Enhancing Large Language Model Reasoning via Latent Space Modeling for Pre-thinking and Pre-answering

Anonymous ACL submission

Abstract

Chain-of-Thought (CoT) has proven effective for eliciting reasoning abilities in large language models, but standard autoregressive decoders treat reasoning and answer tokens uniformly as next-token prediction targets, despite their fundamentally different roles. This homogeneous modeling introduces conflicting inductive biases and significant inference overhead. Latent CoT methods attempt to improve efficiency by removing explicit reasoning tokens, yet often degrade performance, indicating that explicit reasoning supervision remains critical for preserving pretrained reasoning capabilities. To address these challenges, we propose **Pre**^{2}, a phase-aware reasoning framework that decouples thinking and answering while retaining explicit CoT supervision. Our approach introduces independent pre-thinking and pre-answering latent states and integrates them with explicit token representations through a hybrid latent–token decoding paradigm. This design increases computation along both width and depth dimensions with minimal parameter overhead, enabling stronger reasoning without sacrificing inference efficiency. Experiments on one in-domain dataset and four out-of-domain benchmarks demonstrate consistent improvements over strong baselines and robust generalization. Code will be released at `github.com`

1 Introduction

Chain-of-Thought (CoT) prompting, introduced by Wei et al. (2022), has proven effective in enhancing the reasoning abilities of large language models by inducing intermediate reasoning steps, yielding strong gains on mathematical and multi-step reasoning tasks (Guo et al., 2025; OpenAI, 2024; Muennighoff et al., 2025). However, these prompt-based methods rely on explicitly generating reasoning tokens at inference time (Wang et al., 2022; Zhou et al., 2022; Kojima et al., 2022; Zhang et al., 2022). Due to the autoregressive nature of language models, early reasoning errors can propagate through subsequent predictions, often resulting in brittle and unstable reasoning trajectories.

To mitigate this limitation, a line of work has explored test-time scaling strategies that aggregate or refine reasoning paths through increased computation at inference. Representative approaches include self-consistency (Wang et al., 2022), which samples multiple reasoning trajectories and selects the most consistent answer, as well as more structured methods such as Tree-of-Thoughts (ToT, Yao et al. 2023), which explicitly searches over reasoning branches. While effective, these methods significantly increase inference cost, as they require extensive sampling, branching, or search procedures, making them impractical for latency- or budget-constrained settings (Li et al., 2024; Wang et al., 2025).

More recently, latent CoT approaches have emerged as an alternative direction, aiming to bypass explicit reasoning token generation altogether (Wei et al., 2025; He et al., 2025; Liu and Zhang, 2025). Instead of producing natural language reasoning steps, these methods perform reasoning directly in a continuous latent space, thereby reducing inference-time token generation. Representative works such as Coconut (Hao et al., 2024), CoDi (Shen et al., 2025), and SoftCoT (Xu et al., 2025) demonstrate that latent representations can capture certain aspects of reasoning while improving efficiency. Nevertheless, by skipping or discarding explicit reasoning tokens, these approaches often struggle to fully preserve the reasoning capabilities of pretrained models, highlighting a fundamental trade-off between efficiency and reasoning fidelity (Xia et al., 2025; Liu et al., 2025).

Traditional methods uniformly model reasoning and answer tokens, ignoring their fundamentally different patterns (Nye et al., 2021; Lampinen et al., 2022). While latent reasoning approaches eliminate CoT for efficiency, we argue that CoT provides essential supervision signals aligned with pretrain-

ing. Fully eliminating CoT may weaken reasoning ability (Turpin et al., 2023). To address these limitations, we explicitly decouple the modeling of reasoning and answering by introducing independent pre-thinking and pre-answering phases, and our approach preserves explicit reasoning supervision while introducing structured latent phases, balancing inference efficiency and reasoning expressivity. Specifically, instead of forcing the model to autoregressively commit to explicit reasoning tokens, we supervise continuous latent representations that summarize the reasoning-ready and answer-ready states. This separation enables the model to first internalize a structured reasoning abstraction before transitioning into answer generation, thereby strengthening its reasoning capability while reducing reliance on verbose or brittle CoT generation.

We introduce **Pre**^{2}, a novel framework that effectively addresses all of the above-mentioned problems simultaneously. **We model Chain-of-Thought reasoning as a structured latent trajectory composed of a reasoning-ready state and an answer-ready state. Instead of autoregressively generating reasoning tokens, we supervise continuous latent representations aligned with pooled reasoning and answer embeddings, enabling non-autoregressive and controllable reasoning abstraction**. Our contributions are summarized as follows: (1) We propose a phase-aware reasoning framework that independently models the pre-thinking and pre-answering processes, effectively decoupling the conflicting patterns between thinking and answering stages. (2) We introduce a hybrid paradigm that combines latent representations with explicit token-level supervision, requiring only a minimal increase in model parameters while preserving strong performance and high inference efficiency. (3) We conduct extensive experiments on one in-domain dataset and four out-of-domain benchmarks, where our method consistently outperforms strong baselines, demonstrating robust generalization across domains.

2 Related Works

Explicit Chain-of-Thought Reasoning. Explicit Chain-of-Thought (CoT) prompting has become a cornerstone for eliciting reasoning capabilities in large language models (LLMs). Pioneering works demonstrate that explicitly generating intermediate reasoning steps significantly boosts performance on arithmetic and symbolic tasks (Wei et al., 2022; Kojima et al., 2022). Despite the successes, explicit CoT suffers from inherent inefficiency due to the autoregressive generation of long reasoning sequences. Furthermore, treating reasoning tokens and answer tokens as homogeneous prediction targets introduces structural conflicts; reasoning trajectories often require exploratory and verbose patterns, whereas answers demand conciseness (Nye et al., 2021; Lampinen et al., 2022).

Latent Chain-of-Thought Reasoning. To mitigate the inference overhead of explicit tokens, Latent CoT approaches aim to internalize reasoning into continuous hidden states or implicit processes, enabling models to "think before speaking" (Zelikman et al., 2024). Methods like implicit reasoning via stepwise knowledge internalization (Deng et al., 2024) and teacher-student distillation (e.g., CODI, Shen et al. 2025) attempt to transfer reasoning semantics from explicit traces to model embeddings. Other approaches, such as Coconut (Hao et al., 2024) and CCoT (Cheng and Van Durme, 2024), explore reasoning in continuous space by compressing steps into dense vectors or "contemplation tokens". However, purely latent methods face significant challenges: they often suffer from performance degradation on complex tasks due to the lack of fine-grained supervision signals (Hao et al., 2024). Unlike explicit CoT, where every step is verified, methods like Coconut typically rely on answer-level supervision, while distillation methods like CODI focus on trajectory-level alignment, potentially losing critical step-wise logic.

Computational Scaling across Width and Depth Dimensions. Existing computational scaling methods face limitations: width-based approaches (e.g., Pause Tokens, Goyal et al. 2023) often lack explicit semantic guidance, while depth-based methods (e.g., MoR, Bae et al. 2025 and Ouro Zhu et al. 2025) keep reasoning implicit within recursive loops. We bridge these dimensions by scaling width via learnable latent tokens and depth through independent modules.

Our approach leverages a hybrid framework combining explicit and latent Chain-of-Thought reasoning. Unlike pause tokens that operate exclusively with static token-level embeddings, our framework employs learnable sentence-level embeddings to capture sequence representations of complete reasoning steps and answer tokens.

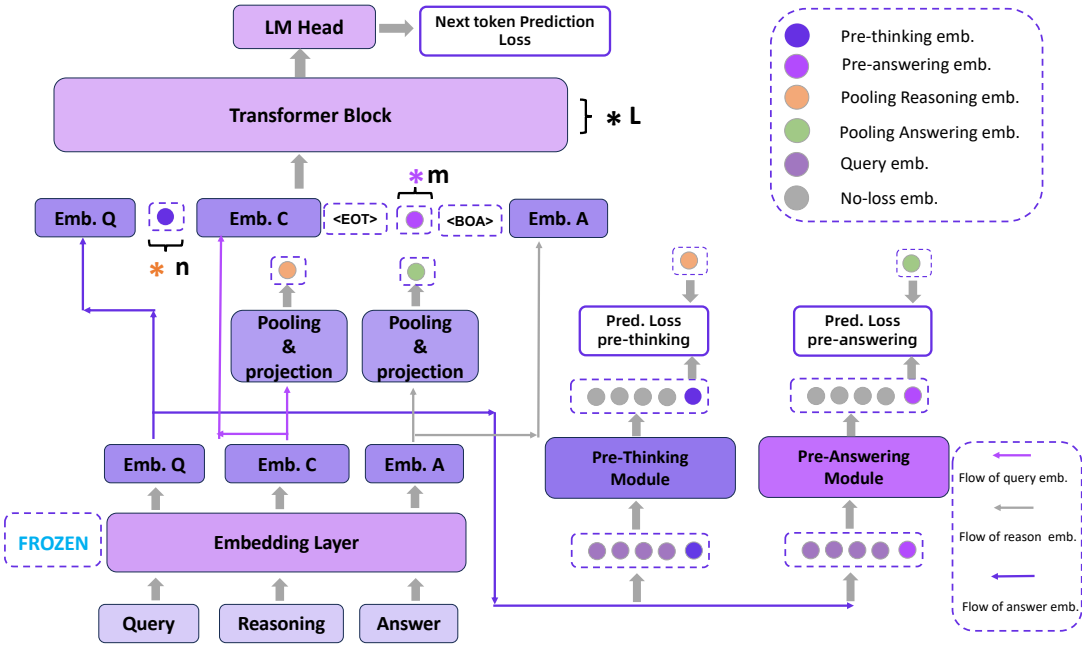

Figure 1: Overview of the proposed framework.

3 Methodology

3.1 Preliminary of Pre^2

Let $\mathcal{V}$ denote a vocabulary of size $V = |\mathcal{V}|$. Given a single training sample, we define a CoT sequence as

$$\mathbf{X} = (\mathbf{Q} := x^{(q)},\ \mathbf{R} := x^{(r)},\ \mathbf{A} := x^{(a)}),$$

where $x^{(q)} = (q_1, \ldots, q_{L_q}) \in \mathcal{V}^{L_q}$, $x^{(r)} = (r_1, \ldots, r_{L_r}) \in \mathcal{V}^{L_r}$, and $x^{(a)} = (a_1, \ldots, a_{L_a}) \in \mathcal{V}^{L_a}$ denote the query, reasoning steps, and answer, respectively.

Our framework introduces a novel approach where three built-in embeddings serve as initialization for the latent states of pre-thinking and pre-answering. We first introduce three continuous latent vectors that have been initialized within our model:

$$\mathbf{z}^{(0)}_{\text{think}} \in \mathbb{R}^d, \mathbf{z}^{(0)}_{\text{ans}} \in \mathbb{R}^d, \mathbf{z}^{(0)}_{\text{boa}} \in \mathbb{R}^d,$$

where $\mathbf{z}^{(0)}_{\text{think}}$, $\mathbf{z}^{(0)}_{\text{ans}}$ and $\mathbf{z}^{(0)}_{\text{boa}}$ are learnable global parameters. Specifically, $\mathbf{z}^{(0)}_{\text{think}}$ is trained to learn a latent representation of the complete reasoning process; $\mathbf{z}^{(0)}_{\text{ans}}$ captures an abstract representation of the answer; and $\mathbf{z}^{(0)}_{\text{boa}}$ serves as an answer indicator that signals the model to transition into answer generation mode.

Our model uses a frozen token embedding matrix $\mathcal{E}$ to get different non-contextualized embeddings for query, reasoning and answer tokens. This approach offers an additional advantage: the frozen embedding layer provides stable training targets while preserving model capacity, thus avoiding performance degradation. Let $\mathcal{E} : \mathcal{V} \to \mathbb{R}^d$ be a shared token embedding function. The embedded sequences are given by:

$$\begin{aligned}\mathbf{Q} &= (\mathbf{q}_1, \ldots, \mathbf{q}_{L_q}) \in \mathbb{R}^{L_q \times d},\\ \mathbf{R} &= (\mathbf{r}_1, \ldots, \mathbf{r}_{L_r}) \in \mathbb{R}^{L_r \times d},\\ \mathbf{A} &= (\mathbf{a}_1, \ldots, \mathbf{a}_{L_a}) \in \mathbb{R}^{L_a \times d}.\end{aligned}$$

We then define a pooling operator $\mathcal{P}(\cdot)$ to obtain sequence-level representations. And the pooled reasoning and answers representations will be regarded as gold latent representations because we assume that these pooled representations include all necessary information to guide the model to generate complete reasoning tokens and answering tokens.

$$\begin{aligned}\bar{\mathbf{r}} &= \mathcal{P}(\mathbf{R}) \in \mathbb{R}^d,\\ \bar{\mathbf{a}} &= \mathcal{P}(\mathbf{A}) \in \mathbb{R}^d.\end{aligned}$$

3.2 Pre-thinking and Pre-answering Modeling

The modeling of pre-thinking and pre-answering is centered on a key principle: we aim to obtain embeddings that encapsulate holistic information. Specifically: The **pre-thinking embeddings** are expected to capture complete information about the reasoning steps, encoding the entire reasoning

trajectory in a compact representation. The **pre-answering embeddings** should encapsulate the final answer information. With these global representations, the model can provide top-down guidance and predictions throughout the subsequent autoregressive generation of reasoning tokens and answer tokens, rather than relying solely on local, step-by-step predictions. We now address the key question: how are these compressed embeddings learned?

We make a key empirical observation: for a small proportion of complex reasoning tasks, models can produce correct answers directly from the query even when the generated reasoning steps are unreliable or incorrect. This finding suggests that *the query itself encapsulates sufficient information for answer prediction, independent of the reasoning quality*. Motivated by this insight, we design our approach to leverage query tokens as conditions, using the built-in embeddings as latent state initializations and employing two lightweight Transformer decoder blocks to causally model token-level dependencies. Moreover, recent studies have demonstrated that increasing per-token computation depth yields unique benefits that cannot be achieved by simply scaling the width of sequence modeling, particularly for certain reasoning tasks (Bae et al., 2025; Zhu et al., 2025). To enhance computational and inference efficiency, we selectively allocate additional depth-wise computation exclusively to reasoning and answer tokens.

Let f_{think} denote the pre-thinking module. The input sequence is constructed as

$$\mathbf{H}_{\text{think}}^{(0)} = \left[\mathbf{Q}; \mathbf{z}_{\text{think}}^{(0)}\right],$$

where $[\cdot;\cdot]$ is concatenation operator.

The hidden states are obtained by

$$\mathbf{H}_{\text{think}} = f_{\text{think}}(\mathbf{H}_{\text{think}}^{(0)}).$$

We take the final hidden state as the predicted pre-thinking latent representation:

$$\hat{\mathbf{z}}_{\text{think}} = \mathbf{H}_{\text{think}}[-1].$$

Similarly, we define a pre-answering module f_{ans} with input

$$\mathbf{H}_{\text{ans}}^{(0)} = \left[\mathbf{Q}; \mathbf{z}_{\text{ans}}^{(0)}\right].$$

The predicted pre-answering latent is given by

$$\hat{\mathbf{z}}_{\text{ans}} = f_{\text{ans}}(\mathbf{H}_{\text{ans}}^{(0)})[-1].$$

Specifically, the two modules consist of two Transformer blocks and utilize a causal mask for attention computation. This design enables both $\mathbf{z}_{\text{think}}^{(0)}$ and $\mathbf{z}_{\text{ans}}^{(0)}$ to attend to the complete set of query token embeddings, allowing modeling pre-thinking and pre-answering separately. To enable $\mathbf{z}_{\text{think}}^{(0)}$ and $\mathbf{z}_{\text{ans}}^{(0)}$ to learn representations that can predict the overall architecture encompassing the thinking and answering stages, we let $\bar{\mathbf{r}}$ and $\bar{\mathbf{a}}$ be the supervised signal for pre-thinking modeling and pre-answering modeling, and the loss function will be either smooth L1 or L2 loss. To prevent gradient backpropagation and ensure unidirectional learning flow, we apply a stop-gradient operation ($\mathcal{SG}(\cdot)$) on $\bar{\mathbf{r}}$ and $\bar{\mathbf{a}}$. Thus, the pre-thinking alignment loss (e.g., L2 norm) is defined as

$$\mathcal{L}_{\text{think}} = \|\hat{\mathbf{z}}_{\text{think}} - \mathcal{SG}(\bar{\mathbf{r}})\|_2^2 . \tag{1}$$

The pre-answering alignment loss is defined as

$$\mathcal{L}_{\text{ans}} = \|\hat{\mathbf{z}}_{\text{ans}} - \mathcal{SG}(\bar{\mathbf{a}})\|_2^2 . \tag{2}$$

3.3 Training of Pre^2

In the following section, we describe the training procedure. Let $\mathbf{e}_{\text{eot}} \in \mathbb{R}^d$ denote the end-of-thought embedding. The decoder input sequence is constructed as

$$\mathbf{H}_{\text{dec}}^{(0)} = \left[\mathbf{Q}; \hat{\mathbf{z}}_{\text{think}}; \mathbf{R}; \mathbf{e}_{\text{eot}}; \hat{\mathbf{z}}_{\text{ans}}; \mathbf{z}_{\text{boa}}; \mathbf{A}\right] \in \mathbb{R}^{T_{\text{dec}} \times d},$$

where $T_{\text{dec}} = L_q + 1 + L_r + 1 + 1 + 1 + L_a$.

And the $\mathbf{e}_{\text{eot}}$ is a special token that is already in the vocabulary of the pre-trained backbone model. And this $\mathbf{e}_{\text{eot}}$ is essential for our inference pipeline because it can provide a proper signal that tells us the reasoning stage is completed.

The decoder f_{dec} produces hidden states

$$\mathbf{H}_{\text{dec}} = f_{\text{dec}}(\mathbf{H}_{\text{dec}}^{(0)}).$$

The corresponding supervision labels are defined as

$$\mathbf{Y} = \big[\underbrace{\texttt{-100}}_{\mathbf{Q}},\ \mathbf{R},\ \mathbf{e}_{\text{eot}},\ \underbrace{\texttt{-100}}_{\hat{\mathbf{z}}_{\text{ans}}},\ \underbrace{\texttt{-100}}_{\mathbf{z}_{\text{boa}}},\ \mathbf{A}\big],$$

where `-100` indicates masked positions that do not contribute to the loss.

Let $\mathcal{T}_{\text{sup}} \subset \{1, \ldots, T_{\text{dec}}\}$ denote the set of supervised token positions, given by

$$\mathcal{T}_{\text{sup}} = \mathcal{T}_{\text{cot}} \cup \mathcal{T}_{\text{ans}},$$

where

$$\mathcal{T}_{\text{cot}} = \{\, t \mid x_t \in \mathbf{R} \cup \{\mathbf{e}_{\text{eot}}\} \,\}, \mathcal{T}_{\text{ans}} = \{\, t \mid x_t \in \mathbf{A} \,\}.$$

The masked language modeling loss is defined as

$$\begin{aligned} \mathcal{L}_{\text{LM}} &= -\sum_{t \in \mathcal{T}_{\text{sup}}} \log p_\theta(y_t \mid \mathbf{H}_{\text{dec}}^{(<t)}) \\ &= -\sum_{t \in \mathcal{T}_{\text{cot}}} \log p_\theta(r_t \mid \mathbf{H}_{\text{dec}}^{(<t)}) \\ &\quad -\sum_{t \in \mathcal{T}_{\text{ans}}} \log p_\theta(a_t \mid \mathbf{H}_{\text{dec}}^{(<t)}). \end{aligned} \tag{3}$$

The overall training objective is given by

$$\mathcal{L} = (1-\lambda_{\text{think}}-\lambda_{\text{ans}})\mathcal{L}_{\text{LM}} + \lambda_{\text{think}}\mathcal{L}_{\text{think}} + \lambda_{\text{ans}}\mathcal{L}_{\text{ans}}. \tag{4}$$

3.4 Inference of Pre^2

The inference procedure differs from vanilla autoregressive decoding in several ways. Notably, we must explicitly maintain: (1) the current query length, and (2) an End-of-Thought (EoT) signal to determine when CoT generation concludes and answer generation should begin. We provide a detailed pseudocode description in Algorithm 1, where $\mathbf{q}$ is query embeddings, $\hat{\mathbf{z}}_{\text{t}}$ is predicted pre-thinking embedding, $\hat{\mathbf{z}}_{\text{a}}$ is predicted pre-answering embedding, $\mathbf{z}_{\text{boa}}$ is the answering indicator embedding that initialized in the model. Specifically, during the prefilling phase, we execute the pre-thinking $f_{\text{think}}(\cdot)$and pre-answering modules $f_{\text{answer}}(\cdot)$ to generate $\hat{\mathbf{z}}_{\text{t}}$ and $\hat{\mathbf{z}}_{\text{a}}$ before any token generation begins. In the autoregressive generation stages that follow, we dynamically reconstruct the input sequence $\mathbf{X}$ through concatenation operations, ensuring that the model's behavior remains consistent across training and inference.

Algorithm 1 Inference Process of **Pre**^{2}

Require: Token embeddings $\mathbf{x}$ from frozen embedding layer;
Require: Stage-flags: `is_prefilling`, `is_cot_end`, `is_answering`,
Ensure: Updated embeddings $\mathbf{x}$

if `is_prefilling` **then**
  $\hat{\mathbf{z}}_{\text{t}} = f_{\text{think}}([x; \mathbf{z}_{\text{think}}^{(0)}])[-1]$
  $\hat{\mathbf{z}}_{\text{a}} = f_{\text{answer}}([x; \mathbf{z}_{\text{answer}}^{(0)}])[-1]$
  $\mathbf{x} \leftarrow [\mathbf{x}; \hat{\mathbf{z}}_{\text{t}}]$
else
  if `is_cot_end` **then**
    if ¬`is_answering` **then**
      $\mathbf{x} \leftarrow [\mathbf{q}; \hat{\mathbf{z}}_{\text{t}}; \text{CoT}; \langle\text{eot}\rangle; \hat{\mathbf{z}}_{\text{a}}; \mathbf{z}_{\text{boa}}]$
      `is_answering` $\leftarrow$ `True`
    else
      $\mathbf{x} \leftarrow [\mathbf{q}; \hat{\mathbf{z}}_{\text{t}}; \text{CoT}; \langle\text{eot}\rangle; \hat{\mathbf{z}}_{\text{a}}; \mathbf{z}_{\text{boa}}; \text{Ans}]$
    end if
  else
    $\mathbf{x} \leftarrow [\mathbf{q}; \hat{\mathbf{z}}_{\text{t}}; \text{CoT}]$
  end if
end if
return $\mathbf{x}$

4 Experimental Design

4.1 Datasets

We train the **Pre**^{2} framework on **GSM8K-Aug** (Deng et al., 2024), a large-scale corpus of approximately 385,000 samples, which augments the original GSM8K (Cobbe et al., 2021) training set with questions, step-by-step reasoning chains, and final solutions to provide granular supervision for aligning our pre-thinking and pre-answering modules. For evaluation, we assess reasoning robustness and generalization across five benchmarks categorized into in-domain and out-of-domain settings. The in-domain evaluation uses the standard **GSM8K** (Cobbe et al., 2021) test split, while out-of-domain evaluation employs four datasets: **GSM-Hard** (increased arithmetic difficulty through larger integers) (Gao et al., 2022), **MultiArith** (multi-step sequential reasoning) (Roy and Roth, 2015), **SVAMP** (structural and textual variations) (Patel et al., 2021), and **ASDiv** (diverse problem types and linguistic patterns) (Xu et al., 2025). Full dataset details are provided in the Appendix B.

4.2 Baselines

We evaluate **Pre**^{2} against five representative baselines covering different reasoning paradigms: **(1) Zero-shot Pretrained**, which directly evaluates a pretrained model without fine-tuning; **(2) Few-shot In-Context Learning**, where three demonstration examples are provided as prompts without parameter updates; **(3) SFT with Explicit CoT**, which fine-tunes the model to generate the full query–reasoning–answer sequence; **(4) SFT with Frozen Embeddings**, identical to (3) but with the embedding layer fixed to assess the role of input representations; and **(5) Pause Token Baseline** (Goyal et al., 2023), which inserts fixed special tokens around the reasoning sequence following prior "thinking token" approaches, while keeping

Model	Method	Datasets					Overall	
		In-domain	*Out-of-domain*					
		GSM8K	**GSM_H**	**Multi**	**ASDiv**	**SVAMP**	**Total**	**Avg**
Qwen-0.5B	Zero-shot	43.06	12.15	82.78	77.55	46.33	261.87	52.37
	Few-shot	41.60	10.99	81.83	78.13	45.67	258.22	51.64
	Baseline	51.50	12.74	92.89	82.15	53.22	292.50	58.50
	Freeze-Baseline	51.96	12.36	92.61	82.30	**53.89**	293.12	58.62
	Pause Token	51.98	12.45	93.38	82.75	52.67	293.23	58.65
	Pre^2-1	**53.78**	**12.76**	**94.17**	**83.33**	53.56	**297.60**	**59.52**
Qwen-1.5B	Zero-shot	46.90	12.40	90.00	81.31	51.33	281.94	56.38
	Few-shot	46.62	13.16	88.50	81.70	49.67	279.65	55.93
	Baseline	54.28	13.39	94.39	83.46	58.56	304.08	60.82
	Freeze-Baseline	54.01	13.57	94.40	83.49	58.33	303.80	60.76
	Pause Token	54.56	13.57	94.50	83.27	57.78	303.68	60.73
	Pre^2-1	**54.76**	**14.53**	**94.50**	**83.53**	**59.11**	**306.43**	**61.29**

Table 1: Main performance comparison on in-domain (GSM8K) and out-of-domain reasoning benchmarks.

these tokens non-learnable during training.

4.3 Evaluation Metrics

We evaluate model performance using three metrics. **(1) Accuracy (Acc.)** measures answer correctness, defined as the percentage of correctly predicted answers. **(2) Generated Sequence Length** quantifies reasoning compactness by computing the average number of tokens in the generated reasoning chains across all examples, where shorter sequences indicate higher efficiency. **(3) Inference Throughput** evaluates efficiency by measuring the generation speed in tokens per second, computed as the total number of generated tokens divided by the total inference time. Higher throughput indicates better inference efficiency.

4.4 Research Questions

Our experiments aim to answer the following research questions: (1) Compared to traditional decoder-based models, what magnitude of improvement can our architecture achieve on downstream reasoning tasks? (2) More specifically, do pre-thinking modeling and pre-answering modeling enable the model to predict reasoning tokens and answer tokens with higher accuracy? (3) Does scaling up the pre-thinking module and pre-answering module along the depth dimension yield consistent performance gains? (4) Similarly, does scaling up the pre-thinking scope along the width dimension enhance the model's reasoning capabilities? (5) Compared to the baseline model, we introduce two prediction modules and three internally initialized embeddings; relative to the baseline size, we add only a modest number of parameters, and our inference mixes autoregressive decoding with latent-space prediction. Will these extra parameters and the new inference paradigm materially impact inference efficiency?

We show the experimental results for the five questions in five subsections of Sec. 5 respectively. Refer to the appendix A for implementation details.

5 Results and Discussions

5.1 Main Results

Our model consistently outperforms baselines across one in-domain and four out-of-domain benchmarks. Table 1 presents a comprehensive comparison between Qwen2.5-0.5B and Qwen2.5-1.5B models (Qwen et al., 2025) across various reasoning benchmarks. Several key observations emerge from these results: **(1) Consistent superiority of Pre^2-1:** Our proposed **Pre**^{2}-1 method consistently outperforms all baseline approaches across both model scales. For the 0.5B model, **Pre**^{2}-1 achieves an average accuracy of 59.52%, representing a 1.02 percentage point improvement over the standard Baseline (58.50%). Similarly, for the 1.5B model, **Pre**^{2}-1 attains 61.29% average accuracy, surpassing the Freeze-Baseline by 0.53 percentage points. **(2) Strong generalization to out-of-domain tasks:** Notably, **Pre**^{2}-1 demonstrates robust performance not only on the in-domain GSM8K benchmark but also across all four out-of-domain datasets. For instance, on the 0.5B model, **Pre**^{2}-1 improves Multi-Arith from 98.89% to 94.17%, and ASDIV from 82.15% to 83.33%, while maintaining competitive performance on SVAMP (53.56%). **(3) Comparison with Pause Token baseline:** While the Pause To-

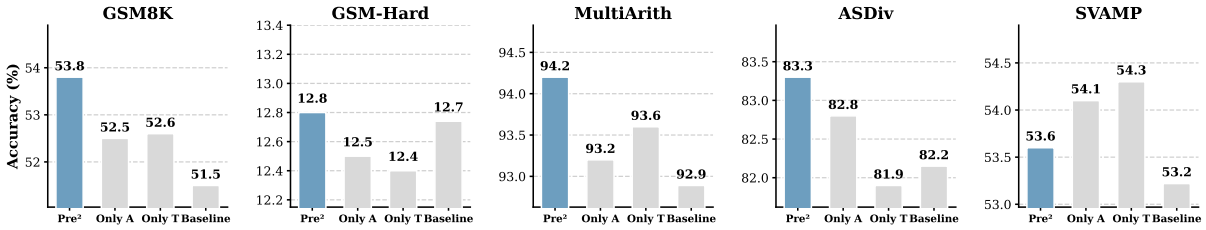

Figure 2: Ablation study on component effectiveness. Avg.acc: Pre^2(59.52), Only A(59.01), Only T(58.97), Baseline(58.50)

ken method shows marginal improvements over the standard Baseline, it is consistently outperformed by our **Pre**^{2}-1 approach. This suggests that our pre-thinking and pre-answering mechanisms provide more effective modeling of the reasoning process than simple pause token insertion. **Although both the pause token method and our approach perform equivalent scaling along the sequence width dimension, we hypothesize that this advantage arises because pause tokens employ static embeddings, whereas our pre-thinking and pre-answering embeddings are learnable and dynamic. Our novel training approach further enables these embeddings to encapsulate rich information about future token sequences, providing stronger guidance for reasoning. (4) Scaling behavior:** The results indicate that our architectural improvements are complementary to model scaling, with benefits observed across both 0.5B and 1.5B parameter scales, demonstrating the generalizability of our approach.

5.2 Impact of Pre-thinking and Pre-answering on Performance

As shown in Figure 2, which presents an ablation study comparing our full model against the baseline and two single-component variants, revealing the individual and synergistic contributions of our architectural components. The full model (**Pre**^{2}) achieves 59.52% average accuracy, outperforming the baseline by +1.02 points. Critically, this exceeds both only_t (pre-thinking only) (58.97%, +0.47) and only_a (pre-answering only) (59.01%, +0.51), demonstrating that the combined gain is substantially larger than either component alone, confirming their complementary nature. Besides, both ablated variants independently improve over the baseline, with only_a marginally outperforming only_t. The full model shows particularly strong gains on GSM8K (+2.28 points) and Multi-Arith (+1.28 points). Interestingly, single-component models slightly outperform the full model on SVAMP (54.33%/54.11% vs. 53.56%), suggesting task-specific sensitivities.

Model	GSM8K	GSM_H	Multi	ASDiv	SVAMP
Pre^2-1	**53.78**	**12.76**	94.17	83.33	53.56
Pre^2-3	52.31	12.11	**95.17**	83.30	**56.33**
Pre^2-5	53.65	12.53	94.17	83.49	54.56
Pre^2-7	52.97	12.26	94.89	**84.01**	55.11

Table 2: Ablation study on the Depth Dimension. Avg.acc: Pre^2-1(59.52), Pre^2-3(59.84), Pre^2-5(59.68), Pre^2-7(59.85)

5.3 Exploring the Depth Scaling of Pre-thinking and Pre-answering Modules

Table 2 investigates the impact of scaling the pre-thinking and pre-answering modules along the depth dimension by varying the number of Transformer layers (1, 3, 5, 7). Increasing module depth consistently improves overall performance, with **Pre**^{2}-7 achieving the highest average accuracy of 59.85%, followed closely by **Pre**^{2}-3 (59.84%). This represents a +0.33 point improvement over the shallowest **Pre**^{2}-1 variant (59.52%), demonstrating that allocating additional depth-wise computation to the reasoning process yields meaningful gains. While deeper variants improve average performance, the benefits are not uniform across all datasets. **Pre**^{2}-1 excels on GSM8K (53.78%) and GSM_hard (12.76%), whereas **Pre**^{2}-3 achieves the best results on Multi-Arith (95.17%) and SVAMP (56.33%). **Pre**^{2}-7 demonstrates the strongest performance on ASDiv (84.01%), suggesting that different reasoning tasks may benefit from varying computational depths.

5.4 On the Width Scaling of Pre-thinking Scope

Figure 3 examines the impact of scaling the pre-thinking scope (here we fix the depth at 1 transformer layer) (width dimension) from 1 to 5 to-

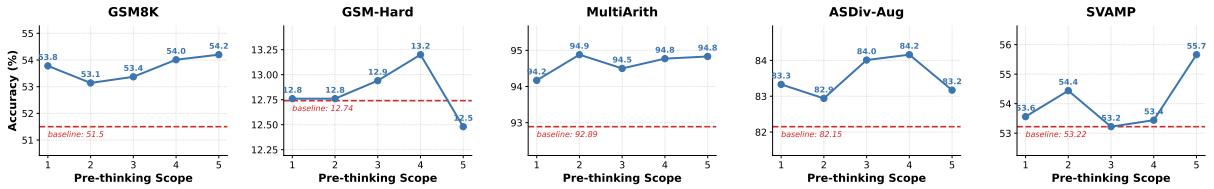

Figure 3: Ablation study on the Width Dimension (Pre-thinking Scope) Avg.acc: Scope-1(59.52), Scope-2(59.63), Scope-3(59.61), Scope-4(59.91), Scope-5(60.06), Baseline(58.50).

Model	GSM8K		GSMHard		MultiArith		SVAMP		ASDiv-Aug	
	Tok	T/s	Tok	T/s	Tok	T/s	Tok	T/s	Tok	T/s
Pre^2-0.5b	47.6	76.7 (1.00)	78.9	76.6 (0.36)	23.6	77.2 (1.31)	25.9	76.5 (1.07)	15.6	75.9 (1.45)
Pause Token	44.5	76.8 (0.96)	77.2	76.5 (0.78)	24.6	77.1 (0.55)	24.7	76.6 (1.00)	15.6	75.1 (1.52)
Pre^2-0.5b-1-1	46.1	76.9 (0.49)	78.6	76.5 (0.85)	24.6	76.3 (1.61)	24.4	76.3 (1.57)	15.6	75.4 (2.36)
Pre^2-0.5b-3-1	45.1	75.0 (1.55)	71.3	75.8 (0.81)	24.6	75.3 (1.99)	23.3	75.7 (0.90)	15.6	77.0 (0.92)
Pre^2-0.5b-5-1	44.4	74.5 (0.40)	74.9	74.4 (0.43)	24.6	74.6 (0.99)	28.5	74.3 (1.93)	15.6	75.3 (0.87)
Pre^2-0.5b-7-1	46.2	74.1 (1.59)	72.3	73.6 (1.37)	24.6	74.5 (1.29)	28.5	73.8 (2.19)	15.6	73.1 (0.88)
Pre^2-0.5b-1-2	45.0	75.8 (1.31)	77.8	76.1 (1.24)	24.6	75.7 (1.16)	26.3	76.0 (1.04)	15.6	74.2 (1.68)
Pre^2-0.5b-1-3	45.3	75.3 (1.46)	75.5	76.5 (0.88)	24.6	75.6 (1.38)	25.8	75.6 (1.23)	16.4	75.1 (0.79)
Pre^2-0.5b-1-4	45.3	75.1 (1.63)	71.0	75.2 (0.76)	24.6	75.3 (0.41)	26.4	75.4 (1.38)	15.6	75.3 (2.04)
Pre^2-0.5b-1-5	49.0	74.6 (1.04)	73.8	74.5 (0.71)	24.6	75.1 (1.31)	25.7	75.6 (0.95)	15.6	73.8 (1.69)

Table 3: Inference efficiency analysis comparing **Pre**^{2} variants against the Pause Token baseline.

kens, which means that we will expand the width of pre-thinking scope during training and inference time. Note that we fix the pre-answering scope at 1, as all benchmark answers are sufficiently short to be compressed into a single token representation. Expanding the pre-thinking scope demonstrates progressive performance improvements, with **Pre**^{2}-1-5 achieving the highest average accuracy of 60.06%, representing a 0.54 point gain over **Pre**^{2}-1 and 1.56 points over the baseline (58.50%). This trend suggests that allocating wider latent space for reasoning compression enhances the model's capacity to capture complex thought processes. Different benchmarks respond variably to scope expansion. GSM8K and GSMHard show steady improvements with wider scope (GSM8K: 53.78%→54.20%, GSMHard: 12.76%→12.48% with fluctuations). SVAMP exhibits notable gains at scope-5 (55.66%), suggesting that certain reasoning tasks benefit more from expanded latent capacity. Unlike depth scaling, width expansion doesn't follow a strictly monotonic pattern. Scope-2 through scope-4 show incremental improvements (59.63%→59.61%→59.92%), with scope-3 showing a slight dip. This suggests potential trade-offs between compression capacity and optimization difficulty. These findings indicate that width-wise scaling of the pre-thinking module provides complementary benefits to depth scaling, with wider scopes enabling richer compression of reasoning chains.

5.5 Model Efficiency

Table 3 presents a comprehensive evaluation of inference throughput (tokens/second) across all model variants and benchmarks, demonstrating that our architectural do not significantly degrade inference performance compared to the baseline.

6 Conclusion

We presented a phase-aware reasoning framework that decouples thinking and answering in large language models by combining latent representations with explicit chain-of-thought supervision. By independently modeling pre-thinking and pre-answering phases, our approach mitigates the limitations of autoregressive reasoning while preserving pretrained reasoning capabilities. Experiments across in-domain and out-of-domain benchmarks demonstrate consistent improvements in both performance and inference efficiency. This work highlights the importance of structured reasoning phases as a promising direction for building more efficient and robust reasoning models.

Limitations

Despite its effectiveness, our approach has several limitations. First, the proposed phase-aware design

introduces additional architectural components and training objectives, which slightly increase implementation complexity compared to standard autoregressive decoders. Second, while our method improves inference efficiency relative to explicit chain-of-thought decoding, it still relies on explicit reasoning supervision during training, limiting its applicability to domains where high-quality reasoning annotations are available. Finally, our evaluation focuses on reasoning-centric benchmarks; the impact of phase-aware modeling on general-purpose language generation and non-reasoning tasks remains to be explored.

Ethical Considerations

Our use of publicly available datasets and developed models in this research raises no ethical concerns, and we confirm compliance with the ACL Code of Ethics.

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A Experiment Details

We provide the full hyperparameter settings, training procedures, and additional analysis used in our experiments.

A.1 Implementation Details

Our framework, **Pre**^{2}, was implemented using PyTorch and the Litgpt framework. We selected the **Qwen2.5** series (Qwen et al., 2025) (0.5B and 1.5B) as our backbone models due to their strong mathematical reasoning capabilities.

Module Initialization As described in Section 3, we freeze the original embedding layer of the backbone model to maintain a stable semantic space. The pre-thinking (f_{think}) and pre-answering (f_{ans}) modules are initialized as lightweight Transformer decoder blocks. Specifically, their hidden size and number of attention heads align with the backbone model to ensure compatibility with the frozen embeddings. For the latent vectors $\mathbf{z}_{\text{think}}^{(0)}$ and $\mathbf{z}_{\text{ans}}^{(0)}$, we use a Gaussian initialization with mean 0 and standard deviation 0.02.

Training Setup All models were trained on a cluster of 8 NVIDIA A100 (80GB) GPUs using the Distributed Data Parallel (DDP) strategy. The

training process utilized the AdamW optimizer. To stabilize the learning of the newly introduced latent modules alongside the backbone, we applied a linear warmup strategy followed by a cosine decay scheduler. For the baselines (Freeze-Baseline and Pause Token), we strictly followed the same training configurations (batch size, learning rate, and scheduler) to ensure a fair comparison. For all evaluations, we use greedy decoding to ensure deterministic generation.

A.2 Hyperparameters

Table 4 lists the detailed hyperparameters used for the main experiments. We determined the loss weights λ_{think} and λ_{ans} through a preliminary grid search on a hold-out validation set from GSM8K-Aug. We found that setting the weights to 1.0 provided a balanced gradient flow between the alignment objectives ($\mathcal{L}_{\text{think}}, \mathcal{L}_{\text{ans}}$) and the autoregressive prediction objective ($\mathcal{L}_{\text{LM}}$).

B Dataset Details

In this section, we provide detailed specifications of the datasets used in our experiments. We first present the statistical overview and then detail the specific characteristics and examples for each benchmark.

B.1 Dataset Overview and Metadata

Table 5 summarizes the general metadata for each dataset. All selected datasets focus on the *Math* domain and require *Arithmetic* reasoning capabilities. The answer types are consistently numerical values.

B.2 Dataset Statistics

Table 6 presents the statistical details regarding the length of inputs and reasoning paths. We report the average length of questions (tokens), the average length of the Chain-of-Thought (CoT) rationale, and the average number of reasoning steps.

B.3 Benchmark Detail

In this section, we describe the specific characteristics of each evaluation benchmark and the training data.

GSM8K-Aug (Deng et al., 2024) GSM8K-Aug is an augmented dataset derived from GSM8K (Cobbe et al., 2021). It expands the original 8.5k training problems to roughly 385k examples through paraphrasing, numerical resampling, and synthetic generation using GPT-4. The dataset exhibits a distribution of reasoning steps where the majority of problems require two to four steps, accompanied by a long tail of instances requiring six or more steps. This balance of common and complex instances makes the dataset suitable for training models to generalize across various reasoning difficulty levels.

GSM-Hard (Gao et al., 2022) GSM-Hard is a challenging variant of the GSM8K dataset constructed to test the ability of models to cope with harder numerical values. It retains the same problem statements as the original GSM8K but replaces many of the numbers with larger and less common numerical values, thereby increasing the difficulty of superficial arithmetic computation and reasoning. The dataset consists of approximately 1,319 examples.

MultiArith (Roy and Roth, 2015) MultiArith is a dataset of multi-step arithmetic word problems designed to challenge systems in correctly sequencing multiple operations. It contains 600 problems collected from educational sources, each solvable by a single equation involving two or more of the four basic operations (addition, subtraction, multiplication, division). The problems require reasoning over multiple sentences to extract and combine numeric quantities, understand implied operations, and compute the result. It is widely used as a benchmark for evaluating arithmetic reasoning generalization, particularly for handling compositional and sequential numerical reasoning.

SVAMP (Patel et al., 2021) SVAMP (Simple Variations on Arithmetic Math Word Problems) is a benchmark dataset designed to test the robustness of math word problem solvers to superficial changes. It contains 1,000 elementary-level arithmetic word problems (grade 4 and below), each involving a single unknown and solvable by an arithmetic expression with no more than two operators. The problems are generated through controlled variations in wording, structure, and number values applied to existing datasets (such as MAWPS and ASDiv-A) to reduce artifacts and superficial cues.

ASDiv-AUG (Xu et al., 2025) ASDiv (Academia Sinica Diverse MWP Dataset) is a diverse corpus designed to evaluate the capability of solvers across a wide range of language patterns and problem types. The original dataset contains approximately 2,305 math word problems, each annotated with its

Hyperparameter	*0.5b*										*1.5b*		
	1-1	**1-2**	**1-3**	**1-4**	**1-5**	**3-1**	**5-1**	**7-1**	pt	sft	**1-1**	pt	sft
Pre-thinking Scope	1	2	3	4	5	5	5	5	1	-	1	1	-
Thinking Layers	1	1	1	1	1	3	5	7	-	-	1	-	-
Answering Layers	1	1	1	1	1	3	5	7	-	-	1	-	-
Epochs	50										11	-	-
Micro Batch Size	32										16	-	-
Max Gradient Norm	1.0												
Optimizer	AdamW												
Learning Rate	2.0e-4												
Min LR	6.0e-5												
LR Warmup Steps	2000												
Precision	bf16-mixed												
Weight Decay	0.01												
Global Batch Size	256												
β_1	0.9												
β_2	0.999												
λ_{think}	0.15												
λ_{ans}	0.15												

Table 4: Hyperparameters for all model variants. (For the format 1-1, the first 1 means we use 1-layer for Pre-thinking module and Pre-answering module and the latter 1 means Pre-thinking scope.)

Dataset	**Properties**			**Size**		**License**
	Domain	**Reasoning**	**Ans. Type**	**Train**	**Test**	
GSM8K-AUG	Math	Arithmetic	Number	385,620	1,319	MIT
GSM-Hard	Math	Arithmetic	Number	1,319	-	MIT
MultiArith	Math	Arithmetic	Number	420	180	CC BY 4.0
SVAMP	Math	Arithmetic	Number	700	300	MIT
ASDiv-AUG	Math	Arithmetic	Number	4,183	1,038	CC BY 4.0

Table 5: Overview of the datasets. We categorize the datasets by their domain properties and sample sizes.

Dataset	**Avg. Q Len.**	**Avg. CoT Len.**	**Avg. Steps**
GSM8K-AUG	61.3	36.2	3.26
GSM-Hard	65.8	154.2	-
MultiArith	41.3	-	-
SVAMP	41.5	15.0	1.28
ASDiv-AUG	36.6	12.3	1.23

Table 6: Detailed statistics of the **test split** used in our experiments. Lengths are calculated based on token count, and steps represent the reasoning depth of the gold solutions.

problem type and grade level to indicate difficulty. ASDiv-AUG serves as an augmented or hardened version of this benchmark, typically constructed by filtering for higher-complexity problems or generating challenging variations to demand stronger mathematical reasoning capabilities from the models.

Dataset	Example Question	Rationale / Steps	Answer
GSM8K-Aug	Out of 600 employees in a company, 30% got promoted while 10% received bonus. How many employees did not get either a promotion or a bonus?	["«600*30/100=180»", "«600*10/100=60»", "«180+60=240»", "«600-240=360»"]	360
GSM-Hard	Janet's ducks lay 16 eggs per day. She eats three for breakfast every morning and bakes muffins for her friends every day with 4933828. She sells the remainder at the farmers' market daily for $2 per fresh duck egg. How much in dollars does she make every day at the farmers' market?	def solution(): """Janet's ducks lay 16 eggs per day. She eats three for breakfast every morning and bakes muffins for her friends every day with four. She sells the remainder at the farmers' market daily for $2 per fresh duck egg. How much in dollars does she make every day at the farmers' market?""" eggs_per_day = 16 eggs_eaten = 3 eggs_baked = 4933828 eggs_sold = eggs_per_day - eggs_eaten - eggs_baked price_per_egg = 2 money_made = eggs_sold * price_per_egg result = money_made return result	-9,867,630
MultiArith	There are 64 students trying out for the school's trivia teams. If 36 of them didn't get picked for the team and the rest were put into 4 groups, how many students would be in each group?	-	7
SVAMP	There are 87 oranges and 290 bananas in Philip's collection. If the bananas are organized into 2 groups and oranges are organized into 93 groups How big is each group of bananas?	(290.0 / 2.0)	145
ASDiv-AUG	gino has 64 popsicle sticks . i have 100 popsicle sticks . what is the sum of our popsicle sticks ?	«64+100=164»	164

Table 7: Examples from the evaluation and training datasets. Each entry shows a representative question, the reasoning steps (Rationale) required to solve it, and the final numerical answer.