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Self-recovery effect of orbital angular momentum mode of circular beam in weak non-Kolmogorov turbulence

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Abstract: It is generally true that the orbital angular momentum (OAM) mode persistently degenerate when a vortex beam propagates in the atmospheric turbulence. Here, however, we unveil an interesting self-recovery effect of OAM mode of the circular beam (CiB) in weak non-Kolmogorov turbulence. We show that the CiB displays the self-focusing effect and has clear focus in the weak non-Kolmogorov turbulence if we choose proper complex parameters, and the detection probability of the original OAM mode reaches the maximum at the focus. Our study proposes a method to alleviate the turbulent effects on OAM-based communication.

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1. Introduction

In general, the OAM-carrying beam has the helical phase front with regard to a phase term of $\exp(\mathrm{i}l\varphi)$, where l is an arbitrary integer and φ refers to azimuthal coordinate [1]. Different coaxial OAM eigen-modes of vortex beam are orthogonal, which has important application in free space communication [2,3]. While, it is well known that the effect of atmosphere turbulence is one of the key factors that limit the development of OAM-based communication, because the turbulence effects cause the spreading of the spiral spectrum [4–6]. Recently, the effect of Kolmogorov turbulence on single photon and entangled photon pairs has been studied in detail [7,8]. Moreover, the effect of non-Kolmogorov turbulence on some vortex beams, such as hypergeometric-Gaussian Beam [9], Hankel-Bessel beam [10] and Whittaker-Gaussian beam [11], has been discussed as well.

Very recently, some attention is paid to circular beam (CiB) [12, 13], which is a very general solution of the paraxial wave equation. The equivalent form, special cases, normalization, Laguerre-Gaussian expansion, free-space divergence and parameter constraints of CiB have been discussed in previous works [12,13]. By assigning particular values to three complex beam parameters, the CiB is reduced to some well-known vortex beams such as standard Laguerre-Gaussian beam, elegant Laguerre-Gaussian beam, generalized Laguerre-Gaussian beam, hypergeometric beams, fractional-order elegant Laguerre-Gaussian beams, hypergeometric-Gaussian Beam and Whittaker-Gaussian beam [12].

The focusing property of beam can be employed in many fields, such as trapping and guiding microparticles [14] and realizing optical micromanipulation [15]. To our knowledge, currently, there is no further research to the influence of focusing property on the propagation of vortex beams in atmosphere turbulence. In this work, we discuss how the beam parameters affect the self-focusing property of CiB. Furthermore, we characterize the influence of non-Kolmogorov turbulence on the OAM modes of self-focusing CiB and reveal its self-recovery effect.

2. Power weight of OAM state for CiB in weak non-Kolmogorov turbulence

The normalized CiB propagating along z axis in cylindrical coordinates reads [13]

$$\begin{aligned}\mathrm{CiB}_{p,l_0}^{(q_0,q_1)}(r,\varphi,z) &= \left(\mathrm{i}\sqrt{2}\frac{z_0}{W_0}\right)^{|l_0|+1}\left[\pi\,|l_0|!\Psi_{p,l_0}^{(\xi)}\right]^{-\frac{1}{2}}\frac{1}{q(z)}\exp\left[-\frac{\mathrm{i}kr^2}{2q(z)}\right]\\ &\times\left[(1+\xi)\frac{\widetilde{q}(z)}{q(z)}\right]^{\frac{p}{2}}\left[\frac{r}{q(z)}\right]^{|l_0|}{}_1\mathrm{F}_1\left(-\frac{p}{2},|l_0|+1;\frac{r^2}{\chi^2(z)}\right)\\ &\times\exp(\mathrm{i}l_0\varphi)\end{aligned}\quad (1)$$

In Eq. (1), there are three complex parameters q_0, q_1 and p, where q_0 is given by $q_0 = \mathrm{i}z_0 - d_0$. W_0 is a real number similar to the waist radii of Gaussian beam and hence $z_0 = kW_0^2/2$ is similar to the Rayleigh range of wave number k. d_0 is the position of waist of Gaussian envelope. l_0 is an integer corresponding to the initial OAM quantum number. Besides, the other four complex parameters in Eq. (1) are defined as $q(z) = q_0 + z$, $\widetilde{q}(z) = q_1 + z$, $\xi = (q_1 - q_0)/(q_0^* - q_1)$ and $1/\chi^2(z) = \mathrm{i}k\left[1/q(z) - 1/\widetilde{q}(z)\right]/2$. $q(z)$ and $\widetilde{q}(z)$ are similar to the q-parameter of

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Gaussian beam and $1/\chi^2(z)$ is the scale factor. $\Psi_{p,l_0}^{(\xi)} = {}_2\mathrm{F}_1\left(-p/2, -p^*/2, |l_0|+1; |\xi|^2\right)$ is the normalization factor. ${}_1\mathrm{F}_1$ and ${}_2\mathrm{F}_1$ denote the confluent hypergeometric function and hypergeometric function, respectively.

In weak atmospheric turbulence, the intensity fluctuation is very small that can be neglected, so we only consider the phase aberration on the complex amplitude of CiB. Supposing propagation distance $z > 0$, the complex amplitude of distorted CiB here can be expressed as

$$\Psi_{p,l_0}^{(q_0,q_1)}(r,\varphi,z) = \mathrm{CiB}_{p,l_0}^{(q_0,q_1)}(r,\varphi,z)\cdot\exp[\mathrm{i}\psi(r,\varphi,z)], \tag{2}$$

where $\psi(r,\varphi,z)$ is the term corresponding to complex phase perturbation caused by the turbulence. To elucidate the weight of all OAM modes, the complex amplitude of distorted CiB in weak turbulence can be expressed as the superposition of spiral harmonics [16]

$$\Psi_{p,l_0}^{(q_0,q_1)}(r,\varphi,z) = \frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty}\beta_{p,l_0,l}^{(q_0,q_1)}(r,z)\exp(\mathrm{i}l\varphi) \tag{3}$$

with the expansion coefficient

$$\beta_{p,l_0,l}^{(q_0,q_1)}(r,z) = \frac{1}{\sqrt{2\pi}}\int_0^{2\pi}\mathrm{CiB}_{p,l_0}^{(q_0,q_1)}(r,\varphi,z)\exp[\mathrm{i}\psi(r,\varphi,z)]\exp(-\mathrm{i}l\varphi)\mathrm{d}\varphi. \tag{4}$$

Instead of the random variable $\beta_{p,l_0,l}^{(q_0,q_1)}(r,z)$, we are usually interested in the ensemble average over the turbulence statistics, i.e.

$$\begin{aligned}\left\langle\left|\beta_{p,l_0,l}^{(q_0,q_1)}(r,z)\right|^2\right\rangle = {} & \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi}\mathrm{CiB}_{p,l_0}^{(q_0,q_1)}(r,\varphi_1,z)\mathrm{CiB}_{p,l_0}^{(q_0,q_1)*}(r,\varphi_2,z)\\ & \times\exp[-\mathrm{i}l(\varphi_1-\varphi_2)]\\ & \times\langle\exp\{\mathrm{i}\,[\psi(r,\varphi_1,z)-\psi(r,\varphi_2,z)]\}\rangle\,\mathrm{d}\varphi_1\mathrm{d}\varphi_2\end{aligned}. \tag{5}$$

The ensemble average in right hand of Eq. (5) reads [17]

$$\langle\exp\{\mathrm{i}\,[\psi(r,\varphi_1,z)-\psi(r,\varphi_2,z)]\}\rangle = \exp\left[-\frac{2r^2-2r^2\cos(\varphi_1-\varphi_2)}{\rho_0^2}\right], \tag{6}$$

where ρ_0 denotes the spatial coherence radius of a spherical wave propagating in non-Kolmogorov turbulence [18]

$$\rho_0 = \left[\frac{8}{\alpha-2}\Gamma\left(\frac{2}{\alpha-2}\right)\right]^{\frac{1}{2}}\cdot\left[\frac{2(\alpha-1)\Gamma\left(\frac{3-\alpha}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{2-\alpha}{2}\right)k^2\mathrm{C}_n^2 z}\right]^{\frac{1}{\alpha-2}} \quad (3<\alpha<4), \tag{7}$$

where C_n^2 denotes refractive-index structure constant and α denotes non-Kolmogorov turbulence parameter. Equation (7) implies that $\rho_0 \to \infty$ if $\alpha \to 3$, and $\rho_0 \to 0$ if $\alpha \to 4$.

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Following Eqs. (5)-(7) and using Eq. 8.411.1 in [19], we can obtain

$$\begin{aligned}\left\langle\left|\beta_{p,l_0,l}^{(q_0,q_1)}(r,z)\right|^2\right\rangle = & \frac{1}{2^{|l_0|}\,|l_0|!\Psi_{p,l_0}^{(\xi)}}\left(\frac{kW_0}{|q(z)|}\right)^{2|l_0|+2}\left|\left[(1+\xi)\frac{\widetilde{q}(z)}{q(z)}\right]^p\right| \\ & \times \exp\left(-\frac{k^2r^2W_0^2}{2\,|q(z)|^2}\right)\left|{}_1F_1\left(-\frac{p}{2},|l_0|+1;\frac{r^2}{\chi^2(z)}\right)\right|^2 , \\ & \times r^{2|l_0|}\exp\left(-\frac{2r^2}{\rho_0^2}\right)\mathrm{I}_{l-l_0}\left(\frac{2r^2}{\rho_0^2}\right)\end{aligned} \tag{8}$$

where I_{l-l_0} is the modified Bessel function of the first kind with order $l - l_0$. By normalizing the detected power $P_l = \int_0^{+\infty}\left\langle\left|\beta_{p,l_0,l}^{(q_0,q_1)}(r,z)\right|^2\right\rangle r\mathrm{d}r$ in spiral harmonics with azimuthal number l, the corresponding power weight reads [16]

$$C_l = \frac{P_l}{\sum_{m=-\infty}^{\infty} P_m}. \tag{9}$$

Here C_l means the power weight of OAM mode with azimuthal number l and in OAM-based optical communication it's also the detection probability of OAM state in receiver. Especially, C_{l_0} is the power weight corresponding to the original OAM mode with azimuthal number l_0. Without turbulence and with infinite aperture, $C_{l_0} = 1$ but $C_{l\neq l_0} = 0$. In the presence of turbulence, the transmitted power leak into near modes and therefore $C_{l\neq l_0}$ is called as *crosstalk power weight* in context.

Because the crosstalk power weight is symmetric about l_0 due to $\mathrm{I}_n(x) = \mathrm{I}_{-n}(x)$, it's convenient to discuss the case of $l \geq l_0$ only. Under the condition of $l \geq l_0$, the infinite series form of C_l can be obtained using Eq. 6.622 in [19] as

$$\begin{aligned}C_l = & \frac{\mathrm{i}(-1)^{|l_0|+1}kW_0\rho_0}{\sqrt{2\pi}\,|q(z)|\cdot|l_0|!\Psi_{p,l_0}^{(\xi)}}\left(\frac{\eta-1}{\eta+1}\right)^{\frac{|l_0|}{2}+\frac{1}{4}}\left|\left[(1+\xi)\frac{\widetilde{q}(z)}{q(z)}\right]^p\right| \\ & \times\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\left(-\frac{p^*}{2}\right)_m\left(-\frac{p}{2}\right)_n}{(|l_0|+1)_m\,(|l_0|+1)_n\,n!m!} , \\ & \times\left[-\frac{\rho_0^2}{2(\chi^2)^*\,(\eta^2-1)^{\frac{1}{2}}}\right]^m\left[-\frac{\rho_0^2}{2\chi^2\,(\eta^2-1)^{\frac{1}{2}}}\right]^n \\ & \times Q_{l-l_0-\frac{1}{2}}^{|l_0|+n+m+\frac{1}{2}}(\eta)\end{aligned} \tag{10}$$

where $\eta = \frac{k^2W_0^2\rho_0^2}{4|q(z)|^2}+1$, $(*)_n$ denotes the Pochhammer symbol and $Q_\nu^\mu(*)$ denotes the associated Legendre functions of the second kind.

3. Numerical results and discussion

In this section, we discuss the relation of self-focusing effect to the parameters of CiB at first and then investigate the influence of non-Kolmogorov turbulence on the original OAM mode. W_0 relates to the scale of beam and is taken as 0.04m here. The imaginary part of q_1, which is similar to that of q_0, is taken as 1200m here. Besides, wavelength is taken as $\lambda = 1550$nm in the paper. For quantitative analysis of the focusing property of CiB, we use the second order

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momentum to describe the beam width as

$$W^2(z) = 4\left\langle r^2 \right\rangle = 4 \iint r^2 \left| \mathrm{CiB}_{p,l_0}^{(q_0,q_1)}(\vec{r}, z) \right|^2 \mathrm{d}^2\vec{r} \quad (z \geq 0), \tag{11}$$

where $\vec{r}$ denotes the point vector on transverse plane and $\langle * \rangle$ denotes the ensemble average. The waist, i.e. the minimal beam width, is found at $z = z_e$. z_e is also the position where the energy of beam is most concentrated. If $z_e > 0$m, the CiB converges at z_e and we call the beam is self-focusing in context. If z_e=0m, the CiB is not self-focusing.

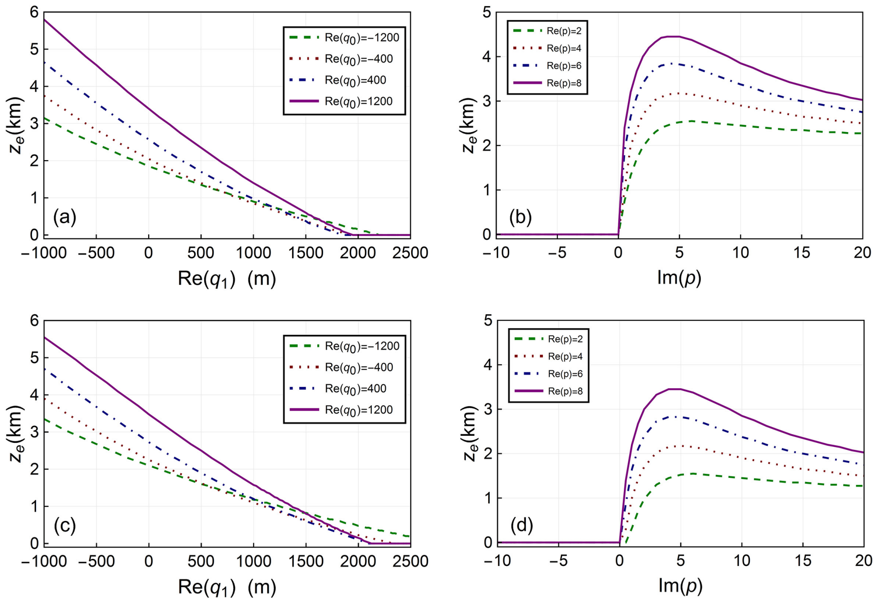

Fig. 1. The waist position z_e against (a) Re(q_0) and Re(q_1) with p=2+20i; (b) p with Re(q_0)=Re(q_1)=0m; (c) Re(q_0) and Re(q_1) with p=2+10i; (d) p with Re(q_0)=Re(q_1)=1000m. Other parameters: $l_0 = 1$, $C_n^2 = 10^{-15}\mathrm{m}^{3-\alpha}$ and $\alpha = 3.67$, respectively.

In Fig. 1, We investigate the effect of some parameters of the CiB on z_e. This curves In Figs. 1(a)-1(d) are non-differentiable at the joint point of z_e=0m and $z_e > 0$m due to the mandatory condition $z \geq 0$m in Eq. (11) resulting in $z_e \geq 0$m. In Figs. 1(a) and 1(c), we plot the waist position z_e against beam parameters Re(q_0) and Re(q_1). z_e decreases as Re(q_1) increases or Re(q_0) decreases. We observe an approximate linear relation between Re(q_1) and z_e, which shows how the waist of Gaussian envelope affects the waist of the CiB. If the value of Re(q_1) is greater than about 2000m, z_e reaches 0m that means the CiB is not self-focusing. From Figs. 1(a) and 1(c), We suppose that the value of z_e can be changed greatly if Re(q_1) is varied adequately. In Figs. 1(b) and 1(d), we plot the waist position z_e against beam parameter p. As Im(p) increases from 0 to 20, z_e increases at first and then decreases. z_e increases as Re(p) increases. It is clear that when the value of Im(p) is smaller than about 0, z_e is 0m, that is seemingly independent of Re(p). It seems that the value of z_e can but be changed into a limited range if only Im(p) is varied. The information contained in Fig. 1 indicate that one can manipulate z_e by varying p , Re(q_0) and Re(q_1).

Figure 2(a) shows the location of waist of one kind of self-focusing CiB in weak non-Kolmogorov turbulence. The location of waist of this CiB is at z_e=2264m. The difference between the beam width at z_e and the one at z=0m is about 100mm. The phase patterns of distorted CiB in weak non-Kolmogorov turbulence are shown in Figs. 2(b)-2(d). In these phase patterns, The gray scales from black to white correspond to the phase varying from 0 to 2π. Figure 2(b) shows the phase pattern at z=0m. This undistorted azimuthal phase increases counterclockwise while the phase contour lines twine clockwise. Figure 2(c) shows the distorted phase pattern at the waist position $z = z_e$=2264m. A serious distorted azimuthal spiral phase is difficult to recognize at farther position such as z=5000m shown in Fig. 2(d) .

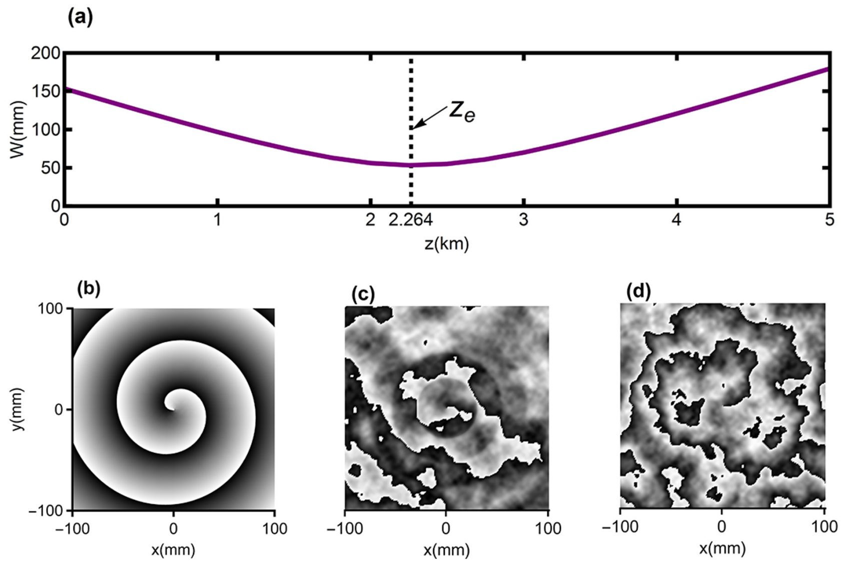

Fig. 2. (a) beam width $W(z)$ of the CiB against propagation distance z in non-Kolmogorov turbulence. Phase patterns of the CiB on $x - y$ plane of (b) z=0m, (c) z=2264m and (d) z=5000m in non-Kolmogorov turbulence. Other parameters: $C_n^2 = 10^{-15}\text{m}^{3-\alpha}$, α=3.67, q_1=1200i m, l_0=1 and p=2+20i, respectively.

In Fig. 3, we plot the received power weight C_{l_0} against propagation distance z in different cases. Figure 3(a) is plotted for different values of parameter p. All the lines show the similar character that as z increases, C_{l_0} decreases except for an arch near z_e. Since the previous works show that detection probability of initial OAM state decreases as the relative beam width $W(z)/r_0$ increases [7], where r_0 is Fried parameter, here the presence of arch can be explained as the case that the ratio $W(z)/\rho_0$ decreases as z increases. Around z_e, C_{l_0} varies slowly. This mechanism proposes a novel method to alleviate the effect of turbulence on free space communication. In other words, one can manipulate z_e according to the position of the receiver to obtain maximal C_{l_0} or alleviate the influence of atmospheric turbulence.

Figure 3(a) also shows that as $\text{Re}(p)$ increases, z_e increases, but meanwhile the value of C_{l_0} at z_e decreases. For example, $\text{Re}(p)$ increases from 2 to 8, z_e expands from about 2.25km to about 3.0km while C_{l_0} at z_e decreases from about 0.93 to 0.89. Parameter $\text{Re}(p)$ of CiB has similar meaning to the radial index of LGB, and bigger value of $\text{Re}(p)$ results in larger beam width that makes the OAM of CiB be more vulnerable to spatial aberrations in atmospheric turbulence.

Figure 3(b) shows that C_{l_0} decreases as α increases. With different α, the curves also show an arch near z_e, But with smaller α the curve is flatter and the arch is not clear. Figure 3(c) shows similar character that C_{l_0} decreases as C_n^2 increases. While turbulence is weak, e.g. $C_n^2 = 7.5 \times 10^{-16}\text{m}^{3-\alpha}$, C_{l_0} is almost 1 from 0km to 5km. In Figs. 3(b) and 3(c), it is obvious that the self-recovery effect of self-focusing CiB is more clear in the strong perturbation case i.e. large C_n^2 or α than that in the weak perturbation case. Figures 3(b) and 3(c) also show that z_e is almost

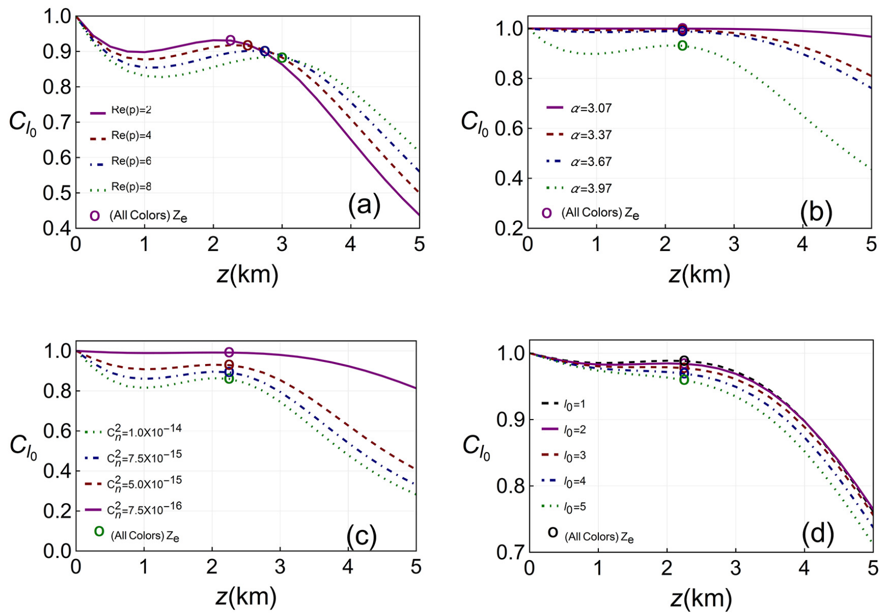

Fig. 3. The received power weight C_{l_0} for the CiB against propagation distance z (a) with different Re(p) where $C_n^2 = 10^{-15}\mathrm{m}^{3-\alpha}$, α=3.97, Im(p) = 20 and l_0=1; (b) with different α where $C_n^2 = 10^{-15}\mathrm{m}^{3-\alpha}$, l_0=1, p=2+20i; (c) with different C_n^2 where α=3.67, l_0=1, p=2+20i; (d) with different l_0 where $C_n^2 = 10^{-15}\mathrm{m}^{3-\alpha}$, α=3.67, p=2+20i. Other parameter: q_1=1200i m.

the same and not relevant to the weak atmospheric turbulence. This conclusion is approximative because the weak turbulence is presupposed to affect the phase of the beam only in Eq. (2). But this presupposition is unreasonable in strong turbulence.

Figure 3(d) is plotted for different l_0. C_{l_0} decreases as l_0 increases, because bigger l_0 means larger beam width $W(z)$ that makes the OAM of beam be more vulnerable to turbulence, which is similar to LGB propagating in atmospheric turbulence in [20]. From Fig. 3(d) we also find that as l_0=1 and 2, the arch emerges and the recovery effect of CiB arises. As l_0=3,4 and 5, the local arches are not so clear and C_{l_0} descends very slowly around z_e. Therefore, the resistance ability of OAM mode is weaker with larger l_0. To obtain better signal quality, it is better to employ lower OAM modes as channels in OAM-based free space communication. For example, the OAM modes with l_0=-4,-3...3,4 can be employed for coaxial transmission in condition of Fig. 3(d).

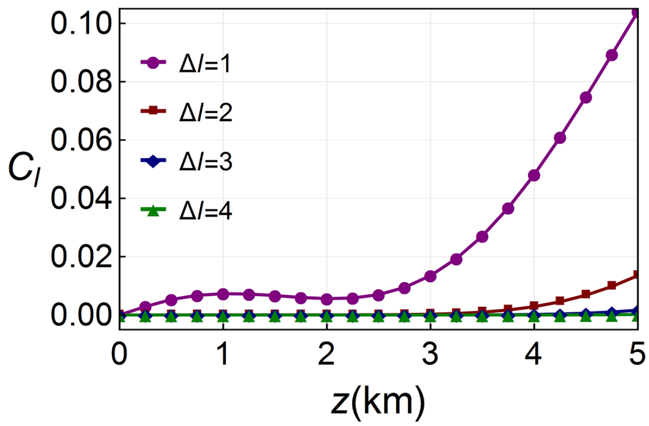

Fig. 4. The crosstalk power weight C_l for the CiB against propagation distance z with Δl=$|l - l_0|$=1,2,3,4. Other parameters: l_0=1, p=2+20i, q_1=1200i m, α=3.67 and C_n^2 = $10^{-15}\text{m}^{3-\alpha}$.

In Fig. 4, we consider the crosstalk power weight C_l of CiB mode. C_l decreases as $\Delta l = |l - l_0|$ increases. As z increases, C_l increases except a slight declining near z_e=2.264km. The reason for this declining is that received power weight C_{l_0} reaches its maximum near z_e and then less power from initial mode falls into other modes. As $\Delta l = 1$, this declining is most obvious. It is clear that the crosstalk power weights for Δl=2,3 or 4 are very small within short propagation distance and they can be omitted comparing with the crosstalk power weights for Δl=1. If some communication systems utilize CiB modes for short distance message transmission, the crosstalk is very small when nonadjacent modes are employed.

4. Conclusions

In this paper, we have revealed the self-focusing property and the self-recovery effect of the CiB in weak non-Kolmogorov turbulence. This work is carried out theoretically and numerically. Our results show that the appearance of self-focusing property of the CiB strongly depends on Re(q_1) and Im(p), and the location of waist of the CiB can be changed by varying Re(q_0),Re(q_1), Re(p) and Im(p). The key conclusion shows that the self-recovery effect may arise while the CiB has self-focusing property, which implies that self-focusing CiB may have more excellent resistance ability to turbulence effect on the position of waist than vortex beam with no self-focusing property. It is noticeable that the self-recovery effect is only available for the CiB with small azimuthal number l_0. Anyway, it proposes an opportunity to alleviate the turbulent effects on OAM-based optical communication.

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