance comparison between PHP and EoP with different prompts. Both methods update their responses by utilizing hints drawn from previously generated answers. The difference is that EoP aims to incorporate external perspectives to overcome the intrinsic capacity constraints of LLMs. The results are from GPT-3.5-Turbo with greedy decoding and PEC redefinition.

Prompt	Method	Arithmetic Dataset						Avg.
		AddSub	MultiArith	SingleEQ	SVAMP	GSM8K	AQuA	
Standard	PHP	89.1	87.3	89.8	77.8	32.7	32.3	68.2
	EoP (ours)	90.9	85.2	90.7	80.2	57.2	40.9	74.2
	Δ Absolute gain	(+1.8)	(-2.1)	(+0.9)	(+2.4)	(+24.5)	(+8.6)	(+6.0)
CoT	PHP	88.3	96.9	92.8	82.5	79.7	59.8	83.3
	EoP (ours)	90.4	97.7	94.5	82.1	80.4	64.2	84.9
	Δ Absolute gain	(+2.1)	(+0.8)	(+1.7)	(-0.4)	(+0.7)	(+4.4)	(+1.6)
Complex CoT	PHP	85.3	98.0	92.9	83.1	85.1	60.6	84.2
	EoP (ours)	87.3	98.2	93.6	84.6	84.2	64.2	85.3
	Δ Absolute gain	(+2.0)	(+0.2)	(+0.7)	(+1.5)	(-0.9)	(+3.6)	(+1.1)

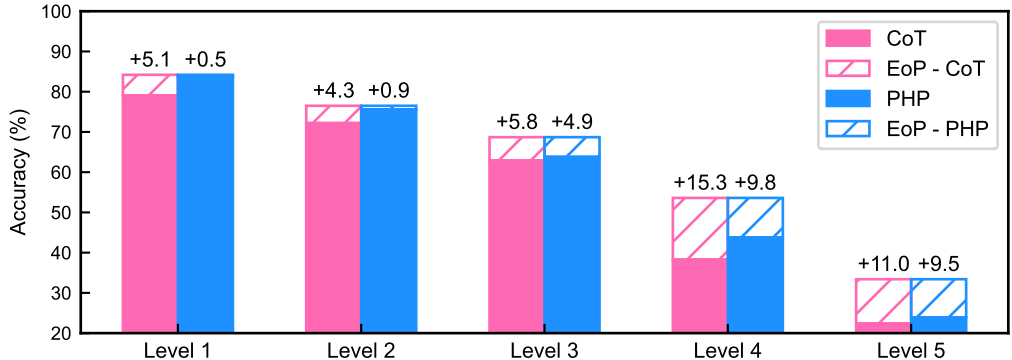


Figure 4: Performance comparison between CoT, PHP and EoP on math dataset with varying difficulty levels. It shows that EoP achieves the best performance across all difficulty levels. The enhancement in performance becomes more evident when tackling more challenging problems. The results are based on GPT-4 with greedy decoding and PEC redefinition.

4 RELATED WORK

Chain-of-Thought prompting Since CoT underscores the value of multi-step logical pathways in deriving conclusive answers, a series of enhancement strategies has been proposed. One noteworthy strategy is the careful selection of demonstrations. For instance, Lu et al. (2023) introduced a framework known as PromptPG, which employs policy gradients to choose demonstrations. Research conducted by Wang et al. (2024) focused on training dense retrievers to select high-quality demonstrations. For complex tasks, a two-stage method is recommended, involving the breakdown of tasks into manageable sub-tasks that are solved sequentially before integration (Zhou et al., 2023; Khot et al., 2023). Additionally, approaches such as Program-of-Thought (PoT; Chen et al., 2023b), Program-Aided Language models (PAL; Gao et al., 2023b) propose to generate intermediate reasoning programs and employ external interpreters for execution. Moreover, equipping LLMs with external tools, such as scratch pads for intermediate computations, search engines for information retrieval, QA systems for clarifying inquiries, and calculators for performing mathematical operations, can further enhance task performance (Nye et al., 2021; Shuster et al., 2022; Schick et al., 2023). However, these methods rely on a single reasoning pathway, if the initial reasoning steps exhibit inaccuracies or inherent biases, these defects may propagate through the reasoning process, culminating in erroneous conclusions (Bai et al., 2022; Lyu et al., 2023).

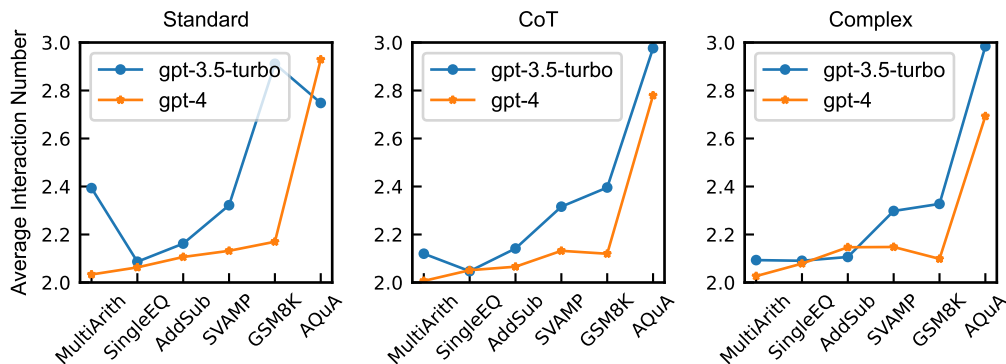
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Figure 5: The Interaction Number with different prompt settings. The result reveals two key findings: (1) A more complex problem increases the interaction number; (2) A stronger model reduces the interaction number.

Self-Correction Self-correction represents a method that allows LLMs to refine their outputs based on feedback from prior responses. Two main branches exist for self-correction: fine-tuning and prompting. In the field of fine-tuning, McAleese et al. (2024) advocate for developing a critic model that assesses the responses of LLMs, which is subsequently employed to improve their responses. Conversely, research by An et al. (2023) and Kumar et al. (2024) recommends directly fine-tuning the LLM rather than training an additional critic model. However, this fine-tuning method demands considerable effort and resources. In the field of prompting, research conducted by (Kim et al., 2023) and (Madaan et al., 2023) utilizes the LLM’s self-reflection and past errors to enhance reasoning. Zheng et al. (2023) developed the PHP method, which employs hints from earlier responses to guide models toward more accurate outcomes. Nonetheless, recent studies (Huang et al., 2024; Qu et al., 2024) illustrate that even top-tier models frequently struggle with self-correcting reasoning mistakes and may experience performance declines without external feedback. This external feedback may originate from humans, other models (Wang et al., 2023b; Paul et al., 2024), or external tools and knowledge sources (Gou et al., 2024a; Chen et al., 2024; Gao et al., 2023a). However, in practice, high-quality external feedback is often unavailable. This study proposes a simple yet effective external feedback approach that incorporates different perspectives by exchanging answers to the same question framed with varied definitions.

5 CONCLUSION

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We have introduced the EoP framework, a novel and impactful approach aimed at enhancing the reasoning capabilities of LLMs. By integrating external perspectives from various definitions of the question, EoP overcomes the limitations of current chain-of-thought and self-correction methods, which rely heavily on the model’s internal perspective. This method highlights the significance of comprehensively grasping the question prior to developing a solution, and sharing diverse perspectives can cultivate more resilient and nuanced reasoning. Our experimental results across a variety of complex reasoning tasks demonstrate that the EoP framework significantly outperforms existing benchmarks, highlighting its potential to improve the problem-solving skills of LLMs. In conclusion, our work emphasizes the importance of integrating external perspectives obtained from diverse problem definitions, setting a new direction for the development of more reliable and accurate language models.

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702 A PROMPT

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704 A.1 PROMPT FOR REDEFINITION FUNCTION

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706 PEC Prompt

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708 Extract premises and clear question from input, output a dictionary with 'premise' and 'question' as
709 keys.

710 **[Demonstration]**

711 Input:

712 There are 96 fourth-graders at Small Tree School. 43 of them are girls. On Friday, 5 fourth-grade
713 girls and 4 fourth grade boys were absent. How many fourth grade boys were at Small Tree School on
714 Friday?

715 Output:

```
716 {
717   "premises": [
718     "Small Tree School has a total of 96 fourth-graders.",
719     "Out of these, 43 are girls.",
720     "On Friday, 5 girls and 4 boys from the fourth grade were absent."
721   ],
722   "question": "How many fourth-grade boys were present at Small Tree School on Friday?"
723 }
```

724 **[Question to be answered]**

725 Input:

726 {{question}}

727 Output:

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728 {
729   "premises": ["string", ...], // all premises extracted from input
730   "question": string, //core question from input
731 }
```

732
733 QR Prompt

734 Revise and improve the given question while retaining all its original premises and final result:

735 Original question:

736 {{question}}

737 New question:

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739 A.2 PROMPT FOR REASONING

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741 Reasoning Prompt

742 Follow the given demonstration and answer the question.

743 **[Demonstration]**

744 {{demonstrations}}

745 **[Question to be answered]**

746 Question: {{question}}

747 **[Note]**

748 {% if dataset == 'aqua' -%}

749 The final answer in the format of "the answer is ANSWER" should be included, where ANSWER is
750 one from the options ["(a)", "(b)", "(c)", "(d)", "(e)"]. For example, "the answer is (a)", "the answer
751 is (b)", "the answer is (c)"... If the answer is not in the options, select the most possible option.

752 {%- else -%}

753 The final answer in the format of "the answer is" should be included.

754 {%- endif %}

755 Answer:

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A.3 REASONING DEMONSTRATIONS

Demonstrations for AQuA dataset.

Base Standard

Question:

John found that the average of 15 numbers is 40. If 10 is added to each number then the mean of the numbers is?

Options: (a) 50 (b) 45 (c) 65 (d) 78 (e) 64

Answer:

The answer is (a).

Question:

If $a/b = 3/4$ and $8a + 5b = 22$, then find the value of a.

Options: (a) $1/2$ (b) $3/2$ (c) $5/2$ (d) $4/2$ (e) $7/2$

Answer:

The answer is (b).

Question:

A person is traveling at 20 km/hr and reached his destiny in 2.5 hr then find the distance?

Options: (a) 53 km (b) 55 km (c) 52 km (d) 60 km (e) 50 km

Answer:

The answer is (e).

EoP Standard

Question:

John found that the average of 15 numbers is 40. If 10 is added to each number then the mean of the numbers is? (Hint: The answer is near to (a)).

Options: (a) 50 (b) 45 (c) 65 (d) 78 (e) 64

Answer:

We know the Answer Hints: (a). With the Answer Hints: (a), we will answer the question.

The answer is (a).

Question:

If $a/b = 3/4$ and $8a + 5b = 22$, then find the value of a. (Hint: The answer is near to (b)).

Options: (a) $1/2$ (b) $3/2$ (c) $5/2$ (d) $4/2$ (e) $7/2$

Answer:

We know the Answer Hints: (b). With the Answer Hints: (b), we will answer the question.

The answer is (b).

Question:

A person is traveling at 20 km/hr and reached his destiny in 2.5 hr then find the distance? (Hint: The answer is near to (d), (c)).

Options: (a) 53 km (b) 55 km (c) 52 km (d) 60 km (e) 50 km

Answer:

We know the Answer Hints: (d), (c). With the Answer Hints: (d), (c), we will answer the question.

The answer is (e).

Base CoT

Question:

John found that the average of 15 numbers is 40. If 10 is added to each number then the mean of the numbers is?

Options: (a) 50 (b) 45 (c) 65 (d) 78 (e) 64

Answer:

If 10 is added to each number, then the mean of the numbers also increases by 10. So the new mean would be 50.

The answer is (a).

Question:

If $a/b = 3/4$ and $8a + 5b = 22$, then find the value of a.

Options: (a) $1/2$ (b) $3/2$ (c) $5/2$ (d) $4/2$ (e) $7/2$

Answer:

$a/b = 3/4$, then $b = 4a/3$. So $8a + 5(4a/3) = 22$. This simplifies to $8a + 20a/3 = 22$, which means $44a/3 = 22$. So a is equal to $3/2$.

The answer is (b).

Question:

A person is traveling at 20 km/hr and reached his destiny in 2.5 hr then find the distance?

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Options: (a) 53 km (b) 55 km (c) 52 km (d) 60 km (e) 50 km

Answer:

The distance that the person traveled would have been $20 \text{ km/hr} * 2.5 \text{ hrs} = 50 \text{ km}$.
The answer is (e).

EoP CoT

Question:

John found that the average of 15 numbers is 40. If 10 is added to each number then the mean of the numbers is? (Hint: The answer is near to (a)). Options: (a) 50 (b) 45 (c) 65 (d) 78 (e) 64

Answer:

We know the Answer Hints: (a). With the Answer Hints: (a), we will answer the question.
If 10 is added to each number, then the mean of the numbers also increases by 10. So the new mean would be 50.
The answer is (a).

Question:

If $a/b = 3/4$ and $8a + 5b = 22$, then find the value of a. (Hint: The answer is near to (b)). Options: (a) $1/2$ (b) $3/2$ (c) $5/2$ (d) $4/2$ (e) $7/2$

Answer:

We know the Answer Hints: (b). With the Answer Hints: (b), we will answer the question.
 $a/b = 3/4$, then $b = 4a/3$. So $8a + 5(4a/3) = 22$. This simplifies to $8a + 20a/3 = 22$, which means $44a/3 = 22$. So a is equal to $3/2$.
The answer is (b).

Question:

A person is traveling at 20 km/hr and reached his destiny in 2.5 hr then find the distance? (Hint: The answer is near to (d), (c)). Options: (a) 53 km (b) 55 km (c) 52 km (d) 60 km (e) 50 km

Answer:

We know the Answer Hints: (d), (c). With the Answer Hints: (d), (c), we will answer the question.
The distance that the person traveled would have been $20 \text{ km/hr} * 2.5 \text{ hrs} = 50 \text{ km}$.
The answer is (e).

Base Complex CoT

Question:

there were 35 students in a hostel . due to the admission of 7 new students the expenses of the mess were increased by rs . 84 per day while the average expenditure per head diminished by re 1 . what was the original expenditure of the mess ?

Options: (a) rs 450 (b) rs 920 (c) rs 550 (d) rs . 630 (e) none of these

Answer:

Let's think step by step let the original average expenditure be rs . x then , $42(x - 1) - 35x = 84$ $7x = 126$ $x = 18$ therefore original expenditure = rs . ($35 * 18$) = rs . 630.
The answer is (d).

Question:

the compound and the simple interests on a certain sum at the same rate of interest for two years are rs . 11730 and rs . 10200 respectively . find the sum

Options: (a) rs . 17037 (b) rs . 17000 (c) rs . 17276 (d) rs . 170287 (e) rs . 171881

Answer:

Let's think step by step the simple interest for the first year is $10200 / 2$ is rs . 5100 and compound interest for first year also is rs . 5100. the compound interest for second year on rs . 5100 for one year so rate of the interest = $(100 * 1530) / (5100 * 1) = 30\%$ p so $p = (100 * 10200) / (30 * 2) = rs . 17000$.
The answer is (b).

Question:

solution x contains 20 % of material a and 80 % of material b . solution y contains 30 % of material a and 70 % of material b . a mixture of both these solutions contains 22 % of material a in the final product . how much solution x is present in the mixture ?

Options: (a) 40 % (b) 60 % (c) 80 % (d) 100 % (e) 110 %

Answer:

Let's think step by step we can assume the total weight of the mixture = 100 conc of a in the final mixture = 22 let weight of a in the mixture be x. conc given = $20\% = 0.2$ therefore , weight of b = $100 - x$. conc given = $30\% = 0.3$ now , according to the problem, $0.2x + 0.3(100 - x) = 22$ solving , we get $x = 80$ since we assumed the weight of the mixture = 100, therefore presence of a in the mixture = 80%.

The answer is (c).

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EoP Complex CoT

Question:

there were 35 students in a hostel . due to the admission of 7 new students the expenses of the mess were increased by rs . 84 per day while the average expenditure per head diminished by re 1 . what was the original expenditure of the mess ? (Hint: The answer is near to (d)).

Options: (a) rs 450 (b) rs 920 (c) rs 550 (d) rs . 630 (e) none of these

Answer:

We know the Answer Hints: (d). With the Answer Hints: (d), we will answer the question.

Let's think step by step let the original average expenditure be rs . x then , $42(x - 1) - 35x = 84$ $7x = 126$ $x = 18$ therefore original expenditure = rs . ($35 * 18$) = rs . 630.

The answer is (d).

Question:

a train 200 m long passes a man , running at 5 km / hr in the same direction in which the train is going , in 10 seconds . the speed of the train is ? (Hint: The answer is near to (c)).

Options: (a) 28 (b) 50 (c) 77 (d) 22 (e) 12

Answer:

We know the Answer Hints: (c). With the Answer Hints: (c), we will answer the question.

Let's think step by step speed of the train relative to man = $(200 / 10)$ m / sec = (20) m / sec. [$(20) * (18 / 5)$] km / hr = 72 km / hr. let the speed of the train be x km / hr. then , relative speed = $(x - 5)$ km / hr. $x - 5 = 72$, $x = 77$ km / hr .

The answer is (c).

Question:

solution x contains 20 % of material a and 80 % of material b . solution y contains 30 % of material a and 70 % of material b . a mixture of both these solutions contains 22 % of material a in the final product . how much solution x is present in the mixture ? (Hint: The answer is near to (a)).

Options: (a) 40 % (b) 60 % (c) 80 % (d) 100 % (e) 110 %

Answer:

We know the Answer Hints: (a). With the Answer Hints: (a), we will answer the question.

Let's think step by step we can assume the total weight of the mixture = 100 conc of a in the final mixture = 22 let weight of a in the mixture be x. conc given = 20% = 0.2 therefore , weight of b = 100 - x. conc given = 30% = 0.3 now , according to the problem, $0.2x + 0.3(100 - x) = 22$ solving , we get $x = 80$ since we assumed the weight of the mixture = 100, therefore presence of a in the mixture = 80%.

The answer is (c).

Demonstrations for AddSub, MultiArith, SingleEQ, SVAMP and GSM8K.

Base Standard

Question:

There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today?

Answer:

The answer is 6.

Question:

If there are 3 cars in the parking lot and 2 more cars arrive, how many cars are in the parking lot?

Answer:

The answer is 5.

Question:

Leah had 32 chocolates and her sister had 42. If they ate 35, how many pieces do they have left in total?

Answer:

The answer is 39.

EoP Standard

Question:

There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today? (Hint: The answer is near to 6).

Answer:

We know the Answer Hints: 6. With the Answer Hints: 6, we will answer the question.

The answer is 6.

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Question:

If there are 3 cars in the parking lot and 2 more cars arrive, how many cars are in the parking lot? (Hint: The answer is near to 10, 8).

Answer:

We know the Answer Hints: 10, 8. With the Answer Hints: 10, 8, we will answer the question. The answer is 5.

Question:

Leah had 32 chocolates and her sister had 42. If they ate 35, how many pieces do they have left in total? (Hint: The answer is near to 30, 35).

Answer:

We know the Answer Hints: 30, 35. With the Answer Hints: 30, 35, we will answer the question. The answer is 39.

Base CoT

Question:

There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today?

Answer:

There are 15 trees originally. Then there were 21 trees after the Grove workers planted some more. So there must have been $21 - 15 = 6$ trees that were planted. The answer is 6.

Question:

If there are 3 cars in the parking lot and 2 more cars arrive, how many cars are in the parking lot?

Answer:

There are originally 3 cars. Then 2 more cars arrive. Now $3 + 2 = 5$ cars are in the parking lot. The answer is 5.

Question:

Leah had 32 chocolates and her sister had 42. If they ate 35, how many pieces do they have left in total?

Answer:

Originally, Leah had 32 chocolates and her sister had 42. So in total they had $32 + 42 = 74$. After eating 35, they had $74 - 35 = 39$ pieces left in total. The answer is 39.

EoP CoT

Question:

There are 15 trees in the grove. Grove workers will plant trees in the grove today. After they are done, there will be 21 trees. How many trees did the grove workers plant today? (Hint: The answer is near to 6).

Answer:

We know the Answer Hints: 6. With the Answer Hints: 6, we will answer the question.

There are 15 trees originally. Then there were 21 trees after the Grove workers planted some more. So there must have been $21 - 15 = 6$ trees that were planted. The answer is 6.

Question:

If there are 3 cars in the parking lot and 2 more cars arrive, how many cars are in the parking lot? (Hint: The answer is near to 10, 8).

Answer:

We know the Answer Hints: 10, 8. With the Answer Hints: 10, 8, we will answer the question.

There are originally 3 cars. Then 2 more cars arrive. Now $3 + 2 = 5$ cars are in the parking lot. The answer is 5.

Question:

Leah had 32 chocolates and her sister had 42. If they ate 35, how many pieces do they have left in total? (Hint: The answer is near to 30, 35).

Answer:

We know the Answer Hints: 30, 35. With the Answer Hints: 30, 35, we will answer the question.

Originally, Leah had 32 chocolates and her sister had 42. So in total they had $32 + 42 = 74$. After eating 35, they had $74 - 35 = 39$ pieces left in total. The answer is 39.

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Base Complex CoT

Question:

Angelo and Melanie want to plan how many hours over the next week they should study together for their test next week. They have 2 chapters of their textbook to study and 4 worksheets to memorize. They figure out that they should dedicate 3 hours to each chapter of their textbook and 1.5 hours for each worksheet. If they plan to study no more than 4 hours each day, how many days should they plan to study total over the next week if they take a 10-minute break every hour, include 3 10-minute snack breaks each day, and 30 minutes for lunch each day?

Answer:

Let's think step by step. Angelo and Melanie think they should dedicate 3 hours to each of the 2 chapters, 3 hours x 2 chapters = 6 hours total. For the worksheets they plan to dedicate 1.5 hours for each worksheet, 1.5 hours x 4 worksheets = 6 hours total. Angelo and Melanie need to start with planning 12 hours to study, at 4 hours a day, $12 / 4 = 3$ days. However, they need to include time for breaks and lunch. Every hour they want to include a 10-minute break, so 12 total hours x 10 minutes = 120 extra minutes for breaks. They also want to include 3 10-minute snack breaks, 3×10 minutes = 30 minutes. And they want to include 30 minutes for lunch each day, so 120 minutes for breaks + 30 minutes for snack breaks + 30 minutes for lunch = 180 minutes, or $180 / 60$ minutes per hour = 3 extra hours. So Angelo and Melanie want to plan 12 hours to study + 3 hours of breaks = 15 hours total. They want to study no more than 4 hours each day, $15 \text{ hours} / 4 \text{ hours each day} = 3.75$. They will need to plan to study 4 days to allow for all the time they need. The answer is 4.

Question:

Mark's basketball team scores 25 2 pointers, 8 3 pointers and 10 free throws. Their opponents score double the 2 pointers but half the 3 pointers and free throws. What's the total number of points scored by both teams added together?

Answer:

Let's think step by step Mark's team scores 25 2 pointers, meaning they scored $25 \times 2 = 50$ points in 2 pointers. His team also scores 8 3 pointers, meaning they scored $8 \times 3 = 24$ points in 3 pointers They scored 10 free throws, and free throws count as one point so they scored $10 \times 1 = 10$ points in free throws. All together his team scored $50 + 24 + 10 = 84$ points Mark's opponents scored double his team's number of 2 pointers, meaning they scored $50 \times 2 = 100$ points in 2 pointers. His opponents scored half his team's number of 3 pointers, meaning they scored $24 / 2 = 12$ points in 3 pointers. They also scored half Mark's team's points in free throws, meaning they scored $10 / 2 = 5$ points in free throws. All together Mark's opponents scored $100 + 12 + 5 = 117$ points The total score for the game is both team's scores added together, so it is $84 + 117 = 201$ points. The answer is 201.

Question:

Bella has two times as many marbles as frisbees. She also has 20 more frisbees than deck cards. If she buys $2/5$ times more of each item, what would be the total number of the items she will have if she currently has 60 marbles?

Answer:

Let's think step by step When Bella buys $2/5$ times more marbles, she'll have increased the number of marbles by $2/5 \times 60 = 24$ The total number of marbles she'll have is $60 + 24 = 84$ If Bella currently has 60 marbles, and she has two times as many marbles as frisbees, she has $60 / 2 = 30$ frisbees. If Bella buys $2/5$ times more frisbees, she'll have $2/5 \times 30 = 12$ more frisbees. The total number of frisbees she'll have will increase to $30 + 12 = 42$ Bella also has 20 more frisbees than deck cards, meaning she has $30 - 20 = 10$ deck cards If she buys $2/5$ times more deck cards, she'll have $2/5 \times 10 = 4$ more deck cards. The total number of deck cards she'll have is $10 + 4 = 14$ Together, Bella will have a total of $14 + 42 + 84 = 140$ items. The answer is 140.

EoP Complex CoT

Question:

Angelo and Melanie want to plan how many hours over the next week they should study together for their test next week. They have 2 chapters of their textbook to study and 4 worksheets to memorize. They figure out that they should dedicate 3 hours to each chapter of their textbook and 1.5 hours for each worksheet. If they plan to study no more than 4 hours each day, how many days should they plan to study total over the next week if they take a 10-minute break every hour, include 3 10-minute snack breaks each day, and 30 minutes for lunch each day? (Hint: The answer is near to 4).

Answer:

We know the Answer Hints: 4. With the Answer Hints: 4, we will answer the question.

Let's think step by step. Angelo and Melanie think they should dedicate 3 hours to each of the 2 chapters, 3 hours x 2 chapters = 6 hours total. For the worksheets they plan to dedicate 1.5 hours for each worksheet, 1.5 hours x 4 worksheets = 6 hours total. Angelo and Melanie need to start with planning 12 hours to study, at 4 hours a day, $12 / 4 = 3$ days. However, they need to include time for

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breaks and lunch. Every hour they want to include a 10-minute break, so 12 total hours x 10 minutes = 120 extra minutes for breaks. They also want to include 3 10-minute snack breaks, 3 x 10 minutes = 30 minutes. And they want to include 30 minutes for lunch each day, so 120 minutes for breaks + 30 minutes for snack breaks + 30 minutes for lunch = 180 minutes, or 180 / 60 minutes per hour = 3 extra hours. So Angelo and Melanie want to plan 12 hours to study + 3 hours of breaks = 15 hours total. They want to study no more than 4 hours each day, 15 hours / 4 hours each day = 3.75 They will need to plan to study 4 days to allow for all the time they need.
The answer is 4.

Question:

Mark's basketball team scores 25 2 pointers, 8 3 pointers and 10 free throws. Their opponents score double the 2 pointers but half the 3 pointers and free throws. What's the total number of points scored by both teams added together?(Hint: The answer is near to 201).

Answer:

We know the Answer Hints: 201. With the Answer Hints: 201, we will answer the question. Let's think step by step. Mark's team scores 25 2 pointers, meaning they scored $25*2=50$ points in 2 pointers. His team also scores 8 3 pointers, meaning they scored $8*3=24$ points in 3 pointers They scored 10 free throws, and free throws count as one point so they scored $10*1=10$ points in free throws. All together his team scored $50+24+10=84$ points Mark's opponents scored double his team's number of 2 pointers, meaning they scored $50*2=100$ points in 2 pointers. His opponents scored half his team's number of 3 pointers, meaning they scored $24/2=12$ points in 3 pointers. They also scored half Mark's team's points in free throws, meaning they scored $10/2=5$ points in free throws. All together Mark's opponents scored $100+12+5=117$ points The total score for the game is both team's scores added together, so it is $84+117=201$ points.
The answer is 201.

Question:

Bella has two times as many marbles as frisbees. She also has 20 more frisbees than deck cards. If she buys 2/5 times more of each item, what would be the total number of the items she will have if she currently has 60 marbles? (Hint: The answer is near to 160,145).

Answer:

We know the Answer Hints: 160, 145. With the Answer Hints: 160, 145, we will answer the question. Let's think step by step. When Bella buys 2/5 times more marbles, she'll have increased the number of marbles by $2/5*60=24$ The total number of marbles she'll have is $60+24=84$ If Bella currently has 60 marbles, and she has two times as many marbles as frisbees, she has $60/2=30$ frisbees. If Bella buys 2/5 times more frisbees, she'll have $2/5*30=12$ more frisbees. The total number of frisbees she'll have will increase to $30+12=42$ Bella also has 20 more frisbees than deck cards, meaning she has $30-20=10$ deck cards If she buys 2/5 times more deck cards, she'll have $2/5*10=4$ more deck cards. The total number of deck cards she'll have is $10+4=14$ Together, Bella will have a total of $14+42+84=140$ items.
The answer is 140.

B CASE STUDY

B.1 SUCCESSFUL CASE

Question: The zookeeper feeds all the apes in the zoo. He orders all the bananas from a local farm every 2 months. If the monkeys need 200 bananas, the gorillas need 400 bananas, and the baboons need 100 bananas every month, how many bananas does he need to order to last for 2 months? (Correct answer is 1400)



Original Question (1-th iteration): The zookeeper feeds all the apes in the zoo. He orders all the bananas from a local farm every 2 months. If the monkeys need 200 bananas, the gorillas need 400 bananas, and the baboons need 100 bananas every month, how many bananas does he need to order to last for 2 months?

Original Solution (1-th iteration):

Let's think step by step. The monkeys need 100 bananas every month, so for 2 months, they will need $100*2=200$ bananas. The gorillas need 200 bananas every month, so for 2 months, they will need $200*2=400$ bananas. The baboons need 50 bananas every month, so for 2 months, they will need $50*2=100$ bananas. To calculate the total number of bananas needed for 2 months, we add up the bananas needed by each group: $200+400+100=700$ bananas. Therefore, the zookeeper needs to order 700 bananas to last for 2 months. **The answer is 700.**

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Augmented Question (1-th iteration): The monkeys need 200 bananas every month. The gorillas need 400 bananas every month. The baboons need 100 bananas every month. The zookeeper orders all the bananas from a local farm every 2 months. How many bananas does the zookeeper need to order to last for 2 months?

Augmented Solution (1-th iteration):

Let's think step by step. The monkeys need 200 bananas every month, so for 2 months, they will need $200 \text{ bananas} \times 2 = 400 \text{ bananas}$. The gorillas need 400 bananas every month, so for 2 months, they will need $400 \text{ bananas} \times 2 = 800 \text{ bananas}$. The baboons need 100 bananas every month, so for 2 months, they will need $100 \text{ bananas} \times 2 = 200 \text{ bananas}$. To calculate the total number of bananas needed for 2 months, we add up the bananas needed by each group: $400 \text{ bananas} + 800 \text{ bananas} + 200 \text{ bananas} = 1400 \text{ bananas}$. Therefore, the zookeeper needs to order 1400 bananas to last for 2 months. **The answer is 1400.**



Original Question (2-th iteration): The zookeeper feeds all the apes in the zoo. He orders all the bananas from a local farm every 2 months. If the monkeys need 200 bananas, the gorillas need 400 bananas, and the baboons need 100 bananas every month, how many bananas does he need to order to last for 2 months? (Hint: The answer is near to 1400).

Original Solution (2-th iteration):

We know the Answer Hints: 1400. With the Answer Hints: 1400, we will answer the question. Let's think step by step. The monkeys need 200 bananas every month, so for 2 months, they will need $200 * 2 = 400 \text{ bananas}$. The gorillas need 400 bananas every month, so for 2 months, they will need $400 * 2 = 800 \text{ bananas}$. The baboons need 100 bananas every month, so for 2 months, they will need $100 * 2 = 200 \text{ bananas}$. To calculate the total number of bananas needed for 2 months, we add up the bananas needed by each group: $400 + 800 + 200 = 1400 \text{ bananas}$. The zookeeper needs to order 1400 bananas to last for 2 months. **The answer is 1400.**



Augmented Question (2-th iteration): The monkeys need 200 bananas every month. The gorillas need 400 bananas every month. The baboons need 100 bananas every month. The zookeeper orders all the bananas from a local farm every 2 months. How many bananas does the zookeeper need to order to last for 2 months? (Hint: The answer is near to 700).

Augmented Solution (2-th iteration):

We know the Answer Hints: 700. With the Answer Hints: 700, we will answer the question. Let's think step by step. The monkeys need 200 bananas every month, so for 2 months, they will need $200 * 2 = 400 \text{ bananas}$. The gorillas need 400 bananas every month, so for 2 months, they will need $400 * 2 = 800 \text{ bananas}$. The baboons need 100 bananas every month, so for 2 months, they will need $100 * 2 = 200 \text{ bananas}$. To calculate the total number of bananas needed for 2 months, we add up the bananas needed by each animal: $400 + 800 + 200 = 1400 \text{ bananas}$. Therefore, the zookeeper needs to order 1400 bananas to last for 2 months. **The answer is 1400.**

B.2 FAILURE CASE

Question: Carlos is planting a lemon tree. The tree will cost \$90 to plant. Each year it will grow 7 lemons, which he can sell for \$1.5 each. It costs \$3 a year to water and feed the tree. How many years will it take before he starts earning money on the lemon tree? **(Correct answer is 13.)**

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Original Question (1-th iteration): Carlos is planting a lemon tree. The tree will cost \$90 to plant. Each year it will grow 7 lemons, which he can sell for \$1.5 each. It costs \$3 a year to water and feed the tree. How many years will it take before he starts earning money on the lemon tree?

Original Solution (1-th iteration):

Let's think step by step. Carlos will spend \$90 to plant the lemon tree. Each year, the lemon tree will produce 7 lemons, which he can sell for \$1.5 each. So, he will earn $7 * \$1.5 = \10.5 per year from selling lemons. However, he will also have to spend \$3 each year to water and feed the tree. To calculate the net earnings each year, we subtract the cost of watering and feeding the tree from the earnings from selling lemons: $\$10.5 - \$3 = \$7.5$. Carlos will start earning money on the lemon tree when his net earnings per year are greater than or equal to \$0. So, we need to find out how many years it will take for his net earnings to reach \$0 or more. We divide the initial cost of planting the tree (\$90) by the net earnings per year (\$7.5) to find out how many years it will take: $\$90 / \$7.5 = 12$. Therefore, it will take Carlos 12 years before he starts earning money on the lemon tree. **The answer is 12.**

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Augmented Question (1-th iteration): Carlos is planting a lemon tree. The cost to plant the tree is \$90. The tree will produce 7 lemons each year. The lemons can be sold for \$1.5 each. It costs \$3 a year to water and feed the tree. How many years will it take before Carlos starts earning money on the lemon tree?

Augmented Solution (1-th iteration):

Let's think step by step. Carlos needs to calculate the total cost of maintaining the tree each year. This includes the cost of watering and feeding the tree, which is \$3. Carlos also needs to calculate the total revenue he will earn each year from selling the lemons. This is the number of lemons the tree produces, which is 7, multiplied by the price per lemon, which is \$1.5. So the total revenue is $7 * \$1.5 = \10.5 . To determine when Carlos will start earning money on the lemon tree, we need to find the number of years it takes for the revenue to exceed the cost. So we need to solve the equation $\$10.5 * x > \90 , where x is the number of years. Dividing both sides of the equation by \$10.5, we get $x > \$90 / \$10.5 = 8.57$. Since we can't have a fraction of a year, Carlos will start earning money on the lemon tree after 9 years. **The answer is 9.**

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Original Question (2-th iteration): Carlos is planting a lemon tree. The tree will cost \$90 to plant. Each year it will grow 7 lemons, which he can sell for \$1.5 each. It costs \$3 a year to water and feed the tree. How many years will it take before he starts earning money on the lemon tree? (Hint: The answer is near to 9).

Original Solution (2-th iteration):

We know the Answer Hints: 9. With the Answer Hints: 9, we will answer the question. Let's think step by step. Carlos will spend \$3 a year to water and feed the tree, so he will spend $\$3 * 1 \text{ year} = \3 in the first year. He will earn $\$1.5 * 7 \text{ lemons} = \10.5 from selling the lemons in the first year. So in the first year, Carlos will earn $\$10.5 - \$3 = \$7.5$. Since the tree cost \$90 to plant, Carlos will need to earn $\$90 / \$7.5 = 12$ years to earn back the initial cost. Therefore, it will take Carlos 12 years before he starts earning money on the lemon tree. **The answer is 12.**

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Augmented Question (2-th iteration): Carlos is planting a lemon tree. The cost to plant the tree is \$90. The tree will produce 7 lemons each year. The lemons can be sold for \$1.5 each. It costs \$3 a year to water and feed the tree. How many years will it take before Carlos starts earning money on the lemon tree? (Hint: The answer is near to 12).

Augmented Solution (2-th iteration):

1188 We know the Answer Hints: 12. With the Answer Hints: 12, we will answer the question.
1189 Let's think step by step. Carlos needs to calculate the total cost of maintaining the tree each
1190 year. This includes the cost to water and feed the tree, which is \$3. Carlos also needs to
1191 calculate the total revenue he will earn each year from selling the lemons. This is calculated
1192 by multiplying the number of lemons produced each year (7) by the price per lemon (\$1.5).
1193 So the total revenue earned each year is 7 lemons x \$1.5/lemon = \$10.5. To determine when
1194 Carlos will start earning money on the lemon tree, we need to find the number of years it
1195 takes for the revenue to exceed the cost. Let's set up an equation: Revenue - Cost = 0. In this
1196 case, Revenue = \$10.5 and Cost = \$3. So the equation becomes: $\$10.5 - \$3 = \$7.5$. Carlos
1197 will start earning money on the lemon tree when the revenue exceeds the cost, which is when
1198 $\$7.5 > 0$. To find the number of years it takes for this to happen, we divide the initial cost of
1199 planting the tree (\$90) by the annual revenue (\$7.5). So the number of years it takes is $\$90 /$
1200 $\$7.5 = 12$. **The answer is 12.**

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