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From Calculation to Adjudication: Examining LLM Judges on Mathematical Reasoning Tasks

Anonymous ACL submission

Abstract

To reduce the need for human annotations, large language models (LLMs) have been proposed as judges of the quality of other candidate models. The performance of LLM judges is typically evaluated by measuring the correlation with human judgments on generative tasks such as summarization or machine translation. In contrast, we study LLM judges on mathematical reasoning tasks. These tasks require multi-step reasoning, and the correctness of their solutions is verifiable, enabling a more objective evaluation. We perform a detailed performance analysis and find that easy samples are easy to judge, and difficult samples are difficult to judge. Our analysis uncovers a strong correlation between judgment performance and the candidate model task performance, indicating that judges tend to favor higher-quality models even if their answer is incorrect. As a consequence, we test whether we can predict the behavior of LLM judges using simple features such as part-of-speech tags and find that we can correctly predict 70%-75% of judgments. We conclude this study by analyzing practical use cases, showing that LLM judges consistently detect the on-average better model but largely fail if we use them to improve task performance. 1

1 Introduction

The automatic evaluation of machine learning models promises to reduce the need for human annotations. Specifically, the LLM-as-a-judge paradigm (Zheng et al., 2023) has gained traction, aiming to assess or compare the quality of generated texts automatically. This approach is beneficial for automated data labeling (Tan et al., 2024), self-improvement of LLMs (Wu et al., 2024), and ranking the capabilities of LLMs, potentially concerning specific tasks (Zheng et al., 2023). Much like judges in the real world, who are expected to be exact, fair, and unbiased (Bangalore Principles, 2002),

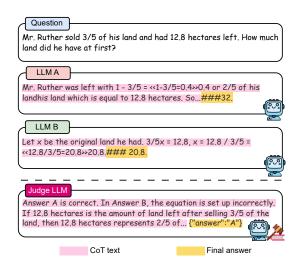


Figure 1: In our problem setup two LLMs (A and B), provide candidate answers for a math problem, and a judge LLM has to decide which one is correct. All three use chain-of-thought (CoT) reasoning (Wei et al., 2022).

LLM judges, should be unbiased and logical. Previous works investigate properties and biases of LLM judges on generative tasks such as translation or summarization (Kim et al., 2024b; Liu et al., 2024), typically evaluated using correlation with human annotators, and thus being inherently subjective.

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In this work, we investigate LLM judges on mathematical reasoning datasets. Such tasks require complex multi-step reasoning and judgments can be analyzed through the lense of verifiable solutions, allowing us to investigate the relationship between judge and candidate models in a principled manner. In our setup, LLM Judges are given two answers and they have to classify whether both answers, one of them (which one), or none is correct (see Figure 1. We base our analysis on four large (> 27B parameters) LLMs and four small (< 10B parameters) LLMs on three mathematical reasoning datasets.

Our experiments contain a detailed performance examination. We find that the best tested judge is LLama 3.1 70B, reaching 60% to 90% judgment

¹Code will be made available upon acceptance.

performance, depending on the dataset. Our results confirm the intuition that judgment performance is aligned with task difficulty.

We perform a statistical analysis of judgment performance and model quality. We find that the individual task performances of judge and candidate models are highly indicative features of judgment performance, as they explain most of the variance in a linear model (as measured by \mathbb{R}^2). On the subset of questions where the candidate models give one correct and one incorrect answer, we uncover an intriguing correlation between judge performance and candidate models' task performance, indicating that LLM judges tend to select incorrect answers from better models.

We hypothesize that judges partially base their judgment on linguistic cues rather than solely on the reasoning withinin the answers. We follow literature analysing machine-generated text (Shaib et al., 2024) and find that 70%-75% of the judgments can be predicted using simple linguistic features, highlighting the systematicity behind the judge decisions.

Lastly, we analyze practical use cases and discuss usage recommendations. Our experiments suggest that LLM judges reliably detect the model of higher task performance but can not reliably improve task performance. Rather, we find that it is more sensible to use the judge model as an answer generator, and subsequently take the majority vote of all three answers.

In summary, our contributions are as follows:

- 1. We perform an in-depth performance analysis of LLM judges on three diverse mathematical reasoning tasks.
- We identify a correlation between model quality, as measured by task performance, and judgment performance, indicating that LLM judges are biased towards higher-quality models.
- 3. We are able to predict the LLM judgment with 70%-75 accuracy using only stylistic patterns, e.g. N-grams of POS-tags. This indicates that LLMs, to a large degree, judge independently of the reasoning.
- 4. We find that judges reliably detect the model of higher quality but are not able to reliably improve task performance.

2 Related Work

2.1 LLM as Judges

Using *LLMs* as judges to evaluate text generated by LLMs, including their own outputs, has recently attracted significant interest because it reduces the need for human annotation (Zheng et al., 2023). Typically, large state-of-the-art models are used as judges. Applications include the automatic assessment of language model capabilities and, such as ranking models with respect to their competence on a given task (Zheng et al., 2023), and reinforcement learning from AI feedback by automatically generating data for preference optimization (Bai et al., 2022; Wu et al., 2024).

Various methods exist to make judgments (Zheng et al., 2023; Liusie et al., 2024). One approach is pairwise selection (Wang et al., 2024b), where two answers are presented, and the model is asked to select the better one. Another approach is pointwise grading (Li et al., 2024), where the model is asked to assign a grade based on a predefined scale, and the answer with a better grade is chosen. Judgment prompts may involve reference solutions or not. Another body of research explicitly trains models to act as judges (Kim et al., 2024a; Wang et al., 2024b) or closely related, as reward models (Wang et al., 2024c; Li et al., 2024).

The effectiveness of LLMs as judges is typically assessed by measuring the correlation or overlap with human judgments (Zheng et al., 2023; Kim et al., 2024b). In contrast, we focus on tasks with a concrete final answer. Finally, we want to stress that several works caution against the use of LLM judges as experts (Bavaresco et al., 2024; Koo et al., 2023; Raina et al., 2024; Doddapaneni et al., 2024).

2.2 Biases in LLM-as-a-judge

Human-annotated data inherently reflects the annotators' biases and opinions. These biases can be detrimental or (intentionally) beneficial, depending on the goals of the annotation process (Plank, 2022). Similarly, several studies have explored the biases present in LLM judges:

One linguistic bias is ordering bias (Zheng et al., 2023; Koo et al., 2023; Wang et al., 2024a), where a judge gives a different answer depending on the order in which answers are presented. Panickssery et al. (2024) note that it is possible to interpret position bias as a sign that the model is unsure. There are multiple works (Xu et al., 2024; Panickssery et al., 2024; Liu et al., 2024) that find evidence for

self-bias or self-preference. Koo et al. (2023) provide a benchmark for analyzing cognitive biases. West et al. (2024) and Oh et al. (2024) explore the "Generative AI Paradox" where it is easier for LLMs to generate solutions rather than analyzing them, unlike humans who often find analysis easier than generation.

In this work, we aim to establish a better understanding of underlying patterns that relate judgments to interpretable factors, such as task performance or stylistic patterns.

3 General Setup

In the following, we describe the problem setting, including the used notation, and the general experimental setting including used models and datasets.

3.1 Problem Description

In this work, we use an LLM judge, referred to as J, to assess answers produced by two other candidate LLMs, A and B, in response to math questions (see Figure 1 for an illustrative example). The two candidate answers may both be correct, both incorrect, or either the answer of model A or B correct. The judge's task is to determine which of these cases applies by reviewing both the CoT reasoning and the final responses provided in candidate answers.

Thus, the judge engages in a four-class classification task. We denote the judge's accuracy by the score $S_{A,B}^J$ and call this metric *judgment performance*. Further, we define the *task performance* of an individual model X on a specific dataset as S_X , e.g. S_A , S_B or S_J .

3.2 Datasets

The experiments encompass three mathematical reasoning datasets where models highly benefit from multi-step CoT reasoning. For all datasets, we use accuracy as the performance metric.

AQUA-RAT (Ling et al., 2017) is a dataset to test the quantitative reasoning ability of LLMs. Unlike the other two datasets, the questions are multiple-choice. GSM8K (Cobbe et al., 2021) consists of grade school math word problems. The answers are free-form numbers. MATH (Hendrycks et al., 2021) contains challenging competition mathematics problems. Find more details in Appendix A.1

3.3 Models

We evaluate the performance of openly available LLMs, including four large models including

Qwen 2.5 72B (Yang et al., 2024), Llama 3.1 70B (AI@Meta, 2024), Yi 1.5 34B (Young et al., 2024), Mixtral 8x7B (Jiang et al., 2024) and four small models, namely Llama 3 8B (AI@Meta, 2024), Gemma 1.1 7B (Gemma Team et al., 2024), Mistral 7B v0.3 (Jiang et al., 2023), and Mistral 7B v0.1 (Jiang et al., 2023). We use the chat- or instruction-tuned model variants and test each model as a candidate answer generator and as a judge. More information is in Appendix A.2.

3.4 Text Generation

This section describes the generation of candidate answers and judgments. Find more information on prompts and hardware details in Appendix A.

Candidate answer generation. For each model we sample two CoT solutions using 4-shot prompting by setting the temperature to 0.9. By generating two answers a_1, a_2 from the same model, we can also evaluate judgments of two different answers by the same model.

Judgements. For all 36 unique model combinations $(A, B)^2$, each model as judge J and each sample of a dataset, we generate a zero-shot judgment. In the case of self-pairing, i.e., A = B, we use both generated candidate answers, a_1 and a_2 . Otherwise, for consistency, we always use the same sampled candidate answer a_1 . We accommodate positional bias (Zheng et al., 2023; Koo et al., 2023) by prompting in both possible orders. We obtain the judgment performance by averaging how often the judgments were correctly classified across orderings.

4 Performance Analysis

The experiments have multiple degrees of freedom, such as judges, candidate models, and datasets. To gain a comprehensive understanding of judges' behavior, we consider two perspectives. First, we investigate judge performance for each dataset, aiming to associate judge performance with task difficulty. Second, for a fixed dataset, we analyze judge performance across different pairs of candidate models.

4.1 General Performance

First, we compare how often the judges make a correct classification across different datasets and

 $^{^2}$ We consider all pairs from the eight LLMs, including self-pairing, yielding $\binom{8+2-1}{2}=36$ combinations.

		Llama 3.1 70B	Qwen 2.5 72B	Qwen 2.5 14B	Gemma 2 27B	Qwen 2.5 7B	Gemma 2 9B	Llama 3.1 8B	Gemma 2 2B
(1) $S_{A,B}^{J}$	GSM8K	90.05	85.39	89.2	81.96	81.92	83.60	79.96	64.33
11,12	AQUA-RAT	74.47	69.26	72.26	68.48	65.09	67.48	66.26	60.97
	MATH	61.18	58.03	62.36	55.34	50.70	52.92	50.96	50.35
(2) Same answer	GSM8K	95.27	95.46	95.02	92.27	92.86	94.70	88.13	75.50
	AQUA-RAT	79.8	77.74	77.19	<u>78.67</u>	77.21	77.94	74.52	76.01
	MATH	79.09	77.91	76.86	75.39	73.14	75.65	71.05	77.85
(3) Different Answer	GSM8K	70.04	55.67	68.43	47.09	49.93	49.50	51.41	31.71
	AQUA-RAT	<u>57.44</u>	48.92	57.64	45.55	40.45	46.31	44.10	30.76
	MATH	48.95	45.40	51.47	42.66	36.70	38.86	36.41	32.50
(4) 1-correct	GSM8K	78.18	64.08	76.92	52.25	56.19	58.10	59.26	27.78
	AQUA-RAT	66.43	57.10	69.22	44.44	47.13	54.43	52.93	24.05
	MATH	71.92	70.19	79.62	41.80	57.79	60.97	60.73	22.62

Table 1: Performance of judge LLMs (1) on all samples, (2) on samples where A and B agree, (3) on samples where A and B disagree and (4) on samples where exactly one given answer is correct. Results are averaged over all candidate model pairs (A, B). The highest accuracy is **bold** and the second highest <u>underlined</u>.

different subsets of the datasets.

Setup. We analyze multiple cases, each corresponding to a specific subset of the data. Case(1) investigates the observed judgment performance $S_{A,B}^{J}$ on the full dataset and Case(2) analyzes the subset where both models give the same answer (A=B). Case(3) shows the performance where both models give a different answer $(A \neq B)$ and Case(4) describes the performance on the subset where exactly one answer is correct. The results are shown in Table 1. Further, we show the class confusion matrix for the four best-performing judges in Figure 2.

Results. In general, we observe in Table 1 that larger models outperform smaller models, with LLama 3.1 70B performing the best. Interestingly, Qwen 2.5 14B outperforms Qwen 2.5 72B. As shown in Figure 2, the LLM judges have a performance of larger than 95% if both answers are correct. Conversely, the most challenging situation is when both answers are incorrect. It seems that the difficulty of a problem also transcends the difficulty of making a judgment. This is not necessarily intuitive. For instance, humans may find it easier to detect individual wrong reasoning steps and identify wrong answers, respectively.

In cases where one answer is correct and one answer is incorrect, we observe a moderate performance of the judges, reaching up to 80% accuracy (see Case (4) in Table 1 and Figure 2).

In Case (3) where both answers disagree, we observe moderate performance for large models of up to 70%. Here, the smallest model Gemma 2 2B, has a low performance of around 35%. In what follows, we mostly focus on the analysis of the four largest LLMs as judges.



Figure 2: Class confusion matrices per model. We observe that it is difficult for judges to detect that both answers are incorrect.

4.2 Performance per model combination

Each model has unique strengths and weaknesses and often answers different questions correctly. In this section, we analyze the judgment performance per model pair to gain a better understanding of the impact of candidate model combinations on judgment performance.

Setup. Figure 3 illustrates the judgement performance $S_{A,B}^J$ across model pairs (A,B), indicating the probability of a correct judgement. The results are averaged over datasets and presented as an upper triangular matrix due to symmetry (we always present the answers in both possible orders and average performance). We report the performance of all models used as judges in the Appendix B in Table 9.

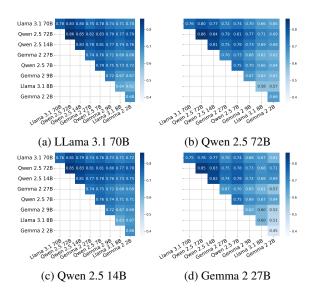


Figure 3: Judgment Performance $S_{A,B}^{J}$ of LLM judges on model pairs, averaged across datasets.

Results. The highest performance is achieved when two answers of Qwen 2.5 72B are compared which is the highest performing model (see task performance in Appendix B.1) In general, we observe that it is easier for the judge to make a correct judgment if candidate models are of higher quality. This seems intuitive because such models likely structure and present their reasoning well, allowing a judge to compare solutions more easily. Figure 2 gives an additional explanation. It shows that judges very reliably detect whether both answers are correct. When both models are capable, it is more likely that both give a correct answer, which makes it easier for LLM judges to classify correctly.

Interestingly, the judgment performances of Qwen 2.5 72B, Qwen 2.5 14B, and LLama 3.1 70B are very similar across pairs. The former possibly agree on a lot because of the similarity of training data and knowledge distillation (Hinton et al., 2014). The largest performance difference is that LLama 3.1 70B performs 10% better when Qwen 2.5 72B and Gemma 2 2B are compared.

These results show that there is a relationship between the task performance of a candidate model and judgment performance. The following section will provide further analysis.

5 Population-level Analysis: Judgements and Model Quality

In this section, we investigate the relationship between LLM judgments and candidate LLM quality.

	Llama 3.1 70B	Qwen 2.5 72B	Qwen 2.5 14B	Gemma 2 27B
R^2	0.89	0.87	0.85	0.93
(p-value)	(0.0)	(0.0)	(0.0)	(0.0)

Table 2: \mathbb{R}^2 values for the regression models *per judge* (first row) and corresponding p-values of the Overall F-Test (second row). All \mathbb{R}^2 values are statistically significant on the 5% level.

First, we provide a statistical analysis where we use LLM task performance to explain the variance in LLM judgment performance. Further, we focus on the subsets where the candidate models make exactly one correct and one incorrect prediction. We observe a strong statistical relationship between the difference in candidate task performances and judgment performances.

5.1 Can we explain Judgement Performance using Task Performance?

A good indicator of the competence of a model on a specific dataset is its task performance. Clearly, there is a relationship between the quality of the involved models and the made judgments. We investigate the relationship between task performances (of candidate and judge models) and judgment performance.

Setup. We fit multiple different linear regression models using the judgment performances as the target variables Y, including all variations of judges, model pairs (A, B), and datasets D. Regarding the covariates \mathbf{X} in the model, we solely use the task performances $S_X, X \in \{J, A, B\}$ of judge and candidate models, to predict judgment performance. Since we are not specifically interested in the individual features' effects, but rather in their ability to explain the variation of judgment performance, we rely on the coefficient of determination, R^2 , for evaluation (Fahrmeir et al., 2013, see Appendix D).

Results. The results are shown in Table 2 (excluding data sets from the probability formulas for simplicity). We observe that the performance-related features of the models can explain the variation in the judgment performance (all R^2 values greater or equal than 0.85), very well. Logically, S_A and S_B , have significant³ explanatory power for judgment performance, as they encompass all correct answers.

³We test statistical significance using an Overall-F-Test for each fitted model. Further details are in Appendix D.

5.2 Are LLM judges biased towards LLMs of higher quality?

To get a better understanding of whether there is a bias of LLM judges towards LLMs of higher quality, we investigate the subset where one candidate answer is correct and the other candidate answer is incorrect. This subset is of the highest practical relevance. The goal is to investigate the relationship between the task performances of the candidate models and the judge's performance.

Setup. For all model pairs (A, B), $A \neq B$ we analyze subsets where A's solutions are correct, and B's solutions are incorrect, and call it 1-correct. Note that we can always order A and B this way. Each plot in Figure 4 shows the relationship between judge performance on the 1-correct subset (Y-axis) and candidate model performance gap of A and B, i.e., $S_A - S_B$ (X-axis). The color of the points indicate the size of the particular subset of samples. Examples of these subsets and their corresponding performances are in Appendix C.1.

Results. The analysis reveals a strong correlation (Pearson's $r^2 > 0.78$) between judgement performance and candidate model performance gap. For the rest of this section, we call the model of higher performance on a dataset the *more competent model*. I.e., if the performance gap is larger than 0, the model giving the correct answer (A) is the more competent model. If the correct model is the more competent model, the judgment performance on the subset is higher, e.g., for LLama 3.1 70B, sometimes approaching 100%. If the performance gap is more positive, it is easier to choose the correct answer. On the other hand, if the less competent model gives the correct answer, judgment performance is low, often lower than 20%.

We infer that LLM judges are biased towards models of higher task performance. This finding aligns with previous research identifying self-bias (Xu et al., 2024; Panickssery et al., 2024; Liu et al., 2024), as judge LLMs are typically of higher quality than the judged models. We hypothesize that this bias arises because more competent models articulate their responses more convincingly and exhibit a specific writing style, thereby misleading the judges.

However, models of higher task performance typically answer correctly more often (as indicated by the color of the points in Figure 4.

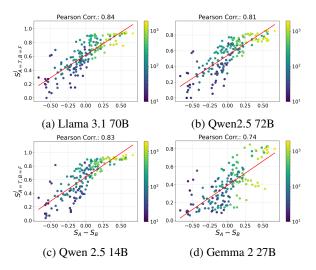


Figure 4: Judges' accuracy vs. performance gap between two candidate models A and B. Each point represents a subset where A is correct, and B is incorrect. The color reflects the size of these subsets.

6 Sample-level analysis: Judgments and Stylistic Patterns

In Section 5, we found that the quality of a candidate LLM (as indicated by the task performance) correlates with the made judgment. We hypothesize that models of higher quality exhibit a particular style of expressing themselves and judges partially base their judgment on the incorporated textual cues. Motivated by recent work in machinegenerated text detection which finds that LLMs often exhibit certain styles (Wu and Aji, 2025) or patterns (Shaib et al., 2024), we aim to gain a deeper understanding of whether shallow or even content-independent patterns affect the final judgment.

Setup. We separate all judgments each judge made into training and test splits and train two classifiers. The test accuracy is reported in Table 3. We use two types of features. First, we use TF-IDF embeddings. Secondly, we use N-Grams of part-of-speech (POS) tags, motivated by Shaib et al. (2024) who show and investigate the distinct occurrence of such in LLM-generated text. Given two candidate answers, we create two independent feature sets and concatenate those. Then a logistic regression and a RandomForest classifier (Breiman, 2001) are trained on these concatenated features. Find more information in Appendix E.

Results. We observe that the models achieve a performance between approximately 70% and 75%. This indicates that structural information (POS

Features Model		Llama 3.1 70B	Qwen 2.5 72B	Qwen 2.5 14B	Gemma 2 27B
POS	LR	72.79	69.66	72.33	70.19
	RF	71.71	69.77	71.89	69.18
TF-IDF	LR	75.75	73.65	75.12	72.27
	RF	75.65	71.05	75.79	70.58

Table 3: Accuracy of predicting LLM judges' decisions using Logistic Regression (LR) and Random Forest (RF) classifiers based on N-Grams of either POS tags or TF-IDF features.

tags) and word choice (TF-IDF) are important factors in understanding the patterns behind the behavior of LLM judges. The ground truth judgment distribution is shown in Appendix E.

Nevertheless, these results suggest that decision-making is a multi-faceted process. While specific shallow cues hold influence, a substantial portion of the decision-making process (25%-30%) can not be predicted this way and is based on other contextual factors which could include reasoning or noise.

7 Usage recommendations

Lastly, we aim to give some usage recommendations. We start by analyzing two applied questions, namely, whether LLM judges can identify models of higher task performance and whether LLMs should be used to improve task performance. In the end, we discuss those results, connecting them to the overall insights of this paper.

7.1 Do judges identify better models?

An essential application of LLM judges is whether they can accurately identify which model performs better for a given task. This is crucial if we want to rank LLMs by their capabilities or if a practitioner wants to decide which model to deploy.

Setup. We evaluate which model a judge perceives as better by measuring the frequency of how often a judge selects the answer of a specific model. Formally, let (A, B) be a candidate model pair where we assume that A has higher task performance, i.e. $S_A > S_B$. If the judge chooses A more often, we say a judge correctly determines A to be better than B. For this analysis, we determine the proportion of model pairs (A, B) for which the judge chooses A over B for all pairs $(A, B), S_A > S_B$ as shown in Figure 5.

Results. We observe that all tested large models consistently select the more competent model, i.e., the model with higher task performance. Also, the

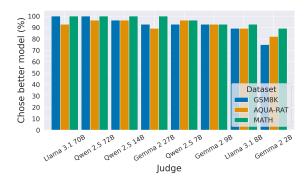


Figure 5: Percentage of model pairs (A, B) where a judge picks a better model A (meaning $S_A > S_B$), by selecting more answers of A than from B.

small models with 7-9B parameters choose the correct model in over 90% of the cases. In general, it seems to be the hardest on the AQUA-RAT dataset. This is also the hardest dataset in Case (4), in Table 1, where exactly one answer is correct. Note that the bias found in Section 5.2 is not necessarily problematic for this specific use case, because a bias towards the more competent model supports a correct outcome of this experiment.

7.2 Do judges elicit task improvement?

Another interesting question of practical relevance is whether it makes sense to use LLM judges to improve task performance. One use case is the application of LLM judges in agentic systems where LLM judges might serve as a dedicated unit in a system. Another use case is the subsequent usage of the answers chosen by the judge for self-training (Yuan et al., 2024).

Setup. We separate the analysis into two questions. In Case (1), we evaluate whether the answers chosen by the judge result in a better performance than the individual models. Formally, for all pairs of models (A, B), we plot the difference of performance of chosen answers, $C_{A,B}^J$ and maximal single candidate model performance $\max\{S_A, S_B\}$ in blue in a bar chart in Figure 6. Secondly, in Case (2), we test whether it makes more sense to use the judge model to generate a candidate answer J and then take the majority vote across all three answers. Therefore we plot the performance difference of $C_{A,B}^J - \text{MV}(A,B,J)$ in orange in a bar chart, where MV(A,B,J) is the performance of the majority vote across all three answers.

Results. In general, we observe that the performance differences are almost following a normal

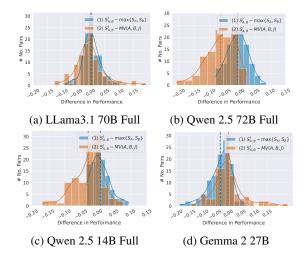


Figure 6: The Y-axis describes the number of model pairs A,B where the answers chosen by the judge achieve a higher task performance than the performances of the individual models (blue) or than the majority vote (MV) of answers of A,B and J as candidate answer generator (red). The X-axis describes the performance difference. A value of, e.g., x=0.05 means the answers chosen by the judge result in a 5% (absolute) performance increase.

distribution. In Case (1), the distribution has a mean value (dashed line) slightly larger than 0 for LLama 3.1 70B (0.3) and Qwen 2.5 14B (0.9). That means that, on average, the answers chosen by the judge result in slightly increased performance, e.g., an increase from 40% accuracy to 40.9% accuracy. In Case (2), the mean value is never larger than 0, meaning that the majority vote is more likely to be better than the answer chosen by the judge. Especially for Qwen 2.5 14B and Qwen 2 72B, it is more viable to use the majority voting strategy.

7.3 Discussion

Our analysis of LLM judges on mathematical reasoning tasks reveals several insights for practitioners, which we discuss in the following. We separate our discussion into the sample level, i.e., the interpretation of a single prediction, and aggregate level, i.e., the interpretation of a set of predictions.

Sample level. In Table 1, we find that LLM judges often achieve a strong judgment performance ($S_{A,B}^{J} > 80\%$ accuracy) across tasks. While this is a solid classification performance, it means that the prediction is wrong in 20% of the cases which limits practical applicability. In Section 4, we observe that LLM judges demonstrate high precision when identifying correct answers from both models. This might be valuable for filtering sam-

ples and curating training data or, e.g., self-training. Nevertheless, one has to be careful how to use these because correctly judged samples are biased towards simple samples. In summary, we do not recommend fully relying on individual LLM judgments, especially not in high-stakes domains such as legal or health care.

Aggregate level. As shown in Figure 5, we find that LLM judges are consistently able to select or rank models by their task performance. This is supported by Section 5.1 where we show that a simple linear model can explain a high share of the variance in judgment performance, given individual task performances, suggesting that the performance difference of two candidate models is linearly linked to the judgment outcome.

In summary, our results suggest that LLM judges are more effective and consistent at aggregate-level comparisons than instance-level judgments, for example when ranking or selecting which LLM is better for a particular task when no ground truth data is available.

8 Conclusion

We conduct a thorough analysis of LLM judges on mathematical reasoning tasks. We evaluate the judgment performance of eight models of different sizes on three datasets. We find that larger judge models generally outperform smaller judge models and that judges can reliably detect whether both answers are correct. Our analysis reveals a strong correlation between judgments and task performance, indicating that judges tend to choose models of higher quality even if their answers are incorrect. We hypothesize that LLM judges partially base their decisions on linguistic cues in contrast to the reasoning within the answers. We support this hypothesis with our experiments showing that 70% of the judges' decisions can be predicted using simple linguistic features such as N-grams of partof-speech tags. Lastly, our analysis finds that LLM judges reliably detect LLMs of higher task performance but are not reliably useable to improve task performance. Our results show that LLM judges contain biases and suggest that practitioners should not blindly trust LLM judges. We advise practitioners to carefully decide whether LLM judges should be used in their particular application.

With this work, we set the stage for further research to investigate how to understand, use, and improve LLM judges.

9 Limitations

Our analysis is primarily focused on mathematical reasoning datasets, which allows us to explore judgments through the lens of verifiability, i.e., problems that have a definitely correct answer. While this approach provides valuable insights, it limits the generalizability of our findings to other tasks or domains. Nevertheless, we want to emphasize the importance of the class of verifiable tasks. For instance, there is currently a focus on training so-called large reasoning models, which demonstrate significant progress in solving complex problems such as coding or maths. It is a possibility that an increased capability of LLMs on verifiable tasks fuels scientific progress.

In our experiments, we focus on testing a single, specific prompt. It is common knowledge that LLMs are highly sensitive to variations in prompt phrasing, which can substantially influence their performance. However, the resources available to us do not allow us to meet the computational demands necessary to run our experiments with multiple prompts. Further, our impression is that it is a custom approach to conduct LLM studies using single prompts, as they are typically indicative of behavior. Therefore we decided to run our analysis on full datasets with a single prompt instead of using subsets of datasets with variations of the prompt with mostly the same content.

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A Experimental Setup

We provide further details on the general setup described in Section 3. Specifically, we include statistics and examples of the datasets, additional information on the models used, and the exact prompts employed in this study.

A.1 Datasets

Additional information about the datasets is given in Table 4, which presents an overview of the dataset statistics. Note that for the MATH dataset, we only include the most challenging questions, called levels 4 and 5, in the dataset. Notably, it has ground truth answer sequences that are, on average, almost three times longer than those in other datasets.

	# questions	Avg. # question characters	Avg. # answer characters
AQUA-RAT	254	239.1	203.1
MATH	1516	216.5	643.9
GSM8K	1319	239.9	292.9

Table 4: An overview of dataset size and text length.

User

You are a reasoning assistant. Always answer exactly in the same format. Use '####' to separate the final answer (without additional comments) from the reasoning.

« Few-Shot Question 1 »

Assistant

« Few-Shot Answer 1 »

...

User

« Few-Shot Question 4 »

Assistant

« Few-Shot Answer 4 »

User

« Sample Question »

Figure 7: The prompt to solve tasks. Few-shots and actual questions are filled in within "«" and "»" symbols.

In Table 5, we provide examples of questions and their corresponding answers from the ground truth. Note that these examples were used for few-shot prompting.

A.2 Models

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We execute all models using the VLLM software for LLM serving (Kwon et al., 2023). The weights for all models are accessible through Huggingface Transformers (Wolf et al., 2020). Table 6 includes hyperlinks to each model for easy reference.

A.3 Prompts

We used two different prompts within this project. In general, we designed the prompts to be minimial, by assigning a minimal personality, a quick task description, and description of the output format.

```
Question:
« question »

Answer A:
« answer A »

Answer B:
« answer B »

Compare both answers in detail and decide whether both answers are correct, both answers are incorrect or whether answer 1 or answer 2 is correct.

Conclude with a JSON in Markdown format indicating your choice between "answer_1", "answer_2", "both_correct" or "both_incorrect": "'json {
   "answer": "..."
}
```

Figure 8: Judge Prompt. Candidate answers are filled in within "«" and "»" symbols.

The prompt shown in Figure 7 is used for the candidate solution generation for all datasets. Examples of the few-shots are in Table 5. The prompt for the judges is given in Figure 8. Note that we run experiments for both orders of the answers of the models A and B.

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A.4 Infrastructure

The experiments were run on NVIDIA A100 and NVIDIA H100. The judgments used in Section 4 took around 3 day equivalents on 4 A100 40GB. Using 2 H100 90GB and 4 A100 40 GB it took less than 2 days.

B General Performance

This section provides additional information related to Section 4. Specifically, we present the task performance of all models across all datasets, as well as the judging performance of all models when used as judges.

B.1 Task Performance

In various contexts in this work, the task performance of the individual models is essential. Therefore, we provide the accuracy of all models and all datasets in Table 7.

	Question	Answer
AQUA-RAT	Two friends plan to walk along a 43-km trail, starting at opposite ends of the trail at the same time. If Friend P's rate is 15% faster than Friend Q's, how many kilometers will Friend P have walked when they pass each other? Options: A)21 B)21.5 C)22 D)22.5 E)23	If Q complete x kilometers, then P completes $1.15x$ kilometers. $x + 1.15x = 43$ $2.15x=43$ $x = 43/2.15 = 20$ Then P will have have walked $1.15*20=23$ km. The answer is E. #### E
GSM8K	Natalia sold clips to 48 of her friends in April, and then she sold half as many clips in May. How many clips did Natalia sell altogether in April and May?	Natalia sold 48/2 = «48/2=24»24 clips in May. Natalia sold 48+24 = «48+24=72»72 clips altogether in April and May. #### 72
MATH	Mr. Madoff invests 1000 dollars in a fund that compounds annually at a constant interest rate. After three years, his investment has grown to 1225 dollars. What is the annual interest rate, as a percentage? (Round your answer to the nearest integer.)	Let r be the annual interest rate. Then after three years, Mr. Madoff's investment is $1000 \cdot \left(1 + \frac{r}{100}\right)^3$, so $1000 \cdot \left(1 + \frac{r}{100}\right)^3 = 1225$. Then $\left(1 + \frac{r}{100}\right)^3 = 1.225$, so $\left[1 + \frac{r}{100} = \sqrt[3]{1.225} = 1.069987\ldots$, which means $r = \boxed{7}$, to the nearest integer. #### 7.0

Table 5: Example of ground truth answers used for few-shot prompting.

Model	URL
Llama 3.1 70B	https://huggingface.co/meta-llama/Llama-3.1-70B-Instruct
Qwen 2.5 72B	https://huggingface.co/Qwen/Qwen2.5-72B-Instruct
Qwen 2.5 14B	https://huggingface.co/Qwen/Qwen2.5-14B-Instruct
Gemma 2 27B	https://huggingface.co/google/gemma-2-27b-it
Qwen 2.5 7B	https://huggingface.co/Qwen/Qwen2.5-7B-Instruct
Gemma 2 9B	https://huggingface.co/google/gemma-1.1-9b-it
Llama 3.1 8B	https://huggingface.co/meta-llama/Llama-3.1-8B-Instruct
Gemma 2 2B	https://huggingface.co/google/gemma-1.1-2b-it

Table 6: Used models and corresponding hyperlinks.

	GSM8K	AQUA-RAT	MATH
Llama 3.1 70B	93.25	78.57	47.11
Qwen 2.5 72B	95.07	83.73	73.86
Qwen 2.5 14B	93.48	<u>82.54</u>	<u>64.47</u>
Gemma 2 27B	85.97	67.46	38.80
Qwen 2.5 7B	88.10	75.40	60.31
Gemma 2 9B	80.52	61.51	31.31
Llama 3.1 8B	72.40	61.51	20.69
Gemma 2 2B	37.53	26.98	7.15

Table 7: Task performance of the investigated models.

B.2 Judging performance per model pair

We conduct experiments with all eight models serving as judges. We present the performance metrics of all judges across all model pairs in Figure 9.

C Examples

C.1 Example Subset Performance

To better understand the correlation observed in Figure 4, we provide examples of these subsets, which can be seen in Table 8. These examples include the following details: the judge, the compared models, the dataset, the performance of the correct model on the dataset (denoted by S_A), the performance of the incorrect model on the dataset S_B , the judgment performance on the subset (denoted by $S_{A,B}^J$), and the size of the subset. We provide the three subsets with the highest performance, the three subsets with the lowest performance, and three random subsets where Llama 3.1 70B is the judge.

D Statistical Methodology

We describe the statistical background for the tests applied in Section 6. All predictions and statistical tests in Section 6 were performed using the statsmodels library (Seabold and Perktold, 2010).

Judge	model A	model B	dataset	S_A	S_B	$S_{A,B}^{J}$	No. Samples
Llama 3.1 70B	Qwen 2.5 14B	Gemma 2 2B	MATH	64.50	7.10	94.60	1655
Llama 3.1 70B	Qwen 2.5 72B	Gemma 2 2B	AQUA-RAT	83.70	27.00	94.50	309
Llama 3.1 70B	Qwen 2.5 7B	Gemma 2 2B	MATH	60.30	7.10	94.00	1520
Llama 3.1 70B	Qwen 2.5 72B	Qwen 2.5 7B	AQUA-RAT	83.70	75.40	62.50	64
Llama 3.1 70B	Qwen 2.5 7B	Qwen 2.5 14B	AQUA-RAT	75.40	82.50	42.30	26
Llama 3.1 70B	Qwen 2.5 72B	Qwen 2.5 72B	MATH	73.90	73.90	49.50	206
Llama 3.1 70B	Gemma 2 27B	Qwen 2.5 14B	AQUA-RAT	67.50	82.50	15.00	20
Llama 3.1 70B	Gemma 2 2B	Qwen 2.5 14B	GSM8K	37.50	93.50	15.00	20
Llama 3.1 70B	Gemma 2 2B	Llama 3.1 70B	MATH	7.10	47.10	14.50	62

Table 8: Examples of judgement performances on subsets where model A is correct and model B is incorrect.

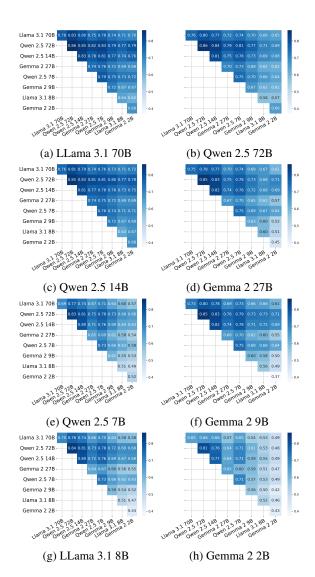


Figure 9: Performance $S_{A,B}^{J}$ of LLM judges on model pairs, averaged across datasets.

D.1 Coefficient of Determination

The coefficient of determination, R^2 , for evaluation of linear regression models (Fahrmeir et al., 2013) is defined as follows:

$$R^{2} = \frac{\sum_{i=1}^{n} (\hat{y}_{i} - \bar{y})^{2}}{\sum_{i=1}^{n} (y_{i} - \bar{y})^{2}}$$

 R^2 measures the share of the variance in Y explained by its covariation with the features \mathbf{X} included in the model by dividing the variation of the *predicted* values \hat{y}_i by the variation of the true target values y_i . If the features \mathbf{X} have high explanatory power for Y, the \hat{y}_i will be close to the y_i and R^2 will be close to 1, while in the extreme case of no correlation between \mathbf{X} and Y the arithmetic mean is the best estimate (i.e., $\hat{y}_i = \bar{y} \ \forall \ i = 1, \ldots, n$) resulting in $R^2 = 0$.

D.2 Overall-F-Test

The Overall-F-Test is built upon \mathbb{R}^2 and tests whether the overall model is of any significant value for explaining the variation of the target variable. The F-distributed test statistic is calculated as

$$\frac{R^2}{1-R^2} \cdot \frac{n-p-1}{p},$$

where \mathbb{R}^2 is the coefficient of determination, n is the number of observations, and p is the number of covariates included in the model (i.e., the number of estimated coefficients excluding the intercept). The hypotheses that can be tested this way are

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_n = 0$$

vs.

 $H_1: \beta_j \neq 0$ for at least one $j \in \{1, \dots, p\}$.

Model	Both correct	A correct	B correct	Both incorrect
Llama 3.1 70B	51.8	18.1	21.7	8.4
Qwen 2.5 72B	54.9	19.6	19.8	5.6
Qwen 2.5 14B	50.2	20.0	23.0	6.9
Gemma 2 27B	52.6	15.9	15.4	16.0

Table 9: Percentage of predictions individual models made.

So from a rejection of H_0 , it can be concluded that at least one of the included features exhibits explanatory power for the variation of the target variable.

D.3 Multiple Testing

Since we conduct multiple statistical tests within the scope of one research project, it is important to consider multiple testing as a potential problem resulting in false positive findings. The p-values from our tests, however, also satisfy a significance level resulting from a Bonferroni Correction of the typical significance level of 5%.

E Sample-level Analysis

We utilize Scikit-learn (Pedregosa et al., 2011) library to train and evaluate the Logistic Regression and Random Forest Model. We use the standard settings for the Logistic Regression model. We use the Random Forests model with 1500 estimators, and standard settings apart from that.

During preprocessing, we use simple word splittling by spaces. We employ the english stop word removal integreated into Scikit-learn. We set the maximum number of features to 5,000, for the N-Gram of part-of-speech tags, we set the N-gram range from 5-grams up to 13-grams, following settings of Shaib et al. (2024). For training, we use the Scikit-learn (Pedregosa et al., 2011) library. The running time was negligible.

In Table 9