Solving Kuramoto Oscillator Model using Physics Informed Neural Network

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Abstract

Physics informed machine learning has been emerged as a powerful tool with the help of deep learning as the latter has been instrumental as a data-driven function approximator. Many recent works have been focusing on solving hard to solve differential equations with the help of physics informed neural network (PINN), a tremendously simple approach which blends physics and deep learning. We explore the application of PINN in solving Kuramoto system of coupled differential equations as well as in decision making problem of synchronization state of the system. The experimental results illustrate that PINN can not only be used to solve the coupled differential equations but be applied to figure out the synchronization capability of the oscillator system in consideration.

1 Introduction

The Kuramoto model [1] of oscillatory systems have been widely used for its capability to model the synchronized behaviour through a system of coupled differential equations. More recently, it is getting attention in the field of optimal experiment design [2, 3]. For Kuramoto system, the goal of the optimal experiment design is to reduce the effect of uncertainty on selecting a control oscillator to make the system synchronized. The uncertainty originates from the unknown interaction strengths between oscillators. If the coupling strengths are known accurately, then we would have chosen the optimum control oscillator. But, due to the lack of information, we suffer a cost by choosing a suboptimal solution. To quantify the effects of this uncertainty, we need to calculate the expected cost by solving a large number of Kuramoto systems which are generated by sampling from the uncertainty class for interaction strengths. Solving these large number of systems in numerical method requires a lot of computational time. To address this challenge, parallel computation of differential equation solver [2], surrogate model for estimating the average cost [3] are already proposed. In this project, we have tried to study the feasibility of the physics informed neural network (PINN) to tackle the computational complexity in optimal experiment design for controlling the Kuramoto system.

The PINN [4] has laid out the data-driven approach powered by the automatic differentiation to solve the system of differential equations. Although the training of PINN is not understood well enough, it is showing promising results [5], specially for the failure cases of numerical solvers. For the Kuramoto model, the numerical method works very well but takes long time. The PINN based approach may have potential to be used as accurate as well as fast solver. With this goal in mind, we have focused on whether the PINN solver can accurately approximate the synchronized and unsynchronized Kuramoto system.

In section 2, we provide an introductory description of the Kuramoto model and how we have applied the physics informed neural network to solve this coupled system. For each case of experiments under section 3, we have highlighted our findings that point out the successes as well as the existing challenges for PINN.

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2 Methods

2.1 Kuramoto Model

A Kuramoto model of N oscillators can be represented by the system of coupled differential equations, 1 with the initial conditions in equation 2 for the angular position θ_i. The set of interaction strengths or coupling coefficients between each pair of oscillators, $a_{i,j}$ governs the dynamics of this system.

$$\dot{\theta}_i(t) = \omega_i + \sum_{j=1}^{N} a_{i,j} \sin(\theta_j(t) - \theta_i(t)), \quad i = 1, 2, \cdots, N \tag{1}$$

$$\theta_i(0) = \theta_i^0 \tag{2}$$

Along with the intrinsic angular frequencies, $\{\omega_i\}$, the coupling strengths can make a system synchronized or unsynchronized. A system is frequency synchronized when the instantaneous angular frequencies of all the oscillators converge to a single value. (equation 3)

$$\lim_{t\to\infty} |\dot{\theta}_i(t) - \dot{\theta}_j(t)| = 0, \quad \forall i, j \tag{3}$$

2.2 Synchronization condition for $N = 2$

For Kuramoto model with $N = 2$ oscillators, it is proved in [3] that equation 3 is satisfied if and only if $|w_1 - w_2| \leq 2a$ where $a = a_{1,2} = a_{2,1}$. We use this condition to build a synchronized and unsynchronized system of 2 oscillators and apply the PINN solver, described in the following section, to get the angular positions of each oscillator.

2.3 Solving with PINN

We model the solution provided by the PINN as $\boldsymbol{\theta}(t, \mathbf{W}) \in \mathbb{R}^N$ where $\mathbf{W}$ represents the parameters of the neural network. Also, we have $\boldsymbol{\theta}^0 \in \mathbb{R}^N$, the initial angular positions of the oscillators. We train the PINN by combining the loss at the initial time-point, $t = 0$ and the residual loss at N_r time-points, $t_1, t_2, \cdots, t_{N_r}$.

$$\mathcal{L}(\mathbf{W}) = \lambda_b \mathcal{L}_b(\mathbf{W}) + \lambda_r \mathcal{L}_r(\mathbf{W}) \tag{4}$$

$$\mathcal{L}_b(\mathbf{W}) = \left|\left|\boldsymbol{\theta}(0, \mathbf{W}) - \boldsymbol{\theta}^0\right|\right|^2 \tag{5}$$

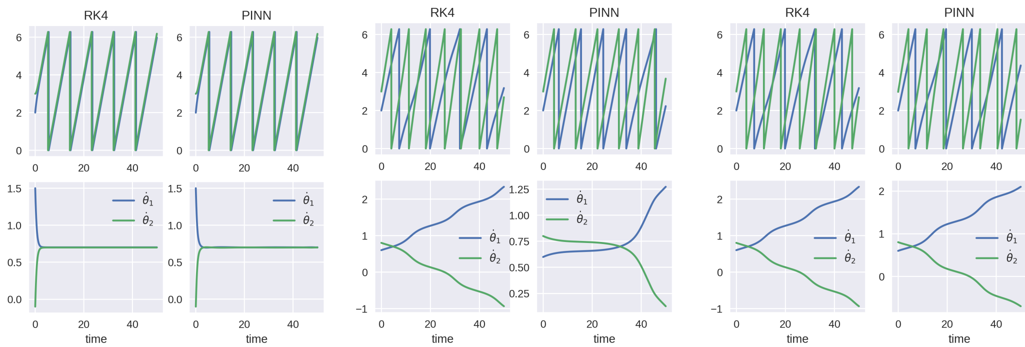
$$\mathcal{L}_r(\mathbf{W}) = \frac{1}{N_r} \sum_{k=1}^{N_r} \left|\left|r\left(\boldsymbol{\theta}(t_k, \mathbf{W})\right)\right|\right|^2 \tag{6}$$

$$r\left(\boldsymbol{\theta}(t_k, \mathbf{W})\right) = \begin{bmatrix} \frac{\partial \theta_1(t_k,\mathbf{W})}{\partial t} - \omega_1 - \sum_{j=1}^{N} a_{1,j} \sin(\theta_j(t_k, \mathbf{W}) - \theta_1(t_k, \mathbf{W})) \\ \frac{\partial \theta_2(t_k,\mathbf{W})}{\partial t} - \omega_2 - \sum_{j=1}^{N} a_{2,j} \sin(\theta_j(t_k, \mathbf{W}) - \theta_2(t_k, \mathbf{W})) \\ \vdots \\ \frac{\partial \theta_N(t_k,\mathbf{W})}{\partial t} - \omega_N - \sum_{j=1}^{N} a_{N,j} \sin(\theta_j(t_k, \mathbf{W}) - \theta_N(t_k, \mathbf{W})) \end{bmatrix} \tag{7}$$

3 Results

3.1 Experiment for two oscillator system

We set the intrinsic frequencies as $\omega_1 = 0.5, \omega_2 = 0.9$. For synchronized system, we choose 1 as the coupling coefficient, and 0.1 in case of unsynchronized system. We use a PINN with 8 hidden layers, 20 neurons per layer to approximate the solutions of the system. $N_r = 2000$ points are randomly selected from $0 \leq t \leq 50$ seconds. We empirically set $\lambda_b = 1$ and $\lambda_r = 2$ and train the PINN for 4000 iterations. Figure 1a and 1b show the approximated angular positions (top row) by the fourth order Runge–Kutta (RK4) method and the trained PINN and also the corresponding instantaneous angular frequencies (bottom row). It appears that for synchronized system, the PINN's solution is very close to that from RK4 method. In unsynchronized system, the solution from PINN is way off from the RK4's solutions. To resolve this issue, we increase λ_r to 20, and the PINN's accuracy gets much better (Figure 1c).

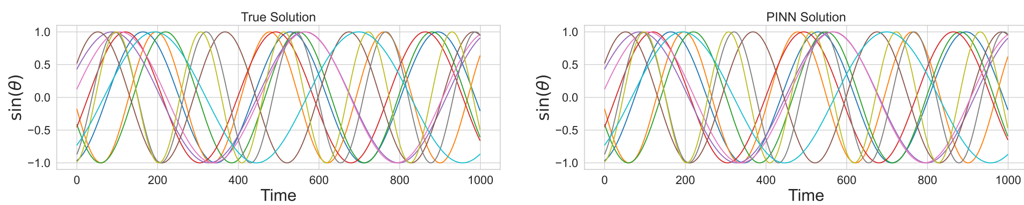


(a) Synchronized (b) Unsynchronized ($\lambda_r = 2$) (c) Unsynchronized ($\lambda_r = 20$)

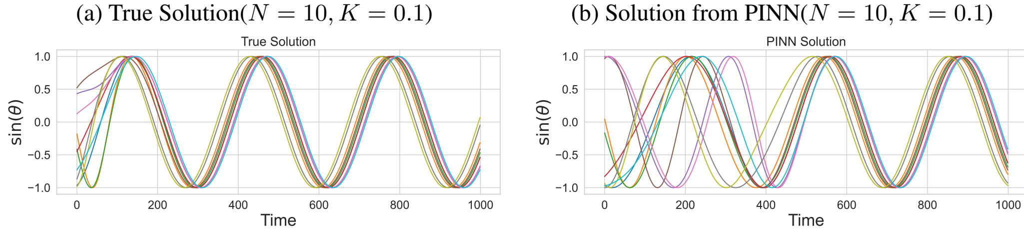
Figure 1: Experiments with two oscillator Kuramoto network.

3.2 Effect of Coupling Coefficient on PINN's accuracy

We considered Kuramoto models with $10, 15$ and 20 oscillators. For simplicity, we assumed the whole Kuramoto model has a single coupling coefficient, $a_{i,j} = K/(N-1)$ instead of different coupling coefficients for each connection of the oscillator model. We used $\lambda_r = 10, \lambda_b = 1$ and $1,000$ collocation points. We trained the models for $20,000$ iterations for each case. Figures 2a and 2b and Figures 2c and 2d show the true solution and the predicted solution by PINN for the two networks ($K = 0.1, 2$) with $N = 10$. We observed that the PINN fails to learn the true solution with the increase in coupling coefficient. Similar results for $N = 15, 20$ are included in Appendix C.



(a) True Solution($N = 10, K = 0.1$) (b) Solution from PINN($N = 10, K = 0.1$)



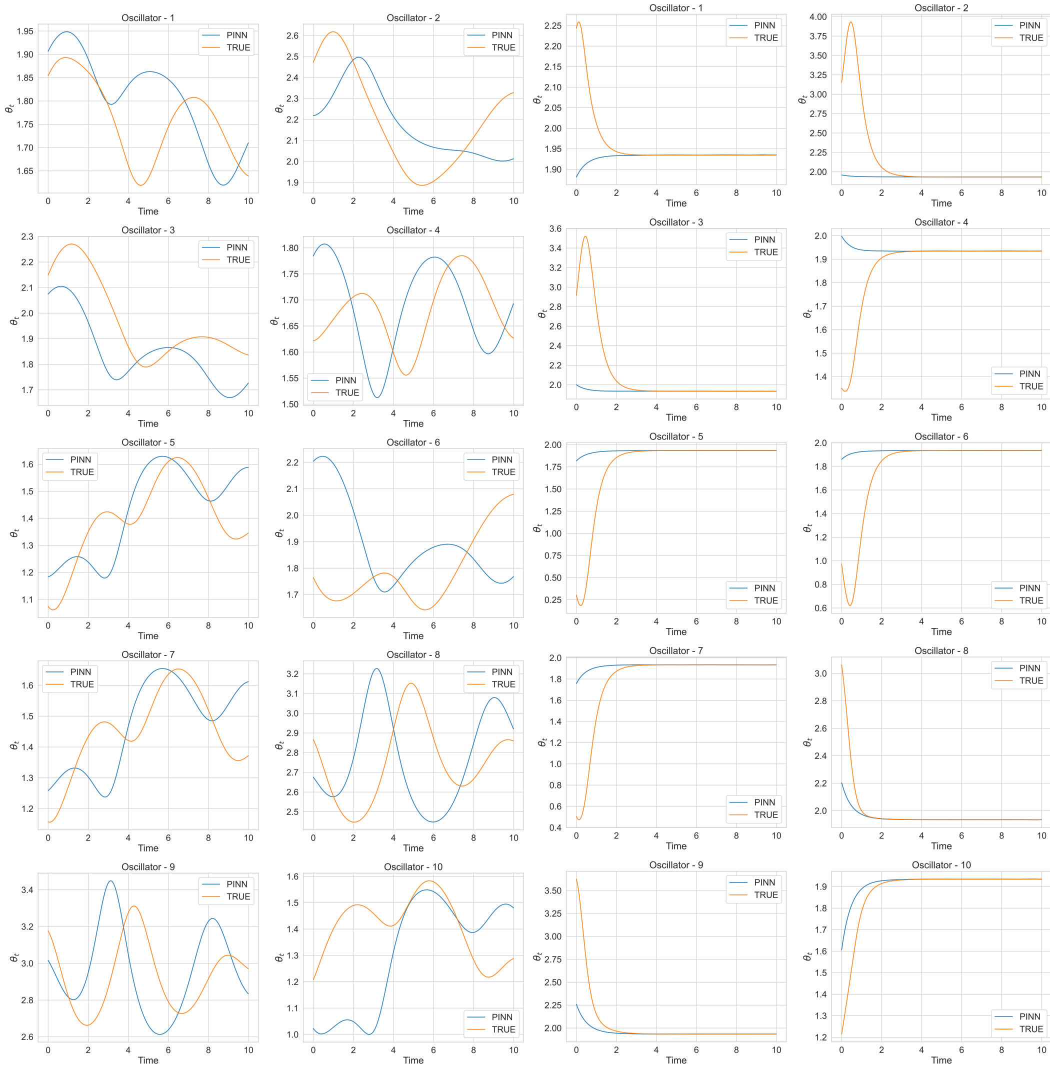
(c) True Solution($N = 10, K = 2.0$) (d) Solution from PINN($N = 10, K = 2.0$)

Figure 2: True and PINN solution for weakly and strongly coupled Kuramoto network.

3.3 Training PINN without Initial Condition

In this section, we trained PINN without the initial conditions to examine whether the PINN can learn the synchronization capability of the Kuramoto model. We experimented with two different coupling coefficients $K = 0.5, 2.0$ for three Kuramoto networks with $N = 10, 15$ and 20. Here we only used $1,000$ collocation points to train the model.

Figures 3a and 3b show the comparison of instantaneous frequencies between two different coupling coefficient values for $N = 10$. For higher K, the Kuramoto model is supposed to be locked up to a single instantaneous frequency for all oscillators. For $K = 2$, the PINN can learn that instantaneous frequency quite successfully. As the initial phases are assumed by the PINN, they differ from the true solution near initial timepoints. For $K = 0.5$, the true solution does not synchronize to a single frequency, neither does the PINN. Additional results for Kuramoto model with 15 and 20 oscillators are added in Appendix D. For $N = 20$, the PINN is able to identify the synchronous frequency while the numerical solver is slow to reach the synchronized states in the given time duration.



(a) $N = 10, K = 0.5$

(b) $N = 10, K = 2.0$

Figure 3: Instantaneous frequencies of 10 oscillator systems. PINN was trained without providing the initial conditions.

4 Conclusion

In this work, we examined the potential of PINNs to solve coupled differential equations. Our main objective was twofold: firstly, examine whether the PINN can accurately solve the coupled differential equation when all the necessary initial conditions are known, and secondly, whether the PINN can figure out whether a given Kuramoto system will synchronize or not without utilizing any initial conditions. We saw that PINN can learn the solution accurately with low coupling strength. With higher coupling strength, PINN seems to fail in following the abrupt change of some of the oscillators. This is a well-known phenomenon of PINNs to fail to learn the high frequencies of the true solution. The self-adaptive training approach [6] might be a solution to resolve this issue. For the second objective, PINN was able to predict the locking frequency of a synchronized Kuramoto model. For models with a higher number of oscillators, the results (Section 3.3 and Appendix D) show that the PINN (without initial conditions) can find the locking frequency quicker than the numerical solver with random initial phases. This demonstrates the PINN's advantage over the numerical approaches in identifying a synchronous network. One natural extension of our work can be the exploration of PINN in Kuramoto systems with arbitrarily different interaction strengths for each pair of oscillators.

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A Quantification of Approximation Accuracy of PINN Solver

To quantify the the approximation accuracy of the PINN solver, we have used the metric of expected maximum relative L_2 error:

$$\text{Expected maximum relative } L_2 \text{ error} = \mathrm{E}\left[\max_{i\in\{1,2,\cdots,N\}} \frac{\sqrt{\sum_{k=1}^{N_q}\left(\theta_i(t_k,\mathbf{W})-\theta_i^{true}(t_k)\right)^2}}{\sqrt{\sum_{k=1}^{N_q}(\theta_i^{true}(t_k))^2}}\right] \tag{8}$$

In equation 8, the true solution for each oscillator is obtained by numerical solver. The N_q test points

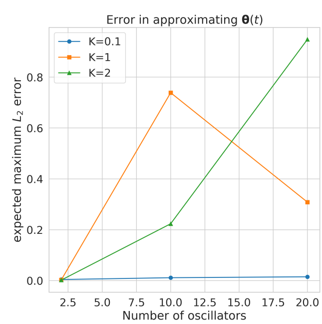
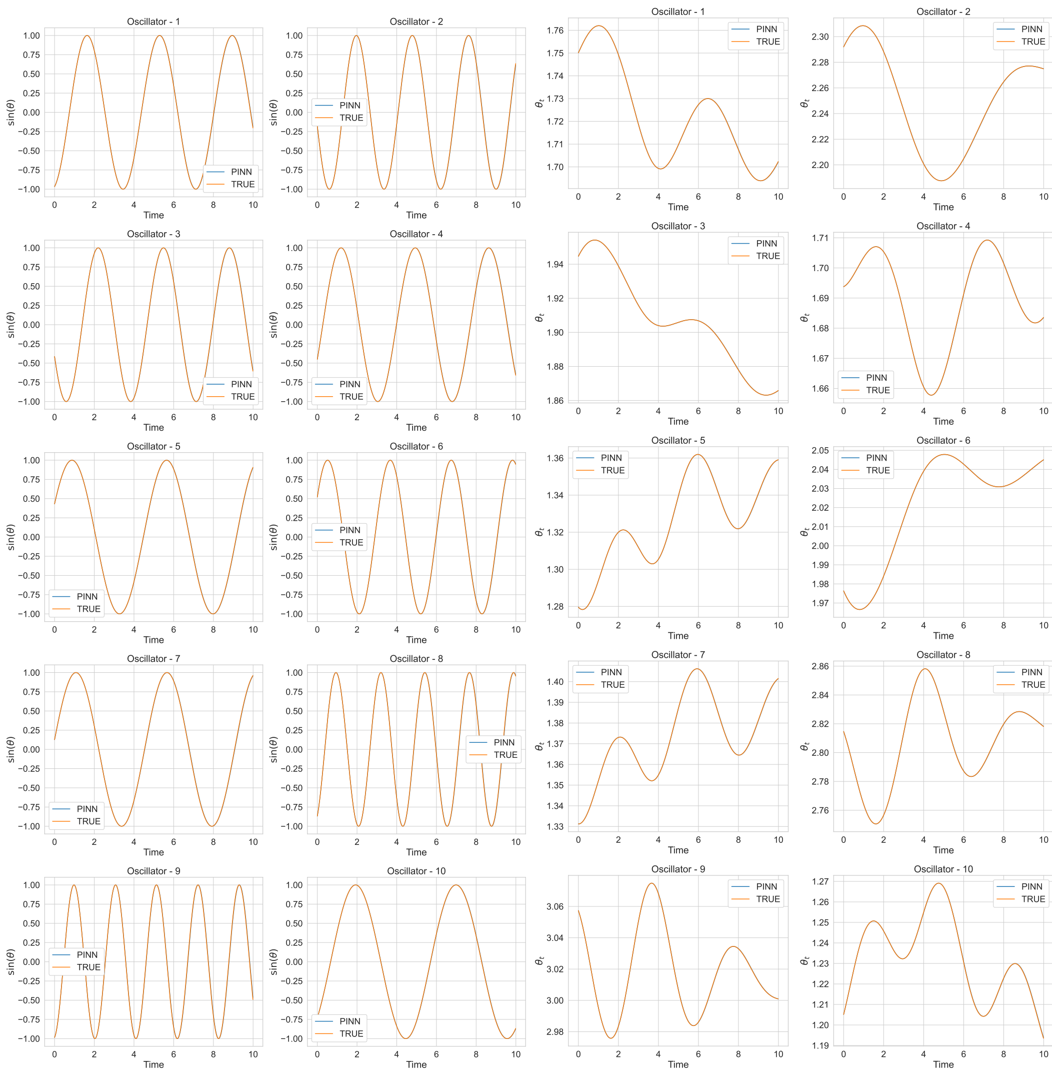


Figure 4: Expected maximum relative L_2 error for different network sizes and coupling strengths.

are taken from the domain with uniform separation. For performing the expectation operation, we train 5 PINNs with 8 hidden layers and 50 neurons per layer and take the average maximum L_2 error for Kuramoto systems with $N = 2, 10, 20$ oscillators and $K = 0.1, 1, 2$. Each PINN is trained with 1000 collocation points for 5000 iterations with $\lambda_r = 10$ and $\lambda_i = 1$. Figure 4 demonstrates that the PINN's accuracy decreases for Kuramoto model with larger number of oscillators with stronger coupling.

B PINN's solutions for $N = 10$

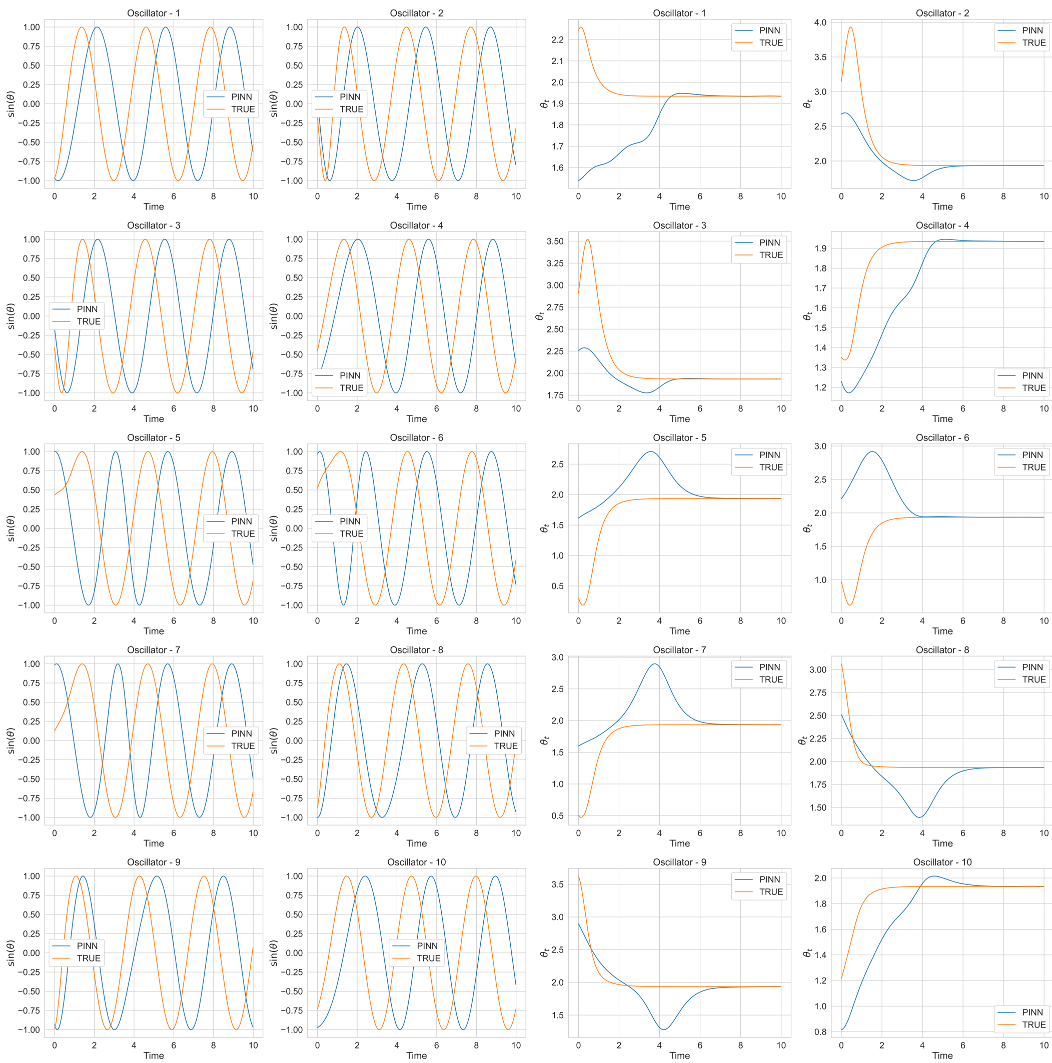
Figure 5a and 5b show the phases and instantaneous frequencies of each oscillator of a weakly coupled Kuramoto network. These figures show that the PINN quite successfully learns the true solution. Figure 6a and 6b show the phases and instantaneous frequencies of each oscillator of a strongly coupled Kuramoto network. These figures show that the PINN could not learn the true solution.



(a) Phase

(b) Instantaneous frequency

Figure 5: Phases and instantaneous frequency of each oscillator($N = 10, K = 0.1$)



(a) Phase

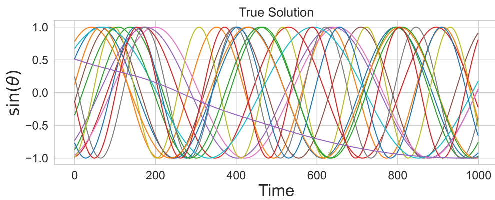
(b) Instantaneous frequency

Figure 6: Phases and instantaneous frequency of each oscillator($N = 10, K = 2.0$)

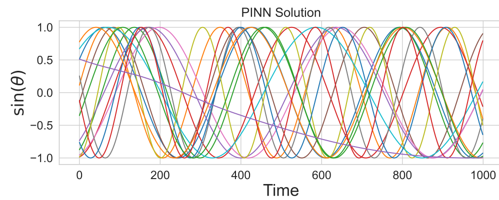
C Kuramoto Oscillator Network with 15 and 20 Oscillators

Figure 7 shows the true and PINN solution for both a weakly and strongly coupled Kuramoto oscillator network with 15 oscillators. Figure 8 shows the phase and instantaneous frequencies of each oscillator for weakly coupled Kuramoto oscillator network. Figure 9 shows similar results for a strongly coupled Kuramoto oscillator network.

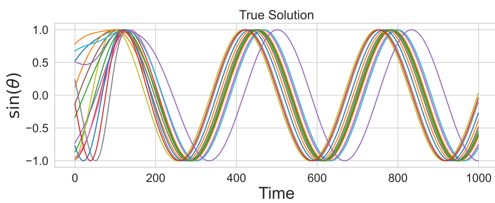
Figures 10 to 12 are the corresponding results for the Kuramoto oscillator network with 20 oscillators.



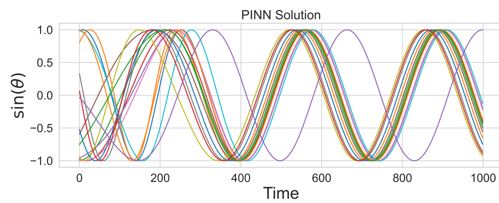
(a) True Solution($N = 15, K = 0.1$)



(b) Solution from PINN($N = 15, K = 0.1$)

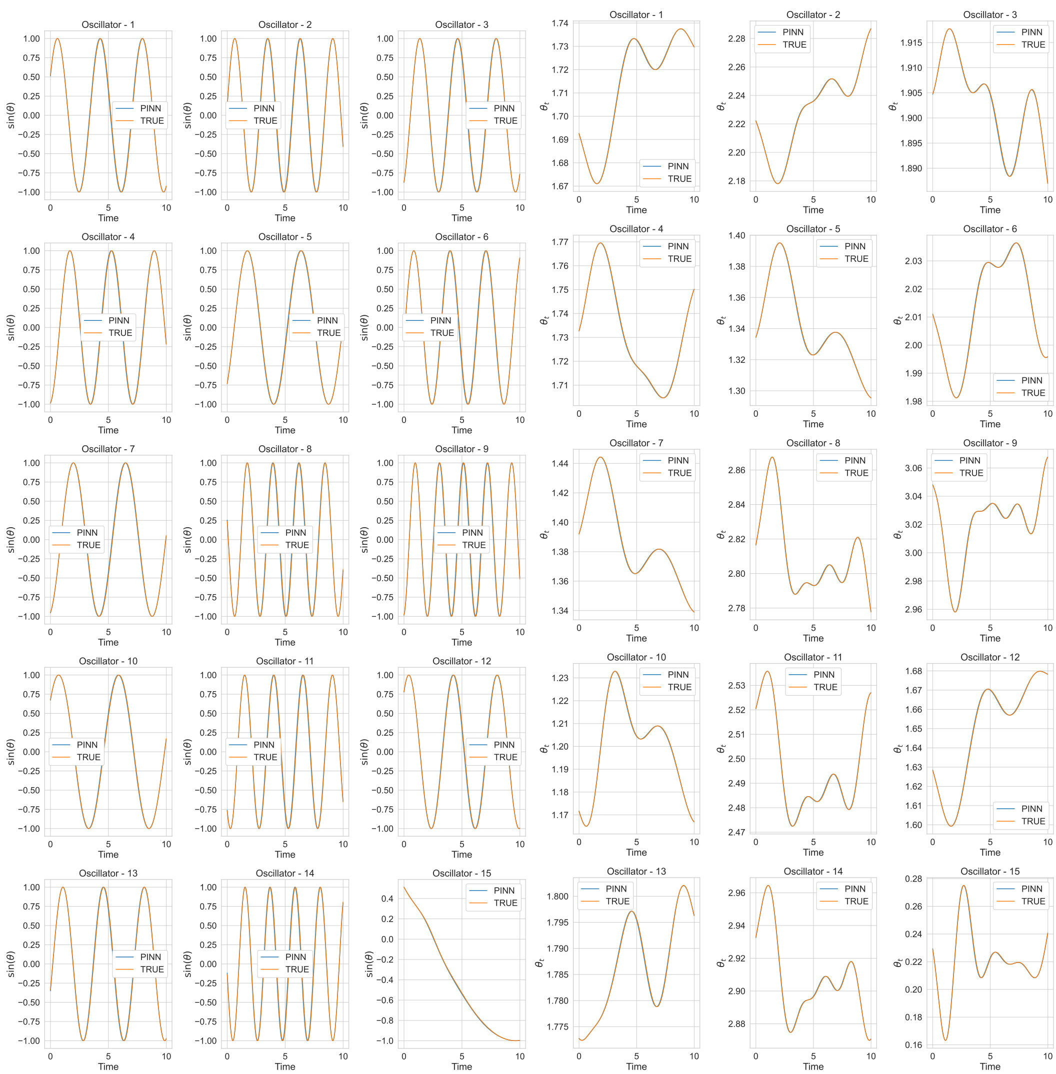


(c) True Solution($N = 15, K = 2.0$)



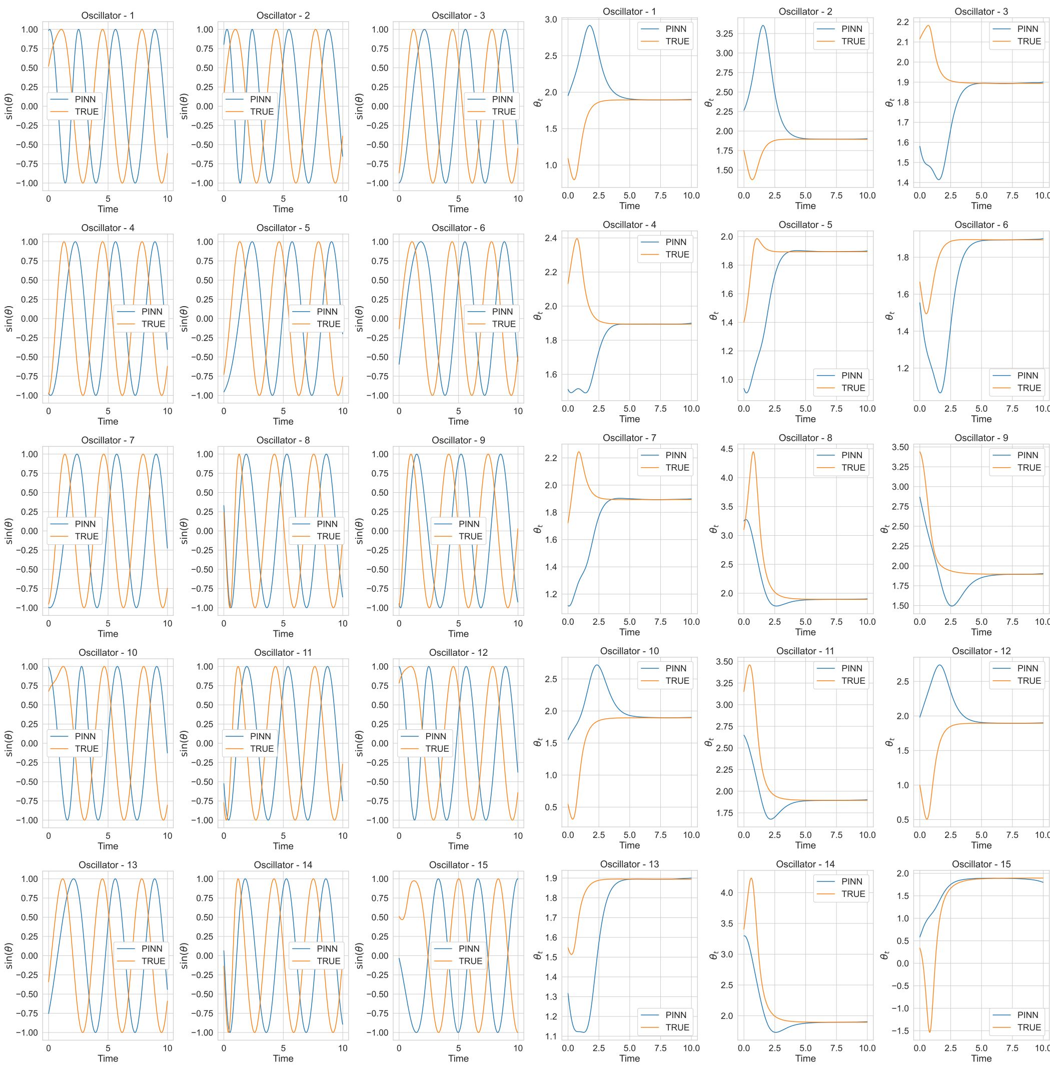
(d) Solution from PINN($N = 15, K = 2.0$)

Figure 7: True and PINN solution for weakly and strongly coupled Kuramoto network with 15 oscillators.



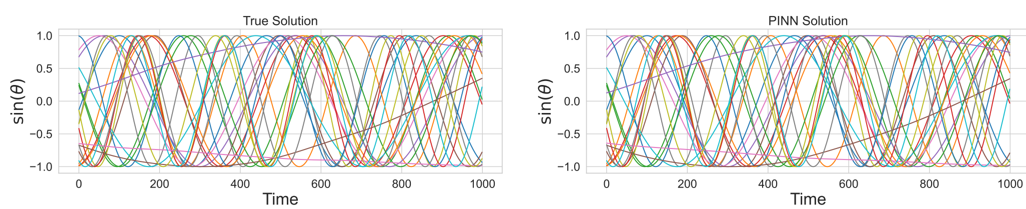
(a) Phase (b) Instantaneous frequency

Figure 8: Phases and instantaneous frequency of each oscillator($N = 15, K = 0.1$)

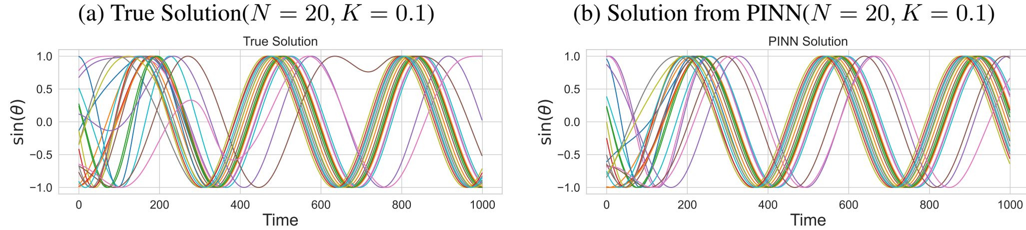


(a) Phase (b) Instantaneous frequency

Figure 9: Phases and instantaneous frequency of each oscillator($N = 15, K = 2.0$)

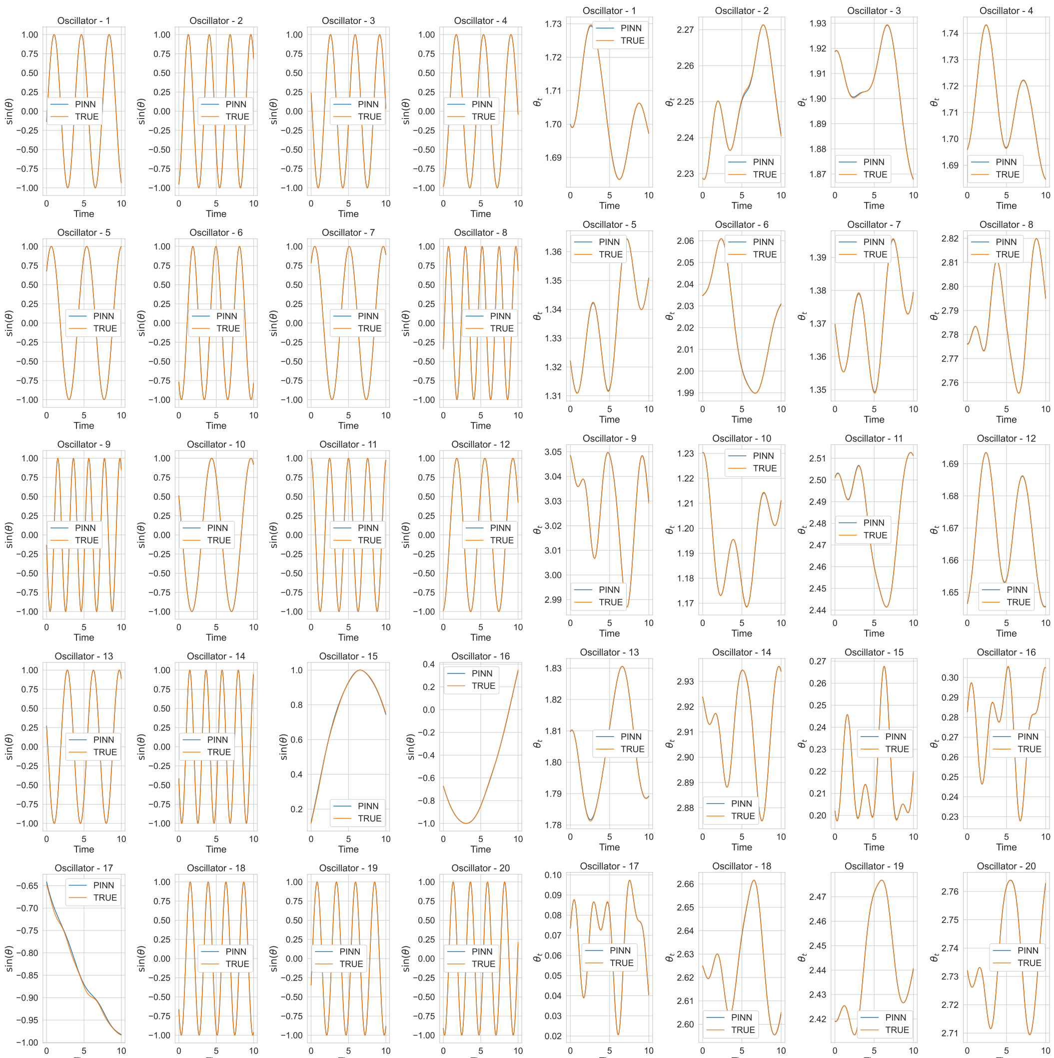


(a) True Solution($N = 20, K = 0.1$) (b) Solution from PINN($N = 20, K = 0.1$)



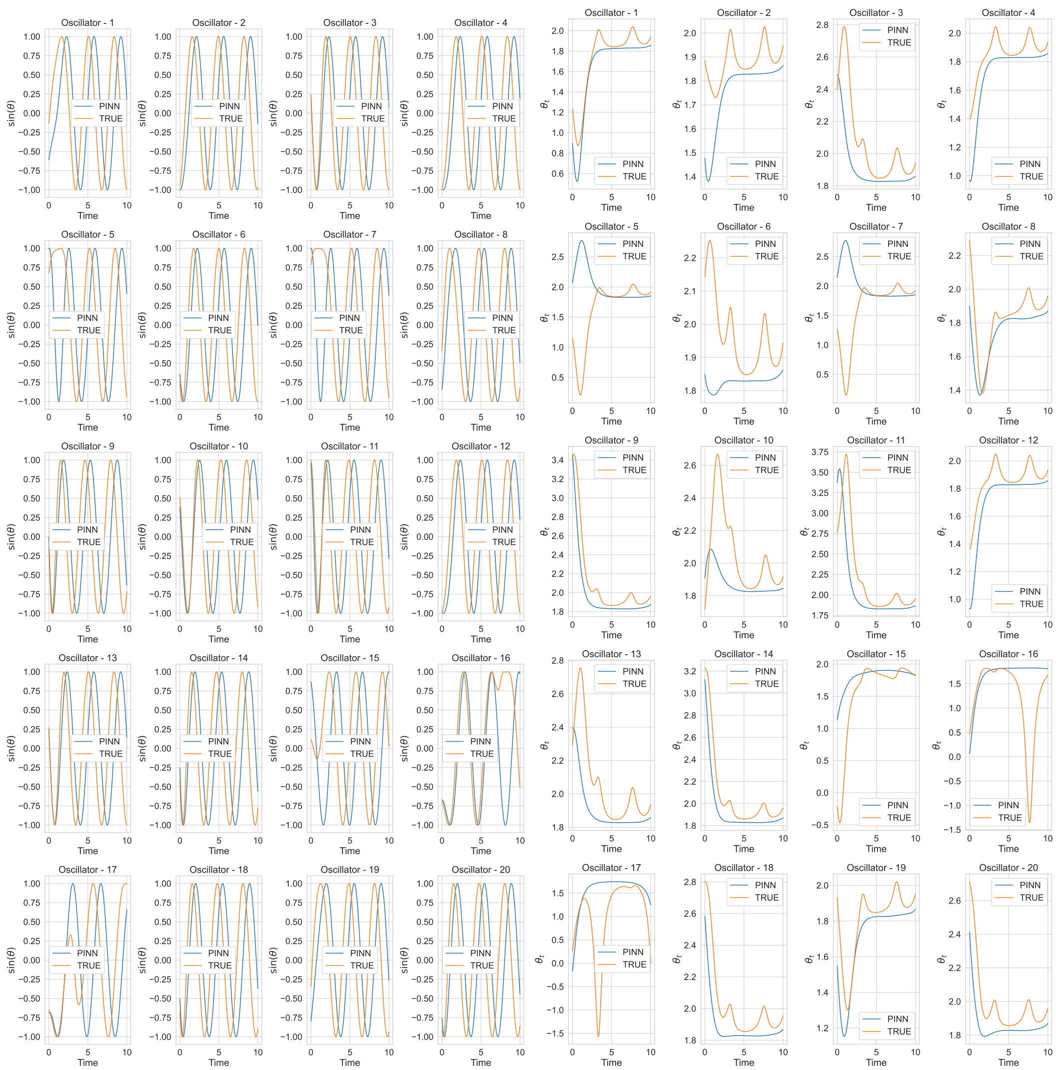
(c) True Solution($N = 20, K = 2.0$) (d) Solution from PINN($N = 20, K = 2.0$)

Figure 10: True and PINN solution for weakly and strongly coupled Kuramoto network with 20 oscillators.



(a) Phase (b) Instantaneous frequency

Figure 11: Phases and instantaneous frequency of each oscillator($N = 20, K = 0.1$)



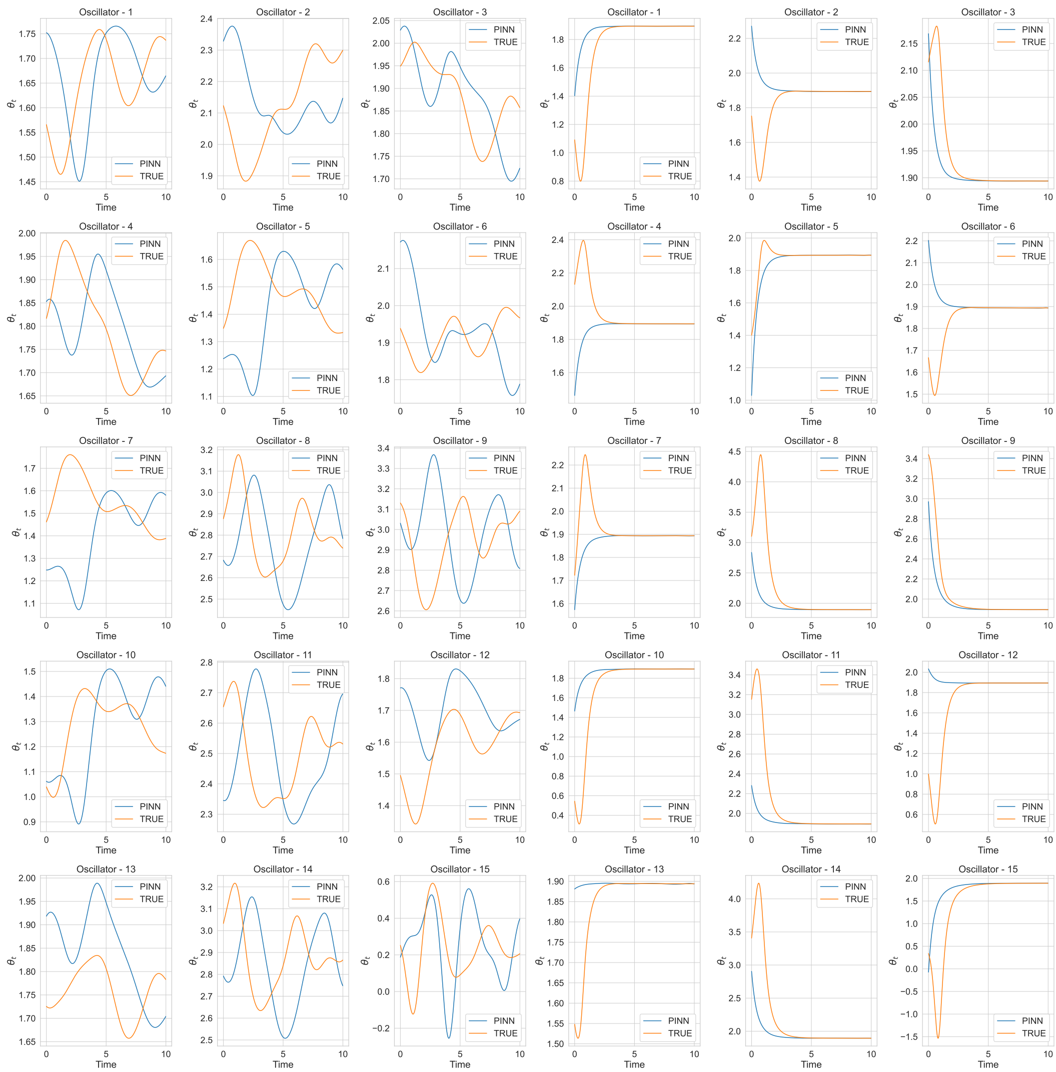
(a) Phase (b) Instantaneous frequency

Figure 12: Phases and instantaneous frequency of each oscillator($N = 20, K = 2.0$)

D Additional Experiments without Initial Condition

Here we have the results with the higher number of oscillators, i.e. $N = 15, 20$ mentioned in Section 3.3. Figures 13a and 13b show the comparison of instantaneous frequencies between two different coupling coefficient values for $N = 15$. Figures 14a and 14b show similar comparison for $N = 20$.

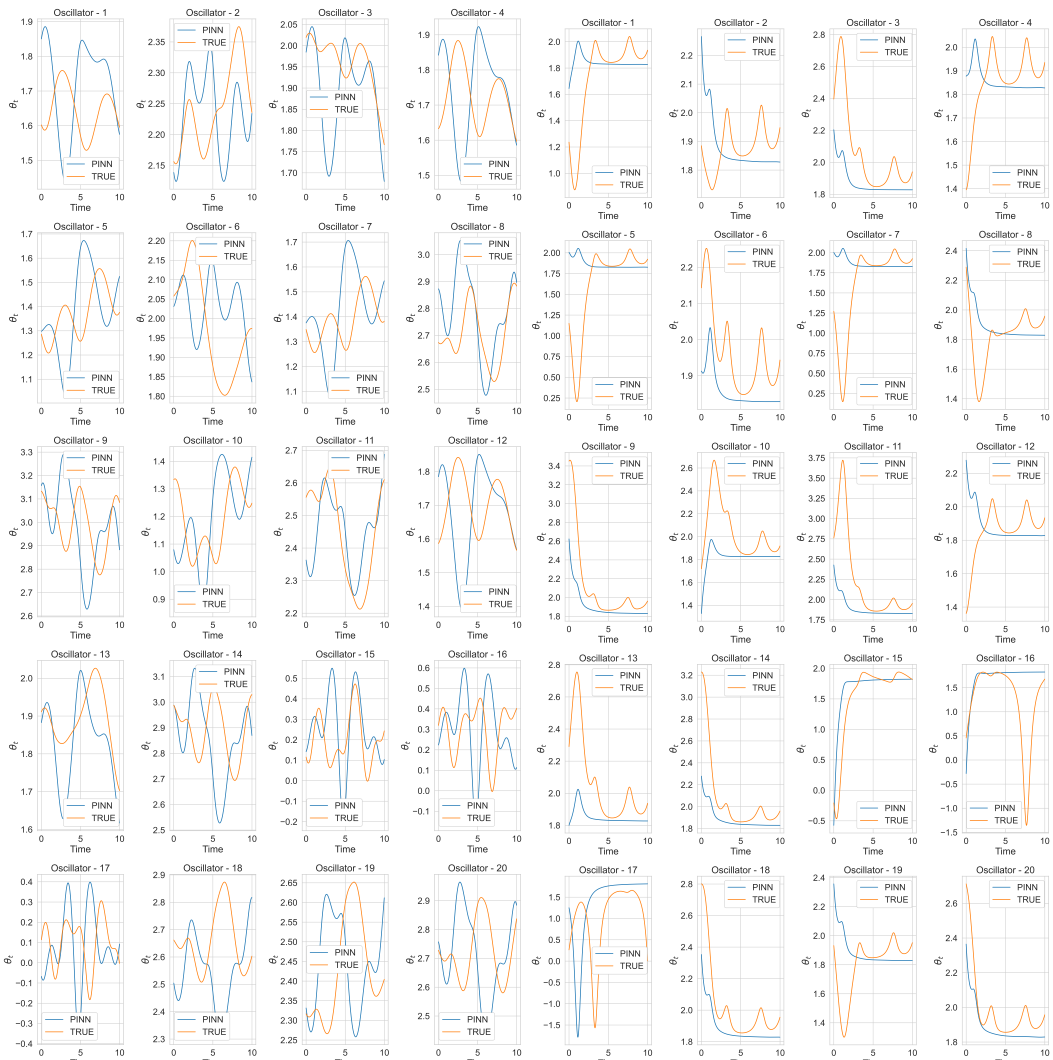
One interesting observation in Figure 14b is that the true solution is yet to reach a locking frequency in the given time duration, while the PINN has already reached it. This shows that PINN can be utilized to identify whether a system is synchronized or not without the initial conditions, whereas the numerical solver requires a longer time to reach the synchronous frequency due to random initialization of the phases of the oscillators.



(a) $N = 15, K = 0.5$

(b) $N = 15, K = 2.0$

Figure 13: Instantaneous frequencies of 15 oscillator systems. PINN was trained without providing the initial conditions.



(a) $N = 20, K = 0.5$

(b) $N = 20, K = 2.0$

Figure 14: Instantaneous frequencies of 20 oscillator systems. PINN was trained without providing the initial conditions.

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