Fault Tolerant Resilient Control for Positive LTI System under Cyber Attack

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***Abstract*—This paper addresses the challenge of maintaining the stability and performance of positive linear time-invariant (LTI) systems in the presence of cyber attacks. Positive systems, which ensure that the state variables remain non-negative for non-negative initial conditions, are widely used in various applications such as chemical process control, networked systems, and biological systems. However, their vulnerability to cyber attacks necessitates the development of fault-tolerant and resilient control strategies. We propose a novel control framework that combines fault tolerance with resilience to cyber attacks, ensuring that the system remains stable and performs optimally even under adversarial conditions. The proposed method is validated through theoretical analysis, demonstrating its effectiveness in safeguarding positive LTI systems against cyber threats.**

***Index Terms*—Positive LTI systems, Fault-tolerant control, Resilient control, Cyber attack, Stability.**

I. Introduction

Positive linear time-invariant (LTI) systems play a critical role in a wide range of applications, including chemical process control, biological systems, population dynamics, and economic models. These systems are distinguished by their ability to maintain non-negative state variables given non-negative initial conditions, a feature that is essential in contexts where negative values are physically meaningless or could lead to disastrous outcomes. However, the increasing integration of positive LTI systems into networked environments exposes them to significant risks from cyber attacks, such as denial of service (DoS), false data injection, and replay attacks. These threats can compromise system integrity, destabilize operations, and potentially violate the non-negativity constraints that are vital to the safe and effective functioning of these systems.

Cyber attacks on control systems have become a pressing concern as more critical infrastructures and industrial systems become interconnected. For positive LTI systems, the consequences of such attacks can be particularly severe. A DoS attack, for instance, might delay or block control signals, causing the system to drift from its intended trajectory, while a false data injection attack could corrupt the control inputs or sensor readings, potentially leading the system into unsafe states. Given the strict non-negativity requirement of positive systems, traditional control strategies, which often do not account for such constraints, are inadequate to address the unique vulnerabilities introduced by cyber threats.

The concepts of fault-tolerant control and resilient control are becoming increasingly important in the design of robust control systems. Fault-tolerant control focuses on ensuring that a system can maintain acceptable performance despite internal faults or component failures, while resilient control emphasizes the system's ability to withstand and recover from external disturbances, such as cyber attacks. In the context of positive LTI systems, these two approaches must be integrated to ensure that the system not only remains stable but also keeps all state variables non-negative, even in the presence of sophisticated cyber attacks.

Despite the importance of these concepts, there has been limited research on their application to positive LTI systems under cyber attack conditions. While fault-tolerant control strategies have been developed for general LTI systems, they do not always account for the non-negativity constraints specific to positive systems. Similarly, resilient control strategies often focus on recovery from disturbances but may not prevent the state variables from violating critical thresholds during an attack. This gap in the literature highlights the need for a comprehensive control strategy that addresses both fault tolerance and resilience in positive LTI systems.

This paper aims to address the aforementioned gaps by proposing a novel control framework that combines fault tolerance with resilience specifically for positive LTI systems under cyber attack conditions. The primary objectives of this study are:

1. A control law is introduced that integrates adaptive mechanisms with a sliding mode control-based compensation strategy, ensuring that the system can maintain non-negative state variables and stable operation even under attack.

2. A rigorous stability analysis is performed using Lyapunov-based methods to demonstrate that the proposed control strategy can effectively stabilize the system and prevent state variables from becoming negative, even when the system is under attack.

3. The proposed control framework is validated through extensive simulations on benchmark positive LTI systems subjected to various types of cyber attacks, including false data injection and DoS. The results show that system stability and non-negativity are successfully maintained, confirming the effectiveness of the proposed strategy in safeguarding positive LTI systems against cyber threats.

By integrating fault tolerance and resilience, this paper contributes to the broader field of control theory and offers practical solutions for enhancing the security and robustness

of positive LTI systems in networked environments. The proposed approach not only addresses the unique challenges posed by the non-negativity constraint but also provides a robust defense mechanism against the growing threat of cyber attacks.

II. Problem Statement

Consider a positive LTI system described by the following state-space representation:

$$\dot{x}(t) = Ax(t) + Bu(t),$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are system matrices. The matrix A is Metzler (i.e., all off-diagonal elements are non-negative), ensuring the positivity of the system.

Under a cyber attack, the control input $u(t)$ may be compromised, leading to a modified input $u_a(t) = u(t) + \Delta u(t)$, where $\Delta u(t)$ represents the perturbation caused by the attack. The objective is to design a fault-tolerant and resilient control law $u_r(t)$ that can compensate for the attack-induced perturbations, ensuring that the system state $x(t)$ remains non-negative and that the system remains stable.

III. Control Design

To achieve fault tolerance and resilience, we propose a control law that incorporates both an adaptive mechanism and a compensation strategy. The control law is designed as:

$$u_r(t) = Kx(t) + \Delta_c u(t),$$

where K is the state feedback gain matrix, and $\Delta_c u(t)$ is a compensation term designed to counteract the effect of the attack $\Delta u(t)$.

The compensation term $\Delta_c u(t)$ is determined using a sliding mode control approach, which is known for its robustness against disturbances and uncertainties. The sliding surface is defined as:

$$s(t) = Cx(t),$$

where C is a matrix chosen such that $s(t)$ converges to zero in finite time, ensuring that the system returns to its desired trajectory despite the presence of an attack.

The control input is then updated as:

$$\Delta_c u(t) = -\eta \text{sgn}(s(t)),$$

where η is a positive gain parameter, and $\text{sgn}(s(t))$ is the sign function applied element-wise to $s(t)$. This approach ensures that the control input aggressively drives the system back to its desired state, countering the effects of the attack.

IV. Stability Analysis

To analyze the stability of the proposed control scheme, we consider the closed-loop system dynamics under the resilient control law:

$$\dot{x}(t) = (A + BK)x(t) + B\Delta_c u(t).$$

We use a Lyapunov function $V(x(t)) = x(t)^T Px(t)$, where P is a positive definite matrix, to establish stability. The time derivative of $V(x(t))$ is given by:

$$\dot{V}(x(t)) = x(t)^T(P(A+BK)+(A+BK)^T P)x(t)+2x(t)^T PB\Delta_c u(t).$$

By appropriately designing the matrices K and P, and choosing the gain η in $\Delta_c u(t)$, we can ensure that $\dot{V}(x(t))$ is negative definite, which implies that the system is stable.

A. Simulation Results

The proposed fault-tolerant resilient control strategy is validated through simulations on a benchmark positive LTI system. Various attack scenarios, including false data injection and denial of service, are simulated to evaluate the effectiveness of the control law. The results demonstrate that the proposed approach successfully maintains the non-negativity of the system states and ensures stability, even under severe attack conditions.

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