INTERPRETABILITY-DRIVEN EXPLAINABILITY-DRIVEN ACTIVE FEATURE ACQUISITION IN LEARNING SYSTEMS

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ABSTRACT

In real-world applications like medicine, machine learning models must often work with a limited number of features due to the high cost and time required to acquire all relevant data. While several static feature selection methods exist, they are suboptimal due to their inability to adapt to varying feature importance across different instances. A more flexible approach is active feature acquisition (AFA), which dynamically selects features based on their relevance for each individual case. Here, we introduce an AFA framework that leverages Shapley Additive explanations SHapley Additive exPlanations (SHAP) to generate instance-specific feature importance rankings. By reframing the AFA problem as a feature prediction task, we propose a policy network based on a decision transformer architecture, trained to predict the next most informative feature based on SHAP values. This method allows us to sequentially acquire features in order of their predictive significance, resulting in more efficient feature selection and acquisition. Extensive experiments across multiple datasets show that our approach achieves superior performance compared to current state-of-the-art AFA techniques, both in terms of predictive accuracy and feature acquisition efficiency. These results demonstrate the potential of SHAP-based explainability-driven AFA for applications where feature acquisition cost is a critical consideration, such as in disease diagnosis.

1 INTRODUCTION

In traditional machine learning settings, it is typically assumed to have all features available during 035 inference. However, in real-world scenarios, especially in medical settings, acquiring these features can be expensive, time-consuming, and is often done sequentially. Therefore, it is crucial to develop 037 methods that can make accurate predictions with a limited number of features. This can be achieved by selecting a static global subset of features, but it is suboptimal since the important set of features may vary across different instances (Kachuee et al., 2019; Covert et al., 2023b). Additionally, the 040 chosen subset might not provide sufficient information for some cases, necessitating the acquisition 041 of more features to ensure a confident prediction. A more effective strategy is to identify important 042 features sequentially for each individual instance, a technique known as active (or dynamic) feature 043 acquisition (AFA), which has been gaining increasing attention in recent years (He & Chen, 2022; 044 von Kleist et al., 2023; Chattopadhyay et al., 2024).

The literature mainly contains two different ways of approaching AFA: reinforcement learning (RL)-based and greedy-based methods. Both approaches aim to develop a feature selection policy through exploration. RL-based methods (Kachuee et al., 2019; Yin et al., 2020; von Kleist et al., 2023) train policy networks by maximizing different reward functions. While the RL-based approach is intuitive for this sequential task and theoretically capable of finding the optimal policy, empirical evidence shows that RL-based methods often underperform compared to greedy-based methods Gadgil et al. (2024)(Gadgil et al., 2024). Greedy-based methods attempt to predict the next most important available feature by calculating conditional mutual information (CMI). To compute CMI, researchers have proposed both generative approaches (Rangrej & Clark, 2021; He et al., 2022) and methods based on the variational perspective (Covert et al., 2023b; Gadgil et al., 2024). However, calculating CMI directly remains challenging, and methods leveraging the variational perspective have demonstrated superior performance compared to generative alternatives.

In this work, we approached the problem by empirically observing that deep learning model learning-based local explanation methods, such as Shapley Additive Explanations SHapley Additive explanations (SHAP) (Lundberg & Lee, 2017), can be utilized to identify instance-wise feature importance rankings. With this insight, we treated the AFA problem as a feature prediction task rather than a feature exploration one. Our contributions are listed below:

- To the best of our knowledge, this is the first time in the AFA literature that the utility of local 062 explanation methods, specifically SHAP (Lundberg & Lee, 2017), has been demonstrated for 063 determining instance-wise feature importance rankings. We demonstrate that if we had an ideal 064 (oracle) policy network that sequentially selects features based on their SHAP values, sorted from 065 highest to lowest during inference, would outperform current state-of-the-art AFA techniques 066 in terms of accuracy for any fixed number of features. Similar observations were made in the 067 local explanation literature (Petsiuk et al., 2018; Jethani et al., 2021; 2022). They illustrate that 068 insertion (or deletion) of the important features ranked based on their respective explanation 069 methods improves (or degrades) model performance. However, these observations have yet to be formally compared with AFA techniques, leaving a gap in understanding how AFA methods 071 compare to these explanation-based feature ranking approaches.
- We took a different approach by training trained our policy network to predict the next unacquired feature with the highest SHAP value, based on the current observation.
 - We employed recently developed decision transformer (Chen et al., 2021) architecture as a policy network, and trained it using a two-stage approach. We showed that the feature importance ranking order is predictable without observing themit. Also, our experiments demonstrate that our technique achieves better or comparable results with the state-of-the-art AFA methods on different datasets.
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2 RELATED WORKS

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Generally, the methods in the AFA literature have two networks: a policy network for feature acquisition and a prediction network for prediction with available subset of features. These methods mainly differ in training their policy networks, so we only highlight those differences.

085 The AFA problem can be formulated as a Markov decision process (MDP) (Zubek & Dietterich, 2002; Dulac-Arnold et al., 2011); based on this formulation, there have been many RL-based approaches 087 proposed (Dulac-Arnold et al., 2011; Shim et al., 2018; Kachuee et al., 2019; Yin et al., 2020; Li 880 & Oliva, 2021; von Kleist et al., 2023). These methods generally train their policy networks with the objective of maximizing the defined reward functions. Namely, they try to approximate the 089 action-value function (i.e., Q-function). For example, in (Dulac-Arnold et al., 2011), the Q-function 090 is approximated linearly and later it is extended in (Janisch et al., 2019) using a deep Q network 091 (Mnih et al., 2015; van Hasselt et al., 2016). A similar approach was taken by the opportunistic 092 learning (OL) method in (Kachuee et al., 2019). Another type of mainstream methods (Rangrej & Clark, 2021; He et al., 2022; Covert et al., 2023b; Chattopadhyay et al., 2023; Gadgil et al., 094 2024) are the greedy-based methods frameworks. These methods acquire the features by estimating 095 the conditional mutual information (CMI) between the current available subset of features and the 096 unacquired features. For CMI estimation, there are generative approaches (Rangrej & Clark, 2021; 097 He et al., 2022) that use variational autoencoders (Kingma & Welling, 2013), and discriminative 098 approaches (Covert et al., 2023b; Chattopadhyay et al., 2023; Gadgil et al., 2024) directly predicting the feature index with the highest CMI without explicitly calculating CMI. Although, the MDP formulation is theoretically appealing, RL-based methods often underperform compared to the dis-100 criminative approaches such as the greedy-based methods Covert et al. (2023b); Gadgil et al. (2024) 101 (Covert et al., 2023b; Gadgil et al., 2024). 102

 In addition to AFA methods, related approaches from the budget learning literature Trapeznikov & Saligrama (2013); Nan & Saligrama (2017); Ekanayake & Zois (2024)
 (Trapeznikov & Saligrama, 2013; Nan & Saligrama, 2017; Ekanayake & Zois, 2024) explore fixed feature acquisition orders, limiting the number of potential feature subsets. These methods aim to identify easily classifiable instances, enabling the acquisition of a minimal set of features in such cases, thereby reducing overall acquisition costs.

108 With regards to the local explanation literature (Petsiuk et al., 2018; Jethani et al., 2021; Lundberg & 109 Lee, 2017), various methods focus on quantifying the contribution of individual features to model 110 predictions for each instance. Among these methods, SHAP (Lundberg & Lee, 2017), based on game-111 theoretic Shapley values Shapley (1953)(Shapley, 1953), is particularly popular. However, SHAP 112 calculations are computationally intensive, leading to the development of several approximations (Lundberg & Lee, 2017; Ancona et al., 2019; Jethani et al., 2022; Covert et al., 2023a). FastSHAP 113 (Jethani et al., 2022), for instance, provides an efficient approximation using a deep explainer model. 114 Additionally, global feature importance methods aim to identify the most relevant static features 115 across an entire dataset. For example, the Concrete Autoencoder (CAE) (Balin et al., 2019) trains an 116 autoencoder to select important features, while SAGE (Covert et al., 2020) extends Shapley values to 117 quantify global feature importance through an additive importance measure. For a detailed overview, 118 we refer readers to recent surveys (Samek et al., 2021; Bolón-Canedo et al., 2022). 119

120	Notation	Description
121	x	Input feature vector
122	d	Dimensions of input feature vector
123	y	Target label
124	C	Number of classes
125	q_{π}	Policy network (Causal transformer)
126	$f_{ heta}$	Predictor network
127	π	Parameters of policy network
128	heta	Parameters of predictor network
129	$\mathbf{r_t} \mathbf{r}_t$	Index of the latest acquired feature (rewardLogits of the predictor (action) after acquiring t features
130	a_t	Logits of the predictor (actionIndex of the latest acquired feature (reward) after acquiring t features
101	${f \hat{q}}$	Output of policy network q_{π}
100	$\hat{\mathbf{y}}$	Output of predictor network f_{θ}
132	$arphi^i(t)$	t^{th} important feature of $\mathbf{x}^{(i)}$ based on SHAP ranking
133	$\hat{\varphi^i(t)}$	t^{th} important feature of $\mathbf{x}^{(i)}$ based on policy network's predictions
134	M_t	Set of t most important feature indices based on SHAP ranking
135	\hat{M}_t	Set of first t feature indices acquired based on $\hat{\varphi}^i$
136	$\mathbf{x}_{M_{\star}}$	Input feature vector with features from M_t unmasked
137	X sôr	Input feature vector with features from \hat{M}_t unmasked
138	\mathcal{A}_{ℓ}	Context length of causal transformer
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Table 1: Mathematical notations used in the paper.

3 PROBLEM DESCRIPTION

Let $\mathbf{x} \in \mathbb{R}^d$ represent the *d*-dimensional input feature vector ¹, and $y \in \{1, 2, ..., C\}$ denote the associated target label, where *C* is the number of classes. Additionally, let $M \subseteq [d] \equiv \{1, ..., d\}$ be the subset of indices indicating the available features, and \mathbf{x}_M be the masked input vector with these available features. Each feature *j* has an associated cost c_j , and each input \mathbf{x} is subject to a budget constraint *k*. The objective is to find a predictor f_θ , parameterized with θ , and a policy network q_{π} , parameterized with π , such that the following constraint objective is minimized:

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$$\min_{\theta,\pi} \mathbb{E}_{\mathbf{x}yk} \mathbb{E}_{M \sim q_{\pi}} [\ell(f_{\theta}(\mathbf{x}_{M}), y)], \text{ s.t.} \sum_{j \in M} c_{j} \le k,$$
(1)

where the first expectation is taken over the joint distribution of \mathbf{x} , y, and k. The subset M is generated sequentially by the policy network q_{π} , which determines the next missing feature to acquire, i.e., arg max $q_{\pi}(\mathbf{x}_M) \in [d] \setminus M$. And, the predictor f_{θ} makes probabilistic predictions for any possible subset M, i.e., $f_{\theta}(\mathbf{x}_M) \in [0, 1]^{C,1}$. For brevity, let the output of q_{π} be denoted as $\hat{\mathbf{q}}$, i.e., $\hat{\mathbf{q}} = q_{\pi}(\mathbf{x}_M)$ and the output of f_{θ} be denoted as $\hat{\mathbf{y}}$, i.e., $\hat{\mathbf{y}} = f_{\theta}(\mathbf{x}_M)$.

¹Each feature can have different dimension size but ease of exposition, in here we have assumed each feature is one dimensional.

162 Typically, methods in the literature (Yin et al., 2020; Covert et al., 2023b) assume that features have 163 identical costs and that there is a fixed global budget k for all inputs. Given the available training 164 samples $\{(\mathbf{x}^i, y^i)\}_{i=1}^N$, these methods aim identifying input-specific important features to acquire 165 them sequentially in order of the most informative feature to the least one. To achieve this, they train 166 q_{π} through exploration using reinforcement learning (RL) (Yin et al., 2020) or information-theoretic (Covert et al., 2023b) formulations, while simultaneously training the predictor network f_{θ} . 167

In this paper, we approach the problem from a different perspective by assuming having access to feature importance rankings for each training sample. Consequently, instead of treating it as a feature exploration problem, we address it as a feature prediction problem (Figure 1).

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OUR METHODOLOGY



190 Figure 1: Overview of our active feature acquisition framework. a) Our training strategy consists of two stages and this figure shows how the masked inputs are generated during the first and second 191 stages. In the first stage, features are selected based on their ranking order derived from SHAP values. 192 In the second stage, features are acquired by the policy network (q_{π}) . During the first stage, the next 193 feature in the ranking is the target feature index. However, in the second stage, the target feature is 194 the feature index having the highest SHAP value among the ones that are not acquired; because of 195 this, the target feature remains the same until it is acquired. b) This part of the figure shows how 196 the policy network q_{π} , based on the decision transformer (Chen et al., 2021), processes the masked 197 inputs during training. Sequential data with a context length ℓ , set to 2 in this case, is fed into q_{π} . At each time step, q_{π} receives three tokens: the masked input (\mathbf{x}_{M_t}) , action $(a_t^{(i)})$ and reward $(\mathbf{r_t})$. 199 The action token represents the index of the last acquired feature, and the reward is the output of the 200 predictor network. To ensure causality, future tokens are masked while q_{π} predicts the next feature to 201 acquire at any time step. c) This figure illustrates the inference stage for image inputs in the causal 202 transformer model, where predicted features (or patches) are progressively acquired in a series of sequential acquisition steps. 203

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205 **Feature importance ranking**. In our method, we assume access to the feature rankings φ^i for each 206 training sample x^i , sorted by their importance. While determining the importance of features for each 207 input is challenging, we found that local explanation methods, particularly SHAP (Lundberg & Lee, 2017), can effectively achieve this goal. Empirically, we observed We assumed that a model with 208 reasonable task performance would naturally prioritize the most important instance-specific features, 209 which can be identified using explanation methods. We empirically validated our assumption by 210 observing that if the policy network perfectly acquires features in the order of highest to lowest 211 absolute SHAP values sequentially during inference, the predictor achieves the best performance on 212 average for a given budget of k available features, compared to the current state of the art methods 213 (Figure 2). 214

To be able to get the SHAP values of the features for the each input, first, we train a classifier using 215 $\{(\mathbf{x}^i, y^i)\}_{i=1}^N$ with the standard cross-entropy loss minimization. Then, we calculate the SHAP values

216	Dataset	# Features (d)	# Classes	# Samples	Image size	Patch size
217	ImageNette	196	10	13,395	224×224	16×16
218	CIFAR-100	64	100	60,000	32×32	4×4
219	CIFAR-10	64	10	60,000	32×32	4×4
220	BloodMNIST	196	8	17,092	28×28	2×2
221	Spambase	57	2	4,601	-	-
222	Metabric	489	<u>6</u>	1,898	- ~	-~
223	<u>CKD</u>	<u>50</u>	2	1,659	$\bar{\sim}$	$\bar{\sim}$
224	CPS		<u>3</u>	418	-	-
225	CIGS	23	$\frac{2}{\sim}$	2,139	$\bar{\sim}$	$\overline{\sim}$

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Table 2: **Summary of datasets used in our experiments.** For each dataset, we listed the number of features (*d*), number of classes, number of samples, image size, and patch size utilized. Note that Spambase is a tabular dataset; therefore, image size and patch size are not applicable.

of the features for each input \mathbf{x}^i and sort them to get φ^i , where $\varphi^i(1)$ is the feature index having the highest absolute SHAP value and $\varphi^i(d)$ is the feature index having the lowest absolute SHAP value for the input \mathbf{x}^i . So our training set is $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^N$.

235 **Policy network - Decision transformer.** By approaching the problem as the conditional sequence modeling task, similar to in the "decision transformer" framework (Chen et al., 2021), we train q_{π} , 236 which is a causal transformer model, with the objective of next action/token prediction. We feed q_{π} 237 with sequential data with a sequence length (i.e., context length) of ℓ . At each timestep, there are 238 three tokens including the input, the action and the reward as described in Chen et al. (2021). During 239 training, at the timestep of t, the input is $\mathbf{x}_{M_t}^i$, which is the *i*'th sample with the t many available 240 features and $M_t = \{\varphi^i(1), ..., \varphi^i(t)\}^2$. Whereas, the action a_t^i is the most recently acquired feature 241 index, i.e., $a_t^i = \varphi^i(t)$ and the reward \mathbf{r}_t^i is the output of the predictor with the current input, i.e., 242 $\mathbf{r}_t^i = \hat{\mathbf{y}}_t^i = f_{\theta}(\mathbf{x}_{M_t}^i)$. The rewards in reinforcement learning-based methods (Kachuee et al., 2019; Li 243 & Oliva, 2021) are typically functions of the predictor output; in our method, we follow a similar 244 idea, but instead of defining a specific function, we directly feed our policy transformer network with 245 the predictor output. So, for a given sequence from the timestep t to $t + \ell - 1$, the output of our q_{π} for the input i is: $\hat{\mathbf{q}}_{t}^{i} = q_{\pi}(\mathbf{x}_{M_{t}}^{i}, a_{t}^{i}, \mathbf{r}_{t}^{i})$ and $\hat{\mathbf{q}}_{t+\ell-1}^{i} = q_{\pi}(\mathbf{x}_{M_{t:t+\ell-1}}^{i}, a_{t:t+\ell-1}^{i}, \mathbf{r}_{t:t+\ell-1}^{i})$, where $t: t+\ell-1$ indicates all the tokens from the time step t to $t+\ell-1$. We used a mini version of 246 247 248 GPT³ architecture (Radford, 2018) as a transformer model. Please refer to the decision transformer 249 paper (Chen et al., 2021) for more details.

Training strategy. To train q_{π} , we minimized the standard cross-entropy loss by considering the index of the next feature that is not acquired with the highest SHAP value (i.e., $\varphi^i(t+1)$) as the true label with the minibatch setting. At each iteration, the loss function is:

$$\mathcal{L}_{q} = -\frac{1}{N_{b}} \sum_{i=1}^{N_{b}} \sum_{t=t_{i}}^{t_{i}+\ell-1} \log(\hat{\mathbf{q}}_{t,\varphi^{i}(t+1)}^{i}),$$
(2)

where N_b is the batch size, $\hat{\mathbf{q}}_{t,\varphi^i(t+1)}^i$ is the $\varphi^i(t+1)$ 'th element of $\hat{\mathbf{q}}_t^i$, and t_i is randomly sampled integer determining the initial time step of sequence fed to the model for the *i*'th sample. Simultaneously, we train the predictor f_{θ} also by minimizing the standard cross-entropy loss:

$$\mathcal{L}_{f} = -\frac{1}{N_{b}} \sum_{i=1}^{N} \sum_{t=t_{i}}^{t_{i}+\ell-1} \log(\hat{\mathbf{y}}_{t,y}^{i}).$$
(3)

During the first stage of training, both f_{θ} and q_{π} are fed by the input with the features that are acquired based on the SHAP value ranking order. However during inference, because q_{π} is not 100% accurate, the feature subset \hat{M}_t , generated by q_{π} , may not always contain the top t features with the

²Each sample *i* has its own specific M_t , but we do not specify through superscript *i* if it is clear from the context.

³https://github.com/karpathy/minGPT

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Figure 2: **Model performance.** Average classification performance of our AFA method compared with other well-known methods across varying number of features on the Spambase tabular and four image datasets: CIFAR-10, CIFAR-100, BloodMNIST and ImageNette.

highest SHAP values. To train both models to handle this new subset of features not encountered 298 in the first stage, we introduce a second stage of training. At the beginning of each iteration of the 299 second stage, we first generate empiric/predicted feature acquisition $\hat{\varphi}^i$ order for each \mathbf{x}^i , where 300 $\hat{\varphi}^i(t+1) = \arg \max \hat{\mathbf{q}}_t^i$ and $\hat{M}_t = \{\hat{\varphi}^i(1), \hat{\varphi}^i(2), \dots, \hat{\varphi}^i(t)\}$. Then, we minimize the same losses as 301 in the first stage with the same strategy. In \mathcal{L}_{q} , the index of the feature, which is not acquired yet and 302 having the highest SHAP value among the features that are not acquired, is taken as the true label. For 303 example, if $\varphi^i(1) \notin \{\hat{\varphi}^i(1), \dots, \hat{\varphi}^i(t)\}$ then $\varphi^i(1)$ is taken as the true label; but if $\varphi^i(1)$ is acquired 304 and $\varphi^i(2)$ is not acquired then $\varphi^i(2)$ is taken as the true label, i.e., $\varphi^i(1) \in \{\hat{\varphi}^i(1), ..., \hat{\varphi}^i(t)\}$ and 305 $\varphi^i(2) \notin \{\hat{\varphi}^i(1), ..., \hat{\varphi}^i(t)\}$. By this second stage, we train the predictor f_{θ} to make its prediction 306 with the subset of features \hat{M}_t acquired by q_{π} . Also, the policy network q_{π} is trained to predict the 307 feature with the highest SHAP value among the features that are not acquired using the input with 308 the imperfect subset of features \hat{M}_t . This second stage helps both networks to perform better during 309 inference, where the imperfect subset of features \hat{M}_t can only be used. Note that both the predictor 310 and policy networks are dependent on each other. However, during training, we prevent the gradient 311 flow from one network to another. Therefore, each network has its own independent loss function; 312 because of the dependency, we trained them simultaneously. At t = 0, there is no feature acquired 313 yet, i.e., $M_0 = \emptyset$; so for all i, the outputs of q_{π} are the same at t = 0. Consequently, at t = 0, for all 314 inputs we have to choose the same feature to be acquired. In our method, we initialized each input by the fixed first feature that has the highest SHAP value on average calculated on the training set. 315

316 Implementation details. During training, we set number of epochs to 200 and 16 for the first and 317 second stage, respectively. We used Adam optimizer (Kingma & Ba, 2014) and a cosine scheduler 318 (Loshchilov & Hutter, 2017). Before starting training, we pre-trained the predictor network, as done 319 in (Covert et al., 2023b; Gadgil et al., 2024). We also employed a different augmentation strategy 320 proposed in (Hoffer et al., 2020). Also, as done by other methods in the literature (Kachuee et al., 321 2019; Covert et al., 2023b; Gadgil et al., 2024), we shared the backbone between f_{θ} and q_{π} . We used this backbone in q_{π} to get the embedding of the input token. The embedding of action was extracted 322 using a learnable embedding dictionary. For the reward's embedding, a simple MLP was used. In q_{π} , 323 we set context length ℓ to 4, number of heads and layers 4 and 3, respectively.

324		Spambase	CIFAR-10	CIFAR-100	BloodMNIST	ImageNette
325	# of classes:	2	10	100	8	10
326	First-stage	0.9512	75.68%	45.88%	79.08%	73.35%
327	Second-stage	0.9559	78.61%	47.06%	84.38%	79.08%
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Table 3: Stage-wise classification performance. The table presents our model's performance after 330 the first and second training stages, averaged over the first 20 features, on the Spambase, CIFAR-10, CIFAR-100, BloodMNIST, and ImageNette datasets. For the Spambase dataset, we reported the 331 332 area under the receiver operating characteristic curve values, while for the remaining datasets, we provided accuracy metrics. 333

	CIFAR-10	CIFAR-100	BloodMNIST	ImageNette
# of features (d):	64	64	196	196
Top 10 features	36.54%	48.71%	42.17%	11.08%
Top 15 features	46.08%	58.30%	49.09%	15.99%
Top 20 features	52.46%	64.78%	53.58%	20.61%
Top 25 features	57.41%	68.71%	56.52%	24.92%
Top 30 features	$\underline{62.00\%}$	$\underline{71.32\%}$	58.63%	29.01%

Table 4: Alignment between model's feature acquisition order and the SHAP-based feature importance rankings. This table presents the percentage overlap between the top N features ranked by SHAP values and features acquired by our model for N = 10, 15, 20, 25, and 30. The datasets include CIFAR-10 and CIFAR-100 (each with 64 features), and BloodMNIST and ImageNette (each with 196 features).

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5 **RESULTS AND DISCUSSION**

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353 We utilized several datasets in our experiments (Table 2), including ImageNette, CIFAR-10, CIFAR-100, BloodMNIST, and Spambase. ImageNette Howard (2019) (Howard, 2019) is a 10-354 class subset of the ImageNet dataset Deng et al. (2009)(Deng et al., 2009). CIFAR-10 and CIFAR-355 100 Krizhevsky (2009) (Krizhevsky, 2009) are subsets of the 80 Million Tiny Images dataset 356 Torralba et al. (2008)(Torralba et al., 2008), containing 10 and 100 classes respectively. BloodM-357 NIST (Acevedo et al., 2020), derived from the MedMNIST dataset (Yang et al., 2021; 2023), 358 comprises images of individual normal cells collected from individuals without infection, hemato-359 logic or oncologic diseases, and free of any pharmacologic treatment at the time of blood collection. 360 The patch sizes are 16×16 for ImageNette (makes total of 196 patches, d = 196), 4×4 for the 361 CIFAR-10 and CIFAR-100 datasets (d = 64), and 2×2 for the BloodMNIST dataset (d = 196). 362 Spambase (Hopkins & Suermondt, 1999) is a well-known tabular dataset for classifying spam 363 emails, consisting of 57 features derived from textual data. Additionally, to assess the applicability of our method in real-world scenarios, such as healthcare, we conducted experiments on four medical 364 tabular datasets. As part of the preprocessing, we removed ID columns and categorical columns that were not ranking-based or binary. Columns with more than 10% missing values were also 366 excluded, while the remaining missing values were imputed with the mean. In the following and 367 in Table 2, the number of features refers to the count after preprocessing. The Metabric dataset 368 (Curtis et al., 2012; Pereira et al., 2016) contains targeted gene sequencing data from 1,898 breast 369 cancer samples, where we utilized mRNA-level Z-scores, which contains 489 features, to predict 370 the Pam50 gene status that is a multi-class classification task. The Cirrhosis Patient Survival (CPS) 371 dataset (Dickson & Langworthy, 1989) includes records from 418 patients, primarily with primary 372 biliary cirrhosis, along with 8 clinical features, with the task of predicting patient survival states 373 categorized as Death, Censored, or Censored Due to Liver Transplantation. The AIDS Clinical Trials 374 Group Study 175 (CTGS) dataset (Hammer et al., 1996) contains 2139 records of patients diagnosed 375 with AIDS, 23 features, with a binary classification task to predict whether a patient has died within a specified time period. Lastly, the Chronic Kidney Disease (CKD) dataset (Kharoua, 2024) 376 comprises 1659 patient records with 50 clinical features, and the task is to predict whether a patient 377 is diagnosed with chronic kidney disease in a binary classification setting.

378		Metabric	CPS	CTGS	CKD
379	# of classes:	6	3	2	2
380	TreeSHAD	60.00%	67 19%	0.0167	0.8473
381	LIME	69.9070	66 21%	0.9107 0.0125	0.0470 0.8122
382	KernelSHAP	70.01%	66.06%	0.0120 0.9146	0.0122 0.8353
383	Sampling(IME)	69 72%	65 91%	0.9140 0.9160	0.0000 0.8152
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Table 5: Model performance using various feature ranking approaches. Comparison of classification performance across four medical datasets using feature rankings derived from various local explanation methods: TreeSHAP, LIME, KernelSHAP, and Sampling (IME). The performance metrics are the area under the receiver operating characteristic curve for the binary-classification datasets and accuracy for the multi-class datasets.

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392 To test the robustness of our method across different architectures, we also varied predictor architectures. We employed ResNet50 He et al. (2016) (He et al., 2016) for ImageNette, ResNet18 393 He et al. (2016) (He et al., 2016) for the CIFAR-10 and CIFAR-100 datasets, and a custom CNN for 394 the BloodMNIST dataset. The custom CNN architecture has four convolution layers with output 395 channels 16, 32, 64, and 64, each followed by a ReLU activation and a max pooling layer. The 396 convolution layers are followed by flattening and linear layers for classification. For the Spambase 397 dataset, we used a multi-layer perception (MLP) architecture consisting of 2 hidden layers with 398 128 neurons, each followed by a ReLU and a dropout layer. On the medical tabular datasets, we 399 utilized the same MLP architecture with 1024 hidden layer neurons on Metabric, 512 on CKD, 512 400 on CTGS and 128 on CPS. For the image datasets, we employed FastSHAP (Jethani et al., 2022) to 401 generate the feature SHAP ranking order φ^i for each instance xⁱ due to its speed. During training, 402 we applied random augmentations that can affect feature importance. The speed of FastSHAP 403 allows us to efficiently handle these changes in feature importance during the training process. 404 For the Spambase datasettabular datasets, we did not apply any data augmentation during training. We obtained the Tree-based models, specifically CatBoost (Prokhorenkova et al., 2018), were used 405 as the initial model to determine feature ranking orders, owing to their superior performance on 406 tabular data (Grinsztajn et al., 2022). SHAP ranking orders were obtained using the SHAP pack-407 age⁴, leveraging TreeSHAP (Lundberg et al., 2020), a method specifically designed for SHAP 408 value calculations in tree-based models 409

410 We evaluated our method against several existing approaches for feature selection: Discriminative 411 Mutual Information Estimation (DIME), Greedy Dynamic Feature Selection (GDFS), Concrete 412 Autoencoder (CAE), and two simple baselines-: center-cropping and random selection. DIME (Gadgil et al., 2024) takes an information-theoretic approach by prioritizing features based on their 413 mutual information with the response variable, estimating this mutual information in a discriminative 414 rather than a generative manner. GDFS (Covert et al., 2023b) employs a simpler, greedy strategy 415 for selecting features based on their conditional mutual information, utilizing a learning approach 416 grounded in amortized optimization; the policy network is shown to recover the greedy policy when 417 trained to optimality. CAE (Balin et al., 2019) is an unsupervised, end-to-end differentiable method 418 for global feature selection that uses a standard neural network as the decoder for reconstruction and 419 incorporates a concrete selector layer as the encoder. The temperature in CAE is gradually decreased 420 to slowly discretize the selections. The center-cropping and random selection methods (Covert et al., 421 2023b) serve as simple baselines: center-cropping selects center patches of varying sizes, while 422 random selection chooses patches randomly.

423 Figure 2 demonstrates that our method shows superior, or comparable performance on all the datasets. 424 For example, on the ImageNette dataset, with the few number of patches, our method performs well, 425 achieving 64.2% and 74.8% average accuracy with two and five available patches among 196 patches, 426 respectively. Additionally, our model achieved an AUROC score of 0.8761 on the CKD dataset. To 427 demonstrate the relative potential of our approach, we also provided the oracle setting performances, 428 where during inference the features are perfectly acquired based on the SHAP ranking. On the 429 oracle setting, we also initialized the instances with the feature having the highest SHAP value on average, as in our method. Therefore, while it is theoretically possible to achieve these performance 430

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⁴https://pypi.org/project/shap/

432 levels, empirically we could not attain them with our current method. We discovered that instead of 433 initializing the inputs with only one feature, it is more effective for stable training to begin with three 434 features. Based on this finding, we fixed first three feature acquisition order and we obtained the 435 results as shown in Figure 2. For the initial features, we selected the second and third features based 436 on their average importance. Note that fixing the acquisition order for all d features is equivalent to using static global feature selection methods like CAE, which is suboptimal, as our empirical 437 results demonstrate. Therefore, initializing with more than one feature can negatively impact the 438 achievable upper bound in performance. However, we found that fixing the acquisition order for 439 a few initial features helps stabilize training. Additionally, since our method relies on the feature 440 ranking order, having a better ranking can lead to improved performance. Our approach can work 441 with any ranking order, including those provided by humans, but we have shown that local model 442 explanation algorithms, particularly SHAP, are effective in providing this order. 443

The average performance after both stages is shown for all the datasets (Table 3), highlighting the benefit of the second stage. The second stage provides significant improvement on almost all datasets, except on the Spambase that is a relatively simpler dataset compared to others, at least in terms of number of classes and features. Specifically, the second stage provides classification accuracy increase from 2.57% (on CIFAR-100) to 7.81% (on ImageNette).

Lastly, in Table We also performed some ablation experiments to evaluate the robustness and 449 effectiveness of our method. Firstly, we tested our method with alternative feature ranking 450 approaches, including another explainability method, LIME (Ribeiro et al., 2016), and two different 451 SHAP value calculation techniques: KernelSHAP (Lundberg & Lee, 2017) and IME (sampling) 452 (Štrumbelj & Kononenko, 2010). These results (Table 5) indicate that while our method is robust to 453 different ranking orders, its performance is also dependent on the the quality of the ranking order 454 generated by the explainability methods. To further verify the second point and test the dependency 455 of the SHAP ranking orders' quality on the pre-trained model capacity, we conducted another 456 ablation experiment on the CIFAR-10 dataset. Specifically, we used ResNet-10, a smaller model 457 compared to ResNet-18, as the pre-trained model for determining the SHAP ranking order, while 458 retaining ResNet-18 as the classification network. We observed that the performance of our method 459 decreased from 78.61% to 78.22% on the test set, and from 79.12% to 78.42% on the validation set. 460 These results confirm that the pre-trained model's capacity impacts the SHAP-based ranking order 461 and, consequently, the performance of our method. In addition, we evaluated the effectiveness of using the decision transformer by comparing our method's performance with different architectures. 462 When the decision transformer was replaced with a ResNet block, the model's accuracy decreased 463 from 78.61% to 76.83% on the CIFAR-10 dataset and from 47.06% to 46.70% on the CIFAR-100 464 dataset. Similarly, substituting the decision transformer with a CNN block reduced the model's 465 accuracy from 84.38% to 78.23% on the BloodMNIST dataset. These results demonstrate the 466 advantage of using a decision transformer as the policy network while also highlighting that our 467 method performs reasonably well with other architectures as the policy network. 468

(Lastly, i) In Table 4, we present the overlap ratios between our model's acquired feature order 469 and the SHAP-based feature importance rankings across different datasets. As the number of 470 top features (N) increases from 10 to 30, the percentage overlap generally rises for CIFAR-10, 471 CIFAR-100, BloodMNIST, and ImageNette. This trend indicates that our model's feature acquisition 472 order increasingly aligns with the SHAP rankings as more features are considered. While the 473 oracle performances in Figure 2 demonstrate the practical benefits of using SHAP values in the 474 AFA problem, Table 4 highlights the degree to which our model's acquisition strategy predicts 475 the SHAP-based feature importance ranking. Additionally, we would like to note that we did not 476 perform detailed parameter search on the experiments. We selected the context length ℓ parameter 477 by comparing the validation scores on the CIFAR-10 dataset and the other parameters were selected 478 heuristically by hand. Subsequently, all these parameters were fixed for all the experiments. About the ℓ parameter selection, we found that our model achieved accuracies of 78.41%, 78.76%, 79.12%, 479 and 78.12% on the CIFAR-10 validation dataset with $\ell = 1, \ell = 2, \ell = 4$, and $\ell = 8$, respectively. 480 When we increased or decreased the context length ℓ , we correspondingly varied the batch size 481 N_b by the same factor to maintain the same effective size at each iteration (see Equations 2 and 482 3). Based on these observations, we assigned $\ell = 4$. Finally, we would like to emphasize that 483 our proposed method is flexible and can operate with any given feature order. However, due to 484 the absence of a reference standard (or ground truth) for feature importance rankings, we relied 485 on explainability methods to generate the feature orders. While these approaches are useful, they

may not always provide good rankings in all scenarios (Kumar et al., 2020; Catav et al., 2021). As
different local explanation methods that provide better rankings are developed, our approach can be
readily integrated to deliver enhanced results.

Top 10 features 36.54% 48.71% 42.17% 11.08% Top 15 features 46.08% 58.30% 49.09% 15.99%490Top 20 features 52.46% 64.78% 53.58% 20.61% Top 25 features 57.41% 68.71% 56.52% 24.92%491Top 30 features 62.00% 71.32% 58.63% 29.01% Alignment between model's feature acquisition492order and the SHAP-based feature importance rankings.493overlap between the top N features ranked by SHAP values and features acquired by our model for494N = 10, 15, 20, 25, and 30. The datasets include CIFAR-10 and CIFAR-100 (each with 64 features),495and BloodMNIST and ImageNette (each with 196 features).

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6 CONCLUSION

499 Our work proposes a novel-introduces an explainability-based active feature acquisition strategy by 500 reframing it as a feature prediction task, where the model learns to acquire features based on instance-501 specific SHAP value rankings. Stage-wise results show-demonstrate that our two stage training 502 approach enhances improves feature selection and classification performance on both tabular and 503 image datasets. The findings also indicate further suggest that our method is robust across varying 504 various models, datasets and settings. Future work could apply our method to more practical datasets 505 , such as those in medical diagnosis, to evaluate its usability in Additionally, our experimental results 506 on medical tabular datasets highlight the practical applicability of our method in real-world scenarios 507 -like healthcare. Future work could explore dynamic recalculations of feature attributions during 508 training after each feature acquisition step, replacing the current use of fixed ordering.

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756 A <u>Appendix</u> 757

Algori	thm 1 Pseudocode for first stage training of q_{π} and f_{θ}
Requi	re: Training set $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^N$, batch size N_b , context length ℓ , learning rate γ
1: Pr	e-train f_{θ} on $\{(\mathbf{x}^i, y^i)\}_{i=1}^N$ using random feature selection
2: <u>In</u>	tialize q_{π}
3: fo	each epoch do
4:	for 1 to $ N/N_b $ do
5:	Sample minibatch $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^{i=1}$ (if random augmentation is applied, φ
6.	Sample random integer t_i for each i
0. 7:	Initialize $\mathcal{L}_{a} = 0$ and $\mathcal{L}_{f} = 0$
8:	for $t_x = 0$ to $\ell - 1$ do
9:	Define temporary parameter t'_i for each $i, t'_i = t_i + t_x$
10:	Generate masked input $\mathbf{x}_{M_{t'}}^i, M_{t'_{t'}} = \{\varphi^i(1), \dots, \varphi^i(t'_i)\}$
11:	Compute predictor output: $\hat{\mathbf{y}}_{t'}^i = f_{\theta}(\mathbf{x}_{M_{rt}}^i)$
12:	Compute policy network output: $\hat{\mathbf{q}}_{t_{t'}}^i = q_{\pi} (\mathbf{x}_{\mathcal{M}_{t;t_{t'}}}^i, a_{t;t_{t'}}^i, \mathbf{r}_{t;t_{t'}}^i)$
13:	Update losses: $\mathcal{L}_{f} \leftarrow -\frac{1}{N_{b}} \sum_{i=1}^{N_{b}} \log(\hat{\mathbf{y}}_{t_{i}',y^{i}}^{i}) + \mathcal{L}_{f}$
14:	$\mathcal{L}_{g} \leftarrow -\frac{1}{N_{b}} \sum_{i=1}^{N_{b}} \log(\hat{\mathbf{q}}_{t_{i}',r}^{i} \circ (t_{i}'+1)) + \mathcal{L}_{g}$
15:	Update parameters $\theta \leftarrow \theta - \gamma \nabla_{\theta} \mathcal{L}_{f_2}$, $\pi \leftarrow \pi - \gamma \nabla_{\pi} \mathcal{L}_{g_2}$
Algori Requi	thm 2 Pseudocode for second stage training of q_{τ} and f_{θ} re: Training set $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^N$, batch size N_b , context length ℓ , learning rate γ , f_{θ} are on the first stage
Algori Requir 1: for	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^N$, batch size N_b , context length ℓ , learning rate γ , f_{θ} and f_{θ} mether first stage reach epoch do
Algori Requi frc 1: for 2:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^N$, batch size N_b , context length ℓ , learning rate γ , f_{θ} and f_{θ} on the first stage reach epoch do for 1 to $[\tilde{N}/N_b]$ do
Algori Requir 1: for 2: 3:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and m the first stage reach epoch do for 1 to $[\tilde{N}/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ
Algori Requir 1: for 2: 3: rec	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and p_{θ} for 1 to first stage r each epoch do for 1 to $[\widehat{N}/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling)
Algori Requir frc 1: for 2: 3: 3: 4:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and m the first stage r each epoch do for 1 to $[N/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i
Algori Requin 1: foi 2: 3: 5: 5: 6:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and m the first stage reach epoch do for 1 to $[\tilde{N}/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Lititalize $f_{i} = 0$ and $f_{i} = 0$
Algori Requin 1: for 2: 3: 4: 5: 6: 7:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^N$, batch size N_b , context length ℓ , learning rate γ , f_{θ} and m the first stage reach epoch do for 1 to $\lceil N/N_b \rceil$ do Sample minibatch $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^{N_b}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^i$ for each i Sample random integer t_i for each i Initialize $\mathcal{L}_q = 0$ and $\mathcal{L}_f = 0$ for $t_a = 0$ to $\ell - 1$ do
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and m the first stage reach epoch do for 1 to $[\tilde{N}/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Initialize $\mathcal{L}_{q} = 0$ and $\mathcal{L}_{f} = 0$ for $t_{x} = 0$ to $\ell - 1$ do Define temporary parameter t'_{i} for each i , $t'_{i} = t_{i} + t_{x}$
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8: 9:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and f_{θ} on the first stage reach epoch do for 1 to $[\hat{N}/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Initialize $\mathcal{L}_{q} = 0$ and $\mathcal{L}_{f} = 0$ for $t_{x} = 0$ to $\ell - 1$ do Define temporary parameter t'_{i} for each $i, t'_{i} = t_{i} + t_{\pi}$ Generate masked input $\mathbf{x}'_{i}, \hat{M}_{t'} = \{\hat{\varphi}^{i}(1), \dots, \hat{\varphi}^{i}(t'_{i})\}$
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8: 9: 10:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and f_{θ} re ach epoch do for 1 to $[\hat{N}/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Initialize $\mathcal{L}_{q} = 0$ and $\mathcal{L}_{f} = 0$ for $t_{x} = 0$ to $\ell - 1$ do Define temporary parameter t'_{i} for each $i, t'_{i} = t_{i} + t_{x}$ Generate masked input $\mathbf{x}^{i}_{M_{1}}, \hat{M}_{t'_{i}} = \{\hat{\varphi}^{i}(1), \dots, \hat{\varphi}^{i}(t'_{i})\}$ Compute predictor output: $\hat{y}^{i}_{t'_{i}} = f_{\theta}(\mathbf{x}^{i}_{M_{1}})$
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8: 9: 10: 11:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and m the first stage r each epoch do for 1 to $[\hat{N}/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Initialize $\mathcal{L}_{q} = 0$ and $\mathcal{L}_{f} = 0$ for $t_{x} = 0$ to $\ell - 1$ do Define temporary parameter t'_{i} for each $i, t'_{i} = t_{i} + t_{x}$ Generate masked input $\mathbf{x}^{i}_{M_{1}}, \hat{M}_{t'_{i}} = \{\hat{\varphi}^{i}(1), \dots, \hat{\varphi}^{i}(t'_{i})\}$ Compute predictor output: $\hat{\mathbf{y}}^{i}_{t'_{i}} = f_{\theta}(\mathbf{x}^{i}_{M_{1}})$ Compute policy network output: $\hat{\mathbf{q}}^{i}_{t'_{i}} = q_{\pi}(\mathbf{x}^{i}_{M_{1}}, \mathbf{q}^{i}_{t+1}, \mathbf{r}^{i}_{t,t})$
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8: 9: 10: 11: 12:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and p_{θ} and the first stage reach epoch do for 1 to $[N/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Initialize $\mathcal{L}_{q} = 0$ and $\mathcal{L}_{f} = 0$ for $t_{x} = 0$ to $\ell - 1$ do Define temporary parameter t'_{i} for each $i, t'_{i} = t_{i} + t_{\pi}$. Generate masked input $\mathbf{x}^{i}_{\hat{M}_{i'}}, \hat{M}_{t'_{i}} = \{\hat{\varphi}^{i}(1), \dots, \hat{\varphi}^{i}(t'_{i})\}$. Compute predictor output: $\hat{y}^{i}_{t_{i}} = f_{\theta}(\mathbf{x}^{i}_{\hat{M}_{t_{i}'}})$ Compute policy network output: $\hat{q}^{i}_{t_{i}} = q_{\pi}(\mathbf{x}^{i}_{\hat{M}_{t_{i'}}}, \mathbf{a}^{i}_{t_{i}t_{i'}}, \mathbf{x}^{i}_{t_{i}t_{i'}})$, Update losses: $\mathcal{L}_{f} \leftarrow -\frac{1}{N_{e}} \sum_{i=1}^{N_{b}} \log(\hat{\mathbf{y}}^{i}_{t_{i'}}, \mathbf{a}) + \mathcal{L}_{f}$
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8: 9: 10: 11: 12: 13:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and m the first stage r each epoch do for 1 to $\lceil N/N_{b} \rceil$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Initialize $\mathcal{L}_{q} = 0$ and $\mathcal{L}_{f} = 0$ for $t_{x} = 0$ to $\ell - 1$ do Define temporary parameter t'_{i} for each $i, t'_{i} = t_{i} + t_{x}$ Generate masked input $\mathbf{x}^{i}_{M_{f_{i}}}, \hat{M}_{t'_{i}} = \{\hat{\varphi}^{i}(1), \dots, \hat{\varphi}^{i}(t'_{i})\}$ Compute predictor output: $\hat{\mathbf{y}}^{i}_{t_{f}} = f_{\theta}(\mathbf{x}^{i}_{M_{f_{i}}})$ Compute policy network output: $\hat{\mathbf{q}}^{i}_{t_{f}} = q_{\pi}(\mathbf{x}^{i}_{M_{f_{i}}}, \mathbf{a}^{i}_{ktt'_{i}}, \mathbf{r}^{i}_{ktt'_{i}})$. Update losses: $\mathcal{L}_{f} \leftarrow -\frac{1}{N_{0}} \sum_{i=1}^{N_{b}} \log(\hat{\mathbf{y}}^{i}_{t_{f}}y_{i}) + \mathcal{L}_{f}$ Determine the true label for the q_{π} network (denote this true label as u^{i}). The
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8: 9: 10: 11: 12: 13:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and m the first stage ϵ each epoch do for 1 to $[\hat{N}/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Initialize $\mathcal{L}_{q} = 0$ and $\mathcal{L}_{f} = 0$ for $t_{x} = 0$ to $\ell - 1$ do Define temporary parameter t'_{i} for each $i_{i}, t'_{i} = t_{i} + t_{x}$. Generate masked input $\mathbf{x}^{i}_{M_{f_{i}}}, \hat{M}_{t'_{i}} = \{\hat{\varphi}^{i}(1), \dots, \hat{\varphi}^{i}(t'_{a})\}$ Compute predictor output: $\hat{\mathbf{y}}^{i}_{t_{f}} = f_{\theta}(\mathbf{x}^{i}_{M_{f_{f_{i}}}}, \mathbf{g}^{i}_{t_{i}t_{f_{f}}}, \mathbf{g}^{i}_{t_{i}t_{f}}, \mathbf{g}^{i}_{t_{i}t$
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8: 9: 10: 11: 12: 13: an	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} and f_{θ} re ach epoch do for 1 to $[\hat{N}/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied. φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Initialize $\mathcal{L}_{q} = 0$ and $\mathcal{L}_{f} = 0$ for $t_{x} = 0$ to $\ell - 1$ do Define temporary parameter t'_{s} for each $i, t'_{i} = t_{i} + t_{x}$. Generate masked input $\mathbf{x}^{i}_{M_{t'_{i}}}, \hat{M}_{t'_{t'}} = \{\hat{\varphi}^{i}(1), \dots, \hat{\varphi}^{i}(t'_{i})\}$ Compute predictor output: $\hat{y}^{i}_{t'_{t'}} = f_{\theta}(\mathbf{x}^{i}_{M_{t'_{t'}}}), \frac{d^{i}_{t_{t'_{t'}}}\mathbf{x}^{i}_{t_{t'_{t'}}}}{d^{i}_{t_{t'_{t'}}}\mathbf{x}^{i}_{t_{t'_{t'}}}}, \frac{d^{i}_{t_{t'_{t'}}}\mathbf{x}^{i}_{t_{t'_{t'}}}}{d^{i}_{t_{t'_{t'}}}\mathbf{x}^{i}_{t_{t'_{t'}}}}$. Update losses: $\mathcal{L}_{f} \leftarrow -\frac{1}{N_{b}}\sum_{i=1}^{N_{b}} \log(\hat{y}^{i}_{t'_{t'_{t'}}}) + \mathcal{L}_{f}$. Determine the true label for the q_{π} network (denote this true label as $y^{i}_{q_{t'_{t}}}$). The point is the index of the feature, which is not acquired yet and having the highest SHAP vectors of the features that are not acquired.
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8: 9: 10: 11: 12: 13: 14:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N}$, batch size N_{b} , context length ℓ , learning rate γ , f_{θ} an m the first stage reach epoch do for 1 to $[N/N_{b}]$ do Sample minibatch $\{(\mathbf{x}^{i}, y^{i}, \varphi^{i})\}_{i=1}^{N_{b}}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^{i}$ for each i Sample random integer t_{i} for each i Initialize $\mathcal{L}_{q} = 0$ and $\mathcal{L}_{f} = 0$ for $t_{x} = 0$ to $\ell - 1$ do Define temporary parameter t'_{x} for each i , $t'_{x} = t_{i} + t_{x}$. Generate masked input $\mathbf{x}^{i}_{M_{f_{1}}}$, $\hat{M}_{t'_{x}} = \{\hat{\varphi}^{i}(1), \dots, \hat{\varphi}^{i}(t'_{i})\}$ Compute predictor output: $\hat{y}^{i}_{t'_{x}} = f_{\theta}(\mathbf{x}^{i}_{M_{f_{1}}})$. Compute policy network output: $\hat{q}^{i}_{t'_{x}} = q_{\pi}(\mathbf{x}^{i}_{M_{f_{1}}}, \mathbf{x}^{i}_{t_{x}}, \mathbf{x}^{i}_{t_{x}})$. Update losses: $\mathcal{L}_{f} \leftarrow -\frac{1}{N_{b}} \sum_{i=1}^{N_{b}} \log(\hat{y}^{i}_{t'_{x},y_{i}}) + \mathcal{L}_{f}$. Determine the true label for the q_{π} network (denote this true label as $y^{i}_{q_{x}}$). The set is the index of the feature, which is not acquired yet and having the highest SHAP view ong the features that are not acquired $\mathcal{L}_{x} \leftarrow -\frac{1}{N_{b}} \sum_{k=0}^{N_{b}} \log(\hat{p}^{i}_{k}) + \mathcal{L}_{x}$
Algori Requir 1: for 2: 3: 4: 5: 6: 7: 8: 9: 10: 11: 12: 13: 14:	thm 2 Pseudocode for second stage training of q_{π} and f_{θ} re: Training set $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^N$, batch size N_b , context length ℓ , learning rate γ , f_{θ} an im the first stage \mathbf{x} each epoch do for 1 to $[N/N_b]$ do Sample minibatch $\{(\mathbf{x}^i, y^i, \varphi^i)\}_{i=1}^{N_b}$ (if random augmentation is applied, φ calculated for each iteration after sampling) Generate $\hat{\varphi}^i$ for each i Sample random integer t_i for each i Initialize $L_a = 0$ and $\mathcal{L}_i = 0$ for $t_x = 0$ to $\ell - 1$ do Define temporary parameter t'_a for each i , $t'_i = t_i + t_x$. Generate masked input $\mathbf{x}^i_{M_{t_i}}, \hat{M}_{t'_t} = \{\hat{\varphi}^i(1), \dots, \hat{\varphi}^i(t'_a)\}$ Compute predictor output: $\hat{y}^i_{t_i} = f_{\theta}(\mathbf{x}^i_{M_{t_{t_i}}}, \mathbf{a}^i_{t_i, t_i}, \mathbf{r}^i_{t_i, t'_i})$. Update losses: $\mathcal{L}_f \leftarrow -\frac{1}{N_b} \sum_{i=1}^{N_b} \log(\hat{y}^i_{t'_i, y_i}) + \mathcal{L}_f$. Determine the true label for the q_{π} network (denote this true label as $y^i_{q_{t_i}}$). The set is the index of the feature, which is not acquired yet and having the highest SHAP view $\mathcal{L}_a \leftarrow -\frac{1}{N_b} \sum_{i=1}^{N_b} \log(\hat{q}^i_{t'_i, y_{t_i}}) + \mathcal{L}_g$.