

# 000 001 002 003 004 005 006 007 008 009 010 011 012 013 014 015 016 017 018 019 020 021 022 023 024 025 026 027 028 029 030 031 032 033 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 ENHANCING GENERATIVE AUTO-BIDDING WITH OFFLINE REWARD EVALUATION AND POLICY SEARCH

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## ABSTRACT

Auto-bidding serves as a critical tool for advertisers to improve their advertising performance. Recent progress has demonstrated that AI-Generated Bidding (AIGB), which learns a conditional generative planner from offline data, achieves superior performance compared to typical offline reinforcement learning (RL)-based auto-bidding methods. However, existing AIGB methods still face a performance bottleneck due to their inherent inability to explore beyond the static offline dataset. To address this, we propose AIGB-Pearl (*Planning with EvaluAtor via RL*), a novel method that integrates generative planning and policy optimization. The core of AIGB-Pearl lies in constructing a trajectory evaluator for scoring generation quality and designing a provably sound KL-Lipschitz-constrained score maximization scheme to ensure safe and efficient exploration beyond the offline dataset. A practical algorithm incorporating the synchronous coupling technique is further devised to ensure the model regularity required by the proposed scheme. Extensive experiments on both simulated and real-world advertising systems demonstrate the state-of-the-art performance of our approach.

## 1 INTRODUCTION

The increasing demand for commercial digitalization has facilitated the development of the auto-bidding technique in online advertising. Distinguished from traditional manual bidding products, auto-bidding provides advertisers with an efficient and flexible scheme to automatically optimize bids in dynamic and competitive environments (Balseiro et al., 2021a; Deng et al., 2021; Balseiro et al., 2021b). Technically, auto-bidding can be viewed as an offline sequential decision-making problem, which aims to maximize the advertising performance over the bidding episode, relying solely on a static offline dataset due to system limitations (Mou et al., 2022), such as safety concerns.

As a standard approach to offline decision-making problems, offline reinforcement learning (RL) (Kumar et al., 2020) is widely adopted to solve the auto-bidding problem. By employing conservative policy search schemes, offline RL mitigates the infamous *out-of-distribution* (OOD) problem (Fujimoto et al., 2019), enabling reliable generalization beyond the offline dataset. However, due to the employment of bootstrapped value estimations, offline RL methods typically suffer from the training instability issue (Peng et al., 2024), risking policy performance degradation and instability.

Recent advances in generative models shed new light on offline decision-making problems (Zhu et al., 2023; Kang et al., 2023). Specifically, AI-generated bidding (AIGB) models the auto-bidding as a trajectory generation task and employs a generative model to approximate the conditional trajectory distribution of the offline dataset (Guo et al., 2024). AIGB avoids the requirement for bootstrapping and exhibits more stable training and superior performance. However, the modeling approach in AIGB does not explicitly align with the performance optimization objective of the auto-bidding problem, preventing it from actively improving toward higher performance. As a result, AIGB primarily imitates well-behaved trajectories from the offline dataset, lacking the ability to explore higher-quality trajectories beyond the offline data (Ajay et al., 2023).

Hence, there arises a question: *built on AIGB, the latest state-of-the-art auto-bidding method, can we devise a plausible scheme to involve policy optimization in its generative model?* To this end, a natural idea is to integrate offline RL methods into AIGB. However, it is nontrivial to implement in the auto-bidding problem since (i) there is a lack of reward signals in AIGB to guide the generative model. Specifically, the generation quality of the generative model remains unknown during

054 training, making it infeasible to explore new trajectories beyond the offline dataset; (ii) no dedicated  
 055 offline RL algorithm exists for AIGB. In particular, theoretical analysis that guarantees safe general-  
 056 ization and mitigates OOD issues for generative models in auto-bidding remains largely unexplored.  
 057

058 To address these critical challenges, we propose **AIGB-Pearl** (Planning with EvaluAtor via RL),  
 059 an RL-enhanced version of AIGB that learns a *trajectory evaluator* to score generation quality and  
 060 drive exploration of the generative model through continuous interaction. The evaluator is trained  
 061 through supervised learning on the offline dataset. Crucially, to mitigate the OOD problem and en-  
 062 sure reliable evaluator utilization, we examine the theoretical upper bound on the evaluator’s bias.  
 063 Then, guided by this analysis, we establish a KL-Lipschitz-constrained score maximization objec-  
 064 tive with a provable sub-optimality bound, enabling safe and effective exploration beyond the offline  
 065 data. Moreover, to perform constrained score maximization, we design a practical algorithm incor-  
 066 porating the *synchronous coupling* technique, which helps ensure the Lipschitz requirement of the  
 067 generative model. In addition, we note that AIGB-Pearl operates without the need for bootstrapping,  
 068 exhibiting greater training stability compared to offline RL methods.

069 To summarize, our contributions in this paper are fourfold: (i) we propose a novel generative auto-  
 070 bidding method, AIGB-Pearl, that enables continuous improvement in generation quality through  
 071 exploration beyond the offline dataset; (ii) we propose a provable KL-Lipschitz constrained score  
 072 maximization objective with a sub-optimality bound, ensuring a safe and effective generalization  
 073 beyond the offline dataset; (iii) we devise a practical algorithm with synchronous coupling that  
 074 effectively ensures the Lipschitz requirement for the generative model; (iv) extensive simulated and  
 075 real-world experiments demonstrate that AIGB-Pearl achieves SOTA performance and verify the  
 076 effectiveness of the developed techniques in enhancing safe and effective generalization.

## 077 2 PRELIMINARIES

### 079 2.1 PROBLEM STATEMENT

081 This work studies the auto-bidding problem for a single advertiser subject to a budget constraint.  
 082 The auction mechanism follows a sealed-bid, second-price rule. The objective is to devise a bidding  
 083 policy that maximizes the cumulative value of the impressions won over a finite bidding episode  
 084 (e.g., a day) within a budget  $B > 0$ . As established in (He et al., 2021), the optimal bid for each  
 085 impression  $i$  is proportional to its intrinsic value  $v_i > 0$ , scaled by a non-negative factor  $a \geq 0$  that  
 086 remains consistent across all impressions. Under this strategy, the advertiser wins an impression  $i$  if  
 087  $av_i \geq p_i$  and pays  $p_i$  upon winning, where  $p_i > 0$  is the market price. The Return on Investment  
 088 (ROI) of impression  $i$  is defined as  $v_i/p_i$ , and we denote its upper bound as  $R_m \triangleq \max_i v_i/p_i$ .

089 However, the scaling factor is unknown in advance, and the volatility of impressions drives its con-  
 090 tinual change throughout the bidding process. Hence, a standard practice involves recalibrating the  
 091 scaling factor  $a$  at fixed intervals of  $T \in \mathbb{N}_+$  time steps (Guo et al., 2024; He et al., 2021; Mou et al.,  
 092 2022). This casts the auto-bidding to a sequential decision-making problem.

093 Specifically, the auto-bidding problem can be modeled as a Markov Decision Process (MDP)  $< \mathcal{S}, \mathcal{A}, \mathcal{R}, \mathcal{P} >$ . The state  $s_t \triangleq [t, \bar{c}_{t-1}, x] \in \mathcal{S}$  is composed of the current time step  $t \in [T]$ , the  
 094 cost ratio  $\bar{c}_{t-1} = c_{t-1}/B > 0$  where  $c_{t-1}$  is the advertiser’s cost for impressions won between time  
 095 step  $t-1$  and  $t$ , and a static advertiser-specific feature  $x$  that includes the budget and many other  
 096 individual information. The action  $a_t \in \mathcal{A}$  denotes the calibrated scaling factor at time step  $t$ . The  
 097 reward  $r_t \geq 0$  describes the value of the impressions won between time steps  $t$  and  $t+1$ , and  $\mathcal{P}$   
 098 denotes the state transition rule. The auto-bidding problem can be formulated as:

$$101 \quad \max_{a_1, a_2, \dots, a_T} \mathbb{E}_{s_{t+1} \sim \mathcal{P}(\cdot | s_t, a_t)} \left[ \sum_{t=1}^T r_t \right], \quad \text{s.t.} \quad \sum_{t=1}^T c_t \leq B. \quad (1)$$

104 **Offline Setting.** Due to safety concerns—common in real-world advertising systems—we are re-  
 105 stricted to learning the optimal bidding policy from a static *offline dataset*  $\mathcal{D}$  composed of historical  
 106 states and actions along with associated rewards. This makes the considered auto-bidding problem  
 107 an offline sequential decision-making task.

108 2.2 OFFLINE RL METHODS  
109110 RL constitutes a standard approach for auto-bidding problems, seeking an optimal bidding policy  
111  $\pi : \mathcal{S} \rightarrow \mathcal{A}$  that maximizes cumulative rewards. Specifically, this is typically achieved by learning a  
112 Q-value function,  $Q(s_t, a_t) \triangleq \mathbb{E}_\pi[\sum_{t'=t}^T r_{t'}]$ , through temporal difference (TD) error minimization:  
113

114 
$$\min_Q \mathbb{E}_{(s_t, a_t, r_t, s_{t+1}) \sim \mathcal{D}} [Q(s_t, a_t) - r_t - \max_{a_{t+1}} \hat{Q}(s_{t+1}, a_{t+1})]^2, \quad (2)$$
  
115

116 where  $\hat{Q}$  is a target Q-value function with parameters updated via Polyak averaging (Mnih et al.,  
117 2015). Upon convergence, the optimal bidding policy is derived as  $\pi(s_t) = \arg \max_{a_t} Q(s_t, a_t)$ .  
118119 Due to the offline setting of the considered auto-bidding problem, directly employing Eq. 2 results  
120 in the infamous *out-of-distribution* (OOD) problem (Fujimoto et al., 2019), making the policy er-  
121 roneously deviate from the offline dataset  $\mathcal{D}$ . As a standard solution, offline RL (Yu et al., 2020;  
122 Kumar et al., 2020; Kidambi et al., 2020; Wang et al., 2022) constrains the policy’s behavior near  $\mathcal{D}$   
123 during TD learning, enabling reliable generalization beyond the offline dataset.  
124125 However, offline RL methods notoriously suffer from training instability caused by TD-learning  
126 (Peng et al., 2024), where the bootstrapped value of the Q function serves as its training label,  
127 resulting in an erroneous ground truth. Training stability is critical in auto-bidding due to the absence  
128 of an accurate offline policy evaluation method and the high cost of online policy examination in a  
129 real-world advertising system (Mou et al., 2022).  
130131 2.3 GENERATIVE AUTO-BIDDING METHODS  
132133 **Definition 1** (Trajectory and Trajectory Quality). *The trajectory is formalized as the state sequence  
134 throughout the bidding episode, i.e.,  $\tau \triangleq [s_1, s_2, \dots, s_T]$ . The trajectory quality is defined as the  
135 normalized cumulative reward of the trajectory, i.e.,  $y(\tau) \triangleq \sum_{t=1}^T \bar{r}_t^{-1}$ , where  $\bar{r}_t = r_t/B$ .*  
136137 Unlike RL methods, the AI-generated auto-bidding (AIGB) (Guo et al., 2024) treats the auto-bidding  
138 problem as a sequence generation task. Specifically, a conditional generative model is employed to  
139 fit the conditional trajectory distribution  $p_\theta(\tau|y(\tau))$  within the offline dataset  $\mathcal{D}$ , i.e.,  
140

141 
$$\max_\theta \mathbb{E}_{(\tau, y(\tau)) \sim \mathcal{D}} [\log p_\theta(\tau|y(\tau))], \quad (3)$$
  
142

143 where  $\theta$  denotes the parameter. Let  $y_m > 0$  be the maximum trajectory quality in  $\mathcal{D}$ , we have  $\forall y \in \mathcal{D}, y \in [0, y_m]$ . During inference, AIGB follows a *planning-and-control* architecture. Specifically,  
144 at each time step, a trajectory is sampled from the trained generative model that acts as the *planner*,  
145 with a manually set condition  $y^* \triangleq (1 + \epsilon)y_m$ , where  $\epsilon > 0$  is a hyper-parameter with usually a  
146 small value. Then, an extra off-the-shelf inverse dynamic model (Agrawal et al., 2016), acting as  
147 the *controller*, is employed to compute the action. See Appendix B for detailed descriptions. AIGB  
148 avoids TD learning and generally outperforms offline RL methods (Guo et al., 2024).  
149150 However, the modeling approach in AIGB modeling does not explicitly align with the performance  
151 optimization objective of the auto-bidding problem. As a result, AIGB primarily relies on imitating  
152 trajectories from the offline dataset, lacking the ability to actively explore higher-quality trajectories  
153 and improve its generation quality from the performance feedback. This limitation imposes a funda-  
154 mental performance ceiling. Particularly, as generative models (e.g., diffusion models) are prone to  
155 overfitting with limited data (Ajay et al., 2022), their generalization ability beyond the offline dataset  
156 can be restricted, especially when there is a lack of high-quality trajectories in the offline dataset.  
157158 3 METHOD  
159160 Enabling AIGB to explore higher-quality trajectories actively beyond the offline dataset can enhance  
161 its performance and generalization ability. To this end, we propose **AIGB-Pearl** (Planning with  
162 **EvaluAtoR** via **RL**) that constructs a *trajectory evaluator* (referred to as the evaluator for simplicity)  
163164 <sup>1</sup>Note that, as in real-world advertising systems, the bidding process will automatically suspend once the  
165 advertiser’s budget runs out, and thereby, any action sequence will not violate the budget constraint. Hence, the  
166 trajectory quality can be directly defined as the cumulative reward of the trajectory.  
167

162 to integrate RL methods into AIGB’s planner training. Specifically, the evaluator learns a *score*  
 163  $\hat{y}_\phi(\tau)$  to estimate the trajectory quality  $y(\tau)$  via supervised learning based on the offline dataset  $\mathcal{D}$ ,  
 164 i.e.,  $\min_\phi \mathbb{E}_{\tau \sim \mathcal{D}}[(\hat{y}_\phi(\tau) - y(\tau))^2]$ , where  $\phi$  denotes the evaluator parameter. Then, with  $\phi$  fixed, the  
 165 planner tries to maximize the score of its generation through iterative interactions with the evaluator,  
 166 as shown in Fig. 1. Formally, this can be formulated as:

$$\max_{\theta} L(\theta) \triangleq \mathbb{E}_{\tau \sim p_\theta(\tau|y^*)}[\hat{y}_\phi(\tau)], \quad (4)$$

169 where the condition is fixed to  $y^*$  in both training and inference stages to ensure consistency.

170 As can be seen, the effectiveness of AIGB-Pearl fundamentally hinges on the evaluator’s reliability.  
 171 However, given the offline nature of the considered auto-bidding problem, evaluator training is con-  
 172 fined to the fixed dataset  $\mathcal{D}$ . Directly solving Eq. 4 can induce the infamous OOD problem due to the  
 173 evaluator’s generalization limits, potentially degrading the planner’s true performance. Importantly,  
 174 there is a notable lack of theoretical approaches to this OOD problem.

175 To address this challenge, we examine the theoretical bounds on the evaluator’s bias. Then, guided  
 176 by this analysis, we propose a KL-Lipschitz-constrained score maximization objective for the plan-  
 177 ner in Section 3.1 to ensure reliable utilization of the evaluator. Notably, this objective is theoreti-  
 178 cally justified by a sub-optimality bound established in Section 3.1.1. Finally, a practical algorithm is  
 179 presented in Section 3.2, where a synchronous coupling method is employed to realize the planner’s  
 180 Lipschitz constraint.

### 182 3.1 KL-LIPSCHITZ-CONSTRAINED SCORE MAXIMIZATION

184 This section focuses on the reliable exploitation of the evaluator-guided score maximization.

185 **Our basic idea** is to optimize  $\theta$  within a domain where the gap between the planner’s score  $L(\theta)$   
 186 and its true performance  $J(\theta) \triangleq \mathbb{E}_{\tau \sim p_\theta(\tau|y^*)}[y(\tau)]$  is bounded by a small certifiable upper bound.  
 187 This ensures the score maximization occurs only in regions where the evaluator is reliable.

$$189 |J(\theta) - L(\theta)| = |\mathbb{E}_{\tau \sim p_\theta(\tau|y^*)}[y(\tau)] - \mathbb{E}_{\tau \sim p_\theta(\tau|y^*)}[\hat{y}_\phi(\tau)]|. \quad (5)$$

190 In the following, we investigate this gap. Specifically, we find that the trajectory quality  $y(\tau)$  is a  
 191 Lipschitz continuous function as stated in Theorem 1, and the proof is given in Appendix C.1.

192 **Theorem 1** (Lipschitz Continuous of  $y(\tau)$ ). *The trajectory quality  $y(\tau)$  is  $\sqrt{T}R_m$ -Lipschitz con-  
 193 tinuous with respect to the Frobenius norm.*

195 Motivated by Theorem 1, we enforce a  $\sqrt{T}R_m$ -Lipschitz regularity on the evaluator’s training to  
 196 inherit the Lipschitz continuity of the true trajectory quality  $y(\tau)$  (as described in Section 3.2.1).  
 197 Since the Lipschitz constant of the trained evaluator may not be exactly  $\sqrt{T}R_m$ , we denote its  
 198 value as  $k\sqrt{T}R_m$ , where  $k \geq 0$  reflects the violation degree. Note that a tighter satisfaction of the  
 199 Lipschitz constraint by the evaluator results in  $k$  closer to 1.

200 Equipped with Theorem 1 and the Lipschitz property of the evaluator, we derive the following upper  
 201 bound on the performance gap between  $J(\theta)$  and  $L(\theta)$ , and the proof is given in Appendix C.2.

202 **Theorem 2** (Evaluator Bias in Planning Performance Bound). *Let the upper bound of the evaluator’s  
 203 bias on its training dataset  $\mathcal{D}$  be  $\delta_D > 0$ , i.e.,  $\mathbb{E}_{\tau \sim \mathcal{D}}|y(\tau) - \hat{y}_\phi(\tau)| \leq \delta_D$ , and let the Lipschitz  
 204 constant of  $\hat{y}_\phi(\tau)$  be  $k\sqrt{T}R_m$ . The gap between the planner’s score  $L(\theta)$  and its true performance  
 205  $J(\theta)$  can be bounded by:*

$$207 |J(\theta) - L(\theta)| \leq \delta_D + (1 + k)\sqrt{T}R_m \mathbb{E}_{y \sim p_{\mathcal{D}}(y)} \left[ \underbrace{W_1(p_\theta(\tau|y^*), p_\theta(\tau|y))}_{\text{Lipschitz sensitivity to } y} + \underbrace{W_1(p_\theta(\tau|y), p_{\mathcal{D}}(\tau|y))}_{\text{imitation error on } \mathcal{D}} \right],$$

210 where  $W_1$  denotes the 1-Wasserstein distance.

211 Note that  $\delta_D$  could be regulated to a small value via supervised training of the evaluator, and  $k$   
 212 depends on the Lipschitz property of the resulting evaluator<sup>2</sup>. Consequently, bounding the evaluator  
 213 bias in the planner’s performance requires constraining the following two factors:

215 <sup>2</sup>Note that  $k$  cannot approach zero without compromising  $\delta_D$ , as excessively small  $k$  prevents the evaluator  
 from fitting the offline dataset  $\mathcal{D}$ .

216     • the planner’s Lipschitz sensitivity to condition  $y$  (the first Wasserstein term)  
 217     • the planner’s imitation error on the offline dataset (the second Wasserstein term).

219     Formally, we establish that the expectation of the first Wasserstein term can be bounded by the  
 220     Lipschitz constant  $\text{Lip}_{W_1}(p_\theta(\tau|y))$  of the planner with respect to the condition  $y$  measured by  $W_1$ :

$$222 \quad \mathbb{E}_{y \sim p_D(y)}[W_1(p_\theta(\tau|y^*), p_\theta(\tau|y))] \leq (1 + \epsilon)y_m \text{Lip}_{W_1}(p_\theta(\tau|y)). \quad (6)$$

223     The proof is given in Appendix C.3. Therefore, we constrain the planner’s Lipschitz constant to  
 224     a positive value  $L_p > 0$  to ensure the boundedness of the first Wasserstein term, where  $L_p$  is a  
 225     hyperparameter whose lower bound is provided later in Eq. 10.

226     Moreover, we establish that a constrained KL divergence  $\mathbb{E}_{y \sim p_D(y)}[D_{\text{KL}}(p_D(\tau|y) \| p_\theta(\tau|y))] \leq \delta_K$   
 227     could bound the expectation of the second Wasserstein distance term as follows, where  $\delta_K > 0$  is a  
 228     hyperparameter and can be set to a small value, close to zero. See Appendix C.4 for the proof. Note  
 229     that the KL divergence constraint here inherently makes the planner perform conditional behavior  
 230     cloning on the offline dataset  $\mathcal{D}$  (Guo et al., 2025).

$$232 \quad \mathbb{E}_{y \sim p_D(y)}[W_1(p_\theta(\tau|y), p_D(\tau|y))] \leq \sqrt{\delta_K}. \quad (7)$$

234     Collectively, to effectively perform score maximization with a small and certifiable evaluator bias,  
 235     we enforce Lipschitz continuity of the planner with respect to the condition  $y$  while preserving its  
 236     behavior cloning fidelity to the offline dataset  $\mathcal{D}$ . Formally, Eq. 4 is transformed to:

$$237 \quad \max_{\theta} L(\theta) \quad (\text{Score Maximization}) \quad (8)$$

$$239 \quad \text{s.t. } \mathbb{E}_{y \sim p_D(y)}[D_{\text{KL}}(p_D(\tau|y) \| p_\theta(\tau|y))] \leq \delta_K \quad (\text{KL Constraint}) \quad (8a)$$

$$240 \quad \text{Lip}_{W_1}(p_\theta(\tau|y)) \leq L_p \quad (\text{Lipschitz Constraint}) \quad (8b)$$

241     Eq. 8 forms the score maximization objective in AIGB-Pearl.

243     **Remark 1.** *Intuitively, the KL and Lipschitz constraints jointly ensure the planner’s generation  
 244     under condition  $y^*$  remains within a certified neighborhood of the offline dataset  $\mathcal{D}$ . Meanwhile, the  
 245     evaluator trained on  $\mathcal{D}$  maintains high accuracy within this  $\mathcal{D}$ -proximal region, and the Lipschitz  
 246     regularization on the evaluator bounds its sensitivity to input perturbations, preventing drastic value  
 247     fluctuations in OOD regions and promoting more reliable extrapolation. Therefore, Eq. 8 with a  
 248     Lipschitz-regularized evaluator can realize reliable evaluator-guided score maximization.*

### 249     3.1.1 SUB-OPTIMALITY GAP BOUND

251     This section focuses on presenting and analyzing the sub-optimality bound of the solution to the proposed Eq. 8. Specifically, denote the solution to the true performance  $J(\theta)$  as  $\theta^* \triangleq \arg \max_{\theta} J(\theta)$ ,  
 252     and denote the solution to the proposed Eq. 8 as  $\hat{\theta}$ . The following theorem gives the sub-optimality  
 253     bound of the planner’s performance, and the proof is given in Appendix C.5.

255     **Theorem 3** (Sub-optimality Gap Bound). *Let  $\delta_M \triangleq \mathbb{E}_{y \sim p_D(y)}[D_{\text{KL}}(p_D(\tau|y) \| p_{\theta^*}(\tau|y))]$  be the  
 256     expected distance between the optimal trajectory distribution and the trajectory distribution of the  
 257     offline dataset  $\mathcal{D}$ . The true performance gap between the optimal parameter  $\theta^*$  and the solution  $\hat{\theta}$   
 258     to Eq. 8 is bounded by:*

$$260 \quad J(\theta^*) - J(\hat{\theta}) \leq 2\delta_D + (1 + 2k)\sqrt{T}R_m \left[ \sqrt{\delta_M} + \sqrt{\delta_K} + (1 + \epsilon)y_m L_p \right]. \quad (9)$$

263     **Theoretical Result Analysis.** In Theorem 3, the constants  $\delta_M, R_m, T, \epsilon, y_m$  characterize domain-  
 264     specific properties of the auto-bidding task and the offline dataset  $\mathcal{D}$ . Nonetheless, a lower training  
 265     error  $\delta_D$  of the evaluator and a closer  $k$  to 1 correspond to a smaller sub-optimality gap. Note that  
 266      $k$  cannot be smaller than 1 without compromising  $\delta_D$ , as excessively small  $k$  prevents the evaluator  
 267     from fitting the offline dataset  $\mathcal{D}$ .

268     Moreover, in Theorem 3, a lower behavior cloning error  $\delta_K$  and a lower Lipschitz constant  $L_p$  of the  
 269     planner lead to a smaller sub-optimality gap. However, an excessively small  $L_p$  prevents the planner  
 270     from behavior cloning the offline dataset  $\mathcal{D}$  (as required by the KL constraint), resulting in a large

270  $\delta_K$ . Actually, a theoretical lower bound for  $L_p$  is given by the Lipschitz constant of the conditional  
 271 trajectory distribution of the offline dataset  $p_D(\tau|y)$ :

$$273 \quad L_p \geq \sup_{y_1 \neq y_2} \frac{W_1(p_D(\tau|y_1), p_D(\tau|y_2))}{|y_1 - y_2|}. \quad (10)$$

275 where  $y_1, y_2 \in \mathcal{D}$ . Consequently, we leverage this lower bound of  $L_p$  in AIGB-Pearl.  
 276

## 277 3.2 PRACTICAL ALGORITHM DESIGN

279 This section focuses on the practical algorithm implementation of Eq. 8. Section 3.2.1 first presents  
 280 our reliability-enhanced evaluator architecture, followed by the synchronous-coupling-based Lips-  
 281 chitz planner design in Section 3.2.2.  
 282

### 283 3.2.1 LIPSCHITZ TRAJECTORY EVALUATOR

285 As shown in Fig. 1, the evaluator processes the trajectory  $\tau$  to predict a score  $\hat{y}_\phi(\tau)$  for quality  
 286 estimation. The evaluator is trained via supervised learning using the offline dataset  $\mathcal{D}$ . Besides, to  
 287 satisfy  $\sqrt{T}R_m$ -Lipschitz constraint requirement according to Theorem 2, we add Lipschitz regular-  
 288 ization term to the training loss of the evaluator, which can be expressed as:

$$289 \quad l_e(\phi) = \underbrace{\mathbb{E}_{\tau \sim \mathcal{D}} [(\hat{y}_\phi(\tau) - y(\tau))^2]}_{\text{fitting the ground truth}} + \underbrace{\beta_1 \mathbb{E}_{\tau_1, \tau_2} [|\hat{y}_\phi(\tau_1) - \hat{y}_\phi(\tau_2)| - \sqrt{T}R_m \|\tau_1 - \tau_2\|_F]}_{\text{Lipschitz penalty}}_+, \quad (11)$$

293 where  $\beta_1 > 0$  is a hyper-parameter,  $[\cdot]_+ \triangleq \max\{0, \cdot\}$ . Moreover, to further enhance the accuracy of  
 294 the evaluator, we devise two specific techniques, including the LLM Embedding enhancement and  
 295 pair-wise learning, whose details are given in Appendix D.1.  
 296

### 297 3.2.2 LIPSCHITZ PLANNER WITH SYNCHRONOUS COUPLING

299 As shown in Fig. 1, the planner is implemented by a Causal Transformer (Chen et al., 2021b)  
 300 that generates trajectories in an auto-regressive manner. Specifically, the model takes the  
 301 condition  $y$  and history states  $s_{1:t}$  as input tokens, and predicts the next state as a Gaussian distribution,  
 302  $p_\theta(s_{t+1}|s_{1:t}, y) = \mathcal{N}(\mu_\theta(s_{1:t}, y, t), \sigma_\theta^2(s_{1:t}, y, t))$ , where  $\mu_\theta$  denotes the mean and  $\sigma_\theta > 0$  the stand-  
 303 ard deviation. During the auto-regressive generation process, each output state is sampled from the  
 304 Gaussian distribution using the reparameterization trick, i.e.,  $s_{t+1} = \mu_\theta(s_{1:t}, y, t) + \sigma_\theta(s_{1:t}, y, t) \cdot \eta_t$ ,  
 305 where  $\eta_t \sim \mathcal{N}(0, I)$ <sup>3</sup>.

306 **Regularized Planner Training Loss.** To perform the score maximization in Eq. 8, we involve two  
 307 regularization terms in the planner’s training loss  $l_p(\theta)$ , including a conditional behavior cloning  
 308 loss, corresponding to the KL constraint Eq. 8a, and a Lipschitz penalty loss, corresponding to the  
 309 Lipschitz constraint Eq. 8b, i.e.,

$$310 \quad l_p(\theta) = - \underbrace{\mathbb{E}_{\tau \sim p_\theta(\tau|y^*)} [\hat{y}_\phi(\tau)]}_{\text{planner score } L(\theta)} - \beta_2 \underbrace{\mathbb{E}_{(\tau, y) \sim p_D} [\log p_\theta(\tau|y)]}_{\text{conditional behavior clone}} \\ 311 \\ 312 \quad + \underbrace{\beta_3 \mathbb{E}_{y_1, y_2 \in \mathcal{D} \cup \{y^*\}} \left[ W_1(p_\theta(\tau|y_1), p_\theta(\tau|y_2)) - L_p |y_1 - y_2| \right]}_{\text{Lipschitz penalty, where } W_1(p_\theta(\tau|y_1), p_\theta(\tau|y_2)) \text{ is replaced by } \hat{W}_1(y_1, y_2; \theta)}_+, \quad (12)$$

317 where  $\beta_2, \beta_3 > 0$  are two hyper-parameters. With prior RL works (Sutton et al., 1999), we can  
 318 derive the closed-form expression of planner’s score gradient  $\nabla_\theta L(\theta)$  as shown in Appendix C.6.  
 319 The core of the planner loss lies in the computation of  $W_1(p_\theta(\tau|y_1), p_\theta(\tau|y_2))$ .

320 **Wasserstein Upper Bound as Surrogate.** Accurate computation of this Wasserstein distance  
 321 term is challenging, as it requires finding the optimal coupling between  $p_\theta(\tau|y_1)$  and  $p_\theta(\tau|y_2)$   
 322

323 <sup>3</sup>Note that the time step  $t$  and the static advertiser feature  $x$  in the state  $s_t = [t, c_{t-1}, x]$  do not need to be  
 324 generated. We only generate the next cost ratio  $c_t$  in practice.

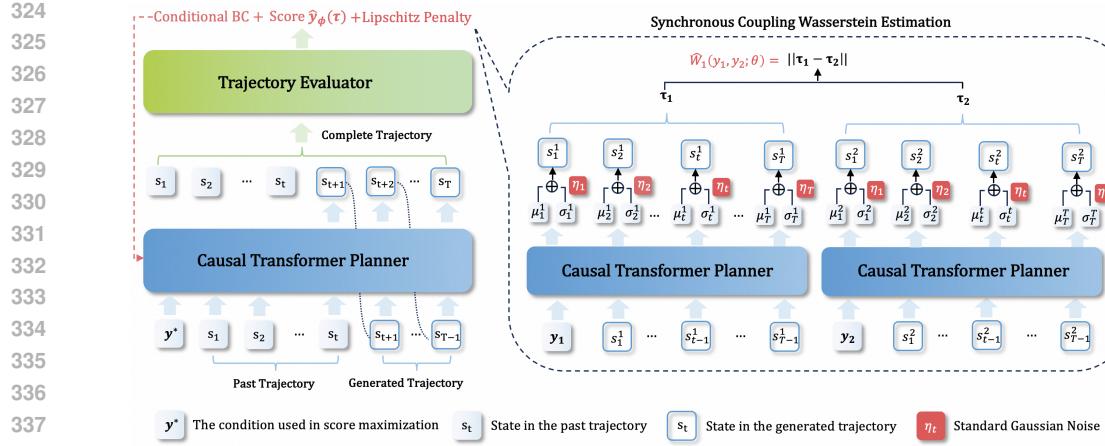


Figure 1: **AIGB-Pearl** (*Planning with EvaluAtor via RL*) constructs a trajectory evaluator to score the trajectory quality and let the planner maximize the obtained score under the KL-Lipschitz constraint through continuous interaction with the evaluator. A synchronous coupling method is used to estimate the Wasserstein term in the Lipschitz penalty.

that minimizes the expected transportation cost. Nonetheless, we can choose a certain coupling  $\gamma \in \Gamma(p_\theta(\tau|y_1), p_\theta(\tau|y_2))$  to obtain an upper bound of this Wasserstein term, i.e.,

$$\begin{aligned} W_1(p_\theta(\tau|y_1), p_\theta(\tau|y_2)) &\triangleq \inf_{\gamma \in \Gamma(p_\theta(\tau|y_1), p_\theta(\tau|y_2))} \mathbb{E}_{(\tau_1, \tau_2) \sim \gamma} \left[ \sum_t \|s_t^1 - s_t^2\| \right] \\ &\leq \mathbb{E}_{\eta_{1:T} \sim \mathcal{N}(0, I)} \left[ \sum_t \|s_t^1 - s_t^2\| \right] \triangleq \hat{W}_1(y_1, y_2; \theta). \end{aligned} \quad (13)$$

where  $\hat{W}_1(y_1, y_2; \theta)$  denotes the upper bound, and  $s_t^i$  is the  $t$ -th state in trajectory  $\tau_i$ . It can be seen that  $\hat{W}_1(y_1, y_2; \theta) \leq L_p|y_1 - y_2|$  acts as a sufficient condition to make the planner  $L_p$ -Lipschitz continuous. Thus, we replace  $W_1(p_\theta(\tau|y_1), p_\theta(\tau|y_2))$  by this upper bound in the planner loss.

**Synchronous Coupling Wasserstein.** Instead of using random couplings, we employ a *synchronous coupling*  $\gamma_{\text{sync}}$  to make the upper bound tighter. Specifically, two trajectories  $\tau_1$  and  $\tau_2$ —conditioned on  $y_1$  and  $y_2$ , respectively—are generated using the same sequence of Gaussian noise  $\{\eta_1, \eta_2, \dots, \eta_T\}$ . The definition of  $\hat{W}_1(y_1, y_2; \theta)$  is given in Eq. 13. Compared to random couplings, the synchronous coupling reduces spurious variance in the trajectory comparison by aligning stochasticity through shared noise, resulting in a tighter upper bound on the Wasserstein distance (Lindvall, 2002).

Moreover, if we make the predicted variance  $\sigma_\theta$  of the planner as a fixed constant, then the expression of  $\hat{W}_1(y_1, y_2; \theta)$  can be further simplified to  $\hat{W}_1(y_1, y_2; \theta) = \sum_t \|\mu_\theta(s_{1:t}^1, y_1, t) - \mu_\theta(s_{1:t}^2, y_2, t)\|$ . The overall AIGB-Pearl algorithm is summarized in Algorithm 1 in Appendix D due to page limits.

## 4 EXPERIMENTS

We conduct both simulated and real-world experiments to validate the effectiveness of our approach. In the experiments, we mainly investigate the following Research Questions (RQs): (1) Does enhancing AIGB with policy optimization improve overall performance, and can it generalize better to unseen data compared to existing AIGB methods? (Section 4.2) (2) How does the KL-Lipschitz constraint affect the performance of the planner? (Section 4.3) (3) Can the proposed method guarantee the Lipschitz property of the evaluator and the planner? (Section 4.4). (4) **What is the evaluator's accuracy on the training data, and how well does it generalize to unseen data?** (Section 4.5). The training stability of AIGB-Pearl is studied in Appendix E.5.

### 4.1 EXPERIMENT SETUP

**Experiment Environment.** We conduct simulated experiments in an open-source offline advertising system with 30 advertisers of four budget levels (1.5k, 2.0k, 2.5k, and 3.0k), as in (Mou et al., 2022; Guo et al., 2024). The offline dataset comprises 5k trajectories generated by 20 advertisers.

378 Table 1: Overall performance (GMV) in simulated experiments with 30 advertisers.  $\Delta$  indicates the  
 379 relative improvement of AIGB-Pearl against the most competitive baseline (which is underlined).  
 380 Note that the absolute values are normalized without specific meanings; only  $\Delta$  matters.

Budget	USCB	BCQ	CQL	IQL	Diff-QL	MOPO	DT	DiffBid	AIGB-Pearl	$\Delta$
1.5k	454.25	454.72	461.82	456.80	469.73	470.38	477.39	480.76	502.98	+4.62%
2.0k	482.67	483.50	475.78	486.56	487.91	489.27	507.30	511.17	521.84	+2.09%
2.5k	497.66	498.77	481.37	518.27	510.83	523.91	527.88	531.29	545.03	+2.59%
3.0k	500.60	501.86	491.36	549.19	552.73	549.01	550.66	556.32	574.17	+3.21%

386 Table 2: Overall performance in real-world A/B tests, involving 6k advertisers over 19 days.

Methods	GMV	BuyCnt	ROI	Cost	Methods	GMV	BuyCnt	ROI	Cost
<b>DiffBid</b>	76,390,174	650,962	5.31	14,395,290	<b>USCB</b>	52,182,805	516,994	4.92	10,598,486
<b>AIGB-Pearl</b>	78,676,009	665,173	5.41	14,551,054	<b>AIGB-Pearl</b>	53,973,101	520,796	5.13	10,515,772
$\Delta$	+3.00%	+2.20%	+1.89%	+1.10%	$\Delta$	+3.43%	+0.74%	+4.24%	-0.78%
Methods	GMV	BuyCnt	ROI	Cost	Methods	GMV	BuyCnt	ROI	Cost
<b>DT</b>	34,808,665	341,995	5.61	6,205,665	<b>MOPO</b>	51,674,071	579,332	3.08	16,771,892
<b>AIGB-Pearl</b>	35,957,933	344,194	5.77	6,246,512	<b>AIGB-Pearl</b>	53,292,945	591,741	3.23	16,475,670
$\Delta$	+3.30%	+0.64%	+0.16%	+0.66%	$\Delta$	+3.13%	+2.14%	+4.87%	-1.77%

396 Extra detailed settings of simulated experiments are given in Appendix E.1. For real-world experiments,  
 397 we conduct online A/B tests on one of the world’s largest E-commerce platforms, TaoBao.  
 398 The offline dataset comprises 200k trajectories of 10k advertisers. See Appendix E.2 for extra de-  
 399 tailed settings of real-world experiments. In both simulated and real-world experiments, we employ  
 400 the same inverse dynamics model from Agrawal et al. (2016) as used in AIGB to serve as the con-  
 401 troller. **Moreover, the evaluator is trained on the entire offline dataset, and its generalization ability**  
 402 **is evaluated using  $K$ -fold cross-validation with  $K = 5$ .**

403 **Baselines.** We compare our method with the state-of-the-art AIGB methods, including **DiffBid**  
 404 (Guo et al., 2024) and **DT** (Chen et al., 2021a) that learn from conditional behavior cloning of the  
 405 offline dataset with a diffusion model and a Causal Transformer, respectively. We also compare  
 406 our method with RL auto-bidding methods, including **USCB** (He et al., 2021) that learns the auto-  
 407 bidding policy in a manually constructed advertising system with DDPG (Silver et al., 2014); and  
 408 offline RL auto-bidding methods, including model-free offline RL methods **BCQ** (Fujimoto et al.,  
 409 2019), **CQL** (Kumar et al., 2020), **IQL** (Kostrikov et al., 2022) and **Diff-QL** (Wang et al., 2022),  
 410 and model-based offline RL method **MOPO** (Yu et al., 2020).

411 **Performance Index.** The objective in the auto-bidding problem Eq. 1, i.e., the cumulative rewards  
 412 over the bidding episode, acts as the main performance index in our experiments and is referred to  
 413 as the *gross merchandise volume*, **GMV**. In addition, we utilize three other metrics commonly used  
 414 in the auto-bidding problem to evaluate the performance of our approach. The first metric is the total  
 415 number of impressions won over the bidding episode, referred to as the **BuyCnt**. The second metric  
 416 is the **Cost** over the bidding episode, and the third one is the *return on investment* **ROI** defined as  
 417 the ratio between the GMV and the Cost. Note that larger values of GMV, BuyCnt, and ROI with a  
 418 Cost oscillating within an acceptable tolerance ( $\pm 2\%$ ) indicate a better performance.

## 420 4.2 OVERALL PERFORMANCE

422 **To answer RQ(1):** Table 1 shows that our method consistently outperforms all baselines in GMV  
 423 across all four budget levels in simulated experiments. In real-world experiments, the results in Table  
 424 2 show that our method also achieves superior performance in GMV, BuyCnt, and ROI, with Cost  
 425 fluctuations within 2%. Notably, both simulated and real-world experiments consistently demon-  
 426 strate that AIGB-Pearl achieves a +3% improvement in GMV over the AIGB, the state-of-the-art  
 427 auto-bidding method. Since our method and DiffBid share the same controller, the performance gain  
 428 stems solely from the planner. This provides strong empirical evidence that the proposed conserva-  
 429 tive RL learning for score maximization effectively enhances overall performance.

430 Notably, we also apply AIGB-Pearl to another important auto-bidding problem, named TargetROAS.  
 431 As shown in Appendix E.3, real-world experiments show that AIGB-Pearl achieves a +5% improve-  
 432 ment in GMV compared to AIGB.

432 Table 3: Generalization performance in real-world A/B tests with unseen advertisers against AIGB  
 433 methods, involving 4k advertisers over 19 days.

Methods	GMV	BuyCnt	ROI	Cost	Methods	GMV	BuyCnt	ROI	Cost
<b>DiffBid</b>	67,092,973	553,020	5.39	12,444,306	<b>DT</b>	30,562,007	300,271	5.61	5,450,573
<b>AIGB-Pearl</b>	69,252,539	565,776	5.53	12,534,379	<b>AIGB-Pearl</b>	31,502,309	305,202	5.74	5,484,473
$\Delta$	+3.32%	+2.31%	+2.48%	+0.72%	$\Delta$	+3.08%	+1.64%	+2.32%	+0.62%

438  
 439 Table 4: Ablation Study. The effectiveness of the KL constraint and the Lipschitz constraint in Real-  
 440 world A/B tests, involving 6k advertisers over 8 days.

AIGB-Pearl	GMV	BuyCnt	ROI	Cost	Methods	GMV	BuyCnt	ROI	Cost
w/o KL	30,906,963	292,605	4.25	7,269,018	w/o Lipschitz	32,284,972	268,551	5.73	5,634,304
with KL	31,243,688	292,783	4.26	7,342,485	with Lipschitz	32,869,329	281,979	5.79	5,678,252
$\Delta$	+1.09%	+0.06%	+0.08%	+1.01%	$\Delta$	+1.81%	+0.50%	+1.05%	+0.78%

445  
 446 It is worth noting that in auto-bidding systems, a **+3% GMV uplift** is highly significant, translating  
 447 to millions of RMB in additional **daily GMV** on Taobao-scale advertising platforms.

448  
 449 **Generalization Ability.** We examine the performance of AIGB-Pearl on advertisers that are not  
 450 used to generate trajectories in the offline dataset, comparing it against existing AIGB methods. For  
 451 simplicity, we refer to these advertisers as *advertisers outside the offline dataset*. Table 3 reports  
 452 the performance on 4k advertisers outside the offline dataset in real-world experiments. AIGB-  
 453 Pearl demonstrates consistently better results in terms of GMV (+3%), BuyCnt, and ROI, while  
 454 maintaining Cost fluctuations within 2% compared to baselines. These results indicate that the  
 455 proposed method has better generalization ability than existing AIGB methods.

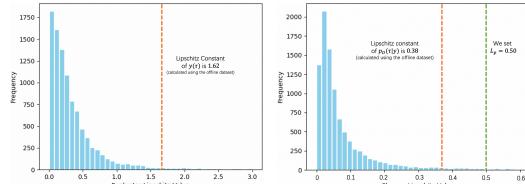
#### 4.3 ABLATION STUDY

477 To answer RQ(2): We remove the KL constraint and the Lipschitz constraint from AIGB-Pearl indi-  
 478 videntally and evaluate the model’s performance in each ablated variant with real-world A/B tests. The  
 479 results are presented in Table 4. It can be seen that the KL constraint contributes **+1.1%** improve-  
 480 ment in GMV, and the Lipschitz constraint provides **+1.8%** improvement in GMV, demonstrating  
 481 their respective roles in enhancing AIGB-Pearl’s performance.

482 **Visualization.** Three AIGB-Pearl-generated trajectory examples are presented in Fig. 2. As can  
 483 be observed, the trajectories generated by AIGB-Pearl are plausible. In contrast, the ablated variant  
 484 without the KL and Lipschitz constraints produces trajectories that significantly deviate from the  
 485 optimal trajectory in the offline dataset<sup>4</sup> and exhibit clear pathological behaviors—such as exces-  
 486 sive budget consumption, backward-trending pacing, and underutilization of available budgets (see  
 487 Appendix E.4 for explanation)—which further validate the KL-Lipschitz constraint necessity.

#### 4.4 LIPSCHITZ VALUE EXAMINATION

488 To answer RQ(3): We report that the Lips-  
 489 chitz value of the trajectory quality  $y(\tau)$  and the  
 490 conditional trajectory distribution  $p_D$  of the of-  
 491 fline dataset are 1.62 and 0.38, respectively. We  
 492 set  $L_p = 0.50$ , which is near its lower bound  
 493 estimation 0.38<sup>5</sup>. To calculate the Lipschitz  
 494 constants of the evaluator and the planner, we  
 495 sample 8,000 pairs of trajectories and compute  
 496 their Lipschitz constants. The results are shown  
 497 in Fig. 3 and Fig. 4. It can be observed that most sample values satisfy the Lipschitz constraint, and  
 498 the Lipschitz constants of the evaluator  $\hat{y}_\phi(\tau)$  and planner  $p_\theta(\tau|y)$  are 2.2 and 0.56, respectively,  
 499 near 1.62 and 0.50. This indicates that the Lipschitz constraints of the evaluator and the planner are  
 500 successfully satisfied.



488 Figure 3: Examination of Evaluator Lipschitz. Figure 4: Examination of Planner Lipschitz.

489 <sup>4</sup>The optimal trajectory in the offline dataset can act as a reference trajectory, and empirically, optimal  
 490 trajectories should not deviate from this reference largely.

491 <sup>5</sup>Considering that the lower bound estimation is computed solely in a data-driven manner and may be  
 492 underestimated, we conservatively increase its value accordingly.

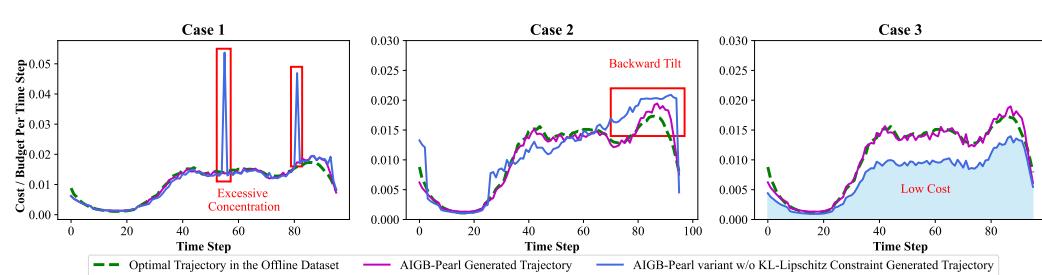


Figure 2: Trajectory Generation Visualization. Three cases are presented. Here, the AIGB-Pearl generates plausible trajectories, whereas its variant without the KL-Lipschitz constraint produces generations that significantly deviate from the reference and exhibit evident issues.

Table 5: Evaluator accuracy for simulated and real-world experiments. Results are reported for training data and OOD data evaluated using 5-fold cross-validation.

Simulated Exp	Training Data	OOD Data (Cross-Validation)	Real-world Exp	Training Data	OOD Data (Cross-Validation)
MAE ↓ AUC ↑	0.6 89.9%	0.7 ± 0.06 85.5% ± 0.5%	MAE ↓ AUC ↑	1.0 77.4%	1.2 ± 0.03 75.1% ± 0.2%

#### 4.5 EVALUATOR ACCURACY EXAMINATION

**Accuracy Metrics.** The evaluator’s accuracy is assessed along two dimensions, including the *absolute accuracy* measured by mean absolute error (MAE) metrics, reflecting how close the predicted scores are to ground truth scores, and the *ranking accuracy* by AUC metric, reflecting the correctness of relative rankings between trajectory pairs. Note that the MAE of each advertiser’s data sample is normalized by its budget to ensure comparability across advertisers. A lower MAE, together with a higher AUC, indicate better evaluator accuracy.

**To answer RQ(4):** We report the accuracy of the trained evaluator in both simulated and real-world experiments in Table 5. We evaluate the evaluator’s accuracy on the training data and its generalization ability using  $K$ -fold cross-validation, where  $K = 5$ . To the best of our knowledge, we are the first to introduce the trajectory evaluator into the generative auto-bidding framework. The reasonableness of our evaluator is evidenced by its pairwise ranking accuracy of 86% AUC and 75% AUC on OOD trajectories in the simulated and real-world experiments, respectively, which are substantially above the 50% random chance level, despite the high complexity and dynamic nature of the bidding environment. Importantly, with the guidance of the trained evaluator, the planner outperforms state-of-the-art AIGB methods even on OOD data, as demonstrated in the Table. 3.

## 5 CONCLUSIONS

This paper proposes AIGB-Pearl to enhance AIGB with reward evaluation and policy optimization. By introducing a trajectory evaluator and a provable KL-Lipschitz-constrained score maximization objective, our approach ensures safe and efficient generalization beyond the offline dataset, supported by theoretical guarantees. Extensive simulated and real-world experiments validate the SOTA performance of our approach.

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702 A RELATED WORKS  
703704 A.1 RL-BASED AUTO-BIDDING METHODS  
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706 Auto-bidding plays a critical role in online advertising by automatically placing bids, allowing ad-  
707 vertisers to participate efficiently in real-time auctions (Balseiro et al., 2021a; Deng et al., 2021;  
708 Balseiro et al., 2021b). The auto-bidding problem can be modeled as a Markov Decision Process  
709 and addressed using reinforcement learning techniques. USCB (He et al., 2021) proposes a uni-  
710 fied solution to the constrained bidding problem, employing an RL method, DDPG (Silver et al.,  
711 2014), to dynamically adjust parameters to an optimal bidding strategy. Mou et al. (2022) design a  
712 sustainable online reinforcement learning framework that iteratively alternates between online ex-  
713 plorations and offline training, alleviating the sim2rel problem. A few studies explore multi-agent  
714 RL for auto-bidding (Jin et al., 2018; Guan et al., 2021; Wen et al., 2022), while several focus  
715 on budget allocation and bidding strategies in multi-channel scenarios using RL-based approaches  
716 (Wang et al., 2023; Deng et al., 2023; Duan et al., 2025). Importantly, offline RL methods such as  
717 BCQ (Fujimoto et al., 2019), CQL (Kumar et al., 2020), IQL (Kostrikov et al., 2022), and MOPO  
718 (Yu et al., 2020) have demonstrated significant potential in this domain. These methods allow pol-  
719 icy learning from pre-collected datasets without requiring online interaction. Moreover, offline RL,  
720 such as Diffusion-QL (Wang et al., 2022), adopts generative models as the policy model architecture  
721 for better expressive capacity.

721 However, RL-based methods often suffer from training instability caused by bootstrapping and al-  
722 ternating training paradigms between critics and actors. Training instability typically deteriorates  
723 policy performance (Sutton et al., 1998). Moreover, training stability is even more critical in auto-  
724 bidding considering two domain-specific challenges: the absence of an accurate offline policy eval-  
725 uation method and the high cost of online policy examination in a real-world advertising system  
726 (Mou et al., 2022). Therefore, stable convergence to a well-performed policy is essential to ensure  
727 deployment reliability and system safety.

728  
729 A.2 GENERATIVE AUTO-BIDDING METHODS  
730

731 Generative models exhibit strong capabilities in capturing and replicating the underlying data distri-  
732 butions across a wide range of fields (Kingma & Welling, 2022; Goodfellow et al., 2020; Pan et al.,  
733 2023; Sohl-Dickstein et al., 2015; Ho et al., 2020; Vaswani et al., 2017). They can be effectively  
734 incorporated into decision-making systems by generating complete trajectories to guide agents to-  
735 ward high reward behaviors (Zhu et al., 2023; Kang et al., 2023; Li et al., 2025). In particular,  
736 Decision Transformer (DT) (Chen et al., 2021a) reframes RL as a conditional sequence modeling  
737 problem and leverages transformer architectures to generate actions conditioned on desired returns,  
738 historical states, and actions. AIGB (Guo et al., 2024) extends the generative perspective to the auto-  
739 bidding domain by formulating auto-bidding as a conditional generative modeling problem. DiffBid  
740 generates a state trajectory based on the desired return utilizing a conditional diffusion model, and  
741 then generates actions aligned with the optimized trajectory. These methods achieve superior per-  
742 formance in auto-bidding and offer distinct advantages over traditional RL methods. They do not  
743 rely on the bootstrapping mechanism commonly used in RL, thereby avoiding the instability caused  
744 by the deadly triad. Even so, these generative auto-bidding methods still encounter a performance  
745 bottleneck due to their neglect of fine-grained generation quality evaluation and inability to explore  
746 beyond static datasets. In contrast, our method facilitates both reward evaluation and policy search  
747 through a learned trajectory evaluator.

748 B AIGB METHOD DETAILS  
749

750 AIGB models the sequential decision-making problem through conditional diffusion modeling, en-  
751 abling effective trajectory generation for auto-bidding scenarios. Specifically, AIGB utilizes the  
752 denoising diffusion probabilistic model (DDPM) (Ho et al., 2020) for generation. The forward and  
753 reverse processes are modeled as:

$$q(\tau_{k+1}|\tau_k), \quad p_\theta(\tau_k|\tau_{k+1}, y(\tau)), \quad (14)$$

754 respectively, where  $q$  represents the forward noising process while  $p_\theta$  the reverse denoising process.  
755

756 **Forward Process.** In the forward process, the noise is gradually added to the latent variable by a  
 757 Markov chain with pre-defined variance schedule  $\beta_k$ :

$$759 \quad q(\tau_k | \tau_{k-1}) = \mathcal{N}(\tau_k; \sqrt{1 - \beta_k} \tau_{k-1}, \beta_k I) \quad (15)$$

760 where  $k \in [K]$  refers to the diffusion step,  $\tau_k \triangleq [s_1, s_2, \dots, s_T]_k$  represents the latent variable in  
 761 the  $k$ -th diffusion step, and  $\tau_0$  is the original trajectory. A notable property of the forward process is  
 762 that  $\tau_k$  at an arbitrary time-step  $k$  can be sampled in closed form as:

$$764 \quad q(\tau_k | \tau_0) = \mathcal{N}(\tau_k; \sqrt{\bar{\alpha}_k} \tau_0, (1 - \bar{\alpha}_k) I), \quad (16)$$

765 where  $\alpha_k = 1 - \beta_k$  and  $\bar{\alpha}_k = \prod_{i=1}^k \alpha_i$ . When  $k \rightarrow \infty$ ,  $\tau_k$  approaches a standard Gaussian  
 766 distribution. In particular, AIGB employs a cosine noise schedule (Nichol & Dhariwal, 2021) to  
 767 control the schedule  $\beta_k$ .

768 **Reverse Process.** In the reverse process, diffusion models aim to remove the added noise on  $\tau_K$  and  
 769 recursively recover  $\tau_0$ . This process is governed by the conditional model  $p_\theta(\tau_{k-1} | \tau_k, y(\tau))$ , which  
 770 is parameterized through a noise prediction model  $\epsilon_\theta(\tau_k, y(\tau), k)$ . AIGB adopts a temporal U-Net  
 771 (Ronneberger et al., 2015) for the noise prediction model, a common choice in diffusion-based  
 772 decision-making methods (Ajay et al., 2022).

## 774 B.1 TRAINING STAGE

776 The training of the diffusion model is typically formulated as minimizing the mean squared error  
 777 between the predicted noise  $\epsilon_\theta$  and the true noise applied during the forward diffusion process.  
 778 Specifically, during each iteration, we randomly sample a trajectory from the offline dataset  $\mathcal{D}$  and  
 779 pick a time step  $t \in [T]$ . We recursively add the Gaussian noise  $\epsilon$  to the states in  $\tau$  with time steps  
 780 bigger than  $t$  and predict the added noises with  $\epsilon_\theta(\tau_k, y(\tau), k)$ , where the states between 0 and  $t$   
 781 in  $\tau_k$  are set to real history states  $s_1, s_2, \dots, s_t$ . In addition to this standard objective, AIGB also  
 782 incorporates a supervised loss that measures the discrepancy between the true actions and the actions  
 783 predicted by an inverse dynamics model  $\hat{f}_\phi(s_t, \hat{s}_{t+1})$ . Overall, the complete training objective of  
 784 AIGB can be expressed as:

$$785 \quad \mathcal{L}(\theta, \phi) = \mathbb{E}_{k, \tau \in \mathcal{D}} [||\epsilon - \epsilon_\theta(\tau_k, y(\tau), k)||^2] + \mathbb{E}_{(s_t, a_t, \hat{s}_{t+1}) \in \mathcal{D}} [||a_t - \hat{f}_\phi(s_t, \hat{s}_{t+1})||^2]. \quad (17)$$

786 During training, the condition  $y(\tau)$  is randomly dropped to enhance model robustness. This technique  
 787 ensures that both the unconditional model  $\epsilon_\theta(\tau_k, k)$  and the conditional model  $\epsilon_\theta(\tau_k, y(\tau), k)$   
 788 are effectively trained together.

## 790 B.2 INFERENCE STAGE

792 Starting with Gaussian noise, trajectories are iteratively generated through a series of denoising  
 793 steps. Specifically, AIGB uses a classifier-free guidance strategy (Ho & Salimans, 2021) to guide  
 794 the generation of bidding and extract high-likelihood trajectories in the dataset. During generation,  
 795 AIGB combines conditional and unconditional score estimates linearly:

$$796 \quad \hat{\epsilon}_k := \epsilon_\theta(\tau_k, k) + \omega (\epsilon_\theta(\tau_k, y(\tau), k) - \epsilon_\theta(\tau_k, k)), \quad (18)$$

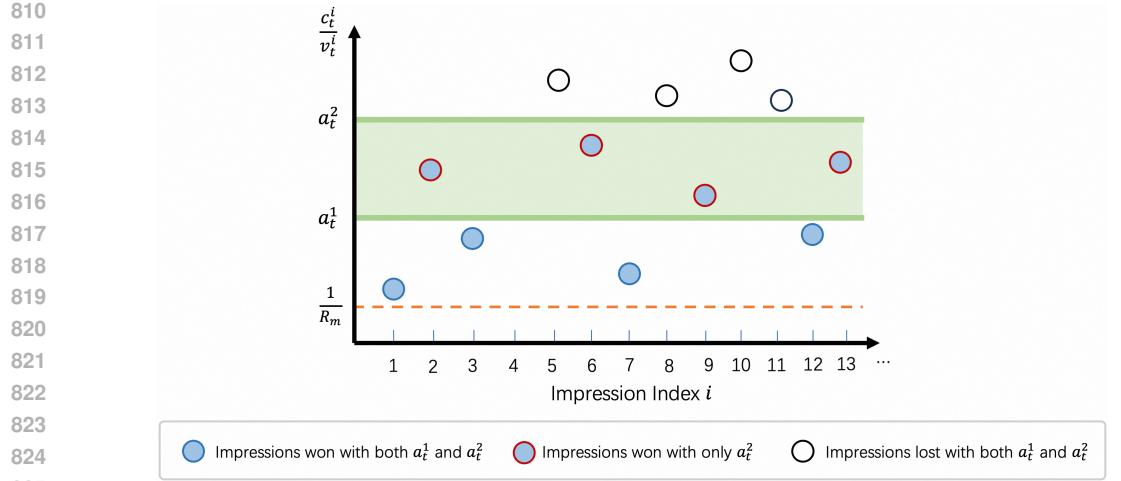
798 where  $\omega$  is the guidance scale that controls the influence of the condition  $y(\tau)$ . This formulation  
 799 effectively steers the trajectory generation towards regions of the data distribution most consistent  
 800 with the given condition. The predicted state at each step is sampled from  $p_\theta(\tau_{k-1} | \tau_k, y(\tau))$ :

$$801 \quad \tau_{k-1} \sim \mathcal{N}(\tau_{k-1} | \mu_\theta(\tau_k, y(\tau), k), \Sigma_\theta(\tau_k, k)), \quad (19)$$

802 with mean and variance defined as  $\mu_\theta(\tau_k, y(\tau), k) = \frac{1}{\sqrt{\alpha_k}} (\tau_k - \frac{\beta_k}{\sqrt{1 - \bar{\alpha}_k}} \hat{\epsilon}_k)$  and  $\Sigma_\theta(\cdot) = \beta_k$ . Note  
 803 that the initial noisy trajectory  $\tau'_K \sim \mathcal{N}(0, I)$  is assigned with history states  $s_{1:t}$  for the first  $t$  states  
 804 to ensure history consistency. This is consistent with the training process. By recursively applying  
 805 the reverse diffusion process using:

$$807 \quad \tau'_{k-1} = \mu_\theta(\tau'_k, y(\tau), k) + \sqrt{\beta_k} z, \quad (20)$$

808 where  $z \sim \mathcal{N}(0, I)$ , we obtain the final denoised trajectory  $\tau'_0$ , from which the next state  $\hat{s}_{t+1}$  is  
 809 derived. Then the action is generated through an inverse dynamics  $\hat{a}_t = \hat{f}_\phi(s_t, \hat{s}_{t+1})$ .



## C THEORETICAL PROOFS

### C.1 PROOF OF THEOREM 1.

**Theorem 1** (Lipschitz Continuous of  $y(\tau)$ ). *The trajectory quality  $y(\tau)$  is  $\sqrt{TR_m}$ -Lipschitz continuous with respect to the Frobenius norm.*

*Proof.* Recall from Section 2.1 that the cost  $c_t$  and reward  $r_t$  under action  $a_t$  between time step  $t$  and  $t+1$  can be written as:

$$c_t = \sum_i \mathbb{1} \left\{ a_t \geq \frac{p_t^i}{v_t^i} \right\} p_t^i \quad \text{and} \quad r_t = \sum_i \mathbb{1} \left\{ a_t \geq \frac{p_t^i}{v_t^i} \right\} v_t^i, \quad (21)$$

where  $p_t^i$  and  $v_t^i$  denote the market price and the value of the  $i$ -th impression between time step  $t$  and  $t+1$ . Accordingly, the cost ratio  $\bar{c}_t$  and the normalized reward  $\bar{r}_t$  can be written as:

$$\bar{c}_t = \frac{1}{B} \sum_i \mathbb{1} \left\{ a_t \geq \frac{p_t^i}{v_t^i} \right\} p_t^i \quad \text{and} \quad \bar{r}_t = \frac{1}{B} \sum_i \mathbb{1} \left\{ a_t \geq \frac{p_t^i}{v_t^i} \right\} v_t^i, \quad (22)$$

Consider two different trajectories  $\tau_1$  and  $\tau_2$  with actions, cost ratios and normalized rewards sequences  $\{a_{1,t}, \bar{c}_{1,t}, \bar{r}_{1,t}\}_{t=1}^T$  and  $\{a_{2,t}, \bar{c}_{2,t}, \bar{r}_{2,t}\}_{t=1}^T$ , respectively. The trajectory quality gap between  $\tau_1$  and  $\tau_2$  holds that:

$$|y(\tau_1) - y(\tau_2)| = \left| \sum_t \bar{r}_{1,t} - \sum_t \bar{r}_{2,t} \right| \leq \sum_t |\bar{r}_{1,t} - \bar{r}_{2,t}|. \quad (23)$$

Consider the reward gap between time step  $t$  and  $t+1$ , as shown in Fig.5. Without loss of generality, let  $a_{2,t} \geq a_{1,t}$ . We have:

$$\begin{aligned} |\bar{r}_{1,t} - \bar{r}_{2,t}| &= \frac{1}{B} \sum_i \left[ \mathbb{1} \left\{ a_{2,t} \geq \frac{p_t^i}{v_t^i} \right\} - \mathbb{1} \left\{ a_{1,t} \geq \frac{p_t^i}{v_t^i} \right\} \right] v_t^i \\ &= \frac{1}{B} \sum_i \mathbb{1} \left\{ a_{2,t} \geq \frac{p_t^i}{v_t^i} \geq a_{1,t} \right\} v_t^i \\ &= \frac{1}{B} \sum_i \mathbb{1} \left\{ a_{2,t} \geq \frac{p_t^i}{v_t^i} \geq a_{1,t} \right\} \frac{v_t^i}{p_t^i} p_t^i \\ &\leq \frac{R_m}{B} \sum_i \mathbb{1} \left\{ a_{2,t} \geq \frac{p_t^i}{v_t^i} \geq a_{1,t} \right\} p_t^i. \end{aligned} \quad (24)$$

Note that the cost ratio gap between time step  $t$  and  $t + 1$  can be written as:

$$|\bar{c}_{1,t} - \bar{c}_{2,t}| = \frac{1}{B} \sum_i \left[ \mathbb{1} \left\{ a_{2,t} \geq \frac{p_t^i}{v_t^i} \right\} - \mathbb{1} \left\{ a_{1,t} \geq \frac{p_t^i}{v_t^i} \right\} \right] p_t^i = \frac{1}{B} \sum_i \mathbb{1} \left\{ a_{2,t} \geq \frac{p_t^i}{v_t^i} \geq a_{1,t} \right\} p_t^i. \quad (25)$$

Therefore, combining Eq. 24 and Eq. 25, we have:

$$|\bar{r}_{1,t} - \bar{r}_{2,t}| \leq R_m |\bar{c}_{1,t} - \bar{c}_{2,t}|. \quad (26)$$

We examine the Frobenius norm of the gap between  $\tau_1$  and  $\tau_2$ :

$$\begin{aligned}
\|\tau_1 - \tau_2\|_F &= \left\| \begin{bmatrix} 1 & \bar{c}_{1,0} & x \\ 2 & \bar{c}_{1,1} & x \\ \vdots & \vdots & \vdots \\ T & c_{1,T-1} & x \end{bmatrix} - \begin{bmatrix} 1 & \bar{c}_{2,0} & x \\ 2 & \bar{c}_{2,1} & x \\ \vdots & \vdots & \vdots \\ T & \bar{c}_{2,T-1} & x \end{bmatrix} \right\|_F \\
&= \sqrt{\sum_t (\bar{c}_{1,t} - \bar{c}_{2,t})^2} \\
&\geq \frac{1}{\sqrt{T}} \sum_t |\bar{c}_{1,t} - \bar{c}_{2,t}| \quad \text{(Cauchy-Schwarz Inequality)} \quad (27)
\end{aligned}$$

Combining Eq. 23, Eq. 26 and Eq. 27, we can obtain that:

$$\begin{aligned}
|y(\tau_1) - y(\tau_2)| &\leq \sum_t |\bar{r}_{1,t} - \bar{r}_{2,t}| \\
&\leq R_m \sum_t |\bar{c}_{1,t} - \bar{c}_{2,t}| \\
&\leq \sqrt{T} R_m \frac{1}{\sqrt{T}} \sum_t |\bar{c}_{1,t} - \bar{c}_{2,t}| \\
&\leq \sqrt{T} R_m \|\tau_1 - \tau_2\|_F.
\end{aligned} \tag{28}$$

This concludes the proof.

## C.2 PROOF OF THEOREM 2

Here, we list two lemmas used in the proof of Theorem 2.

**Lemma 1** (Additivity of the Lipschitz). *Let  $f_1(x)$  and  $f_2(x)$  be two Lipschitz continuous functions with Lipschitz constants  $L_1 > 0$  and  $L_2 > 0$ , respectively. Then  $|f_1(x) + f_2(x)|$  is also a Lipschitz continuous function, with Lipschitz constant at most  $L_1 + L_2$ .*

*Proof.* Recall the Reverse Triangle Inequality states that  $\forall a, b$ , we have  $||a| - |b|| \leq |a - b|$ . Then,  $\forall x, y$ , we have:

$$\begin{aligned}
||f_1(x) + f_2(x)| - |f_1(y) + f_2(y)|| &\leq |f_1(x) + f_2(x) - f_1(y) - f_2(y)| \\
&\leq |f_1(x) - f_1(y)| + |f_2(x) - f_2(y)| \\
&< (L_1 + L_2)|x - y|. \tag{29}
\end{aligned}$$

This concludes the proof.

**Lemma 2** (Kantorovich-Rubinstein Duality Theorem (Villani, 2021)). *Let  $(X, d)$  be a metric space, and let  $p$  and  $q$  be two probability distributions on  $X$ . Let  $f : X \rightarrow \mathbb{R}$  be an  $L$ -Lipschitz function, and  $W_1(p, q)$  denotes the  $1$ -Wasserstein distance between  $p$  and  $q$ . Then we have:*

$$|\mathbb{E}[f(x)] - \mathbb{E}[f(x)]| \leq L \cdot W_1(p, q) \quad (30)$$

We next give the proof of Theorem 2.

**Theorem 2** (Evaluator Bias in Planning Performance Bound). *Let the upper bound of the evaluator’s bias on its training dataset  $\mathcal{D}$  be  $\delta_D > 0$ . The gap between the planner’s score  $L(\theta)$  and its true performance  $J(\theta)$  can be bounded by:*

$$|J(\theta) - L(\theta)| \leq \delta_D + (1+k)\sqrt{T}R_m \mathbb{E}_{y \sim p_D(y)} \left[ \underbrace{W_1(p_\theta(\tau|y^*), p_\theta(\tau|y))}_{\text{Lipschitz sensitivity to } y} + \underbrace{W_1(p_\theta(\tau|y), p_D(\tau|y))}_{\text{imitation error on } \mathcal{D}} \right],$$

where  $W_1$  denotes the 1-Wasserstein distance.

*Proof.* The evaluator bias in the planner’s performance can be written as:

$$|J(\theta) - L(\theta)| = |\mathbb{E}_{\tau \sim p_\theta(\tau|y^*)}[y(\tau) - \hat{y}_\phi(\tau)]| \leq \mathbb{E}_{\tau \sim p_\theta(\tau|y^*)} \underbrace{|y(\tau) - \hat{y}_\phi(\tau)|}_{\triangleq f(\tau)} \quad (31)$$

Let  $f(\tau) \triangleq |y(\tau) - \hat{y}_\phi(\tau)|$  be the evaluator bias in trajectory  $\tau$ . From Theorem 1 and Lemma 1, we know that  $f(\tau)$  is a  $(1+k)\sqrt{T}R_m$ -Lipschitz continuous function. Then, we have:

$$\begin{aligned} |J(\theta) - L(\theta)| &\leq \mathbb{E}_{\tau \sim p_\theta(\tau|y^*)} f(\tau) \\ &= \mathbb{E}_{y \sim p_D(y)} \left[ \mathbb{E}_{\tau \sim p_\theta(\tau|y^*)} f(\tau) - \mathbb{E}_{\tau \sim p_D(\tau|y)} f(\tau) + \mathbb{E}_{\tau \sim p_D(\tau|y)} f(\tau) \right] \\ &= \underbrace{\mathbb{E}_{y \sim p_D(y)} \mathbb{E}_{\tau \sim p_D(\tau|y)} f(\tau)}_{\leq \delta_D} + \underbrace{\mathbb{E}_{y \sim p_D(y)} \left[ \mathbb{E}_{\tau \sim p_\theta(\tau|y^*)} f(\tau) - \mathbb{E}_{\tau \sim p_D(\tau|y)} f(\tau) \right]}_{\leq (1+k)\sqrt{T}R_m W_1(p_\theta(\tau|y^*), p_D(\tau|y))}, \text{ (Lemma 2)} \\ &\leq \delta_D + (1+k)\sqrt{T}R_m \mathbb{E}_{y \sim p_D(y)} [W_1(p_\theta(\tau|y^*), p_D(\tau|y))]. \end{aligned} \quad (32)$$

To further enhance physical intuition and simplify constraint computation, we decompose the Wasserstein distance term  $W_1(p_\theta(\tau|y^*), p_D(\tau|y))$  via the triangle inequality:

$$W_1(p_\theta(\tau|y^*), p_D(\tau|y)) \leq W_1(p_\theta(\tau|y^*), p_\theta(\tau|y)) + W_1(p_\theta(\tau|y), p_D(\tau|y)). \quad (33)$$

Therefore, we have:

$$|J(\theta) - L(\theta)| \leq \delta_D + (1+k)\sqrt{T}R_m \mathbb{E}_{y \sim p_D(y)} \left[ W_1(p_\theta(\tau|y^*), p_\theta(\tau|y)) + W_1(p_\theta(\tau|y), p_D(\tau|y)) \right], \quad (34)$$

This concludes the proof.  $\square$

### C.3 PROOF OF EQ. 6

We give the proof of Eq. 6 as follows. Denote  $\text{Lip}_{W_1}(p_\theta(\tau|y))$  as the planner’s Lipschitz constant with respect to  $y$  regarding the Wasserstein distance  $W_1$ , we have:

$$\begin{aligned} \mathbb{E}_{y \sim p_D(y)} [W_1(p_\theta(\tau|y^*), p_\theta(\tau|y))] &\leq \text{Lip}_{W_1}(p_\theta(\tau|y)) \mathbb{E}_{y \sim p_D(y)} [((1+\epsilon)y_m - y)] \\ &= \text{Lip}_{W_1}(p_\theta(\tau|y)) \int_0^{y_m} p_D(y) [(1+\epsilon)y_m - y] dy \\ &\leq \text{Lip}_{W_1}(p_\theta(\tau|y)) \int_0^{y_m} p_D(y) [(1+\epsilon)y_m] dy \\ &= (1+\epsilon)y_m \text{Lip}_{W_1}(p_\theta(\tau|y)), \end{aligned} \quad (35)$$

where we leverage the non-negativity property of the condition  $y \geq 0, \forall y \in \mathcal{D}$ . This completes the proof.

### C.4 PROOF OF EQ. 7

**Lemma 3** (Pinsker’s Inequality (Tsybakov, 2008)). *Let  $P$  and  $Q$  be two probability measures defined on the same measurable space, and assume that  $P$  is absolutely continuous with respect to  $Q$ , i.e.,  $P \ll Q$ . Then the total variation distance between  $P$  and  $Q$  is bounded above by the KL divergence from  $P$  to  $Q$  as follows:*

$$\|P - Q\|_{TV} \leq \sqrt{\frac{1}{2} D_{KL}(P\|Q)}. \quad (36)$$

972 **Lemma 4** (Wasserstein–Total Variation Inequality on Bounded Metric Spaces (Villani et al., 2008)).  
 973 Let  $(\mathcal{Z}, d)$  be a metric space with diameter  $\text{diam}(\mathcal{Z}) \triangleq \sup_{z_1, z_2 \in \mathcal{Z}} d(z_1, z_2)$ . Let  $P$  and  $Q$  be two  
 974 probability measures on  $\mathcal{Z}$ . Then the 1-Wasserstein distance between  $P$  and  $Q$  satisfies:  
 975

$$976 \quad W_1(P, Q) \leq \text{diam}(\mathcal{Z}) \|P - Q\|_{\text{TV}}. \quad (37)$$

977 We give the proof of Eq. 7 as follows. Equipped with the above two lemmas, we have:  
 978

$$979 \quad W_1(p_\theta(\tau|y), p_D(\tau|y)) \leq \text{diam}(\mathcal{T}) \|p_\theta(\tau|y) - p_D(\tau|y)\|_{\text{TV}} \\ 980 \quad \leq \text{diam}(\mathcal{T}) \sqrt{\frac{1}{2} D_{\text{KL}}(p_D(\tau|y) \| p_\theta(\tau|y))}, \quad (38)$$

983 where  $\mathcal{T}$  is the trajectory space. Note that due to the budget constraint  $\sum_t c_t \leq B$ <sup>6</sup>, we have the  
 984 sum of the cost ratio satisfies  $\sum_t \bar{c}_t \leq 1$ . The trajectory space can be expressed as:  
 985

$$986 \quad \mathcal{T} = \left\{ \left[ [1, \bar{c}_0, x], [2, \bar{c}_1, x], \dots, [T, \bar{c}_{T-1}, x] \right] \middle| \bar{c}_t \geq 0, \forall t, \text{ and } \sum_t \bar{c}_t \leq 1 \right\} \quad (39)$$

988 We next prove that the diameter of the trajectory space,  $\text{diam}(\mathcal{T})$ , can be bounded by a constant.  
 989 Specifically, the diameter only depends on the largest possible distance between the cost ratio se-  
 990 quences in two trajectories since:  
 991

$$992 \quad \text{diam}(\mathcal{T}) = \sup_{\tau_1, \tau_2 \in \mathcal{T}} \|\tau_1 - \tau_2\|_F \\ 993 \\ 994 \quad = \sup_{\tau_1, \tau_2 \in \mathcal{T}} \left\| \begin{bmatrix} 1 & \bar{c}_{1,0} & x \\ 2 & \bar{c}_{1,1} & x \\ \vdots & \vdots & \vdots \\ T & \bar{c}_{1,T-1} & x \end{bmatrix} - \begin{bmatrix} 1 & \bar{c}_{2,0} & x \\ 2 & \bar{c}_{2,1} & x \\ \vdots & \vdots & \vdots \\ T & \bar{c}_{2,T-1} & x \end{bmatrix} \right\|_F \\ 995 \\ 996 \\ 997 \\ 998 \\ 999 \quad = \sup_{\tau_1, \tau_2 \in \mathcal{T}} \sqrt{\sum_t (\bar{c}_{1,t} - \bar{c}_{2,t})^2}. \quad (40)$$

1000 For convenience, we let  $\mathbf{c}_i \triangleq [\bar{c}_{i,0}, \bar{c}_{i,1}, \dots, \bar{c}_{i,T-1}]$ ,  $i \in \{1, 2\}$ . Then, the key part in the above  
 1001 result,  $\sum_t (\bar{c}_{1,t} - \bar{c}_{2,t})^2$ , can be written as:  
 1002

$$1003 \quad \sum_t (\bar{c}_{1,t} - \bar{c}_{2,t})^2 = \sum_t (\bar{c}_{1,t}^2 - 2\bar{c}_{1,t}\bar{c}_{2,t} + \bar{c}_{2,t}^2) \\ 1004 \\ 1005 \quad = \|\mathbf{c}_1\|_2^2 + \|\mathbf{c}_2\|_2^2 - 2\langle \mathbf{c}_1, \mathbf{c}_2 \rangle \\ 1006 \\ 1007 \quad \leq \|\mathbf{c}_1\|_2^2 + \|\mathbf{c}_2\|_2^2, \quad (41)$$

1008 where  $\langle \mathbf{c}_1, \mathbf{c}_2 \rangle \geq 0$ . As  $\bar{c}_{i,t} \geq 0$  and  $\sum_t \bar{c}_{i,t} \leq 1$ , we have  $0 \leq \bar{c}_{i,t} \leq 1$ . Therefore, it holds that:  
 1009

$$1010 \quad \|\mathbf{c}_i\|_2^2 = \sum_t \bar{c}_{i,t}^2 \leq \sum_t \bar{c}_{i,t} \leq 1. \quad (42)$$

1011 Combining Eq. 41 and Eq. 42, we have:  
 1012

$$1013 \quad \sqrt{\sum_t (\bar{c}_{1,t} - \bar{c}_{2,t})^2} \leq \sqrt{\|\mathbf{c}_1\|_2^2 + \|\mathbf{c}_2\|_2^2} \leq \sqrt{2}. \quad (43)$$

1014 Therefore, we have  $\text{diam}(\mathcal{T}) = \sqrt{2}$ . According to Eq. 38, we have:  
 1015

$$1016 \quad W_1(p_\theta(\tau|y), p_D(\tau|y)) \leq \sqrt{D_{\text{KL}}(p_D(\tau|y) \| p_\theta(\tau|y))}. \quad (44)$$

1017 Recall that we impose the KL-constraint as:  
 1018

$$1019 \quad \mathbb{E}_{y \sim p_D(y)} [D_{\text{KL}}(p_D(\tau|y) \| p_\theta(\tau|y))] \leq \delta_K, \quad (45)$$

1020 <sup>6</sup>As explained in Footnote 1, the budget constraint is guaranteed to be satisfied in real-world advertising  
 1021 systems thanks to an automatic suspension mechanism that halts bidding once the budget is exhausted.  
 1022

1026 Taking the expectation over  $y \sim p_D(y)$  on both sides of Eq. 44, we have:  
 1027

$$\begin{aligned} 1028 \mathbb{E}_{y \sim p_D(y)}[W_1(p_\theta(\tau|y), p_D(\tau|y))] &\leq \mathbb{E}_{y \sim p_D(y)}\left[\sqrt{D_{KL}(p_D(\tau|y)\|p_\theta(\tau|y))}\right] \\ 1029 &\leq \sqrt{\mathbb{E}_{y \sim p_D(y)}[D_{KL}(p_D(\tau|y)\|p_\theta(\tau|y))]} \quad (\text{Jensen Inequality}) \\ 1030 &\leq \sqrt{\delta_K} \quad (\text{KL constraint Eq.45}). \\ 1031 &\quad \quad \quad (46) \\ 1032 &\quad \quad \quad (46) \\ 1033 \end{aligned}$$

1034 This completes the proof.  
 1035

### 1036 C.5 PROOF OF THEOREM 3

1037 **Theorem 3** (Sub-optimality Gap Bound). *Let  $\delta_M \triangleq \mathbb{E}_{y \sim p_D(y)}[D_{KL}(p_D(\tau|y)\|p_{\theta^*}(\tau|y))]$  be the  
 1038 expected distance between the optimal trajectory distribution and the trajectory distribution of the  
 1039 offline dataset  $D$ . The true performance gap between the optimal parameter  $\theta^*$  and the solution  $\hat{\theta}$   
 1040 to Eq. 8 is bounded by:*

$$1042 \quad J(\theta^*) - J(\hat{\theta}) \leq 2\delta_D + (1 + 2k)\sqrt{T}R_m\left[\sqrt{\delta_M} + \sqrt{\delta_K} + (1 + \epsilon)y_mL_p\right]. \quad (47)$$

1043 *Proof.* The sub-optimality gap can be expressed as follows:  
 1044

$$\begin{aligned} 1045 \quad J(\theta^*) - J(\hat{\theta}) &= (J(\theta^*) - L(\theta^*)) + (L(\theta^*) - L(\hat{\theta})) + (L(\hat{\theta}) - J(\hat{\theta})) \\ 1046 &\leq \underbrace{|J(\theta^*) - L(\theta^*)|}_{\text{evaluator bias in } p_{\theta^*}} + \underbrace{|L(\theta^*) - L(\hat{\theta})|}_{\text{score gap}} + \underbrace{|L(\hat{\theta}) - J(\hat{\theta})|}_{\text{evaluator bias in } p_{\hat{\theta}}}. \\ 1047 &\quad \quad \quad (48) \\ 1048 &\quad \quad \quad (48) \\ 1049 &\quad \quad \quad (48) \\ 1050 &\quad \quad \quad (48) \end{aligned}$$

1051 We examine the above three terms accordingly.  
 1052

1053 **(1) Evaluator Bias in  $p_{\theta^*}$ .** Denote the evaluator bias on trajectory  $\tau$  as  $f(\tau) \triangleq |y(\tau) - \hat{y}_\phi(\tau)|$ .  
 1054 Following the derivation process in Eq. 32, we have:

$$\begin{aligned} 1055 \quad |J(\theta^*) - L(\theta^*)| &\leq \mathbb{E}_{y \sim p_D(y)}\mathbb{E}_{\tau \sim p_D(\tau|y)}f(\tau) + \mathbb{E}_{y \sim p_D(y)}\left[\mathbb{E}_{\tau \sim p_{\theta^*}(\tau|y^*)}f(\tau) - \mathbb{E}_{\tau \sim p_D(\tau|y)}f(\tau)\right] \\ 1056 &\leq \delta_D + (1 + k)\sqrt{T}R_m\mathbb{E}_{y \sim p_D(y)}[W_1(p_{\theta^*}(\tau|y^*), p_D(\tau|y))], \\ 1057 &\quad \quad \quad (49) \\ 1058 &\quad \quad \quad (49) \\ 1059 &\quad \quad \quad (49) \end{aligned}$$

1060 where  $W_1(p_{\theta^*}(\tau|y^*), p_D(\tau|y))$  denotes the probability distribution distance between the optimal  
 1061 planner and the offline dataset. Based on the derivation in Appendix C.4, we have:  
 1062

$$\mathbb{E}_{y \sim p_D(y)}[W_1(p_{\theta^*}(\tau|y^*), p_D(\tau|y))] \leq \sqrt{\mathbb{E}_{y \sim p_D(y)}[D_{KL}(p_D(\tau|y)\|p_{\theta^*}(\tau|y))]} \quad (50)$$

1063 Let  $\delta_M \triangleq \mathbb{E}_{y \sim p_D(y)}[D_{KL}(p_D(\tau|y)\|p_{\theta^*}(\tau|y))]$  be the distance between the optimal trajectory distribution  
 1064 and the offline dataset trajectory distribution. We have:  
 1065

$$|J(\theta^*) - L(\theta^*)| \leq \delta_D + (1 + k)\sqrt{T}R_m\sqrt{\delta_M}. \quad (51)$$

1066 **(2) Score Gap.** Recall that the trained evaluator  $\hat{y}_\phi(\tau)$  is a  $k\sqrt{T}R_m$ -Lipschitz continuous function  
 1067 with the Lipschitz constraint design. With Lemma 2, we have:  
 1068

$$\begin{aligned} 1069 \quad |L(\theta^*) - L(\hat{\theta})| &= |\mathbb{E}_{\tau \sim p_{\theta^*}(\tau|y^*)}\hat{y}_\phi(\tau) - \mathbb{E}_{\tau \sim p_{\hat{\theta}}(\tau|y^*)}\hat{y}_\phi(\tau)| \\ 1070 &\leq k\sqrt{T}R_mW_1(p_{\theta^*}(\tau|y^*), p_{\hat{\theta}}(\tau|y^*)) \\ 1071 &\leq k\sqrt{T}R_m\mathbb{E}_{y \sim p_D(y)}\left[W_1(p_{\theta^*}(\tau|y^*), p_D(\tau|y)) + W_1(p_D(\tau|y), p_{\hat{\theta}}(\tau|y^*))\right] \\ 1072 &\leq k\sqrt{T}R_m\left[\sqrt{\delta_M} + (1 + \epsilon)y_mL_p + \sqrt{\delta_K}\right], \\ 1073 &\quad \quad \quad (52) \\ 1074 &\quad \quad \quad (52) \\ 1075 &\quad \quad \quad (52) \\ 1076 &\quad \quad \quad (52) \\ 1077 &\quad \quad \quad (52) \\ 1078 &\quad \quad \quad (52) \end{aligned}$$

1079 where we leverage the fact that  $\hat{\theta}$  is a solution to Eq. 8 which satisfies the KL and Lipschitz constraint, and we leverage the results in Eq. 6, Eq. 7 and Eq. 50.

1080 (3) **Evaluator Bias in  $p_{\hat{\theta}}$ .** Since  $\hat{\theta}$  satisfies the KL and Lipschitz constraint in Eq. 8, we can use the  
 1081 results in Theorem 2, Eq. 6 and Eq. 7 to obtain:

$$1082 \quad |J(\hat{\theta}) - L(\hat{\theta})| \leq \delta_D + (1 + k)\sqrt{T}R_m[(1 + \epsilon)y_mL_p + \sqrt{\delta_K}], \quad (53)$$

1084 Overall, combining the results in Eq. 51, Eq. 52, and Eq. 53, we have:

$$1086 \quad J(\theta^*) - J(\hat{\theta}) \leq 2\delta_D + (1 + 2k)\sqrt{T}R_m\left[\sqrt{\delta_M} + \sqrt{\delta_K} + (1 + \epsilon)y_mL_p\right]. \quad (54)$$

1088 This concludes the proof.  $\square$

## 1091 C.6 PROOF OF SCORE GRADIENT

1093 The probability of the trajectory generated by the Causal Transformer can be decomposed into:

$$1094 \quad p_{\theta}(s_{1:T}|y) = \Pi_t p_{\theta}(s_t|s_{1:t-1}, y). \quad (55)$$

1095 Then, we have:

$$\begin{aligned} 1097 \quad \nabla_{\theta} L(\theta) &= \nabla_{\theta} \int_{\tau} p_{\theta}(\tau|y^*) \hat{y}_{\phi}(\tau) d\tau \\ 1098 &= \nabla_{\theta} \int_{s_1, \dots, s_T} p_{\theta}(s_{1:T}, \tau|y^*) \hat{y}_{\phi}(\tau) ds_1 \dots ds_T \\ 1100 &= \int_{s_{1:T}} p_{\theta}(s_{1:T}|y^*) \frac{\nabla_{\theta} p_{\theta}(s_{1:T}|y^*)}{p_{\theta}(s_{1:T}|y^*)} \hat{y}_{\phi}(\tau) ds_1 \dots ds_T \\ 1101 &= \mathbb{E}_{s_{1:T} \sim p_{\theta}(s_{1:T}|y^*)} \left[ \nabla_{\theta} \log p_{\theta}(s_{1:T}|y^*) \hat{y}_{\phi}(\tau) \right] \\ 1102 &= \mathbb{E}_{s_{1:T} \sim p_{\theta}(s_{1:T}|y^*)} \left[ \nabla_{\theta} \log \Pi_t p_{\theta}(s_t|s_{1:t-1}, y^*) \hat{y}_{\phi}(\tau) \right] \\ 1103 &= \mathbb{E}_{s_{1:T} \sim p_{\theta}(s_{1:T}|y^*)} \left[ \sum_t \nabla_{\theta} \log p_{\theta}(s_t|s_{1:t-1}, y^*) \hat{y}_{\phi}(\tau) \right]. \end{aligned} \quad (56)$$

## 1111 D AIGB-PEARL ALGORITHM SUMMARY

1113 Algorithm 1 summarizes the training process of AIGB-Pearl. The development of AIGB-Pearl is  
 1114 supported by ROLL (Wang et al., 2025).

### 1116 D.1 ADDITIONAL DESIGNS FOR EVALUATOR ACCURACY ENHANCEMENT

1118 To further enhance the reliability of the trajectory evaluator, we design two specific techniques.  
 1119 Specifically, as described in the following, we (i) integrate an LLM into its model architecture for  
 1120 better representational capacity; and (ii) mix pair-wise and point-wise losses for better score estima-  
 1121 tion accuracy.

## 1123 E ADDITIONAL EXPERIMENTS

### 1125 E.1 SIMULATED EXPERIMENT SETTINGS

1127 We include the detailed simulated experiment settings in Table 6. Specifically, we consider the  
 1128 bidding process in a day, where the bidding episode is divided into 96 time steps. Thus, the duration  
 1129 between two adjacent time steps  $t$  and  $t + 1$  is 15 minutes. The number of impression opportunities  
 1130 between time steps  $t$  and  $t + 1$  fluctuates from 100 to 500. The **minimum and maximum** budgets  
 1131 of advertisers are 1000 Yuan and 4000 Yuan, respectively. The upper bound of the bid price is 10  
 1132 Yuan, and the values of impressions are positive.

1133 **Hardware Resource.** The simulated experiments are conducted based on an NVIDIA T4 Tensor  
 Core GPU. We use 10 CPUs and 200G memory.

```

1134
1135 Algorithm 1: AIGB-Pearl (Planning with EvaluAtor via RL)
1136 Input : Offline dataset  $\mathcal{D}$ , desired condition  $y^*$ , hyper-parameters  $\beta_1, \beta_2, \beta_3$ .
1137 Output : Optimized  $\theta$  and  $\phi$ 
1138 Initialization: randomly initialized planner parameter  $\theta$ , trajectory evaluator parameters  $\phi$ 
1139 // Determining the Lipschitz Value
1140 Calculate the Lipschitz value of  $y(\tau)$  and  $p_D(\tau|y)$  using the offline dataset  $\mathcal{D}$ .
1141 Set the Lipschitz constraint value  $L_e$  for the evaluator and  $L_p$  for the planner to be bigger than
1142 the Lipschitz value of  $y(\tau)$  and  $p_D(\tau|y)$ , respectively.
1143 // Training the trajectory evaluator
1144 while not converged do
1145   | Update  $\phi$  by minimizing Eq. 11;
1146 end
1147 // Training the generative planner
1148 Warm start with pretrained planner  $p_\theta$ ;
1149 while not converged do
1150   | Generate bidding trajectories  $\tau \sim p_\theta(\tau|y^*)$ ;
1151   | Score generated trajectories with frozen  $\phi$ :  $\hat{y}_\phi(\tau)$ ;
1152   | Update  $\theta$  by maximizing Eq. 12.
1153 end

```

Table 6: Settings of the simulated experiments.

Parameters	Values
Number of advertisers	30
Time steps in an episode, $T$	96
Minimum number of impressions within a time step	50
Maximum number of impressions within a time step	300
Minimum budget	1000 Yuan
Maximum budget	4000 Yuan
Value of impressions	$> 0$
Minimum bid price, $\min\{av_i\}$	0 Yuan
Maximum bid price, $\max\{av_i\}$	10 Yuan
Maximum market price, $p_M$	10 Yuan

## E.2 REAL-WORLD EXPERIMENT SETTINGS

We include the detailed real-world experiment settings in Table 7. Specifically, we consider the bidding process in a day, where the bidding episode is divided into 96 time steps. Thus, the duration between two adjacent time steps  $t$  and  $t + 1$  is 15 minutes. The number of impression opportunities between time steps  $t$  and  $t + 1$  fluctuates from 100 to 2,500. The **minimum and maximum** budgets of advertisers are 50 Yuan and 10,000 Yuan, respectively. The upper bound of the bid price is 25 Yuan, and the values of impressions **are positive**.

**Hardware Resource.** The training process in the real-world experiments is conducted using 10 NVIDIA T4 Tensor Core GPUs in a distributed manner. For each distributional worker, we use 10 CPUs and 200 GB of memory.

### E.3 REAL-WORLD EXPERIMENTS ON TARGETROAS BIDDING PROBLEM

In addition to the budget-constrained auto-bidding problem, we also apply the proposed AIGB-Pearl algorithm to a more challenging type of auto-bidding problem, named TargetROAS, with an extra ROI (Return on Investment) constraint. We evaluate our method in a real-world experiment on TaoBao involving 300k advertisers over 22 days. The offline dataset comprises 16 million trajectories of 800k advertisers. The results are given in Table 8. AIGB-Pearl achieves a **+5.1%** improvement in GMV compared to the SOTA auto-bidding method, DiffBid, demonstrating its effectiveness in managing more complex and realistic constraints.

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Table 7: Settings of the real-world experiments.

Parameters	Values
Number of advertisers	6,000
Time steps in an episode, $T$	96
Minimum number of impressions within a time step	100
Maximum number of impressions within a time step	2,500
Minimum budget	50 Yuan
Maximum budget	10,000 Yuan
Value of impressions	$> 0$
Minimum bid price, $\min\{av_i\}$	0 Yuan
Maximum bid price, $\max\{av_i\}$	25 Yuan
Maximum market price, $p_M$	25 Yuan

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Table 8: Overall performance of TargetROAS in real-world A/B test, involving 300k advertisers over 22 days.

Methods	GMV	BuyCnt	ROI	Cost
<b>DiffBid</b>	779,642,891	11,519,082	4.68	166,544,918
<b>AIGB-Pearl (ours)</b>	819,550,812	11,886,501	4.70	174,234,673
$\Delta$	<b>+5.1%</b>	<b>+3.2%</b>	+0.5%	<b>+4.6%</b>

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1211  
E.4 PATHOLOGICAL TRAJECTORY BEHAVIOR EXPLANATION

1212 In industrial practice, stable and effective metrics have been developed to evaluate pathological  
1213 behaviors. For the case of the budget-constrained auto-bidding problem with bidding cycles structured  
1214 as 24-hour episodes ( $T = 96$  time steps), the following three key metrics are commonly used to  
1215 identify pathological behaviors:

- 1217 • **Excessive budget consumption:** there exists a time step  $t$  such that the cost between time  
1218 step  $t$  and  $t + 1$  exceeds 10% of the budget  $B$ ;
- 1219 • **Forward- (or Backward-) trending pacing:** the cost between time step 1 and 24 (or  
1220 between time step  $T - 24$  and  $T$ ) exceeds 40% (or 40%) of the budget  $B$ ;
- 1221 • **Underutilization of available budgets:** the total cost over the bidding episode is lower  
1222 than 90% of the budget  $B$ .

1224  
1225 E.5 TRAINING STABILITY

1226 We present additional comparisons between the training curves of the offline RL with bootstrapping  
1227 and those of AIGB-Pearl in Fig. 6, Fig. 7, Fig. 8, and Fig. 9 concerning:

- 1229 • **Cumulative Rewards:** the main performance index of the considered auto-bidding prob-  
1230 lem;
- 1231 • **Online Rate:** the ratio between the bidding period before the budget runs out and the total  
1232 bidding period. A larger Online Rate indicates a better performance.
- 1233 • **Bad Case Rate:** the ratio between the number of “bad” trajectories and the total number  
1234 of generated trajectories. A lower Bad Case Rate indicates a better performance.
- 1235 • **Cost Rate:** the ratio between the cost and the budget. A larger cost rate indicates a better  
1236 performance.

1237 We can see that the offline RL method tends to suffer from significant instability throughout training,  
1238 showing high variance across different seeds. In contrast, AIGB-Pearl achieves much smoother and  
1239 more consistent learning progress, demonstrating the improved training stability.

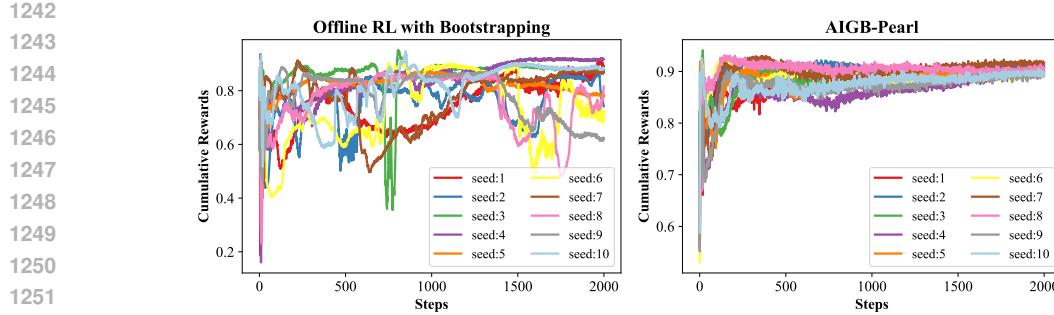


Figure 6: Learning curves of cumulative rewards between offline RL with bootstrapping method and AIGB-Pearl under 10 seeds.

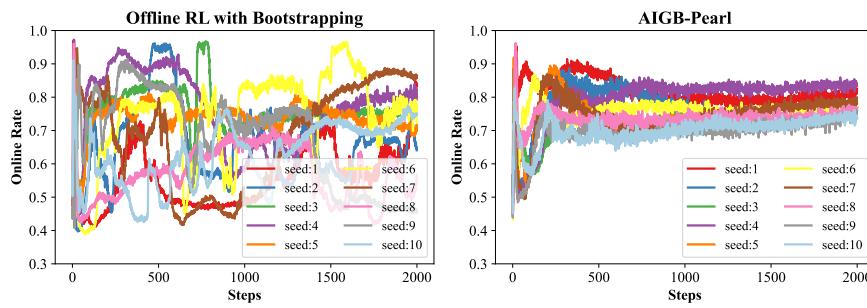


Figure 7: Learning curves of online rate between offline RL with bootstrapping method and AIGB-Pearl under 10 seeds.

## E.6 EMPIRICAL PERFORMANCE WITH GENERAL OFFLINE DATA DISTRIBUTIONS

Note that in many real-world auto-bidding systems, including the one considered in the paper, due to operational safety constraints, the online-deployed bidding policy is typically a single fixed model, and the offline dataset is collected exclusively from this single policy over multiple days, where an advertiser contributes a single trajectory per day. For example, in the considered auto-bidding system, the online-deployed baseline policy is a conditional generative model that generates identical trajectory plans for a given advertiser under identical conditions each day. The variation across different trajectories of the same advertiser in the offline dataset is solely due to stochastic environmental factors (e.g., traffic fluctuations). Since these exogenous perturbations are typically the sum of many independent impression-level sources of noise, the resulting trajectory deviations can be reasonably approximated as a Gaussian distribution.

To demonstrate the broad applicability of the proposed algorithm, we evaluate its performance in settings where multiple policies are used for data collection. Specifically, we collect trajectories in the simulated environment using nine distinct bidding policies, thereby constructing an offline dataset with a multi-modal distribution that violates the Gaussian distribution. The empirical results are presented in Table 9.

Table 9: Empirical performance with multiple data-collection policies.

Methods	GMV	ROI	Cost
<b>DiffBid</b>	548.5	5.00	109.4
<b>AIGB-Pearl (ours)</b>	575.7	5.04	114.6
$\Delta$	+4.9%	+0.7%	+4.2%

We observe that AIGB-Pearl still outperforms AIGB by +4.9%, indicating that its performance is robust to the specific distribution of the offline dataset.

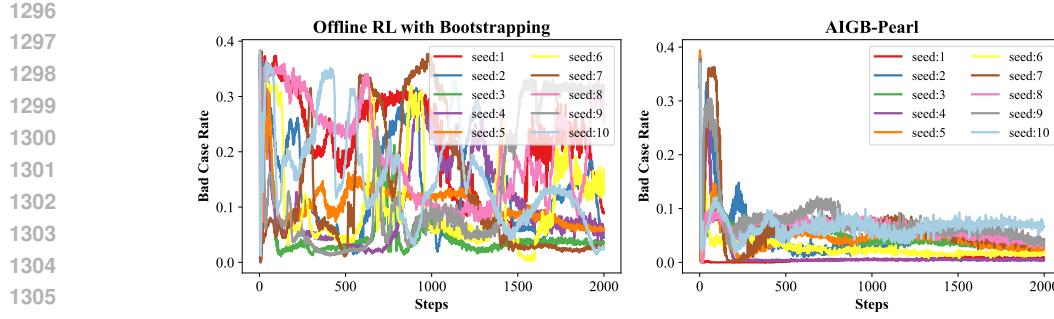


Figure 8: Learning curves of bad case rate between offline RL with bootstrapping method and AIGB-Pearl under 10 seeds.

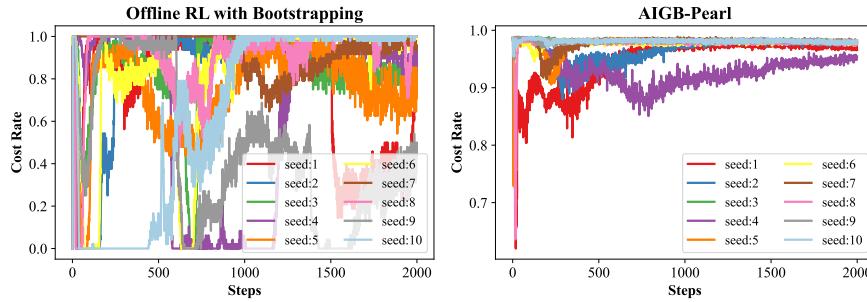


Figure 9: Learning curves of cost rate between offline RL with bootstrapping method and AIGB-Pearl under 10 seeds.

## F EXTENDING AIGB-PEARL TO FIRST-PRICE AUCTIONS

We note that the proposed method remains effective in first-price auctions with a proper adaptation. Specifically, unlike second-price auctions where the optimal bid for impression  $i$  takes the form  $\text{bid}_i = av_i$ , in first-price auctions, the optimal bid for impression  $i$  is given by  $\text{bid}_i = \min(av_i, p_i)$ , which typically involves an extra bid shading method to predict the winning price (Gligorijevic et al., 2020; Wu et al., 2015). Equipped with an off-the-shelf bid shading method (whose design is beyond the scope of this work), the auto-bidding problem in a first-price auction remains an offline sequential decision problem, i.e., making decisions over an  $a$ -sequence, to which the proposed method applies directly.

To validate the effectiveness of the proposed method under first-price auctions, we additionally conduct a simulated first-price auction experiment against the state-of-the-art AIGB method, where all methods are equipped with the same bid shading method. The results are presented in Table 10, demonstrating the effectiveness of our proposed method.

## G EXTENDING AIGB-PEARL TO ONLINE SETTING DISCUSSION

We note that AIGB-Pearl can be extended to online settings when equipped with a safe online exploration policy. Specifically, due to safety constraints, the auto-bidding policy during training cannot interact directly with the live advertising system; only safe exploration policies are permitted to collect data online (Mou et al., 2022). Consequently, an online auto-bidding framework typically involves two parts:

- a safe online exploration policy, which is a well-established component in existing work (Mou et al., 2022) and beyond the scope of this paper;
- an offline policy training method that leverages the data collected online.

1350 Table 10: Empirical performance in the simulated experiments with a first-price auction.  
1351

Methods	GMV	ROI	Cost
<b>DiffBid</b>	1,546	5.13	301
<b>AIGB-Pearl (ours)</b>	1,611	5.18	311
$\Delta$	+4.2%	+1.0%	+3.3%

1352  
1353 AIGB-Pearl can be directly applied as the offline policy training method within the online framework  
1354 without modification. In practice, due to the safety and stability concerns, many industrial auto-  
1355 bidding systems adopt an offline optimization paradigm. For this practical reason, we focus on the  
1356 offline setting in this work.  
1357

## 1358 H LLM USAGE

1359 The authors have used Large Language Models (LLMs) exclusively for grammar checking and lexical  
1360 refinement during the writing process. No LLM-generated content, data analysis, or substantive  
1361 contributions to the research methodology, results, or conclusions are involved in this work.  
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