A STATISTICAL METHOD FOR ATTACK-AGNOSTIC Adversarial Attack Detection with Compressive Sensing Comparison

Anonymous authors

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Abstract

Adversarial attacks present a significant threat to modern machine learning systems. Yet, existing detection methods often lack the ability to detect unseen attacks or detect different attack types with a high level of accuracy. In this work, we propose a statistical approach that establishes a detection baseline before a neural network's deployment, enabling effective real-time adversarial detection. We generate a metric of adversarial presence by comparing the behavior of a compressed/uncompressed neural network pair. Our method has been tested against state-of-the-art techniques, and it achieves near-perfect detection across a wide range of attack types. Moreover, it significantly reduces false positives, making it both reliable and practical for real-world applications.

1 INTRODUCTION

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Neural networks (NNs) are being used in numerous use cases across many disciplines. They have
almost become an integral part of our lives. One variation of them that we use very commonly is
the Convolutional Neural Network (CNN). They are widely used in image recognition, and they are
considerably reliable in this application, and they keep getting better virtually every day. A challenge
in using CNNs in sensitive applications is the existence of adversarial attacks (Goodfellow et al., 2015).

Adversarial attacks inject a small, ideally human-imperceptible perturbation in the input image before being fed into a CNN classifier. This perturbation usually takes the form of random noise and is applied at a practically imperceptible level. Although this modification looks harmless or even completely invisible to the human eye, it wreaks havoc inside the workings of the CNN classifier. It ultimately pushes the detection to an invalid class. This can lead to many unseemly outcomes, ranging from loss of accuracy to failure of safety-critical systems.

Several methods exist to detect and suppress adversarial attacks. However, these methods suffer
 from inherent flaws, such as the requirement of apriori knowledge of the attack type, the high number
 of false positives, low overall accuracy, and scaling issues with different network architectures.

In this paper, we present a simple attack-agnostic detection method that does not require prior knowl edge of attack models. It requires a simple training process before the deployment to generate a set
 of class identities and, during runtime, uses those identities to match every incoming sample.

Compression suppresses adversarial noise to an extent, as shown in Aydemir et al. (2018) among others. While this effect is less than ideal for reliably suppressing all adversarial perturbations, we can observe a difference when we take the same input and run it through a pair of slightly different networks. The pair of networks will be almost identical, except that to the second network, we feed a compressed version of the image, and the network itself is pre-trained on compressed images after regular training. We leverage a secondary denoising network that operates on compressed images and check how far the matching of the distributions skew between the two networks. We propose a metric that can be calculated at runtime for each sample that measures this disparity and a threshold T that can be empirically determined pre-deployment. We use the threshold on the metric to determine the presence of adversarial perturbations and filter out the adversarial samples. The metric calculation uses the feature maps generated in both networks' last feature layer (the layer before the winner-takes-all/softmax layer) and represents how much they disagree. This disagreement is more pronounced in adversarial images, thus allowing the discrimination between them and clean images. Our method consistently gives accurate detections for every adversarial attack tested, while existing work performs well in some attacks and poorly in others.

We developed this method without considering any of the attack models and their behavior since compressive networks suppress almost any adversarial signal presence. This led us to believe this method should perform well on any attack universally. We claim that our method is an attackagnostic adversarial attack detector.

Several mathematical/statistical operations are used in this work. One of the notable operators is the KL divergence, which is defined for two vectors $\mathbf{a} = (a_1, a_2, ..., a_n)$ and $\mathbf{b} = (b_1, b_2, ..., b_n)$ as follows, where ln stands for the natural logarithmic operator.

$$KL(\mathbf{a}, \mathbf{b}) = \sum_{i=1}^{n} a_i . ln(\frac{a_i}{b_i})$$
(1)

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One important point to note here is that the KL operator is not commutative, and therefore, KL(a, b)and KL(b, a) are not necessarily equal.

We utilize the L2 norm to measure the difference between two vectors. The L2 norm between two vectors *a* and *b* are defined as,

$$L2_norm(a,b) = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$$
(2)

Also, we used the Mann-Whitney U test to compare two smaller distributions and decide whether
 the data belongs to a common distribution. This method is outlined in Mann & Whitney (1947)

In the rest of the paper, section 2 briefly outlines related work. Section 3 presents the details of the technique and how it works. Our experimental results are showcased in section 4, and we compare our approach with existing methods. Finally, section 5 concludes the paper.

2 RELATED WORK

We use several adversarial attack models to verify our theory of attack agnostic detection. They are contained within the Adversarial Robustness Toolbox (ART) by Nicolae et al. (2018) Python package. The attack models that are used here are the Fast Gradient Sign Method (FGSM) (Goodfellow 091 et al., 2015), the Projected Gradient Descent (PGD) (Madry et al., 2017) approach, the Square At-092 tack (Andriushchenko et al., 2020) method, the DeepFool (Moosavi-Dezfooli et al., 2015) method and the Carlini-Wagner(CW) (Carlini & Wagner, 2016) attack. These methods use an approximately 094 similar CNN model (black box attack) or the exact CNN model used in detection (white box attack) 095 to generate an attack. The exact way they create the attack varies by the attack method. Still, they 096 usually use gradient-based methods, where the adversary calculates the gradient of the model's loss 097 function with respect to the input and adjusts the input accordingly to maximize the loss, as opposed 098 to minimizing the loss when the goal is to predict the image content accurately. A noise vector is calculated using these gradients that skew the prediction in a way that increases the chance of mis-099 classification. This noise vector is then added to the image, making it fool the detector. The idea of 100 our work is to identify images that have been tampered with with such a malicious noise vector. 101

Current methods of detecting the presence of adversarial attacks include attack agnostic methods
such as the Least Significant Component Feature (LCSF) (Cheng et al., 2022) method, the Energy
Distance/Maximum mean discrepancy (Saha et al., 2019) method, the Mahalanobis distance-based
classifier (Lee et al., 2018) method. Also, there exist attack specific methods such as the Feature
Squeezing (Xu et al., 2018) method, using Latent Neighborhood Graphs (Abusnaina et al., 2021),
using Influence Functions/Nearest Neighbors (Cohen et al., 2020), using Bayesian Neural Networks
(Deng et al., 2021), by random input responses (Huang et al., 2019) and by Random Subspace

108 Analysis (Drenkow et al., 2021). These methods have their shortcomings, such as poor performance 109 in some attack models, requiring extra training data, high false positive rates, and limited flexibility. 110

One major part of our method is the secondary denoising network. In theory, this can be accom-111 plished in numerous methods, but we have chosen compressive sensing using JPEG2000 compres-112 sion, as demonstrated by Aydemir et al. (2018). JPEG compression is typically used to reduce the 113 file size of images by reducing unnecessary and imperceptible information contained in an image. 114 However, this provides a benefit when attempting to suppress adversarial attacks since the JPEG 115 compression algorithm treats the adversarial noise signals as imperceptible information and disre-116 gards them, restoring the original image to an extent. The downside of this in practice is that the 117 accuracy improvement is not perfect and, in most cases, only restores about 20-30% of the accuracy. 118 So, this alone is not a comprehensive defense strategy against adversarial attacks. However, since we know that a compressed network treats adversarial noise differently, we can take advantage of 119 that to implement a more sophisticated detection method. 120

121 In order to properly build class identities and match new samples, we need a proper comparison 122 system. This is accomplished using the method proposed by Pentsos & Tragoudas (2023). The 123 idea is to partition the pre-provided train-test sets and use the feature vectors of those partitions to build an identity. Then, the new samples are augmented using benign noise vectors to form a rich 124 representation of the sample, which is compared against the pre-built class identities. We use this 125 underlying concept to check how the new samples match our known class baselines. 126

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3 METHODOLOGY

130 Here, we describe a method to detect adversarial attacks in an attack-agnostic way, using a pre-built 131 class identity and the deviation from it on a new sample. The sample is run through the regular net-132 work and a redundant network, which uses a denoising method such as JPEG compression Aydemir 133 et al. (2018). Before the deployment, we run the system through a known dataset and build each 134 class's identities on both networks. Then, in the field, we match each example to the class identity 135 on both networks and extract a measure of how much the two networks disagree on the image. If 136 they disagree beyond a certain threshold, the sample is marked adversarial. This effect is evident in 137 the sample images shown in Figure 1



(a) Raw Clean

Figure 1: Sample image of the class 'Cat' from CIFAR-10 at the various stages of the method

 $(\epsilon = 0.08)$

 $(\epsilon = 0.08)$

152 Our method is derived from the technique from Pentsos & Tragoudas (2023) to build and match class identities for a classification problem. This allows us to have an apriori knowledge of the data, 153 how each class behaves, and a way to identify when they misbehave, i.e., an adversarial attack. We 154 used the Mann-Whittney U test and the KL divergence to generate and match distributions with each 155 other. For the identity creation phase, we run Algorithm 1 to create the class identity for each class. 156 This takes the form of a histogram, one for each class. The second phase generates a distribution 157 from the input image using Algorithm 2, matching them with each class distribution, getting the 158 distance to the closest distribution, comparing how the redundant network agrees with this measure, 159 and making the decision. 160

- We preserve the name convention used in Pentsos & Tragoudas (2023) for better clarity. The algo-161 rithm 1 is used to calculate the distribution identity of each class and store it for later use. It takes
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¹⁶² In	iput : Class c, test set of class c, training set of c, I parameter
¹⁶³ O	utput: Distribution identity $MW_{-i}d(c)$ of class c
¹⁶⁴ fo	$\mathbf{r} \ \mathbf{i} \leftarrow 1 \ \mathbf{to} \ I \ \mathbf{do}$
165	Randomly sample N images from the test set of c to create sample S_T ;
166	Partition S_T into subsets S_i , each containing k images;
167	Randomly sample N images from the train set of c to create sample S_R ;
168	Perform a forward pass of S_R through the CNN;
169	for each subset S_i of S_T do
170	Perform a forward pass of S_i through the CNN;
171	Extract the average feature vectors $f_n^{S_i}$ and fn^{S_R} of the examined samples using the output of the penultimate layer <i>n</i> of the CNN:
172	Normalize the two extracted average feature vectors;
173	Calculate the KL divergence $D_{i}^{(I)}$ between \hat{f}^{S_i} and $\hat{f}n^{S_R}$
174	end
175	Partition $S_{\mathcal{P}}$ into subsets S_{i} each containing k images:
176	Perform a forward pass of S_T through the CNN:
177	for each subset S_i of S_B do
178	Perform a forward pass of S_i through the CNN;
179	Extract the average feature vectors $\hat{f}_{n}^{S_{i}}$ and $\hat{f}n^{S_{T}}$ of the examined samples using the
180	output of the penultimate layer n of the CNN;
181	Normalize the two extracted average feature vectors;
182	Calculate the KL divergence $D_{DT}^{(I)}$ between $\hat{f}_{r}^{S_i}$ and $\hat{f}_{DT}^{S_T}$
183	end
184	Calculate the p value of the Mann Whitney II test between $D^{(i)}$ and $D^{(i)}$.
185	Store the n-metric as the <i>i</i> -th element of the distribution identity of class c:
186 er	nd
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Algorithm 1: Calculating the distribution identity MW_{-id} of a class using the KL divergence

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the train set and test set for each class as input, along with the parameter I, and outputs a dictionary of class maps for each class. The parameter I is empirically determined by experimentation.

3.1 BUILDING THE DISTRIBUTION IDENTITY

When building the distribution identity for a class, we first take the test set S_T for that class and 196 partition it into subsets of size k each, which we will call S_i . Then, we select N random images 197 from the train set to create the sample batch S_R . These image sets S_R and each S_i are passed 198 through the neural network in question, as well as the feature vector of the layer before the softmax 199 layer is extracted. We calculate the average vector of each of these sets, which we call \hat{f}^{S_R} and 200 \hat{f}^{S_i} respectively. The average feature vector here is the element-wise average of each vector from 201 the neural network output. These vectors are then normalized, and their KL divergence $D_{S_i S_R}$ is 202 calculated as shown below. 203

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$$D_{S_i S_R} = \frac{KL(\hat{f}^{S_i}, \hat{f}^{S_R}) + KL(\hat{f}^{S_R}, \hat{f}^{S_i})}{2}$$
(3)

208 We repeat this process for each subset S_i of S_T , and then we are left with a series of 'divergence 209 points', namely D_{TR} . We need to calculate D_{RT} , which involves the same procedure but with S_R 210 and S_T interchanged. We then take these two populations and perform a Mann-Whitney U test 211 (Mann & Whitney, 1947) between them. The resulting p-value is stored as the *i*th value of the 212 class distribution identity. This process is repeated over I iterations to build the complete distri-213 bution identity. The samples generated during the algorithm's execution are saved as a dictionary, class_distributions. This step is executed before the actual deployment of the neural network, and 214 we need to make sure that the train and test sets are strictly free of perturbations, adversarial or 215 random. This process is also performed separately on the redundant network.

216 3.2 DETECTING ADVERSARIAL SAMPLES

218 After the neural network is deployed, we use the second procedure, as outlined in algorithm 2, to 219 determine whether the given image is adversarial. The algorithm takes in the unknown image q, the sample dictionary $class_distributions$, and the class identity dictionary MW_{-id} . It will return the 220 distance metric, which we can use to compare with a predetermined threshold, where if it is higher, 221 the image will be tagged as adversarial and as clean otherwise. We take the input image and generate 222 a sequence of N images. To do this, we add various random noise signals to the image, save it as a new image, and generate N-1 images, which gives us N images when combined with the original 224 image. We call this batch of N images as S_q , and it is a rich representation of the image itself. 225 Then, to match this new representation with the classes, we pick N images from the class distribution 226 for a given class c. These images will form our S_T for this execution. We run this S_T and S_q through 227 the same partitioning, p-value calculation, and KL divergence calculation we performed in algorithm 228 1. The resulting KL divergence value for each class is stored as a vector with respect to the class. 229 Two such vectors are extracted, one from the standard network and the other from the redundant 230 network. 231 232 **Input** : Input image q, dictionary class_distributions, list of class distribution identities 233 MW_id 234 **Output:** Adversarial Possibility Metric P_A 235 Run Instance(q, N), which creates N - 1 uncorrelated instances of q and form sample S_q 236 with these N images; Partition S_q into subsets of size k; 237 for each class label c do do 238 Randomly select m samples from the distribution $class_distributions[c]$; 239 Forward pass images in m through the deployed network and extract their average feature 240 vector, denoted by f_c^m ; 241 for each subset S of S_a do do 242 Forward pass images in S through the deployed network and extract their average 243 feature vector, denoted by f_n^{sq} ; 244 Append p_value between \hat{f}_c^m and \hat{f}_n^{sq} to the class-sample signature D_c 245 end 246 Compute the KL divergence between D_c and $MW_{-i}d(c)$ and append it to distance vector 247 V;248 end 249 Get V for the raw network V_R and for the compressed network V_C ; 250 **Return** $L2_Norm(V_R, V_C)$ as P_A ; 251 Algorithm 2: Calculating the adversarial possibility metric 252 253 The two networks are used here because the denoising network will try to push back against the 254 adversarial features and diminish them while pushing the image toward its original class and its 255 resultant features. This will cause a disparity between the distribution distances between the class 256 identities. The distance vector will match very closely on clean images but will take a significant 257 value on the adversarial, giving a clean separation between the two. 258

We decide the detection threshold T empirically. It is chosen so that all the clean samples score below this threshold and still give good results on the adversarial data. Once this universal threshold is established, it does not need to vary by the type of attack. Anything below is classified as clean, and anything above is classified as an adversarial sample. In this way, we can achieve true attackagnostic detection.

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4 EXPERIMENTAL RESULTS

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We performed experiments using the proposed method on three datasets across five attacks. The cho sen datasets are CIFAR-10, CIFAR-100, and a truncated version of Imagenet, which contains only
 50 classes. FGSM, PGD, Square Attack, DeepFool, and Carlini-Wagner attacks were performed on
 them. Most of the evaluation was conducted on a workstation running Ubuntu 22.04 LTS, an Intel

Dataset	Raw Clean	Compressed Clean	Raw FGSM	Compressed FGSM
CIFAR-10	85.63	83.65	62.72	81.59
CIFAR-100	82.34	79.86	53.92	71.82
ImageNet	78.98	73.47	51.65	67.44

Table 2: Performance Comparison of Various Defenses

Table 1: Effect of Compression on Adversarial Images

280 281	Attack	Dataset	Dist. Matching ₁ (ours)	Cheng ¹	Saha ¹	Mahalanobis	Feat. Squeezing	Abus- naina	Cohen	LiBre	Huang
282	FGSM	CIFAR-10	100 00 ²	99 90	75.90	99 94	20.80	99 88	87 75	_3	77.20
283	10500	CIFAR-100	100.00	100.00	-	99.86	-	-	87.23	-	-
200		ImageNet	100.00	-	-	-	99.60	99.53	-	100.00	-
284	PGD	CIFAR-10	100.00	100.00	-	-	-	91.39	99.34	-	96.40
285		CIFAR-100	100.00	99.90	-	-	-	-	81.87	-	-
000		ImageNet	98.80 ⁴	-	-	-	-	99.35	-	99.40	-
280	Square Attack	CIFAR-10	98.00	-	-	-	-	98.82	-	-	-
287		CIFAR-100	100.00	-	-	-	-	-	-	-	-
288		ImageNet	99.50	-	-	-	-	82.20	-	-	-
200	DeepFool	CIFAR-10	100.00	84.60	-	83.41	//.40	-	97.98	-	99.80
289		CIFAR-100	97.50	/3.30	-	11.57	- 78 60	-	/8.82	-	-
290	Elastic Net	CIEAR 10	99.50	-	-	-	/8.00	-	- 86.05	-	- 05 10
200	Elastic Net	CIFAR-100	97 30	-	-		-	-	70.49	-	-
291		ImageNet	97.80	_	_	-	_	_	-	_	_
292	CW	CIFAR-10	97.80	94.30	100.00	87.31	98.10	91.51	98.98	-	98.70
000		CIFAR-100	99.50	81.60	-	91.77	-	-	93.16	-	-
293		ImageNet	97.10	-	-	-	97.90	86.05	-	98.50	-
294	JSMA	CIFAR-10	98.60	-	-	-	83.70	-	98.95	-	98.40
205		CIFAR-100	98.10	-	-	-	-	-	80.76	-	-
290		ImageNet	97.00	-	-	-	-	-	-	-	-

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Xeon E5 CPU, and triple Nvidia GTX980 GPUs. Some time-intensive tasks were offloaded to a high-performance cluster containing multiple GPU nodes.

301 CIFAR-10 was used as the primary benchmark to evaluate the method. The ResNet18 (He et al., 2015) network was used and trained to achieve an accuracy of 91.39% on unmodified raw images of 302 the test set, which is a set of 10,000 images, 1000 images from each class. JPEG2000 compression 303 with a quality factor of 80% was used to duplicate the train and test sets, and the network was re-304 trained on the JPEG-compressed images. When evaluated on the compressed test set, an accuracy of 305 90.27% was obtained. The same test set was then run through the ART adversarial attack generation 306 tool to create an FGSM attack with a perturbation strength (ϵ) of 0.02, which was run through raw 307 and JPEG networks, which resulted in accuracies of 62.72% and 81.59% respectively, indicating 308 adversarial suppression. A complete summary of the effect of JPEG compression on FGSM attack 309 across multiple datasets can be seen in Table 1. 310

The pre-deployment information was calculated using 10,000 images randomly selected from the 311 train set, and algorithm 1 was used to extract 10 class signatures from them. The I parameter was 312 set to 50 through experimentation. The evaluation was then performed on the 10,000 image test set. 313 These images were taken from the test set of each data set and were run as clean samples through 314 the method. The threshold of 5 was selected such that 100% of the clean images were marked as 315 clean by the technique, with a margin added to it for the CIFAR-10 dataset. Then, each image was 316 used to create one adversarial attack of each type, resulting in 10,000 test images per attack. The 317 scores given in the result summary are the percentages of images in that set marked adversarial by 318 the method, i.e., had a P_A higher than the threshold T. 319

¹The method names in boldface are attack agnostic

²The values in boldface are the best overall result for the given dataset-attack combination

³The dash (-) indicates that this result has not been reported

⁴The values in italics are the best attack agnostic method result for the given dataset-attack combination, but there is a better attack-specific result

324 The Experiment was repeated for CIFAR-100 and a subset of TinyImageNet that consists of only 50 325 classes to facilitate the attack generation, which is very time/memory intensive for complex attacks 326 such as JSMA and Elastic Net. Separate thresholds were calculated for each dataset (8.3 for CIFAR-327 100 and 8.7 for ImageNet). Compared with other methods, the complete result set can be seen in 328 Table 2.

5 CONCLUSION

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332 To conclude this work, we have presented a method that can detect adversarial attacks without the requirement for attack-specific training and maintain a high detection accuracy across multiple attack models without compromising the false positive rate. The results show that our method is 335 reliable across multiple datasets and attack models, maintaining almost perfect accuracy. These 336 characteristics make our method ideal for sensitive applications needing reliable defense against potential adversarial threats. 338

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