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# AdEMAMix: Better and Faster Training with Older Gradients

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## Abstract

Momentum based optimizers are central to a wide range of machine learning applications. These typically rely on an Exponential Moving Average (EMA) of gradients, which decays exponentially the present contribution of older gradients. This accounts for gradients being local linear approximations which lose their relevance as the iterate moves along the loss landscape. This work questions the use of a single EMA to accumulate past gradients and empirically demonstrates how this choice can be sub-optimal: a single EMA cannot simultaneously give a high weight to the immediate past, and a non-negligible weight to older gradients. Building on this observation, we propose AdEMAMix, a simple modification of the Adam optimizer with a mixture of two EMAs to better take advantage of past gradients. Our experiments on language modeling show—quite surprisingly—that gradients can stay relevant for tens of thousands of steps. They help to converge faster, and often to lower minima: e.g., a $1.3$B parameter AdEMAMix LLM trained on $101$B tokens performs comparably to an AdamW model trained on $197$B tokens ($+95\%$). Moreover, our method significantly slows-down model forgetting during training. Our work motivates further exploration of different types of functions to leverage past gradients, beyond EMAs. For an extended version of this work, see: https://arxiv.org/abs/2409.03137.

## 1. Introduction

With large neural networks, deep-learning has revolutionized numerous fields, such as computer vision and natural language processing. At the heart of this paradigm lies the challenge of optimizing complex, non-convex loss functions using noisy gradient estimates. This optimization process is typically carried out using variants of Stochastic Gradient Descent (SGD) [28] or adaptive methods such as Adam and AdamW [16, 19], which have become ubiquitous in training state-of-the-art models [1, 5–8, 27, 33, 36].

A key component in many of these iterative optimization algorithms is momentum, which has long been shown to accelerate convergence [21] and often leads to solutions with superior generalization properties [31]. By accumulating gradient vectors over successive optimization steps, momentum helps overcome small local variations of the loss landscape, potentially escaping shallow local minima, and accelerate in plateau regions [12, 26, 29]. Both SGD with momentum (SGD+M) and Adam incorporate momentum under the form of Exponential Moving Averages (EMAs) of past

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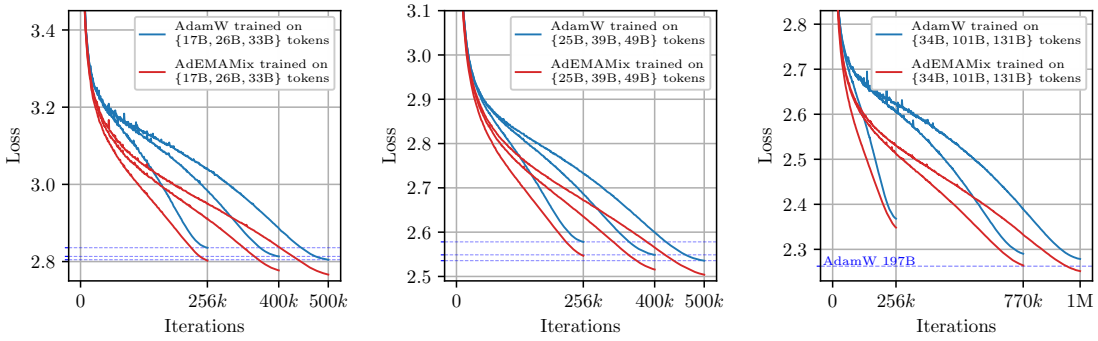



(*a*) 110M parameters. (*b*) 330M parameters. (*c*) 1.3B parameters.

Figure 1: **Comparing AdamW and AdEMAMix on language modeling.** In **(a,b,c)**, we plot the loss obtained using AdamW and AdEMAMix (our optimizer) to train Transformer models of various sizes on the Redpajama dataset. In **(a)**, we train multiple baselines for $256k$, $400k$, and $500k$ iterations, resulting in processing from 17B to 33B tokens. Two AdamW runs with different number of iterations look very different as we use a cosine-decay for the learning rate. We compare those baselines to training AdEMAMix for $256k$ iterations. We observe that our method reaches a similar loss as an AdamW model trained on nearly twice the number of tokens. Analogous comparisons can be derived from **(b)** and **(c)**. Notably, in **(c)**, a 1.3B parameter AdEMAMix model trained on 101B tokens performs comparably to an AdamW model trained on 197B tokens (95% more, blue horizontal line).

gradients $\mathcal{G}^T = \{\boldsymbol{g}^{(0)}, \dots, \boldsymbol{g}^{(T)}\}$:

$$\text{EMA}(\beta, \mathcal{G}^T) \triangleq \beta \cdot \text{EMA}(\beta, \mathcal{G}^{(T-1)}) + (1-\beta)\boldsymbol{g}^{(T)} = \sum_{i=0}^{T} \beta^i (1-\beta)\boldsymbol{g}^{(T-i)}. \tag{EMA}$$

Two considerations support the use of EMAs. From a practical standpoint, the recursive formula of EMA allows for efficient implementations, which do not require maintaining a buffer of past gradients. From a theoretical standpoint, gradient descent with momentum leads to optimal convergence rates for quadratics [22, 25]. However, those results do not guarantee any optimality for general non-quadratic cases [13].

The use of momentum in optimization is grounded in the varying nature of gradients. As local linear approximations of the loss landscape, their information can quickly become outdated as the optimization process progresses [24]. For this reason, practitioners typically employ moderate momentum values (i.e. $\beta \approx 0.8$ or $0.9$), effectively creating a moving average of recent gradients while discarding older information. Selecting larger $\beta$ values seems counter-intuitive, as it would suggest that older gradients maintain their relevance over extended periods of training. While it is tempting to see the use of small $\beta$s as a confirmation of the limited temporal relevance of gradients, our work reveals instead that older gradients can efficiently be used. When we increase $\beta$, we decrease the relative importance of recent gradients, and the iterate now fails to respond to local changes in the loss landscape. We observe that a single EMA cannot both give a significant weight to recent gradients, and give a non-negligible weight to older gradients. However, a linear combination between a "fast-changing" (e.g. $\beta = 0.9$ or $\beta = 0$) and a "slow-changing" (e.g. $\beta = 0.9999$) EMA

allows the iterate to beneficiate from (i) the great speedup provided by the larger (slow-changing) momentum, while (ii) still being reactive to small changes in the loss landscape (fast-changing). More precisely, we find the following statement to convey an important intuition behind this approach:

> *While changing the direction of the slow momentum is difficult, any adjustment orthogonal to that direction is easy—which favors fast progress in sinuous canyon-like landscapes.*

A toy illustration of this can be seen in Fig. 3. Based on this idea, we propose AdEMAMix (**Ad**aptive **EMA Mix**ture), a novel Adam based optimizer which successfully leverages very old gradients to reach better solutions.

**Contributions.** Our contributions can be summarized as follows: **(i)** We propose AdEMAMix, a novel optimizer which better leverages past gradients by avoiding a common pitfall of EMA-based optimizers (see § 3). **(ii)** We empirically demonstrate the superiority of our method over Adam by training Transformer language models of up to 1.3B parameters (see § 4). **(iii)** We show AdEMAMix forgets the training data slower when compared to Adam (see Fig. 2). **(iv)** More broadly, our findings contribute to a deeper understanding of the optimal balance between using historical gradients and adapting to the rapidly changing loss landscape. Our work invites further research in methods combining old and recent gradients, beyond EMAs.

## 2. Related Work

**Works on understanding momentum.** From the seminal work of [25], many publications analyzed the effect of gradient descent + momentum (GD+M) in both convex and non-convex settings [4, 9, 10, 15, 30]. While the acceleration in the noise-free setting has been long theorized for convex functions, several publications indicate this effect might not necessarily extend to stochastic settings [15, 17, 35], emphasizing instead a link between momentum and effective learning rate. Recent work have been seeking to understand the impact of momentum on generalization through studying the implicit bias of momentum methods [11, 23], exposing a preference of SGD+M for lower norm solutions. Those further exposed a link between higher momentum and higher effective learning rate and higher variance reduction. Despite the volume of prior work on the subject, our understanding of momentum methods in non-convex stochastic settings is still incomplete [35]. Oscillatory behaviours, and the sometimes ambiguous effect of variance on optimization render the analysis tedious. From a theoretical standpoint, our work raises several questions. First, given that we gain from averaging very old gradients, what can it reveal of the loss landscape and the consistency of one batch's gradient during training? Second, would our approach not decrease the variance up to a point that is harming generalization [11]? While no answer to those questions is given in this work, we provide a toy justification which indicates that large momentums can have a positive impact in noise-free non-convex settings (see Fig. 3)—indicating the improvement of our approach is at least partially explainable without considering variance-reduction effects. We moreover expose a link between momentum and forgetting the training data (see Fig. 2), which to our knowledge is novel.

**Works on deep-learning optimizers.** Despite the popularity of Adam and AdamW [16, 19] in training deep neural networks, optimizer design is a rich field of research and we focus on a few of the works most relevant to this study. Chen et al. [2] use algorithm discovery to derive the Lion optimizer. Contrary to Adam, Lion uses a single momentum term and the sign function to produce updates with the same magnitude across dimensions. Interestingly, Chen et al. [2] also report better scores are obtained when using a slightly larger momentum term ($\beta = 0.99$). In this work we show how increasing the momentum well beyond this value can still be beneficial. Recently,

Liu et al. [18] introduced Sophia, a scalable second-order optimizer designed for LLM training. Sophia uses a Hessian-based pre-conditioner which better normalizes the step size, penalizing steps in high curvature direction and accelerating in low curvature directions. Understanding in which circumstances those novel optimizers bring improvements is still being investigated [14], and Adam's dominance remains mostly unchallenged. Most relevant to us, Lucas et al. [20, AggMo] also observe that using a combination of EMAs can enable the use of larger $\beta$s, and incorporates a sum of $K$ momentum terms into *GD* . They show their approach reaches similar performances as baseline optimizers, with a faster convergence. In contrast, we modify *Adam*, which is the workhorse algorithm for large-scale optimization, and introduce schedulers that are critical to reaching good performances at larger scales. As a result, we not only converge faster, but better, and outperform Adam by a significant margin. Finally, Szegedy et al. [32] propose a general framework to derive and study optimizers with linear combinations of memory vectors—which encompasses EMA mixtures.

## 3. Our method: AdEMAMix

**Setup & notations.** Let $\mathcal{L}_{\boldsymbol{\theta}} : \mathcal{X} \mapsto \mathbb{R}$ be a loss function parameterized by $\boldsymbol{\theta}$, and mapping inputs $\boldsymbol{x} \in \mathcal{X}$ to $\mathbb{R}$. Given a sampled batch $\boldsymbol{x}$, let $\boldsymbol{g} = \nabla_{\boldsymbol{\theta}} \mathcal{L}_{\boldsymbol{\theta}}(\boldsymbol{x})$ be a stochastic gradient of the loss w.r.t. $\boldsymbol{\theta}$. To minimize the empirical loss, the Adam optimizer [16] relies on first and second moments, resp. $\boldsymbol{m}$ and $\boldsymbol{\nu}$, estimated via two EMAs parametrized by $(\beta_1, \beta_2) \in [0, 1[^2$. A weight-decay parameter $\lambda \in \mathbb{R}^+$ is often used as in Loshchilov and Hutter [19]:

$$\begin{cases} \boldsymbol{m}^{(t)} = \beta_1 \boldsymbol{m}^{(t-1)} + (1-\beta_1)\boldsymbol{g}^{(t)}, & \hat{\boldsymbol{m}}^{(t)} = \frac{\boldsymbol{m}^{(t)}}{1-\beta_1^t} \\ \boldsymbol{\nu}^{(t)} = \beta_2 \boldsymbol{\nu}^{(t-1)} + (1-\beta_2){\boldsymbol{g}^{(t)}}^2, & \hat{\boldsymbol{\nu}}^{(t)} = \frac{\boldsymbol{\nu}^{(t)}}{1-\beta_2^t} \\ \boldsymbol{\theta}^{(t)} = \boldsymbol{\theta}^{(t-1)} - \eta\big(\frac{\hat{\boldsymbol{m}}^{(t)}}{\sqrt{\hat{\boldsymbol{\nu}}^{(t)}}+\epsilon} + \lambda \boldsymbol{\theta}^{(t-1)}\big). \end{cases} \tag{AdamW}$$

With $t > 0$ being the timestep, $\eta$ being the learning rate, and $\epsilon$ a small constant. Initially $\boldsymbol{m}^{(t=0)} = \boldsymbol{\nu}^{(t=0)} = \mathbf{0}$. To prevent the bias induced by the initial $\boldsymbol{m}^{(t=0)}$ and $\boldsymbol{\nu}^{(t=0)}$, the outputs of the two EMAs are corrected into $\hat{\boldsymbol{m}}^{(t)}$ and $\hat{\boldsymbol{\nu}}^{(t)}$. Those are used to compute the final weight update, scaled by the learning rate.

**How far to look into the past?** A typical value for $\beta_1$ is $0.9$. The larger the $\beta$, the more uniform the average is. To put this in perspective—observing that $\sum_{i=0}^{\infty} \beta^i(1-\beta) = 1$ for $\beta \in [0, 1[$—the number of successive previous steps receiving a cumulative weight of $0.5$, is $t_{half} = \frac{\ln(0.5)}{\ln(\beta)} - 1$. For $\beta = 0.9$, $t_{half} \approx 6$, meaning that half of the weight is given to the previous six gradients. This observation can also be extended to SGD with e.g. polyak or nesterov momentums [22, 25], which typically relies on similar $\beta$ values. The value of $\beta_1$ is rarely increased beyond $\sim 0.9$. In our experiments with AdamW, increasing $\beta_1$ further degraded the performance. Does this mean older gradients are outdated? We show that this is not the case, rather, increasing beta is reducing the sensitivity to recent gradients too much. We design AdEMAMix such that the sensitivity to recent gradients is kept, while also incorporating information from much older gradients using an additional momentum term. This allows for the use of much larger $\beta$ values e.g. $0.9999$. To compare, for $\beta = 0.9999$, $t_{half} \approx 6{,}930$, spreading half of the mass over the previous $6{,}930$ past gradients.

**AdEMAMix.** To keep a high sensitivity to recent gradients, while also incorporating information from older gradients, we add a second EMA (changes compared to AdamW are in Blue):

$$\begin{cases} \boldsymbol{m}_1^{(t)} = \beta_1 \boldsymbol{m}_1^{(t-1)} + (1-\beta_1)\boldsymbol{g}^{(t)}, & \hat{\boldsymbol{m}}_1^{(t)} = \frac{\boldsymbol{m}_1^{(t)}}{1-\beta_1^t} \\ \boldsymbol{m}_2^{(t)} = \beta_3 \boldsymbol{m}_2^{(t-1)} + (1-\beta_3)\boldsymbol{g}^{(t)} & \\ \boldsymbol{\nu}^{(t)} = \beta_2 \boldsymbol{\nu}^{(t-1)} + (1-\beta_2){\boldsymbol{g}^{(t)}}^2, & \hat{\boldsymbol{\nu}}^{(t)} = \frac{\boldsymbol{\nu}^{(t)}}{1-\beta_2^t} \\ \boldsymbol{\theta}^{(t)} = \boldsymbol{\theta}^{(t-1)} - \eta\big(\frac{\hat{\boldsymbol{m}}_1^{(t)} + \alpha \boldsymbol{m}_2^{(t)}}{\sqrt{\hat{\boldsymbol{\nu}}^{(t)}}+\epsilon} + \lambda \boldsymbol{\theta}^{(t-1)}\big). \end{cases} \quad \text{(AdEMAMix)}$$

In our experiments, while the values of $\beta_1, \beta_2$ remain similar to those of equation AdamW, we often use $\beta_3 = 0.9999$. We find $\alpha \in [4, 10]$ to work well in practice.

**Tackling early training instabilities.** Early training instabilities are commonplace when training deep models, and empirically motivated the introduction of methods such as learning rate warmup and gradient clipping. While we use learning rate warmup in all our experiments, we still noticed AdEMAMix models using a large $\beta_3$ would diverge early. This, despite not using bias correction over $\boldsymbol{m}_2$, which lets the momentum buffer fill itself slowly. Those failed runs are characterized by updates of large magnitudes in the early phase of training. For this reason, we progressively increase the values of $\beta_3$ and $\alpha$ using schedulers. For $\alpha$ we use a linear scheduler. A linear scheduler for $\beta_3$ would be ill-fitted as the same increment of $\beta_3$ have a different impact for different values of $\beta_3$. For instance, observe that an increase of $\beta$ of $\delta_\beta = 0.0001$ barely increases the $t_{half}$ for $\beta = 0.9$, while $0.999 \rightarrow 0.999 + \delta_\beta$ increases the $t_{half}$ of 77. For this reason, we design the $\beta_3$ scheduler to *increase* $t_{half}$ *linearly*. The two schedulers are summarized below:

$$\alpha^{(t)} = f_\alpha(t, \alpha, T_\alpha) = \min(\frac{t\alpha}{T_\alpha}, \alpha), \quad (f_\alpha)$$

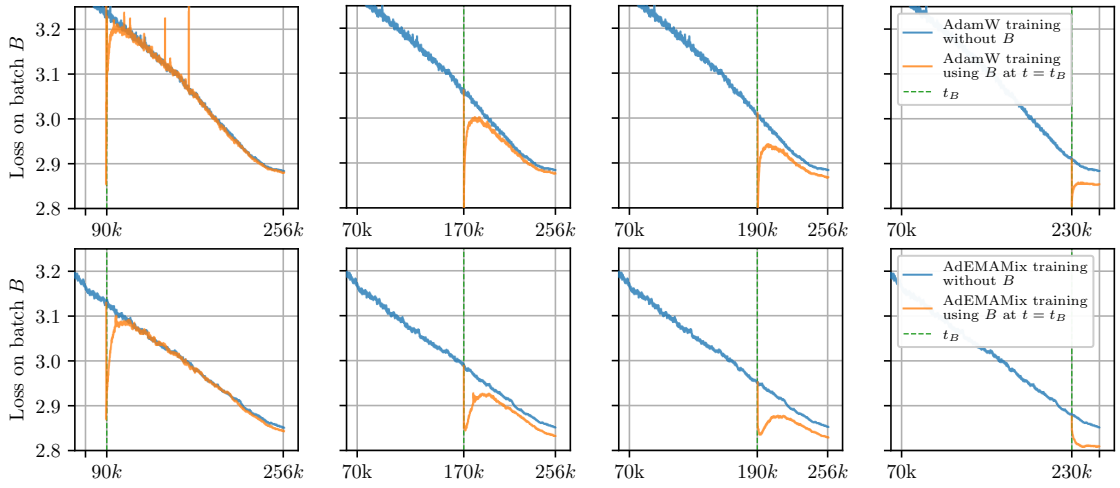
$$\beta_3^{(t)} = f_{\beta_3}(t, \beta_3, \beta_{start}, T_{\beta_3}) = \min\big(\exp\big(\frac{\ln(\beta_{start})\ln(\beta_3)}{(1-\frac{t}{T_{\beta_3}})\ln(\beta_3) + \frac{t}{T_{\beta_3}}\ln(\beta_{start})}\big), \beta_3\big). \quad (f_{\beta_3})$$

With $T_\alpha$ and $T_{\beta_3}$ are resp. the warmup times for $\alpha^{(t)}$ and $\beta_3^{(t)}$ to reach their final and maximal values. In practice we always set those two to the same value: $T_\alpha = T_{\beta_3} = T_{\alpha,\beta_3}$, and we typically use $T_{\alpha,\beta_3} = T$, with $T$ being the total number of iterations. $\beta_{start}$ is always set to $\beta_1$ in our experiments.

**Hyperparameter sensitivity.** While we introduce up to four new hyperparameters: $\alpha$, $\beta_3$, $T_\alpha$, and $T_{\beta_3}$. In practice we always set $T_\alpha = T_{\beta_3} = T_{\alpha,\beta_3}$, and use $T_{\alpha,\beta_3} = T$ in most cases. While all of our experiments on language modeling use $\beta_3 = 0.9999$, other values such as $0.999$ or even $0.99999$ still can outperform the AdamW baseline. Overall, we find the ranges of values of $\alpha$, $\beta_3$ and $T_{\alpha,\beta_3}$ providing improvements over AdamW to be wide.

## 4. Results

**Experimental setup.** We use a transformer architecture [34]. Our experiments use sequences of 1,024 tokens. We experiment with three model sizes: 110M, 335M, and 1.3B. We use 3k warmup steps followed by a cosine decay. We extensively tuned the hyperparameters for both AdamW and AdEMAMix models. We use the RedPajama v2 [3] dataset for all of our experiments. We use batch sizes of 64, 96 and 128 for respectively our 110M, 335M, and 1.3B parameter models. For AdEMAMix, we use $\beta_3 = 0.9999$ and $\alpha \in \{5, 8, 10\}$ depending on the model.




(*a*) $t_B = 90k$. (*b*) $t_B = 170k$. (*c*) $t_B = 190k$. (*d*) $t_B = 230k$.

Figure 2: **Measuring forgetting using a held-out batch $B$.** The top row is for AdamW, the bottom row is for AdEMAMix. We trained one AdamW and AdEMAMix model on a RedPajama dataset *not* containing the batch $B$, those runs are in blue. We then run multiple experiments where we inject $B$ in the training data at a specific timestep $t_B$. Those runs are in orange. To inspect how much influence $B$ had when it is injected at $t_B$, we can observe the evolution of the gap between the blue and the orange curves. For both optimizers, we observe a rapid decrease of the loss on $B$ right after training on $B$. The sharpness of this decrease in loss is more pronounced for AdamW compared to AdEMAMix. However, when using AdamW, the loss on $B$ then increases faster, which we interpret as the model forgetting $B$ faster. In contrast, the curves for AdEMAMix are smoother, the loss on $B$ goes back up slower, and ultimately $B$ had a bigger impact on the training when using AdEMAMix—as can be seen by looking at the larger gap between the orange and blue curves for the last iteration. Finally, the forgetting behaviour evolve during training, with the later training batches being remembered better.

**Why not simply increasing AdamW's $\beta_1$?** We train multiple 110M models using Adam with large $\beta_1 \in \{0.99, 0.999, 0.9999, 0.99999\}$. When we use a large $\beta_1$ from the beginning of training, we observe instabilities for larger $\beta_1$ values and no $\beta_1 > 0.9$ improves upon the AdamW baseline. One can imagine this to be due to increasing $\beta_1$ too early. Therefore, we also modify AdamW and add the same scheduler on $\beta_1$ as we use on AdEMAMix's $\beta_3$. $\beta_1$ is now increased steadily over the entire training duration. While this mostly stabilizes the training, none of the experiments outperformed the baseline using $\beta_1 = 0.9$. Those experiments show that simply increasing the $\beta_1$ value in AdamW is not enough, which justifies our design of AdEMAMix.

**Better perplexity for the same number of steps.** For all model sizes, AdEMAMix always outperforms the AdamW baseline. In Fig. 1, we show the validation loss curves for AdamW and AdEMAMix models trained on various numbers of tokens. For 110M parameter models, training for $256k$ iterations gives similar results as training an AdamW model for $500k$ iterations. For 1.3B parameter models, training using $770k$ steps is on par with training the baseline for 1.5M iterations.

**AdEMAMix models forget the training data slower.** When comparing the forgetting curves for AdamW and AdEMAMix in Fig. 2, we see striking differences. AdamW models forget much faster—the loss over $B$ increases faster—than AdEMAMix models. Moreover, at the end of training, batches processed by AdEMAMix see their loss being improved over many thousands of iterations.

## 5. Conclusion

In this work, we propose a novel optimizer which combines two momentum terms. A slow (large $\beta$) momentum gathers information over many timestep, while a fast (slow $\beta$) momentum can adapt the trajectory of the iterates to the rapidly changing loss landscape. We demonstrate the superiority of our optimizer over AdamW through a set of experiments on text modeling. We moreover reveal how our optimizer forgets the training data at a slower pace.

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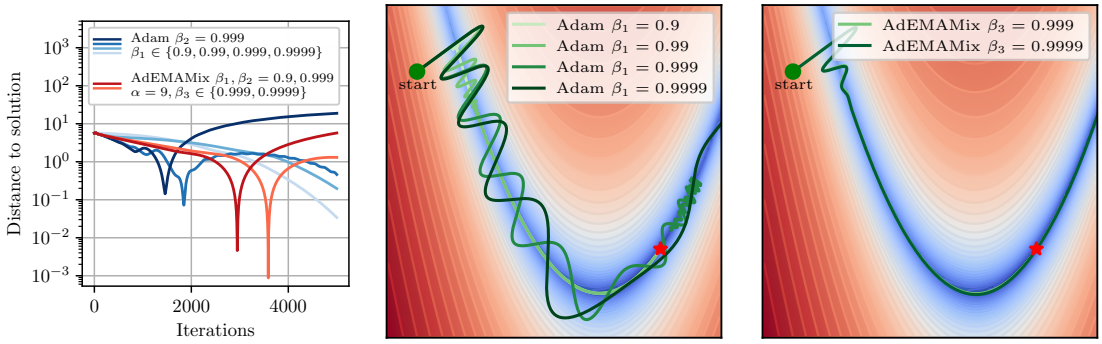
## Appendix A. Toy example




(*a*) $\|\boldsymbol{x}^{(t)} - \boldsymbol{x}^\star\|_2$. (*b*) Adam trajectories. (*c*) AdEMAMix trajectories.

Figure 3: **Comparing Adam and AdEMAMix on the Rosenbrock function.** Starting from $\boldsymbol{x}^{(0)} = [-3, 5]$, we minimize the Rosenbrock function $f(x_1, x_2) = (1 - x_1)^2 + 100(x_2 - x_1^2)^2$. The global minimum ($\star$) is $\boldsymbol{x}^\star = [1, 1]$. We use $\beta_2 = 0.999$ for Adam and $(\beta_1, \beta_2, \alpha) = (0.9, 0.999, 9)$ for AdEMAMix (see § 3). We reduce the learning rate for AdEMAMix to compensate for the influence of $\alpha$. We do a sweep over $\beta_1$ (resp. $\beta_3$) for Adam (resp. for AdEMAMix). In **(b)**, When Adam's $\beta_1$ is small (e.g. $0.9$), the iterates do not oscillate but convergence is slow. Increasing $\beta_1$ makes the iterates move faster but with large oscillations. In contrast, for AdEMAMix in **(c)**, we observe that despite $\beta_3$ being large, the iterates moves fast and without oscillations. This results in reaching better solutions faster as can be seen in **(a)**.

## Appendix B. Deriving the $\beta_3$ scheduler

Let's consider $S(t)$, the sum of the weights given to the last $t$ gradients by an EMA parameterized by $\beta \in [0, 1[$:

$$S(t) = (1 - \beta) \sum_{i=0}^{t} \beta^i$$

We want to know which timestep $t$ would correspond to a cumulative weight of $0.5$:

$$(1 - \beta) \sum_{i=0}^{t} \beta^i = 0.5 \Leftrightarrow \beta^{t+1} = 0.5 \Leftrightarrow t = \frac{\ln(0.5)}{\ln(\beta)} - 1$$

Let $f(\beta) = \frac{\ln(0.5)}{\ln(\beta)} - 1$. This function provides how many past consecutive gradients receive a cumulative weight of $0.5$.

Its inverse is:

$$f^{-1}(t) = 0.5^{\frac{1}{t+1}}$$

We want a scheduler which increases $\beta$ from $\beta_{start}$ to $\beta_{end}$ such that $f(\beta)$ increases linearly. Given an interpolating parameter $\mu \in [0, 1]$, this scheduler can be written as:

$$\beta(\mu) = f^{-1}((1 - \mu) f(\beta_{start}) + \mu f(\beta_{end}))$$

By replacing $f$ and $f^{-1}$ by their respective formula, one can arrive to:

$$\beta(\mu) = \exp\left(\frac{\ln(\beta_{start})\ln(\beta_{end})}{(1-\mu)\ln(\beta_{end}) + \mu\ln(\beta_{start})}\right)$$

By setting $\beta_{end} = \beta_3$ and $\mu = \frac{t}{T_{\beta_3}}$, we arrive to the $\beta_3$-scheduler introduced in § 3. We show the shape of our scheduler and compare it to a linear scheduler in Fig. 4.

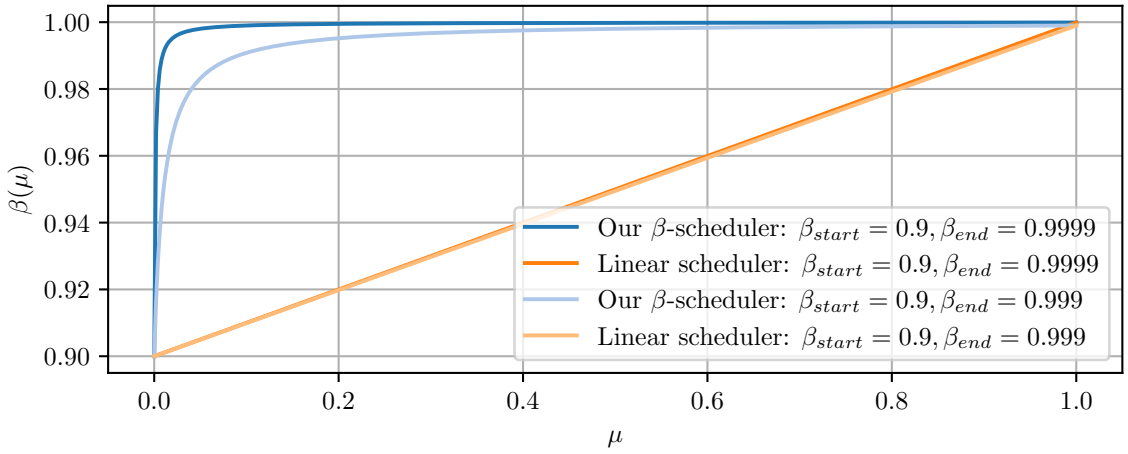



Figure 4: **AdEMAMix's $\beta_3$ scheduler.** We compare our scheduler to a linear scheduler for $\beta_{start} = 0.9$ and $\beta_{end} \in \{0.999, 0.9999\}$. While our scheduler looks more aggressive at first glance, it increases fast for smaller values of $\beta$, and slowly for larger ones associated. This makes sense as the same increase of $\beta$ for larger $\beta$ values has a bigger impact than that same increase applied to a smaller value of $\beta$. The two linear schedulers look practically the same, despite values of $\beta_{end}$ differing by one order of magnitude. This is not the case with our scheduler.