PHYSICS-BASED SYMBOLIC REGRESSION FOR POWER FLOW MODELING AND ANALYSIS

Anonymous authors
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ABSTRACT

Symbolic regression searches the space of mathematical expressions to find the model that best fits a given dataset. Therefore, it can successfully integrate mathematical expressions and underlying physical laws to improve data-driven power flow modeling and analysis. We introduce physics-based symbolic regression for power flow modeling and analysis by taking inspiration from 'AI Feynman' (Udrescu & Tegmark, 2020). A physics-informed neural network is integrated into the proposed symbolic regression algorithm in Udrescu & Tegmark (2020). The results show that, for power flow analysis and modeling of a low-voltage distribution network, the physics-based symbolic regression outperforms the original algorithm, where a black-box neural network is used as a part of the algorithm. The contribution of this paper is, therefore to introduce the idea of integrating physical laws and constraints into the physics-inspired symbolic regression algorithm using physics-informed neural networks.

1 INTRODUCTION

Data-driven approaches, e.g., supervised machine learning and reverse engineering, have been successfully used for the modeling and analysis of modern power systems (e.g., Zafar et al. (2022); Mishra & Rout (2018)). Recently, research has shown that the integration of prior knowledge, i.e., physical laws and mathematical models, into data-driven approaches improves accuracy and efficiency but also reduces the computational burden (e.g., Hu et al. (2021); Yang et al. (2020)). In this perspective, symbolic regression offers the possibility of simultaneously integrating the physical laws and mathematical models in order to unveil a physics-based expression of the system under study, such as modern power systems (e.g, Sarić et al. (2021)).

Among other data-driven approaches, symbolic regression algorithms are used to search the space of mathematical expressions and find a mathematical model that can best describe underlying physical laws based on a given dataset (Vaddireddy et al., 2020). However, increasing the space of mathematical expressions and/or the dataset size makes symbolic regression algorithms challenging or even intractable (ref.). Research has shown that evolutionary algorithms and neural networks are promising to address the issue (e.g., Lu et al. (2016); Petersen et al. (2021)). Despite recent advances in training neural networks to solve complex problems, the integration of neural networks and symbolic regression has not yet been fully explored Udrescu & Tegmark (2020).

Therefore, in this paper, we introduce physics-based symbolic regression for power flow modeling and analysis. We integrate a physics-informed neural network to the proposed algorithm in (Udrescu & Tegmark, 2020) for the application of power flow analysis and modeling. The approach is also applied to a small distribution network.

2 METHODOLOGY

In this paper, a small-size distribution network is modeled from data using a modified version of the symbolic regression algorithm 'AI Feynman' Udrescu & Tegmark (2020). The proposed approach in this paper, physics-based symbolic regression, involves three main parts: (i) power flow analysis, (ii) symbolic regression, and (iii) physics-informed neural network. Detail information about each part is provided in the following subsections:

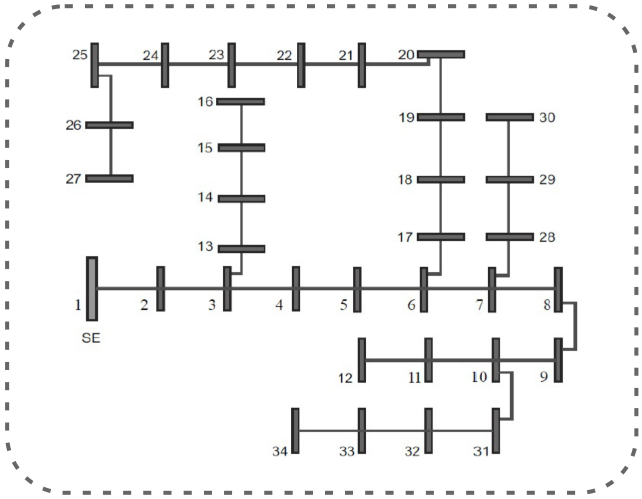

Figure 1: 34-node distribution network.

2.1 POWER FLOW ANALYSIS

In this paper, a small-size distribution network consisting of 34 nodes that operates at a low voltage distribution level is selected as the case study, as shown in figure 2.1. Note that the state of distribution networks can be described once the voltages at all nodes are calculated. Traditionally, numerical methods are used to calculate the voltages based on the given variables, e.g., active power consumption/generation. The given variables are different for different types of nodes. For example, in the distribution network shown in figure 2.1, a reference node and 33 PQ nodes are involved, which means that the given variables for the PQ nodes are active and reactive power consumption. The state of the distribution network can be represented by the balanced linear power flow equations (e.g., Pinzon et al. (2019)). The active and reactive power balance are described by equations (1) and (2), respectively:

$$\sum_{ji \in \mathcal{L}} P_{ji} - \sum_{ij \in \mathcal{L}} \left(P_{ij} + R_{ij} I_{ij}^2\right) + P_i^S = P_i^L \quad \forall ij \tag{1}$$

$$\sum_{ji \in \mathcal{L}} Q_{ji} - \sum_{ij \in \mathcal{L}} \left(Q_{ij} + X_{ij} I_{ij}^2\right) + Q_i^S = Q_i^L \quad \forall ij \tag{2}$$

where $P_{i,t}^L$ and $Q_{i,t}^L$ are the active and reactive loads, respectively, and $P_{i,t}^S$ and $Q_{i,t}^S$ represents the active and reactive generations, respectively. The voltage drop in lines and the current magnitude through lines are represented by equations (3) and (4), respectively:

$$V_i^2 - V_j^2 = 2\left(R_{ij} P_{ij} + X_{ij} Q_{ij}\right) - Z_{ij}^2 I_{ij}^2 \quad \forall ij \tag{3}$$

$$V_i^2 I_{ij}^2 = P_{ij}^2 + Q_{ij}^2 \quad \forall ij \tag{4}$$

Finally, the voltage and current magnitude limits are represented by equations (5) and (6), respectively:

$$0 \leq V_i^2 \quad \forall ij \tag{5}$$

$$0 \leq I_{ij}^2 \quad \forall ij \tag{6}$$

Based on the proposed model equations, a dataset of 1000 data points is generated to develop the symbolic regression but also the physics-based neural network. For each data point, the amount of active and reactive power is considered as input variables, and the voltage magnitude at each node is considered as the output.

2.2 Symbolic regression

Taking inspiration from 'AI Feynman' (Udrescu & Tegmark, 2020), a physics-based symbolic regression approach is introduced for power flow analysis and modeling[1]. AI Feynman is a physics-inspired method for symbolic regression that involves different steps to discover hidden simplicity, such as symmetry and separability, in a given dataset. It uses black-box neural networks to recursively break harder problems into simpler ones with fewer variables. The algorithm is applied to 100 equations from the Feynman Lectures on Physics, and it discovers all of them. More information about AI Feynman can be found in Ref. Udrescu & Tegmark (2020).

According to Udrescu & Tegmark (2020), for power flow analysis and modeling, the function and the variables, i.e., active and reactive power and voltage magnitude, have known physical units. It is also continuous and can be written as a composition of a small set of functions and/or a sum or product of two parts with no variables in common. Based on the definition, the polynomial fit step is skipped in this paper. Therefore, the flowchart of the approach is modified, as shown in figure 2.2:

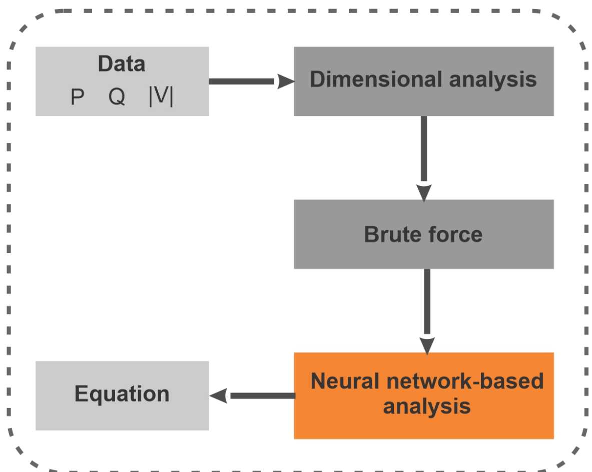

Figure 2: Schematic illustration of the modified AI Feynman algorithm.

2.3 Physics-informed neural network

The black-box neural network developed for the AI Feynman algorithm is replaced with a physics-informed neural network where the underlying physical laws of the application under study, i.e., power flow analysis and modeling, are incorporated into the neural network training process. Figure 2.3 shows the physics-informed neural network architecture used in this paper for power system analysis and modeling. The simplified equations of branch flows, i.e., physical loss terms, are added to the supervised loss term to improve the prediction accuracy. Equation (7) and () represents the supervised loss term and physical loss terms, respectively. Detailed information about the loss function can be found in (Yang et al., 2020).

$$\sum_{i \in \mathcal{L}} (V_i - f(P_i, Q_i))^2 \tag{7}$$

$$\sum_{ij \in \mathcal{L}} (\frac{\partial P_{ij}}{\partial V_i} + \frac{\partial P_{ji}}{\partial V_j})^2 \tag{8}$$

[1]Data is available on `https://github.com/SJ001/AI-Feynman`

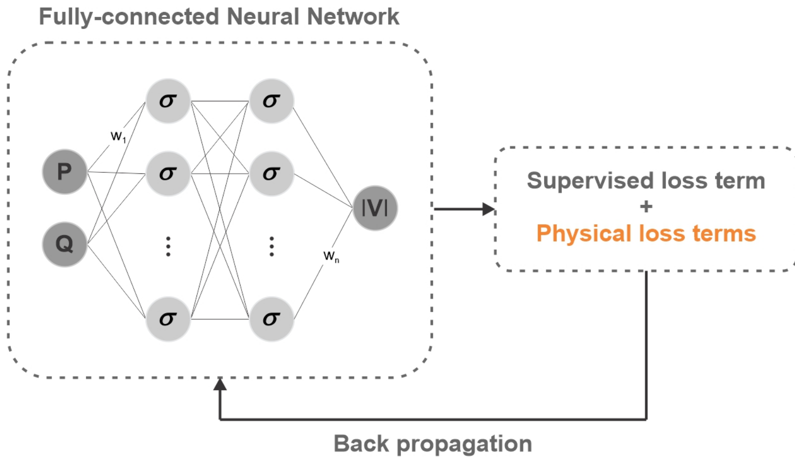

Figure 3: Physics-informed neural network architecture for power flow analysis and modeling of distribution networks.

3 Results

The performance of the proposed algorithm is compared with the original algorithm AI Feynman for power flow analysis and modeling of a distribution network. Equation (9) represents the result obtained from the original algorithm. Numbers are rounded for a better representation. Using this expression, the average deviation from the actual values is 32.7% for the test dataset, which means that the original algorithm is not applicable to complex problems, such as ad power flow analysis and modeling, even for small cases.

$$|V| = (32.57 \times P^2 - 9.54 \times P \times Q + 4872.88 \times Q^2)^{0.001} + 163.69 + 3.27 * 10^{-11} \tag{9}$$

By adding the physical loss terms to the training process of the neural network in the original algorithm, the overall performance of the algorithm improved substantially. Equation (10) represents the result obtained from the modified algorithm. Using this expression, the average deviation from the actual values is reduced to 9.4% for the test dataset, which means an improvement of more than 3 times is achieved by replacing the black-box neural network with the physics-informed neural network.

$$\begin{aligned} |V| = (26.55 \times P^2 - 9.46 \times P \times Q + 4826.93 \times Q^2)^{0.23} + \\ 158.68 + \frac{(\exp(P) + 1) \times (\exp(\sqrt{\pi}) + 1)}{exp(P) + 1} \end{aligned} \tag{10}$$

4 Discussion and Conclusion

In this work, the physics-inspired symbolic regression algorithm (AI Feynman) is modified by replacing the black-box neural network with a physics-informed neural network where the loss function involves physical loss terms. The algorithm is applied to a low-voltage distribution network for power flow modeling and analysis. The results show that the prediction accuracy of the modified algorithm with a physics-informed neural network is improved by up to 3 times compared to the original algorithm with a black-box neural network. Future work can investigate the applicability of the proposed idea, physics-based symbolic regression, for larger and more complicated grid topologies.

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