

# 000 001 002 003 004 005 006 007 FLY-CL: A FLY-INSPIRED FRAMEWORK FOR EN- HANCING EFFICIENT DECORRELATION AND REDUCED TRAINING TIME IN PRE-TRAINED MODEL-BASED CON- TINUAL REPRESENTATION LEARNING

008 **Anonymous authors**  
009 Paper under double-blind review

## 010 011 012 ABSTRACT 013

014 Using a nearly-frozen pretrained model, the continual representation learning  
015 paradigm reframes parameter updates as a similarity-matching problem to miti-  
016 gate catastrophic forgetting. However, directly leveraging pretrained features for  
017 downstream tasks often suffers from multicollinearity in the similarity-matching  
018 stage, and more advanced methods can be computationally prohibitive for real-time,  
019 low-latency applications. Inspired by the fly olfactory circuit, we propose Fly-CL, a  
020 bio-inspired framework compatible with a wide range of pretrained backbones. Fly-  
021 CL substantially reduces training time while achieving performance comparable to  
022 or exceeding that of current state-of-the-art methods. We theoretically show how  
023 Fly-CL progressively resolves multicollinearity, enabling more effective similarity  
024 matching with low time complexity. Extensive simulation experiments across  
025 diverse network architectures and data regimes validate Fly-CL’s effectiveness in  
026 addressing this challenge through a biologically inspired design.

## 027 028 1 INTRODUCTION 029

030 Artificial neural networks have exhibited remarkable capabilities across various domains in recent  
031 years. Nevertheless, real-world applications often require continuous model adaptation to handle  
032 progressively emerging unseen scenarios, making updates based on sequential incoming data essential.  
033 This need has led to the development of Continual Learning (CL). Earlier research primarily focused  
034 on training models from scratch (Aljundi et al., 2018; Kirkpatrick et al., 2017; Li & Hoiem, 2017;  
035 Zenke et al., 2017). Pretrained models have recently become prominent in CL, owing to their robust  
036 generalization in downstream tasks for downstream tasks (Wang et al., 2022a;b) .

037 Popular CL methods utilizing pre-trained models can generally be classified into three categories:  
038 (1) prompt/adapter-based approaches (Jung et al., 2023; Smith et al., 2023; Tang et al., 2023; Wang  
039 et al., 2022a;b; 2025; Liang & Li, 2024; Yu et al., 2024), (2) mixture-based approaches (Chen et al.,  
040 2023; Gao et al., 2023; Wang et al., 2023a;b; Zhou et al., 2023b), and (3) representation-based  
041 approaches (McDonnell et al., 2023; Sun et al., 2025; Zhou et al., 2023a; 2024; Zhuang et al., 2024).  
042 All three paradigms operate without exemplars and significantly outperform traditional training-  
043 from-scratch methods. Despite their strengths, each approach has limitations. Prompt/adapter-based  
044 methods are constrained to transformer architectures, and updating prompts/adapters inherently  
045 risks propagating forgetting within the prompt/adapter space. Mixture-based approaches require  
046 storing previous models, resulting in significant storage overhead and increased computational  
047 complexity during model fusion. Compared with parameter-learning approaches, **representation-  
048 based methods perform better by reframing learning as similarity matching and avoiding  
049 dependence on a specific backbone**. They form class prototypes (CPs) by averaging features  
050 extracted by a frozen pretrained network, with each prototype acting as the centroid of its class.  
051 However, **insufficient separation between features can lead to ambiguous decision boundaries  
052 when distinguishing between prototypes, complicating the similarity matching process**. This  
053 challenge is formally recognized as the multicollinearity problem. Previous studies (McDonnell  
et al., 2023) have attempted to address this issue, but they still incur substantial computational costs,  
hindering real-world deployment.

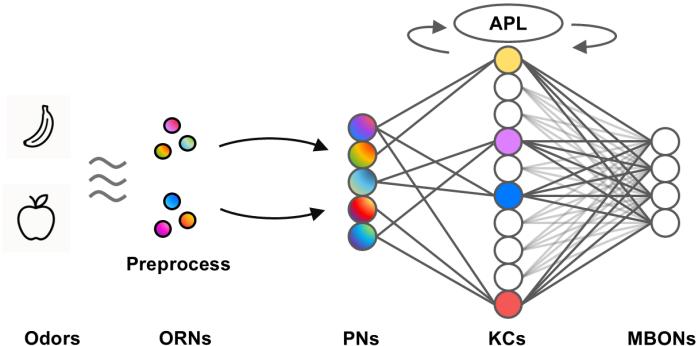


Figure 1: **Schematic of the Fly Olfactory Circuit.** Odors are first detected and pre-processed by olfactory receptor neurons (ORNs) in the antennal lobe, where feature extraction and normalization take place, before being transmitted to projection neurons (PNs). Expansion coding occurs as signals move from PNs to Kenyon cells (KCs), with an expansion ratio of approximately 40. Each KC connects to a fixed number of PNs (about 6). Lateral inhibition, mediated by an anterior paired lateral (APL) neuron, suppresses most weakly activated KCs, exemplifying a winner-take-all strategy. Finally, signals from KCs to mushroom body output neurons (MBONs) involve a dense down-projection that reduces dimensionality to select specific actions.

In biological systems, pattern separation is a well-recognized circuit motif across cerebellum-like and related networks, including the cerebellum, the hippocampus, and the fly olfactory system (Lin et al., 2014; Papadopoulou et al., 2011; Stevens, 2015; Zang & De Schutter, 2023). These neural circuits implement efficient decorrelation of overlapping sensory inputs. Figure 1 illustrates the information processing mechanism in the fly olfactory circuit. The process unfolds in several steps: in response to an odor stimulus, olfactory neurons extract and pre-process odor information represented as a 50-dimensional vector (PNs). These pre-extracted features are then randomly projected into expanded dimensions (KCs), selectively activating a small subset of KCs that receive the strongest excitation while zeroing out others. The high-dimensional features in KCs subsequently converge to low-dimensional MBONs for classification. Over the years, numerous studies have highlighted the role of the PN  $\rightarrow$  KC transformation in producing decorrelated representations, and many strategies have been proposed to model this process. By contrast, the downstream KC  $\rightarrow$  MBON transformation has received far less attention, and a clear theory of its contribution to decorrelation has yet to be established. To address this gap, **we examine whether the KC  $\rightarrow$  MBON pathway also supports decorrelation.**

Inspired by the fly olfactory circuit, we propose Fly-CL, an efficient framework for progressive decorrelation in representation-based learning with pre-trained models. The pipeline starts with feature extraction and normalization using a pre-trained model, followed by random sparse projection into a high-dimensional space and a top- $k$  operation for collective decorrelation, mimicking the PN-to-KC process. We then implement a similarity matching mechanism between class prototypes that mimics the KC-to-MBON structure during inference, and an efficient streaming ridge classification method to decorrelate parameter weights during training. Empirical results show substantially reduced time consumption versus baseline methods.

Our main contributions are as follows:

1. We propose an efficient and biologically plausible decorrelation framework that significantly reduces computational costs compared to current SOTA methods while achieving comparable or improved performance in CL.
2. Our method's effectiveness and robustness in decorrelation are validated by extensive experiments under various data setups and model architectures, supported by theoretical and empirical analyses.
3. The alignment of our framework with the fly olfactory circuit suggests that biological structures can inspire effective and efficient solutions to AI problems.

## 2 RELATED WORK

**Representation-based methods in CL using Pre-trained Models:** Representation-based methods (McDonnell et al., 2023; Sun et al., 2025; Zhou et al., 2023a; 2024) demonstrate superior performance

108 over parameter-learning approaches by leveraging features extracted from a frozen pre-trained  
 109 model to compute similarities with class prototypes. They are also more practical for resource-  
 110 constrained deployments (e.g., edge computing scenarios), since they avoid updating the entire model.  
 111 For instance, in a smart camera system, such methods enable the efficient addition of new object  
 112 recognition capabilities without retraining the entire model. While showing promising progress,  
 113 their computational overhead remains substantial, which may limit their applicability in scenarios  
 114 requiring real-time responses.

115  
 116 **Fly Olfactory Circuit:** Information processing in cerebellum-like circuits, including the fly olfactory  
 117 circuit, involves several stages (Lin et al., 2014; Papadopoulou et al., 2011; Stevens, 2015; Zang  
 118 & De Schutter, 2023). Theoretical neuroscience studies suggest that most stages exhibit progressive  
 119 feature separation effects (Hige et al., 2015). Algorithms inspired by the fly olfactory circuit have  
 120 been applied across various AI domains, including locality-sensitive hashing (Dasgupta et al., 2017;  
 121 Ryali et al., 2020; Sharma & Navlakha, 2018), word embedding (Liang et al., 2021), and federated  
 122 learning (Ram & Sinha, 2022).

### 123 3 BACKGROUND

#### 124 3.1 PROBLEM STATEMENT

125 In this paper, we focus on CL within the context of image classification tasks. We denote sequentially  
 126 arriving tasks as  $\mathcal{D} = \{\mathcal{D}_1, \dots, \mathcal{D}_T\}$ , where each task  $\mathcal{D}_t = \{(\mathbf{x}_t^i, y_t^i)\}_{i=1}^{n_t}$  consists of  $n_t$  samples.  
 127 Each sample  $\mathbf{x}_t^i$  within a task is drawn from the input space  $\mathcal{X}_t$ , and its corresponding label  $y_t^i$  belongs  
 128 to the label space  $\mathcal{Y}_t$ . The training process involves sequential learning from  $\mathcal{D}_1$  to  $\mathcal{D}_T$ , followed by  
 129 class prediction on an unseen test set spanning the full label space.

130 To demonstrate the effectiveness and efficiency of our framework, we adopt Class Incremental  
 131 Learning (CIL), a widely used experimental setup. Unlike traditional Task Incremental Learning,  
 132 **CIL does not provide access to the task ID during the testing process**, making it more challenging. In  
 133 CIL, the model learns mutually exclusive classes within each task, ensuring that the intersection of  
 134 label spaces satisfies  $\mathcal{Y}_i \cap \mathcal{Y}_j = \emptyset$ . We denote that there are  $c_t$  classes for the first  $t$  tasks.

#### 135 3.2 REPRESENTATION-BASED PARADIGM IN CL

136 We study on the basis of the recently popular representation-based paradigm using pre-trained models  
 137 (McDonnell et al., 2023; Sun et al., 2025; Zhou et al., 2023a; 2024), which demonstrates superior  
 138 performance for CL. Given an input image  $\mathbf{x}_t^i$ , it is first compressed into a  $d$ -dimensional feature  
 139  $\mathbf{v}_t^i = f_\theta(\mathbf{x}_t^i) \in \mathbb{R}^d$  using a pre-trained encoder  $f_\theta$ . For each class  $i$  in task  $t$ , we compute its prototype  
 140 by averaging features over all training samples belonging to this class:

$$141 \quad \mathbf{\mu}_t^i = \frac{1}{N_i} \sum_{j=1}^{|\mathcal{D}_t|} \mathbb{I}(y_t^j = i) f_\theta(\mathbf{x}_t^j) \in \mathbb{R}^d, \quad (1)$$

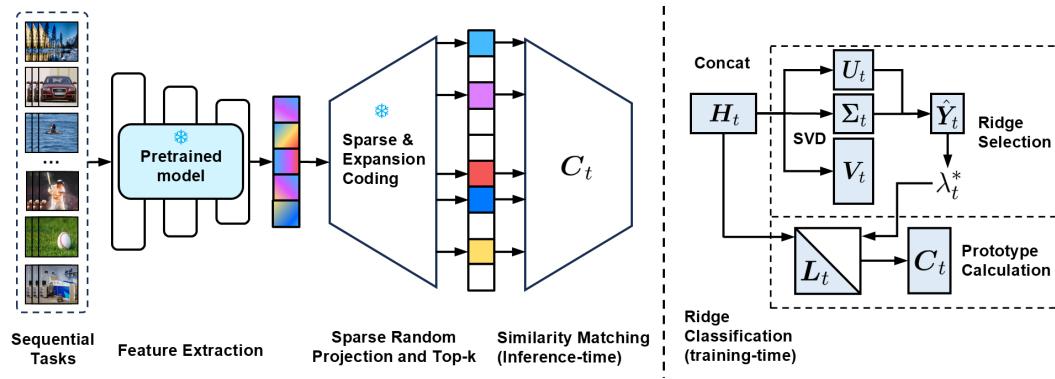
142 where  $N_i = \sum_{j=1}^{|\mathcal{D}_t|} \mathbb{I}(y_j = i)$  denotes the cardinality of class  $i$ 's training set, and  $\mathbb{I}(\cdot)$  is the indicator  
 143 function. During inference, for a test sample with feature vector  $\mathbf{v}$ , the predicted class  $\hat{y}$  is determined  
 144 by finding the maximum cosine similarity between  $\mathbf{v}$  and all learned class prototypes:

$$145 \quad \hat{y} = \arg \max_{t,i} \frac{\mathbf{v}^\top \mathbf{\mu}_t^i}{\|\mathbf{v}\| \cdot \|\mathbf{\mu}_t^i\|}. \quad (2)$$

146 However, significant inter-prototype correlations ( $\mathbb{E}[\mathbf{\mu}_{t,i}^{m_i} \top \mathbf{\mu}_{t,j}^{m_j}] \gg 0$ , see Figure 3(a)), can severely  
 147 compromise the discriminative power of the similarity measurement (Belsley et al., 2005). This  
 148 phenomenon arises because high correlations reduce the effective angular separation between classes  
 149 in the embedding space, leading to ambiguous decision boundaries. Specifically, when prototypes  
 150 cluster near a dominant direction in  $\mathbb{R}^d$ , the cosine similarity metric becomes less sensitive to subtle  
 151 but critical inter-class distinctions, thereby degrading classification performance.

162 4 FLY-CL  
163

164 In this section, we detail the design motivation and functionality of each component of our Fly-CL  
165 framework. A schematic of the overall framework is provided in Figure 2, along with the pseudocode  
166 for the training and inference pipeline in Appendix A. The decorrelation effect of each component is  
167 visualized in Figure 3 using Pearson correlation coefficients of different class prototypes.



181 **Figure 2: Schematic of the Fly-CL Framework. Left:** Our framework extracts image embeddings  
182 using a frozen pre-trained model, projects them into a higher-dimensional space via a fixed sparse  
183 random projection, and filters them through a top- $k$  operation (PNs  $\rightarrow$  KCs). Then we utilize a  
184 learned down-projection for similarity matching during inference time (KCs  $\rightarrow$  MBONs). **Right:**  
185 During the training phase, the parameter  $C_t$  is learned via a streaming ridge classification scheme.

186 4.1 SPARSE RANDOM PROJECTION AND TOP-K OPERATION  
187

188 Building upon the representation-based paradigm, it is necessary to decouple different class prototypes.  
189 Inspired by the decorrelation mechanism of the fly olfactory circuit, we emulate the sparse expansion  
190 projection from PNs to KCs, followed by winner-take-all inhibition mediated by APL neurons. Given  
191 a feature embedding  $v \in \mathbb{R}^d$  extracted from the pre-trained encoder, we formulate the transformation  
192  $Z(v) : \mathbb{R}^d \rightarrow \mathbb{R}^m$  as:

$$h' = Z(v) = \text{top-}k(h) = \text{top-}k(Wv), \quad (3)$$

193 where the fixed projection matrix  $W \in \mathbb{R}^{m \times d}$  (with  $m \gg d$ ) implements weight sparsity: each row  
194 contains exactly  $p$  ( $p < d$ ) non-zero entries independently sampled from  $\mathcal{N}(0, 1)$ . The top- $k$  operator  
195 implements activation sparsity by preserving only the  $k$  largest components ( $k < m$ ) while zeroing  
196 out others, formally defined as:

$$[h']_i = \begin{cases} [h]_i & \text{if the magnitude of } [h]_i \text{ is among the top-}k \text{ values of } h, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

200 This two-stage process achieves effective decorrelation through the following properties, and its  
201 empirical effect is visualized in the transformation from Figure 3(a) to (b).

203 **1. High-Dimensional Embedding Enhances Linear Separability:** Random projection of low-  
204 dimensional features into an extremely high-dimensional space can improve the linear separability of  
205 the feature representations (Litwin-Kumar et al., 2017).

206 **2. Powerful Inhibition Suppresses Noisy Components:** The top- $k$  operation imposes sparsity by  
207 suppressing noisy dimensions that may interfere with discrimination through dimensional competition,  
208 while enhancing separation by keeping the most discriminative dimensions (Metwally et al., 2006).

210 Considering computational efficiency,  $W$ 's sparse pattern reduces the time complexity for random  
211 projection from  $\mathcal{O}(mn_td)$  to  $\mathcal{O}(mn_tp)$  while preserving the core representational capacity compared  
212 to dense projection. Similarly, the top- $k$  operation reduces similarity matching complexity from  
213  $\mathcal{O}(mn_tc_t)$  to  $\mathcal{O}(kn_tc_t)$ , while simultaneously improving performance.

214 We further propose two theorems to demonstrate that strong sparsity does not significantly degrade  
215 performance. According to Theorem 4.1, as long as  $p$  and  $d$  are not extremely small, the matrix  $W$   
retains full column rank with probability  $1 - o(1)$ , which is a common approach to demonstrate that

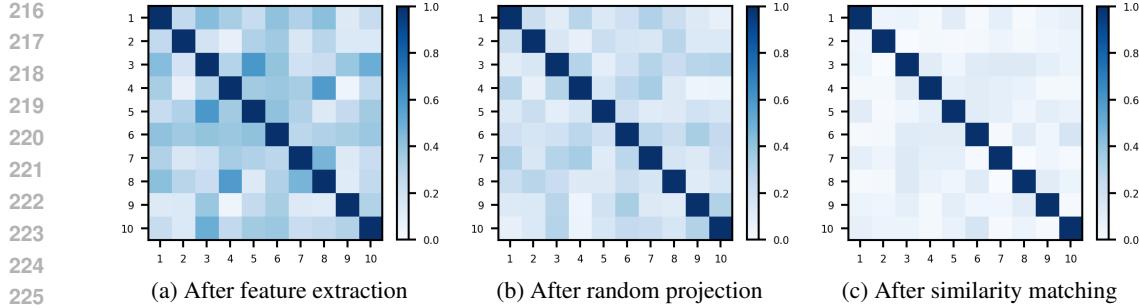


Figure 3: **Pearson Correlation Coefficients of Prototypes at Different Decorrelation Stages in Fly-CL.** Heatmaps display Pearson correlation coefficients for 10 randomly selected class prototypes at each stage of our pipeline (consistent across visualizations).

a sparse random projection does not result in severe information loss. Furthermore, in Theorem 4.2, by proving that the performance degradation between this sparsification operation and the original vector is bounded, we show that if  $k$  is not extremely small, it can preserve most of its performance. For a complete proof, please refer to Appendix B.

**Theorem 4.1.** *Given the matrix  $\mathbf{W} \in \mathbb{R}^{m \times d}$ , where  $m > d$ , with each row having exactly  $p$  non-zero entries, which are randomly sampled from  $\mathcal{N}(0, 1)$ . Let  $\mathcal{W} \in \mathbb{R}^{d \times d}$  be any square submatrix of  $\mathbf{W}$ . Then, for any  $\epsilon > 0$ , it holds that*

$$\mathbb{P} \left( |\det(\mathcal{W})| \geq \left( \frac{p}{d} \right)^{d/2} \sqrt{d!} \exp(-d^{1/2+\epsilon}) \right) = 1 - o(1).$$

Thus, for sufficiently large  $p$  and  $d$ , any submatrix  $\mathcal{W}$  is invertible with probability at least  $1 - o(1)$ .

**Theorem 4.2.** *For top- $k$  sparsification in the expanded dimension  $m$ , the performance degradation is bounded by:*

$$\mathbb{E} [|L(\mathbf{h}, y) - L(\mathbf{h}', y)|] \leq M \cdot \sqrt{\frac{C}{k} \cdot \mathbb{E}[\|\mathbf{h}\|_2^2]},$$

where  $L(\cdot)$  is a performance loss function for downstream tasks and  $C, M$  are constants. To ensure negligible performance degradation, we require:

$$\sqrt{\frac{C}{k} \cdot \mathbb{E}[\|\mathbf{h}\|_2^2]} \leq \mathcal{O} \left( \frac{1}{\sqrt{m^\alpha}} \right),$$

i.e., when  $k = \Omega(m^\alpha)$  ( $0 < \alpha < 1$ ), the error bound decays polynomially with increasing dimension.

## 4.2 STREAMING RIDGE CLASSIFICATION

Previous studies on decorrelation in the fly olfactory circuit have primarily focused on the transformation from PNs to KCs, where sparse and decorrelated representations have been experimentally observed. In contrast, the downstream transformation from KCs to MBONs has received little attention in this regard, and the physiological evidence remains inconclusive. This gap motivates us to investigate whether the KC  $\rightarrow$  MBON pathway can also facilitate decorrelation. To better align with the goal of CL, we model it using a streaming ridge classification framework with adaptive regularization, which naturally achieves decorrelation while ensuring computational efficiency and compatibility with sequential data. Ridge classification (Hoerl & Kennard, 1970) mitigates feature collinearity through  $\ell_2$ -regularization, trading increased bias for reduced variance by shrinking correlated feature weights, thereby stabilizing prototype estimation in non-i.i.d. sequential learning scenarios. Let  $\mathbf{H}_t \in \mathbb{R}^{n_t \times m}$  denote the concatenation of high-dimensional features  $\mathbf{h}'$  for  $n_t$  samples in task  $t$ , and  $\mathbf{Y}_t \in \{0, 1\}^{n_t \times c_t}$  represent the corresponding one-hot label matrix for the total  $c_t$  classes. We maintain two streaming statistics: a Gram matrix  $\mathbf{G} \in \mathbb{R}^{m \times m}$ , which captures self-correlated statistics, and a matrix  $\mathbf{S} \in \mathbb{R}^{m \times c_t}$ , which accumulates cross-dimensional weights for each class prototype. During each task iteration  $t$ , these are updated as follows:

$$\mathbf{G}_t \leftarrow \mathbf{G}_{t-1} + \mathbf{H}_t^\top \mathbf{H}_t, \quad \mathbf{S}_t \leftarrow \mathbf{S}_{t-1} + \mathbf{H}_t^\top \mathbf{Y}_t. \quad (5)$$

270 The classifier matrix  $\mathbf{C} \in \mathbb{R}^{m \times c_t}$  is updated via regularized least squares accordingly:  
 271

$$272 \quad \mathbf{C}_t = (\mathbf{G}_t + \lambda \mathbf{I}_m)^{-1} \mathbf{S}_t. \quad (6)$$

273 Prediction for preprocessed new samples  $\mathbf{h}' \in \mathbb{R}^m$  follows:  
 274

$$275 \quad \hat{y} = \arg \max_{i \in \{1, \dots, c_t\}} \mathbf{h}'^\top \mathbf{C}_{\cdot, i}, \quad (7)$$

276 where  $\mathbf{C}_{\cdot, i}$  denotes the  $i$ -th column of modulated prototypes.  
 277

278 **Adaptive Regularization:** Due to the inherent heterogeneity of different tasks, a fixed penalty  
 279 coefficients  $\lambda$  will cause suboptimal performance. For adaptive regularization, vanilla  $\lambda$  selection via  
 280 grid search and cross-validation incurs prohibitive computational costs of  $\mathcal{O}(lm^3)$  for  $l$  candidates  
 281 where the expanded feature dimension  $m$  is extremely large (McDonnell et al., 2023). To achieve our  
 282 efficiency desideratum, we draw inspiration from an adaptive Generalized Cross-Validation (GCV)  
 283 (Golub et al., 1979) framework that analytically approximates cross-validation error without explicit  
 284 validation steps that require calculating large matrix inverses.  
 285

286 Given new task data  $\mathbf{H}_t \in \mathbb{R}^{n_t \times m}$ , we first obtain its singular value decomposition (SVD) as  
 287  $\mathbf{H}_t = \mathbf{U}_t \Sigma_t \mathbf{V}_t^\top$ , where  $\mathbf{U}_t \in \mathbb{R}^{n_t \times r}$  and  $\mathbf{V}_t \in \mathbb{R}^{m \times r}$  are semi-orthogonal column matrices that  
 288 satisfy  $\mathbf{U}_t \mathbf{U}_t^\top = \mathbf{I}_{n_t}$ ,  $\mathbf{V}_t \mathbf{V}_t^\top = \mathbf{I}_m$ , and  $\Sigma_t = \text{diag}(s_1, \dots, s_r) \in \mathbb{R}^{r \times r}$  contains non-zero singular  
 289 values with  $r = \text{rank}(\mathbf{H}_t) = \min(n_t, m)$  (typically it's of full rank, since numerical computation  
 290 is usually precise). The time complexity for SVD is  $\mathcal{O}(n_t rm)$ . For  $l$  candidate regularization  
 291 coefficients  $\lambda \in \Lambda = \{\lambda_{\min}, \dots, \lambda_{\max}\}$  on a log scale, we use the following steps to compute the  
 292 GCV criterion for each one:  
 293

294 First, in  $\mathcal{O}(lr)$  time, we get the shrinkage matrix and calculate the effective degrees-of-freedom by:  
 295

$$296 \quad \mathbf{D}_t = \frac{\Sigma_t^2}{\Sigma_t^2 + \lambda \mathbf{I}_r}, \quad \text{df}(\lambda) = \text{tr}(\mathbf{D}_t) = \sum_{i=1}^r \frac{s_i^2}{s_i^2 + \lambda}. \quad (8)$$

297 Then, we reconstruct the prediction value of ridge regression in  $\mathcal{O}(ln_t r c_t)$  time by  
 298

$$299 \quad \hat{\mathbf{Y}}_t = \mathbf{U}_t (\text{vecdiag}(\mathbf{D}_t) \otimes \mathbf{1}_c^\top) \odot \mathbf{U}_t^\top \mathbf{Y}_t, \quad (9)$$

300 where the  $\otimes$  denotes the outer product,  $\odot$  denotes the Hadamard product,  $\text{vecdiag}$  denotes extracting  
 301 the diagonal elements and concatenating them into a column vector. Finally, we can get the GCV  
 302 value by  
 303

$$304 \quad \text{GCV}(\lambda) = \frac{\|\mathbf{Y}_t - \hat{\mathbf{Y}}_t(\lambda)\|_F^2}{n_t \left(1 - \frac{\text{df}(\lambda)}{n_t}\right)^2}, \quad (10)$$

305 with time complexity being  $\mathcal{O}(ln_t c_t)$ . The optimal regularization parameter is then selected by:  
 306

$$307 \quad \lambda_t^* = \arg \min_{\lambda \in \Lambda} \text{GCV}(\lambda). \quad (11)$$

308 Considering projected dimension  $m$  is extremely large, we can make a mild assumption  $m > n_t$ , and  
 309  $l c_t \ll m$ , thus  $r = \min(n_t, m) = n_t$ . The original  $l$  loop complexity is  $\mathcal{O}(ln_t r c_t) = \mathcal{O}(ln_t^2 c_t) \ll$   
 310  $\mathcal{O}(n_t^2 m) = \mathcal{O}(n_t r m)$ . Hence, the time complexity is reduced to being determined by SVD, at  
 311  $\mathcal{O}(n_t^2 m)$ . Compared to vanilla cross-validation that takes  $\mathcal{O}(lm^3)$ , time consumption is greatly  
 312 reduced.  
 313

314 **Accelerated Prototype Calculation:** Upon determining the optimal regularization parameter  $\lambda$   
 315 through GCV, we solve Eq. 6 to obtain class prototypes  $\mathbf{C}_t$ . While vanilla matrix inversion via LU  
 316 decomposition provides a baseline implementation, for the sake of computational efficiency, we  
 317 exploit the inherent positive-definiteness of  $\mathbf{G}_t + \lambda_t \mathbf{I}_t$  to achieve computational acceleration through  
 318 Cholesky factorization by:  
 319

$$320 \quad \mathbf{L}_t \mathbf{L}_t^\top = \mathbf{G}_t + \lambda_t^* \mathbf{I}_m, \quad \mathbf{C}_t = \mathbf{L}_t^{-\top} (\mathbf{L}_t^{-1} \mathbf{S}_t), \quad (12)$$

321 where  $\mathbf{L}_t$  denotes the lower-triangular Cholesky factor. This approach reduces theoretical complexity  
 322 from  $\mathcal{O}(\frac{2}{3}m^3)$  to  $\mathcal{O}(\frac{1}{3}m^3)$  for factorization, with triangular solves requiring half the FLOPs of  
 323 general linear system solutions. The numerical stability of this method is ensured by the condition  
 324 number bound  $\kappa(\mathbf{L}_t) \leq \kappa(\mathbf{G}_t + \lambda_t^* \mathbf{I}_m)$ , making it particularly suitable for ill-conditioned streaming  
 325 scenarios where  $\mathbf{G}_t$  may accumulate numerical noise over tasks. The decorrelation effect of the  
 326 streaming ridge classification is visualized via the transformation from Figure 3(b) to (c).  
 327

324  
 325 **Table 1: Performance Comparison on Pre-trained ViT-B/16 Models.** We report the average  
 326 training time per task ( $\tau_{\text{train}}$ ), average post-extraction training time ( $\tau_{\text{post}}$ ), and overall accuracy ( $\bar{A}$ )  
 327 across three benchmark datasets: CIFAR-100, CUB-200-2011, and VTAB.

328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377	328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377	328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377			328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377			328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377		
		<b>CIFAR-100</b>		<b>CUB-200-2011</b>			<b>VTAB</b>			
		$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$
L2P		263.54 $\pm$ 0.10	183.56 $\pm$ 0.36	87.74 $\pm$ 0.46	52.02 $\pm$ 0.04	37.03 $\pm$ 0.07	77.48 $\pm$ 1.43	42.10 $\pm$ 0.04	36.45 $\pm$ 0.02	81.24 $\pm$ 0.67
DualPrompt		231.89 $\pm$ 0.63	153.66 $\pm$ 0.47	87.47 $\pm$ 0.58	46.28 $\pm$ 0.05	31.64 $\pm$ 0.06	79.89 $\pm$ 1.44	38.14 $\pm$ 0.16	31.64 $\pm$ 0.06	80.85 $\pm$ 1.34
InfLoRA		220.82 $\pm$ 0.44	140.31 $\pm$ 0.41	91.10 $\pm$ 0.36	45.31 $\pm$ 0.19	30.97 $\pm$ 0.22	80.65 $\pm$ 0.73	35.80 $\pm$ 0.28	29.26 $\pm$ 0.17	88.73 $\pm$ 0.57
SEMA		241.60 $\pm$ 0.82	160.87 $\pm$ 0.69	92.04 $\pm$ 0.25	48.21 $\pm$ 0.23	32.96 $\pm$ 0.20	84.31 $\pm$ 0.37	45.50 $\pm$ 0.13	39.82 $\pm$ 0.26	91.18 $\pm$ 0.46
MoE-Adapter		187.91 $\pm$ 0.61	106.37 $\pm$ 0.53	90.43 $\pm$ 0.46	37.89 $\pm$ 0.15	22.61 $\pm$ 0.12	79.65 $\pm$ 0.32	32.28 $\pm$ 0.19	26.06 $\pm$ 0.14	86.39 $\pm$ 0.77
<b>Fly-CL</b>		<b>19.07<math>\pm</math>0.07</b>	<b>5.38<math>\pm</math>0.01</b>	93.89 $\pm$ 0.12	<b>4.43<math>\pm</math>0.11</b>	<b>0.35<math>\pm</math>0.01</b>	<b>93.84<math>\pm</math>0.18</b>	<b>2.48<math>\pm</math>0.13</b>	<b>0.34<math>\pm</math>0.03</b>	<b>96.54<math>\pm</math>0.38</b>

## 5 EXPERIMENTS

### 5.1 EXPERIMENTAL SETUP

**Datasets and Backbones:** We conduct experiments using various architectures, including transformer-based and CNN-based backbones. Specifically, we utilize the Vision Transformer (ViT-B/16) (Dosovitskiy et al., 2020) and ResNet-50 (He et al., 2016) as representative architectures. We test our method on five widely used datasets: CIFAR-100 (Krizhevsky et al., 2009), CUB-200-2011 (Wah et al., 2011), VTAB (Zhai et al., 2019), ImageNet-R (Hendrycks et al., 2021a), and ImageNet-A (Hendrycks et al., 2021b). Further details of the data setup are provided in Appendix E.

**Baselines:** We compare Fly-CL against eight baselines, including two prompt-based approaches: L2P (Wang et al., 2022b) and DualPrompt (Wang et al., 2022a), three lora/adapter-based approaches: InfLoRA (Liang & Li, 2024), SEMA (Wang et al., 2025), and MoE-Adapter (Yu et al., 2024), as well as three representation-based methods: EASE (Zhou et al., 2024), RanPAC (McDonnell et al., 2023), and F-OAL (Zhuang et al., 2024). In Fly-CL, we find that applying data normalization according to the specific combination of backbone and dataset is beneficial; a detailed analysis can be found in Appendix C.3. For a fair comparison, all baselines use the same data normalization strategy as Fly-CL. Comparisons among the different baselines and implementation details are provided in Appendices D and E, respectively.

**Evaluation Metrics:** To assess CL performance, we employ four metrics: average accuracy ( $A_t$ ), last stage accuracy ( $A_T$ ), backward transfer ( $BWT$ ), and overall accuracy ( $\bar{A}$ ). The average accuracy at stage  $t$  is defined as:  $A_t = \frac{1}{t} \sum_{i=1}^t a_{t,i}$ , where  $a_{t,i}$  denotes the test accuracy on the  $i$ -th task after training on the  $t$ -th task. Specially, we refer to the average accuracy at the last stage  $T$  as the last stage accuracy ( $A_T$ ). The backward transfer is denoted as  $BWT = \frac{1}{T-1} \sum_{i=1}^T (a_{t,i} - a_{i,i})$ . The overall accuracy is computed as the mean of  $A_t$  across all  $T$  tasks:  $\bar{A} = \frac{1}{T} \sum_{i=1}^T A_t$ . To evaluate computational efficiency, we introduce two time-related metrics: average training time per task ( $\tau_{\text{train}}$ ) and average post-extraction training time ( $\tau_{\text{post}}$ ). Here,  $\tau_{\text{train}}$  represents the total training time amortized across all tasks, while  $\tau_{\text{post}}$  is derived by subtracting the average feature extraction time for each task (using the pre-trained model) from  $\tau_{\text{train}}$ .  $\tau_{\text{post}}$  is a more precise metric for evaluating algorithm-specific time consumption, as it excludes the shared preprocessing overhead.<sup>1</sup>

### 5.2 LOW LATENCY AND HIGH ACCURACY

The main CL results across various datasets, architectures, and task settings are summarized in Tables 1, 2, and 5. Our framework’s key strength is achieving CL accuracy comparable to or exceeding SOTA performance with significantly lower computational costs, as measured by both  $\tau_{\text{train}}$  and  $\tau_{\text{post}}$ . In Table 1, using ViT-B/16, Fly-CL reduces  $\tau_{\text{post}}$  by 91% on CIFAR-100 with only a marginal accuracy drop of 0.32% compared to SOTA methods. On CUB-200-2011 and VTAB, Fly-CL achieves 83% and 67% reductions in  $\tau_{\text{post}}$  versus the most efficient baseline while improving overall accuracy by

<sup>1</sup>  $\tau_{\text{train}}$  and  $\tau_{\text{post}}$  are measured in seconds (wall clock time);  $A_t$  and  $\bar{A}$  are measured in %.

378  
 379 **Table 2: Performance Comparison on Pre-trained ResNet-50 Models.** We report the average  
 380 training time per task ( $\tau_{\text{train}}$ ), average post-extraction training time ( $\tau_{\text{post}}$ ), and overall accuracy ( $\bar{A}$ )  
 381 across three benchmark datasets: CIFAR-100, CUB-200-2011, and VTAB. The best results are  
 382 highlighted in **bold**.

Method	CIFAR-100			CUB-200-2011			VTAB		
	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$
RanPAC	55.68 $\pm$ 0.97	46.65 $\pm$ 0.88	82.72 $\pm$ 0.22	58.74 $\pm$ 0.84	54.35 $\pm$ 0.99	78.72 $\pm$ 0.40	50.15 $\pm$ 0.36	47.94 $\pm$ 0.38	92.80 $\pm$ 0.40
F-OAL	80.74 $\pm$ 0.35	71.78 $\pm$ 0.35	66.63 $\pm$ 0.71	5.19 $\pm$ 0.09	1.69 $\pm$ 0.01	60.84 $\pm$ 1.67	2.76 $\pm$ 0.03	0.55 $\pm$ 0.01	26.15 $\pm$ 2.50
<b>Fly-CL</b>	<b>14.28<math>\pm</math>0.04</b>	<b>5.25<math>\pm</math>0.01</b>	<b>84.61<math>\pm</math>0.16</b>	<b>3.90<math>\pm</math>0.31</b>	<b>0.44<math>\pm</math>0.08</b>	<b>80.25<math>\pm</math>0.10</b>	<b>2.53<math>\pm</math>0.10</b>	<b>0.34<math>\pm</math>0.02</b>	<b>94.00<math>\pm</math>0.15</b>

383  
 384 **Table 3: Performance Comparison on Pre-trained ViT-B/16 Models using Online Learning**  
 385 **Setting.**  $\circ$  denotes methods in online mode. We report the average training time per task ( $\tau_{\text{train}}$ ),  
 386 average post-extraction training time ( $\tau_{\text{post}}$ ), and overall accuracy ( $\bar{A}$ ) across three benchmark datasets:  
 387 CIFAR-100, CUB-200-2011, and VTAB. The best results are highlighted in **bold**.

Method	CIFAR-100			CUB-200-2011			VTAB		
	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$
RanPAC $\circ$	1236.74 $\pm$ 1.36	1223.56 $\pm$ 1.07	92.48 $\pm$ 0.31	242.54 $\pm$ 1.56	238.36 $\pm$ 1.47	91.89 $\pm$ 0.26	122.89 $\pm$ 0.46	120.69 $\pm$ 0.42	93.41 $\pm$ 0.57
F-OAL $\circ$	164.58 $\pm$ 0.71	151.27 $\pm$ 0.64	91.48 $\pm$ 0.42	31.34 $\pm$ 0.32	27.20 $\pm$ 0.28	91.60 $\pm$ 0.22	11.49 $\pm$ 0.14	9.47 $\pm$ 0.16	95.28 $\pm$ 0.21
<b>Fly-CL<math>\circ</math></b>	<b>25.46<math>\pm</math>0.32</b>	<b>12.57<math>\pm</math>0.26</b>	<b>92.96<math>\pm</math>0.14</b>	<b>6.44<math>\pm</math>0.08</b>	<b>2.33<math>\pm</math>0.05</b>	<b>92.59<math>\pm</math>0.13</b>	<b>3.17<math>\pm</math>0.05</b>	<b>1.09<math>\pm</math>0.04</b>	<b>96.38<math>\pm</math>0.24</b>

398  
 399 1.17% and 2.38% over the best-performing methods, respectively. In Table 2, with ResNet-50, Fly-CL  
 400 improves overall accuracy by 1.89%, 1.53%, and 1.20% on CIFAR-100, CUB-200-2011, and VTAB,  
 401 respectively, while reducing  $\tau_{\text{post}}$  by 93%, 74%, and 38% versus the most efficient baselines. These  
 402 improvements align with transformer-based backbone trends. Notably, F-OAL exhibits significant  
 403 performance degradation on CNN backbones, presumably due to error accumulation in its iterative  
 404 update mechanism, but Fly-CL does not suffer from this issue. These results highlight Fly-CL’s  
 405 ability to balance computational efficiency and accuracy across diverse CL scenarios, demonstrating  
 406 its robustness. Results on datasets with severe domain shifts are presented in Table 6.

407 Additionally, the time difference between  $\tau_{\text{train}}$  and  $\tau_{\text{post}}$  in Tables 1 and 2 indicates that feature  
 408 extraction becomes the dominant time consumer in Fly-CL. For a fair comparison with the baselines,  
 409 we do not apply additional acceleration techniques here. However, in practical applications, tech-  
 410 niques like model quantization (e.g., INT8) can further reduce feature extraction time by around 4 $\times$   
 411 without significant accuracy degradation, thereby enhancing the speedup ratio. For hardware-specific  
 412 deployment, frameworks like TVM (Chen et al., 2018) can be utilized to maximize efficiency.

413 Furthermore, Fly-CL can be easily adapted to Online CL setups by updating the  $G$  and  $S$  matrices  
 414 and solving Eq. 6 for each batch, without concatenating all batch embeddings within a task. The  
 415 results in Table 3 indicate that batch-mode Fly-CL remains superior to other baselines in training  
 416 time and is also competitive in accuracy.

### 417 5.3 FACTORS CONTRIBUTING TO COMPUTATIONAL SPEEDUP

418 Our analysis in Sections 4.1 and 4.2 demonstrates that the proposed framework achieves significant  
 419 speedup over the vanilla implementation through component-level optimizations. To quantify these  
 420 improvements precisely, we split the post-extraction training time into three key components (as  
 421 illustrated in Figure 2) and evaluate Fly-CL against its vanilla implementation under the CUB-200-  
 422 2011 setting in Table 4. The components include: (1) Random Projection: Acceleration via weight  
 423 sparsity induced by sparse projection versus the dense version. (2) Ridge Selection: Time reduction  
 424 achieved by GCV, which eliminates the need for explicit cross-validation. (3) Prototype Calculation:  
 425 Optimization from LU decomposition to Cholesky factorization. Additionally, the inference stage  
 426 also benefits from the activation sparsity induced by the top- $k$  operation in similarity comparisons.

### 427 5.4 ABLATION STUDY AND HYPERPARAMETER SENSITIVITY ANALYSIS

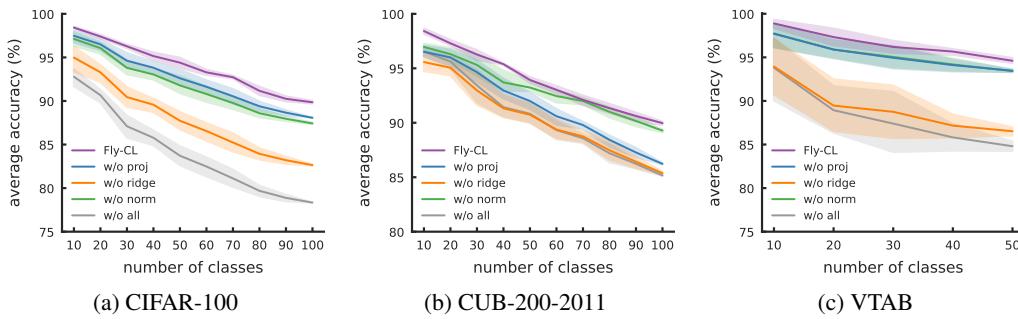
428 Using ViT-B/16 as the backbone, we conduct ablation studies by individually removing the projection  
 429 layer (w/o proj), the streaming ridge classification (w/o ridge), and the data normalization components

432  
 433 **Table 4: Time Savings for Post-Extracting Components on CUB-200-2011.** We compare the theo-  
 434 retical time complexity per task ( $T_{\text{theory}}$ ) and the actual runtime per task ( $T_{\text{actual}}$ ) for each component  
 435 on an NVIDIA GeForce RTX 3090 GPU. The optimized implementations demonstrate significant  
 436 speedups across all components.

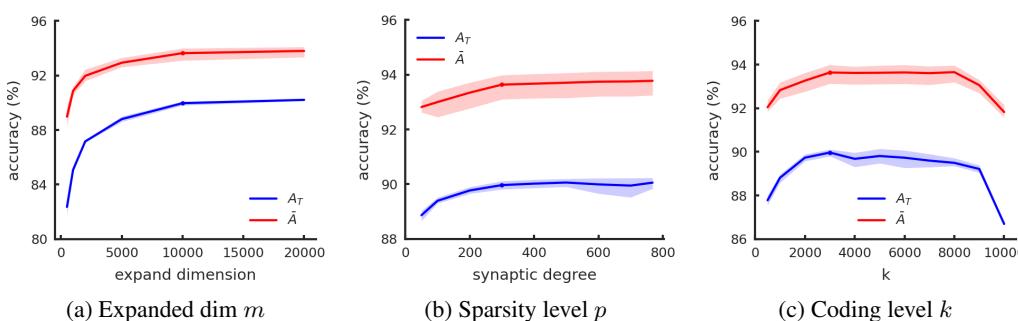
Method	Random Projection		Ridge Selection		Prototype Calculation		Similarity Comparison	
	$T_{\text{theory}}$	$T_{\text{actual}}$	$T_{\text{theory}}$	$T_{\text{actual}}$	$T_{\text{theory}}$	$T_{\text{actual}}$	$T_{\text{theory}}$	$T_{\text{actual}}$
vanilla	$\mathcal{O}(mn_t d)$	$0.22 \pm 0.03$	$\mathcal{O}(lm^3)$	$7.34 \pm 0.12$	$\mathcal{O}(\frac{2}{3}m^3)$	$0.20 \pm 0.01$	$\mathcal{O}(mn_t c_t)$	$0.21 \pm 0.01$
optimized	$\mathcal{O}(mn_t p)$	<b><math>0.08 \pm 0.02</math></b>	$\mathcal{O}(mn_t^2)$	<b><math>0.14 \pm 0.01</math></b>	$\mathcal{O}(\frac{1}{3}m^3)$	<b><math>0.10 \pm 0.01</math></b>	$\mathcal{O}(kn_t c_t)$	<b><math>0.08 \pm 0.01</math></b>

441  
 442  
 443 (w/o norm). The results in Figure 4 demonstrate that each component contributes significantly to  
 444 overall performance. Removing any of these components, or all of them (w/o all), leads to noticeable  
 445 performance degradation.

446 We also analyze the sensitivity of the key hyperparameters in Fly-CL:  $m$  (projection dimension),  
 447  $p$  (weight sparsity), and  $k$  (activation sparsity) in Figure 5. Increasing  $m$  improves accuracy, with  
 448 performance saturating beyond  $m = 10,000$ . Notably, Fly-CL does not suffer from the curse of  
 449 dimensionality, which can be attributed to the fact that random projection into a higher-dimensional  
 450 space preserves pairwise distances between data points, as guaranteed by the Johnson-Lindenstrauss  
 451 Lemma (Johnson et al., 1984). CL performance increases monotonically with  $p$ , and no significant  
 452 performance drop occurs as long as  $p$  does not take an excessively small value. A sufficiently large  
 453  $k$  value avoids information loss, while a smaller value suppresses noisy dimensions. Thus, finding  
 454 an appropriate trade-off can lead to optimal accuracy. Encouragingly, Figure 5(c) shows a broad  
 455 plateau for optimal  $k$  selection. Based on our empirical results, we set  $m = 10,000$ ,  $p = 300$ , and  
 456  $k = 3,000$  as default values.



471 **Figure 4: Accuracy Curves from Ablation Studies on Three Datasets.** We report average accuracy  
 472 ( $A_t$ ) for each stage. w/o refers to the removal of the specific component.



483 **Figure 5: Sensitivity Analysis for Expanded dim  $m$ , Weight Sparsity  $p$ , and Activation Sparsity**  
 484  **$k$  on CUB-200-2011.** We report average accuracy in last task ( $A_T$ ) and overall accuracy ( $\bar{A}$ ). The  
 485 dots denote the default values we use across experiments.

486 **6 CONCLUSION**

487

488 In this work, inspired by the decorrelation mechanism in the fly olfactory circuit, we propose  
 489 an efficient CL framework, Fly-CL. Fly-CL significantly reduces computational overhead during  
 490 training while achieving competitive performance compared to SOTA methods. This framework  
 491 integrates several key components: data normalization, feature extraction, sparse random projection  
 492 with top- $k$  operation, and streaming ridge classification, each contributing to the overall efficiency  
 493 and effectiveness of the system. This work establishes that neurobiological principles—particularly  
 494 sparse coding and progressive decorrelation—can effectively address fundamental efficiency-accuracy  
 495 trade-offs in artificial continual learning systems.

496

497 **REFERENCES**

498

499 Rahaf Aljundi, Francesca Babiloni, Mohamed Elhoseiny, Marcus Rohrbach, and Tinne Tuytelaars.  
 500 Memory aware synapses: Learning what (not) to forget. In *Proceedings of the European conference*  
 501 *on computer vision (ECCV)*, pp. 139–154, 2018.

502 Shuai Bai, Keqin Chen, Xuejing Liu, Jialin Wang, Wenbin Ge, Sibo Song, Kai Dang, Peng Wang,  
 503 Shijie Wang, Jun Tang, et al. Qwen2. 5-vl technical report. *arXiv preprint arXiv:2502.13923*,  
 504 2025.

505 Peter L Bartlett and Shahar Mendelson. Rademacher and gaussian complexities: Risk bounds and  
 506 structural results. *Journal of Machine Learning Research*, 3(Nov):463–482, 2002.

508 David A Belsley, Edwin Kuh, and Roy E Welsch. *Regression diagnostics: Identifying influential data*  
 509 *and sources of collinearity*. John Wiley & Sons, 2005.

511 Haoran Chen, Zuxuan Wu, Xintong Han, Menglin Jia, and Yu-Gang Jiang. Promptfusion: Decoupling  
 512 stability and plasticity for continual learning. *arXiv preprint arXiv:2303.07223*, 2023.

514 Shoufa Chen, Chongjian Ge, Zhan Tong, Jiangliu Wang, Yibing Song, Jue Wang, and Ping Luo.  
 515 Adaptformer: Adapting vision transformers for scalable visual recognition. *Advances in Neural*  
 516 *Information Processing Systems*, 35:16664–16678, 2022.

517 Tianqi Chen, Thierry Moreau, Ziheng Jiang, Lianmin Zheng, Eddie Yan, Haichen Shen, Meghan  
 518 Cowan, Leyuan Wang, Yuwei Hu, Luis Ceze, et al. {TVM}: An automated {End-to-End}  
 519 optimizing compiler for deep learning. In *13th USENIX Symposium on Operating Systems Design*  
 520 *and Implementation (OSDI 18)*, pp. 578–594, 2018.

522 Stuart Coles, Joanna Bawa, Lesley Trenner, and Pat Dorazio. *An introduction to statistical modeling*  
 523 *of extreme values*, volume 208. Springer, 2001.

524 Sanjoy Dasgupta, Charles F. Stevens, and Saket Navlakha. A neural algorithm for a fundamental  
 525 computing problem. *Science*, 358(6364):793–796, 2017. doi: 10.1126/science.aam9868.

527 Alexey Dosovitskiy, Lucas Beyer, Alexander Kolesnikov, Dirk Weissenborn, Xiaohua Zhai, Thomas  
 528 Unterthiner, Mostafa Dehghani, Matthias Minderer, Georg Heigold, Sylvain Gelly, et al. An  
 529 image is worth 16x16 words: Transformers for image recognition at scale. *arXiv preprint*  
 530 *arXiv:2010.11929*, 2020.

531 Ronald Aylmer Fisher and Leonard Henry Caleb Tippett. Limiting forms of the frequency distribution  
 532 of the largest or smallest member of a sample. In *Mathematical proceedings of the Cambridge*  
 533 *philosophical society*, volume 24, pp. 180–190. Cambridge University Press, 1928.

535 Qiankun Gao, Chen Zhao, Yifan Sun, Teng Xi, Gang Zhang, Bernard Ghanem, and Jian Zhang.  
 536 A unified continual learning framework with general parameter-efficient tuning. In *ICCV*, pp.  
 537 11483–11493, October 2023.

538 Gene H Golub, Michael Heath, and Grace Wahba. Generalized cross-validation as a method for  
 539 choosing a good ridge parameter. *Technometrics*, 21(2):215–223, 1979.

540 Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image  
 541 recognition. In *Proceedings of the IEEE conference on computer vision and pattern recognition*,  
 542 pp. 770–778, 2016.

543 Donald Olding Hebb. *The organization of behavior: A neuropsychological theory*. Psychology press,  
 544 2005.

545 Dan Hendrycks, Steven Basart, Norman Mu, Saurav Kadavath, Frank Wang, Evan Dorundo, Rahul  
 546 Desai, Tyler Zhu, Samyak Parajuli, Mike Guo, et al. The many faces of robustness: A critical  
 547 analysis of out-of-distribution generalization. In *Proceedings of the IEEE/CVF international  
 548 conference on computer vision*, pp. 8340–8349, 2021a.

549 Dan Hendrycks, Kevin Zhao, Steven Basart, Jacob Steinhardt, and Dawn Song. Natural adversarial  
 550 examples. In *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*,  
 551 pp. 15262–15271, 2021b.

552 Toshihide Hige, Yoshinori Aso, Gerald M Rubin, and Glenn C Turner. Plasticity-driven individ-  
 553 ualization of olfactory coding in mushroom body output neurons. *Nature*, 526(7572):258–262,  
 554 2015.

555 Arthur E Hoerl and Robert W Kennard. Ridge regression: Biased estimation for nonorthogonal  
 556 problems. *Technometrics*, 12(1):55–67, 1970.

557 Menglin Jia, Luming Tang, Bor-Chun Chen, Claire Cardie, Serge Belongie, Bharath Hariharan, and  
 558 Ser-Nam Lim. Visual prompt tuning. In *European Conference on Computer Vision*, pp. 709–727.  
 559 Springer, 2022.

560 William B Johnson, Joram Lindenstrauss, et al. Extensions of lipschitz mappings into a hilbert space.  
 561 *Contemporary mathematics*, 26(189–206):1, 1984.

562 Dahuin Jung, Dongyoon Han, Jihwan Bang, and Hwanjun Song. Generating instance-level prompts  
 563 for rehearsal-free continual learning. In *Proceedings of the IEEE/CVF International Conference  
 564 on Computer Vision*, pp. 11847–11857, 2023.

565 Pentti Kanerva. *Sparse distributed memory*. MIT press, 1988.

566 James Kirkpatrick, Razvan Pascanu, Neil Rabinowitz, Joel Veness, Guillaume Desjardins, Andrei A  
 567 Rusu, Kieran Milan, John Quan, Tiago Ramalho, Agnieszka Grabska-Barwinska, et al. Overcoming  
 568 catastrophic forgetting in neural networks. *Proceedings of the national academy of sciences*, 114  
 569 (13):3521–3526, 2017.

570 Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images. 2009.

571 Zhizhong Li and Derek Hoiem. Learning without forgetting. *IEEE transactions on pattern analysis  
 572 and machine intelligence*, 40(12):2935–2947, 2017.

573 Dongze Lian, Daquan Zhou, Jiashi Feng, and Xinchao Wang. Scaling & shifting your features: A  
 574 new baseline for efficient model tuning. *Advances in Neural Information Processing Systems*, 35:  
 575 109–123, 2022.

576 Yan-Shuo Liang and Wu-Jun Li. Inflora: Interference-free low-rank adaptation for continual learning.  
 577 In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp.  
 578 23638–23647, 2024.

579 Yuchen Liang, Chaitanya K Ryali, Benjamin Hoover, Leopold Grinberg, Saket Navlakha, Mo-  
 580 hammed J Zaki, and Dmitry Krotov. Can a fruit fly learn word embeddings? *arXiv preprint  
 581 arXiv:2101.06887*, 2021.

582 Andrew C Lin, Alexei M Bygrave, Alix De Calignon, Tzumin Lee, and Gero Miesenböck. Sparse,  
 583 decorrelated odor coding in the mushroom body enhances learned odor discrimination. *Nature  
 584 neuroscience*, 17(4):559–568, 2014.

585 Ashok Litwin-Kumar, Kameron Decker Harris, Richard Axel, Haim Sompolinsky, and LF Abbott.  
 586 Optimal degrees of synaptic connectivity. *Neuron*, 93(5):1153–1164, 2017.

594 Mark D McDonnell, Dong Gong, Amin Parvaneh, Ehsan Abbasnejad, and Anton van den Hengel.  
 595 Ranpac: Random projections and pre-trained models for continual learning. *arXiv preprint*  
 596 *arXiv:2307.02251*, 2023.

597 Ahmed Metwally, Divyakant Agrawal, and Amr El Abbadi. An integrated efficient solution for  
 598 computing frequent and top-k elements in data streams. *ACM Transactions on Database Systems*  
 599 (*TODS*), 31(3):1095–1133, 2006.

600 Maria Papadopoulou, Stijn Cassenaer, Thomas Nowotny, and Gilles Laurent. Normalization for  
 601 sparse encoding of odors by a wide-field interneuron. *Science*, 332(6030):721–725, 2011.

602 Ameya Prabhu, Shiven Sinha, Ponnurangam Kumaraguru, Philip HS Torr, Ozan Sener, and Puneet K  
 603 Dokania. Random representations outperform online continually learned representations. *arXiv*  
 604 *preprint arXiv:2402.08823*, 2024.

605 Parikshit Ram and Kaushik Sinha. Federated nearest neighbor classification with a colony of fruit-  
 606 flies. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 36, pp. 8036–8044,  
 607 2022.

608 Chaitanya Ryali, John Hopfield, Leopold Grinberg, and Dmitry Krotov. Bio-inspired hashing for  
 609 unsupervised similarity search. In *International conference on machine learning*, pp. 8295–8306.  
 610 PMLR, 2020.

611 Jaiyam Sharma and Saket Navlakha. Improving similarity search with high-dimensional locality-  
 612 sensitive hashing. *arXiv preprint arXiv:1812.01844*, 2018.

613 James Seale Smith, Leonid Karlinsky, Vyshnavi Gutta, Paola Cascante-Bonilla, Donghyun Kim, Assaf  
 614 Arbelle, Rameswar Panda, Rogerio Feris, and Zsolt Kira. Coda-prompt: Continual decomposed  
 615 attention-based prompting for rehearsal-free continual learning. In *CVPR*, pp. 11909–11919, 2023.

616 Charles F Stevens. What the fly’s nose tells the fly’s brain. *Proceedings of the National Academy of*  
 617 *Sciences*, 112(30):9460–9465, 2015.

618 Hai-Long Sun, Da-Wei Zhou, Han-Jia Ye, and De-Chuan Zhan. Pilot: A pre-trained model-based  
 619 continual learning toolbox. *arXiv preprint arXiv:2309.07117*, 2023.

620 Hai-Long Sun, Da-Wei Zhou, Hanbin Zhao, Le Gan, De-Chuan Zhan, and Han-Jia Ye. Mos: Model  
 621 surgery for pre-trained model-based class-incremental learning. In *Proceedings of the AAAI*  
 622 *Conference on Artificial Intelligence*, volume 39, pp. 20699–20707, 2025.

623 Yu-Ming Tang, Yi-Xing Peng, and Wei-Shi Zheng. When prompt-based incremental learning does not  
 624 meet strong pretraining. In *Proceedings of the IEEE/CVF International Conference on Computer*  
 625 *Vision*, pp. 1706–1716, 2023.

626 Terence Tao and Van Vu. On random  $\pm 1$  matrices: singularity and determinant. *Random Struct.*  
 627 *Algor.*, 28(1):1–23, 2006.

628 Catherine Wah, Steve Branson, Peter Welinder, Pietro Perona, and Serge Belongie. The caltech-ucsd  
 629 birds-200-2011 dataset. 2011.

630 Huiyi Wang, Haodong Lu, Lina Yao, and Dong Gong. Self-expansion of pre-trained models with  
 631 mixture of adapters for continual learning. In *Proceedings of the Computer Vision and Pattern*  
 632 *Recognition Conference*, pp. 10087–10098, 2025.

633 Liyuan Wang, Jingyi Xie, Xingxing Zhang, Mingyi Huang, Hang Su, and Jun Zhu. Hierarchical  
 634 decomposition of prompt-based continual learning: Rethinking obscured sub-optimality. *arXiv*  
 635 *preprint arXiv:2310.07234*, 2023a.

636 Yabin Wang, Zhiheng Ma, Zhiwu Huang, Yaowei Wang, Zhou Su, and Xiaopeng Hong. Isolation  
 637 and impartial aggregation: A paradigm of incremental learning without interference. In *AAAI*,  
 638 volume 37, pp. 10209–10217, 2023b.

639 Zifeng Wang, Zizhao Zhang, Sayna Ebrahimi, Ruoxi Sun, Han Zhang, Chen-Yu Lee, Xiaoqi Ren,  
 640 Guolong Su, Vincent Perot, Jennifer Dy, et al. Dualprompt: Complementary prompting for  
 641 rehearsal-free continual learning. *arXiv preprint arXiv:2204.04799*, 2022a.

648 Zifeng Wang, Zizhao Zhang, Chen-Yu Lee, Han Zhang, Ruoxi Sun, Xiaoqi Ren, Guolong Su, Vincent  
 649 Perot, Jennifer Dy, and Tomas Pfister. Learning to prompt for continual learning. In *CVPR*, pp.  
 650 139–149, 2022b.

651 Jiazu Yu, Yunzhi Zhuge, Lu Zhang, Ping Hu, Dong Wang, Huchuan Lu, and You He. Boosting  
 652 continual learning of vision-language models via mixture-of-experts adapters. In *Proceedings of  
 653 the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 23219–23230, 2024.

654 Yunliang Zang and Erik De Schutter. Recent data on the cerebellum require new models and theories.  
 655 *Current Opinion in Neurobiology*, 2023.

656 Friedemann Zenke, Ben Poole, and Surya Ganguli. Continual learning through synaptic intelligence.  
 657 In *International conference on machine learning*, pp. 3987–3995. PMLR, 2017.

658 Xiaohua Zhai, Joan Puigcerver, Alexander Kolesnikov, Pierre Ruyssen, Carlos Riquelme, Mario  
 659 Lucic, Josip Djolonga, Andre Susano Pinto, Maxim Neumann, Alexey Dosovitskiy, et al. A  
 660 large-scale study of representation learning with the visual task adaptation benchmark. *arXiv  
 661 preprint arXiv:1910.04867*, 2019.

662 Da-Wei Zhou, Han-Jia Ye, De-Chuan Zhan, and Ziwei Liu. Revisiting class-incremental learn-  
 663 ing with pre-trained models: Generalizability and adaptivity are all you need. *arXiv preprint  
 664 arXiv:2303.07338*, 2023a.

665 Da-Wei Zhou, Yuanhan Zhang, Jingyi Ning, Han-Jia Ye, De-Chuan Zhan, and Ziwei Liu. Learning  
 666 without forgetting for vision-language models. *arXiv preprint arXiv:2305.19270*, 2023b.

667 Da-Wei Zhou, Hai-Long Sun, Han-Jia Ye, and De-Chuan Zhan. Expandable subspace ensemble for  
 668 pre-trained model-based class-incremental learning. In *CVPR*, 2024.

669 Huiping Zhuang, Yuchen Liu, Run He, Kai Tong, Ziqian Zeng, Cen Chen, Yi Wang, and Lap-Pui  
 670 Chau. F-oal: Forward-only online analytic learning with fast training and low memory footprint in  
 671 class incremental learning. *Advances in Neural Information Processing Systems*, 37:41517–41538,  
 672 2024.

673  
 674  
 675  
 676  
 677  
 678  
 679  
 680  
 681  
 682  
 683  
 684  
 685  
 686  
 687  
 688  
 689  
 690  
 691  
 692  
 693  
 694  
 695  
 696  
 697  
 698  
 699  
 700  
 701

702 **A ALGORITHM PSEUDOCODE**  
703704 **Algorithm 1** Fly-CL Training Pipeline

---

705 **Input:** Sequentially arriving data  $\mathcal{D}_t = \{(\mathbf{x}_i^t, y_i^t)\}_{i=1}^{n_t}$  where  $t = 1, \dots, T$ . Pre-trained encoder  
706  $f_\theta$ . Projection operator  $Z(\cdot) : \mathbb{R}^d \rightarrow \mathbb{R}^m$ . Penalty coefficient candidates  $\Lambda = \{\lambda_1, \dots, \lambda_l\}$ .  
707 Zero-initialized matrices  $\mathbf{G}_0 \in \mathbb{R}^{m \times m}$  and  $\mathbf{S}_0 \in \mathbb{R}^{m \times c_t}$ .  
708 **Output:** Modulated Prototypes  $\mathbf{C} \in \mathbb{R}^{m \times c_t}$ .

709 1: **for**  $t = 1, \dots, T$  **do**  
710 2:    $r = \min(n_t, m)$   
711 3:   Get compressed embedding for each datum  $\mathbf{v}_i^t = f_\theta(\mathbf{x}_i^t) \in \mathbb{R}^d$   
712 4:   Transform to high-dim sparse embedding  $Z(\mathbf{v}_i^t) = \text{top-}k(\mathbf{W}\mathbf{v}_i^t)$   $\triangleright \mathcal{O}(mn_tp)$   
713 5:   Concatenate  $Z(\mathbf{v}_i^t)$  to get  $\mathbf{H}_t \in \mathbb{R}^{n_t \times m}$   
714 6:    $\mathbf{G}_t \leftarrow \mathbf{G}_{t-1} + \mathbf{H}_t^\top \mathbf{H}_t$   $\triangleright \mathcal{O}(n_t m^2)$   
715 7:    $\mathbf{S}_t \leftarrow \mathbf{S}_{t-1} + \mathbf{H}_t^\top \mathbf{Y}_t$   $\triangleright \mathcal{O}(n_t c_t m)$   
716 8:    $\mathbf{U}_t, \Sigma_t, \mathbf{V}_t = \text{svd}(\mathbf{H}_t)$   $\triangleright \mathcal{O}(n_t rm)$   
717 9:   **for**  $\lambda \in \Lambda$  **do**  
718 10:     $\mathbf{D}_t = \frac{\Sigma_t^2}{\Sigma_t^2 + \lambda I_r}$   $\triangleright \mathcal{O}(lr)$   
719 11:     $\text{df}(\lambda) = \text{tr}(\mathbf{D}_t) = \sum_{i=1}^r \frac{s_i^2}{s_i^2 + \lambda}$   $\triangleright \mathcal{O}(lr)$   
720 12:     $\hat{\mathbf{Y}}_t = \mathbf{U}_t (\text{vecdiag}(\mathbf{D}_t) \otimes \mathbf{1}_c^\top) \odot \mathbf{U}_t^\top \mathbf{Y}_t$   $\triangleright \mathcal{O}(ln_t r c_t)$   
721 13:     $\text{GCV}(\lambda) = \frac{\|\mathbf{Y}_t - \hat{\mathbf{Y}}_t(\lambda)\|_F^2}{n_t (1 - \frac{\text{df}(\lambda)}{n_t})^2}$   $\triangleright \mathcal{O}(ln_t c_t)$   
722 14:   **end for**  
723 15:   Select  $\lambda_t^* = \arg \min_{\lambda \in \Lambda} \text{GCV}(\lambda)$   
724 16:    $\mathbf{L}_t \mathbf{L}_t^\top = \mathbf{G}_t + \lambda_t^* \mathbf{I}_m$   $\triangleright \mathcal{O}(\frac{1}{3} m^3)$   
725 17:    $\mathbf{C}_t = \mathbf{L}_t^{-\top} (\mathbf{L}_t^{-1} \mathbf{S}_t)$   $\triangleright \mathcal{O}(m^2 c_t)$   
726 18: **end for**

---

727 **Algorithm 2** Fly-CL Inference Pipeline

---

728 **Input:** Sequentially arriving data  $\mathcal{D}_t = \{(\mathbf{x}_i^t, y_i^t)\}_{i=1}^{n_t}$  where  $t = 1, \dots, T$ . Pre-trained encoder  $f_\theta$ .  
729 Projection operator  $Z$ . Modulated class prototypes  $\mathbf{C}_t \in \mathbb{R}^{m \times c_t}$ .  
730 **Output:** Predicted labels  $\hat{y}$ .

731 1: Get compressed embedding for each datum  $\mathbf{v} = f_\theta(\mathbf{x}) \in \mathbb{R}^d$   
732 2: Transform to high-dim sparse embedding  $Z(\mathbf{v}) = \text{top-}k(\mathbf{W}\mathbf{v})$   $\triangleright \mathcal{O}(mp)$   
733 3: Compute prediction  $\hat{y} = Z(\mathbf{v})^\top \mathbf{C}_t$   $\triangleright \mathcal{O}(k c_t)$ 

---

734 **B COMPLETE THEORETICAL ANALYSIS**  
735

736 In this section, we present a comprehensive theoretical analysis of the sparsification effects on both  
737 the weights and activations in the random projection operation, and demonstrate the consistency  
738 between the biologically plausible Hebbian learning rule and ridge classification in modeling the  
739 KC-MBON transformation..

740 **B.1 INFORMATION PRESERVING FOR SPARSE CONNECTIONS IN RANDOM PROJECTION  
741 MATRIX**

742 A common approach to demonstrate that sparse random matrix multiplication preserves information  
743 equivalently to its dense counterpart lies in proving the matrix's near-preservation of full column rank.  
744 For our sparse random matrix  $\mathbf{W} \in \mathbb{R}^{m \times d}$  where  $m > d$ , we prove that  $\mathbf{W}$  almost surely maintains  
745 rank  $d$ .

746 **Theorem B.1.** *Given the matrix  $\mathbf{W} \in \mathbb{R}^{m \times d}$ , where  $m > d$ , with each row having exactly  $p$  non-zero  
747 entries, which are randomly sampled from  $\mathcal{N}(0, 1)$ . Let  $\mathcal{W} \in \mathbb{R}^{d \times d}$  be any square submatrix of  $\mathbf{W}$ .*

756 Then, for any  $\epsilon > 0$ , it holds that  
 757

$$758 \mathbb{P}\left(|\det(\mathcal{W})| \geq \left(\frac{p}{d}\right)^{d/2} \sqrt{d!} \exp(-d^{1/2+\epsilon})\right) = 1 - o(1). \\ 759$$

760 Thus, for sufficiently large  $p$  and  $d$ , any submatrix  $\mathcal{W}$  is invertible with probability at least  $1 - o(1)$ .  
 761

762 *Proof.* According to aformentioned definition, we have  $\mathbb{E}(\mathbf{W}_{ij}) = 0$  and  $\text{Var}(\mathbf{W}_{ij}) = \frac{p}{d}$ . Considering  
 763  $\mathbf{R} = \frac{1}{\sigma} \mathcal{W}$ , which satisfies  $\mathbb{E}(\mathbf{R}_{ij}) = 0$ ,  $\text{Var}(\mathbf{R}_{ij}) = 1$ , we can conclude, based on (Tao & Vu,  
 764 2006, Theorem 8.9), that

$$765 \mathbb{P}\left(|\det(\mathbf{R})| \geq \sqrt{d!} \exp(-d^{1/2+\epsilon})\right) = 1 - o(1). \quad (13) \\ 766$$

767 By using  $\sigma = \sqrt{\frac{p}{d}}$  and  $\det(\mathbf{R}) = \sigma^{-d} \det(\mathcal{W})$  for substitution, we complete the proof.  $\square$   
 768

769 To better demonstrate the information preserv-  
 770 ing property of our construction, another line  
 771 of validation is to utilize the random matrix's  
 772 distance preservation property, which is theore-  
 773 tically guaranteed by the Johnson-Lindenstrauss  
 774 (JL) lemma (Johnson et al., 1984). Specifi-  
 775 cally, we conduct an empirical simulation to  
 776 verify that the normalization of our sparse pro-  
 777 jection matrix  $\Phi = \sqrt{\frac{d}{mp}} \mathbf{W}$  preserves pair-  
 778 wise Euclidean distances between feature vec-  
 779 tors with high probability. For every pair of vec-  
 780 tors  $(\mathbf{x}_1, \mathbf{x}_2)$ , we calculate the Distortion Ratio  
 781 defined as  $\text{Ratio} = \frac{\|\Phi(\mathbf{x}_1 - \mathbf{x}_2)\|_2^2}{\|\mathbf{x}_1 - \mathbf{x}_2\|_2^2}$ . We randomly  
 782 select 500 data points from the extracted fea-  
 783 tures of CIFAR-100 dataset and compute the  
 784 pairwise distances between all point pairs, with  
 785 the results illustrated in Figure 6. By analyzing  
 786 the distribution of these ratios across all vector  
 787 pairs, the resulting histogram shows a strong  
 788 concentration around the ideal value of 1. If we set  $\epsilon$  to 0.03, then the vast majority of the ratios  
 789 are empirically confined within the bounds of  $[1 - \epsilon, 1 + \epsilon]$ . This concentration strongly validates  
 790 that our sparse projection structure-comprising only  $p$  non-zero  $\mathcal{N}(0, 1)$  entries per row-effectively  
 791 maintains the geometric structure of the high-dimensional data.

## 792 B.2 ROBUSTNESS OF TOP- $k$ SPARSIFICATION ON HIGH-DIMENSIONAL EMBEDDINGS

794 Let the high-dimensional embedding vector be  $\mathbf{h} \in \mathbb{R}^m$ . After applying the top- $k$  operation, we  
 795 obtain a sparsified vector  $\mathbf{h}' \in \mathbb{R}^m$ , where only the  $k$  largest absolute values in  $\mathbf{h}$  are retained, and  
 796 the remaining elements are set to zero. We aim to prove that when  $k = \Omega(m^\alpha)$  (with  $0 < \alpha < 1$ , i.e.,  
 797 not overly sparse), the performance degradation is negligible.

798 According to statistic learning theory (Bartlett & Mendelson, 2002) and extreme value theory (Coles  
 799 et al., 2001; Fisher & Tippett, 1928), we start with the following two widely-accepted assumption:

800 **Assumption B.2.** Assume that the “energy” (squared  $\ell_2$ -norm) of the embedding vector  $x$  is concen-  
 801 trated in a few dimensions, i.e., there exists a constant  $C > 0$  such that:

$$803 \mathbb{E} \left[ \frac{\sum_{i=1}^k \mathbf{h}_i^2}{\|\mathbf{h}\|_2^2} \right] \geq 1 - \frac{C}{k}, \\ 804$$

805 where where  $\mathbf{h}_i$  denotes the  $i$ -th largest value in  $\mathbf{h}$ .

806 **Assumption B.3.** Assume that the performance loss function  $L(\mathbf{h}, y)$  of the downstream task (e.g.,  
 807 classifier) is Lipschitz continuous with respect to input perturbations, i.e., there exists a constant  
 808  $M > 0$  such that:

$$809 |L(\mathbf{h}, y) - L(\mathbf{h}', y)| \leq M \cdot \|\mathbf{h} - \mathbf{h}'\|.$$

810 Regarding the approximation error of top- $k$  operation, we show it is bounded.  
 811

812 **Theorem B.4.** *Under the Assumption B.2, the sparsification error satisfies:*

$$813 \quad 814 \quad E [\|\mathbf{h} - \mathbf{h}'\|_2^2] \leq \frac{C}{k} \cdot \mathbb{E} [\|\mathbf{h}\|_2^2]. \\ 815$$

816 *Proof.* By Assumption B.2:

$$817 \quad 818 \quad \mathbb{E} \left[ \sum_{i=k+1}^d \mathbf{h}_i^2 \right] \leq \frac{C}{k} \cdot \mathbb{E} [\|\mathbf{h}\|_2^2]. \\ 819 \\ 820$$

821 Thus,

$$822 \quad 823 \quad \mathbb{E} [\|\mathbf{h} - \mathbf{h}'\|_2^2] = \mathbb{E} \left[ \sum_{i=k+1}^m \mathbf{h}_i^2 \right] \leq \frac{C}{k} \cdot \mathbb{E} [\|\mathbf{h}\|_2^2]. \\ 824$$

825  $\square$

826 Then, we can quantify the upper bound of possible performance degradation.

827 **Theorem B.5.** *Under Assumption B.3, the performance degradation due to sparsification satisfies:*

$$828 \quad 829 \quad \mathbb{E} [|L(\mathbf{h}, y) - L(\mathbf{h}', y)|] \leq M \cdot \sqrt{\frac{C}{k} \cdot \mathbb{E} [\|\mathbf{h}\|_2^2]} \\ 830$$

831 *Proof.* By the Cauchy-Schwarz inequality and Theorem B.4, we can derive that

$$832 \quad 833 \quad \mathbb{E} [|L(\mathbf{h}, y) - L(\mathbf{h}', y)|] \leq M \cdot \mathbb{E} [\|\mathbf{h} - \mathbf{h}'\|_2] \\ 834 \quad 835 \quad \leq M \cdot \sqrt{\mathbb{E} [\|\mathbf{h} - \mathbf{h}'\|_2^2]} \\ 836 \quad 837 \quad \leq M \cdot \sqrt{\frac{C}{k} \cdot \mathbb{E} [\|\mathbf{h}\|_2^2]}. \\ 838 \\ 839 \\ 840 \\ 841$$

842  $\square$

843 From Theorem B.5, we establish in Theorem B.6 that moderate sparsity does not result in significant  
 844 performance degradation, as the error decreases exponentially with the increasing expanded dimension  
 845  $m$ .

846 **Theorem B.6.** *For top- $k$  sparsification in the expanded dimension  $m$ , the performance degradation  
 847 is bounded by:*

$$848 \quad 849 \quad \mathbb{E} [|L(\mathbf{h}, y) - L(\mathbf{h}', y)|] \leq M \cdot \sqrt{\frac{C}{k} \cdot \mathbb{E} [\|\mathbf{h}\|_2^2]},$$

850 where  $L(\cdot)$  is a performance loss function for downstream tasks and  $C, M$  are constants. To ensure  
 851 negligible performance degradation, we require:

$$852 \quad 853 \quad \sqrt{\frac{C}{k} \cdot \mathbb{E} [\|\mathbf{h}\|_2^2]} \leq \mathcal{O} \left( \frac{1}{\sqrt{m^\alpha}} \right),$$

854 *i.e., when  $k = \Omega(m^\alpha)$  ( $0 < \alpha < 1$ ), the error bound decays exponentially with increasing dimension.*

855 For example:

856 

- 857 • If  $k = \Omega(d^{0.5})$ , the performance degradation is  $\mathcal{O}(d^{-0.25})$ .
- 858 • If  $k = \Omega(d^{0.8})$ , the performance degradation is  $\mathcal{O}(d^{-0.4})$ .

864 B.3 CONSISTENCY BETWEEN RIDGE CLASSIFICATION AND HEBBIAN LEARNING RULES  
865

866 In this section, we show that a biologically plausible *local learning rule*—constructed from Hebbian  
867 and anti-Hebbian plasticity (Hebb, 2005)—converges to the same stationary point as ridge classifica-  
868 tion. This provides a theoretical bridge between synaptic dynamics in the KC→MBON pathway and  
869 the streaming ridge classification algorithm used in Fly-CL.

870 We begin by defining a synaptic update rule that depends only on variables locally available at each  
871 synapse. Let  $\mathbf{h}' \in \mathbb{R}^m$  denote the pre-synaptic activity vector and  $\hat{\mathbf{y}} = \mathbf{C}_t^\top \mathbf{h}'$  the post-synaptic  
872 response predicted by the current weights  $\mathbf{C}_t \in \mathbb{R}^{m \times c_t}$ . The synaptic update is defined as:

$$873 \Delta \mathbf{C} = \mathbf{C}_{t+1} - \mathbf{C}_t = \eta(\mathbf{h}' \mathbf{y}^\top - \mathbf{h}' \hat{\mathbf{y}}^\top - \lambda \mathbf{C}_t), \quad (14)$$

874 where  $\eta > 0$  is a small learning rate,  $\mathbf{y} \in \mathbb{R}^{c_t}$  is the one-hot ground-truth label. This rule consists of  
875 three biologically meaningful components:

- 876 1. **Hebbian term**  $\mathbf{h}' \mathbf{y}^\top$ , which strengthens synapses when both pre-synaptic activity and the  
877 target post-synaptic signal co-activate.
- 878 2. **Anti-Hebbian term**  $-\mathbf{h}' \hat{\mathbf{y}}^\top$ , which suppresses correlations between the input and the  
879 model’s own prediction, implementing an error-correcting mechanism.
- 880 3. **Weight decay**  $-\lambda \mathbf{C}_t$ , corresponding to metabolic cost or homeostatic constraints.

881 This formulation is biologically plausible: it requires only pre-synaptic activity, post-synaptic activity,  
882 and a modulatory teaching signal—i.e., a standard *three-factor learning rule* commonly observed in  
883 neuromodulated plasticity.

884 Taking the expectation over data samples  $(\mathbf{h}', \mathbf{y})$ , we obtain:

$$885 \mathbb{E}[\Delta \mathbf{C}] = \eta(\mathbb{E}[\mathbf{h}' \mathbf{y}^\top] - \mathbb{E}[\mathbf{h}' \hat{\mathbf{y}}^\top] - \lambda \mathbf{C}_t) = \eta(\mathbf{S}_t - \mathbf{G}_t \mathbf{C}_t - \lambda \mathbf{C}_t), \quad (15)$$

886 where we define

$$887 \mathbf{G}_t = \mathbb{E}[\mathbf{h}' \mathbf{h}'^\top] \in \mathbb{R}^{m \times m}, \quad \mathbf{S}_t = \mathbb{E}[\mathbf{h}' \mathbf{y}^\top] \in \mathbb{R}^{m \times c_t}.$$

888 Thus, the expected synaptic dynamics follow the linear recursion:

$$889 \mathbf{C}_{t+1} = \mathbf{C}_t + \eta(\mathbf{S}_t - (\mathbf{G}_t + \lambda \mathbf{I}_m) \mathbf{C}_t). \quad (16)$$

890 Taking the continuous-time limit  $\eta \rightarrow 0$  yields the ordinary differential equation:

$$891 \frac{d\mathbf{C}_t}{dt} = \kappa(\mathbf{S}_t - (\mathbf{G}_t + \lambda \mathbf{I}_m) \mathbf{C}_t), \quad (17)$$

892 where  $\kappa > 0$  rescales time. The stationary solution satisfies:

$$893 (\mathbf{G}_t + \lambda \mathbf{I}_m) \mathbf{C}_t^* = \mathbf{S}_t,$$

894 and therefore:

$$895 \mathbf{C}_t^* = (\mathbf{G}_t + \lambda \mathbf{I}_m)^{-1} \mathbf{S}_t, \quad (18)$$

896 which is *exactly* the closed-form solution of ridge regression.

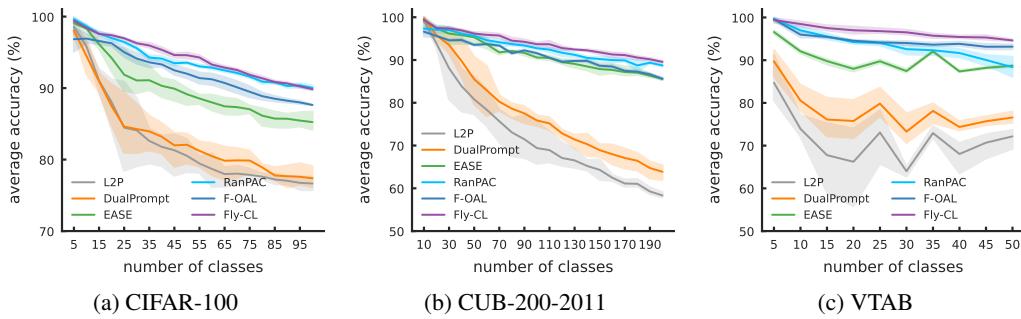
897 In our representation-based CIL setting, since all samples in the same task are available simultaneously,  
898 we do not need to integrate the continuous-time dynamics in Eq. 17. Instead, we can directly compute  
899 the ridge solution to obtain the optimal stationary classifier.

900 C ADDITIONAL RESULTS  
901902 C.1 EXPERIMENTS IN LONGER TASK SEQUENCES  
903

904 To evaluate the long-term stability of Fly-CL, we conduct experiments with task sequences twice as  
905 long as those in Table 1. Results are presented in Table 5 and Figure 7. Overall, both  $\tau_{\text{train}}$  and  $\tau_{\text{post}}$   
906 are shorter than those in Table 1 due to fewer samples per task. Fly-CL improves overall accuracy  
907 by 0.54%, 1.21%, and 1.58% compared to SOTA methods, while significantly reducing average  
908 post-extraction training time by 89%, 74%, and 59% compared to the most efficient baselines. These  
909 results are consistent with the trends observed in Table 1 and Figure 8, demonstrating the robustness  
910 of Fly-CL across different task lengths.

918  
919 **Table 5: Performance Comparison on Pre-trained ViT-B/16 Models with Longer Task Sequence.**  
920 We report the average training time per task ( $\tau_{\text{train}}$ ), average post-extraction training time ( $\tau_{\text{post}}$ ), and  
921 overall accuracy ( $\bar{A}$ ) across three benchmark datasets: CIFAR-100, CUB-200-2011, and VTAB. The  
922 best results are highlighted in **bold**.

923 Method	924 <b>CIFAR-100</b>			925 <b>CUB-200-2011</b>			926 <b>VTAB</b>		
	927 $\tau_{\text{train}}(\downarrow)$	928 $\tau_{\text{post}}(\downarrow)$	929 $\bar{A}(\uparrow)$	930 $\tau_{\text{train}}(\downarrow)$	931 $\tau_{\text{post}}(\downarrow)$	932 $\bar{A}(\uparrow)$	933 $\tau_{\text{train}}(\downarrow)$	934 $\tau_{\text{post}}(\downarrow)$	935 $\bar{A}(\uparrow)$
L2P	147.47 $\pm$ 0.36	107.59 $\pm$ 0.26	82.69 $\pm$ 0.91	30.93 $\pm$ 0.25	23.39 $\pm$ 0.26	72.83 $\pm$ 1.45	32.17 $\pm$ 0.39	29.31 $\pm$ 0.38	71.84 $\pm$ 1.42
Dualprompt	130.12 $\pm$ 0.09	91.05 $\pm$ 0.07	83.42 $\pm$ 0.86	27.66 $\pm$ 0.18	20.28 $\pm$ 0.18	77.93 $\pm$ 0.91	29.44 $\pm$ 0.24	26.62 $\pm$ 0.23	78.46 $\pm$ 1.14
InfLoRA	<b>124.80<math>\pm</math>0.39</b>	<b>84.36<math>\pm</math>0.12</b>	<b>88.18<math>\pm</math>0.34</b>	<b>27.12<math>\pm</math>0.14</b>	<b>19.85<math>\pm</math>0.09</b>	<b>75.67<math>\pm</math>0.16</b>	<b>29.71<math>\pm</math>0.21</b>	<b>26.92<math>\pm</math>0.17</b>	<b>81.09<math>\pm</math>0.73</b>
SEMA	128.95 $\pm$ 0.46	88.57 $\pm$ 0.19	88.93 $\pm$ 0.52	27.98 $\pm$ 0.25	20.52 $\pm$ 0.18	80.82 $\pm$ 0.11	30.36 $\pm$ 0.27	27.55 $\pm$ 0.20	84.56 $\pm$ 0.48
MoE-Adapter	<b>132.67<math>\pm</math>0.62</b>	<b>92.31<math>\pm</math>0.45</b>	<b>88.42<math>\pm</math>0.28</b>	<b>30.25<math>\pm</math>0.19</b>	<b>22.82<math>\pm</math>0.15</b>	<b>78.62<math>\pm</math>0.35</b>	<b>29.25<math>\pm</math>0.18</b>	<b>26.81<math>\pm</math>0.13</b>	<b>83.24<math>\pm</math>0.57</b>
EASE	350.92 $\pm$ 0.39	331.76 $\pm$ 0.29	90.14 $\pm$ 0.51	83.38 $\pm$ 0.22	75.05 $\pm$ 0.16	91.47 $\pm$ 0.65	64.68 $\pm$ 0.73	57.24 $\pm$ 0.62	90.26 $\pm$ 0.41
RanPAC	44.53 $\pm$ 0.58	37.03 $\pm$ 0.59	93.68 $\pm$ 0.24	20.68 $\pm$ 0.32	18.18 $\pm$ 0.31	92.65 $\pm$ 0.26	41.49 $\pm$ 0.74	40.02 $\pm$ 0.59	93.62 $\pm$ 0.32
F-OAL	16.32 $\pm$ 0.02	8.91 $\pm$ 0.02	92.63 $\pm$ 0.46	3.65 $\pm$ 0.02	1.10 $\pm$ 0.01	91.48 $\pm$ 0.23	2.17 $\pm$ 0.16	0.71 $\pm$ 0.03	94.96 $\pm$ 0.27
<b>Fly-CL</b>	<b>8.31<math>\pm</math>0.04</b>	<b>1.00<math>\pm</math>0.01</b>	<b>94.22<math>\pm</math>0.09</b>	<b>2.76<math>\pm</math>0.06</b>	<b>0.29<math>\pm</math>0.01</b>	<b>93.86<math>\pm</math>0.27</b>	<b>1.70<math>\pm</math>0.10</b>	<b>0.29<math>\pm</math>0.02</b>	<b>96.54<math>\pm</math>0.38</b>



944  
945 **Figure 7: Accuracy Curves of Different Methods on Pre-trained ViT-B/16 with Longer Task**  
946 **Sequence.** The average accuracy ( $A_t$ ) is reported for each dataset. These results align with and  
947 extend the quantitative analysis presented in Table 5.

## 948 C.2 EXPERIMENTS ON DATASETS WITH SEVERE DOMAIN SHIFT

949 Table 6 summarizes the results on ImageNet-R and ImageNet-A under severe domain shift. Compared  
950 with existing continual learning baselines, Fly-CL achieves the best overall accuracy on both datasets  
951 (83.19% on ImageNet-R and 67.98% on ImageNet-A), comparable with the previous SOTA RanPAC.  
952 More importantly, Fly-CL attains these improvements with substantially lower computation cost.  
953 Its average training time per task is reduced by an order of magnitude compared to prompt-based  
954 methods (e.g., L2P, DualPrompt) and much faster than EASE, while its post-extraction training time  
955 is almost negligible (0.21s vs. 67.71s for RanPAC on ImageNet-R). These results demonstrate that  
956 Fly-CL is not only robust to severe distribution shifts but also highly efficient, making it especially  
957 suitable for practical continual learning scenarios where both accuracy and efficiency are critical.

## 958 C.3 DATA NORMALIZATION STRATEGY

959 While data normalization is a well-established technique for improving classification performance  
960 in i.i.d. scenarios, its effectiveness in facilitating CL with frozen pre-trained encoders remains  
961 unclear. Our results indicate that applying proper architecture-specific normalization to input images  
962 significantly improves the learning performance compared to baseline CL methods (Table 7). The  
963 optimal normalization strategies for the included backbones differ. Across all tested datasets, ViT-  
964 B/16 (Dosovitskiy et al., 2020) benefits more from standard normalization that projects inputs into  
965 the  $[-1, 1]$  range, while ResNet-50 (He et al., 2016) achieves optimal performance when normalized  
966 using ImageNet statistics.

967 We hypothesize that the improved performance arises from a reduced feature distribution shift across  
968 tasks. Proper normalization preserves the geometry of the pre-trained feature manifold, which is  
969 crucial for prototype-based classification, where cosine similarity measures depend on the angular

972 **Table 6: Performance Comparison on Pre-trained ViT-B/16 Models with Severe Domain Shift.**  
 973 We report the average training time per task ( $\tau_{\text{train}}$ ), average post-extraction training time ( $\tau_{\text{post}}$ ), and  
 974 overall accuracy ( $\bar{A}$ ) across two benchmark datasets: ImageNet-R and ImageNet-A. The best results  
 975 are highlighted in **bold**.

Method	ImageNet-R			ImageNet-A		
	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$	$\tau_{\text{train}}(\downarrow)$	$\tau_{\text{post}}(\downarrow)$	$\bar{A}(\uparrow)$
L2P	131.97 $\pm$ 0.46	110.56 $\pm$ 0.42	76.13 $\pm$ 0.21	56.28 $\pm$ 0.32	48.92 $\pm$ 0.27	48.86 $\pm$ 0.08
Dualprompt	117.80 $\pm$ 0.34	96.58 $\pm$ 0.30	73.92 $\pm$ 0.46	49.60 $\pm$ 0.28	42.31 $\pm$ 0.26	57.05 $\pm$ 0.13
<b>InfLoRA</b>	<b>62.32<math>\pm</math>0.33</b>	<b>41.05<math>\pm</math>0.23</b>	<b>82.15<math>\pm</math>0.41</b>	<b>25.71<math>\pm</math>0.30</b>	<b>18.50<math>\pm</math>0.24</b>	<b>62.32<math>\pm</math>0.28</b>
SEMA	68.85 $\pm$ 0.21	47.34 $\pm$ 0.19	81.89 $\pm$ 0.17	28.65 $\pm$ 0.26	21.23 $\pm$ 0.22	61.79 $\pm$ 0.32
<b>MoE-Adapter</b>	<b>66.37<math>\pm</math>0.29</b>	<b>45.02<math>\pm</math>0.24</b>	<b>81.76<math>\pm</math>0.33</b>	<b>26.27<math>\pm</math>0.41</b>	<b>18.89<math>\pm</math>0.37</b>	<b>61.72<math>\pm</math>0.21</b>
EASE	311.00 $\pm$ 0.29	274.36 $\pm$ 0.25	81.69 $\pm$ 0.24	80.80 $\pm$ 0.19	73.47 $\pm$ 0.22	65.03 $\pm$ 0.28
RanPAC	76.25 $\pm$ 0.35	67.71 $\pm$ 0.28	83.02 $\pm$ 0.12	32.43 $\pm$ 0.13	28.86 $\pm$ 0.11	67.28 $\pm$ 0.09
F-OAL	16.51 $\pm$ 0.11	8.80 $\pm$ 0.04	80.62 $\pm$ 0.25	3.99 $\pm$ 0.07	1.05 $\pm$ 0.02	63.99 $\pm$ 0.30
<b>Fly-CL</b>	<b>7.55<math>\pm</math>0.04</b>	<b>0.21<math>\pm</math>0.02</b>	<b>83.19<math>\pm</math>0.14</b>	<b>3.10<math>\pm</math>0.03</b>	<b>0.15<math>\pm</math>0.01</b>	<b>67.98<math>\pm</math>0.17</b>

988 relationships between features. Our empirical results suggest that input normalization may serve  
 989 as a fundamental defense against forgetting by anchoring the feature space topology to the original  
 990 pre-training distribution.

992 **Table 7: Comparison of CL Performance across Pre-trained Models and Normalization Strategies.** We report overall accuracy ( $\bar{A}$ ). Normalization methods includes: “None” (no data normalization),  
 993 “ImageNet” (ImageNet statistics), and “Standard”(scaled to the  $[-1, 1]$ ).

Backbone	CIFAR			CUB			VTAB		
	None	ImageNet	Standard	None	ImageNet	Standard	None	ImageNet	Standard
ViT-B/16	91.64 $\pm$ 0.62	87.87 $\pm$ 0.62	<b>93.89<math>\pm</math>0.12</b>	93.04 $\pm$ 0.37	90.68 $\pm$ 0.42	<b>93.84<math>\pm</math>0.18</b>	95.26 $\pm$ 0.68	95.47 $\pm$ 0.52	<b>96.54<math>\pm</math>0.38</b>
ResNet-50	80.66 $\pm$ 0.48	<b>84.61<math>\pm</math>0.16</b>	83.09 $\pm$ 0.48	75.08 $\pm$ 1.23	<b>80.25<math>\pm</math>0.10</b>	76.78 $\pm$ 1.08	92.45 $\pm$ 0.71	<b>94.00<math>\pm</math>0.15</b>	92.76 $\pm$ 0.54

#### C.4 MEMORY CONSUMPTION

1004 We also compare the memory consumption of Fly-CL against other methods in Table 8 using ViT-  
 1005 B/16 with the same task sequence as in Table 1. For fairness, we use a batch size of 128 across all  
 1006 methods and datasets. The results show Fly-CL also has the minimal memory cost, strengthening the  
 1007 efficiency of our method.

1008 **Table 8: Memory Usage (GB) of Different Methods on Pre-trained ViT-B/16.** We report highest  
 1009 peak memory usage of each methods. The best results are highlighted in **bold**.

Method	CIFAR-100	CUB-200-2011	VTAB
L2P	16.4GB	16.4GB	16.4GB
DualPrompt	13.6GB	13.6GB	13.6GB
<b>InfLoRA</b>	<b>14.1GB</b>	<b>14.1GB</b>	<b>14.1GB</b>
SEMA	<b>12.9GB</b>	<b>12.9GB</b>	<b>12.9GB</b>
<b>MoE-Adapter</b>	<b>20.2GB</b>	<b>20.2GB</b>	<b>20.2GB</b>
EASE	12.2GB	12.2GB	12.2GB
RanPAC	12.2GB	12.2GB	22.8GB
F-OAL	12.2GB	4.9GB	4.5GB
<b>Fly-CL</b>	<b>6.7GB</b>	<b>4.6GB</b>	<b>4.3GB</b>

#### C.5 MEMORY-TIME TRADEOFF IN HIGH-DIM PROJECTIONS

1023 We present the trade-off between memory, training time, and overall accuracy in Table 9. The  
 1024 overall accuracy gradually saturates as the dimension increases, while memory and training time  
 1025

grow quadratically. Therefore, we chose 10,000 as the dimension in our simulations. As long as the dimension does not exceed 10,000, both memory and training time consumption remain lower than those of previous methods, as summarized in Table 1.

Table 9: **Memory-Time-Accuracy comparison with increasing projection dimension.** It's conducted on the CUB dataset using ViT B/16. We report highest peak memory usage of each methods. The best results are highlighted in **bold**.

Dimension	1000	2000	5000	10000	20000
Memory	2.8G	2.8G	3.0G	4.6G	9.3G
$\tau_{\text{train}}$	$4.19 \pm 0.02$	$4.20 \pm 0.05$	$4.25 \pm 0.07$	$4.43 \pm 0.11$	$5.13 \pm 0.08$
$\bar{A}$	$90.87 \pm 0.49$	$91.97 \pm 0.52$	$92.93 \pm 0.41$	$93.84 \pm 0.18$	$93.90 \pm 0.52$

### C.6 ADDITIONAL VISUALIZED FIGURES AND EVALUATION METRIC DURING THE TRAINING PROCESS

Here, we present a more detailed breakdown of the training processes for ViT-B/16 and ResNet-50. Results from Tables 1 and 2 are visualized in Figures 8 and 9. We list the average accuracy of different methods at different stages across three datasets. **We additionally report the last-stage accuracy ( $A_T$ )**

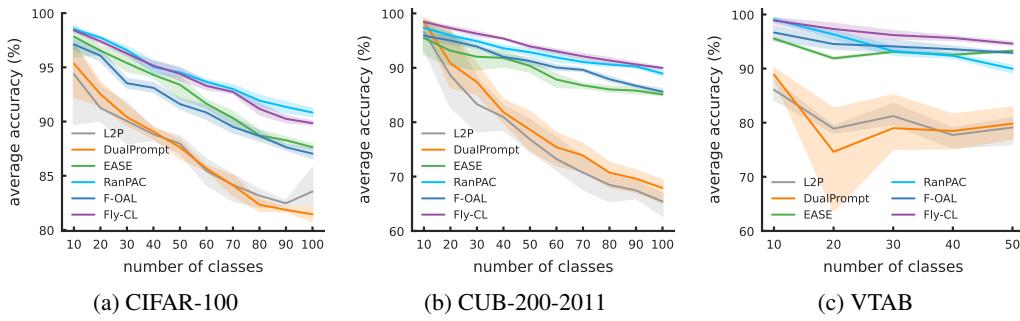


Figure 8: **Accuracy Curves of Different Methods on Pre-trained ViT-B/16.** The average accuracy ( $A_t$ ) is reported for each dataset. These results align with and extend the quantitative analysis presented in Table 1.

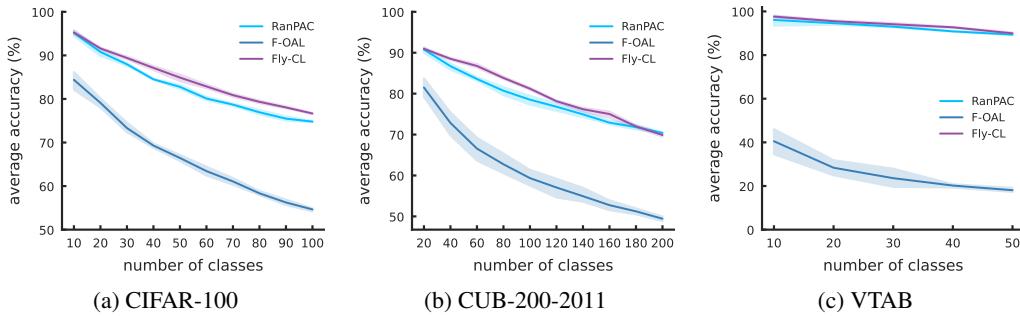


Figure 9: **Accuracy Curves of Different Methods on Pre-trained ResNet-50.** The average accuracy ( $A_t$ ) is reported for each dataset. These results align with and extend the quantitative analysis presented in Table 2.

and the backward transfer score (a representative forgetting metric) of Table 1 in Table 10 to provide a more comprehensive evaluation.

1080 Table 10: **Performance Comparison on Pre-trained ViT-B/16 Models.** We report the last stage  
 1081 accuracy ( $A_T$ ), and backward transfer ( $BWT$ ) across three benchmark datasets: CIFAR-100, CUB-  
 1082 200-2011, and VTAB.

Method	CIFAR-100		CUB-200-2011		VTAB	
	$A_T(\uparrow)$	$BWT(\uparrow)$	$A_T(\uparrow)$	$BWT(\uparrow)$	$A_T(\uparrow)$	$BWT(\uparrow)$
L2P	83.55 $\pm$ 1.53	-6.23 $\pm$ 0.41	65.41 $\pm$ 1.84	-13.14 $\pm$ 0.57	79.12 $\pm$ 2.18	-8.04 $\pm$ 0.78
DualPrompt	81.45 $\pm$ 0.52	-7.06 $\pm$ 0.20	67.90 $\pm$ 1.89	-12.12 $\pm$ 0.53	79.83 $\pm$ 2.37	-7.63 $\pm$ 0.75
InfLoRA	86.56 $\pm$ 0.46	-5.07 $\pm$ 0.16	69.45 $\pm$ 0.56	-11.53 $\pm$ 0.25	87.88 $\pm$ 0.73	-4.59 $\pm$ 0.24
SEMA	87.47 $\pm$ 0.43	-4.70 $\pm$ 0.13	73.66 $\pm$ 0.36	-9.94 $\pm$ 0.19	89.28 $\pm$ 0.60	-4.06 $\pm$ 0.22
MoE-Adapter	86.88 $\pm$ 0.32	-4.97 $\pm$ 0.15	68.11 $\pm$ 0.41	-12.11 $\pm$ 0.23	88.06 $\pm$ 0.48	-4.51 $\pm$ 0.19
EASE	87.63 $\pm$ 0.20	-4.66 $\pm$ 0.06	85.10 $\pm$ 0.19	-5.68 $\pm$ 0.08	93.30 $\pm$ 0.07	-2.48 $\pm$ 0.04
RanPAC	<b>90.83<math>\pm</math>0.41</b>	<b>-3.47<math>\pm</math>0.11</b>	88.95 $\pm$ 0.48	-4.16 $\pm$ 0.22	89.97 $\pm$ 0.71	-3.79 $\pm$ 0.26
F-OAL	87.04 $\pm$ 0.50	-4.90 $\pm$ 0.14	85.61 $\pm$ 0.50	-5.43 $\pm$ 0.24	92.91 $\pm$ 0.07	-2.66 $\pm$ 0.04
<b>Fly-CL</b>	<b>89.85<math>\pm</math>0.17</b>	<b>-3.82<math>\pm</math>0.09</b>	<b>89.97<math>\pm</math>0.18</b>	<b>-3.80<math>\pm</math>0.07</b>	<b>94.61<math>\pm</math>0.35</b>	<b>-2.01<math>\pm</math>0.15</b>

### C.7 ADDITIONAL SENSITIVITY ANALYSIS ACROSS TASK COMPLEXITY, NUMBER OF TASKS/CLASSES, AND DIFFERENT PRETRAINED BACKBONES

We further conduct additional sensitivity analyses under three complementary settings: (i) ImageNet-A with 10 tasks and 20 classes per task using ViT-B/16 to examine the effect of task complexity (Figure 10); (ii) CUB-200-2011 with 20 tasks and 10 classes per task using ViT-B/16 to evaluate the impact of a larger number of tasks and classes (Figure 11); and (iii) CUB-200-2011 with 10 tasks and 20 classes per task using ResNet-50 to assess the influence of different backbone architectures (Figure 12). Across all settings, the observed trends remain consistent with those reported in Figure 5.

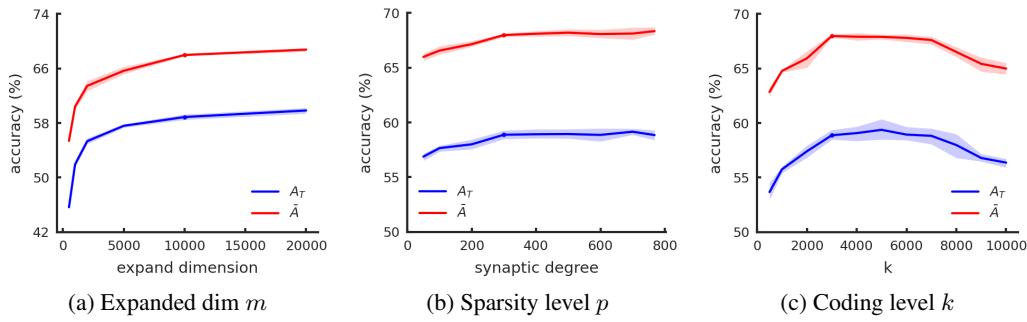


Figure 10: **Sensitivity Analysis for Expanded dim  $m$ , Weight Sparsity  $p$ , and Activation Sparsity  $k$  on ImageNet-A.** We report average accuracy in last task ( $A_T$ ) and overall accuracy ( $\bar{A}$ ). The dots denote the default values we use across experiments.

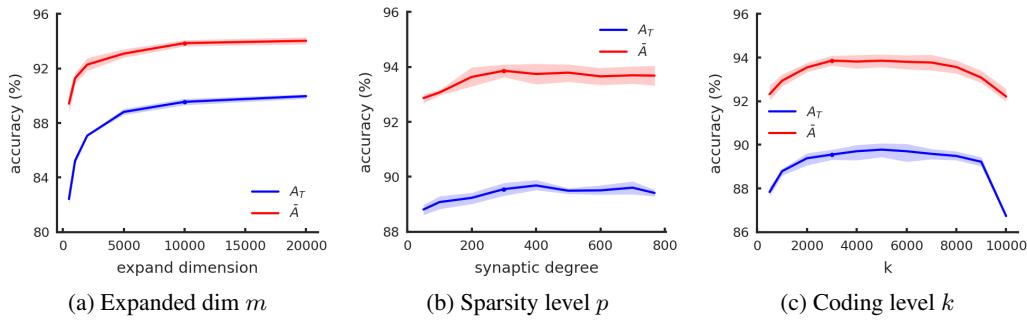


Figure 11: **Sensitivity Analysis for Expanded dim  $m$ , Weight Sparsity  $p$ , and Activation Sparsity  $k$  on CUB-200 with Longer Task Sequence.** We report average accuracy in last task ( $A_T$ ) and overall accuracy ( $\bar{A}$ ). The dots denote the default values we use across experiments.

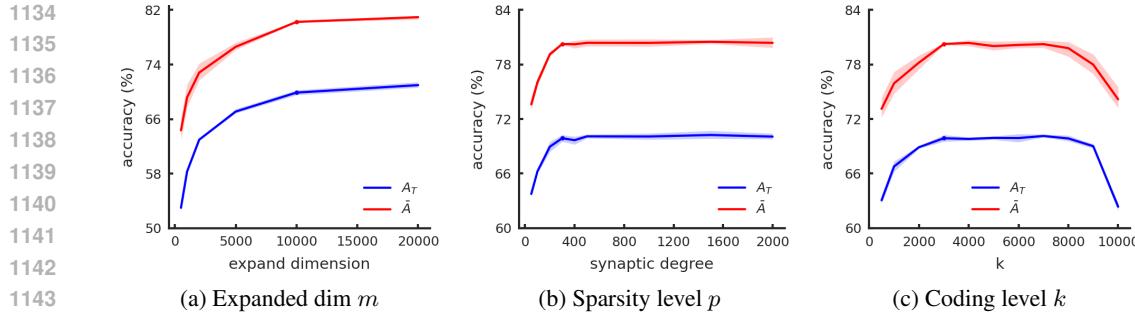


Figure 12: **Sensitivity Analysis for Expanded dim  $m$ , Weight Sparsity  $p$ , and Activation Sparsity  $k$  on CUB-200 with ResNet-50 as Backbone.** We report average accuracy in last task ( $A_T$ ) and overall accuracy ( $\bar{A}$ ). The dots denote the default values we use across experiments.

### C.8 ADDITIONAL EXPERIMENTS ON LARGER PRE-TRAINED MODELS

In our previous experiments, we primarily adopted ResNet-50 and ViT-B/16 as backbones for feature extraction. To further evaluate the scalability of Fly-CL on modern foundation models, we employ the vision encoder of Qwen2.5-VL-7B (Bai et al., 2025) to extract visual features. As shown in Table 11, increasing the scale of the pre-trained backbone consistently leads to improved performance. Furthermore, in Figure 13, we analyze the effect of expanding the projection dimension when using Qwen2.5-VL. The trend remains consistent with our earlier findings: performance improves steadily as the expanded dimension grows, but begins to saturate around 10,000.

Table 11: **Performance Comparison on Different Scale Pre-trained Models.** We report overall accuracy ( $\bar{A}$ ) across three benchmark datasets: CIFAR-100, CUB-200-2011, and VTAB.

Model	CIFAR-100	CUB-200-2011	VTAB
ResNet-50	$84.61 \pm 0.16$	$80.25 \pm 0.10$	$94.00 \pm 0.15$
ViT-B/16	$93.89 \pm 0.12$	$93.84 \pm 0.18$	$96.54 \pm 0.38$
Qwen2.5-VL-7B	$95.06 \pm 0.23$	$94.68 \pm 0.21$	$97.45 \pm 0.24$

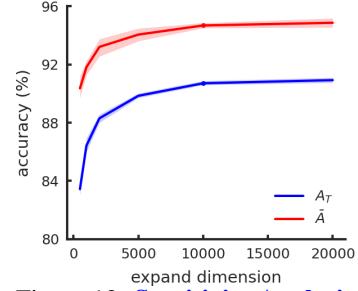


Figure 13: **Sensitivity Analysis for Expanded dim  $m$  on CUB-200 with Qwen2.5-VL-7B as Backbone.**

## D DETAILED DISCUSSION OF RELATED WORK

### D.1 COMPARISON WITH SEVERAL REPRESENTATION-BASED METHODS

We highlight the main advantages of our proposed Fly-CL over several related representation-based methods, including RanPAC (McDonnell et al., 2023), F-OAL (Zhuang et al., 2024), and RanDumb (Prabhu et al., 2024).

**Comparison with RanPAC.** RanPAC employs several Parameter-Efficient Transfer Learning (PETL) approach (Chen et al., 2022; Jia et al., 2022; Lian et al., 2022) to adapt the pre-trained model to the downstream domain in the first task, alongside a ridge classification with explicit cross-validation for all ridge candidates. Although effective, these two components make the entire pipeline computationally expensive (see Table 1, 2, and 5). In contrast, Fly-CL eliminates the need for PETL and significantly optimizes the ridge classification process. Additionally, we introduce a sparse projection layer with a top- $k$  operation, replacing the dense projection with ReLU, and analyze the impact of data normalization techniques. The speedup for each components can be referred to Table 4.

1188 **Comparison with F-OAL.** F-OAL is originally designed for online CL and shares similarities with  
 1189 Fly-CL in feature extraction, random projection, and decorrelation. Although it can also be adapted to  
 1190 the CIL setting with batched data, it has several flaws under this circumstance. For instance, F-OAL  
 1191 lacks the top- $k$  operation to filter noisy components after random projection, and its iterative analytic  
 1192 classifier may accumulate errors, leading to significant performance degradation on ResNet-50 (see  
 1193 Table 2). Moreover, while F-OAL is efficient on CUB-200-2011 and VTAB, its computational cost  
 1194 scales more rapidly with sample size compared to Fly-CL, making it less efficient on CIFAR-100  
 1195 (see Table 1, 2, and 5).

1196 **Comparison with RanDumb.** RanDumb shares a similar pipeline with F-OAL and is also designed  
 1197 for online CL. Like F-OAL, it does not utilize a top- $k$ -like operation, and its fixed penalty coefficient  
 1198  $\lambda$  may result in suboptimal performance. Crucially, RanDumb relies on StreamingLDA, which  
 1199 processes samples sequentially and cannot be parallelized for batch processing. This makes RanDumb  
 1200 significantly slower than all baselines evaluated in Table 1, 2, and 5.

## 1202 D.2 SUMMARY OF OTHER COMPARED BASELINES

1204 **L2P** (Wang et al., 2022b) utilizes a prompt pool  $\mathbf{P} = \{P_1, P_2, \dots, P_M\}$  where  $M$  is the size of the  
 1205 pool, to store task-specific knowledge. Each prompt  $P_i$  is associated with a learnable key  $K_i$  for  
 1206 key-value selection. By optimizing the cosine distance  $\gamma(p(x), k_i)$ , where  $p(x)$  is the feature selected  
 1207 by the query function during the training process, L2P can select the most appropriate prompt to  
 1208 provide information that is specific to the task.

1209 **DualPrompt** (Wang et al., 2022a) extends the key-value selection and optimization methods of L2P  
 1210 by further encoding different types of information into a task-invariant prompt  $g$  and a task-specific  
 1211 prompt  $e$ . This is shown to be more effective in encoding the learned knowledge. It also decouples  
 1212 the higher-level prompt space by attaching prompts to different layers, which is crucial for the model  
 1213 to reduce forgetting and achieve effective knowledge sharing.

1214 **EASE** (Zhou et al., 2024) first initializes and trains an adapter for each incoming task to encode  
 1215 task-specific information. It then extracts features of the current task and synthesizes prototypes of  
 1216 former classes to mitigate the subspace gaps between adapters. Finally, EASE constructs the full  
 1217 classifier and reweights the logits for prediction.

1218 **InfLoRA** (Liang & Li, 2024) is a Interference-Free Low-Rank Adaptation method for continual  
 1219 learning. It injects a small set of parameters to constrain weight updates to a specific subspace.  
 1220 Critically, this subspace is designed to be orthogonal to the gradients of all past tasks while containing  
 1221 the gradient subspace of the new task, thereby eliminating interference and achieving an effective  
 1222 balance between model stability and plasticity.

1223 **SEMA** (Wang et al., 2025) introduces a self-expansion mechanism by dynamically adding modular  
 1224 adapters only when significant distribution shifts are detected. Each adapter consists of a functional  
 1225 module and a representation descriptor, which acts as a novelty detector to determine whether existing  
 1226 adapters can handle the new task. SEMA further maintains an expandable mixture router to compose  
 1227 adapters through weighted combination, enabling flexible reuse of old modules while expanding  
 1228 only on demand. This design achieves a sub-linear parameter growth rate while improving the  
 1229 stability–plasticity balance across tasks.

1230 **MoE-Adapter** (Yu et al., 2024) enhances continual learning by injecting a mixture-of-experts adapter  
 1231 structure into transformer blocks. Each MoE-Adapter contains multiple expert adapters, and a learned  
 1232 routing network selects or mixes experts conditioned on the input. This architecture allows the model  
 1233 to capture diverse task-specific patterns while mitigating forgetting through expert specialization. By  
 1234 leveraging the MoE structure, the method improves representational flexibility and achieves stronger  
 1235 adaptation capacity compared to using a single shared adapter.

## 1237 D.3 RELATIONSHIP WITH KANERVA’S SPARSE DISTRIBUTED MEMORY

1239 Kanerva’s Sparse Distributed Memory (SDM) (Kanerva, 1988) is a classical high-dimensional  
 1240 computing framework in which memory addresses are distributed in a large binary space, and  
 1241 read/write operations are performed by activating only those locations within a neighborhood defined  
 by Hamming similarity. To highlight its conceptual connection to Fly-CL, we rewrite the SDM

1242 forward computation using the same notation as Fly-CL:

$$1243 \quad \hat{y} = \mathbf{C}^\top (f(\mathbf{W} \mathbf{v})), \quad (19)$$

1244 where  $\mathbf{W}$  is an address transformation matrix,  $\mathbf{C}$  is a content transformation matrix, and  $f(\cdot)$  denotes  
1245 a binary activation function.

1246 Under this formulation, SDM and Fly-CL share two high-level principles: (i) expansion into a high-  
1247 dimensional representational space, which promotes separability and reduces interference; and (ii)  
1248 sparse activation, which suppresses noises and enhances discrimination through selective addressing.

1249 Despite these conceptual parallels, Fly-CL differs from SDM in several important ways. First,  $\mathbf{W}$  in  
1250 SDM is updated through iterative read/write operations, whereas Fly-CL employs a fixed, randomly  
1251 initialized projection matrix motivated by the fly olfactory circuit. Second, SDM does not incorporate  
1252 any decorrelation mechanism analogous to the Fly-CL ridge-based KC-to-MBON transformation.  
1253 Third, SDM operates in a binary space: the activation function  $f(\cdot)$  produces binary addresses and  
1254 similarity is evaluated via Hamming distance, while Fly-CL performs real-valued projections and  
1255 uses cosine similarity for downstream matching and classification.

1256 These distinctions illustrate that although Fly-CL and SDM share the broad philosophy of high-  
1257 dimensional sparse representations, their architectural assumptions, objectives, and operating regimes  
1258 differ substantially.

## 1261 E TRAINING DETAILS

### 1262 E.1 PRE-TRAINED MODELS

1263 We use pre-trained ViT-B/16 and ResNet-50 models in our experiments. The ViT-B/16 checkpoint  
1264 we used was first pretrained on ImageNet-21K and then fine-tuned on the ImageNet-1K dataset. All  
1265 of which are loaded using the timm library. We list the dimensions of the extracted features and the  
1266 download links for the checkpoints of each model in Table 12.

1267 **Table 12: Information Related to the Pre-trained Models We Used in This Work.** We list the  
1268 dimensions of the extracted features and provide corresponding download links for these pre-trained  
1269 models.

1270 Model	1271 feature dimension	1272 Link
1273 ViT-B/16	1274 768	<a href="#">1275 Link</a>
1276 ResNet-50	1277 2048	<a href="#">1278 Link</a>

### 1279 E.2 DATASETS

1280 We evaluate our method on three benchmark datasets for CL tasks. Detailed information about these  
1281 datasets, including download links, is provided in Table 13. For the experiments summarized in Tables  
1282 1, 2, and 3, we configure the number of training tasks as  $T = 10$  for CIFAR-100 and CUB-200-2011,  
1283 with 10 and 20 classes per task, respectively. For VTAB, we set  $T = 5$  with 10 classes per task. In  
1284 the longer task sequence experiments (Table 5), we double the task sequence length: for CIFAR-100  
1285 and CUB-200-2011, we set  $T = 20$  with 5 and 10 classes per task, respectively, while for VTAB, we  
1286 set  $T = 10$  with 5 classes per task. For experiments in Table 6, we set  $T = 10$  with 20 classes per  
1287 task.

### 1288 E.3 EXPERIMENT SETUP

1289 We reproduce the baseline results for L2P, DualPrompt, EASE, and RanPAC using the code provided  
1290 by PILOT (Sun et al., 2023), ensuring that the learning parameters for each baseline align with  
1291 the description in their original papers. For F-OAL, we adopt their official implementation for  
1292 reproduction.

1293 In our proposed Fly-CL, we set the expanded dimension  $m$  to 10,000,  $p$  to 300, and  $k$  to 3,000 across  
1294 all experiments. For ViT-B/16, we apply standard data normalization, scaling each pixel value to the

1296 Table 13: **Details of CIFAR-100, CUB-200-2011, VTAB Datasets.** We list the number of training,  
 1297 validation samples and classes for the following datasets, along with the download links.  
 1298

1299	Dataset	Training Samples	Validation Samples	Classes	Download Link
1300	CIFAR-100 (Krizhevsky et al., 2009)	50000	10000	100	<a href="#">Link</a>
1301	CUB-200-2011 (Wah et al., 2011)	9430	2358	200	<a href="#">Link</a>
1302	VTAB (Zhai et al., 2019)	1796	8619	50	<a href="#">Link</a>
1303	Imagenet-R Hendrycks et al. (2021a)	24000	6000	200	<a href="#">Link</a>
1304	Imagenet-A Hendrycks et al. (2021b)	5981	1519	200	<a href="#">Link</a>

1305  
 1306 range  $[-1, 1]$ . For ResNet-50, we normalize the input images using ImageNet statistics. Given the  
 1307 prior knowledge of high multicollinearity in this task, we explore the penalty coefficient range starting  
 1308 from larger values, specifically from  $10^6$  to  $10^9$  on a log scale for ViT-B/16 and  $10^4$  to  $10^9$  for ResNet-  
 1309 50. Since prompt-based methods and PETL techniques are limited to transformer-based architectures,  
 1310 we compare Fly-CL only with RanPAC and F-OAL in the ResNet-50 setting. For RanPAC, we remove  
 1311 PETL and incorporate data normalization following their original implementation (McDonnell et al.,  
 1312 2023) for ResNet-50. All experiments are conducted using five different random seeds, and we report  
 1313 the mean  $\pm$  standard deviation.

#### 1315 E.4 ENVIRONMENTS

1317 All experiments were conducted on a Linux server running Ubuntu 20.04.4 LTS, equipped with an  
 1318 Intel(R) Xeon(R) Platinum 8358P CPU at 2.60GHz and 8 NVIDIA GeForce RTX 3090 GPUs, using  
 1319 CUDA version 11.7. For model loading, we employed the timm library (version 0.9.16).

#### 1321 F LIMITATIONS AND FUTURE WORK

1323 Our proposed Fly-CL is theoretically applicable to various scenarios requiring feature separation. Its  
 1324 lightweight design further suggests potential utility in a wide range of Continual Learning and Metric  
 1325 Learning tasks.

1326 Recent neuroscience research (Dasgupta et al., 2017) indicates that the random projection layer in the  
 1327 fly olfactory circuit may not be entirely random. Biological experiments also suggest the presence of  
 1328 certain constraints within this projection layer. Inspired by these findings, a promising direction for  
 1329 future research is to explore structuring the projection layer as an entity with learnable parameters,  
 1330 potentially enhancing its adaptability and performance.

#### 1332 G BROADER IMPACT

1334 Our work provides a new perspective for enhancing the efficiency of CL using pre-trained models,  
 1335 which is crucial for real-world deployment, especially with increasingly large modern models. Fly-CL  
 1336 can help AI researchers and developers create more efficient CL algorithms.

1338 On the other hand, the efficiency improvements in CL could potentially accelerate the development  
 1339 of AI systems that rapidly adapt to new domains without proper safeguards. This might lead to: (1)  
 1340 amplified propagation of biases present in sequential datasets, (2) reduced transparency as models  
 1341 continuously evolve beyond their initial training, and (3) potential misuse for generating tailored  
 1342 content at scale. We recommend implementing rigorous monitoring frameworks to track model  
 1343 behavior across learning phases.

#### 1345 H LLM USAGE DECLARATION

1347 During the preparation of this manuscript, a large language model was employed exclusively for  
 1348 language refinement. Its role was limited to rephrasing certain passages and enhancing the overall  
 1349 clarity and readability of the text. All conceptual contributions, theoretical derivations, experimental  
 design, and analysis were independently developed and verified by the authors. The LLM was not

1350 involved in generating research ideas, shaping methodologies, or producing novel scientific content.  
1351 The authors bear full responsibility for the entirety of the paper.  
1352

1353

1354

1355

1356

1357

1358

1359

1360

1361

1362

1363

1364

1365

1366

1367

1368

1369

1370

1371

1372

1373

1374

1375

1376

1377

1378

1379

1380

1381

1382

1383

1384

1385

1386

1387

1388

1389

1390

1391

1392

1393

1394

1395

1396

1397

1398

1399

1400

1401

1402

1403