Extending Earthquake Network Models with a Magnitude-Based Time-Window Approach

Keywords: earthquakes; visibility graph model; time window model; q-exponential; network topology

Extended Abstract

Earthquakes are inherently complex phenomena, characterized by nonlinear dynamics and long-range correlations across space and time. Understanding these dynamics is crucial for advancing seismic risk assessment and the modeling of geophysical processes. In recent years, complex network theory has emerged as a powerful framework to analyze the spatiotemporal structure of seismicity. Two main approaches have been employed to construct such networks: (*i*) networks of events, where nodes represent individual seismic events, and (*ii*) networks of epicenters, where nodes correspond to spatial cells on the Earth's surface that host these events. Links between nodes are typically established using either the Visibility Graph (VG) model [1], which connects events based on geometric visibility rules in the time-magnitude plane, or the Time-Window (TW) model [2], which connects events that occur within a predefined temporal window. While both models have successfully revealed key properties of seismic networks—such as small-world topology and scale-free degree distributions—they each exhibit limitations. Specifically, the VG model is not well-suited for epicenter-based networks due to its reliance on temporal ordering, and the TW model does not incorporate earthquake magnitude, potentially overlooking important physical features of seismic interactions. In this work, we propose a novel hybrid time-window model that integrates the event magnitude as a key factor in establishing links between nodes within a time window (see Fig. 1). This new model extends the applicability of the TW approach to both networks of events and networks of epicenters, enabling a more physically meaningful representation of seismic correlations. Using global earthquake catalogs, we demonstrate that networks constructed with this hybrid model preserve small-world and scale-free characteristics across different geographic regions and temporal scales. Furthermore, statistical analysis of centrality measures (e.g., degree, betweenness centrality) reveals that their distributions follow Tsallis q-exponential functions [3], indicating the presence of non-extensive statistical features. The networks also exhibit assortative mixing, suggesting that highly connected nodes tend to be linked with others of similar connectivity. Geospatial visualizations further show that high-degree nodes correspond to regions of frequent and/or high-magnitude seismic activity, while certain "bridge" nodes act as hubs linking distant seismic regions. These findings underscore the hybrid model's capacity to capture both local clustering and long-range connectivity in seismic systems. Overall, the proposed hybrid model provides a more robust and comprehensive method for constructing earthquake networks. It enhances our ability to analyze the intricate patterns of seismicity and opens new avenues for the exploration of earthquake dynamics across multiple dimensions—spatial, temporal, and energetic.

References

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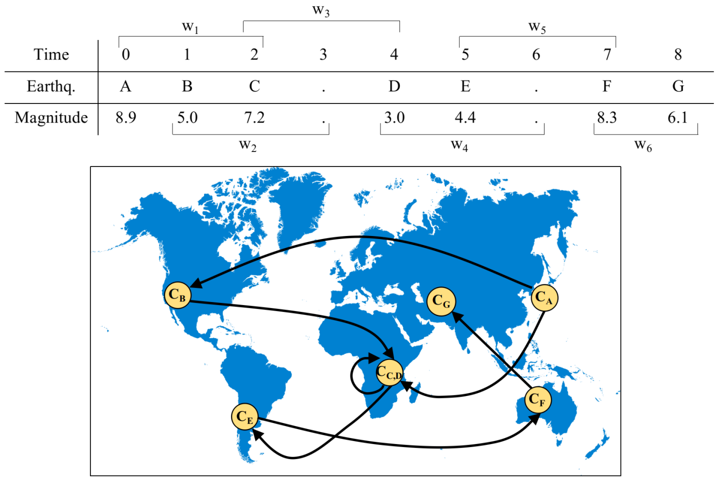
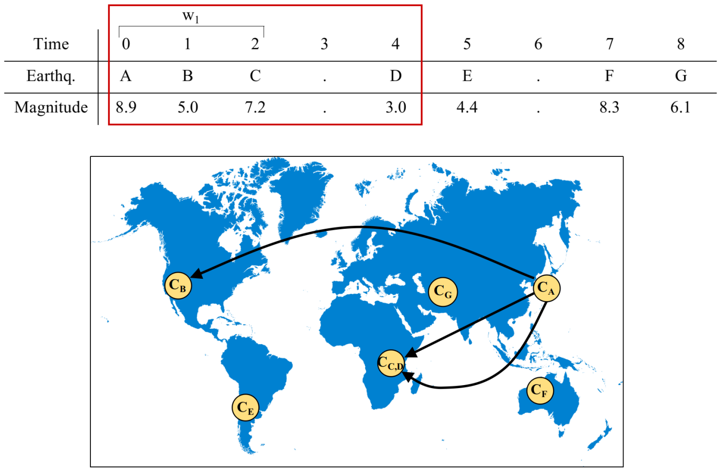


Time	0	1	2	3	4	5	6	7	8
Earthq.	A	B	C	.	D	E	.	F	G
Magnitude	8.9	5.0	7.2	.	3.0	4.4	.	8.3	6.1

(a)



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(b)

Figure 1: Example of network construction. The time of occurrence and magnitude of earthquakes named A to G are presented. C_k is the cell where event k occurred. (a) TW model for the *epicenter network*. Time windows are represented by w_i, with i being the window number, and all windows have the same size $T = 2$ (arbitrary units). The first event within the window is linked to the others englobed. (b) Considering the hybrid model, the average magnitude inside each window is calculated. If the value is greater than the average of the entire magnitude series, the window is increased through a power law relation, $T_{w_i} = \alpha\,(T)^{\gamma_i}$, where γ_i is the ratio between these averages and α is a positive constant (in our analysis, $\alpha = 2$). This procedure allows windows engobling large-magnitude events to establish more links. To exemplify, the calculation for window w_1 is presented. The average magnitude of w_1 is 7.03, which is greater than the 6.13 average magnitude of the series. Therefore, in our methodology, w_1 value is increased to $T_{w_1} = 4.43$, then the new window englobes events A to D (red rectangle). The links for w_1 are shown on the map.