Firewall Routing: Blocking Leads to Better Hybrid Inference for LLMs

Anonymous ACL submission

Abstract

The rapid advancement of Large Language Models (LLMs) has significantly enhanced performance across various natural language processing (NLP) tasks, yet the high computational costs and latency associated with deploying such models continue to pose critical bottlenecks, limiting their broader applicability. To mitigate these challenges, we propose a dynamic hybrid inference framework, **Firewall Routing**, which efficiently selects between a strong and a weak LLMs based on the complexity of the query. A lightweight routing model is trained to optimize resource allocation by learning from response quality and preventing long-tail queries, which are often unsolvable for both LLMs, from being routed to the stronger model. Moreover, our method incorporates multiple sampling to enhance query evaluation reliability while leveraging **Hard Blocking** and **Soft Blocking** to handle long-tail queries along with refining labels for model selection. Extensive experiments show our method outperforms existing routing strategies by up to 5.29% in APGR, demonstrating state-of-the-art performance across multiple benchmarks.

1 Introduction

In recent years, we have witnessed the rapid advancement of artificial intelligence technologies, particularly the rise of large language models (LLMs) such as ChatGPT, which are reshaping the paradigms of our daily work. These models, often containing billions or even trillions of parameters, generate fluent and contextually appropriate responses, enabling natural interactions without requiring specialized user knowledge (OpenAI et al., 2024; Touvron et al., 2023; Grattafiori et al., 2024). However, such remarkable capabilities come at a significant cost: deploying LLMs demands expensive infrastructure, such as multi-GPU systems with high memory capacity, or incurs higher per-token charges in cloud-based LLM services for more capable models (Yu et al., 2022). Moreover, larger models often introduce higher latency, making them less suitable for real-time or resource-constrained applications. Striking a balance among strong model performance, high efficiency, and economical costs remains an "impossible triangle," yet it is precisely this challenge that drives ongoing research efforts in the field.

Making the "impossible triangle" possible requires a paradigm shift in how we allocate computational resources for language model inference. Extensive experiments have demonstrated that not all tasks require the full power of the largest models (Grattafiori et al., 2024). Simpler queries can often be handled effectively by smaller, lower-cost models without compromising quality, whereas more complex queries leverage the advanced capabilities of larger models. This principle forms the foundation of **Hybrid Inference**.

Given the promising potential, **Hybrid Inference** has garnered significant attention from both academia and industry. Existing strategies can be broadly categorized into two main types: **Cascade** methods (Chen et al., 2023; Gupta et al., 2024; Ramírez et al., 2024), and **Route** methods (Shnitzer et al., 2023; Šakota et al., 2024; Lu et al., 2023; Ong et al., 2024; Ding et al., 2024).

Cascade methods first process all queries using a weaker model. If the weaker model's confidence in its response is low, typically determined through an internal evaluation mechanism, the query is escalated to a stronger model for reprocessing. Although this approach is conceptually straightforward, it has several inherent limitations. On the one hand, evaluating response quality before completion in generative tasks is inherently difficult, leading to unreliable decision-making(Gupta et al., 2024). On the other hand, evaluating response quality after completion brings greatly increased latency. These factors make **Cascade** methods less

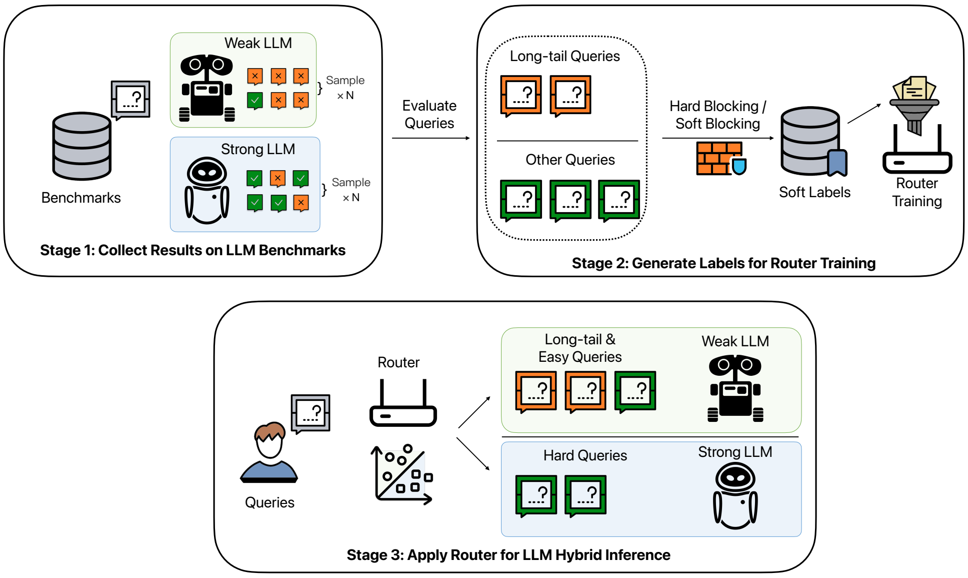

Figure 1: **Firewall Routing** framework for dual-model hybrid inference, comprising a strong model, a weak model, and a router model to balance performance and cost for LLM inference. By blocking unsolvable long-tail queries from being routed to the strong model, the framework achieves state-of-the-art performance.

efficient in real-world applications.

Motivated by these considerations, we focus on **Route** methods, which leverage a lightweight router model to dynamically allocate queries to the most appropriate LLM under a given configuration. However, existing **Route** methods predominantly rely on collected preference data, which are often limited by strict domain-specific constraints (Shnitzer et al., 2023; Šakota et al., 2024; Lu et al., 2023), or heavily depend on model-generated scores (Ong et al., 2024; Ding et al., 2024). Moreover, these methods often depend on preference data or artificially generated labels based on model scoring. In the context of dual-model hybrid inference, where the strong model generally outperforms the weak model, they fail to address long-tail queries that challenge both models, highlighting opportunities for further optimization.

To address these challenges, we propose **Firewall Routing**, a dual-model hybrid inference system that builds on reliable benchmark results and manages to block long-tail queries, enhancing both performance and efficiency.

Specifically, we propose a novel paradigm for training the router model. Unlike existing methods, our approach utilizes multiple sampling during benchmark evaluations to obtain more accurate estimations of the capabilities of both the strong and weak models. These estimations are then used to construct soft labels for router training. Through mathematical derivations, this paradigm highlights the generality of soft label training in the domain of router optimization and demonstrates that the hard label approach is a specific instance of this broader framework.

To further address the challenge of long-tail queries, we propose two novel approaches—**Hard Blocking** and **Soft Blocking**—designed to effectively manage unsolvable cases. **Hard Blocking** utilizes statistical information to identify unsolvable long-tail queries and assigns them the label "route to the weak model," minimizing unnecessary computational overhead. In contrast, **Soft Blocking** leverages the **Pass Rate** (pass@1) to generate refined soft labels with more precise routing conditions, further reducing computational inefficiencies.

To summarize, we make the following contributions:

1. We propose a novel router training paradigm leveraging multiple sampling to generate soft labels, which generalizes router optimization and demonstrates hard label training as a specific case within this framework.

2. We propose **Hard Blocking** and **Soft Blocking** as automated mechanisms to enable our approach to overcome the challenges associated with long-tail queries.

3. We validate our approach through extensive experiments across diverse configurations.

2 Related Works

Hybrid Inference balances response quality and inference cost by dynamically selecting models based on task complexity. For image classification, Kag et al. (2023) explored joint training of a small model, a large model, and a router, while in NLP tasks, the Tryage architecture (Hari and Thomson, 2023) employed a joint-trained router to optimize performance across domains. However, for LLMs, joint training is computationally expensive and deviates from the pre-training paradigm, leading to two main approaches: **Cascade Methods** and **Route Methods**.

Cascade Methods first query a weaker model and escalate the request to a stronger model only when necessary. FrugalGPT (Chen et al., 2023) estimates response confidence using an LLM-based heuristic to decide whether a query should be forwarded to a larger model. Similarly, Gupta et al. (2024) proposed a confidence estimation method based on the conditional probability of the generated response, serving as a reliability metric. By assessing the correctness of the weaker model's responses, these methods effectively reduce the number of strong model invocations while maintaining high response quality. However, this approach introduces significant response time overhead, as the weaker model must first generate an output before determining whether escalation is required.

Margin Sampling (Ramírez et al., 2024) is a different cascade approach without introducing extra response time. Only when the probability difference between the top two predicted tokens is small at the beginning of generation, indicating uncertainty, is the query escalated to the strong model.

While both cascade hybrid inference and speculative decoding involve a weak and strong model processing the same query, their goals differ. Speculative decoding (Kim et al., 2024; Leviathan et al., 2023) speeds up text generation by having a smaller model propose tokens, which are then verified by a larger model, but this frequent validation incurs high computational costs. In contrast, cascade methods prioritize reducing reliance on the strong model, balancing performance and efficiency by minimizing its usage.

Route Methods introduce a router model to determine which model should handle a given query. For example, Shnitzer et al. (2023) frame this as an out-of-distribution (OOD) detection problem, where they predict a model's response correctness and confidence using k-nearest embedded queries. Similarly, Šakota et al. (2024) train a model to predict whether a query can be correctly answered, incorporating a special token to indicate which LLM should be used. Lu et al. (2023) distilled a reward model to predict which LLM serves as the optimal expert for a given query.

Many recent works focus on dual-model hybrid inference systems. For instance, RouteLLM (Ong et al., 2024) uses preference pairs from multiple LLMs in Chatbot Arena to train a Bradley-Terry model (Bradley and Terry, 1952) as the router. Hybrid LLM (Ding et al., 2024) derives Win Rates for queries through a biased comparison of response BARTScores, creating a desired label distribution to train the router. These approaches highlight the potential for training routers with more reliable evidence, such as pass@k (Chen et al., 2021), to improve model selection.

3 Method

3.1 Router Training Criterions

3.1.1 Train with Hard Label

Early works on building up hybrid inference systems usually train a system with the router model as a whole, where the router model learns how to route under a fixed configuration(Kag et al., 2023). Due to the high training costs associated with large-scale models, most works in LLM hybrid inference only train the router model.

In existing evaluation frameworks for large language models, generative tasks typically follow a greedy decoding paradigm, where the model outputs the token with the highest probability while disregarding alternative token possibilities. Based on this setting, existing methods (Ding et al., 2024) adopt a "Hard Label" approach for router training.

Specifically, for a single query $x_i \in Q$, let $S(x_i)$ and $W(x_i)$ represent the responses generated by the strong model S and the weak model W, respectively, using greedy decoding. The correctness of these responses is denoted as $\delta(S(x_i))$ and $\delta(W(x_i))$, where $\delta(\cdot) \in \{0, 1\}$, with 1 indicating a correct response and 0 indicating an incorrect one. The decision on *whether to route the query to the weak model* is determined by the label y_i, defined as $y_i := \mathbb{I}[\delta(S(x_i)) \leq \delta(W(x_i))]$. Here, $y_i = 1$ implies the weak model is capable of performing at least as well as the strong model for query x_i, and thus the query should be routed to the weak model.

The hard-label router is trained by minimizing the binary cross-entropy loss:

$$\mathcal{L}(\theta) = -\frac{1}{|Q|}\sum_{i=1}^{|Q|}((1-y_i)log(1-p_\theta(x_i)) + y_i log(p_\theta(x_i))), \quad (1)$$

where $p_\theta(x)$ is output of router θ toward query x, where larger $p_\theta(x)$ indicates that the queries should more likely to be routed to the weak model.

The hard label approach is limited by its inability to account for the inherent variability in the responses of large models, thereby restricting the router's ability to make fine-grained decisions. This limitation becomes particularly apparent in scenarios where the smaller model's performance is often comparable to that of the larger model. An ideal hybrid inference system should incorporate the inherent variability of model responses into the training labels for the router, enabling it to make more reliable and cost-effective routing decisions, thereby enhancing overall system efficiency.

3.1.2 Train with Soft Label

To more objectively reflect the performance of large models, existing evaluations often involve multiple sampling of model outputs. Inspired by this approach, we extend our approach by incorporating multiple sampling, which allows us to evaluate the models more thoroughly and account for response variability. This enhancement aims to improve the robustness and efficiency of the routing decisions in our hybrid inference framework.

Specifically, for a single query x_i , let $S^1(x_i), \ldots, S^n(x_i)$ and $W^1(x_i), \ldots, W^n(x_i)$ denote the responses generated by the strong model S and the weak model W over n sampling iterations. The correctness of these responses is represented by $\delta(S^j(x_i))$ and $\delta(W^j(x_i))$, where $\delta(\cdot) \in \{0, 1\}$, with 1 indicating a correct response and 0 indicating an incorrect one. Each sampling iteration produces a noisy observation of y_i, denoted as $y_i^j = \mathbb{I}[\delta(S^j(x_i)) \le \delta(W^j(x_i))]$. In this setting, x_i is associated with n data pairs in the training set, denoted as $(x_i, y_i^1), (x_i, y_i^2), \ldots, (x_i, y_i^n)$.

Using this data, the router can still be trained with a hard label-based objective. However, this approach presents two significant challenges: first, the training cost scales proportionally with the number of sampling attempts n; second, a single input can correspond to varying labels, potentially misleading the router's behavior.

Thus, we introduce the concept of the weak-to-strong **Win Rate**, defined as $r_i := \frac{1}{n}\sum_{j=1}^{n} y_i^j$, which represents the probability that the weak model matches or exceeds the performance of the strong model. Furthermore, we demonstrate that optimization objectives based on **Win Rate** exhibit greater generality for router training. Notably, hard label training inherently captures the concept of **Win Rate**, which can be expressed in the following form:

$$\begin{aligned}\mathcal{L}(\theta) &= -\frac{1}{n|Q|}\sum_{i=1}^{|Q|}\sum_{j=1}^{n}((1-y_i^j)log(1-p_\theta(x_i)) + y_i^j log(p_\theta(x_i))) \\ &= -\frac{1}{n|Q|}\sum_{i=1}^{|Q|}((n-\sum_{j=1}^{n} y_i^j)log(1-p_\theta(x_i)) + (\sum_{j=1}^{n} y_i^j)log(p_\theta(x_i))) \\ &= -\frac{1}{|Q|}\sum_{i=1}^{|Q|}((1-r_i)log(1-p_\theta(x_i)) + r_i log(p_\theta(x_i))). \quad (2)\end{aligned}$$

Here, $p_\theta(x)$ represents the output of the router θ for the query x, where a larger $p_\theta(x)$ indicates a higher likelihood that the query should be routed to the weak model. This formula motivates us to explore more refined soft labels that better characterize the behavior of large models through their win rates.

3.2 Blocking Long-tail Queries

Even for large models, there are instances where, despite multiple sampling attempts n, the model is still unable to resolve certain long-tail queries. This limitation arises from the inherent complexity and ambiguity in some queries, which even powerful models may struggle to address consistently, regardless of the number of samples taken. Consequently, such cases highlight the need for more sophisticated handling of unsolvable long-tail queries in hybrid inference systems.

3.2.1 Hard Blocking

To automatically identify long-tail queries, we introduce multiple sample **Pass Rate** (pass@k when k=1) from Chen et al. (2021)'s work to substitute single sample correctness. For a single query $x_i \in Q$ with n sampled responses $R^1(x_i), ..., R^n(x_i)$

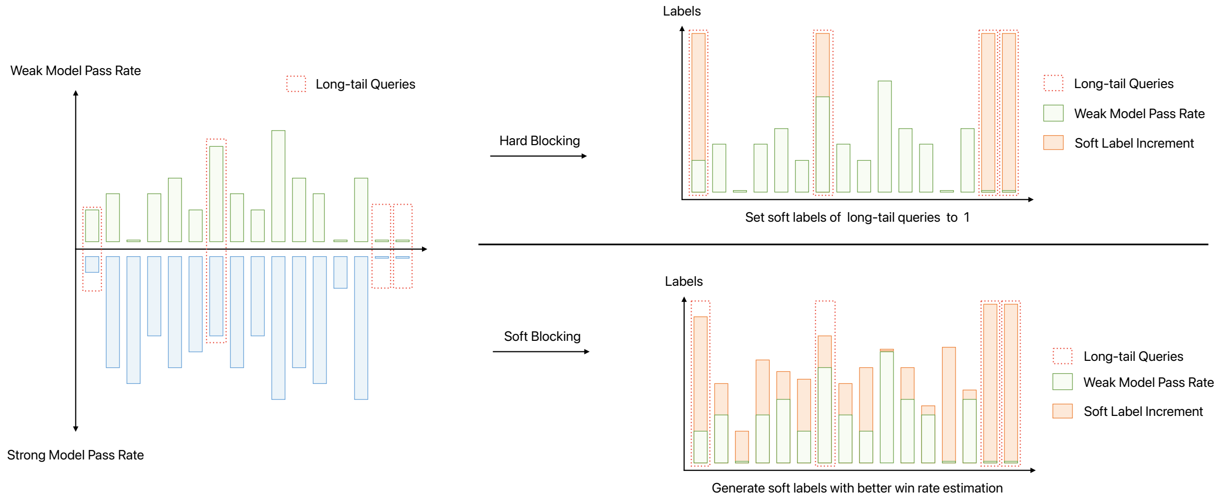

Figure 2: Hard Blocking and Soft Blocking facilitate the automatic handling of unsolvable long-tail queries by generating reliable soft labels for router training. Queries assigned larger soft label values are more likely to be routed to the weak model.

from model R, **Pass Rate** is defined as the average correctness of these responses:

$$pr(x_i) := \frac{1}{n}\sum_{j=1}^{n}\delta(R^j(x_i)). \tag{3}$$

We are able to split queries into two sets, Q_u and $Q_s = Q - Q_u$, representing unsolvable and solvable queries, satisfying:

$$\begin{aligned} &\forall x^u \in Q_u, pr_s(x^u) \le pr_w(x^u), \\ &\forall x^s \in Q_s, pr_s(x^s) > pr_w(x^s). \end{aligned} \tag{4}$$

By addressing unsolvable long-tail queries through routing them to the weak model, the decision to route other queries similarly hinges entirely on the weak model's capability to handle these queries effectively:

$$label_i = \begin{cases} pr_w(x_i), x_i \in Q^s, \\ 1, x_i \in Q^u, \end{cases} \tag{5}$$

where $label_i$ is the soft label used in router training to substitute r_i in Eq.2.

To further reduce the cost associated with label collection in this method, it is also possible to split Q_u and Q_s using only the strong model's greedy-decoding responses, subject to the following restrictions:

$$\begin{aligned} &\forall x^u \in Q_u, \delta(S(x^u)) = 0, \\ &\forall x^s \in Q_s, \delta(S(x^s)) = 1. \end{aligned} \tag{6}$$

3.2.2 Soft Blocking

A closer examination of Eq.2 and the concept of the Pass Rate reveals that r_i functions as a noisy indicator, capturing the behaviors of the two models when processing the same query. A key insight is that the performance of the strong model is independent of whether the weak model answers correctly. Instead of treating the two models' performances as a joint distribution, we can more effectively leverage the distributional information obtained from multiple samplings. By treating the two independent events separately, we can more accurately estimate r_i through **Pass Rate**. To maximize the use of this information, we define the joint event for routing the query to the weak model by combining two conditions: *the weak model is correct* and *even if the weak model is incorrect, the strong model also fails*. This method allows us to offer a more refined and informative estimate of overall performance:

$$\begin{aligned} label_i &= pr_w(x_i) + (1 - pr_w(x_i))(1 - pr_s(x_i)) \\ &= 1 - (1 - pr_w(x_i))pr_s(x_i), \end{aligned} \tag{7}$$

where $label_i$ is the soft label used in router training to substitute r_i in Eq.2, and $label_i$ is the observed frequency that the strong model fail to overperform the weak model.

4 Experiments

4.1 Settings

Datasets In this study, we conduct experiments on generative tasks commonly used to assess the capabilities of large language models (LLMs). These

Datasets	TriviaQA				GSM8K				HumanEval			
Metrics	APGR↑	Pass Rate↑			APGR↑	Pass Rate↑			APGR↑	Pass Rate↑		
		20%	50%	80%		20%	50%	80%		20%	50%	80%
Linear Interpolation	50.00	19.95	34.97	49.99	50.00	11.69	17.16	22.62	50.00	8.12	10.10	12.08
Hybrid LLM	49.17	18.98	34.38	49.99	62.08	14.38	20.75	24.79	51.94	8.10	10.50	**12.35**
RouteLLM (MF)	51.58	20.69	36.27	51.09	49.39	11.37	17.13	22.27	47.08	7.81	9.95	12.23
Margin Sampling	50.02	19.78	35.01	50.15	46.01	10.85	16.02	21.70	44.88	7.74	9.81	11.53
Ours (Hard Block)	53.16	22.09	37.85	50.96	**67.37**	**16.34**	**22.46**	24.67	**54.36**	8.17	**10.77**	12.27
Ours (Soft Block)	**55.00**	**22.48**	**38.99**	**52.88**	66.65	15.53	22.15	**25.32**	53.13	**8.23**	10.69	12.27

Table 1: Zero-shot performance of different methods across selected datasets. The weak model is Llama3.2-1B, and the strong model is Llama3.1-70B. Linear Interpolation represents the combined performance of the two LLMs to simulate random routing. **Bolded values** indicate the best-evaluated results.

tasks include TriviaQA (Joshi et al., 2017) for common sense question answering (QA), GSM8K (Cobbe et al., 2021) for mathematical reasoning, and HumanEval (Chen et al., 2021) for code synthesis. Our training set is derived from the TriviaQA and GSM8K training datasets, while HumanEval is utilized exclusively for evaluation, serving as an Out-Of-Domain task benchmark.

Prompts For all datasets, we employ a straightforward zero-shot prompt format without using any system prompts. Specifically, each query is formatted as: "Question: {question}\nAnswer: ".

Models In this study, we utilize two large language models (LLMs) from the Llama family (Grattafiori et al., 2024) for our experiments: Llama3.2-1B serves as the weak model, while Llama3.1-70B is employed as the strong model for training the router. Furthermore, to assess the generalizability of the trained router, we test it on an alternative model pair, substituting Llama3.2-3B as the weak model, to evaluate its adaptability to varying model configurations.

Routers Aligned with prior studies, we adopt DeBERTa-v3-large (He et al., 2023) as the router model, enhanced with an additional linear layer to predict the probability of routing queries to either the weak or strong model. The router is trained for 10 epochs using the specified loss function, with the best-performing checkpoints selected based on validation set performance for the final evaluation. All experiments are conducted on 8 NVIDIA A100 GPUs, each with 80GB of memory, to facilitate data parallelization; however, the same experiments can be reproduced on a single NVIDIA A100 GPU. The training code will be made publicly available.

Baselines We compare our approach with several state-of-the-art methods, including Hybrid LLM (Ding et al., 2024), RouteLLM (Ong et al., 2024), and Margin Sampling (Ramírez et al., 2024). For Hybrid LLM, we reproduce the methodology and hyperparameter selection as outlined in the original paper. For RouteLLM, we employ the best practices and downloadable pre-trained weights, utilizing Matrix Factorization (MF) with OpenAI's text-embedding-3-small to embed the queries. For Margin Sampling, we treat it as a train-free baseline, where the routing decision is based on detecting the probability difference between the first and second most likely tokens.

Metrics We evaluate the performance of the hybrid inference system using the **Pass Rate**, defined as pass@1 (Chen et al., 2021), based on $n = 32$ sampling iterations. The system's performance is assessed at different proportions (20%, 50%, 80%) of queries routed to the strong model. Furthermore, we incorporate the Average Performance Gap Recovered (APGR) metric from RouteLLM (Ong et al., 2024), which measures the system's ability to recover performance gaps between two LLMs. The APGR is computed across various proportions (0%, 10%, ..., 100%) of queries routed to the strong model, with values ranging from 0% to 100%, reflecting the extent to which performance discrepancies are mitigated through dynamic routing.

4.2 Main Results

4.2.1 Overall Performance

Table 1 summarizes the overall performance of various routing methods within a hybrid inference system utilizing Llama3.2-1B and Llama3.1-70B. Methods achieving higher APGR also exhibit improved performance across different proportions of queries routed to the strong model. Our proposed methods outperform existing approaches, with a notable improvement of **3.72% on TriviaQA, 5.29%**

Datasets	TriviaQA				GSM8K				HumanEval			
Metrics	APGR↑	Pass Rate↑			APGR↑	Pass Rate↑			APGR↑	Pass Rate↑		
		20%	50%	80%		20%	50%	80%		20%	50%	80%
Linear Interpolation	50.00	25.75	38.59	51.44	50.00	12.60	17.72	22.85	50.00	10.27	11.44	12.61
Hybrid LLM	49.15	24.86	38.04	51.50	61.09	14.38	20.75	24.79	51.94	**10.42**	11.47	12.63
RouteLLM (MF)	51.22	26.14	39.45	52.28	49.11	15.11	20.72	24.66	50.33	10.10	11.26	12.65
Margin Sampling	51.21	26.07	39.15	52.49	43.82	11.71	16.11	21.42	44.88	9.97	11.41	12.42
Ours (Hard Block)	53.29	27.61	41.15	52.28	**65.97**	**16.68**	**22.30**	24.54	50.86	10.04	11.62	12.63
Ours (Soft Block)	**55.38**	**27.91**	**42.28**	**54.28**	65.48	16.01	22.04	**25.24**	**52.37**	10.33	**11.72**	**12.80**

Table 2: Zero-shot performance of various methods across selected datasets, generalizing to different model pairs. Trained on the hybrid inference system of Llama3.2-1B and Llama3.1-70B, and evaluated on the hybrid inference system of Llama3.2-3B and Llama3.1-70B. Linear Interpolation simulates random routing by combining the performance of the two LLMs. **Bolded values** indicate the best-evaluated results.

on GSM8K, and 2.42% on HumanEval, demonstrating robustness across diverse query scenarios. Additional visualizations of these results are provided in Appendix B.

On TriviaQA, the Soft Blocking method delivers the best performance, while the Hard Blocking method also outperforms all existing approaches. The poor performance of Hybrid LLM in this context is not surprising, as it relies on win-rate based on BartScore, which has proven unreliable across datasets in Appendix A. Specifically, responses with higher BartScore do not consistently outperform those with lower scores. In contrast, the other methods exceed random routing, demonstrating their effectiveness.

Across both GSM8K and HumanEval, routing methods exhibit consistent patterns, typically either performing well on both datasets or underperforming on both. In contrast, our methods achieve state-of-the-art performance. Although based on the same training data, the difference in training objectives sets our approach apart from Hybrid LLM, underscoring the effectiveness of our method. On the other hand, RouteLLM and Margin Sampling show weaker performance on these datasets, indicating potential generalization challenges. For RouteLLM, these limitations may stem from domain shifts in evaluation tasks and Out-of-Domain challenges associated with LLM selection. As for Margin Sampling, the results indicate that reasoning tasks—such as math, where multiple valid solutions exist—pose difficulties, as they conflict with the fundamental assumption of Margin Sampling, particularly when smaller LLMs are used. Besides, uncertainty in the responses is not the only factor that needs to be considered when deciding whether to cascade to the strong model. This oversight contributes to the failure of Margin Sampling on HumanEval, where multiple valid paths may lead to the correct answer, making the assumption underlying Margin Sampling less reliable in this context.

4.2.2 Generalizing to Different Model Pairs

In Table 2, we evaluate the performance of the hybrid inference system configured with Llama3.2-3B and Llama3.1-70B, utilizing routers trained in prior experiments without any additional retraining. Our methods, particularly Soft Blocking, consistently demonstrate superior performance in this configuration, achieving an APGR improvement of **4.16% on TriviaQA, 4.88% on GSM8K, and 0.43% on HumanEval**, which highlights the generalization capability of our method, where routers trained on one model pair exhibit consistent performance when applied to another, confirming its adaptability. Additional visualizations of these results are provided in Appendix B.

On TriviaQA, both of our methods continue to outperform the other approaches, with Hybrid LLM performing worse than random routing. RouteLLM and Margin Sampling show improved performance in this setting. For Margin Sampling, as the weak model scales up, the probability difference becomes a more reliable indicator of response uncertainty in common-sense QA tasks.

On GSM8K and HumanEval, most methods maintain their performance as observed in Table 1. For RouteLLM, since the weakest model in its training setting is Llama-13B, replacing the weak model with Llama3.2-3B likely reduces the gap between the training and evaluation conditions.

4.3 Ablation Study

4.3.1 Router Models

An alternative choice for the router model backbone is causal LLMs (Ong et al., 2024). However, we argue that using a router model larger than the weak model incurs unnecessary computa-

Datasets	TriviaQA				GSM8K				HumanEval			
Metrics	APGR↑	Pass Rate↑			APGR↑	Pass Rate↑			APGR↑	Pass Rate↑		
		20%	50%	80%		20%	50%	80%		20%	50%	80%
Weak Model Pass Rate	50.96	20.00	35.88	50.94	51.17	12.18	17.48	22.63	53.42	8.19	10.56	12.31
Strong Model Pass Rate	54.31	21.44	38.53	**53.28**	65.98	15.24	21.82	**25.35**	49.16	7.87	9.95	12.27
Hard Label	52.05	20.80	36.51	51.51	63.26	14.68	21.02	24.91	50.63	8.61	10.12	11.97
Hard Blocking w/o SMS	54.48	22.23	38.62	52.61	63.43	14.95	21.09	25.05	**54.44**	**8.86**	10.48	**12.60**
Hard Block	53.16	22.09	37.85	50.96	**67.37**	**16.34**	**22.46**	24.67	54.36	8.17	**10.77**	12.27
Soft Block	**55.00**	**22.48**	**38.99**	52.88	66.65	15.53	22.15	25.32	53.13	8.23	10.69	12.27

Table 3: Zero-shot performance of various label designs across selected datasets. The models were trained and evaluated with the weak model being Llama3.2-1B and the strong model being Llama3.1-70B. **Bolded values** indicate the best-evaluated results.

Datasets	TriviaQA	GSM8K	HumanEval
Metrics		APGR↑	
Hard Blocking (Causal)	51.78	57.16	54.44
Hard Blocking (Deberta)	53.16	**67.37**	54.36
Soft Blocking (Causal)	52.44	58.55	**55.31**
Soft Blocking (Deberta)	**55.00**	66.65	53.13

Table 4: Zero-shot performance of different backbone models (DeBERTa-v3-large, Llama3.2-1B) across selected datasets. Trained and evaluated within the hybrid inference system of Llama3.2-1B and Llama3.1-70B. **Bolded values** indicate the best-evaluated results.

tional costs and impacts response time. As a result, we train the weak model as the router for comparison. As shown in Table 4, DeBERTa-v3-large (with 300M parameters) outperforms Llama3.2-1B, despite its smaller size, demonstrating better performance. Notably, Llama3.2-1B performs better on HumanEval, indicating its superior generalization ability.

4.3.2 Label Designs

We also conduct an ablation study on various label designs, as presented in Table 3. More visualized results can be found in Appendix B.

Pass Rates of the Weak Model Training the router using only the weak model's Pass Rates as a soft label results in performance that is only marginally better than random routing. This outcome suggests that effective routing does not primarily rely on the weak model's capacity to provide correct answers, highlighting the need for additional factors to guide the routing process.

Pass Rates of the Strong Model When trained using only the strong model's Pass Rates, the router achieves solid performance but remains outperformed by the two proposed methods. Interestingly, this labeling strategy demonstrates superior results when only a small fraction of queries are routed to the weak model, as it focuses on the strong model's accuracy and effectively identifies long-tail queries. This suggests that the Pass Rate of the strong model is intrinsically tied to query complexity, as queries that are challenging for the strong model are equally difficult for the weak model. However, in broader scenarios, its performance is surpassed by the more robust Hard Blocking and Soft Blocking techniques.

Hard Label Using hard labels, as defined in Eq 1, derived from greedy sampling leads to improved performance compared to relying solely on the weak model's Pass Rates. This improvement suggests that the router is not simply forwarding unresolved queries from the weak model to the strong model, but instead is capable of learning more sophisticated and nuanced routing strategies.

Hard Blocking without Strong Model Sampling This variant of Hard Blocking, described in Eq 6, is a more economical alternative. By employing greedy decoding on the strong model while performing multiple samplings on the weak model, the router achieves comparable performance. This demonstrates the efficiency of the method, as it reduces computational overhead while maintaining robust routing performance.

5 Conclusions

In this work, we propose **Firewall Routing**, a dual-model hybrid inference framework that leverages multiple sampling and innovative blocking techniques to optimize query routing. Through extensive experiments across various benchmarks, our approach demonstrates state-of-the-art performance, significantly reducing computational costs while maintaining high response quality. These results highlight the effectiveness and robustness of the proposed framework in handling routing tasks.

Limitations

The generalization of the proposed hybrid inference system across different model pairs and datasets remains an area for further exploration. While our approach demonstrates promising results on selected model combinations (e.g., Llama3.2-1B and Llama3.1-70B) and datasets (e.g., TriviaQA, GSM8K, HumanEval), the system's performance may vary when applied to other model pairs or domains. Specifically, the router's ability to generalize across models with varying sizes, architectures, and performance characteristics requires further investigation. Additionally, the limited range of datasets tested raises questions about the system's robustness in more specialized or domain-specific tasks. Future work should include a broader evaluation across diverse models and datasets to assess the scalability and applicability of the proposed approach in real-world, heterogeneous settings.

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A Is BartScore a Reliable Metric of Response?

We calculate the BartScore for the responses of different LLMs on TriviaQA and GSM8K. The responses are sorted based on their BartScore, and the sorted responses are grouped into bins. Average accuracy is then calculated within each bin to assess the performance of the models at different levels of response correctness.

As shown in Figure 3, 4, a correlation between BartScore and accuracy is only observed on TriviaQA with Llama3.1-70B. In other cases, no consistent or discernible pattern is evident.

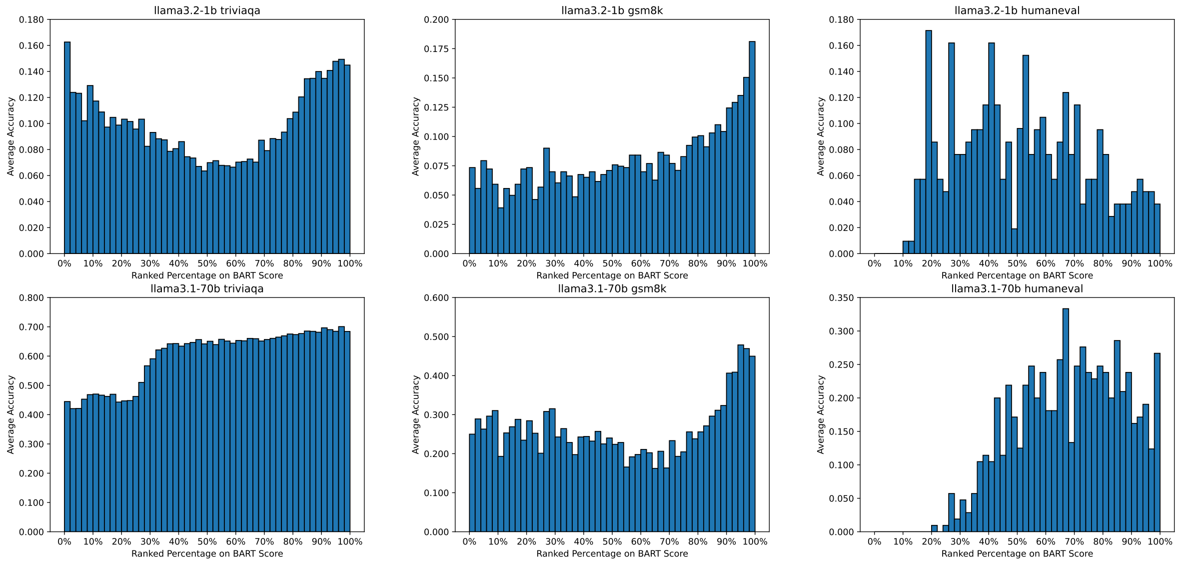

Figure 3: BartScore analysis of LLM responses on TriviaQA, GSM8K, and HumanEval. The responses are sorted by BartScore and grouped into bins, with accuracy calculated within each bin to evaluate performance at varying levels of response quality.

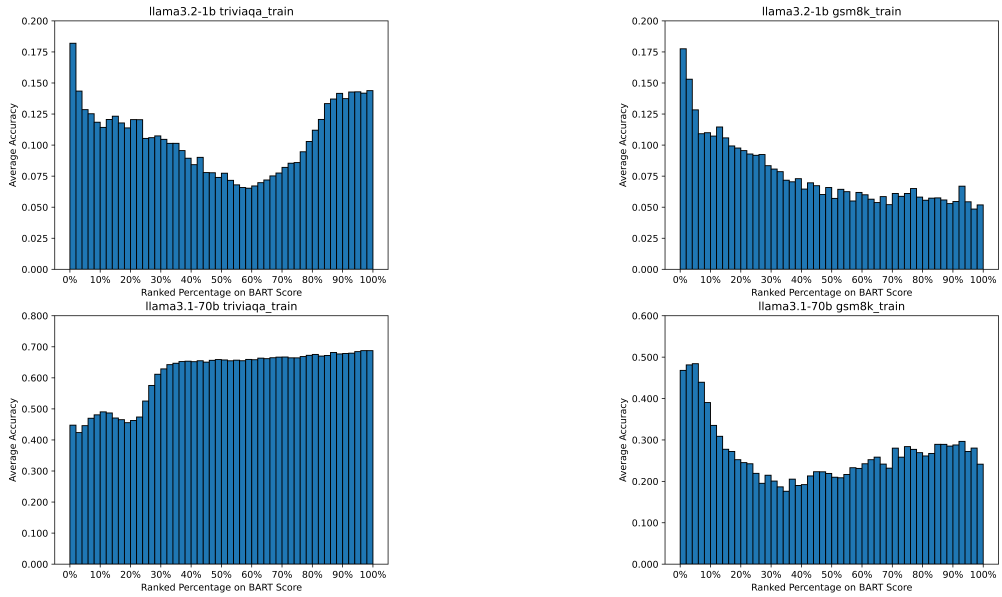

Figure 4: BartScore analysis of LLM responses on training set of TriviaQA and GSM8K. The responses are sorted by BartScore and grouped into bins, with accuracy calculated within each bin to evaluate performance at varying levels of response quality.

B Visualization of Route Method Performance

Similarly, we rank all queries based on the values predicted by the models, and patch them into distinct bins. For each bin, we compute the average pass rate of the strong model and the weak model. Additionally, we evaluate the improvement in pass rate achieved by routing the queries in each bin to the strong model, rather than to the weak model.

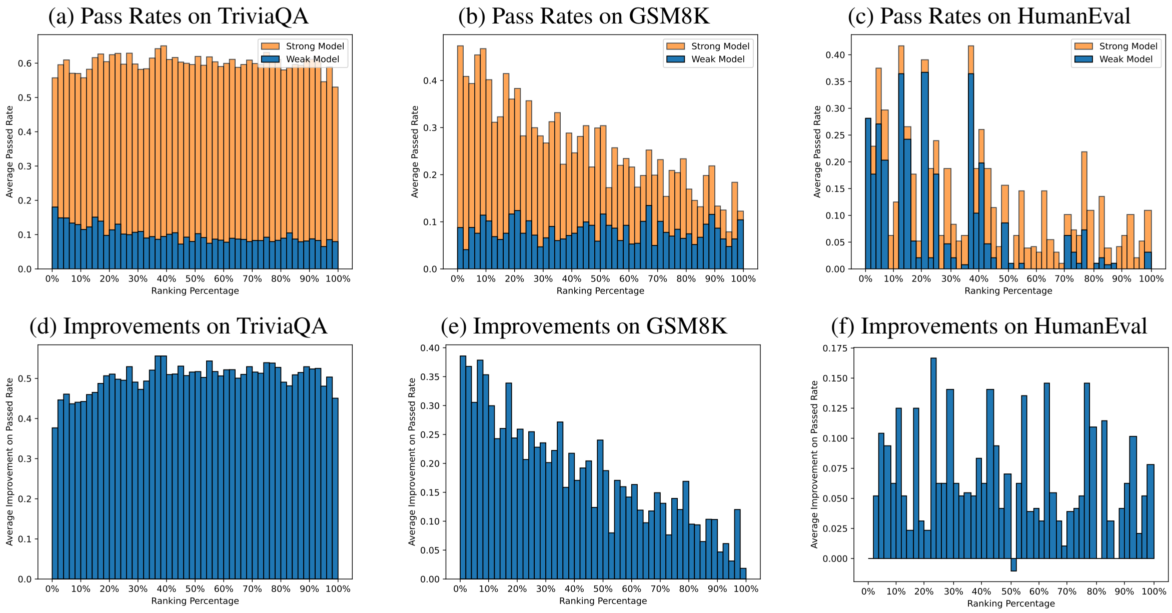

Figure 5: Performance evaluation of reproduced Hybrid LLM on selected datasets. The system utilizes Llama3.2-1B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

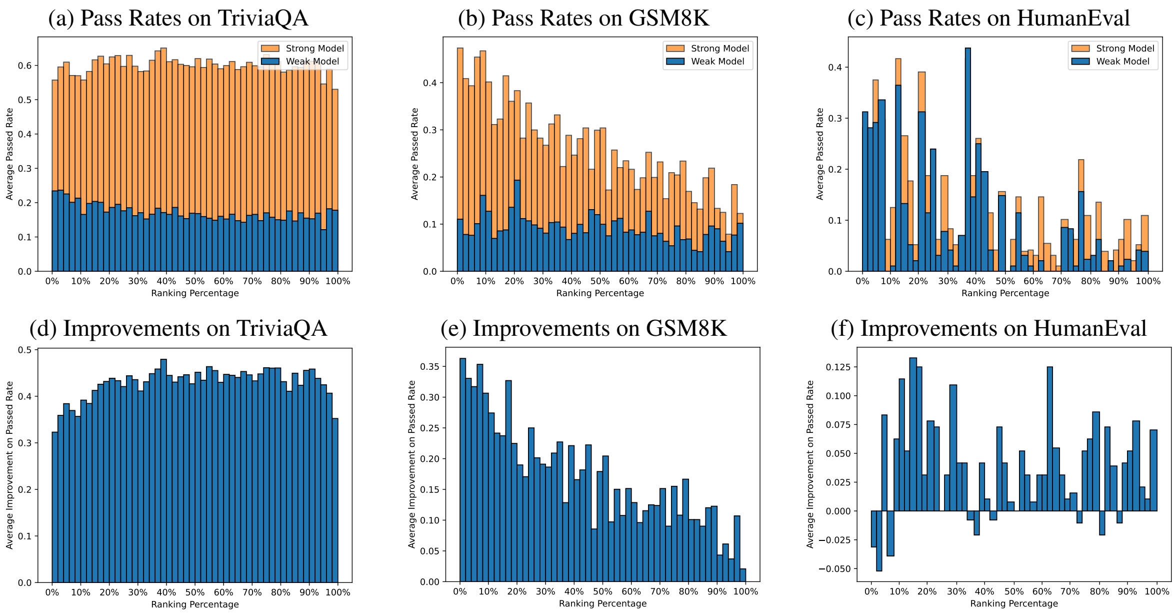

Figure 6: Performance evaluation on generalization of reproduced Hybrid LLM on selected datasets. Evaluated on a system with Llama3.2-3B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

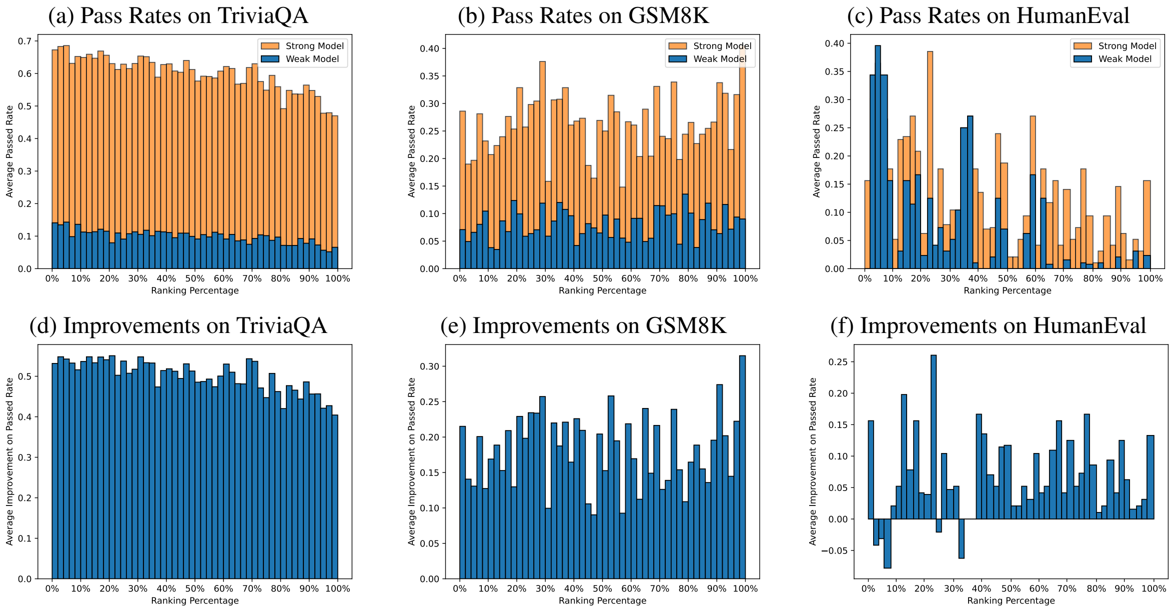

Figure 7: Performance evaluation of Matrix Factorization from RouteLLM on selected datasets. The system utilizes Llama3.2-1B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

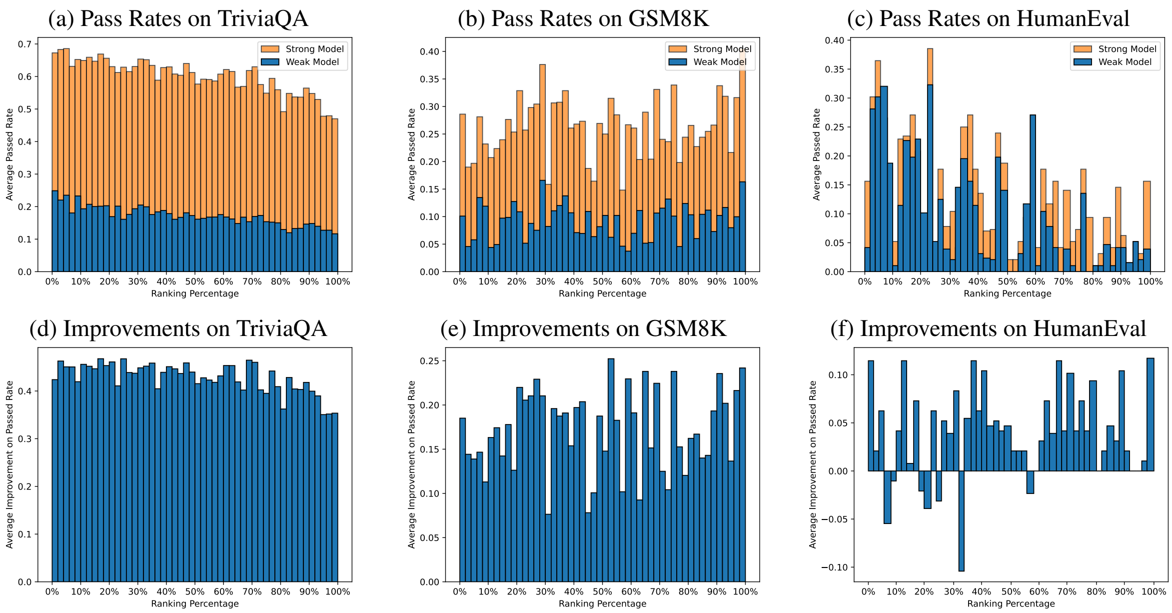

Figure 8: Performance evaluation on generalization of Matrix Factorization from RouteLLM on selected datasets. Evaluated on a system with Llama3.2-3B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

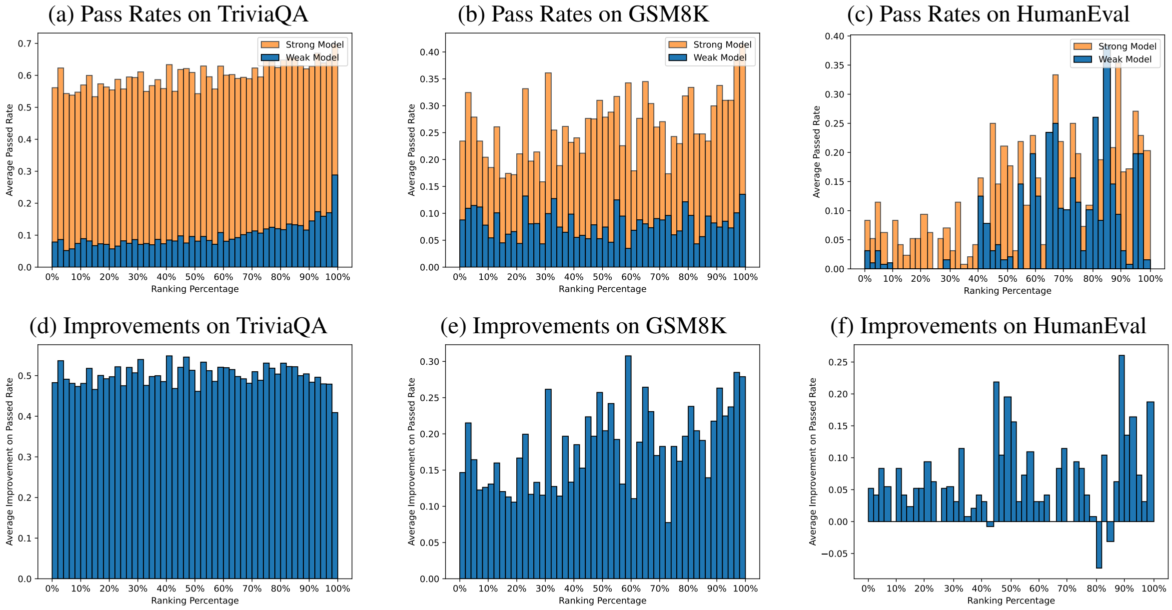

Figure 9: Performance evaluation of Margin Sampling on selected datasets. The system utilizes Llama3.2-1B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

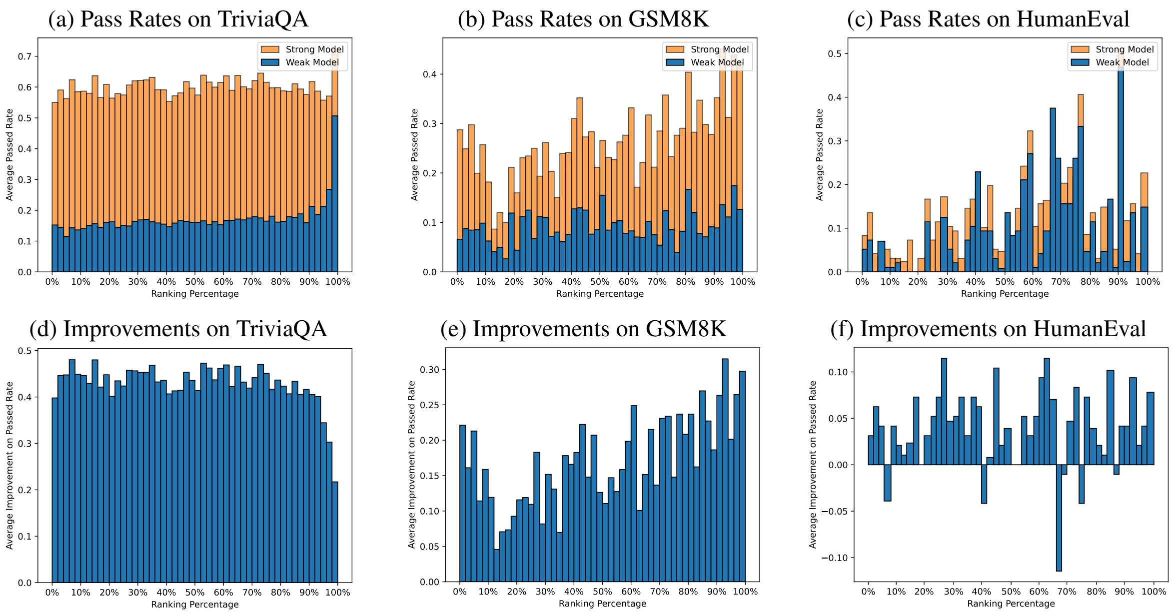

Figure 10: Performance evaluation on generalization of Margin Sampling on selected datasets. Evaluated on a system with Llama3.2-3B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

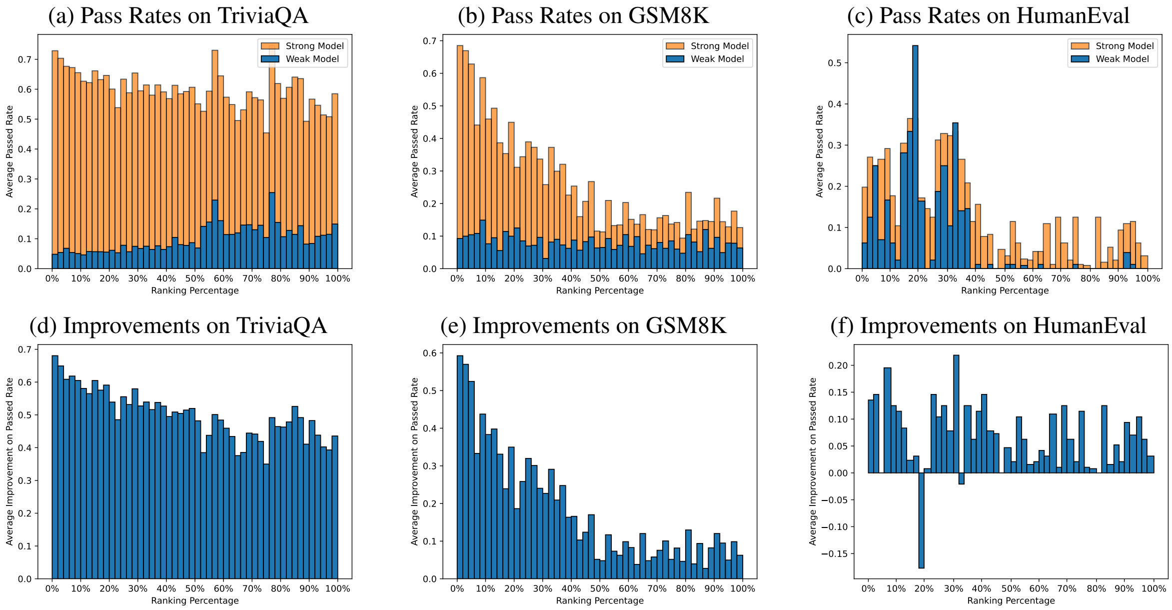

Figure 11: Performance evaluation of Hard Blocking on selected datasets. The system utilizes Llama3.2-1B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

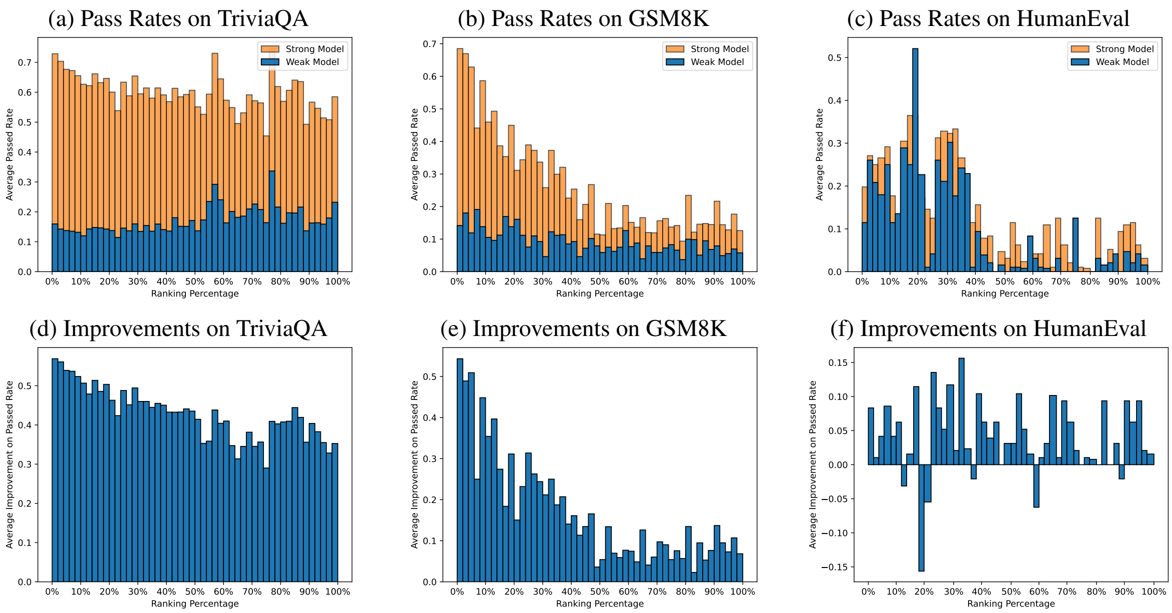

Figure 12: Performance evaluation on generalization of the Hard Blocking on selected datasets. Evaluated on a system with Llama3.2-3B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

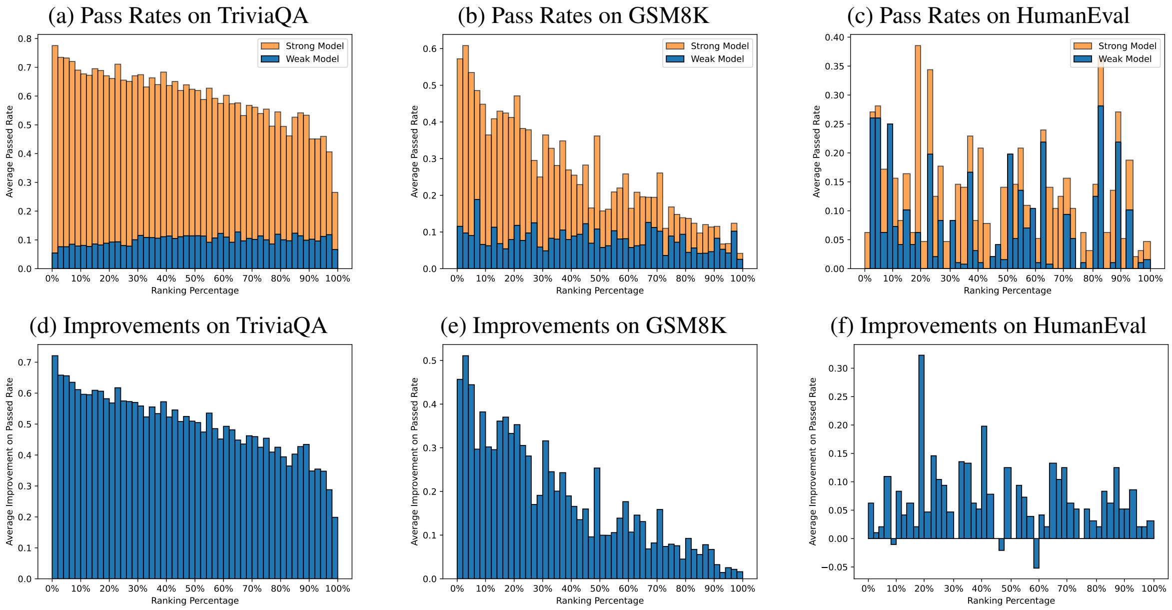

Figure 13: Performance evaluation of Soft Blocking on selected datasets. The system utilizes Llama3.2-1B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

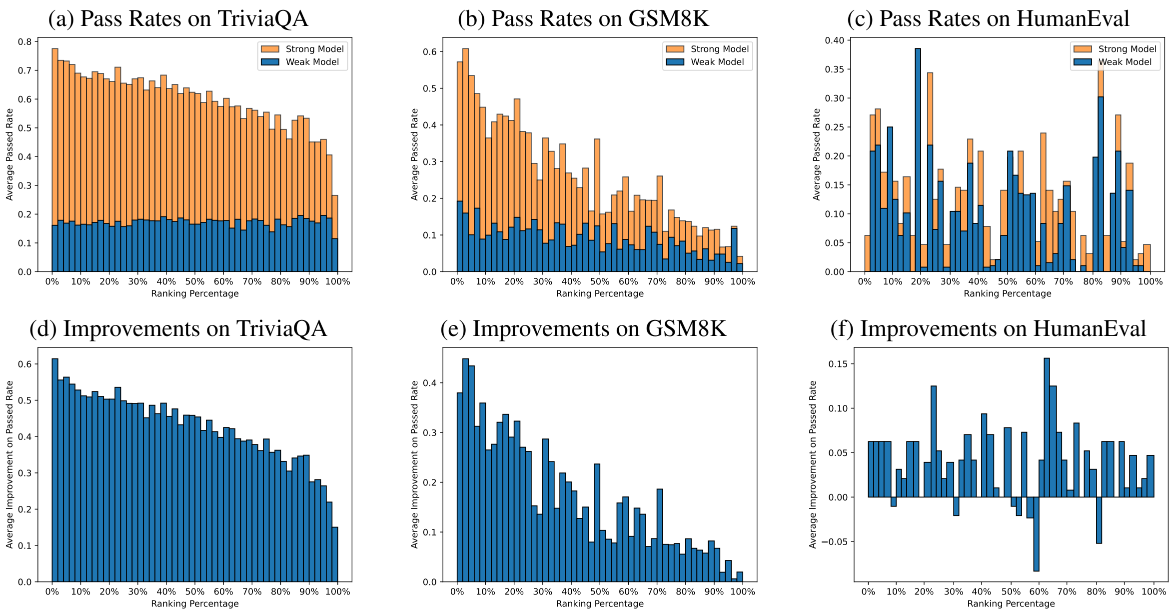

Figure 14: Performance evaluation on generalization of the Soft Blocking on selected datasets. Evaluated on a system with Llama3.2-3B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

Figure 15: Performance evaluation of the router trained on Weak Model's Pass Rates across selected datasets. The system utilizes Llama3.2-1B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

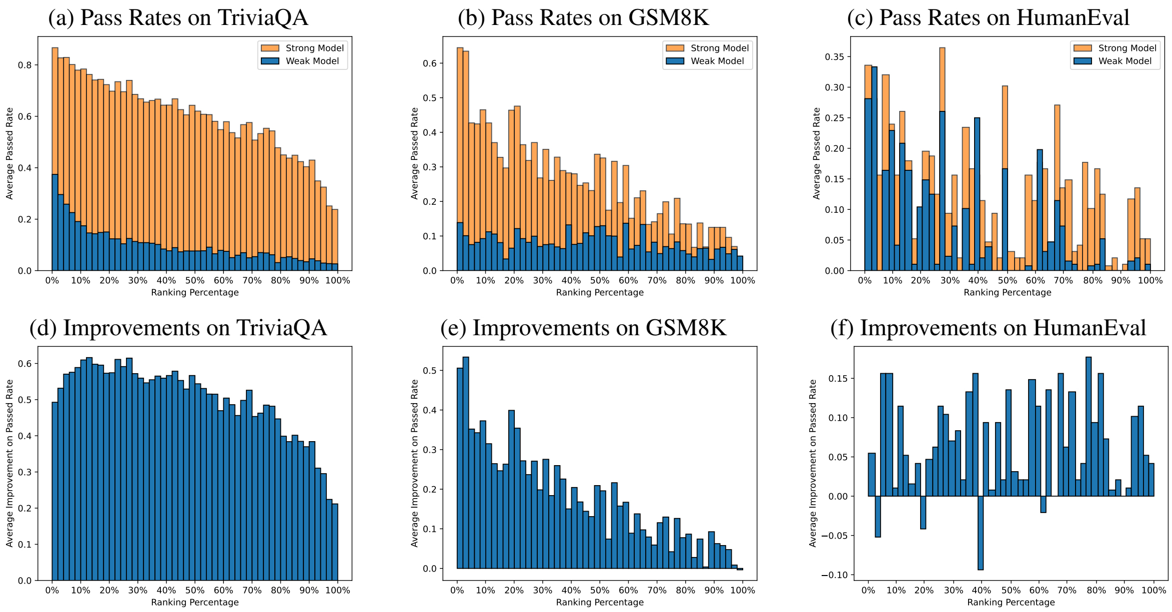

Figure 16: Performance evaluation of the router trained on Strong Model's Pass Rates across selected datasets. The system utilizes Llama3.2-1B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

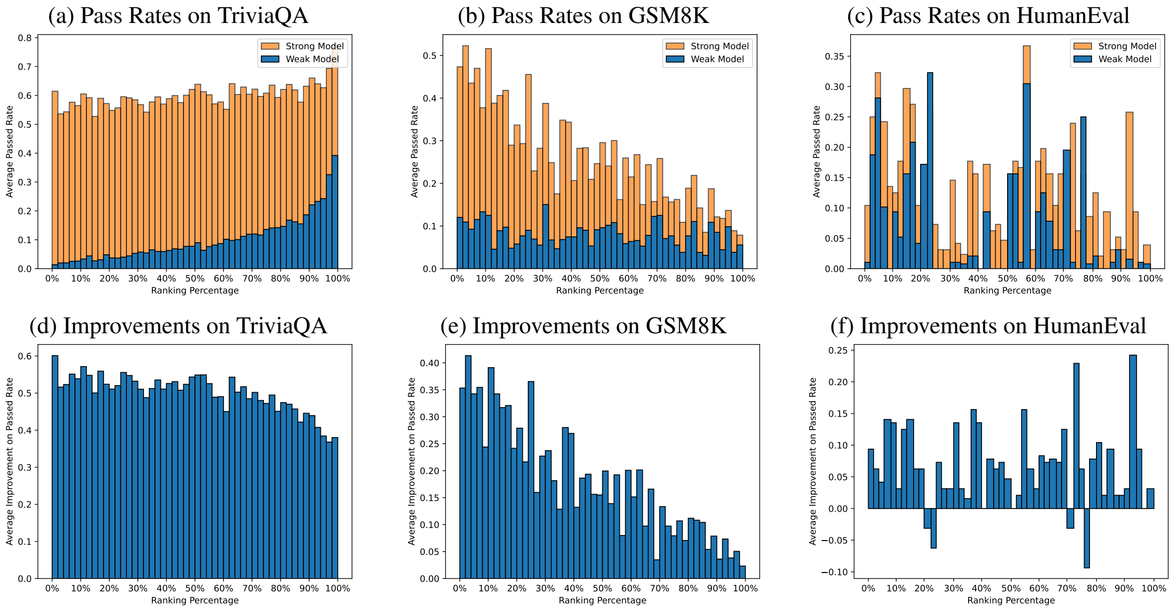

Figure 17: Performance evaluation of the router trained on Hard Labels attained with greedy decoding across selected datasets. The system utilizes Llama3.2-1B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.

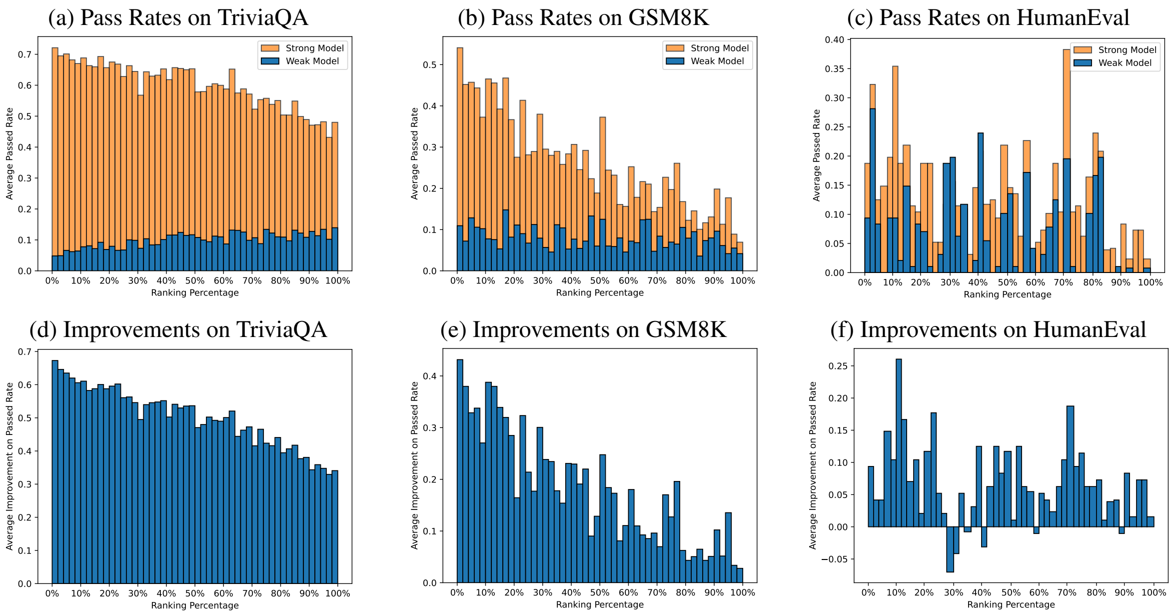

Figure 18: Performance evaluation of the router trained using Hard Blocking without conducting sampling on the strong model across selected datasets The system utilizes Llama3.2-1B as weak model and Llama3.1-70B as strong model. Results are presented in a zero-shot setting.