
Position: Multi-Agent AI for Emergent Behavior Requires Mean-Field Learning

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Abstract

1 As AI agents scale to large populations, traditional coordination mechanisms face
2 intractable complexity. In multi-agent settings, *agentic emergence* is a term com-
3 monly used to characterize unpredictable collective behaviors at scales of multi-
4 agent interactions, but is not well-defined. **This position paper argues that**
5 **grounding agentic emergence in Artificial Social Intelligence and Mean-Field**
6 **Theory is necessary for transforming illusory heuristics to rigorous scaling**
7 **laws.** Current multi-agent interaction paradigms face exponential sample com-
8 plexity barriers that fundamentally limit scalability due to combinatorial interac-
9 tion spaces. We argue that mean-field learning (MFL) is not merely an optimiza-
10 tion technique but a principled and increasingly necessary requirement for reli-
11 able large-population orchestration. We support this position by pointing to recent
12 advances in sample complexity driven by game theory and MFL with example
13 illustrations in agentic coding and robotics. Finally, we offer a rationalized defini-
14 tion of emergence and outline a future research agenda where mean-field learning
15 turns the abstract concept of emergence into a theoretically grounded reality for
16 scaling populations of AI agents.

17 1 Introduction

18 Agentic systems based on foundation models are rapidly moving from single-agent chat assistants
19 to multi-agent populations of autonomous agents that plan, communicate, use tools and act in
20 the world. This shift is fueled by the expectation that scaling the number of agents will unlock
21 performance gains analogous to the scaling laws of large language models Qian et al. [2025],
22 Chaudhari et al. [2025]. However, analytical studies grounding such intuitions to the asymptotic
23 behavior of these multi-agent systems remains limited. Current deployments, including AI-driven
24 social networks like Moltbook Marzo and Garcia [2026] and autonomous personal assistants such
25 as OpenClaw Wang et al. [2026], treat agentic coordination as an engineering challenge to be
26 addressed with more capable models, efficient communication, or improved prompts. We argue
27 instead that coordination at scale is fundamentally an algorithmic challenge, one with a principled
28 solution class: *mean-field learning*. Without it, systems may appear to work for small populations
29 but become brittle as populations grow.

30 When foundation model (FM) agents interact, each agent’s optimal behavior depends on the
31 behaviors of its peers (the tools they use, the subtasks they claim, and the information they
32 produce/consume) Tran et al. [2025]. In an N -agent system, tracking these dependencies exactly
33 requires reasoning over a joint state-action space that grows combinatorially in N . The sample
34 complexity of learning good collective behavior in this space, and the computational complexity
35 of computing coordinated policies scale exponentially with N Blondel and Tsitsiklis [2000]. The
36 multi-round debate, committee, and tournament patterns popular in today’s agentic frameworks
37 impose at least superlinear communication overhead and are structurally mismatched to the large- N

38 regime their proponents envision. Without a principled alternative, agentic multi-agent systems will
39 hit a hard algorithmic wall before the promises of collective capabilities can be realized.

40 **We argue that mean-field learning provides a principled alternative. The mean-field ap-**
41 **proach replaces explicit dependence on all other agents’ states and actions with a compressed**
42 **population-level signal (the empirical distribution of agent states) that serves as a sufficient**
43 **statistic for each agent’s decision problem.** Applying mean-field theory to FM agents, such as
44 LLM-based agents, is not a straightforward translation of existing multi-agent reinforcement learn-
45 ing (MARL) results Yang et al. [2018b], Guo et al. [2021]. Such agents differ fundamentally: their
46 policies are expressed in natural language rather than tabular or neural action spaces, their coordi-
47 nation relies on semantic communication rather than numeric signals, and deployment typically
48 involves pre-training followed by fine-tuning rather than reinforcement learning from scratch, which
49 yields a very different sample-efficiency profile. We show that the mean-field framework extends
50 naturally to accommodate these differences, giving a formal treatment of agentic emergence through
51 the threshold phenomenon by which population-based collective capabilities become reliable as
52 agent population size grows while individual agent capability is held fixed. This framework applies
53 to homogeneous populations of identical agent (FM / LLM) instances, as well as heterogeneous pop-
54 ulations combining orchestrator models, specialized fine-tuned executors, and domain-specific tools.

55 We ground our position in domains where the tension between expected scaling gains and coordi-
56 nation tractability is most acute. In **agentic coding**, large populations of LLM agents collaborating
57 on shared codebases face tool contention, duplicated effort, and cascading dependencies that naive
58 pairwise coordination cannot manage at scale; A mean-field signal may summarize task progress,
59 tool use, and code quality to enable routing and load balancing without centralized management.
60 In **robotics**, the mean-field framework formalizes and extends the population-level conditioning
61 mechanisms already present in systems such as Gemini Robotics 1.5 Team et al. [2025] and $\pi_{0.7}$
62 Intelligence et al. [2026], providing the theoretical grounding those systems currently lack and
63 showing how their population-level abstractions should be generalized to deployed multi-robot
64 fleets. In each domain, the coordination bottleneck that limits scaling is precisely the problem
65 mean-field theory was built to solve, and the population-level abstractions that the most capable
66 systems already employ (compressed behavioral summaries, hierarchical planning-execution
67 decompositions, etc.) are implicit mean-field reasoning waiting to be formalized.

68 2 Background

69 **Agentic Scaling.** With the development and large-scale deployment of LLM-based AI agents has
70 come a sustained effort to scale agentic systems in the number of agents. Researchers and practi-
71 tioners are overcoming the limited functionality of individual agents with interconnected networks
72 of agents. Recent work focuses on improving the orchestration of multi-agent architectures through
73 planning, state management, quality control, and standardized coordination protocols [Adimulam
74 et al., 2026], as well as on improving dynamic topology routing to manage communication over-
75 head while producing interpretable coordination traces [Lu et al., 2026]. The growing capabilities
76 of these agents have motivated research in simulating large-scale agentic systems, such as environ-
77 ments capturing the social-media interactions of over 74,000 agents [Fadaei et al., 2026]. As agent
78 populations grow, systems must accommodate non-uniform and possibly unreliable newcomers by
79 proactively grouping tasks to familiarize them, leading to monotonic performance improvements
80 [Shao et al., 2026, Chaudhari et al., 2025, Anand and Liaw, 2025]; similarly, they extend tool-use
81 capabilities to control contexts available to agents [Gupta et al., 2026], and adapt decisions based on
82 interaction experience across domains such as medicine and robotics [Pati, 2025].

83 These advances have inspired the vision of open-ended generalizable autonomous agents to assist
84 scientists and engineers. Yet as agent populations grow, the need to understand their emergent
85 behavior becomes acute, often left elusive due to a lack of interpretation of agentic interactions. This
86 gap motivates our central position: the brute-force heuristics currently used to scale agent systems
87 must give way to mathematically rigorous orchestration with multi-agent mean-field learning.

88 **Mean-Field Multi-Agent Reinforcement Learning.** MARL has long studied scaling challenges
89 due to the exponential growth of the joint state-action space with the number of agents N [Blondel
90 and Tsitsiklis, 2000]. Mean-field methods address this by replacing the intractable joint space with
91 a population-level aggregate distribution over individual states and actions [Yang et al., 2018b,

92 Carmona et al., 2023, Li et al., 2021, Guo et al., 2021, Chen et al., 2021, Cui et al., 2024, Gu et al.,
 93 2025, Lin et al., 2023]. Under homogeneity assumptions, joint policy optimization can be reduced
 94 to a problem of polynomial complexity whose approximation error vanishes at the rate $O(1/\sqrt{N})$
 95 [Yang et al., 2018a, Gu et al., 2021]. This guarantee is the foundation of our position that mean-field
 96 methods convert unscalable problems into tractable ones.

97 A series of algorithmic advances has made this theoretically sound approach practical. Mean-field
 98 sampling techniques treat agents as part of a latent distribution and enable efficient cooperative
 99 RL without explicit pairwise interactions [Anand and Qu, 2024, Anand et al., 2025, 2026, Anand
 100 and Karmarkar, 2026]. Importantly, the homogeneity assumption can be relaxed. Graphon mean-
 101 field games (GMFGs) model heterogeneous populations by encoding non-uniform interactions via a
 102 graphon, which is a limiting object that captures a diversity of agent types and their couplings [Sub-
 103 ramanian et al., 2022, Cui et al., 2023, Cui and Koeppl, 2022, Caines and Huang, 2018]. Recent work
 104 also extends GMFGs to sequential settings, sparse graphs, and learning from sampled data, while
 105 cooperative graphon mean-field control directly optimizes population-weighted joint objectives [Hu
 106 et al., 2023, Fabian et al., 2023, Zhang et al., 2023, Fabian et al., 2025].

107 These works demonstrate that mean-field theory scales to the kind of heterogenous, structured pop-
 108 ulations that arise in real-world LLM-agent deployments: the success of these methods does not
 109 depend on the specific policy representation, i.e. it can be tabular, neural, or LLM-based, but the
 110 core insight is the same: a compact population signal to replace exhaustive mutual modelling.

111 3 Mean-Field LLM Agents

112 We start by formalizing the concept of LLM agents and multi-agent systems, using the formulations
 113 in Guo et al. [2024] and Kim et al. [2025] as our foundation.

114 **Definition 3.1 (Agents)** *An LLM agent a_i is defined as a tuple $(\Phi_i, \mathcal{A}_i, M_i, \pi_i)$, where Φ_i is the*
 115 *LLM’s reasoning policy, $\mathcal{A}_i = \{\text{ToolCall}(t, \theta) : t \in \mathcal{T}, \theta \in \Theta_t\}$ is the action space of tool calls,*
 116 *where \mathcal{T} is the set of available tools and Θ_t the valid parameters, M_i is the internal memory, and*
 117 *$\pi_i : \mathcal{H}_i \rightarrow \mathcal{A}_i$ is the decision function mapping observation histories to actions.*

118 **Definition 3.2 (Agentic system)** *An agentic system $\mathcal{S} = (A, E, C, \Omega)$ comprises a set of agents*
 119 *$A = \{a_1, \dots, a_N\}$ for $N \geq 1$, a shared environment E , a communication topology C , and an*
 120 *orchestration policy Ω . Each agent a_i operates within the shared environment via iterative feedback.*

121

122 Consider a population of N agents in a shared space, as in Mi et al. [2025]. At time t , a subset
 123 of N_t agents ($\cup_t N_t = \{1, \dots, N\}$) act simultaneously based on an encoding of personal attributes
 124 and observations of the environment, where N_t is the set of active agents at time t . Letting $c_t \in$
 125 $\{1, \dots, N\}$, the collective $\{c_t\}_t$ can be used to define hierarchical or compositional agents [Du et al.,
 126 2023]. Next, the agent states are given by $\vec{s}_t = (s_t^1, \dots, s_t^{N_t})$ with $s_t^i \in \mathcal{X}$, and $\vec{a}_t = (a_t^1, \dots, a_t^{N_t})$
 127 denotes the joint action vector, where each action influences the environment and future behavior.

128 3.1 Mean-Field AI Agent Framework

129 Simulating direct interactions between every pair of agents becomes infeasible as the number of
 130 agents N and the time horizon grow. While full interaction histories can be stored for small groups,
 131 they do not scale to large populations. Mean-field methods address this by modeling the collective
 132 as a whole [Mi et al., 2025]. Rather than tracking each agent individually, the mean-field maintains
 133 a single population-centric signal that evolves over time and summarizes the state of the group.
 134 This is initialized as an empty context m_0 and recursively updated by a dedicated mean-field model
 135 $m_t \leftarrow \mu(m_{t-1}, \vec{s}_{t-1}, \vec{a}_{t-1})$, where $(\vec{s}_{t-1}, \vec{a}_{t-1})$ are the ensemble of agent states and actions. The
 136 update distills the most decision-relevant information from the latest population trajectory.

137 **Decision-Making.** At each step, agent i receives its private state s_t^i and the current mean-field
 138 signal m_t , then selects an action by sampling from its LLM-based policy $a_t^i \sim \pi^i(\cdot | s_t^i, m_t)$. Since
 139 the policy operates over the unbounded space of natural language, it can produce highly adaptive
 140 and context-sensitive actions [Mi et al., 2025, Anand and Liaw, 2025]. Once all N_t agents act, the
 141 system transitions via $\vec{s}_{t+1} \sim P(\cdot | \vec{s}_t, \vec{a}_t, m_t)$, where P captures the dynamics of the shared world.

142 **Mean-Field Learning.** The mean-field signal should be a compact predictor of future agent behav-
 143 ior. We formalize this by applying the information-bottleneck principle [Mi et al., 2025]. Let
 144 $Y = \{a_t^{s_t}\}_{t=1}^{N_t}$ and $X = \{(\vec{s}_\tau^*, \vec{a}_\tau^{s_\tau})\}_{\tau=0}^t$ be the history. Then, the IB objective is $\min_{m_t} I(m_t; X) -$
 145 $\beta \cdot I(m_t; Y)$, where the first term encourages compression of the past and the second term pre-
 146 serves predictive information about upcoming actions. In practice $I(m_t; Y)$ is approximated by
 147 the log-likelihood of observed actions under the policy $\sum_{i=1}^{N_t} \log \pi(a_t^{*i} | s_t^i, m_t)$, and the compres-
 148 sion term $I(m_t; X)$ is upper-bounded using a variational posterior $\mu_\phi(m_t | X)$ and a fixed prior
 149 $r(m_t)$, then $I(m_t; X) \leq \mathbb{E}_{p(X)}[\text{KL}(\mu_\phi(m_t | X) \| r(m_t))]$. Setting $X = \{m_{t-1}, \vec{s}_{t-1}, \vec{a}_{t-1}\}$, for
 150 $m_t \sim \mu_\phi(\cdot | m_{t-1}, \vec{s}_{t-1}, \vec{a}_{t-1})$, balances predictive richness and efficiency, so the combined loss is

$$\mathcal{L}_{\text{mean-field}} = \sum_{t=1}^T \left(\mathbb{E}_{p(X_t)} D_{KL}(\mu_\phi(m_t | X_t) \| r(m_t)) - \beta \sum_{i=1}^{N_t} \log \pi(a_t^{*i} | s_t^i, m_t) \right). \quad (1)$$

151 **Policy Optimization.** The agent policy π is shared across the population and assumes a factorized
 152 form $\pi(\vec{a}_t | \vec{s}_t, m_t) = \prod_{i=1}^{N_t} \pi(a_t^i | s_t^i, m_t)$, where individual behavior differences arises naturally from
 153 the personalized language-expressed state inputs s_t^i , expressed in natural language. The policy is
 154 trained by minimizing the negative log-likelihood of ground-truth actions

$$\mathcal{L}_{\text{policy}} = - \sum_{t=1}^T \sum_{i=1}^{N_t} \log \pi(a_t^{s_t} | s_t^i, m_t), \quad (2)$$

155 which aligns the policy’s output with empirically observed decisions conditioned on both private
 156 state and the population summary.

157 **Heterogeneity.** When agents differ in morphology or role, a single population-wide mean field may
 158 be insufficient. Here, the framework naturally generalizes to multi-class mean fields, where each
 159 agent type has its own aggregate signal, and cross-class coupling is captured by graphon mean-fields
 160 [Caines and Huang, 2021, Cui and Koeppl, 2022, Anand et al., 2026]. Motion transfer across em-
 161 bodiments in robotics (Appendix A) implicitly instantiate this multi-class structure.

162 4 Multi-Agent Emergent Mean-Field LLM Agents

163 Having defined the mean-field orchestration mechanism, we now turn to the phenomenon it is de-
 164 signed to explain and control: agentic emergence. The literature on emergence in large language
 165 models [Berti et al., 2025, Arora and Goyal, 2023] indicates that new capabilities often appear dis-
 166 continuously as model scale increases. However, quantifying emergence is fraught with difficulty:
 167 evaluation metrics can create the illusion of sharp transitions [Schaeffer et al., 2023], different skills
 168 may emerge simultaneously, and the very notion of a “skill” in language agents is hard to formalize.

169 In the multi-agent LLM setting, we are interested in scaling the population of interacting agents
 170 while keeping per-agent capabilities fixed [Riedl, 2026]. A collective behavior, such as reliably
 171 executing a large-scale software migration or generalization across robot embodiments, may appear
 172 only when the number of agents N surpasses some threshold κ . Intuitively, any collective capability
 173 may have an intrinsic minimal population size. We might speak of a behavior being κ -emergent
 174 if it reliably manifests only when $N \geq \kappa$, where κ depends on the task complexity, the structure
 175 of the environment, and the coordination mechanism. For instance, a collective being moral might
 176 require at least two agents (via a debate), so morality might be a 2-emergent behavior. Similarly,
 177 expert coding agents can be κ -Emergent where κ is a function of difficulty of understanding a certain
 178 codebase. Using this notion, we define agentic emergence below:

179 **Agentic Emergence.** Consider an N -agent partially observable game of horizon T . Agent i has
 180 history $h_{i,t} \in \mathcal{H}_t$ and acts via a fixed policy $a_{i,t} \sim \pi_\theta(\cdot | h_{i,t}, u_t)$, where u_t is an optional broadcast
 181 signal. Crucially, the policy class π_θ does not grow in capability with N . We are interested in
 182 capabilities that arise only by increasing N , not from improving individual agents. Let $g : \mathcal{H}_t \rightarrow \mathcal{X}$
 183 be a microstate extractor, with $x_{i,t} = g(h_{i,t})$. Define the empirical population measure

$$\mu_t^{(N)} = \frac{1}{N} \sum_{i=1}^N \delta_{x_{i,t}}, \quad (3)$$

184 which summarizes the distribution of agent states at time t . Let $Z^{(N)} \in \{0, 1\}$ denote a macroscopic
 185 success event, e.g. discovering a valid hypothesis or completing a large software migration.

186 **Definition 4.1 (Agentic emergence)** Fix a per-agent policy class π_θ , an N -agent environment fam-
 187 ily, and a macroscopic success event $Z^{(N)}$. We say that $Z^{(N)}$ is agentially emergent if there exist
 188 $\delta \in (0, 1/2)$, $c \in (0, 1)$, and a threshold $\kappa = N_*(\delta)$ such that

$$\sup_{N \leq cN_*(\delta)} \Pr_{\pi_\theta}^{(N)} [Z^{(N)} = 1] \leq \delta, \quad \inf_{N \geq N_*(\delta)} \Pr_{\pi_\theta}^{(N)} [Z^{(N)} = 1] \geq 1 - \delta. \quad (4)$$

189 Thus, the same per-agent capability cannot reliably realize the macroscopic behavior below a pop-
 190 ulation threshold, but can realize it reliably beyond that threshold. This isolates population-driven
 191 emergence from improvements in individual models by fixing π_θ and defining emergence as a repro-
 192 ducible, thresholded transition in collective performance.

193 **Necessity of Mean-Field Learning.** A naïve N -agent system conditions on the full joint state
 194 $\mathbf{x}_t = (x_{1,t}, \dots, x_{N,t})$ and action $\mathbf{a}_t = (a_{1,t}, \dots, a_{N,t})$. With d possible microstates and k possible
 195 actions per agent, the joint space scales as $d^N k^N$, which is a barrier to explicit reasoning. In contrast,
 196 policies of the form $a_{i,t} \sim \pi_\theta(\cdot | h_{i,t}, m_t)$ where $m_t = \phi(\mu_t^{(N)})$, replaces dependence on all other
 197 agents with dependence on the population distribution, changing the scaling from exponential to
 198 polynomial and making reasoning over the collective tractable.

199 **Emergent Mean-Field Learning.** The mean-field m_t should retain predictive information on future
 200 states or macroscopic success, while discarding inapt individual detail, yielding an objective:

$$\min_{\phi} \sum_{t=1}^T \left[\mathcal{L}_{\text{pred}}(m_t; Z^{(N)}, \mu_{t+1}^{(N)}) + \lambda \mathcal{L}_{\text{comp}}(m_t; \mu_t^{(N)}) \right]. \quad (5)$$

201 Given m_t , the agent policy can be trained via reinforcement learning [Rafailov et al., 2024], using:

$$\mathcal{L}_{\text{policy}}(\theta) = - \sum_{t=1}^T \sum_{i=1}^N \log \pi_\theta(a_{i,t}^* | h_{i,t}, m_t), \quad J_N(\theta, \phi) = \mathbb{E} \left[\sum_{t=0}^T R(\mu_t^{(N)}, m_t, \mathbf{a}_t) \right]. \quad (6)$$

202 The mean-field dynamics/rewards decouple across agents

$$P(x_{i,t+1} | \mathbf{x}_t, \mathbf{a}_t) \approx P_{\text{MF}}(x_{i,t+1} | x_{i,t}, a_{i,t}, \mu_t^{(N)})$$

203 and $r_i(\mathbf{x}_t, \mathbf{a}_t) \approx r_{\text{MF}}(x_{i,t}, a_{i,t}, \mu_t^{(N)})$, moving the dependence on N from the joint state-action
 204 space into the estimation of a population statistic, thereby enabling tractable N -emergence
 205 behaviors.

206 5 Practical Implications of Mean-Field LLMs

207 5.1 Planning agents in Coding

208 Agentic coding tools, such as OpenAI Codex, Claude Code, Windsurf, and Cursor, are reshaping
 209 software engineering [Sapkota et al., 2025, AI Omar et al., 2024, Tufano et al., 2024]. Software engi-
 210 neering is a natural testbed for mean-field agentic coordination, since the environment is structured,
 211 heavily instrumented, and deeply coupled [Bellur et al., 2025, Shirafuji et al., 2023, DePalma et al.,
 212 2024]. A modern codebase is not merely a collection of files; it is a shared dynamical system con-
 213 sisting of modules, dependency graphs, build configurations, and deployment constraints. Agents
 214 operating in this environment must coordinate over both code semantics and shared resources, such
 215 as which files are being edited, which tests are failing, which dependencies are unstable, and which
 216 CI/review bottlenecks are binding. While adding more agents increases parallelism, it also amplifies
 217 duplicated work, conflicting patches, and CI congestion [Li et al., 2025]; however, explicit pairwise
 218 communication, where each agent reads all others’ plans and negotiates task allocation, can resolve
 219 conflicts in small teams, but scales quadratically in communication overhead.

220 This perspective illuminates some challenges: in agentic refactoring, failures often stem from in-
 221 complete global context and missed dependencies [Horikawa et al., 2025]. In the mean-field view,
 222 these are population-state estimation failures: the system lacks a reliable aggregate signal describing
 223 which invariants have been checked, which modules remain inconsistent, and which dependencies
 224 have already been migrated. Similarly, decentralized LLM collaboration methods [Liu et al., 2026]
 225 can be interpreted as learning policies for agents that coordinate through shared signals rather than
 226 exhaustive pairwise deliberation, which is an implicit step toward mean-field LLMs.

227 5.2 Physical embodied agents in Robotics

228 Robotic multi-agent systems present a demanding instantiation of FM / LLM agents. In addition
229 to coarse natural language for representing state and action spaces, embodied agents must also
230 ground population-level coordination in physical dynamics: shared workspace constraints, real-time
231 kinematic coupling, and heterogeneous morphologies that induce structurally distinct action spaces
232 within the same fleet [Fung et al., 2025, Lin et al., 2024, Preiss et al., 2017]. These challenges
233 expose both the necessity and the limits of naive multi-agent scaling, and we argue that mean-field
234 learning provides the only principled pathway to coordination at realistic fleet sizes.

235 We support this argument in Appendix A by examining two independently developed foundation
236 model frameworks for physical robotics: (i) the Gemini Robotics 1.5 framework [Team et al., 2025],
237 comprising the VLA model GR 1.5 and the VLM orchestrator GR-ER 1.5, and (ii) the $\pi_{0.7}$ frame-
238 work from Physical Intelligence, comprising the Hi Robot hierarchical inference framework [Shi
239 et al., 2025] and a generalist VLA model [Intelligence et al., 2026].

240 Our position is that both the cross-embodiment generalization using Motion Transfer in Gemini
241 Robotics 1.5, and the compositional task generalization shown by $\pi_{0.7}$, are empirical validations
242 of an implicit mean-field formalism, albeit along two complementary axes. In the case of Motion
243 Transfer, the learned mean field representation balances the compression versus prediction infor-
244 mation bottleneck in the space of different robot morphologies. On the other hand, to achieve
245 compositional generalization in the space of task description in $\pi_{0.7}$, the mean field representation
246 optimizes the physical grounding of natural language prompts. Neither outline any principled
247 exploration of scaling their agentic frameworks, therefore our analysis in Appendix A attempts to
248 ground their empirical observations in the emergent mean-field learning framework and provides
249 insights into potential avenues for future research.

250 6 Illusion of Scaling Laws for Emergence

251 Much of the practical interest in AI Agents (as in Section 5) stems from ideas of *emergence*: that
252 scaling interactions between agents can often unlock new problem-solving capabilities. A criti-
253 cal study of recent literature reveals that this narrative is largely an illusion due to computational
254 tractability challenges: when analyzed by applying game-theoretic lower bounds and information-
255 theoretic limits, and looking at large-scale controlled ablations, the assumption that scaling the num-
256 ber of agents is “all that you need” dissipates [Li et al., 2024].

257 6.1 The Measurement Artifact: The Mirage of Discontinuous Capability Jumps

258 The assumption driving the search for “emergent abilities” through population scaling is derived
259 from emergence in base AI models, where capabilities suddenly arise once a certain parameter count
260 or compute threshold is reached. It is also intuitive in real world problems where specialized groups
261 of N agents coordinate with each other to solve complicated tasks. If individual models demonstrate
262 this new behavior, prevailing heuristics suggest that similar discontinuous jumps in ability should
263 occur in collectives as N increases.

264 Consider the following complex task: A model (we assume it to be a specialized agent) is asked to
265 output a sequence of L tokens exactly without error. Doing this successfully results in a reward of 1;
266 otherwise the reward is 0. Here, the probability of getting an exact match scales exponentially as p^L ,
267 where p is the model’s per-token accuracy. A smooth improvement with p as the model scales will
268 inevitably produce an exponentially-delayed spike in task performance, giving the illusion of emer-
269 gent capabilities. When continuous linear metrics such as Token Edit Distance or Brier Score [Brier,
270 1950] are used to evaluate these identical outputs, the discontinuous jump in ability vanishes to re-
271 veal a smooth scaling law. Our position is that N agents when giving an output of L tokens each
272 will retain a similar scaling law that grows exponentially with more N and L .

273 This “mirage” is significant for multi-agent systems: the perceived success of a large debate or
274 voting ensemble is almost exclusively the result of a discontinuous evaluation metric amplifying
275 a marginal fractional increase in the underlying probability of success, rather than the collective
276 agents suddenly unlocking a genuinely novel cognitive algorithm. Relying on this pervasive illusion
277 encourages practitioners to endlessly scale population counts, operating under the false assumption
278 that a massive ensemble will eventually cross a hidden threshold for complex tasks.

279 **6.2 Game-Theoretic Limits: The Sample Complexity of Artificial Social Rationality**

280 Beyond mere measurement artifacts, the failure of naive scaling of agents is firmly rooted in funda-
 281 mental, mathematically proven sample complexity limits. A widespread assumption in system de-
 282 sign is that if individual agents are highly capable (i.e., individually rational), their interactions from
 283 their unstructured collective will converge on optimal cooperation. Bandyopadhyay et al. [2025]
 284 weakens this assumption within the framework of socially rational populations.

285 Consider a two-player finitely repeated general-sum game heavily obscured by private types of
 286 agents, defined by the tuple $\mathcal{G} = (\mathcal{I}, \mathcal{A}, \Theta, G, T)$. One of these agents can be selected from a
 287 population of agents at scale, inspired by replicator dynamics Börgers and Sarin [1997], quantifying
 288 the proportion of each type of agents in a population. Agents operate with strictly private utility
 289 functions dictated by their hidden type $\theta \in \Theta$. Utilities signify the reward or incentive for agents
 290 to do a task while type is a model of the agents’ behavior making all actions and resultant payoffs
 291 heavily context-dependent. The target population is defined mathematically as “socially intelligent”
 292 if and only if it satisfies two conditions: (i) **Consistency**: Individual agents are Hannan-consistent
 293 (the no-regret property), meaning their external regret $\frac{1}{T} R_i^{ext}(h_T; \theta) \leq \epsilon$ is tightly bounded with
 294 high probability, and (ii) **Compatibility**: The paired agents successfully achieve an expected utility
 295 comparable to a Pareto-Optimal Nash Equilibrium (PONE) of the underlying game.

296 The critical question is whether an uninitiated algorithm can achieve zero-shot cooperation when
 297 abruptly introduced into this highly structured, socially intelligent population. The learning objec-
 298 tive here is to minimize *Altruistic Regret* R_T , which measures the cumulative deficit between the
 299 partner’s realized payoffs and their lowest-payoff PONE at time T [Bandyopadhyay et al., 2025].

300 In general, individual consistency and compatibility are fundamentally insufficient for zero-shot co-
 301 operation of agentic populations at scale. To illustrate the sample complexity inherent in agentic
 302 cooperation, we examine lower bounds in Bandyopadhyay et al. [2025] which show that for any
 303 $\delta, \epsilon > 0$, there exists a class of games and a class of (δ, ϵ, T) -socially intelligent agents such that,
 304 for any data-dependent meta-strategy informed by a dataset of K interaction histories, the expected
 305 altruistic regret is bounded from below: for any data-dependent meta-strategy informed by K inter-
 306 action histories, the expected regret $\mathbb{E}[R_T]$ scales as:

$$\mathbb{E}[R_T] \geq \Omega \left((T - L) \min \left\{ \frac{1}{2}, \frac{N^{L-2} - 1}{2K + 1} \right\} \right) \quad (7)$$

307 for some required authentication sequence length L .

308 Here, authentication means that the agents have to ensure that their collaborative partner does not
 309 deviate from the optimal strategy, thereby protecting from adversarial agents which try to pose
 310 as a member of the collective. This lower bound shows that the sample complexity required for
 311 cooperative behavior grows exponentially with the interaction history (N^{L-2}). This formally proves
 312 the established intractability of the $d^N k^N$ joint state-action space from Section 4.

313 To avoid this combinatorial explosion in complexity, Bandyopadhyay et al. [2025] gives an upper
 314 bound showing that the exponential lower bound can be circumvented if the agent successfully mod-
 315 els the aggregate behavior of the interactive population over a short horizon \tilde{T} . Though their proof is
 316 applied to create a static “imitate-then-commit” meta-strategy, we argue this same population-level
 317 dynamic can be addressed adaptively using mean-field MARL. Instead of imitation learning, we
 318 capture this aggregate behavior continuously through the mean-field signal m_t . Using the Informa-
 319 tion Bottleneck defined in Section 3.1, we actively compress the interaction history. The expected
 320 altruistic regret under this approach satisfies:

$$\mathbb{E}[R_T] \leq \delta(K) + \epsilon_1 + \delta_1 + \frac{T - \tilde{T}}{T} (\epsilon_0 + \delta_0) \quad (8)$$

321 where $\delta(K)$ decays smoothly and polynomially with the dataset size K , and ϵ_i, δ_i are error bounds
 322 and failure probabilities for $i \in \{0, 1\}$ signifying consistency and compatibility, respectively Bandy-
 323 opadhyay et al. [2025].

324 The implications for scaling in Equations 7 and 8 are absolute: the lower bound proves that scaling
 325 a large number of “rational” agents into a shared environment will never spontaneously yield
 326 emergent cooperation due to the exponential compute barrier of hidden types and complex histories.
 327 In fact, the upper bound proves that sustained success strictly requires analyzing the system at the

328 population level by learning the conventions and statistical behaviors of the aggregate group rather
329 than exhaustively attempting to model individual peers. This validates our position from Section 4:
330 true agentic emergence demands that policies be conditioned on the empirical population measure
331 $\mu_t^{(N)}$. This way, we shift the burden of coordinating from the exponential complexity of joint
332 state-action space modeling to the far more tractable polynomial estimation at the population level.

333 7 Alternative Views

334 Our position is that scaling agentic systems require population-level abstractions, and mean-field
335 learning is the most principled current framework for designing such abstractions. However, there
336 are several credible alternative views. We address them here to clarify the scope of our claim.

337 **A.1: Direct Systems Optimization May Be Enough.** One view is that agent-scaling bottlenecks are
338 engineering problems, solvable by faster inference and cheaper accelerators, rendering new frame-
339 works unnecessary. We agree that system optimization is vital: many near-term gains in agent
340 deployment will come from these improvements. However, it does not remove the underlying inter-
341 action problem. Improving per-agent compute does not change the asymptotic structure of explicit
342 multi-agent planning. A system that requires each agent to track every other agent’s state remains
343 structurally mismatched to the large- N regime even if each inference call is free. The distinction
344 we draw is between making the current paradigm cheaper and changing its scaling law. Mean-field
345 methods address the latter by replacing explicit dependence on individual agents with one on an
346 aggregate population state. It is not a substitute for systems optimization, but a complementary
347 abstraction that determines what the system should compute as N grows.

348 **A.2: Token Efficiency May Matter More Than Algorithmic Efficiency.** Another view is that the
349 dominant cost in LLM-agent systems is token use, so reducing prompt length and training models
350 to use fewer deliberation tokens may make multi-agent systems practical without special orches-
351 tration. While this certainly lowers reasoning costs, multi-agent scaling introduces costs that are
352 irreducible to agent token counts: communication fan-out, conflicting actions, and resource conges-
353 tion. A perfectly token-efficient agent can still be in an inefficient collective if the collective lacks
354 stable population-level control. Scalable systems require both token compression and population-
355 state compression. Mean-field learning provides the latter: instead of each agent reading long peer
356 transcripts, the system compresses collective behavior into a sufficient statistic.

357 **A.3: Hierarchical Orchestration Could Replace Mean-Field Learning.** A third view is that
358 stronger orchestrators (hierarchies of planners, reviewers, and executors) can decompose tasks and
359 resolve conflicts without population-level abstractions, a pattern already common in coding agents.
360 Hierarchy is often the right architecture; however, the critical question is what information flows
361 through it. A top-level orchestrator that explicitly tracks the state of lower-level agents still faces the
362 same scaling problem: as N grows, the orchestrator becomes a centralized bottleneck whose context
363 must either expand with N or discard information in an unprincipled way. Mean-field learning
364 can resolve this by abstracting the missing information, where the upper layer receives population
365 summaries, and the lower layer receives aggregate signals.

366 **A.4: The Measurement Problem.** One might argue that the field moves too fast for asymptotics
367 to be useful: models, tools, and deployment practices change rapidly, and gains from multi-agent
368 systems are often confounded by stronger base models, better prompts, larger context windows, or
369 larger evaluation budgets. In this lens, measurement is the urgent bottleneck rather than the theory.
370 Indeed, many reported improvements in agent systems do not isolate the contribution of multi-agent
371 interactions from simply using more tokens, more wall-clock time, or a better judge. This is a valid
372 concern, and we structure our call to action for the larger community to resolve this.

373 8 Call to Action

374 Current practice often evaluates agent systems by asking whether adding more agents improves a
375 task metric. We believe this is insufficient for understanding when additional agents help, when they
376 interfere, and which abstractions allow the system to remain controllable as N grows. Thus, we call
377 for an agenda centered on mean-field agentic scaling.

378 **A1: Researchers.** Researchers should report agentic emergence as a scaling phenomenon under a
379 fixed agent capability, specifying the base policy, population size, and interaction topology. Without
380 these, it is difficult to distinguish genuine emergence from other factors. In addition to outputs and
381 transcripts, researchers should log population states by recording task stage, tool calls, communi-
382 cation events, and resource use to estimate microstate distributions and broadcast signals, enabling
383 direct tests of whether a mean field signal improves collective outcomes. Researchers should also
384 develop scaling-aware theoretical guarantees to explicitly track how the mean-field approximation
385 error and communication cost depend on population size, heterogeneity, and topology.

386 **A2: Practitioners.** Practitioners should instrument population states early by exposing aggregate
387 metrics (which tasks agents are working on, where failures concentrate, how often work is repeated,
388 and how traffic grows with N). Secondly, they should avoid scaling by all-to-all deliberation: multi-
389 round debate and committee voting may suffice at small scale, but before adding agents, practi-
390 tioners should examine how communication, evaluation, and tool-use costs scale with N . Finally,
391 deployment teams should monitor for negative emergence (synchronous mistakes, correlated hal-
392 lucinations, duplicated tool calls, and collective overconfidence) which are pathologies invisible in
393 small-scale experiments, requiring population-level diagnostics and interventions.

394 **A3: For the Broader Community.** The broader AI agents research community should build more
395 datasets to study agent behavior at the population level. Many contemporary agent traces are kept
396 private, are too task-specific, or cannot be collectively analyzed. Public datasets need multi-agent
397 trajectories with tool calls and intermediate states that cover a wide variety of domains, including
398 coding and embodied control. Moreover, the community should also set a higher standard on the
399 reporting norms for agentic scaling. Papers should report not only the number of agents used,
400 but also the communication topology, total token and per-agent budgets, and marginal benefit of
401 each additional agent. A claim that a system scales should include evidence about how cost and
402 performance change with N , not merely that a larger system outperforms a smaller one.

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600 A Mean Field Learning in Systems for Embodied AI Agents

601 In robotics, multi-agent systems can be used to model a team of individual robots, or even the
602 components of a single robot (for example, a pair of robot arms) that jointly coordinate to execute a
603 task. Practical applications of robotics in multi-agent settings, such as heterogeneous robot fleets or
604 dexterous manipulators, expose both the necessity and the limits of naive scaling. Our position in
605 favor of using mean field learning for principled scaling naturally finds support in the robotics setup.
606 We lay out our arguments in this section by examining two independently developed foundation
607 model families for physical robotics: Gemini Robotics 1.5 [Team et al., 2025] and $\pi_{0.7}$ [Intelligence
608 et al., 2026].

609 **Remark.** We note that the following is our interpretation of the results presented in Team et al.
610 [2025] and Intelligence et al. [2026] through the lens of mean-field learning. We call for further
611 research to the conceptual directions outlined below, to further validate the empirical results
612 reported by the authors of Gemini Robotics 1.5 and $\pi_{0.7}$.

613 A.1 The Embodied Agent System

614 We instantiate the agentic system $\mathcal{S} = (\mathbf{A}, E, \mathcal{C}, \Omega)$ of Definition 3.2 for a robotic fleet as fol-
615 lows. The environment E is a physical workspace with dynamics governed by rigid-body physics
616 and shared resource constraints, such as workspace volume, sensor coverage, communication band-
617 width. The agent set $\mathbf{A} = \{a_1, \dots, a_N\}$ is a fleet of N robots. Following Definition 3.2, each
618 agent $a_i = (\Phi_i, \mathcal{A}_i, M_i, \pi_i)$ instantiates a foundation model Φ_i , which can be a VLM, VLA, or
619 world model, an action space \mathcal{A}_i consisting of tool calls over motor primitives and perception APIs,
620 a memory M_i encoding task history and sensorimotor traces, and a decision function $\pi_i : \mathcal{H} \rightarrow \mathcal{A}_i$
621 mapping observation histories to actions.

622 In this setting, the coordination bottleneck is immediate. In a fleet of N manipulation arms sharing
623 a workspace, or N mobile robots engaged in collaborative coverage, each agent’s optimal action
624 depends on the current configurations of all other agents. Tracking the joint state $\vec{s}_t = (s_t^1, \dots, s_t^N)$
625 exactly scales as $|\mathcal{S}|^N$, placing naive coordination firmly in the intractable regime identified
626 in Section 4. The mean-field signal m_t , updated as $m_t \leftarrow \mu(m_{t-1}, \vec{s}_{t-1}, \vec{a}_{t-1})$ replaces this
627 exponential joint state with a population-level aggregate, enabling each agent’s policy $\pi_i(\cdot | s_t^i, m_t)$
628 to condition on collective behavior without explicit pairwise communication. In the robotic setting,
629 m_t can be thought of as a compressed representation of system dynamics, agent kinematics, failure
630 modes, or policy uncertainty.

631 The most structurally significant observation across both system families [Team et al., 2025, Intelli-
632 gence et al., 2026] is their convergence on an identical two-level hierarchical architecture.

633 In Gemini Robotics 1.5, GR-ER 1.5 is a VLM which serves as a high-level orchestrator that performs
634 spatial reasoning, multi-step task planning, progress estimation, and external tool use including web
635 search. GR 1.5 is a VLA which serves as a low-level executor that translates perceptual observations
636 and natural language instructions from the VLM orchestrator into motor commands, with its own
637 internal Embodied Thinking capability that interleaves reasoning traces with action generation. GR-
638 ER 1.5 can call GR 1.5 as a subagent, alongside any user-defined tool.

639 On the other hand, $\pi_{0.7}$ instantiates precisely the same decomposition under a dual-process framing:
640 a System 2 high-level VLM that reasons through complex prompts and generates intermediate
641 language subtasks, and a System 1 low-level VLA that executes those subtasks reactively. The
642 convergence of both Team et al. [2025] and Intelligence et al. [2026] at this two-level hierarchical
643 structure is strong evidence that it reflects a genuine inductive bias of physical task execution rather
644 than a contingent design choice.

645 In a fleet of N robots, each robot runs one orchestration agent and one execution agent. The or-
646 chestration mean field m_t^{orch} summarizes the population distribution of planning states across the
647 orchestration class. The execution mean field m_t^{exec} summarizes the population distribution of mo-
648 tor contexts and reasoning traces across the execution class. Each class conditions on the other’s
649 mean field rather than on the full joint state of all peer agents, recovering the polynomial sample
650 complexity scaling established in Section 4.

651 This hierarchical nature also hints at the potential for agentic emergence in this setting following
652 a Stackelberg mean-field formulation, in which the VLM class constitutes a population of leaders
653 and the VLA class constitutes a population of followers, with the equilibrium concept incorporat-
654 ing the hierarchical dependency. Such a formulation would also capture the temporal asymmetry
655 of embodied tasks, where high-level planning necessarily precedes and constrains low-level motor
656 commitment, and where the cost of replanning is much higher than the cost of reactive execution
657 adjustment. We believe this could be an important direction of future research in this space.

658 **A.2 Population-Level Conditioning as Empirical Mean Field**

659 Beyond the shared hierarchical architecture, both system families independently instantiate the
660 mean-field signal m_t of Section 3 as a concrete engineering mechanism, though in different forms
661 that together illuminate complementary aspects of the framework.

662 In Gemini Robotics 1.5, the Motion Transfer (MT) mechanism enables a single GR 1.5 checkpoint
663 to learn from heterogeneous data collected across physically distinct robot platforms: ALOHA,
664 Bi-arm Franka, and the Aptronik Apollo humanoid, by abstracting away robot-specific kinematic
665 details into a shared generalized action space. This can be interpreted as a population-level
666 abstraction at training time: MT learns a representation that is sufficient for motor control across
667 the full distribution of embodiment configurations in the training fleet, discarding morphological
668 variation that does not affect the underlying physics of the task. Specifically, MT operationalizes
669 the information-bottleneck objective over the population of embodiments by minimizing $I(m_t; X)$,
670 the compression of embodiment-specific history, while maximizing $I(m_t; Y)$, the predictive value
671 for task-relevant motor behavior.

672 In $\pi_{0.7}$, the same principle is instantiated via multimodal conditioning: $\pi_{0.7}$ is trained with prompts
673 that include not only task descriptions but episode metadata encoding how tasks were performed
674 across the training population (for example, execution quality, speed, strategy) alongside control
675 modality labels and visual subgoal images. Suboptimal trajectories are also incorporated by
676 annotating with appropriate quality and speed metadata. Based on the analysis in Intelligence
677 et al. [2026], the performance improvements observed with the inclusion of the conditioning
678 information seem to indicate to us that the metadata might act as a sufficient statistic for the
679 agent’s behavioral class within the training population. Therefore, our hypothesis suggests that this
680 metadata conditioning is a static, training-time mean field signal: a compressed summary of the
681 population’s behavioral distribution injected into each individual agent’s policy as a conditioning
682 input, instantiating the information-bottleneck objective over the offline training distribution rather
683 than online fleet telemetry.

684 The two mechanisms are thus instantiations of the same underlying principle of encoding
685 population-based distributions via mean-field learning (Section 3), which supports our position. MT
686 abstracts over the heterogeneity of the training population, whereas diverse conditioning abstracts
687 over the behavioral strategy dimension. Both demonstrate empirically that such population-level
688 abstractions exist, are learnable from finite data, and substantially improve generalization, which is
689 precisely the existence condition required for mean-field theory to guarantee tractable policy opti-
690 mization. Taken together, they validate the mean-field framework not as a theoretical imposition on
691 these systems but as the formal structure those systems are already approximating.

692 **A.3 Compositional Task Generalization and Embodiments as Agentic Emergence**

693 Both Team et al. [2025] and Intelligence et al. [2026] provide empirical evidence of agentic emer-
694 gence, manifesting through distinct but structurally related phenomena: compositional task general-
695 ization in $\pi_{0.7}$, and cross-embodiment generalization in GR 1.5.

696 $\pi_{0.7}$ explicitly identifies compositional generalization, the ability to recombine skills learned from
697 distinct training tasks to solve tasks never seen in training, as its central emergent capability. Intelli-
698 gence et al. [2026] reports that $\pi_{0.7}$ can operate a kitchen air fryer having seen only two episodes of
699 closing an air fryer and unrelated DROID dataset manipulation data; it can fold laundry on a biman-
700 ual UR5e platform having collected no laundry folding data for that robot, achieving success rates
701 matching expert human teleoperators attempting the same zero-shot transfer. This emergent capa-
702 bility is threshold-like in the sense of Section 4. It is absent in task-specific fine-tuned specialists
703 operating below the diversity threshold, and reliably present in $\pi_{0.7}$ operating above it, with the same

704 per-agent policy architecture in both cases. The population measure $\mu_t^{(N)}$ in this context is the em-
705 pirical distribution over training-time behavioral strategies and embodiment configurations, which
706 is exactly the distribution that the diverse conditioning mechanism compresses into the metadata
707 signal and that the mean field framework tracks dynamically at deployment.

708 GR 1.5’s Motion Transfer mechanism leads to the same threshold-based emergence phenomenon
709 observed over the morphology: a single VLA checkpoint achieves zero-shot control across ALOHA,
710 Bi-arm Franka, and Apollo humanoid platforms by abstracting the population of embodiments
711 into a shared action space, a capability that does not exist in platform-specific fine-tuned models
712 and emerges only at sufficient embodiment diversity in the training population. Such deployment
713 settings with heterogeneous platform types follow the frameworks of multi-class and graphon mean
714 fields, and we believe could be avenues for future research in proposing a principled formalism of
715 agentic emergence in robotics.