CurviTrack: Curvilinear Trajectory Tracking for High-speed Chase of a USV

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***Abstract*— Heterogeneous robot teams used in marine environments incur time-and-energy penalties when the marine vehicle has to halt the mission to allow the autonomous aerial vehicle to land for recharging. In this paper, we present a solution for this problem using a novel drag-aware model formulation which is coupled with Model Predictive Control (MPC), and therefore, enables tracking and landing during high-speed curvilinear trajectories of an Unmanned Surface Vehicle (USV) without any communication. Compared to the state-of-the-art, our approach yields 40 % decrease in prediction errors, and provides a 3-fold increase in certainty of predictions. Consequently, this leads to a 30 % improvement in tracking performance and 40 % higher success in landing on a moving USV even during aggressive turns that are unfeasible for conventional marine missions. We test our approach in two different real-world scenarios with marine vessels of two different sizes and further solidify our results through statistical analysis in simulation to demonstrate the robustness of our method.**

URL: https://mrs.fel.cvut.cz/curvitrack-boat-landing

I. INTRODUCTION AND RELATED WORKS

The increased demand for exploring, maintaining, and studying the vast open-water habitats of our planet has created a pressing need for a cost-effective unmanned system that can accomplish these tasks autonomously and efficiently. Exploration requires a high vantage point and agility which are primary strengths of an Unmanned Aerial Vehicle (UAV), but UAVs suffer from small payload capabilities and limited flight times. Conversely, USVs can carry higher payloads and conduct long-range missions due to their higher battery capacity. They can also be equipped with more sensors for localisation, water and air sampling, and sea-floor mapping. Therefore, a heterogeneous team of aerial and marine vehicles can accomplish tasks pertaining to exploration, infrastructure maintenance, port security, search-and-rescue, surveillance, and ocean cleanup with greater efficiency than each of the vehicles by itself. Murphy et al. [1] and Lindemuth et al. [2] have demonstrated how a UAV and a USV can be used together to assess structural damages caused by hurricanes and even co-ordinate sub-surface operations. In these applications, current UAV designs are limited by their battery lives and hence, the capability to autonomously land and recharge on mobile platforms in marine environments emerges as a pivotal and essential technology for a seamless and completely autonomous operation.

One of the challenges during such a landing manoeuvre is that a tilted landing deck can result in the UAV rolling or sliding off the platform, which, in turn, can trigger unintended responses from the controller of the UAV, and thus, lead to disastrous consequences. Additionally, a vertically oscillating deck can transfer significant impulse into the aircraft in case of vertical velocity mismatch and wreak havoc on its fuselage during landing. Landing during this wave-induced periodic motion was previously addressed in our work on landing a UAV in harsh winds and turbulent open-waters [3]. However, as mentioned in that work, it was observed that significant time was lost in the motion estimation, and the USV had to be stationary for this duration instead of conducting the manoeuvres necessary for the completion of its mission. These missions usually require sweeping scans, in parallel track patterns [4], which requires constant turning of the USV; therefore, a landing solution limited to the straight-line motion does not aid in the recovery of lost efficiency.

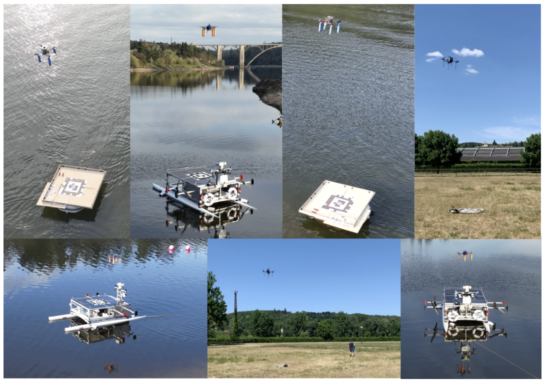


Fig. 1: A collage of various moments from the real-world experiments.

Landing on a marine vessel has been a keenly studied area of research in the recent past. However, fewer works are available that focus on landing on a moving USV and the majority of these focus on straight-line paths for a general platform. Herissé et al. [5] presented one of the first investigations for landing on a general moving platform using optical flow to handle lateral velocities. However, the authors do not consider any predictive model of the underlying vehicle, and therefore, the success of the landing is contingent on the constraints matching between the UAV and the landing platform. Such an approach would be infeasible for a rapidly turning USV. Falanga et al. [6] expanded the state-of-the-art by increasing the velocity of the UAV up to $1.5\,\mathrm{m\,s^{-1}}$, but only in straight lines and following a ground robot. The tracking speed record was presented in the work of Borowczyk et al. [7], where the authors successfully reached a speed of $13.8\,\mathrm{m\,s^{-1}}$ in real-world experiments for straight-line motion and assumed communication between the two robots.

Novák et al. [8] presented the current state-of-the-art in modelling and predicting the motion of a USV using a sophisticated model that considers the hydrodynamic, hydrostatic, and Coriolis forces acting on the USV. They also leverage communication between the vehicles to obtain information from the sensor stack onboard the USV. However, the models rely on multiple sensors and communicated states for state estimation to obtain pose and velocities of the USV, and this requirement leads to poor convergence for challenging turns and curvilinear trajectories, especially when sufficient sensory information cannot be provided without communicating. To this end, we propose a novel decentralized modelling approach which converges reliably on limited information from visual pose estimation, and predicts the curvilinear motion of the USV for performing high-speed tracking and landing on the USV. We present a comparison with the aforementioned approach [8] through statistical analysis in simulation, and bolster our claims by testing our novel approach in several real-world scenarios. In summary, our contributions are:

- a novel drag-aware linear model inside an Linear Kalman Filter (LKF) to predict the future curvilinear motion of a USV using only visual pose estimation, and
- an MPC based control approach for following a high-speed USV during curvilinear trajectories, and for landing with minimal impulse transfer.

A supplementary video of the experiments is attached to this paper and can also be found at https://mrs.fel.cvut.cz/curvitrack-boat-landing.

II. HIGH-SPEED CHASE AND LANDING APPROACH

In this section, we propose our model-based method for predicting, tracking, and landing using two separate models for the USV and the UAV. The UAV model is embedded inside the MPC and is used to make predictions about the state of the UAV for control. Similar to our previous work [3], our proposed controller receives the predicted states of the USV as an input reference and must produce a control reference at a minimum of 20 Hz, within the constraints of the mission (max velocity, max acceleration, and max jerk). The MPC produces control outputs, for desired linear velocities and the desired heading rate of the UAV, which are sent to the underlying MRS-system [9] framework.

A. USV Prediction Model

For a USV moving through a fluid, form drag (also called pressure drag) is the primary factor affecting the curvilinear motion. Therefore, we simulate the form drag experienced by the vessel to generate inputs to a Kalman filter, which causes the predictions to curve. Due to the significant difference between the surface area perpendicular (b_y) and parallel (b_x) to the hull of a USV (see figure 2), we propose this approach based on the assumption that the drag is significantly higher in the b_y axis of the USV compared to the b_x axis and is not compensated by the propulsion of the USV in the b_y axis (see figure 2). As such, the velocity gained by the vessel in the b_x axis before turning transforms into velocity perpendicular to the hull (in b_y axis), which causes the centripetal force required to turn the vehicle.

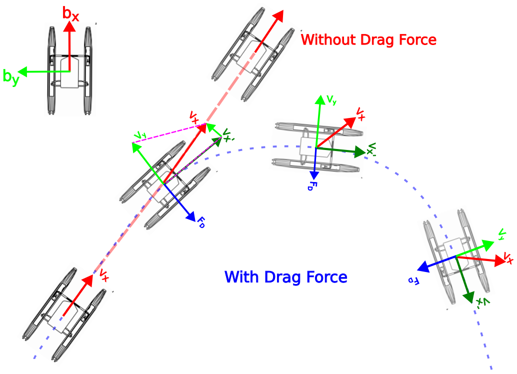


Fig. 2: The red path represents the unchanged path taken by the USV when no drag force is acting on it; whereas the blue path shows how the difference between V_x and $V_{x'}$ produces a V_y vector and therefore, causes significant drag that leads to the turning of the USV.

For this Kalman filter, we write the state vector of the USV as $\mathbf{b} = \begin{bmatrix} x_b & y_b & z_b & \eta_b & \dot{x_b} & \dot{y_b} & \dot{z_b} & \dot{\eta_b} \end{bmatrix}^T$, where x_b, y_b, z_b are the position co-ordinates of the USV in the world frame, η_b is the heading of the USV, $\dot{x_b}$, $\dot{y_b}$, and $\dot{z_b}$ are the translational velocities in the world frame, and $\dot{\eta_b}$ is the heading rate. We then write the discrete state space model of the USV as a point-mass model such that

$$\begin{gathered} \mathbf{b}^{(k+1)} = \mathbf{A}\mathbf{b}^{(k)} + \mathbf{B}\mathbf{u_b}^{(k)} \\ \text{and, } \mathbf{u_b} = \begin{bmatrix} a_x \\ a_y \end{bmatrix}. \end{gathered} \tag{1}$$

Here, dt is the sampling time of the sensor, k is the discrete instance of time where $t = k \cdot dt$, a_x and a_y are the accelerations produced by drag force acting on the USV in the world frame. Since we do not have access to acceleration states, we generate an artificial input, by writing the drag force of a body with an asymmetric projected area, in its own frame, as

$$\mathbf{F_d} = m \begin{bmatrix} a_x \\ a_y \end{bmatrix}_b = -\begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix} \begin{bmatrix} \dot{x_b} \\ \dot{y_b} \end{bmatrix}_b \tag{2}$$

where m is the mass of the USV, $[\bullet]_b$ denotes the vector in the body frame, and $K_x \geq 0$ and $K_y \geq 0$ are the tunable drag coefficients in the x and y axes of the USV frame respectively. For the purpose of our model, we assume that the USV has sufficient propulsion to counteract the drag in the b_x axis so as to maintain a constant velocity along its hull, and therefore, we set $K_x = 0$. We also assume low pitch and roll angles on the USV such that we can write a frame transform as

$${}^{b}\mathbf{R}_w = \begin{bmatrix} \cos\eta_b & \sin\eta_b \\ -\sin\eta_b & \cos\eta_b \end{bmatrix}, \quad {}^{w}\mathbf{R}_b = {}^{b}\mathbf{R}_w^T, \tag{3}$$

where ${}^{w}\mathbf{R}_b$ and ${}^{b}\mathbf{R}_w$ are rotation matrices for rotation transformation from the body frame to the world frame and vice-versa. Hence, we write the input to the system as a function of the velocity of the USV such that

$$m\begin{bmatrix}a_x\\a_y\end{bmatrix}_w = {}^w\mathbf{R}_b\begin{bmatrix}0.0 & 0.0\\0.0 & -K_y\end{bmatrix}{}^b\mathbf{R}_w\begin{bmatrix}\dot{x}\\\dot{y}\end{bmatrix}_w, \qquad (4)$$

where $[\bullet]_w$ denotes the vector in the world frame. Additionally, it was observed that a significant negative correlation exists between the velocity of the USV and its yaw rate due to loss of momentum from the vessel during a turn as the drag component overcomes the thrust provided by the propeller. Therefore, we set $\sigma_{\dot\eta\dot x}, \sigma_{\dot\eta\dot y}, \sigma_{\dot x\dot\eta}, \sigma_{\dot y\dot\eta} < 0$ in the covariance matrix $\mathbf{Q_b}$ to reflect this negative co-relation.

B. *UAV Prediction and Control Model*

The UAV prediction model used in the proposed MPC is based on the Euler approximation of single particle kinematics in world frame, and is identical to our previous work [3].

III. SIMULATION EXPERIMENTS

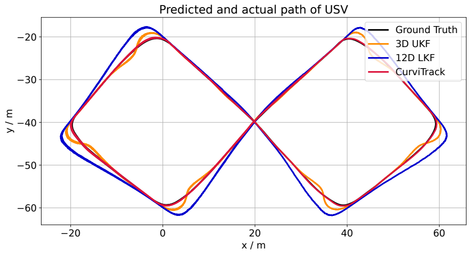
Simulation experiments present comparison with the two types of models presented in [8]. These models are labelled as ‘3D UKF‘ and ‘12D LKF‘ and they use four sensory inputs comprising of two visual pose estimation methods (UltraViolet Direction And Ranging (UVDAR) system [10] and AprilTag [11]) and two communication-reliant sensors (Global Navigation Satellite System (GNSS) and Inertial Measurement Unit (IMU)). The state-of-the-art method can also run solely on visual pose estimation with degraded performance, but we compare their best-case scenario of four sensory inputs to our bare-minimum scenario of a single input via AprilTag pose estimates. For these simulation experiments, we use the Virtual RobotX (VRX) simulation environment in Gazebo [12] to generate the motion of the USV and employ the MRS-system [9] to simulate and control the UAV. For our simulated UAV model, we use a T650 quadrotor frame weighing $3.6\,\mathrm{kg}$, carrying a Garmin LiDAR for altitude measurement, and a camera for live in-simulation video.

A. *Trajectory Prediction Results*

The USV follows an 8-figure trajectory which is specially selected such that the time spent in turning is longer than the chosen 2-second prediction window for these comparisons. Figure 3 shows the predicted paths against the ground truth measured in the simulator. Table I shows the mean error, average error, and standard deviation of the error for predictions in the xy-plane for each technique. We highlight that our technique provides over $70\,\%$ reduction in error compared to ‘12D LKF‘ and $40\,\%$ reduction over the complex model employed in ‘12D Unscented Kalman Filter (UKF)‘. Similarly, our approach also offers a 7-fold decrease in standard deviation (increase in certainty) over the ‘12 LKF‘ and a near 3-fold decrease in standard deviation over the ‘3D UKF‘ method. In conclusion, our efficient curvilinear model outperforms the state-of-the-art by a significant margin without using communication and, therefore, without multi-modal sensory input.

t	Method	Mean (m)	Max (m)	Std. Dev. (m)
1.0 s	*12D LKF*	1.71	4.01	1.31
	3D UKF	0.79	2.03	0.51
	CurviTrack	**0.48**	**0.95**	**0.18**

TABLE I: Mean error, maximum error, and standard deviation of the error for predictions in comparison to [8].



(a) Predictions — 1.0 s into the future

Fig. 3: Comparison between predictions from proposed vs. the state-of-the-art approach [8].

B. *Chasing and Landing Results*

Results for tracking performance can be seen through figure 4 which summarises the horizontal distance between the UAV and the USV during the turns ($\dot\eta_b > 0.1\,\mathrm{rad\,s^{-1}}$), where, it can be seen that our proposed approach doubles the instances where tracking distance is within $0.5\,\mathrm{m}$ of the USV throughout the trajectory. Importantly, this improvement of performance in $0.5\,\mathrm{m}$ distance is approximately the width of our UAV which significantly increases the probability of a successful landing, as highlighted in the next paragraph.

A statistical analysis of landing success was performed by initiating landing at a randomly chosen time, and the attempt was considered successful if all four landing gears touched the deck. If unsuccessful, landings were aborted, and UAV regained altitude. When compared against the ‘12D LKF‘, our approach succeeded in landing 25 times out of 50 attempts ($50\,\%$), whereas the linear approach succeeded only 18 times out of 50 attempts ($36.0\,\%$). These results clearly highlight the advantage of using our curvilinear model for reducing failed landings.

IV. REAL-WORLD EXPERIMENTS

We present two very different real-world conditions on land and water to thoroughly test the robustness of our

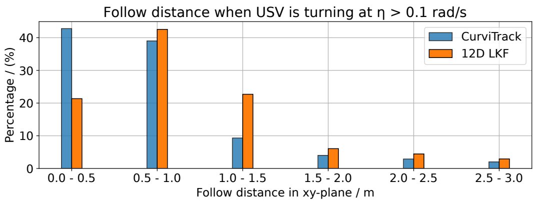


Fig. 4: Statistical analysis on tracking error in simulation for a triangular trajectory when USV is turning at a $\dot\eta_b > 0.1\,\mathrm{rad\,s^{-1}}$.

proposed method in adverse conditions, and to test the limits of estimation and tracking capabilities. For our real-world flights, we employed a 4.5 kg T650 quadrotor equipped with vertical pontoons [13] for emergency landing over water (see figure 1).

For the first set of experiments, we test the predictions over land in a high-speed chase of up to $5\,\mathrm{m\,s^{-1}}$ (the USV moves two body-lengths every second, and UAV moves five body-lengths every second) where the algorithm is challenged by a rapidly curving path over land with unpredictable input from a human operator. For this experiment, we approximate the moving USV using a wooden board with an AprilTag [11] affixed on top, and this board is dragged on the ground by the operator. These experiments present the robustness of our method as the operator can change the magnitude and direction of velocity at their wish and produce turns that are infeasible and unexpected in the marine environment for UAV-USV missions. For example, the assumption of drag in the y-axis is deliberately broken by inducing a side slip in the platform while dragging.

For the second set of experiments, two different kinds of USVs were tested in fresh-water environment to confirm the adaptability of the proposed approach to vessels of different sizes. For the first scenario, an inflatable dinghy is towed behind a paddle-driven boat to represent a low-speed (up to $2\,\mathrm{m\,s^{-1}}$) high-drag marine vessel with a $2\,\mathrm{m}$ by $2\,\mathrm{m}$ landing deck (figure 1). For the second scenario, a sub-$5\,\mathrm{m}$ autonomous USV, capable of moving at speeds of up to $3\,\mathrm{m\,s^{-1}}$, was used to represent a realistic watercraft deployed for missions in fresh-water environments (figure 1).

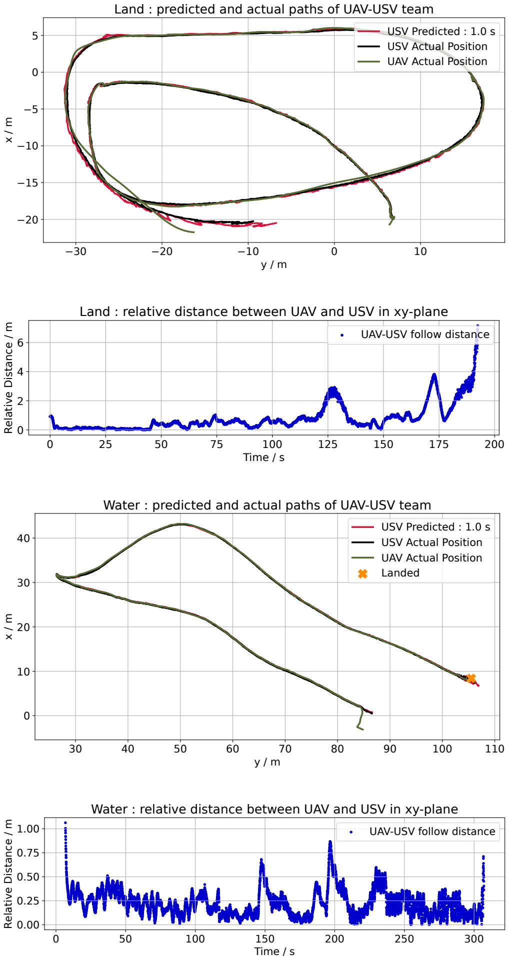


Fig. 5: Predictions and relative distance for chase during land-based and water-based real-world experiments.

A. Trajectory Prediction Results

As seen through figure 5, the prediction method is robust to quick changes in direction and manages to accurately predict the future for both land-based and water-based experiments. In the land-based experiments, unlike the simulation environment, operator made rapid changes in the direction of motion. The only negative impact appears to be the irregularity of the predicted trajectory, which arises from a continuous side slip, causing the projected motion to curve more aggressively than the ground-truth. However, as evident from the figure, the UAV is able to follow the predictions on the curved paths without losing track.

In the fresh-water environment, the predictions are smoother than in land-based experiments due to the uniform drag experienced as per the internal model, and consequently, the predictions line up perfectly with the path taken by the marine vessel.

These experimental runs strongly support the ability of the proposed approach to make reliable predictions about the future of the USV during curvilinear trajectories in real-world scenarios.

B. Chasing and Landing Results

As shown in figure 5, the UAV is able to follow the landing target closely throughout the turns and is able to land on the USV after tracking it through the curvilinear trajectory. These experiments can also be seen in the media attached with the paper and further prove the robustness of our approach to the changes in the environment as well as applicability to real UAVs and USVs of different sizes.

V. CONCLUSION

In this work, we presented a novel approach for the autonomous landing of UAVs onto various moving USV in marine environments for following high-speed curvilinear trajectories. Our proposed decentralized solution leverages visual pose estimation and an efficient motion model for predicting and tracking the moving USV, enabling precise and robust landing manoeuvres. Through statistical analysis in simulation experiments and various real-world experiments, we have demonstrated a decrease in prediction error and an increase in tracking and landing performance for our novel method in comparison to the state-of-the-art. This real-time onboard model can converge and predict for any sensor setup with robust pose estimation without requiring communication.

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