A network analysis of collective motions in a three-species Vicsek model

Keywords: Vicsek model, collective motion, percolating network, non-reciprocal interaction, noise-induced phase separation

Extended Abstract

The conventional Vicsek model [1] describes the collective motion of self-propelled particles that align their direction with neighboring particles while being influenced by noise. Depending on particle density ρ and noise strength η, the system exhibits various macroscopic phenomena such as disordered motion and parallel alignment. This collective behavior has been studied even for systems with two different "species", where the same species follow the conventional model and different species can interact to be aligned or move in opposite directions [2, 3]. The interaction type between different species induces variations in collective motions such as parallel flocking, anti-parallel flocking, and cluster formation. However, real biological systems often exhibit more complex interactions beyond simple alignment, particularly in multi-species ecosystems where cyclic dominance relationships emerge. Examples include allelopathic interactions among competing plant species, bacterial communities with antibiotic production, and predator-prey cycles in ecological food webs. To capture such dynamics, we extend the Vicsek model to incorporate rock-paper-scissors (RPS)-type interactions among three species of self-propelled particles.

To extend the conventional model into a three-species RPS-type system, we introduce the interaction matrix J:

$$J = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix} = \begin{bmatrix} 1 & \alpha & \beta \\ \beta & 1 & \alpha \\ \alpha & \beta & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}, \tag{1}$$

where the cyclic competition among three different species is characterized by the two parameters α and β. These parameters represent the extent to which a species pursues/chases (positive) or avoids/escapes (negative) others [3]. We choose non-reciprocal interactions, $\alpha \neq \beta$, and the same unit interaction strength, resulting in $\alpha = -1, \beta = 1$. We investigate the collective patterns, focusing on the dynamic behavior in the presence of the non-reciprocal interactions [4].

We find three different characteristic patterns as the noise strength increases: i) In the weak noise regime, each species forms dense clusters. ii) At intermediate noise levels, the clusters become scattered, then each species interacts with the others and shows a wave-like pattern. This results in three bands of each species moving in parallel. iii) In the strong noise regime, all the particles move randomly, showing the gaseous phase [see the panels in Figure 1(b)]. By ignoring the species differences and measuring the orientational ordering, one can find the noise-induced ordering in the phase diagram as shown in Figure 1(a). As the noise strength increases, the orientational ordering emerges at a certain threshold, but this ordering disappears at high noise levels. These phases are also detected in the largest cluster size analysis using the percolation scheme [see Figure 1(c)].

Our findings provide insights into predator-prey relationships observed in biological systems and contribute to understanding the formation of collective motion patterns driven by non-reciprocal interactions in heterogeneous species.

References

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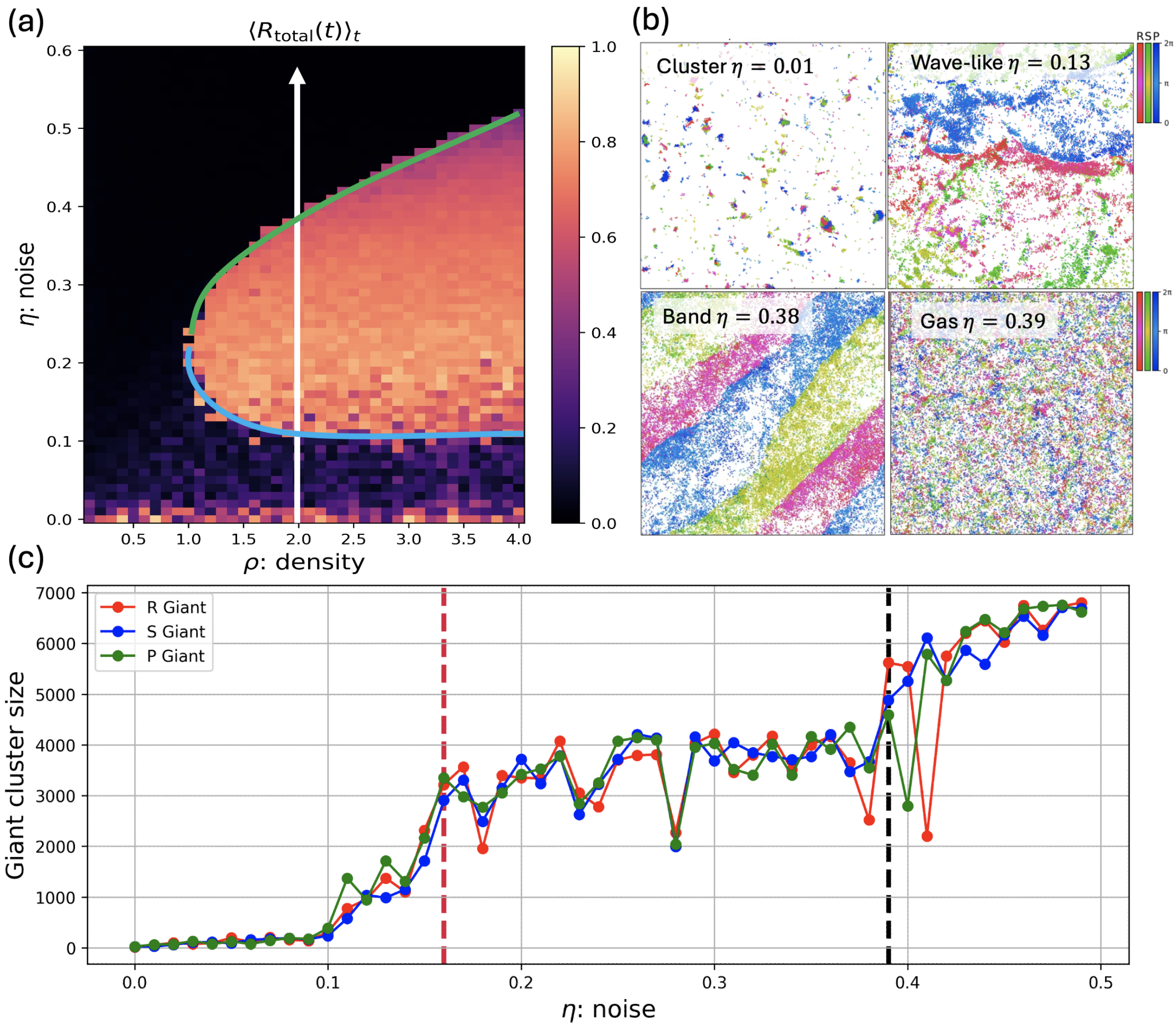

Figure 1: (a) **Phase diagram:** The orientational ordering is measured for different particle density ρ and noise strength η. For fixed $\rho = 2.0$, one can observe various collective patterns according to the noise level. (b) **Three phases:** Cluster phase ($\eta = 0.01$), wave-like intermediate motion at the phase boundary ($\eta = 0.13$), parallel moving band phase ($\eta = 0.38$), and gaseous phase ($\eta = 0.39$) appear as the noise level increases. (c) **Percolation analysis:** The three phases are also detected in the giant cluster size analysis for each species. In the band phase, each giant cluster size remains at 1/3 of the system size.