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## RealMath: A Continuous Benchmark for Evaluating Language Models on Research-Level Mathematics

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### Abstract

Existing benchmarks for evaluating mathematical reasoning in large language models (LLMs) rely primarily on competition problems, formal proofs, or artificially challenging questionsfailing to capture the nature of mathematics encountered in actual research environments. We introduce REALMATH, a novel benchmark derived directly from research papers and mathematical forums that assesses LLMs' abilities on authentic mathematical tasks. Our approach addresses three critical challenges: sourcing diverse research-level content, enabling reliable automated evaluation through verifiable statements, and designing a continually refreshable dataset to mitigate contamination risks. Experimental results across multiple LLMs reveal surprising capabilities in handling research mathematics compared to competition problems, suggesting current models may already serve as valuable assistants for working mathematicians despite limitations on highly challenging problems.

## 1. Introduction

The mathematical capabilities of Large Language Models (LLMs) have become a critical lens for assessing their reasoning and knowledge retention abilities. Although considerable effort has been invested in evaluating LLMs in basic 041 mathematics (Cobbe et al., 2021; Hendrycks et al., 2021), 042 competition-level mathematics (Gao et al., 2024; He et al., 043 2024; Sawada et al., 2024; Sun et al., 2025; Tsoukalas et al., 2024) and formal proof generation (Yang and Deng, 2019; 045 Yang et al., 2023; Zheng et al., 2022), these evaluations 046 may not adequately reflect the potential utility of LLMs in 047 real-world mathematical research contexts.

Current mathematical benchmarks fall predominantly into three categories: (1) those derived from course materials (Cobbe et al., 2021; Hendrycks et al., 2021) or competitive examinations (Gao et al., 2024) (e.g., IMO or AIME) that offer abundant problems with solutions; (2) those centered on formal theorem proving where verification can be automated (Yang et al., 2023; Zheng et al., 2022); (3) those designed by mathematical experts to be as challenging as possible (Glazer et al., 2024; Phan et al., 2025). However, these benchmarks capture only a narrow slice of mathematical practice. The mathematics encountered "in the wild" particularly in research settings—differs substantially from competition problems in structure and topics, rarely relies on formal proofs, and considers statements and results that are not (exclusively) designed to be maximally challenging.

This disconnect raises a fundamental question:

# How effective might LLMs be as assistants for practicing mathematicians today?

To address this question, we introduce a novel benchmark designed to evaluate LLMs on research-level mathematics extracted directly from the literature. Constructing such a benchmark presents three significant challenges:

First, we need to *source authentic content* that faithfully represents the diversity and complexity of contemporary mathematical research.

Second, we require a reliable method for assessing correctness. Unlike competition problems with standardized solutions or problems with formal proofs, research mathematics presents evaluation difficulties: human expert validation is resource-intensive and limits scalability, while using LLMs as judges introduces reliability concerns.

Third, we must address *test set contamination over time*, as mathematical content incorporated in benchmarks might be absorbed into training datasets for future models. In this paper, we present REALMATH, a math benchmark that addresses these challenges through:

1. A data pipeline that extracts verifiable mathematical statements from research papers (e.g., arXiv) and mathematical forums (e.g., Stack Exchange), creating a rich

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Figure 1. LLM performance on a hard subset of REALMATH from arXiv Mathematics papers.

corpus of research-level content.

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- 2. An evaluation methodology focused on verifiable answers rather than proof assessment, allowing for automated correctness checking.
- 3. A continually refreshable dataset design that leverages the vast and growing body of mathematical literature, allowing for regular updates with new content to mitigate contamination concerns.

We systematically transform mathematical statements into 086 question-answer pairs, preserving surrounding context nec-087 essary for comprehension. For example, a theorem stating 088 that "For  $n \ge 1$ , the number of shallow, 123-avoiding cen-089 trosymmetric permutations of length n is  $\frac{n^2}{4} + 1$  when n is even" (Archer et al., 2025) becomes a question, "What 090 091 is the number of 123-avoiding centrosymmetric permuta-092 tions of length n when n is even?", with a verifiable an-093 swer, " $\frac{n^2}{4}$  + 1", with relevant definitions and notation from 094 the source material given as context. 095

From approximately 9,000 mathematics-related academic
papers collected over nine months, our automated pipeline
curated 633 high-quality samples and can generate over 70
new samples each month.

Our evaluation of frontier LLMs reveals interesting patterns in mathematical capabilities that differ from those observed in other recent benchmarks. In particular, models demonstrate stronger performance on our researchmathematics benchmark (see Figure 1) than on deliberately challenging datasets such as math competitions or FrontierMath (Glazer et al., 2024), suggesting that LLMs may already provide valuable assistance in research contexts, even though they cannot solve most advanced problems. Although our approach focuses on statement verification rather than proof generation or verification, it nonetheless provides a valuable signal about LLMs' potential as mathematical assistants. Our findings suggest that current models may already serve as useful tools in mathematical research contexts, even as they continue to struggle with the most challenging mathematical problems.

To summarize, this work makes several contributions: it establishes a new paradigm for evaluating the mathematical capabilities of LLMs using organic research Mathematics; it provides a sustainable methodology for benchmark creation that resists contamination; and it offers insights into the relative strengths of current models on tasks representative of real mathematical practice.

### 2. Related Work

Math benchmarks sourced from exams and competitions. Recent years have seen rapid progress in evaluating LLMs on mathematical tasks. Early data sets were sourced from school materials or entry-level math competitions such as GSM8K (Cobbe et al., 2021), GHOSTS (Frieder et al., 2023), or MATH (Hendrycks et al., 2021), and are approaching saturation. In response, new benchmarks now source questions from advanced math competitions such as the IMO (Balunović et al., 2025; Petrov et al., 2025; Sun et al., 2025), AIME, or the Putnam competition (Tsoukalas et al., 2024).

Math benchmarks sourced from research experts. A growing line of work seeks to evaluate mathematical reasoning on research-level questions. FrontierMath (Glazer et al., 2024) (and part of HLE (Phan et al., 2025)) evaluate LLMs on extremely challenging problems crafted by



*Figure 2.* The data collection pipeline for arXiv papers. The core step is to ensure that each extracted theorem from arXiv papers has a single, exact answer. To maintain data quality, we apply filtering mechanisms, e.g., prompting an LLM to discard trivial samples that can be easily solved.

121 expert mathematicians. These benchmarks focus on a nar-122 row slice of current mathematical practice (at the frontier of 123 human expertise), and are extremely labor-intensive to cu-124 rate. To guard against test set contamination, FrontierMath 125 keeps the test set private, which introduces barriers to reproducibility. In contrast, REALMATH benchmark consists 127 of questions spanning the full range of mathematical re-128 search practice, and can be automatically refreshed as new 129 research is published. 130

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131 Benchmarks for theorem proving. Orthogonal to our 132 work are benchmarks focused on formal proof genera-133 tion and machine-verifiable mathematics, such as Lean-134 Dojo (Yang et al., 2023) and MiniF2F (Zheng et al., 2022). 135 These evaluate LLMs' ability to produce formal proofs 136 within interactive theorem provers, supporting automated 137 verification of correctness. While such work is critical for 138 advancing formal methods and proof automation, it is not 139 representative of most mathematical research that is not 140 fully formalized. 141

Benchmarks for LLM capabilities "in the wild". Our 142 work complements a growing body of research that aims 143 to evaluate the capabilities of LLMs on real-world tasks 144 rather than well-curated proxies. For example, instead of 145 evaluating LLMs' coding abilities on programming com-146 petitions (Huang et al., 2023), projects such as SWE-147 bench (Jimenez et al., 2023), Lancer (Miserendino et al., 148 2025), or BaxBench (Vero et al., 2025) focus on assessing 149 LLMs in real-world software engineering tasks. A similar 150 shift can be observed in the evaluation of cyber-offensive 151 capabilities of LLMs, where a number of recent bench-152 marks focus on evaluating real exploit capabilities (Carlini 153 et al., 2025; Fang et al., 2024) rather than the ability to solve 154 curated capture-the-flag competitions (Shao et al., 2024). 155

### 3. Research-Level Mathematical Benchmark

### 3.1. Design Criteria

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To ensure high quality of our collected data, we establish the following design desiderata:

1. Real-world application focus: Our collection

methodology prioritizes mathematical problems that genuinely represent those encountered in practical scenarios. We source problems from academic research publications (e.g., arXiv preprints), where researchers tackle mathematical challenges to advance scientific knowledge, and from educational platforms (e.g., Mathematics Stack Exchange), where learners engage with mathematical concepts to develop their understanding and problem-solving capabilities. Unlike competition-oriented benchmarks, we focus on representative mathematical tasks encountered in actual practice, emphasizing procedural and techniquedriven aspects rather than contest-style ingenuity.

- 2. Automated verification: We generate *constructive* problems (as in (Balunović et al., 2025)) with clear, unambiguous verification criteria, typically problems with a single, exact numerical or symbolic answer. This approach excludes problems that admit multiple solutions or involve qualitative assessments such as inequalities (e.g., lower bounds, upper bounds) or asymptotic relations, which would complicate the verification process and potentially introduce ambiguity. This choice also means that we omit statements where the main difficulty lies in finding a (non-constructive) proof (e.g., if a paper has a theorem that says "P  $\neq$  NP", then the interesting part is solely the proof and not the construction of the statement itself).
- 3. **Continuous acquisition**: Leading LLMs may be trained on internet data, so it is important to avoid data contamination. To ensure this, we collect data exclusively from publicly available sources such as arXiv and StackExchange, where automatic collection is feasible with minimal human intervention. After each new model release, we can automatically gather fresh data from the internet to evaluate the model, ensuring that the evaluation set remains uncontaminated and up-to-date.

### 3.2. An Automatic and Refreshable Pipeline

We present a detailed walkthrough of our data collection pipeline in Figure 2. Throughout the remainder of this paper, we primarily use data from arXiv papers to illustrate
our methodology. A more detailed discussion of the Stack
Exchange dataset is provided in Appendix A.5. As a concrete example, we use a subset of 4,000 arXiv papers selected from May to September 2022 to illustrate each stage
of the data collection pipeline.

Step	# Usable Samples
Retrieve papers	4,000 papers
Extract LaTeX source	3,922 papers
Extract theorems	14,747 theorems
Identify constructive theorems	407 theorems
Generate question-answer (QA) pairs	401 QA pairs
Filter trivial questions	280 QA pairs

*Table 1.* An example of the pipeline steps and the number of us-able samples at each stage.

183 Our pipeline consists of five stages:

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- Retrieve papers. We collect all mathematics-related arXiv papers within a specified time window. This process involves querying the arXiv API for papers in categories such as Mathematics or Computer Science. To illustrate, we can acquire roughly 4,000 papers from Mathematics over 5 months, from May to September 2022.
- 2. Extract LaTeX source. For each paper, we down-193 load and parse the original LaTeX source files to accurately preserve the mathematical notation. Due to 195 occasional errors in the arXiv API responses or issues 196 with file availability (e.g., missing or corrupted LaTeX 197 sources), the number of successfully processed papers 198 is often lower than the total initially retrieved. In our 199 case, out of 4,000 papers, we obtained 3,922 usable 200 samples.
- 3. Identifying constructive theorems with fixed answers. We use an LLM<sup>1</sup> to identify theorems that in-204 volve the construction of a single exact answer. The LLM analyzes each extracted theorem and classifies 206 it based on whether it presents a clear mathematical statement with an unambiguous result. We specif-208 ically exclude theorems that involve inequalities or 209 those that admit multiple solutions, e.g., theorems of 210 the type "If condition A is satisfied, then relation B 211 happens" but where condition A is not the unique one 212 for B to happen, and *relation* B is not an equality or 213 fixed-answer numerical relation. From the 3,922 pro-214 cessed papers, we extracted 407 theorems that were 215 classified as high-quality by our judge model, out of 216 14,747 theorems detected in total.

- **4. Generate question-answer (QA) pairs.** For each selected theorem, we use an LLM to convert it into a question–answer pair. LLMs will always attempt to generate a QA pair, even if the generated question–answer is easy to answer. For 407 theorems, we obtained 401 QA pairs.
- **5. Filtering trivial questions.** We also implement a post-processing stage where an LLM reviews each generated QA pair. This step filters out low-quality samples, e.g., those with easily guessable answers, or where the answer is obvious from the context fed to the LLM (it is common, for instance, for the paper's introduction to state a theorem's result informally). Finally, we obtained 280 QA pairs that are ready for use in the evaluation phase.

For each test sample, we provide the *context* (i.e., all relevant text preceding the theorem) along with an LLM-generated *question* to frontier LLMs. A response is considered correct only if it exactly matches the ground truth *answer*. We present some detailed examples in Figure 3.

This pipeline ensures that our benchmark remains current, uncontaminated, and representative of real mathematical challenges faced in research. By automating the collection process, we can continuously refresh the dataset with new mathematical problems as they emerge in the research community.

**Discussion on label noise.** The two primary sources of noise in our data pipeline are: (1) the inherent quality of the source materials; and (2) the reliability of using LLMs to evaluate and transform mathematical content. Samples from Mathematics Stack Exchange, in particular, tend to be of lower quality: Questions are often poorly formulated, contain mathematical inaccuracies, or lack answers on the forum. As a result, obtaining high-quality samples from this source requires filtering through a much larger volume of user-submitted content. Additionally, our pipeline relies on LLMs to assess the quality of extracted theorems and generate QA pairs. Thus, any limitations or biases in the LLM's evaluation capabilities may introduce further inaccuracies into the dataset.

Overall, our pipeline yields more than 94% high-quality samples from arXiv papers, without relying on any human annotation. For example, in the Mathematics category, we processed more than 9,000 papers and extracted 633 theorems that were classified as high-quality by our judge model. We then manually reviewed each sample and found that approximately 6% did not meet our quality criteria; these were filtered out. These results suggest that our pipeline can reliably produce high-quality data with min-

 <sup>&</sup>lt;sup>1</sup>By default, we use OpenAI's o3-mini as the judge model.
 Detailed prompts are provided in Appendix A.1.

Theorem to QA Conversion				
Original Theorem	Generated QA Pair			
(Theorem 1.1 in (Bhowmik and Barman, 2024)) Let $q = p^{2t}$ , where $p \equiv 3 \pmod{4}$ is a prime and t is a positive integer. Then, the number of cliques of order 3 in the Peisert graph $P^*(q)$ is given by $k_3(P^*(q)) = \frac{q(q-1)(q-5)}{48}$ .	<b>Q:</b> Let $q = p^{2t}$ , where $p \equiv 3 \pmod{4}$ . What is the number of cliques of order 3 in the Peisert graph $P^*(q)$ , expressed as a function of $q$ ? <b>A:</b> $\frac{q(q-1)(q-5)}{48}$			
(Theorem 22 in (Vishnyakova and Borovoi, 2024)) The automorphism $\varphi_{\alpha} \circ (\Phi, \wedge d(\Phi))$ , where $\alpha \in \mathbb{C}^*$ , can be lifted to the II-symmetric super-Grassmannian $\Pi \text{Gr}_{2k,k}$ , where $k \geq 2$ , if and only if $\alpha = \pm i$ .	<b>Q:</b> Determine the value(s) of $\alpha \in \mathbb{C}^*$ for which the automorphism $\varphi_{\alpha} \circ (\Phi, \wedge d(\Phi))$ can be lifted to the super-Grassmannian $\Pi \operatorname{Gr}_{2k,k}$ for $k \ge 2$ . <b>A:</b> $\alpha = \pm i$			

### **Filtered Out Examples**

### **Example 1:** Let $\mathbb{X}$ be a uniformly convex Banach space. Then $\Gamma(\mathbb{X}) < \frac{1}{2}$ .

> LLMs may respond with weaker but still valid bounds (e.g.,  $\Gamma(X) < 1$ ), which are not equivalent. Stronger answers (e.g., < 0.4) require more advanced verification from the judge model.

**Example 2:** For any convex body K in  $\mathbb{R}^n$ , there exists  $\Phi \in GL(n)$  such that:

$$\frac{S(\Phi K)^n}{|\Phi K|^{n-1}} \le \frac{S(\Delta^n)^n}{|\Delta^n|^{n-1}} = \frac{n^{3n/2}(n+1)^{(n+1)/2}}{n!}.$$

➤ This is a non-constructive existence theorem. It cannot be converted into a fixed-answer QA that could easily be verified by an LLM judge.

*Figure 3.* Illustration of the theorem-to-QA conversion process and samples that are filtered out. The top panel shows examples of highquality question-answer pairs generated from mathematical theorems that contain fixed, verifiable answers. The bottom panel provides examples of theorems that were filtered out due to ambiguity or the lack of a fixed answer.

imal human intervention, even when working with imperfect real-world sources (see Appendix A.3).

Although our data construction inherently assumes that the mathematical theorems sourced from arXiv papers are *correct*, this is, of course, not guaranteed. ArXiv papers have not necessarily been peer-reviewed, and errors or ambiguous claims may be present.<sup>2</sup> On the one hand, incorrect statements could produce some noise in our results (which we hypothesize to be small). On the other hand, we view possible ambiguities in statements or notation as beneficial, since it is truly representative of mathematics "in the wild", compared to the more polished and vetted content found in formal competition settings.

<sup>2</sup>An alternative would be to limit the pipeline to peer-reviewed sources, but this would significantly reduce the diversity and coverage of available material.

### 4. Experiments

### 4.1. Experimental Setup

We use the data pipeline described in Section 3 (and Appendix A) to extract over 1,200 QA pairs from three data sources, Math.arXiv, CS.arXiv, and Math.StackExchange (see Table 2 for summary statistics). We then evaluated multiple frontier models— OpenAI o3 and o4-mini, Claude 3.7 Sonnet, Gemini 2.5 Pro, Grok-3, and DeepSeek-R1—across these datasets. For comparison, we also include several earlier models, such as GPT-40 mini, Claude 3.5 Sonnet, and LLaMA-3.1-405B.

The input format varies slightly across datasets to match their structure. For CS.arXiv and Math.arXiv, each model receives the LLM-generated question along with the full relevant context preceding the target theorem, typically spanning from the introduction up to (but excluding) the

5	Table 2. Frontier LLMs achieve strong performance at answering mathematical questions extracted from research papers and math
, 7	forums. See Figure 4 and Appendix B.2 for a more fine-grained evaluation based on the estimated difficulty of questions.

03	o4 mini	Gemini 2.5-pro	Deepseek R1	Grok 3	Claude 3.7-Sonnet	Claude 3.5-Sonnet	Llama 3.1-405B	GPT 40-mini
49.1	43.4	32.5	30.5	29.5	34.1	18.3	16.4	12.5
44.1	42.3	25.2	31.5	25.2	31.5	16.2	15.3	7.2
70.7	70.8	60.9	62.2	54.8	61.1	37.6	32.1	40.8
	49.1 44.1	49.1       43.4         44.1       42.3	o3         o4 mini         2.5-pro           49.1         43.4         32.5           44.1         42.3         25.2	o3         o4 mini         2.5-pro         R1           49.1         43.4         32.5         30.5           44.1         42.3         25.2         31.5	o3         o4 mini         2.5-pro         R1         Grok 3 <b>49.1</b> 43.4         32.5         30.5         29.5 <b>44.1</b> 42.3         25.2         31.5         25.2	o3         o4 mini         2.5-pro         R1         Grok 3         3.7-Sonnet           49.1         43.4         32.5         30.5         29.5         34.1           44.1         42.3         25.2         31.5         25.2         31.5	o3         o4 mini         2.5-pro         R1         Grok 3         3.7-Sonnet         3.5-Sonnet           49.1         43.4         32.5         30.5         29.5         34.1         18.3           44.1         42.3         25.2         31.5         25.2         31.5         16.2	o3         o4 mini         2.5-pro         R1         Grok 3         3.7-Sonnet         3.5-Sonnet         3.1-405B           49.1         43.4         32.5         30.5         29.5         34.1         18.3         16.4           44.1         42.3         25.2         31.5         25.2         31.5         16.2         15.3

Dataset	Time Span	# QA pairs
Math.arXiv	05/2022 - 09/2022 12/2024 - 03/2025	633
CS.arXiv	05/2022 - 10/2023	111
Math.StackExchange	04/2024 - 03/2025	542

theorem itself. In contrast, for Math.StackExchange, the input consists solely of the LLM-generated question, reflecting the format of the original forum posts.

### 4.2. Main Experimental Results

**Results on frontier LLMs.** Table 2 highlights the performance of the wide range of LLMs that we tested on our three datasets. The ten models under analysis show variation in accuracy, with o3 and o4mini leading on all datasets. More specifically, o3 achieved the highest accuracy on both CS.arXiv and Math.arXiv, whereas o4-mini slightly outperformed it on the Math.StackExchange samples. Other models, such as Deepseek-R1 and Gemini-2.5-pro, showed mid-tier performance, with accuracy percentages ranging between 25.6% – 32.5% on the arXiv datasets, compared to higher accuracy levels of 60.9% – 62.2% on Math.StackExchange.

We acknowledge that our full benchmark comes with relatively high initial performance, which may raise ques-316 tions about its durability. In fact, a common trend in 317 recent benchmarks has been to design problems that are 318 as challenging as possible, resulting in near-zero perfor-319 mance (Glazer et al., 2024; Phan et al., 2025). However, 320 since our benchmark aims to track performance on real, 321 organic mathematical research tasks, we do not view this 322 as an issue. While some benchmarks track LLMs' nascent 323 ability to solve (a few) of the most challenging math prob-324 lems, performance increases on REALMATH may be more 325 indicative of the utility of LLMs on common mathematical research tasks. Nevertheless, in the following experiment, 327 we also show that the difficulty of questions in our datasets 328 is highly nonhomogeneous and that there is a "hard" subset 329

on which current LLMs perform much worse.

**Breakdown by difficulty levels.** We analyze model performance across questions of varying difficulty levels. To determine these difficulty levels, we evaluated how older, weaker models performed on the dataset samples. Based on their performance, we categorized the samples into different difficulty levels, as detailed in Appendix B.2. As shown in Figure 4, accuracy consistently declines as question difficulty increases. For example, the top-performing model, o3, achieves 97.5% accuracy on easy questions, dropping to 81.4% on medium questions and further to just 27.9% on hard questions. A similar downward trend is observed for Gemini-2.5-Pro and DeepSeek-R1. This pattern highlights the significant challenge current models face in handling more complex and research-level mathematics.

Performance analysis across research subcategories. The datasets we collect consist of mathematical problems in a wide variety of subjects. Using human-reported categories (arXiv categories, or StackExchange tags), we can get a more granular evaluation of LLMs' capabilities in specific areas. Table 4 in Appendix B.4 shows the breakdown of categories for the Math.arXiv dataset, with a predominance of samples from Number Theory (math.NT) and Combinatorics (math.CO), each comprising more than 20% of the dataset size. This distribution is consistent with other recent benchmarks, e.g., MathConstruct (Balunović et al., 2025) and FrontierMath (Glazer et al., 2024), and stems from a combination of factors (see Figure 9 in Appendix A.4): (1) a high number of papers on the topic; (2) a higher number of theorems per paper on average; (3) a higher fraction of constructive theorems with a fixed answer.

To illustrate LLM performance on Math.arXiv, Figure 5 shows the accuracy per category for two of the best models on this dataset, o3 and Gemini 2.5-pro. For o3, the subcategories with the highest accuracy are *Representation Theory (math.RT), Number Theory (math.NT)* and *Analysis of PDEs (math.AP)*, each obtaining more than 60% accuracy. The subcategories with the poorest performance are *Optimization and Control (math.OC), Discrete Mathematics (cs.DM)* and *Machine Learning (cs.LG)*, with the last hav-



Figure 4. (a) Accuracy of the best-performing models across different difficulty levels of Math.arXiv and (b) distribution of difficulty levels on Math.arXiv. More details and results in Appendix B.2.



Figure 5. Evaluation of model performance on the Math.arXiv dataset across domain-specific and temporal dimensions. Left: Accuracy across mathematical domains for two models, showing significant variation by topic. Right: Accuracy of GPT-40-mini, Llama-3.1-405B, and Claude-3.5-Sonnet on questions published before vs. after their training cutoff.

ing only 23.8% accuracy.

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However, this capability profile is not uniform across
models. Gemini 2.5-pro performs best on Machine *Learning (cs.LG), Optimization and Control (math.OC)*and Probability (math.PR), and worst on Combinatorics
(math.CO), Representation Theory (math.RT), and Group
Theory (math.GR).

Overall, we observe that o3 performs well on highly theoretical tasks, such as the ones from *Representation Theory*,
whereas Gemini 2.5-pro is best in domains with more practical applicability, such as in machine learning.

### 4.3. Additional Analysis

Analyzing LLM's mistakes. To better assess model
performance, we introduce a pipeline that compares the
LLMs' output with a human-authored solution, taken from
the original paper or verified StackExchange answer. These



*Figure 6.* Error types for o3, Gemini 2.5-pro and DeepSeek R1 on the Math.arXiv dataset.

are provided to the judge model o3-mini, which evaluates whether the LLMs' reasoning aligns with the human solution. Common failures include arithmetic errors, incorrect arguments, missing critical insights, or logical inconsistencies. In Figure 6 we show the error breakdown for o3, **Question 1:** Let  $\mathcal{F}$  be an  $n \times m$  Ferrers diagram with  $m \geq n$ , and let  $1 \leq d \leq n$  be an integer. Assume that the pair  $(\mathcal{F}, d)$  is MDS-constructible and let  $\kappa = \kappa(\mathcal{F}, d)$ . What is the value of  $\lim_{q \to +\infty} \delta_q(\mathcal{F}, \kappa, d)$ ?

 $\overline{ \begin{array}{cccc} \textbf{Question 2: Given parameters } \sigma_1^2, & \sigma_2^2, \\ \text{and } c &\leq c(\sigma_1^2, \sigma_2^2), \\ \lim_{n \to \infty} \frac{C_s(\sigma_1^2, \sigma_2^2, c\sqrt{n}, n)}{n} & \text{expressed in terms} \\ \text{of } \sigma_1^2, & \sigma_2^2, \text{ and } c? \\ \hline \hline \textbf{Question 3: What is the value of} \\ \lim_{\text{odd } k \to \infty} \frac{\alpha(f_{\{(k+1)/2\},k})}{\rho(f_{\{(k+1)/2\},k})}? \end{array}$ 

Figure 7. Questions solvable by LLMs without any context. Interestingly, LLMs can infer the meaning of  $C_s$  in Question 2 and of  $\alpha$ ,  $\rho$ , f in Question 3. Appendix B.5 contains examples of questions LLMs cannot solve without context.

Gemini 2.5-pro and DeepSeek R1 on the Math.arXiv dataset. The majority of errors come from flawed reasoning, followed by conceptual misunderstanding and missed insights. This suggests that even the best-performing models struggle with multistep logical coherence in a solution.

How important is a theorem's context? Recall that for questions in Math.arXiv and CS.arXiv, we provide the LLM with the full context of the theorem, namely the entire research paper preceding the theorem statement. This context may be critical to understanding the notation and statement of the theorem. However, surprisingly, we find that LLMs still perform rather well even when this context is omitted. This suggests that many theorems use concepts or notation that can be understood or inferred in isolation. For example, in the CS.arXiv dataset, the o4-mini model achieves an accuracy of 21.6% despite the absence of any context (compared to 42.3% when the full context is provided). We present some examples of questions solvable without context in Figure 7.

428 **Measuring the impact of data contamination.** One of 429 the core features of REALMATH is that it can be con-430 tinuously updated with new organic samples, keeping the 431 benchmark up to date with contemporary mathematics and 432 preventing data contamination. This characteristic dis-433 tinguishes REALMATH from other research-level bench-434 marks, which either require extensive manual curation by 435 experts (Glazer et al., 2024; Phan et al., 2025), or which can 436 only be extended by a few new samples per year (Balunović 437 et al., 2025; Sun et al., 2025).

438 439 To test for data contamination, we split the Math.arXiv dataset into two batches, according to the cutoff dates of GPT-4o-mini, Llama-3.1-405B, and Claude-3.5-Sonnet. Figure 5 shows the accuracy for papers published before and after the respective cutoff dates (i.e., papers from 2022 vs. 2025). Interestingly, we find that models perform *better* on newer samples from 2025 than on older samples from 2022. One explanation could be that mathematical research became significantly easier (at least for LLMs) from 2022 to 2025, which seems unlikely. Rather, we believe a likely explanation is that current LLMs have been trained on a large amount of math data collected close to their cutoff date, and thus that LLMs may be more familiar with the type of topics studied in current math research.

We note that a nice application of REALMATH would be to study the evolution of mathematical research (and LLMs' performance) over time, an analysis which we leave for future work.

**Fine-tuning impact analysis.** To evaluate whether REALMATH can be used to improve mathematical capabilities of LLMs, we fine-tuned GPT-4o-mini on 500 random samples from the Math.arXiv dataset and evaluated its performance on the remaining 133 samples. Surprisingly, fine-tuning did **not** lead to an improvement in accuracy (see Appendix B.1 for more details). This suggests that the benchmark's difficulty does not stem from questions being out-of-distribution for current LLMs. Rather, current models appear to lack the specialized mathematical knowledge or skills necessary to solve certain problems, and these cannot be effectively acquired through simple fine-tuning.

### 5. Conclusion

Our work introduces REALMATH, a novel benchmark for evaluating LLMs in research-level mathematics. We show that current frontier models demonstrate surprisingly strong capabilities in research mathematics, with leading models achieving accuracy rates of 43–49% on Math.arXiv papers and up to 70% on Math.StackExchange questions. This suggests that these models may already serve as valuable assistants in mathematical research contexts, even as they continue to struggle with the most challenging problems.

A core feature of REALMATH is that it is not a *fixed* benchmark, but rather a refreshable data collection pipeline that reflects current mathematical practice while protecting against data contamination. We believe REALMATH can serve as a valuable tool for both evaluating and improving LLMs' mathematical capabilities. Its design aligns with the actual needs of mathematicians, helping advance AI assistants for research. As models improve on our benchmark, LLMs may see broader use as collaborative tools in mathematical research and education.

### 440 **References**

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## 550 **A. Data Collection and Evaluation**

## **A.1. Prompts for Judge Models**

<sup>553</sup> We used the following two system prompts fed to an LLM (o3-mini) for: (1) filtering theorems in order to retain only the
 <sup>554</sup> ones with unique answers; (2) generating high-quality question-answer pairs from theorems.

5.Đ	SYSTEM_PROMPT_THEOREM = r"""
53	You are an expert in mathematics and computer science. Your task is to verify if a theorem
558	has a single, numerical answer, easy to be verified. The theorems should be at least
55	graduate level.
55	
5.60	The theorems should have a fixed numerical answer, not an approximation. Some common
56	examples:
5र्रु:	- Necessary and Sufficient Conditions: e.g., "X holds if and only if condition A holds" only when at least one of A and X is specific, numerical quantity. We want results of
56:	the form "If condition A holds, then condition X holds" ONLY WHEN X is a NUMERICAL
56	VALUE. We don't want "if some conditions are met, then the quantity satisfies a
56	particular equation, then we can get X" when X is not a strict numerical value
56	relation, because this does not have fixed unique solutions. Please be very strict
56	about these rules!
56	- Existence and Uniqueness Theorems: e.g., "There exists a unique X that satisfies A.",
56	but we don't want "There exists an X that satisfies A", because the latter is not a
	fixed unique solution.
57(	- Exact Formula Calculations: e.g., "The answer of formula (1) is 10", or "The solution
57	for formula (1) is X", then both are fixed unique solutions.
5 <b>%</b> 2	<ul> <li>Unique Maximum/Minimum Points: e.g., "The maximum value of function f is 10 at point x</li> <li>=1", but we don't want "The maximum value of function f is at least 10", because the</li> </ul>
57:	latter is not a fixed unique solution.
57	- Exact Complexity Results in Computational Complexity: e.g., "The time complexity of
57	algorithm A is exactly \$\Theta(n^2)\$" (not \$\Omega(n^2)\$ or \$O(n^2)\$, because big-O
57	and big-omega are not exact).
59	- Explicit number of solutions of an equation: e.g. "X has a unique solution y \in Y" is
57	accepted even if the numerical value of the number of solutions is not specified
57	because it can trivially be deduced that the number of solutions is 1, which is a
58(	fixed answer. We also accept "If X, there are no solutions y \in Y" (implies 0
581	solutions). BUT we DON'T WANT the previous examples if the set Y in which we look for answers is not clear.
	- Equality of two numerical equations: e.g., \sum_{k=1}^n k^2 = \frac{n(n-1)}{2} because
58:	we can assume the numerical fixed answer to be the difference of the 2 which is 0. You
58:	MUST include these equalities even if \$n\$ is not fixed but rather a variable. You
584	MUST also include equations of the form "limit of $f(n) = integral of g(x)$ "
52	
518	Some examples of theorems that we don't want:
581	- We DON'T want the theorems that contain if and only if when neither of the sides is
58	numerical (\$x \in T\$ does not represent a numerical value), e.g. "A graph is bipartite if and only if it contains no cycles of odd length."
58	- We DON'T want theorems of the type "A holds if and only if there exists x such that X(x)
590	holds", but we DO WANT "A holds if and only if for all x, X(x) holds", where X(x) is
59	a fixed numerical value.
59	- We DON'T want the theorems that have any approximations, or any inequalities, or any
59	other non-deterministic statement. e.g. The theorems for which the main result
594	involves the Big-O notation, or where the main result proven in the theorem is that a
59!	certain relation holds "if and only if n \geq x or n \leq y" MUST be rejected. We DO
II	NOT consider any theorems where the answer is not an equality or a fixed answer, i.e.
59	results of the type "n \geq 5" should NOT be considered, so just SKIP these types of theorems.
59	- We DON'T want the theorems that state "X \$\in\$ complexity class Y" since Y can belong to
538	a bigger complexity class Z, so the answer is not unique.
58	- We DON'T want the theorems that state "X is isomorphic or homomorphic with Y", e.g.,
60	Chinese Remainder Theorem.
69	
60:	
21 22 60	Important guidelines:
22 60	- if you cannot find a single, definitive answer, you should return an empty result

69	
60) 80)	"false" - Respond only in the specified JSON format
25	return in this exact JSON format:
26 59 27	{
61	"single_unique_answer": "true" if the theorem has a single, definitive answer, otherwise "false"
69:	"explanation": "explanation of if this theorem has a single, definitive answer,
61 30	otherwise an empty string",
S1	
61 (	
61	SYSTEM_PROMPT_GENERATE_QA = r"""
61	You are a skilled problem setter for graduate-level mathematics and theoretical computer science. You are provided with a set of theorems (called theorems_dataset), each of
61	which has already been verified to contain a single, definitive, and numerical answer.
63	
6£:	Your task is to convert each verified theorem into a precise **question-answer (QA) pair **. MAKE SURE TO NOT MENTION THE ANSWER TO THE QUESTION IN THE QUESTION ITSELF.
62	Your outputs must follow these rules:
6 <u>7</u>	1. The **question** should be a well-posed mathematical or theoretical problem that is **
62! 62	clearly understandable to a graduate-level student**. Do not ask questions that are easy to answer without any mathematical reasoning or easy to guess the answer. **You
62	must never begin your question with "Prove that"**
62	2. The **question must be solvable in principle with a unique numerical or mathematical
62	answer**, based solely on the information in the theorem. 3. The **answer** must be:
69	- Strictly and uniquely determined.
6B: 63:	
63:	clearly by the theorem.
13 63	5. If a question naturally follows the structure of an identity (e.g., "What is the sum of?"), frame it that way.
63	6. All QA pairs must reflect **the exact scope of the theorem**. Do not generalize, weaken
63) Ø\$	
631	theorems where the main result to be proven is an inequality or a Big-O notation. We
63	MUST NOT include any kind of inequalities, questions about the lower/upper bounds, or any asymptotic running time of algorithms, i.e. do not generate QA pairs for theorems
64(	where the main result is of the type "n \geq 5".
66	8. DO NOT include in the question the answer to it, e.g. if a theorem states "The limit of X is equal to Y," where Y is an expression of some parameters defined earlier in the
64: 64:	theorem, phrase the question in the manner "What is the limit of X in terms of the
64	given parameters?", with the associated answer "The limit of X is Y". DO NOT formulate questions in the form "Prove the following relation" since the answer will be
64	already included in the question. Moreover, if the main result of the theorem inquires
64	about the value of a parameter for which a relation holds, do not mention this result in the question itself but rather ask "What is the value of the parameter for which
64	the relation holds?".
648 17 64	9. If you have a theorem where it says that a certain equation has a certain number of
650	solutions (single/unique solution, no solutions, an infinity of solutions, etc.) but the acutal value of the solution is not given, consider the Question-Answer pair to be
65:	of the form "What is the number of solutions to this equation?", i.e. even if the
65:	theorem does not have an explicit numerical expression for the answer, you can consider it to be a theorem with a fixed-answer, where the fixed-answer is the number
65: 65:	of solutions of the equation. However, if the numerical or closed-form of the solution
65!	is mentioned in the theorem statement, it is preferred to formulate the question to be "What is the solution to the following equation?" rather than to inquire about
65	the number of solutions.
98.	10. For theorems of the form "X has a certain property if and only if Y has a certain property", pose the question in such a way so that the answer is the side of the "if
65	and only if" statement which indicates a clear, numerical expression and not an

66		abstract definition, i.e. if a theorem states "X is Pareto optimal if and only if $\Phi$
66		(X) = 0", consider the question to be "If X is Pareto optimal, what is the value of $\setminus$
662		$Phi(X)$ ?" and the answer to be "\Phi(X) = 0", since the relation $Phi(X) = 0$ is a
66:		numerical one, whereas the other part of the "if and only if statement", namely "X is
664		Pareto optimal" is an abstract property. Hence, you must NOT consider the question to
		be "How is X if \Phi(X) = 0?" since the answer "X is Pareto optimal" is not a unique
66!		one.
68	11.	For theorems of the form "The following identity holds: $X = Y$ ", if the identity has
66'		both X and Y as mathematical expressions that are neither closed-form, nor fixed
66		numerical values (i.e. if we have equality between two sums with complex formulas
66		rather than an equality of the type "X=5"), do not ask the question "What is the value $% \left( X_{1}^{2}\right) =0$
		of X?" and the answer to be "Y", since Y might not be the unique answer to the
67(		question. Rather, you must formulate the question to be "What is the value of X-Y?"
67:		and the fixed, clear answer to be $"X-Y = 0"$ . In this pool of theorems with this
671		explicit QA pair, do not consider theorems where X is assumed to be a limit and Y the
67 20		value of the limit, since this case should be treated as in Condition 8.
20 674	12.	For theorems of the form "The following identity holds: $X = Y + ct$ ", if the identity
		has both X and Y as mathematical expressions that are neither closed-form, nor fixed
67!		numerical values, you should ask the question "What is the value of X - Y?" and the
67	10	answer should be "ct" and not "What is the value of X - Y -ct?"
847 (	13.	For theorems of the form "If X holds, then Y", formulate the QA pair to inquire about
67		what happens with Y when relation X holds, and not under what conditions Y holds (
67		since the condition X might not be unique if the theorem is not of the form if and only if).
628	1 /	DO NOT generate a QA pair for theorems where the main result is the belonging to a
681	14.	complexity class.
	15	If the theorem states a result of the type "Y = $ X $ ", where $ X $ indicates the
63:	10.	cardinality of the set X, formulate the question to be "What is the cardinality of X
68:		in terms of?" and the answer to be "Y", but not the other way around, i.e. DO NOT
684		state "What is the value of Y?" and the answer to be "cardinality of X", since it does
68		not make sense from a logical point of view to phrase the question about a numerical
68		quantity rather than about the characteristic of a set that you must determine.
2\$		
25 68	Reti	urn your output strictly in the following JSON format:
26	{	
27		"question": "Clearly stated, unique-answer question derived from the theorem. if the
69(		theorem is not good, return an empty string",
28		"answer": "The single, unique, exact answer derived from the theorem. if the theorem
69		is not good, return an empty string",
28		"is_good_theorem": "true" if the theorem is good, otherwise "false"
30	}	
3P		
69		

#### A.2. Prompts for Evaluation Model

699 We used the following system prompt and user prompt fed to GPT 40 for evaluating whether an LLM's answer to a 700 mathematical question is correct or not.

system\_prompt = r"""You are an expert mathematician tasked with evaluating the correctness 70: of an answer to a mathematical question. Compare the generated answer to the ground truth answer and determine whether the generated answer is mathematically correct and equivalent to the ground truth. 72 Please be very strict and rigorous in your evaluation, mark the answer as incorrect even 7B if it is 80% or 90% correct. Ensure the generated answer can be directly rendered in standard LaTeX without requiring custom command definitions. Be precise and focus on mathematical correctness, not formatting or style differences. Your evaluation should be fair and consider that the same mathematical content can be expressed in different ways.""" 74user\_prompt = f"""QUESTION: {question} GROUND TRUTH ANSWER: {ground\_truth} GENERATED 71 ANSWER: {final\_answer} Carefully evaluate whether the generated answer is mathematically correct and equivalent to the ground truth. Your response should only



### A.3. Manual Review

For the datasets we analyze in Section 4, we rely on a human-based review to ensure the quality of the samples. As noted in Section 3.2, for Math.arXiv we removed 5.2% bad samples, whereas for CS.arXiv we discarded 7.5% samples. Figure 8 shows the percentages of manually discarded items that fall into each category (35 samples in total for Math.arXiv and 9 samples for CS.arXiv). These samples were either theorems or QA pairs that did not have a unique answer but which LLMs failed to filter out in the previous stages of the automated pipeline.

### A.4. Category Distribution Across Construction Stages

Questions in our datasets are grouped into human-labeled categories based on their domain (e.g., arXiv paper categories). Figure 9 shows how category distribution shifts throughout the dataset construction stages. Combinatorics (math.CO) becomes increasingly dominant in later stages, as these papers tend to contain a large number of theorems, many of which are well-suited for the Q&A format.

### A.5. Pipeline for Mathematics Stack Exchange

The pipeline we describe in Section 3 is designed for arXiv papers. To retrieve and process samples from Mathematics Stack Exchange, we need to adapt some steps. We first queried the corresponding API to retrieve users' questions with tags such as *[limits]*, *[definite-integrals]*, *[integration]*—which were expected to have a clear numerical answer—as well as the top-ranked answer, if available. We extracted the HTML-formatted text and used an LLM to process the user's post into a statement and proof format, mirroring the theorems extracted from arXiv papers. We then applied a manual filtering step to ensure statements have a fixed answer and that the answer is indeed correct (for this, we checked cited sources, as well as the top-rated answer posted by other users). Finally, we fed the formulated theorem to an LLM to generate a QA pair and filter trivial samples.



*Figure 9.* Category distribution across different stages of dataset construction for Math.arXiv (05/2022 – 09/2022). The x-axis shows subject categories (e.g., math.CO, math.NT, etc.), and the y-axis represents the percentage of the dataset each category comprises. Notably, combinatorics (math.CO) becomes increasingly dominant in later stages, particularly in the final Q&A dataset.

## A.6. Optional LaTeX Conversion

To improve human readability of our datasets, we incorporated a LaTeX formatting step where we ask a LLM to convert each theorem into a self-contained and valid LaTeX code (this requires, for example, the expansion of any macros). We verified that this step does not negatively impact the language model's ability to formulate a QA pair or filter out low-quality samples.

## **B. Additional Results**

## B.1. Finetuning

In Figure 10, we use OpenAI's fine-tuning service to fine-tune GPT-40-mini on our dataset. We present the detailed training and test loss in Figure 11. To ensure reliability, we repeat the evaluation five times and report the mean and standard deviation of the results. Surprisingly, we observe no statistically significant improvement.

Figure 11 shows the training and testing loss curves for OpenAI fine-tuning. The training loss decreases rapidly and remains consistently low, indicating that the model is fitting the training data well. On the other hand, the testing loss exhibits a fluctuating behavior and increases over time compared to the beginning. This pattern may justify the failure of generalization of the model and further reinforces the idea that fine-tuning does not lead to improved performance.







Figure 11. Training (green) and testing (purple) loss curves from the OpenAI fine-tuning service. Fine-tuning on our dataset does not show a clear improvement.

## **B.2.** Difficulty Levels

We assessed the difficulty level of the samples in the datasets by using older, weaker models and checking their performance
on the samples. More specifically, we evaluated the performance of four older models from distinct families—Qwen3235B, Claude 3.5 Haiku, GPT-3.5 Turbo and Llama 3.3-70B.

Samples in the *Hard* category are the ones which have not been answered correctly by any of the aforementioned models,
whereas samples in the *Medium* and *Easy* categories are the ones which have been answered correctly by less than 50% of
the LLMs (i.e. 1-2 out of 4), and more than 50% of the models (i.e. 3-4 out of 4), respectively.

Figure 12 depicts the distribution of samples across categories in the CS.arXiv data collection, with 69.4% of the samples being considered *Hard*. This distribution suggests that indeed our dataset reflects a hardness level appropriate for the research communities.

Moreover, we show the accuracy per category for some of the best-performing models—03, DeepSeek-R1, Gemini-2.5-Pro—on CS.arXiv. Each of the three LLMs (except o3) has accuracy level decreasing substantially as the difficulty level increases, e.g., DeepSeek-R1 has 100% accuracy on *Easy* samples and only 11.7% accuracy on *Hard* samples. This is an expected pattern which reinforces the correct assessment of our samples in these three categories.

The higher accuracy of o3 on *Medium* samples when compared to that of *Easy* samples—75% versus 71.4%—does not necessarily imply an abnormal behavior and can be justified by the difference in the number of samples from these two categories, where we have 18% of the samples considered *Medium* difficulty and only 12.6% of them labeled as *Easy*. Especially for the *Hard* category, we spot a discrepance between the relatively high accuracy of o3, namely 31.2%, compared to DeepSeek-R1 and Gemini-2.5-Pro, who both achieve less than 12% accuracy.

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- 877
- 878



Figure 12. Accuracy across different difficulty levels for best-performing models on CS.arXiv.

Along the same line of thought, Figure 13 shows the distribution for the Math.StackExchange dataset. We remark that the distribution for this dataset is more spread, with the Medium difficulty questions being the ones that occur the most, as opposed to the Math.arXiv and CS.arXiv distributions, where the questions were dominantly classified as Hard. This reflects the nature of Math.StackExchange, which focuses on clarifications of undergraduate and graduate-level concepts and computations, rather than innovative, research-level inquiries.



Figure 13. Accuracy across different difficulty levels for best-performing models on math.stackExchange.

## 

**B.3.** Performance Across Categories

We have assessed the performance of the models on each subcategory (for CS.arXiv) and on each tag (for Math.StackExchange) in order to understand whether some models perform particularly well on a certain category of questions. In Figure 14, we show the accuracies of o3 and Gemini-2.5-Pro. 

On one hand, we remark for CS. arXiv that, with the exception of *Logic in Computer Science (cs.LO)*, where the models have identical accuracies, o3 outperforms Gemini-2.5-Pro in every category. On the other hand, an important feature that can be observed in Figure 14(a) is that o3 is better at solving questions from applied categories such as Discrete Mathematics (cs.DM) and Machine Learning (cs.LG), while Gemini-2.5-Pro gets its highest accuracies on theoretical topics such as Computational Complexity (cs.CC) and Information Theory (cs.IT). This is consistent with the behavior that these two LLMs have for the Math.arXiv dataset.

For the Math.StackExchange dataset in Figure 14(b) we observe as well that o3 outperforms Gemini-2.5.-Pro in each category. Moreover, the two LLMs follow a similar trend in the sense that their accuracy level per subcategory can be roughly ordered in the same hierarchy structure. Both models perform best on Limits, suggesting this is a relatively easier category where they can get fixed numerical answers correctly with high probability—more than 80%—, whereas their performance decreases on more advanced topics such as Complex Analysis. 



(b) Math.StackExchange

Figure 14. Performance across different math domains in (a) CS.arXiv and (b) Math.StackExchange.

### 990 B.4. Category Distribution of Questions

992 This subsection provides the statistics of how samples are distributed according to subdomains for each of the three datasets 993 under consideration, clearly presented in the subsequent three tables. The subdomains are a good indicator for highlight-994 ing which mathematical domains are best for retrieving theorems along with question-answer pairs with a fixed, unique 995 numerical answer or closed-form expression.

996 An interesting observation is that *Combinatorics (math.CO)* is among the top frequent domains for both Math.arXiv 997 and CS.arXiv data collections. This suggests that problems in combinatorics are not only well-represented but also more 998 likely to have fixed-answer solutions which satisfy our benchmarking criteria and make the automated evaluation process 999 easy to be verified.

Category	arXiv Tag	Percentage (%)
Combinatorics	math.CO	28.89
Number Theory	math.NT	10.16
Algebraic Geometry	math.AG	5.19
Probability	math.PR	3.16
Geometric Topology	math.GT	4.51
Group Theory	math.GR	3.16
Information Theory	math.IT	7.22
Optimization and Control	math.OC	3.61
Discrete Mathematics	cs.DM	4.51
Analysis of PDEs	math.AP	1.13
Commutative Algebra	math.AC	1.35
Machine Learning	cs.LG	3.16
Representation Theory	math.RT	1.35
Mathematical Physics	math.MP	0.68
Rings and Algebras	math.RA	1.35
Symplectic Geometry	math.SG	3.61
Functional Analysis	math.FA	2.93
Classical Analysis and ODEs	math.CA	2.48
Differential Geometry	math.DG	1.58
Complex Variables	math.CV	1.58
	Combinatorics Number Theory Algebraic Geometry Probability Geometric Topology Group Theory Information Theory Optimization and Control Discrete Mathematics Analysis of PDEs Commutative Algebra Machine Learning Representation Theory Mathematical Physics Rings and Algebras Symplectic Geometry Functional Analysis Classical Analysis and ODEs Differential Geometry	Combinatoricsmath.CONumber Theorymath.NTAlgebraic Geometrymath.AGProbabilitymath.PRGeometric Topologymath.GTGroup Theorymath.GRInformation Theorymath.ITOptimization and Controlmath.OCDiscrete Mathematicscs.DMAnalysis of PDEsmath.ACMachine Learningcs.LGRepresentation Theorymath.RTMathematical Physicsmath.MPRings and Algebrasmath.RASymplectic Geometrymath.SGFunctional Analysis and ODEsmath.CADifferential Geometrymath.CA

*Table 4.* Percentage distribution of the top 20 most represented arXiv categories in the Math.arXiv (05/2022 – 09/2022) dataset. 1025 Categories are not mutually exclusive; papers with multiple tags are counted under each corresponding category. The predominance 1026 of questions in combinatorics and number theory is consistent with other recent math benchmarks (e.g., (Balunović et al., 2025; Mis-1027 erendino et al., 2025)) and likely reflects an abundance of constructive statements in these topics.

Category	arXiv Tag	Percentage (%)
Information Theory	cs.IT	49.55
Combinatorics	math.CO	36.04
Data Structures and Algorithms	cs.DS	29.73
Discrete Mathematics	cs.DM	18.92
Computational Complexity	cs.CC	17.12
Machine Learning	cs.LG	10.81
Cryptography and Security	cs.CR	8.11
Logic in Computer Science	cs.LO	7.21
Computer Science and Game Theory	cs.GT	3.60
Optimization and Control	math.OC	3.60
Formal Languages and Automata Theory	cs.FL	2.70
Logic	math.LO	2.70
Databases	cs.DB	1.80
Algebraic Geometry	math.AG	1.80
Neural and Evolutionary Computing	cs.NE	1.80
Commutative Algebra	math.AC	1.80
Distributed, Parallel, and Cluster Computing	cs.DC	1.80
Systems and Control	cs.SY	0.90
Multiagent Systems	cs.MA	0.90
Artificial Intelligence	cs.AI	0.90
Information Retrieval	cs.IR	0.90
Networking and Internet Architecture	cs.NI	0.90
Numerical Analysis	cs.NA	0.90
Rings and Algebras	math.RA	0.90
Probability	math.PR	0.90

*Table 5.* Percentage distribution of arXiv categories in the CS.arXiv dataset. Categories are not mutually exclusive; papers with 1086 multiple tags are counted under each corresponding category.

go	ory	Pe	rcentage (%)
ra	tion		53.32
S			43.91
ılı	us		33.70
it	e Integrals		25.09
A	nalysis		21.77
	nces and Series		12.73
	ex Analysis		6.27
ys	sis		5.72
p	per Integrals		5.3
	on Verification		5.35
v	ariable Calculus		4.4
at	tives		4.4
S	Without Lhopital		4.0
	Form		3.3
n	ometry		2.95
	rgence Divergence		2.7
ιb	oility		2.7
al	l Functions		2.7
	ometric Integrals		2.5
	r Integration		2.0
uı	re Theory		2.0
	ite Integrals		2.0
	ex Integration		1.8
	a Function		1.8
or	n Delta		1.6
	ototics		1.6

1140Table 6. Percentage distribution of the top 26 most represented tags in the Math.stackExchange dataset. Categories are not mutu-1142ally exclusive; questions with multiple tags are counted under each corresponding category.

### 5 B.5. How Context Improves LLM Performance on Mathematical Problems

Figure 15 depicts some examples of questions from CS.arXiv dataset which are correctly solved if the relevant context is provided to the language model, but which are not answered properly when context is not provided.

Based on a manual verification of the samples, we observed that not providing context negatively impacts models in two directions:

- LLMs cannot answer the question since they do not understand the notations of certain quantities, which are only explained in the paper. In these cases, the context is crucial for the LLM since otherwise it either attempts to guess what a notation means, or they simply do not try to answer and specify that the quantities are not known to it.
- LLMs cannot answer the question because it is considered hard if we do not provide context. This situation comprises cases where the context presents some intermediate results or examples of proofs which are used for building up the current theorem or for its proof. Feeding the context to the language model improves its approach into correctly proving the question.

**Question 1:** For  $r \ge 5$ , what is the value of gcover(BF(r)) expressed in terms of r?

 $\blacktriangleright$  LLMs fail to correctly respond because the notation for the quantity gcover(BF(r)) is ambiguous without prior definition.

Question 2: What is the expression for the zero-rate error exponent  $E^{(1)}(0)$  in terms of a maximization over  $q \in \mathcal{P}(\mathcal{X})$  and the function  $d_B(x, x', P)$ ?

 $\blacktriangleright$  LLMs do not have knowledge of the function  $d_B(x, x', P)$  or the set  $\mathcal{P}(\mathcal{X})$  if the context is not provided.

**Question 3:** Let n > 1 be an integer and let  $\mathcal{A} \subseteq \mathbb{Z}_2^n$  be a maximal anticode of diameter one. What is the maximum possible number of codewords in  $\mathcal{A}$ ?

> The LLM knows what an anticode represents, but the question is too hard to be answered correctly.

**Question 4:** Consider the complete bipartite graph  $K_{m,n}$  with parameters satisfying m < n, n even, and (m + n)|mn. What is the ATN of  $K_{m,n}$ ?

The notion of ATN (Alon-Tarsi Number) is a common abreviation for language models, but the difficulty of the question is too high for them to respond correctly.

*Figure 15.* Examples of questions in our CS.arXiv dataset that are not solvable by LLMs without the necessary context, but which become solvable when the context is given.