Hard vs. Soft Contacts in Trajectory Optimization of Legged Robots: An Example with a Compliant Rubber Foot

Maximilian Raff[1]   C. David Remy[1]

***Abstract*— In this contribution to the ICRA 2023 workshop "Embracing Contacts", we compare the use of soft and rigid contact models in the simulation and trajectory optimization of legged locomotion. Specifically, we introduce a soft contact model that closely resembles the compliance of our legged robotic hardware [1] and carefully fit the model parameters to experimentally recorded data. Motion simulations of the robot RAM*bi* were more accurate when computed with this soft model compared to a standard rigid model. Furthermore, we performed a simple gait optimization with both contact models for a one-legged hopper with 7 degrees of freedom, showing that the soft contact model produced results similar to those obtained with a rigid contact model. For this particular hardware example, our work clearly breaks with the common wisdom that rigid contact models are superior in simulation and optimization of legged robotic systems. The results presented in this contribution are taken from [2].**

I. INTRODUCTION

Model-based gait optimization is typically performed on robot models that describe foot-ground interactions through fully rigid contacts (e.g., [3]–[5]). Soft contact models are often considered inferior to their rigid counterparts in this task, due to two main reasons. First, the parameters that describe foot-ground interaction in soft contact models are typically tuned empirically to achieve a desired dynamic behavior [6], which can result in nonphysical models. Second, soft contact models can generate stiff differential equations that are numerically challenging to solve, and their stiff dynamics can be difficult to use for optimization tasks due to the high sensitivity of gradient information.

In this paper, we aim to challenge these arguments using an actual robotic foot design as an example. Specifically, we address the following research questions:

- What is the minimum stiffness required for a soft contact model to achieve realistic dynamics?
- Which model type can better explain experimental data?
- Is it possible to use a soft contact model in trajectory optimization?

To address these questions, we specifically developed a soft contact model to represent the feet that are used in the legged robots Starl*ETH* [7], RAM*one* [8], and RAM*bi* [2]. These feet are spherical air filled racquetballs and made from soft rubber [1]. We then simulated motions of the robot RAM*bi* with this soft contact model as well as with a standard rigid model, and compared the resulting motion to experimentally recorded data. Finally, we performed a simple gait optimization on both models for a one-legged hopper with 7 degrees of freedom.

II. CONTACT MODELS

Our planar contact models were designed to represent spherical feet with radius $r_{\mathrm{foot}} = 2.82$ cm and mass $m_{\mathrm{foot}} = 0.075$ kg. The position of the center of the foot relative to the ground is denoted by c_{T} and c_{N}, respectively (Figure 4).

A. Hard Contact Model

The rigid contact model in the hybrid dynamics description is characterized by a holonomic non-slipping constraint while the foot is in contact with the ground. In particular, the normal contact force λ_{N} and distance to the ground $\delta_{\mathrm{N}} = c_{\mathrm{N}} - r_{\mathrm{foot}}$ are in accordance with Signorini's law: $0 \leq \lambda_{\mathrm{N}} \perp \delta_{\mathrm{N}} \geq 0$. Results of this model are shown in red.

B. Soft Contact Model

In the normal direction, our soft contact model was based on the experimental data of a series of 5 foot-drops reported in [1] and visualized in Figure 1. We identified the following structure:

$$\lambda_{\mathrm{N}}(\delta_{\mathrm{N}}, \dot{\delta}_{\mathrm{N}}) = \begin{cases} 0 & , \delta_{\mathrm{N}} \geq 0 \\ f_{\mathrm{s}}(\delta_{\mathrm{N}}) + f_{\mathrm{d}}(\dot{\delta}_{\mathrm{N}}) s(\delta_{\mathrm{N}}; \hat{\delta}) & , \hat{\delta} < \delta_{\mathrm{N}} < 0 \\ f_{\mathrm{s}}(\delta_{\mathrm{N}}) + f_{\mathrm{d}}(\dot{\delta}_{\mathrm{N}}) & , \delta_{\mathrm{N}} \leq \hat{\delta} \end{cases} \quad (1)$$

with the normal deformation $\delta_{\mathrm{N}} = c_{\mathrm{N}} - r_{\mathrm{foot}}$, quadratic spring force $f_{\mathrm{s}} = k_{\mathrm{N}} \delta_{\mathrm{N}}^2$ and nonlinear damping force $f_{\mathrm{d}} = d_{\mathrm{N},1}(-2(1 + e^{-\dot{\delta}_{\mathrm{N}}/d_{\mathrm{N},2}})^{-1} + 1)$. The stiffness and damping parameters k_{N} and $d_{\mathrm{N},1}$, $d_{\mathrm{N},2}$, respectively, determine the transition function $s = -2(\delta_{\mathrm{N}}/\hat{\delta})^3 + 3(\delta_{\mathrm{N}}/\hat{\delta})^2$ and the smoothing coefficient $\hat{\delta} = -\sqrt{3 d_{\mathrm{N},1}/k_{\mathrm{N}}}$. The transition function s serves two primary purposes. Firstly, it ensures that λ_N is always non-negative, thereby excluding nonphysical adhesive forces (sticking effects) that may be caused by the damping forces f_{d} during lift off. Secondly, the soft contact model is to be used in gradient-based optimization, which requires sufficient differentiability. Figure 2 shows a visual representation of (1).

Our model in the tangential direction is based on the model proposed in [9]. The parameters of this tangential model $\lambda_{\mathrm{T}}(\delta_{\mathrm{N}}, \delta_{\mathrm{T}}, \dot{\delta}_{\mathrm{T}})$ (equation (10) in [9]) were identified in walking experiments of RAM*bi*. Results of this model are shown in blue.

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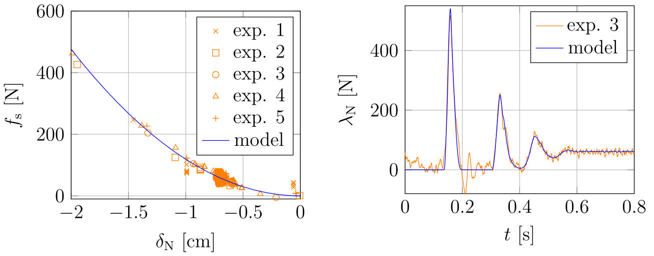
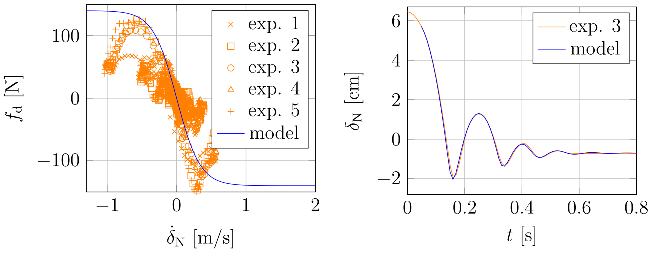
[1] The authors are with the Institute for Nonlinear Mechanics, University of Stuttgart, Germany. `{raff, remy}@inm.uni-stuttgart.de`



(a) spring force (b) normal force



(c) damping force (d) foot deformation

Fig. 1. Postprocessed data of a series of foot-drops of a rubber foot with radius $r_{\text{foot}} = 2.82$ cm [1]. (a) and (c) show how the model is fitted to the data. (b) and (d) compare the normal contact model (1) to a single foot-drop experiment.

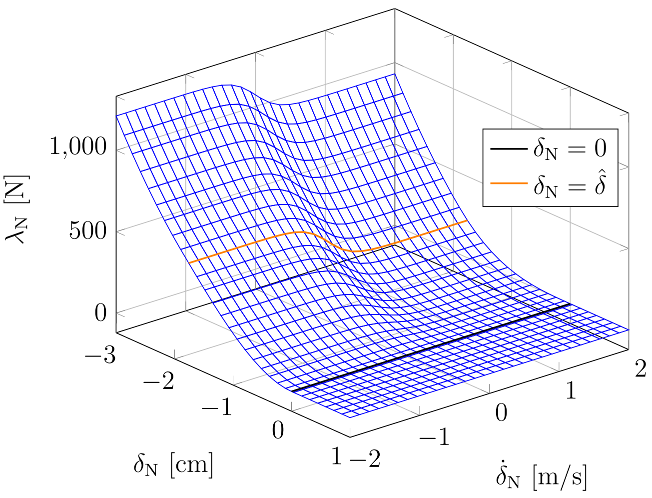
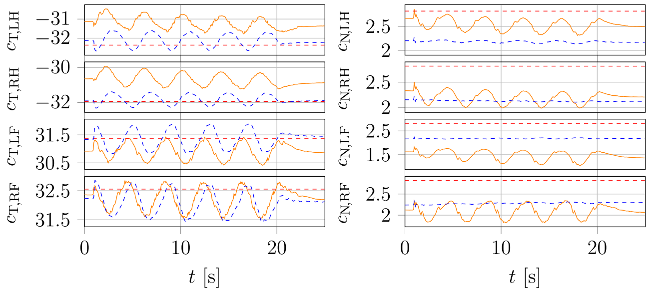


Fig. 2. Visualization of the normal contact model (1). Note, the model is continuous differentiable in δ_{N} and $\dot{\delta}_{\mathrm{N}}$. Furthermore, there are no adhesive forces (sticking effects) at lift off.

III. MODEL EVALUATION

While the normal contact model in [9] is tuned as a non-linear spring-damper model to approximate a hard contact model, our soft contact model (1) is deliberately designed and fitted to replicate experimental data (Figure 1). Still, the identified spring force appears to have the same quadratic stiffening effect as proposed in [9]. However, the identified quadratic stiffness $k_{\mathrm{N}} = 1282\ {}^{m_{\mathrm{foot}}g}/{}_{r^2_{\mathrm{foot}}}$ (gravity g) of our soft contact model is comparably low which clearly separates this model from a numerically smoothed formulation that ultimately seeks to approximate a hard contact.

Both models were implemented in a forward dynamic simulation of the quadrupedal robot RAM*bi* as depicted in Figure 3. While there are still model inaccuracies present (including estimation errors of the robot's geometry), the soft contact model resembled reality much closer.



(a) horizontal foot center location [cm] (b) vertical foot center location [cm]

Fig. 3. Open-loop simulation with the hard (- - -) and soft (- - -) contact model compared to measured data (—) of the quadruped RAM*bi* performing a squatting task. The feet are indexed by left (L) or right (R), followed by front (F) or hint (H).

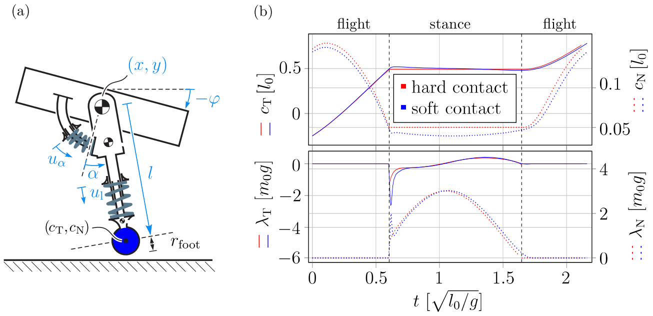


Fig. 4. (a): 7 degrees of freedom robot with series elastic actuators. (b): (Locally) optimal trajectories of both contact models with start and finish at apex-transit $y = 1.2$, $\dot{x}_{\text{periodic}} = 0.5$. The dashed vertical lines indicate the transition between phases of the hard contact model.

Finally, both contact models were employed in the gait optimization task of a monopedal robot (Fig. 4a) [4]. We chose the overall thermal losses in the motors as the objective function. To incorporate the hybrid dynamics due to the hard contact model, we used a direct collocation scheme with Hermite-Simpson splines similar to the one presented in [5]. Our implementation leverages the sparse structure of the problem, utilizes analytic gradients, and incorporates mesh-refinement to improve the accuracy of the system dynamics. For the rigid contact model, we defined a series of phases with discrete transitions, whereas a single phase was sufficient for the soft contact model.

The locally optimal gaits that we obtained for both contact models are presented in Figure 4b, and we observed that they are highly similar to each other.

IV. CONCLUSION

In conclusion, our study has demonstrated that a soft contact model can be a promising alternative to a rigid contact model, even when used for gait optimization. Our analysis has shown that a physically-realistic soft contact model can have substantial compliance, making it (1) more accurate representation of physical contact compared to the rigid model and (2) behaving reasonable well in terms of dynamics. Our optimization results indicate that the performance of the soft contact model is comparable to that of the hard contact model due to the small stiffening effect on the differential equation. Furthermore, the soft contact model is contact invariant, making it particularly advantageous for problems where the optimal contact sequence is unknown beforehand. In contrast, rigid contact models are generally less efficient in automatically discovering diverse gaits with varying contact sequences due to their discrete nature [6].

Our initial findings suggest that a sequential use of the soft and hard contact models in gait optimization may offer an effective approach. Further studies are necessary to validate the effectiveness of the soft contact model in more complex systems and to identify the most effective way to incorporate it into gait optimization algorithms.

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