

# Thinking before You Speak: A Proactive Test-time Scaling Approach

Anonymous ACL submission

## Abstract

Large Language Models (LLMs) often exhibit deficiencies with complex reasoning tasks, such as maths, which we attribute to the discrepancy between human reasoning patterns and those presented in the LLMs’ training data. When dealing with complex problems, humans tend to think carefully before expressing the solutions. However, they often do not articulate their inner thoughts that involve their intentions, chosen methodologies, etc. Consequently, in training data collected from human sources, critical insights essential for bridging reasoning steps may be absent. To bridge this gap, we propose inserting *insights* between consecutive reasoning steps, which review the status and initiate the next reasoning steps. Unlike prior prompting strategies that rely on a single or a workflow of static prompts to facilitate reasoning, *insights* are *proactively* generated to guide reasoning processes. We implement our idea as a reasoning framework, named *Thinking Before You Speak* (TBYS), and design a pipeline for automatically collecting and filtering in-context examples for the generation of *insights*, which alleviates human labeling efforts and fine-tuning overheads. Experiments on challenging mathematical datasets verify the effectiveness of TBYS. *Source code attached will be released upon publication.*

## 1 Introduction

OpenAI’s O1 (OpenAI, 2024) demonstrates the potential of leveraging long chains of thought (CoT) (Wei et al., 2022) to enhance the reasoning capabilities of large language models (LLMs). Through its generated reasoning, O1 exhibits advanced cognitive skills, such as problem decomposition, error identification, and correction – processes that continuously guide thinking toward accurate solutions. Inspired by this, various test-time scaling (Snell et al., 2024; Zhang et al., 2025) approaches were proposed, such as using prompts

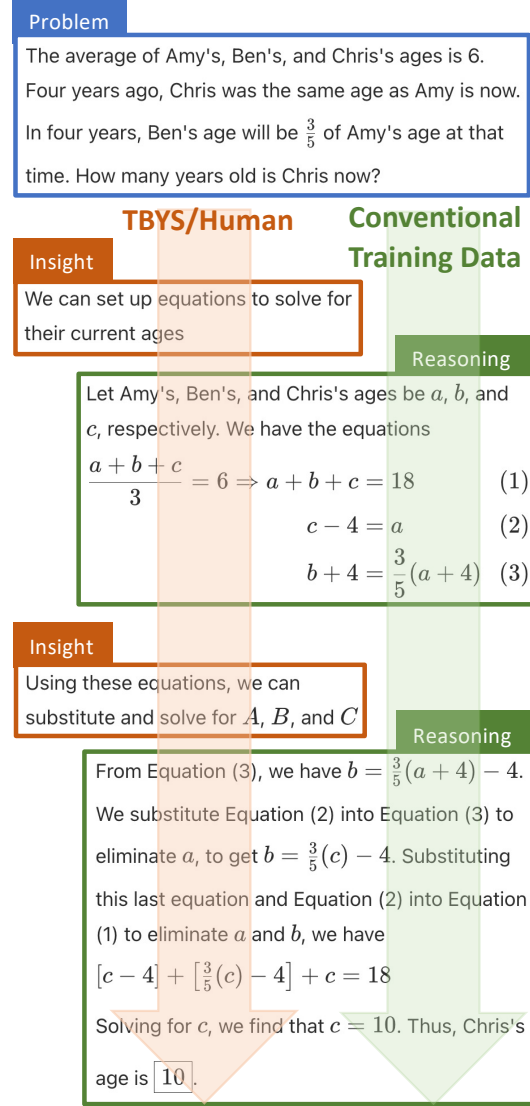


Figure 1: A simplified example to compare the reasoning trace of human (and our TBYS) against one from conventional training data, where human and TBYS excel with a flow of insight-driven reasoning that is more comprehensible. On the other hand, the training set example adds to the difficulty of learning, since it is not always simple to re-engineer the connection between the consecutive steps behind the succinct reasoning logic. TBYS proactively fill reasoning gaps with *insights* representing intention, explanation, or justification, etc.

like “Wait,” (Muennighoff et al., 2025) to stimulate self-correction, “Wait, using Python” to encourage coding (Li et al., 2025a), or fixed workflows of prompts to structure inferences (Hong et al., 2024). However, these methods suffer from task and LLM sensitivity: they rely heavily on specific problem structures and serendipity to succeed. As a result, they are most effective when paired with reinforcement learning techniques (e.g., rejection sampling) to filter suboptimal cases, but are ill-suited for direct application to scale reasoning at test time.

This paper introduces a novel prompting paradigm called **proactive prompting**, where an LLM proactively generates prompts to steer its own reasoning steps, rather than passively reacting to predefined prompting patterns. This approach demonstrates particular advantages in complex reasoning tasks – such as advanced mathematics problems – where the proactive generation of “inner thoughts” (critical for guiding reasoning) is often absent from final reasoning outputs in conventional training data.

To validate this paradigm, we develop a reasoning framework named *Thinking Before You Speak* (TBYS), which iteratively inserts a proactive prompt – termed the *insight* – before each reasoning step to explicitly define the status and the goal of that step. Figure 1 contrasts a TBYS reasoning process with that in conventional training data (with which LLMs are trained). TBYS mirrors human inner-thinking patterns, producing more explainable reasoning traces that facilitate LLM learning and offering greater educational values for human readers.

In the remainder of this paper, we detail the TBYS reasoning framework in Section 2. Since TBYS relies on iteratively generating insights to guide reasoning, the quality of these generated insights is critical to its accuracy. To address this, we employ in-context learning with examples retrieved from a library of insight exemplars. Section 3 describes our pipeline for automatically collecting, filtering, and selecting example *insights* for this library. Section 4 briefly reviews prior related work. Finally, Section 5 evaluates TBYS against strong baselines on challenging datasets, demonstrating significant performance improvements and better accuracy-overhead trade-offs. We further conduct ablation studies to validate the contributions of key components.

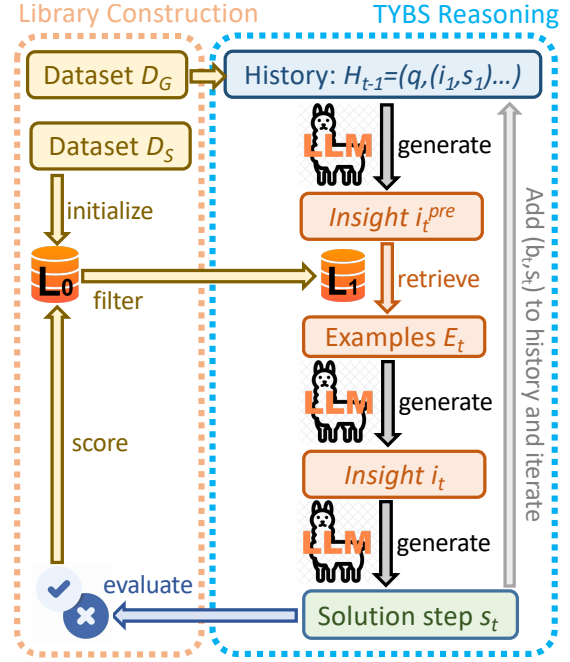


Figure 2: The TBYS reasoning framework (Section 2) and *insight* library construction (Section-3).

## 2 The TBYS Reasoning Framework

TBYS utilizes a library  $L$  of high-quality *insights*. The automatic construction of this library is detailed in Section 3. During inference, examples are retrieved from  $L$  for in-context learning. We also manually define three seed examples  $S$ , each containing a question and the complete reasoning steps for the question with the associated *insights*.

As shown in Figure 2, TBYS employs a multi-round reasoning approach. Each round  $t$  consists of three steps: (1) *Insight Generation*: A preliminary *insight*  $i_t^{pre}$  is generated based on the current reasoning history  $H_{t-1} = (q, (i_1, s_1), (i_2, s_2), \dots, (i_{t-1}, s_{t-1}))$ , where  $q$  is the question, and  $i_i, s_i$  denote the *insight* and solution step in round  $t$ , respectively. (2) *Example Retrieval*: Each *insight* is defined by its two components: *situation* (summarizing the current reasoning status) and *goal* (stating the intention for solution step  $s_t$ ). The situation of  $i_t^{pre}$  is used to retrieve  $k_E = 8$  examples  $E_t$  from library  $L$ . Using these  $k_E$  high-quality *insights* as in-context examples, a refined *insight*  $i_t$  is generated. (3) *Solution Step Generation*: The solution step  $s_t$  is generated using  $H_{t-1}$  and  $i_t$ , then appended to  $H_{t-1}$  to form  $H_t$ . To signal the end of reasoning,  $s_t$  includes a field indicating whether a confident answer to  $q$  has been reached.

### 3 Construction of the *Insight* Library

As shown in Figure 2, we build the library of *insights* in two stages: initialization and filtering.

**Initialization:** We use manually curated seed examples  $S$  and a dataset  $D_S$  containing questions and their chain-of-thought solutions. First, an LLM is prompted to split each solution in  $D_S$  into 1–3 steps. The LLM is then prompted again to generate an *insight*  $i_t$  for each solution step  $s_t$ , consisting of a *situation*, which should represent the reasoning status up to that step, and a *goal*, which should offer a purpose and a guideline to stimulate the LLM to reproduce solution step  $s_t$ . All *insights* and divided solution steps are collected into an initial library  $L_0$ .

**Filtering:** To identify high-quality *insights*, we use a dataset  $D_G$  (containing questions and ground-truth answers) and a scoring mechanism: (1) For each *insight*  $i_i \in L_0$ , maintain counters  $r_i$  (correct uses) and  $w_i$  (wrong uses). (2) Evaluate  $L_0$  by running *TBYS* on each question  $q \in D_G$ . For each reasoning step for  $q$ , retrieve  $k_F = 25$  examples from  $L_0$  and randomly select one as a 1-shot example. If the reasoning yields a correct answer, increment  $r_i$  for each  $i_i$  used; otherwise, increment  $w_i$ . (3) Rank *insights* in  $L_0$  by the score  $\frac{r_i}{r_i + w_i} \log(r_i + w_i)$ , which balances accuracy and usage coverage. Select the top- $k_L$  examples to form  $L_1$ . This process can be iterated (e.g., using  $L_1$  and new data from  $D_G$  to create  $L_2$ ) to progressively improve the library.

In our experiments, the MATH-500 dataset (Lightman et al., 2023) serves as  $D_S$  and the test-set, e.g. MATH-500 or AIME (Zhang et al., 2023a), serves as  $D_G$  in a test-time adaptation (Jang et al., 2023) manner, with  $k_L$  as a variable parameter.

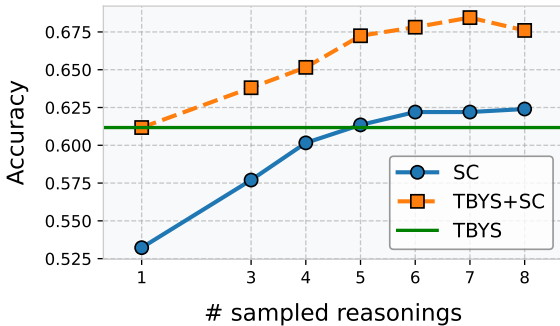


Figure 3: Performance comparison on MATH-500

### 4 Related Work

Extensive research has investigated prompt designs to improve LLM reasoning, including *Chain-of-Thought* (Wei et al., 2022), *Least-to-Most* (Zhou et al., 2023), *Self-Consistency* (Wang et al., 2023b), and *Tree-of-Thoughts* (Cao et al., 2023). Methods to enhance task-specific performance include question rephrasing, subtask decomposition, verification, and symbolic grounding (Lyu et al., 2023; Xu et al., 2024; Wang et al., 2023a; Zelikman et al., 2022; Wang et al., 2024); factuality and faithfulness checking for reasoning chains (Wang et al., 2024); and separating knowledge retrieval from reasoning (Jin et al., 2024).

Iterative prompting techniques rely on pre-defined, hardcoded actions to guide reasoning, such as *Self-Refine* (Madaan et al., 2023), *IRCoT* (Trivedi et al., 2023), *iCAP* (Wang et al., 2022), *MetaGPT* (Hong et al., 2024), and *Chain of Ideas* (Anonymous, 2024b).

Memory-based methods include *Buffer of Thoughts* (Yang et al., 2024c) distills high-level guidelines from previously solved tasks and stores them in a buffer for future reuse, while Skill-based CoT (Didolkar et al., 2024) first predicts skill-based labels for the questions. (Zhang et al., 2023b) identifies key concepts in questions and uses inductive prompting templates to extract related concepts.

*rStar* (Qi et al., 2024) employs a self-play mutual reasoning approach – augmented by Monte Carlo Tree Search (MCTS) with a set of five reasoning-inducing prompts – to enhance reasoning.

Finetuning-based methods, such as *STaR* (Zelikman et al., 2022), *ReST-MCTS* (Zhang et al., 2024), and *AFlow* (Anonymous, 2024a), demonstrate that iterative training on reasoning histories and task-specific workflows of correct answers enables models to tackle increasingly complex problems.

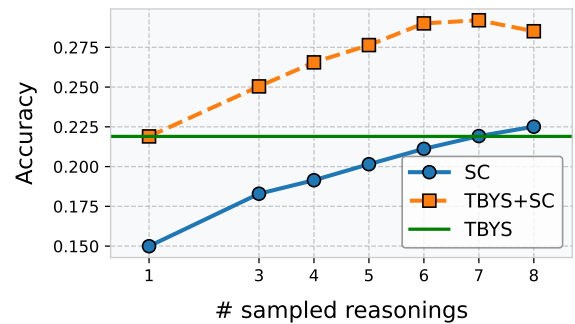


Figure 4: Performance comparison on AIME

## 5 Experiments

### 5.1 Experiment settings

We conducted experiments on two challenging mathematical datasets, *AIME* (Zhang et al., 2023a) and *MATH-500* (Lightman et al., 2023). We compare TBYS against a simple yet very strong baseline: 8-shot *In-context Learning* (Lu et al., 2022) with *Self-Consistency* (Wang et al., 2023b).

For the experiments, we utilize the LLM *Qwen2.5-7B-Instruct* (Yang et al., 2024a) via LLM API provided by Siliconflow (sil), with the following configurations: max\_tokens=1024, temperature=0.2, top\_k=40, top\_p=0.7, and n=1. The *bge-large-en-v1.5* embedding model is employed for *insight* retrieval. Results are reported as the average across 8 experimental runs.

Since coding benefits mathematical problems (Chen et al., 2023), when Python code blocks are detected in the LLMs’ responses, we invoke a customized sandboxed Python interpreter and append the output to the code block.

### 5.2 Comparison

When compared with *Self-Consistency* (SC), TBYS demonstrates comparable performance to SC using 5 reasoning samples (SC@5) on *MATH-500* (Figure 3) and SC@7 on *AIME* (Figure 4). The results further indicate that TBYS integrates effectively with SC: TBYS+SC yields over 5% absolute gains in accuracy on *MATH-500* and 7.5% on *AIME*.

### 5.3 Overhead Analysis

Table 1: Cost comparison to SC under similar accuracy

MATH-500	Acc.	Time	Prompt	Completion
TBYS	0.61	52.82	18163.80	999.57
SC@5	0.61	102.56	13334.62	2217.30
AIME	Acc.	Time	Prompt	Completion
TBYS	0.22	78.15	20686.23	1559.60
SC@7	0.22	322.79	25,242.54	7,102.49

We compare the overhead of TBYS with SC@5 on *MATH-500* and with SC@7 on *AIME*, where the methods achieve comparable accuracies. The metrics analyzed include wall-time, number of prompt tokens, and completion tokens. As shown in Table 1, under similar accuracies, TBYS reduces wall-time and number of completion tokens by approximately  $\frac{1}{2}$  on *MATH-500* and  $\frac{1}{3}$  on *AIME*. While TBYS uses 46% more prompt tokens on *MATH-500*, these can be cached and typically much cheaper than completion tokens.

### 5.4 Ablation Study

Table 2: Ablation Study

	MATH-500	AIME
TBYS	61.17%	21.90%
- Library Construction	58.90%	19.51%
- Coding	57.00%	18.11%
8-shot	53.23%	14.99%

We conducted ablation experiments by using the raw insight library  $L_0$  as  $L_1$  (without filtering, as described in Section 3). Accuracy declines were observed in both datasets. Notably, we only performed one round of insight filtering (i.e., using  $L = L_1$ ), and additional filtering rounds are expected to further improve accuracy. Table 2 also demonstrates that coding contributes half of the accuracy gain compared to simple 8-shot prompting.

### 5.5 Impact of Library Size

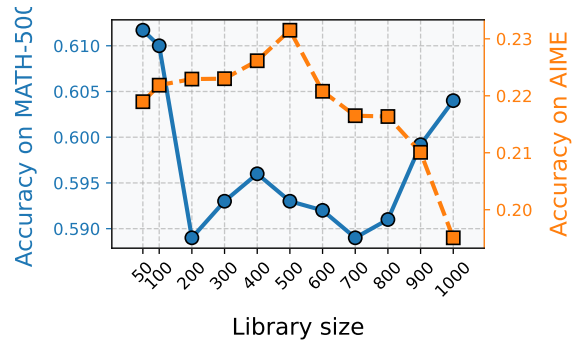


Figure 5: Impact of *insight* library size

In Section 3, we sorted the *insight* library  $L_0$  and selected the top- $k_L$  insights to form  $L_1$ . Figure 5 shows that on *MATH-500*, TBYS achieves peak accuracy with an insight library size of 50, while on *AIME*, the optimal size is 500. This likely arises because *AIME* contains more diverse problem types.

## 6 Conclusion and Future Work

This paper introduces a novel proactive prompting paradigm, instantiates it with the simple TBYS reasoning framework, and verifies the effectiveness of TBYS on challenging advanced mathematics reasoning tasks.

Promising directions for future improvement include: Automated search for optimal *insights* (Yang et al., 2024b); Integration of long-term memory mechanisms (Tang et al.; Anonymous, 2025); Enhancement of programming capabilities (Chen et al., 2023); Enforcement of structured inference processes (Li et al., 2025b; Cao et al., 2023).



## Limitations

Our method incurs higher computational overhead compared to direct prompting, a common drawback among advanced prompting techniques that involve scaling test-time inference.

Due to time and financial constraints (our current experiments take about 50 days with single threaded API calls), we only evaluated the proposed method on two math-domain datasets using a single LLM.

## Ethical Statement

This work fully adheres to the ACL Ethics Policy. To the best of our knowledge, no ethical issues are associated with this research.

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