RoToR: Towards More Reliable Responses for Order-Invariant Inputs

Anonymous ACL submission

Abstract

Mitigating positional bias of language models (LMs) for **listwise** inputs is a well-known and important problem (e.g., lost-in-the-middle). While zero-shot order-invariant LMs have been proposed to solve this issue, their success on practical listwise problems has been limited. In this work, as a first contribution, we identify and overcome two limitations to make zero-shot invariant LMs more practical: **(1)** training and inference distribution mismatch arising from modifying positional ID assignments to enforce invariance, and **(2)** failure to adapt to a mixture of order-invariant and sensitive inputs in practical listwise problems. Then, to overcome these issues we propose **(1)** RoToR, a zero-shot invariant LM for genuinely order-invariant inputs with minimal modifications of positional IDs, and **(2)** Selective Routing, an adaptive framework that handles both order-invariant and order-sensitive inputs in listwise tasks. On the Lost in the middle (LitM), Knowledge Graph QA (KGQA), and MMLU benchmarks, we show that RoToR with Selective Routing can effectively handle practical listwise input tasks in a zero-shot manner.[1]

1 Introduction

Language conveys meaning in part through positional information, such as word placement and sentence structure. Given this nature, Language Models (LMs) that learn from human language are trained sensitive to positional information related to the ordering of segments. However, there are some listwise inputs that require neutrality to positional information. For example, for inputs such as sets, tables, databases, or multiple-choice questions, the ordering of the input **segments**—e.g., rows in a table or elements in an unordered set—require an order-agnostic understanding. We refer to such inputs as "order-invariant inputs," on which LMs reportedly struggle. For example, in LLM-as-a-judge scenarios, LMs exhibit a preference of up to 75% for the first answer in pairwise inputs (Zheng et al., 2024b), and ranking between LMs can change up to 8 positions in different orderings of multiple choice questions on MMLU (Alzahrani et al., 2024). Such results question the reliability of LMs on order-invariant inputs. Meanwhile, existing methods for enforcing invariance to LMs showed limited effectiveness in real-world tasks, which we hypothesize to arise from the following limitations.

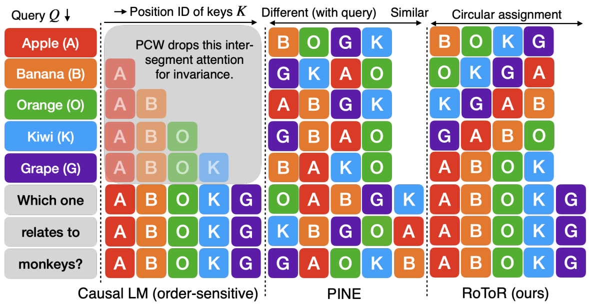


Figure 1: Self-attention alteration from order-invariant models. (a) PCW by elimination (b) PINE by re-assignment of position IDs based on query-based pairwise ordering. In contrast, (c) RoToR minimizes the distribution mismatch by global ordering with circular assignment.

First, training and inference distribution mismatch due to the positional ID re-assignment of zero-shot order-invariant LMs: Fig. 1 illustrates how self-attention is altered in these models. Unlike the original non-invariant model which always assigns position IDs in a causal, ascending manner, order-invariant models either eliminate inter-segment attention, such as PCW (Ratner et al., 2023) in Fig. 1a, or re-assign position IDs as in PINE (Wang et al., 2024) in Fig. 1b, re-ordering segments using pairwise similarity, placing similar segments closer to the query. For each query segment, it computes segment-wise query-key attention (for each attention head in each decoder

[1]The code will be publicly available upon acceptance.

layer) and re-assigns position IDs of segments as keys. This query-dependent segment ordering leads to excessively frequent alterations of positional ID assignments. Frequent re-assignments can also confuse the model and risk collisions which violate the invariance property (e.g., multiple key segments having the same similarity to a query).

To overcome this, we propose a query-agnostic global sorting with circular arrangement for order-invariant positional ID assignment. Ours is named **RoToR**, inspired by the word *rotary* to express circular assignment, and also a palindrome, to reflect order invariance. Fig. 1c contrasts with PINE in Fig. 1c, where RoToR only needs a single global ordering (e.g., `A->B->O->K->G`) with no extra attention computation. The ordering of segments on suffix tokens remains in a fixed order, since it does not rely on their similarity to the query. Finally, we propose three different global sorting algorithms for RoToR, and demonstrate that they consistently outperform previous order-invariant models.

Second, for practical listwise inputs, order-invariant tasks may partially include order-sensitive inputs that require order-specific understanding. For example, the `(d) None of the above` option in MMLU cannot be reordered. Such a "mixed" nature requires handling each of the cases adaptively, for which we propose a simple Selective Routing method. Selective Routing adapts to a given input by routing between two models, invariant and non-invariant (original), based on the confidence scores of their predictions. Experiments on the MMLU benchmark show that Selective Routing effectively handles datasets with order-invariant and sensitive inputs, and achieves better order robustness while maintaining the original performance.

In summary, our contributions are as follows: **1. Clarifying key challenges to robust understanding of listwise inputs.** We pinpoint the distribution mismatch and positional ID assignment complexities that hinder zero-shot order-invariance in LMs, and the need to adaptively handle order-invariant and order-sensitive inputs. **2. A stable, order-invariant solution (RoToR):** We propose a query-agnostic global ordering with minimal positional ID modifications, resulting in stable and efficient order-invariance. **3. Adaptive handling of listwise inputs (Selective Routing):** We introduce a simple routing method that switches between the original and invariant LMs based on confidence. On MMLU, we show that Selective Routing can adaptively deal with both types of input, leading to better stability. To this end, we aim to develop a model that excels at processing a wide range of listwise inputs reliably and efficiently.

2 Related Works

2.1 Positional bias of LLMs

Problem statement. Recent works on (zero-shot) retrieval augmented generation (RAG) with LLMs have found that the models exhibit unwanted bias on the *ordering* of the retrieved documents (Chhabra et al., 2024). Widely known as the lost-in-the-middle problem (Liu et al., 2024), many prior studies (Chen et al., 2024; Gupta et al., 2024; Pezeshkpour and Hruschka, 2023; Zhao et al., 2023; Zhou et al., 2024; Wei et al., 2024; Alzahrani et al., 2024; Zheng et al., 2024a) also investigate the importance of positional bias, extending the domain to structured knowledge grounding (SKG) tasks (Zhao et al., 2023; Zhou et al., 2024) and multiple-choice questions (Gupta et al., 2024) where changing the ordering of rows, schemas, or choices greatly degrades performance.

Considerations for decoder-only LMs. While successful approaches are presented to mitigate this issue for encoder-only (Yang et al., 2022) and encoder-decoder (Yen et al., 2024; Cai et al., 2023) models, they leave decoder-only models, which account for most of the current LLMs, for more consideration. In contrast to transformer encoders that use bidirectional attention which is invariant by nature (Lee et al., 2019), transformer decoders use causal attention to learn causal relation signals, which is not invariant by nature (Haviv et al., 2022a). Therefore, positional bias for decoder-only models is known to stem from *both* positional encoding and causal attention mask (Yu et al., 2024; Wang et al., 2024) and is harder to mitigate.

2.2 Zero-shot order-invariance for LLMs

Long context modeling. Zero-shot approaches for mitigating positional bias in LLMs were first raised in long-context tasks, with a goal to correctly handle relevant information located in the *middle* of lengthy inputs[2]. Nonetheless, these works focus primarily on understanding long texts without losing precision (Li et al., 2023; Zhang et al., 2024a; An et al., 2023; Bai et al., 2024), whereas positional bias is a more general problem that can occur even on multiple-choices questions with relatively short contexts (Alzahrani et al., 2024). Technically, this

[2] github.com/gkamradt/LLMTest_NeedleInAHaystack

line of works modify the attention mechanism by altering the positional encoding to adapt an LLM to longer contexts (Peng et al., 2023; Hsieh et al., 2024; Peysakhovich and Lerer, 2023; Chen et al., 2023; Junqing et al., 2023; Xu et al., 2023; Yu et al., 2024; Zhang et al., 2024b). But since they do not modify the causal mask which also contributes to positional bias, order-invariance is not guaranteed in general (Haviv et al., 2022b).

(Zero-shot) order-invariance. Recent line of works focused on achieving order-invariance by mechanistically altering both positional encoding and causal masking. While several works require training (Junqing et al., 2023; Zhu et al., 2023), we focus on zero-shot approaches for practicality, namely PCW, Set-Based Prompting (Ratner et al., 2023; McIlroy-Young et al., 2024), and PINE (Wang et al., 2024), which we explain in detail at Sec. 3.1. Another line of works based on self-consistency try to mitigate positional bias simply by running inference multiple times with different orderings of contexts (Zheng et al., 2024a). However, in principle, this requires evaluating $n!$ forward passes in total, enforcing Monte Carlo approximations (Tang et al., 2024). More recent work optimizes the number or passes (Lee et al., 2025b) with similar comprehensiveness (Hwang and Chang, 2007), or replaces with contrastive training objective (Lee et al., 2025a). In contrast, our method guarantee invariance with a *single* forward pass, without requiring any approximations.

3 Methodology

3.1 Baseline: Order-invariant causal LMs

In this section, we briefly overview the existing work on endowing decoder-only models on order-invariance by adjusting attention mechanism, and review their limitations.

Isolated parallel processing Prior works like PCW (Ratner et al., 2023) and Set-Based Prompting (McIlroy-Young et al., 2024) have modified the attention mask and positional ID assignments of the language model to isolate the processing of each segment and apply same positional embeddings are applied across segments, and thus achieve order invariance: However, this design completely prevents one segment from attending to the others, and aggregating the information from different segments is solely handled at suffix and generated tokens, significantly hindering the LM's cross-segment contextualized understanding of the text. Yang et al. (2023) have argued that this essentially degenerates to mere ensemble of conditioning on each context separately. Such information bottleneck and train-test time discrepancy limits the applicability, more severely as the number of segments is increased.

Bidirectional processing with Q-K similarity A more recent work, PINE (Wang et al., 2024) has addressed these issues through a bidirectional attention mechanism, by letting each segment attend to all other segments. However, to allow this within decoder-only models with causal attention and still achieve order invariance, PINE modifies the multi-head self-attention procedure to create an '*illusion*': it treats each query segment as if it were the final segment in the input list, so that it can attend to all other segments as keys. For each query segment, the ordering among the key segments is determined by their attention scores (without positional embeddings, $\text{Attn}_{\text{NoPoS}}$), ensuring that segments with stronger relevance to the query appear closer. Meanwhile, the tokens within a query segment still follow causal ordering. Fig. 2 illustrates this with the input `[T1"Given", S1["Apple"], S2["Ban", "ana"], S3["Orange"], T6"which one", .. T10"red?"]`. Prefix tokens "`Given`" and suffix tokens "`which one .. red?`" remain in their original positions and follow normal causal attention, equally attending to all segments. In contrast, the segments in the middle (S1 - S3) require the order-invariant mechanism. PINE dynamically re-assigns positional IDs depending on a token's role as a **query** or a **key**, instead of using a single fixed positional ID. For **Case 1: When "ana" (T4, S2) acts as a query**, S2 is assigned the largest position IDs (5) so it can attend to all previous segments. Within S2, the tokens "`ban`" (4) and "`ana`" (5) still maintain their local causal order (i.e., smaller to larger positions). **Case 2: When "ana" serves as a key,** the placement of S2 depends on its $\text{Attn}_{\text{NoPoS}}$ scores with the query segment. If "`banana`" (S2) is more relevant to the query segment "`apple`" (S1) than "`orange`" (S3), (i.e., $\textbf{Attn}_{\textbf{NoPos}}$(`S1, S2`) > $\textbf{Attn}_{\textbf{NoPos}}$ (`S1, S3`)) S2 is placed nearer to S1, with "ban" and "ana" preserving their internal sequence. For prefix, suffix, and generated tokens, they do not participate in order invariance, so they are always placed at their standard causal positions. However, the same dynamic reordering applies to segment tokens (T2–T5) when computing attention scores of suffix (T6–T10) and the generated token (T11) as query.

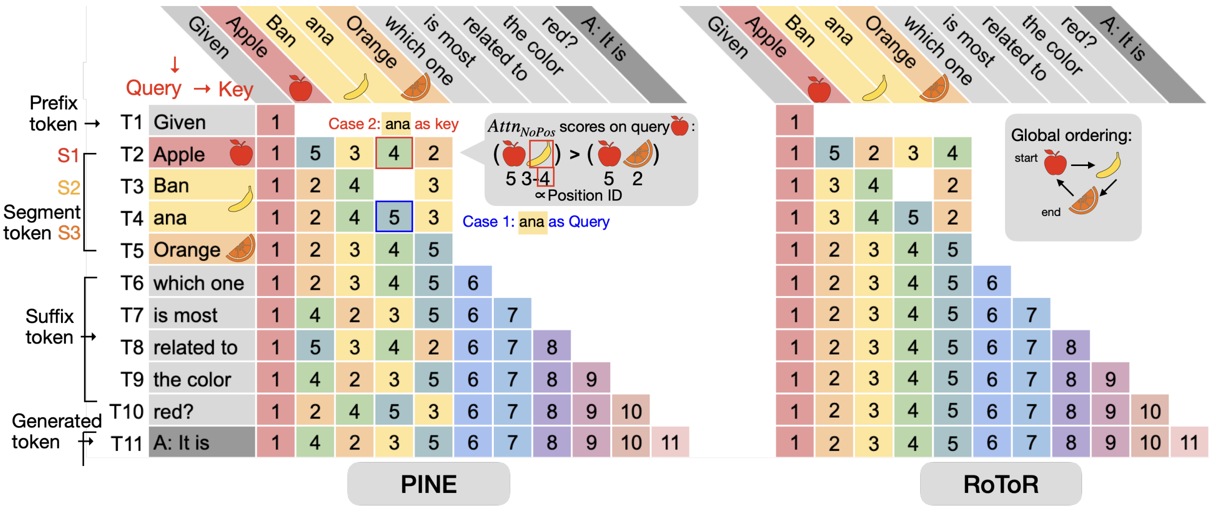


Figure 2: For example input "`Given Apple Banana Orange which one is most related to the color red? A: It is ...`", demonstration of how PINE and RoToR (ours) modify the attention mask and positional IDs for each query-key combination, resulting in segment-wise order invariance. Assume each block represents a single token, and blocks are colored in positional ID assignment. White (no-block) area indicate masked attention.

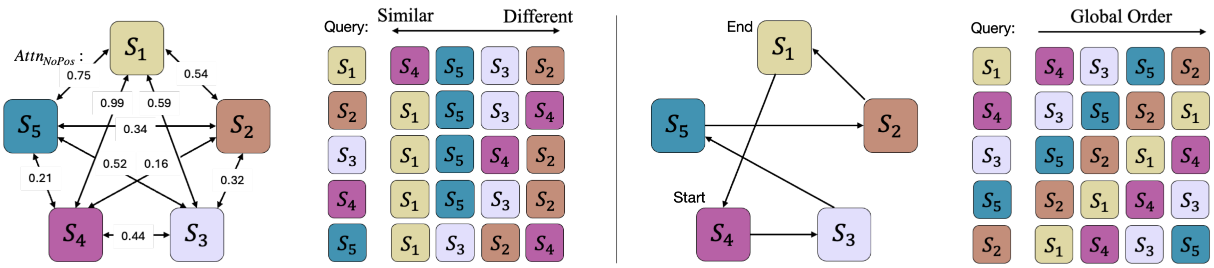


Figure 3: Comparing the ordering of 5 segments (S1 - S5) of PINE (Wang et al., 2024; left) and RoToR (Ours; right). PINE sorts segments using aggregated attention scores. In order to be fully ordering-invariant, segment sorting is changed per token in suffix level, causing confusion. In contrast, we define one global sorting of segments and conduct circular assignment between segments. With this, we simply use the global sorting for position id assignment on suffix tokens, without harming invariance.

3.2 RoToR: minimal OOD from positional ID assignments

While PINE achieves order-invariance by contextualization across segments, its query-specific ordering scheme introduces (1) significant train-test behavior discrepancy as well as (2) unnecessary complexity and numerical instability, which limits its scalability. During decoding with PINE, position IDs are assigned differently for every query token (each token in the suffix), decoder layer, and attention head, as the query-key attention score $\mathrm{Attn}_{\mathrm{NoPos}}$ determines the position IDs. This complexity introduces **excessively frequent alterations on position IDs**: As the base LM is trained with fixed positional IDs and causal masks, this causes hidden activations higher risk of out-of-distribution (OOD) for it to process properly. Moreover, ordering segments based on attention is **computationally expensive** and introduces **numerical instability**. As computing the attention value of one query segment requires computing the KV attention over every other number of segments, PINE invokes $\mathcal{O}(n^2)$ cost overhead for each segment for input length n, which is further multiplied by the number of all combinations of layers, heads, and the number of suffix and generated tokens. Also, in practice, calculating attention without RoPE results in a very narrow range of values. `bfloat16` numeric type lacks precision to distinguish these values, leading to non-determinism originating from several tied values. The outcome may depend on the initial ordering; to address these problems stemming from query-dependent ordering, we instead propose RoToR (Fig. 3), which uses one **global ordering** that is not a function of the initial ordering of segments (e.g., canonical ordering by lexical sorting) and assign IDs for tokens in different segments based on **circular arrangement**.

Global ordering Instead of re-computing the relative order of segments for each query, we reuse a globally shared single ordering, avoiding costly recomputation of numerically unstable attention scores. Moreover, this further reduces the gap between the LLM's pretrained behavior and test-time behavior, as consistent position IDs are assigned across layers/heads/across suffix tokens. Global ordering allows to preserve the relative placement of segments, further closing the gap induced from introducing order invariance to causal LMs. For example, in Figure 3, due to the global ordering, segments S_5 and S_2 are always placed in adjacent positions with RoToR (right side), while it is not satisfied and constantly changed with PINE (left side). We consider three separate global sorting algorithm to be used in RoToR: (1) simple **lexicographical sorting** which can be obtained with minimal overhead based on tokenized sequence of segments, (2) using a **pointwise reranker**[3] to score relevancy of each row with respect to the question, or (3) simple **freq**uency-based sorting which normalizes token ids based on the inverse frequency of each token (Details at Fig. 6). Empirically, we find that using simple lexicographical sorting is sufficient for bringing improvements over PINE.

Circular arrangement To mimic bidirectionality with causal LMs, each segment should be assigned position IDs so that they appear to themselves as being placed at the end of the sequence of segments. To achieve this with a shared global ordering, we employ circular arrangement, each segment taking turns to be placed at the end while their relative ordering is preserved. Given the global ordering, we can construct a single directed graph by combining the front and last parts. Then, we assign orderings for each segment as query by following the path from the graph, starting from the query segment, which is illustrated in Fig. 3. For all suffix and generated tokens, segments are arranged according to the initial front and last part of the global ordering. Compared to PINE where we have to assign different orderings of segments for each suffix and generated tokens, RoToR assign the same positional ID, acting merely the same as the original token. This also accounts for reducing the distributional gap between the original model.

[3] castorini/monot5-base-msmarco-10k

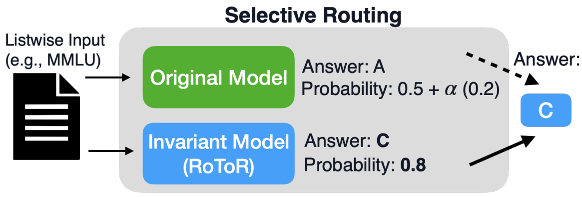


Figure 4: Illustration of Selective Routing (Sec. A.5).

Computational overhead By adopting global sorting, we can avoid extra attention scores computation and thus **improve efficiency**. While the cost with PINE to obtain hidden states is $\mathcal{O}(n^2d + nk\log k)$ for input length n, hidden dimension d, and k segments (Wang et al., 2024), our method with lexicographical sorting achieves $\mathcal{O}(nk\log k)$ cost, being more efficient and faster (lightweight) than PINE. We empirically validate that our method is much faster than PINE as k increases, at Tab. 4.

3.3 Selective Routing for handling order-sensitive inputs

Since many practical benchmarks such as MMLU involves semi-invariant inputs, we propose a routing mechanism that uses the order-invariant model in conjunction with the standard causal model for further applicability. Our design is partly based on the finding from Wei et al. (2024) that there is correlation between task difficulty (which is in turn correlated with confidence values) and the model's sensitivity to ordering. Selective Routing, illustrated in Fig. 4, combines confidence, the model output probability for the generated answer, from two different model versions—the original model and the order-invariant model—on the same input and choose a more confident answer. Both models first produce a maximum probability over possible answer tokens (e.g., `A, B, C, D` for MMLU) and a corresponding answer choice. We then compare the original model's maximum probability, plus a bias term α, to the invariant model's maximum probability. If the original model's adjusted score is higher, we take its answer; otherwise, the invariant model's answer is chosen. α is a hyperparameter that controls how strongly the original model is favored, which was selected as 0.2 according to hyperparameter search on the validation subset (Appendix Sec. A.5).

Total ndoc (segments)	10			20					30						
Gold idx at:	0	4	9	0	4	9	14	19	0	4	9	14	19	24	29
Llama-3.1-8B-Instruct															
Original	54.7	53.0	50.2	54.8	52.6	52.8	52.4	51.0	55.6	51.5	52.4	52.8	52.1	52.3	53.0
PCW	12.4	11.9	12.2	3.7	4.0	4.0	4.0	3.9	2.3	1.8	2.0	2.0	2.1	2.0	2.0
Set-Based Prompting	42.5	42.5	42.5	26.3	26.3	26.3	26.3	26.3	14.1	14.1	14.1	14.1	14.1	14.1	14.1
PINE	58.6	58.8	59.0	56.2	55.7	55.5	55.7	55.5	54.2	54.8	54.3	53.7	54.8	54.2	54.0
RoToR-lexical	61.4	61.6	61.6	**61.4**	59.8	59.6	59.6	59.8	59.2	59.5	59.4	59.1	59.0	59.3	59.1
RoToR-reversed lexical	**61.6**	**61.8**	**61.8**	58.9	59.3	58.8	58.6	58.7	57.9	58.2	57.9	57.4	57.9	57.6	57.5
RoToR-MonoT5	61.2	61.4	61.2	60.9	**61.0**	**61.2**	**61.2**	**61.2**	**60.9**	**60.7**	**60.7**	**60.7**	**60.8**	**60.8**	**60.7**
RoToR-Freq.	61.0	61.1	61.1	60.4	60.3	58.6	60.2	60.0	59.3	60.4	59.7	59.5	59.5	59.6	59.2
Qwen1.5-4B-Chat															
Original	61.3	54.8	53.1	**59.5**	49.1	47.9	45.9	48.3	**56.8**	45.6	44.9	44.6	45.3	43.5	48.3
PINE	57.2	57.4	57.0	48.6	48.2	48.2	48.1	48.9	46.4	-	-	46.6	46.4	46.4	46.3
RoToR	58.5	58.4	58.1	49.9	49.7	49.6	49.8	49.9	44.6	44.8	44.7	44.7	44.9	44.8	44.7
RoToR-MonoT5	**58.9**	**58.5**	**58.7**	52.2	**52.1**	**52.1**	**52.2**	**52.6**	50.6	**50.7**	**50.5**	**50.6**	**50.5**	**50.6**	**50.4**
RoToR-Freq.	56.7	56.9	56.9	51.9	51.5	51.8	51.6	52.4	46.8	46.7	46.7	46.4	47.0	46.8	46.6
Qwen1.5-7B-Chat															
Original	**72.5**	63.3	62.9	**72.5**	58.5	56.1	56.0	58.2	73.1	58.6	55.8	53.3	53.2	52.5	57.5
PINE	65.4	65.5	66.3	59.1	59.4	59.1	58.6	-	-	-	-	56.3	55.1	-	-
RoToR	68.6	68.7	68.6	62.6	62.9	62.7	63.0	62.7	57.0	57.3	-	-	-	-	-
RoToR-MonoT5	68.8	**69.4**	**69.0**	65.2	**65.5**	**65.0**	**64.9**	**65.0**	-	-	-	-	-	62.8	62.5
RoToR-Freq.	68.2	68.4	68.4	62.6	62.9	62.8	62.7	62.3	-	-	-	-	-	-	-

Table 1: The `best_subspan_em` (%) scores on the **lost in the middle (LitM)** benchmark, with indexing bias removed, across varying numbers of documents (ndoc $\in \{10, 20, 30\}$) and models. Results on Llama-3.1-70B-Instruct are on Appendix Tab. 6. RoToR shows the best performance across all setups. Due to resource constraints, some results are unavailable at the moment but will be reported during the author response period.

4 Experiment setup

4.1 Baselines

Original causal LM with no modifications (Orig.) were compared, which processes text sequentially. Also, we compare RoToR against previous zero-shot order-invariant LMs discussed in Sec. 3.1, namely PCW (Ratner et al., 2023), PINE (Wang et al., 2024), and Set-Based Prompting (McIlroy-Young et al., 2024). We use the LLaMA 3.1 (AI, 2024) 8B-Instruct[4], Qwen1.5-4B-Chat and Qwen1.5-7B-Chat[5] as our backbone model for all of our experiments. As our method **doesn't need training**, a single A6000 GPU was sufficient to run all of the experiments except for the Llama-3.1-70B-Instruct model (Appendix, Tab. 6). We also conduct experiments on a subset of benchmarks (LitM and MMLU) on the runtime latency, perplexity, and collision rate of PINE and RoToR, to further **compare RoToR with PINE** and validate our claims on Sec. 3.2.

4.2 Benchmarks with listwise inputs

We experiment with three benchmarks involving real-world listwise input data. Examples of exact inputs and outputs are provided in Appendix A.7.

All reported scores are rounded to the nearest tenth, except for the standard deviation (rounded to the second decimal place).

Knowledge Graph QA (KGQA) In KGQA tasks, the model takes facts over knowledge graphs represented as (subject, relation, object), and answers the given question based on the given facts. We basically follow the KGQA dataset preprocessing and evaluation setup from Baek et al. (2023), which uses Mintaka (Sen et al., 2022) with Wikidata for knowledge source, and use the Exact Match (EM), Accuracy (Acc), and F1 score metric for evaluation. We also use MPNet (Song et al., 2020) as a dense retriever to retrieve top-k facts over each question, and experiment with segment size of 30 and 50. Replication details and example dataset format are at Appendix Sec. A.3 and Fig. 10. Along with measuring the performance of the initial input ordering, we report performance after we **shuffle** the order of segments with 3 different seeds to see shuffle robustness.

Lost in the middle (LitM) We use the Lost in the Middle (LitM) benchmark (Liu et al., 2024), which draws from 2655 queries in the Natural Questions (NQ) dataset. It provides sets of (10, 20, 30) passages, placing the gold passage at predetermined positions (e.g., 0, 4, 9) and filling the remaining slots with irrelevant passages. Following Liu et al.

[4] `meta-llama/Meta-Llama-3.1-8B-Instruct`
[5] `Qwen/Qwen1.5-4/7B-Chat`

	Llama-3.1-8B-Instruct						Qwen1.5-4B-Chat						Qwen1.5-7B-Chat					
	N = 30			N = 50			N = 30			N = 50			N = 30			N = 50		
Method	Acc.	EM	F1	Acc.	EM	F1	Acc.	EM	F1	Acc.	EM	F1	Acc.	EM	F1	Acc.	EM	F1
Initial, no shuffling of segments																		
Original	50.2	44.0	51.9	50.0	44.0	51.7	30.7	27.9	34.9	31.6	28.6	35.8	31.5	27.8	35.4	31.7	28.0	35.7
PINE	51.5	45.0	52.6	51.6	45.1	52.6	31.6	28.7	35.6	31.6	28.8	35.3	32.3	28.8	36.4	32.0	28.5	35.9
RoToR	**53.1**	**46.5**	**54.1**	52.9	46.0	53.6	32.0	29.0	35.7	**32.7**	**29.6**	**36.2**	**34.3**	**29.8**	**37.7**	**34.3**	**30.1**	**38.0**
RoToR-MonoT5	51.6	45.0	52.5	52.4	45.4	52.8	**32.3**	29.1	**36.2**	32.3	29.3	35.9	32.9	28.4	36.3	32.9	28.9	36.6
RoToR-Freq.	52.6	46.1	53.7	**53.1**	**46.4**	**53.7**	**32.3**	**29.2**	36.0	32.3	29.2	35.9	33.7	29.5	37.2	33.5	29.5	37.2
After shuffling segments, averaged over 3 seeds																		
Original	49.5	43.3	51.0	49.7	43.5	51.0	30.1	27.5	34.7	30.3	27.6	35.0	31.4	27.3	35.0	31.6	27.9	35.5
↪ stdev. (±)	0.07	0.14	0.17	0.34	0.28	0.46	0.41	0.34	0.43	0.26	0.24	0.35	0.26	0.28	0.29	0.40	0.56	0.42
PINE	51.8	45.2	52.8	51.8	45.3	52.7	31.5	28.7	35.6	31.5	28.7	35.3	32.3	28.8	35.7	31.7	28.2	35.7
↪ stdev. (±)	0.05	0.07	0.16	0.15	0.16	0.19	0.20	0.18	0.13	0.17	0.20	0.21	0.17	0.20	0.13	0.18	0.16	0.14
RoToR	**52.8**	**46.2**	**53.8**	52.7	45.9	53.5	31.8	28.8	35.5	**32.5**	**29.6**	**36.1**	**34.2**	**29.9**	**37.7**	**34.2**	**30.1**	**38.0**
↪ stdev. (±)	0.05	0.05	0.02	0.05	0.09	0.04	0.05	0.02	0.09	0.11	0.06	0.09	0.09	0.07	0.06	0.06	0.05	0.04
RoToR-MonoT5	51.6	45.0	52.6	52.2	45.2	52.8	**32.4**	29.2	**36.3**	32.3	29.4	35.9	33.0	28.8	36.5	32.8	28.8	36.5
↪ stdev. (±)	0.12	0.06	0.10	0.16	0.18	0.18	0.04	0.02	0.13	0.16	0.13	0.07	0.12	0.09	0.07	0.16	0.09	0.07
RoToR-Freq.	52.5	45.9	53.5	**53.1**	**46.4**	**53.7**	32.3	**29.3**	36.0	32.4	29.3	**36.1**	33.8	29.6	37.4	33.7	29.6	37.4
↪ stdev. (±)	0.10	0.15	0.11	0.02	0.07	0.03	0.13	0.16	0.09	0.09	0.04	0.06	0.04	0.00	0.09	0.04	0.16	0.22

Table 2: Results on the Mintaka (KGQA) dataset on different models, with standard deviation of the average scores with ±. N refers to number of top-k segments per query.

	Llama-3.1-8B-Instruct			**Qwen1.5-4B-Chat**			**Qwen1.5-7B-Chat**		
Method	**Init.**	**Rev.**	**Avg.**	**Init.**	**Rev.**	**Avg.**	**Init.**	**Rev.**	**Avg.**
Orig.	68.3	64.8	65.5 ± 1.0	53.6	51.9	52.6 ± 0.6	**60.1**	56.6	58.6 ± 0.9
PCW	57.0	55.1	56.1 ± 1.1	–	–	–	–	–	–
Set-Based Prompting	31.1	33.0	31.6 ± 0.8	–	–	–	–	–	–
PINE	64.8	63.3	63.6 ± 0.7	50.5	49.3	49.4 ± 0.5	57.0	54.4	55.8 ± 0.9
RoToR	63.2	62.6	62.8 ± 0.7	49.6	47.7	48.3 ± 0.7	56.5	55.8	56.2 ± 0.6
↪ +*S.R.*	**68.5**	65.1	65.7 ± 0.9	53.7	51.8	**52.6 ± 0.6**	**60.1**	**57.4**	**58.8 ± 0.7**
RoToR - MonoT5	64.2	62.9	63.5 ± 0.5	49.7	47.6	48.7 ± 0.7	56.2	54.4	55.5 ± 0.7
↪ +*S.R.*	68.4	65.2	65.8 ± 0.9	**53.8**	51.9	**52.6 ± 0.6**	**60.1**	57.3	58.7 ± 0.8
RoToR - Freq.	64.3	63.6	63.8 ± 0.6	49.9	47.6	48.7 ± 0.5	56.4	54.7	55.7 ± 0.7
↪ +*S.R.*	**68.5**	**65.3**	**65.8 ± 0.8**	53.7	**52.3**	**52.6 ± 0.6**	60.0	57.3	58.6 ± 0.8
RoToR + S.R. (Optim.)	75.0	71.9	72.7 ± 1.0	61.8	60.1	61.1 ± 1.0	68.1	66.2	67.2 ± 0.7

Table 3: Improving applicability to general listwise tasks (MMLU, N=4) with **Selective Routing** (S.R), which includes **both** order-invariant **and** order-sensitive examples. RoToR with Selective Routing shows improved performance and stability across re-orderings of input, and its high upper-bound (Optim.) implies the potential for further improvements.

(2024), the `best_subspan_em` metric is used. Experiments on LitM found that eliminating the effect of index bias is another important detail for measuring true order robustness: (Appendix Sec. B). Thus, we report experiments with index bias eliminated. The exact prompt and full results including index bias is reported at Appendix Fig. 8 and Sec. A.1.

MMLU The Massive Multitask Language Understanding (MMLU) benchmark (Hendrycks et al., 2021) (prompts at Appendix Fig. 11) consists of 57 diverse sub-tasks with a total of 14,015 queries to measure general performance of LMs about the knowledge of the world. Despite its popularity, many works report that performance fluctuates heavily depending on the order of choices (Gupta et al., 2024; Pezeshkpour and Hruschka, 2023; Wei et al., 2024; Alzahrani et al., 2024; Zheng et al., 2024a) and is widely investigated to measure the positional bias of the model. We notice that a lot of proportions consist of ordering-sensitive inputs, which showed the effectiveness of adaptively applying Selective Routing. We additionally report the average performance for all possible (4!-1) re-orderings.

5 Results & Analysis

We report results for KGQA in Tab. 2, and results for MMLU in Tab. 3. Results for LitM are in Tab. 1, with a visualization in Appendix Fig. 5. We use lexical sorting for RoToR unless stated otherwise.

Effectiveness of RoToR We observe that shuffling input segments leads to non-trivial performance degradations in the original model, which exhibits a statistically significant performance drop

	Runtime latency (s)			Perple-xity (↓)	Collision Rate(↓)
	MMLU	LitM		LitM	
data count	14015	2655		2655	
# segments	4	10	30	20	30
PINE	7,371	18,551	41,664	6.91	42.3%
RoToR	6,608	14,264	23,569	**6.65**	**0 (None)**
Reduced %	10.4%	23.1%	**43.4%**		

Table 4: Compared to PINE, RoToR with lexical sort is faster, has lower perplexity, and have zero collision rate between segments. We use a subset of LitM with index bias removed and gold index = 0.

on our experimented dataset (two-tailed t-test, $p < 0.05$, Appendix C). In contrast, our proposed RoToR model does not show a statistically significant difference in performance before and after shuffling, indicating that it is more robust against such perturbations. On LitM (Tab. 1), we notice PCW and Set-Based Prompting has impractical performance, with PINE degrading heavily as number of documents (k) increases, while RoToR is less affected. On KGQA (Tab. 2), we show RoToR outperform PINE with lower standard deviation across shuffled segments, consistent with different model architectures.

Improvements from PINE Experiments against comparing RoToR with PINE (Tab. 4) analyze the following: **Runtime, scalability:** Actual inference times (Appendix Sec. A.6) find that RoToR outperforms PINE substantially, with efficiency gains increasing alongside n. For instance, on LitM (30 docs), RoToR achieves a 43% reduction in total runtime. Practical scalability with increasing k is critical, but we find that previous order-invariant LMs struggle handling larger k (on KGQA and LitM). In contrast, RoToR shows better performance with improved efficiency and robustness. **Perplexity:** Lower generation perplexity indicates input representations are closer to in-distribution. On the same LitM dataset, RoToR's reduced perplexity implies its positional ID assignment effectively mitigates out-of-distribution effects. **Collision Rate:** PINE's similarity-based ordering often collides: on average, only 17.3 of 30 similarity values are unique, causing 42% of the segments to be indistinguishable and thus breaking invariance. In contrast, RoToR with lexical sorting only collides if the segment texts are identical. On LitM, this yields zero collisions, preserving full invariance.

Selective Routing MMLU (Tab. 3) is a representative of a task that involves not only order-invariant, but also order-sensitive (e.g., "None of the above"), inputs. Therefore, single use of order-invariant models does not always outperform the original model, limiting applicability of order-invariant models to practical listwise tasks, i.e., we observe significant performance drops for Set-based Prompting in MMLU, falling short of half the performance of the original model on initial ordering. However, using RoToR with Selective Routing to handle order-sensitive inputs outperforms, or is at least competitive as the original model in all possible orderings of candidate choices. Selective Routing improves the generalizability and extends the applicability on practical listwise tasks by adaptively handling order-sensitive inputs. The RoToR + Selective Routing (Optimal) performance on Tab. 3 was evaluated using a relaxed accuracy metric based on the union of predictions from the original and the invariant (RoToR-lexical) model. This improves significantly, which highlights the potential of Selective Routing for further accuracy gains through optimizing design choices on routing methods, which we plan to explore in future work.

Impact of global ordering algorithm While most of our experiments focus on the simplest lexical sorting method, RoToR supports any global sorting approach. To demonstrate this flexibility, we report experiments with various global sorting strategies, including reversed lexical sorting, MonoT5-based reranking, and token frequency-based sorting. Lexical sort is presented as a baseline (lower bound) - a simple algorithm ensuring global sorting. Our experiments on Tab. 1 show that any type of global sorting, with the use of circular assignment is *superior than PINE*, which relies on pairwise attention arrangements.

6 Conclusion

Our work addresses order-invariance in listwise inputs by identifying core issues in distribution mismatch and adaptive handling of mixed inputs. Our proposed RoToR provides a stable zero-shot order-invariant solution that reduces the complexity of positional ID modification, while Selective Routing adaptively routes between invariant and sensitive LMs to handle real-world scenarios. Together, these methods demonstrate improved performance and reliability on LitM, KGQA, and MMLU benchmarks.

7 Limitations

Our method can utilize any kind of deterministic sorting algorithm, but we have only experimented with limited global sorting algorithms due to time and resource constraints. We plan to investigate potentially better sorting algorithms in the future. Also, current ordering-invariant models are limited to inputs given as prefix + parallel + suffix. It would be beneficial to support more complex structures, such as ability to process multiple order-invariant contexts interleaved with serial text.

References

Meta AI. 2024. Build the future of ai with meta llama 3.

Norah Alzahrani, Hisham Abdullah Alyahya, Yazeed Alnumay, Sultan Alrashed, Shaykhah Alsubaie, Yusef Almushaykeh, Faisal Mirza, Nouf Alotaibi, Nora Altwairesh, Areeb Alowisheq, M Saiful Bari, and Haidar Khan. 2024. When benchmarks are targets: Revealing the sensitivity of large language model leaderboards. *Preprint*, arXiv:2402.01781.

Chenxin An, Shansan Gong, Ming Zhong, Xingjian Zhao, Mukai Li, Jun Zhang, Lingpeng Kong, and Xipeng Qiu. 2023. L-eval: Instituting standardized evaluation for long context language models. *Preprint*, arXiv:2307.11088.

Jinheon Baek, Soyeong Jeong, Minki Kang, Jong C. Park, and Sung Ju Hwang. 2023. Knowledge-augmented language model verification. *Preprint*, arXiv:2310.12836.

Yushi Bai, Xin Lv, Jiajie Zhang, Hongchang Lyu, Jiankai Tang, Zhidian Huang, Zhengxiao Du, Xiao Liu, Aohan Zeng, Lei Hou, Yuxiao Dong, Jie Tang, and Juanzi Li. 2024. Longbench: A bilingual, multitask benchmark for long context understanding. *Preprint*, arXiv:2308.14508.

Tianle Cai, Kaixuan Huang, Jason D. Lee, and Mengdi Wang. 2023. Scaling in-context demonstrations with structured attention. In *Workshop on Efficient Systems for Foundation Models @ ICML2023*.

Xinyun Chen, Ryan A Chi, Xuezhi Wang, and Denny Zhou. 2024. Premise order matters in reasoning with large language models. *arXiv preprint arXiv:2402.08939*.

Yuhan Chen, Ang Lv, Ting-En Lin, Changyu Chen, Yuchuan Wu, Fei Huang, Yongbin Li, and Rui Yan. 2023. Fortify the shortest stave in attention: Enhancing context awareness of large language models for effective tool use. *arXiv preprint arXiv:2312.04455*.

Anshuman Chhabra, Hadi Askari, and Prasant Mohapatra. 2024. Revisiting zero-shot abstractive summarization in the era of large language models from the perspective of position bias. *Preprint*, arXiv:2401.01989.

Leo Gao, Jonathan Tow, Baber Abbasi, Stella Biderman, Sid Black, Anthony DiPofi, Charles Foster, Laurence Golding, Jeffrey Hsu, Alain Le Noac'h, Haonan Li, Kyle McDonell, Niklas Muennighoff, Chris Ociepa, Jason Phang, Laria Reynolds, Hailey Schoelkopf, Aviya Skowron, Lintang Sutawika, Eric Tang, Anish Thite, Ben Wang, Kevin Wang, and Andy Zou. 2024. A framework for few-shot language model evaluation.

Vipul Gupta, David Pantoja, Candace Ross, Adina Williams, and Megan Ung. 2024. Changing answer order can decrease mmlu accuracy. *Preprint*, arXiv:2406.19470.

Adi Haviv, Ori Ram, Ofir Press, Peter Izsak, and Omer Levy. 2022a. Transformer language models without positional encodings still learn positional information. In *Findings of the Association for Computational Linguistics: EMNLP 2022*.

Adi Haviv, Ori Ram, Ofir Press, Peter Izsak, and Omer Levy. 2022b. Transformer language models without positional encodings still learn positional information. *Preprint*, arXiv:2203.16634.

Dan Hendrycks, Collin Burns, Steven Basart, Andy Zou, Mantas Mazeika, Dawn Song, and Jacob Steinhardt. 2021. Measuring massive multitask language understanding. *Preprint*, arXiv:2009.03300.

Cheng-Yu Hsieh, Yung-Sung Chuang, Chun-Liang Li, Zifeng Wang, Long T Le, Abhishek Kumar, James Glass, Alexander Ratner, Chen-Yu Lee, Ranjay Krishna, et al. 2024. Found in the middle: Calibrating positional attention bias improves long context utilization. *arXiv preprint arXiv:2406.16008*.

Seung-won Hwang and Kevin Chen-chuan Chang. 2007. Optimizing top-k queries for middleware access: A unified cost-based approach. *ACM Trans. Database Syst.*, 32(1):5–es.

He Junqing, Pan Kunhao, Dong Xiaoqun, Song Zhuoyang, Liu Yibo, Liang Yuxin, Wang Hao, Sun Qianguo, Zhang Songxin, Xie Zejian, et al. 2023. Never lost in the middle: Improving large language models via attention strengthening question answering. *arXiv preprint arXiv:2311.09198*.

Juho Lee, Yoonho Lee, Jungtaek Kim, Adam R. Kosiorek, Seungjin Choi, and Yee Whye Teh. 2019. Set transformer: A framework for attention-based permutation-invariant neural networks. In *ICML*.

Youngwon Lee, Seung won Hwang, Daniel Campos, Filip Graliński, Zhewei Yao, and Yuxiong He. 2025a. Cord: Balancing consistency and rank distillation for robust retrieval-augmented generation. *NAACL*.

Youngwon Lee, Seung won Hwang, Daniel Campos, Filip Graliński, Zhewei Yao, and Yuxiong He. 2025b. Inference scaling for bridging retrieval and augmented generation. *NAACL*.

Jiaqi Li, Mengmeng Wang, Zilong Zheng, and Muhan Zhang. 2023. Loogle: Can long-context language models understand long contexts? *arXiv preprint arXiv:2311.04939*.

Nelson F Liu, Kevin Lin, John Hewitt, Ashwin Paranjape, Michele Bevilacqua, Fabio Petroni, and Percy Liang. 2024. Lost in the middle: How language models use long contexts. *Transactions of the Association for Computational Linguistics*, 12:157–173.

Reid McIlroy-Young, Katrina Brown, Conlan Olson, Linjun Zhang, and Cynthia Dwork. 2024. Set-based prompting: Provably solving the language model order dependency problem. *Preprint*, arXiv:2406.06581.

Bowen Peng, Jeffrey Quesnelle, Honglu Fan, and Enrico Shippole. 2023. Yarn: Efficient context window extension of large language models. *arXiv preprint arXiv:2309.00071*.

Alexander Peysakhovich and Adam Lerer. 2023. Attention sorting combats recency bias in long context language models. *arXiv preprint arXiv:2310.01427*.

Pouya Pezeshkpour and Estevam Hruschka. 2023. Large language models sensitivity to the order of options in multiple-choice questions. *arXiv preprint arXiv:2308.11483*.

Nir Ratner, Yoav Levine, Yonatan Belinkov, Ori Ram, Inbal Magar, Omri Abend, Ehud Karpas, Amnon Shashua, Kevin Leyton-Brown, and Yoav Shoham. 2023. Parallel context windows for large language models. In *Proceedings of the 61st Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 6383–6402, Toronto, Canada. Association for Computational Linguistics.

Priyanka Sen, Alham Fikri Aji, and Amir Saffari. 2022. Mintaka: A complex, natural, and multilingual dataset for end-to-end question answering. In *Proceedings of the 29th International Conference on Computational Linguistics*, pages 1604–1619, Gyeongju, Republic of Korea. International Committee on Computational Linguistics.

Kaitao Song, Xu Tan, Tao Qin, Jianfeng Lu, and Tie-Yan Liu. 2020. Mpnet: Masked and permuted pre-training for language understanding. *Preprint*, arXiv:2004.09297.

Raphael Tang, Xinyu Zhang, Xueguang Ma, Jimmy Lin, and Ferhan Ture. 2024. Found in the middle: Permutation self-consistency improves listwise ranking in large language models. *Preprint*, arXiv:2310.07712.

Ziqi Wang, Hanlin Zhang, Xiner Li, Kuan-Hao Huang, Chi Han, Shuiwang Ji, Sham M. Kakade, Hao Peng, and Heng Ji. 2024. Eliminating position bias of language models: A mechanistic approach. *Preprint*, arXiv:2407.01100.

Sheng-Lun Wei, Cheng-Kuang Wu, Hen-Hsen Huang, and Hsin-Hsi Chen. 2024. Unveiling selection biases: Exploring order and token sensitivity in large language models. *Preprint*, arXiv:2406.03009.

Peng Xu, Wei Ping, Xianchao Wu, Lawrence McAfee, Chen Zhu, Zihan Liu, Sandeep Subramanian, Evelina Bakhturina, Mohammad Shoeybi, and Bryan Catanzaro. 2023. Retrieval meets long context large language models. *arXiv preprint arXiv:2310.03025*.

Jingfeng Yang, Aditya Gupta, Shyam Upadhyay, Luheng He, Rahul Goel, and Shachi Paul. 2022. Tableformer: Robust transformer modeling for table-text encoding. *Preprint*, arXiv:2203.00274.

Kejuan Yang, Xiao Liu, Kaiwen Men, Aohan Zeng, Yuxiao Dong, and Jie Tang. 2023. Revisiting parallel context windows: A frustratingly simple alternative and chain-of-thought deterioration. *Preprint*, arXiv:2305.15262.

Howard Yen, Tianyu Gao, and Danqi Chen. 2024. Long-context language modeling with parallel context encoding. *Preprint*, arXiv:2402.16617.

Yijiong Yu, Huiqiang Jiang, Xufang Luo, Qianhui Wu, Chin-Yew Lin, Dongsheng Li, Yuqing Yang, Yongfeng Huang, and Lili Qiu. 2024. Mitigate position bias in large language models via scaling a single dimension. *arXiv preprint arXiv:2406.02536*.

Lei Zhang, Yunshui Li, Ziqiang Liu, Jiaxi yang, Junhao Liu, Longze Chen, Run Luo, and Min Yang. 2024a. Marathon: A race through the realm of long context with large language models. *Preprint*, arXiv:2312.09542.

Zhenyu Zhang, Runjin Chen, Shiwei Liu, Zhewei Yao, Olatunji Ruwase, Beidi Chen, Xiaoxia Wu, and Zhangyang Wang. 2024b. Found in the middle: How language models use long contexts better via plug-and-play positional encoding. *arXiv preprint arXiv:2403.04797*.

Yilun Zhao, Chen Zhao, Linyong Nan, Zhenting Qi, Wenlin Zhang, Xiangru Tang, Boyu Mi, and Dragomir Radev. 2023. Robut: A systematic study of table qa robustness against human-annotated adversarial perturbations. *Preprint*, arXiv:2306.14321.

Chujie Zheng, Hao Zhou, Fandong Meng, Jie Zhou, and Minlie Huang. 2024a. Large language models are not robust multiple choice selectors. In *The Twelfth International Conference on Learning Representations*.

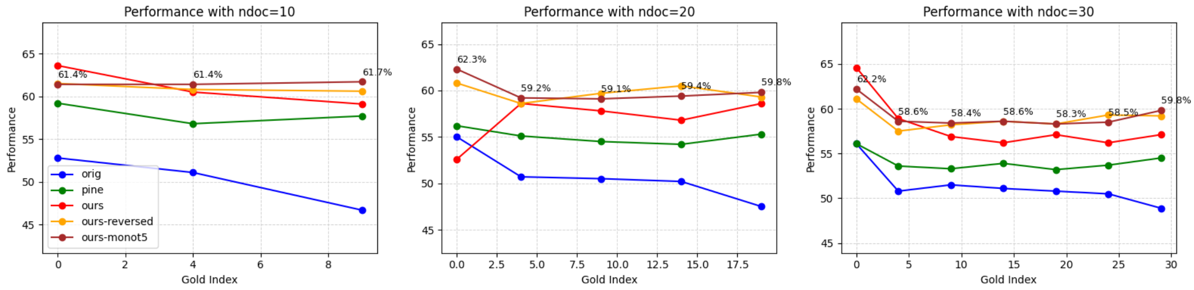
Lianmin Zheng, Wei-Lin Chiang, Ying Sheng, Siyuan Zhuang, Zhanghao Wu, Yonghao Zhuang, Zi Lin, Zhuohan Li, Dacheng Li, Eric Xing, et al. 2024b. Judging llm-as-a-judge with mt-bench and chatbot arena. *Advances in Neural Information Processing Systems*, 36.

Wei Zhou, Mohsen Mesgar, Heike Adel, and Annemarie Friedrich. 2024. Freb-tqa: A fine-grained robustness evaluation benchmark for table question answering. *Preprint*, arXiv:2404.18585.

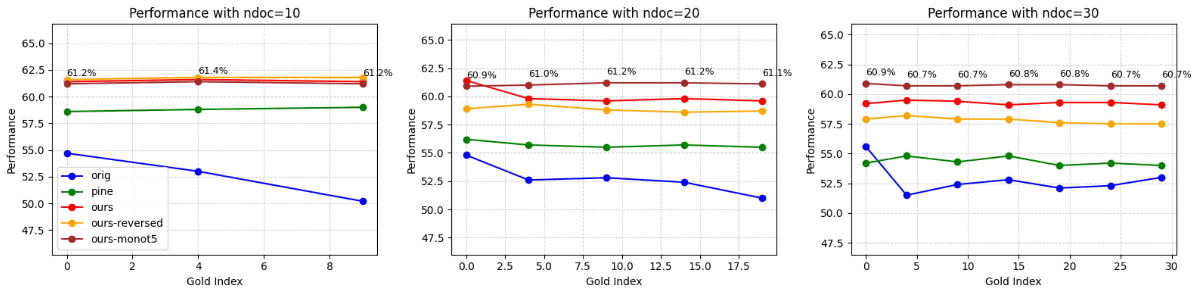
Lianghui Zhu, Xinggang Wang, and Xinlong Wang. 2023. Judgelm: Fine-tuned large language models are scalable judges. *arXiv preprint arXiv:2310.17631*.

A Appendix

A.1 Full results on the Lost in the Middle Benchmark



(a) Results **with index bias** (indexed by numbers). (Example input at Appendix Fig. 7)



(b) Results **without index bias** (indexed by title) (Example input at Appendix Fig. 8)

Figure 5: Results on the Lost-in-the-middle benchmark. Visualization of the `best_subspan_em` results at Appendix Tab. 1. RoToR (dark red, red, yellow) generally performs the best regardless of the position of the gold index, with less fluctuations when we remove index bias. Ours is RoToR with lexical sort, and ours-reversed is the one with the reversed lexical ordering. For brevity, only the performance of RoToR with reranking sort (MonoT5) is annotated as numbers, and the performance of PCW and Set-Based Prompting are reported only at the Table (Appendix Tab. 1) due to its low performance.

Impact of removing index bias on LitM Tab.5 presents the full results on the Lost in the Middle (LitM) benchmark, comparing scenarios where indexing bias is present versus removed. Fig.5 provides a visual representation of these results.

As shown in Appendix Tab.5, invariant LMs exhibit stable performance regardless of the gold index, especially when index bias is removed (as described in Sec.4.2; see also Appendix Fig.5b). However, when index bias is present, performance fluctuations are observed (Appendix Fig.5a). Notably, RoToR achieves the highest performance across all setups, demonstrating its effectiveness in mitigating positional bias in a zero-shot setting while maintaining overall performance.

These findings suggest that index bias acts as an implicit source of additional positional bias and that invariant LMs benefit significantly from its removal.

Results on other models. Table 6 shows results in different model (Llama-3.1-70B-Instruct). We can see that the trend continues for the 70B model, implying the robustness of RoToR.

A.2 Illustration of the global sorting method

We show the two different global sorting algorithms presented in our paper at Fig. 6.

A.3 Details about preprocessing and evaluation of datasets

A.3.1 General.

All inferences were done with a **single** NVIDIA RTX A6000 48GB GPU. Note that all of the baseline models including our method can be applied directly in a zero-shot, training-free manner. For repro-

Total ndoc (segments)	10				20						30							
Gold idx at:	0	4	9	avg.	0	4	9	14	19	avg.	0	4	9	14	19	24	29	avg.
Indexing bias present																		
Original	52.8	51.1	46.7	50.2	55.0	50.7	50.5	50.2	47.5	50.7	56.1	50.8	51.5	51.1	50.8	50.5	48.9	51.4
PCW	12.0	11.9	12.1	12.0	3.4	3.7	3.8	3.9	3.6	5.1	2.1	2.1	2.0	1.9	2.0	2.2	2.0	2.0
Set-Based Prompting	40.8	40.7	40.8	40.8	25.6	25.8	25.7	25.5	25.3	25.6	15.8	15.9	16.1	16.0	16.1	15.7	15.8	15.9
PINE	59.2	56.8	57.7	57.9	56.2	55.1	54.5	54.2	55.3	55.5	56.1	53.6	53.3	53.9	53.2	53.7	54.5	54.0
RoToR-lexical	**63.6**	60.5	59.1	61.1	52.6	58.6	57.8	56.8	58.6	57.6	**64.6**	**58.9**	56.9	56.2	57.1	56.2	57.1	58.1
RoToR-reversed lexical	61.5	60.8	60.6	61.0	60.8	58.6	**59.7**	**60.5**	59.3	60.0	61.1	57.5	58.2	**58.6**	**58.3**	**59.3**	**59.2**	58.9
RoToR-reranking	61.4	**61.4**	**61.7**	**61.5**	**62.3**	**59.2**	59.1	59.4	**59.8**	**60.2**	62.2	58.6	**58.4**	**58.6**	**58.3**	58.5	**59.8**	**59.2**
RoToR-freq	62.8	61.1	59.5	61.1	62.9	58.8	56.7	57.4	58.0	59.1	61.7	58.2	56.9	56.1	56.4	55.4	56.8	57.4
Indexing bias removed (main paper)																		
Original	54.7	53.0	50.2	52.6	54.8	52.6	52.8	52.4	51.0	52.7	55.6	51.5	52.4	52.8	52.1	52.3	53.0	52.8
PCW	12.4	11.9	12.2	12.2	3.7	4.0	4.0	4.0	3.9	3.9	2.3	1.8	2.0	2.0	2.1	2.0	2.0	2.0
Set-Based Prompting	42.5	42.5	42.5	42.5	26.3	26.3	26.3	26.3	26.3	26.3	14.1	14.1	14.1	14.1	14.1	14.1	14.1	14.1
PINE	58.6	58.8	59.0	58.8	56.2	55.7	55.5	55.7	55.5	55.7	54.2	54.8	54.3	53.7	54.8	54.2	54.0	54.3
RoToR-lexical	61.4	61.6	61.6	61.5	**61.4**	59.8	59.6	59.6	59.8	60.0	59.2	59.5	59.4	59.1	59.0	59.3	59.1	59.2
RoToR-reversed lexical	**61.6**	**61.8**	**61.8**	**61.8**	58.9	59.3	58.8	58.6	58.7	58.8	57.9	58.2	57.9	57.4	57.9	57.6	57.5	57.8
RoToR-reranking	61.2	61.4	61.2	61.3	60.9	**61.0**	**61.2**	**61.2**	**61.2**	**61.1**	**60.9**	**60.7**	**60.7**	**60.7**	**60.8**	**60.8**	**60.7**	**60.8**
RoToR-freq	61.0	61.1	61.1	61.1	60.4	60.3	58.6	60.2	60.0	59.9	59.3	60.4	59.7	59.5	59.5	59.6	59.2	59.6

Table 5: The `best_subspan_em` (%) scores on the lost in the middle (LitM) benchmark, with results on Llama-3.1-70B-Instruct on Appendix Tab. 6. For RoToR, we test three different global ordering strategies (lexical, reversed lexical, and MonoT5-base reranking) across varying numbers of documents (ndoc $\in \{10, 20, 30\}$). Appendix Fig. 5 visualizes the fluctuations across different gold positions. RoToR shows the best performance across all setups, and is especially more stable when indexing bias in the input is removed.

ndoc =	10				20
gold idx =	0	4	9	avg.	0
Orig.	66.2%	65.7%	65.7%	65.9%	65.2%
PINE	67.9%	67.8%	67.5%	67.7%	65.9%
RoToR-lex.	**69.6%**	**69.5%**	**69.3%**	**69.5%**	**67.6%**

Table 6: Reporting the performance with the **Llama-3.1-70B-Instruct** model on a subset of lost-in-the middle benchmark (without index bias) performance. We see that the gains are consistant with the models we mainly investigated (Llama-3.1-8B-Instruct)

ducibility, we fix the seed and disabled random sampling (i.e., used greedy decoding), set the maximum number of new generated tokens to 500, and set the pad_token_id to the same value as the eos_token_id for all experiments. For all datasets we tested, we separate the input to 3 parts: prefix, parallel contexts, and suffix and feed them accordingly to the positional-invariant model. For the case of the **original** model, we simply join the prefix, context, and suffix text to make one sequential text. For testing with the **PCW** model, we concat each prefix to the parallel context due to the architectural limitations of the PCW model, which doesn't have the prefix part (which is processed casually before parallel contexts). For example, on testing the PCW model on MMLU, we append the question and each answer options, which generally result in longer input sequences. For PCW, we use the `pcw_generate` function, and we additionally utilized the `RestrictiveTokensLogitsProcessor` provided at the official PCW repository for MMLU classification, to have a similar setup with the log_likelihood option used for other models.

A.3.2 Knowledge Graph Question Answering

For evaluation with Mintaka, we follow the same setup as Baek et al. (2023). Given the gold answer and model generated answer, the EM score counts if both are exactly the same; Accuracy measures if the generated answer includes the gold answer, and F1 score measures the precision and recall among overlapping words. Since we are testing on a non-trained zero-shot version of the model, we enforce the model to output in json format to make it easier to parse. For the row shuffling setup, we fix the seed to 0, 1, 2 on shuffling rows and report the average scores.

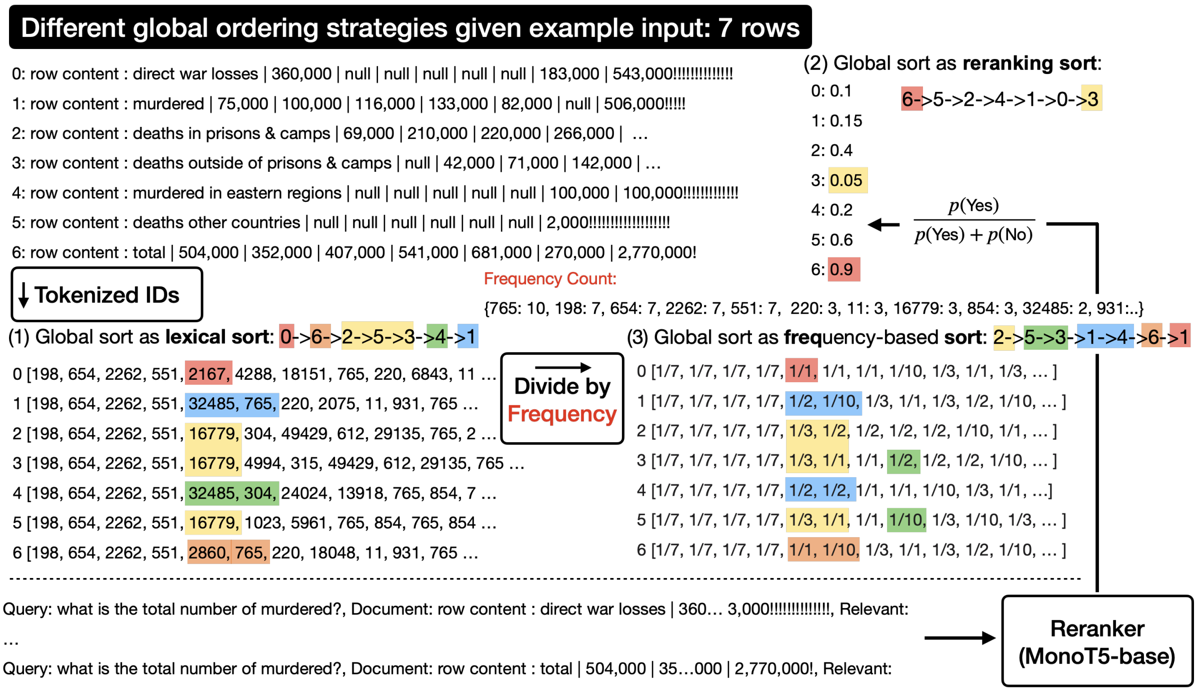


Figure 6: Illustration of ordering 7 rows by 2 different global sort options, (1) lexical sort based on token ids, or (2) reranking sort based on a reranker model (MonoT5-base in this case). (3) frequency based sorting applies lexical sorting, but the definitive token ids are mapped into the inverse frequency.

A.3.3 Lost in the Middle

Specifically, we use the dataset provided in the official repository[6], use the same prompt as the llama 2 chat model with only the instruction tokens adjusted to llama 3 (removed [Inst] and changed to <|begin_of_text|> and etc.,), and evaluate using the best_supspan_em metric.

A.3.4 MMLU

We follow the publicly acknowledged lm-evaluation harness (Gao et al., 2024) prompt design by eluther.ai. We measure accuracy between the gold answer and the token with the highest likelihood (probability) among possible answer tokens [' A', ' B', ' C', ' D'].

A.4 Further impact scenarios on general conversation.

We shortly discuss about how this method may be applied to general conversational scenarios of LLMs. For processing contexts such as chronological history of conversations, the ordering is important, and the original LLM remains the better choice for this case. However, in subsets of conversational tasks requiring order invariance (e.g., Sets, Tables, or RAG contexts), our method enhances unbiased understanding, as demonstrated mainly in Lost-in-the-Middle benchmark. Here, RoToR achieves a significant 7-9% average accuracy gain over the original LLM, very consistently across all setups (doc indexing and ndoc) for all choices of the ordering algorithm, with lower standard deviation than the original model.

A.5 Selection of α for Selective Routing on MMLU

We report α is a hyperparameter that can be tuned per-dataset. We searched its value in the range of -0.5 to 0.5 with a step size of 0.1 using the validation split of MMLU[7] on RoToR with lexical sorting, and applied the found value (0.2) on the test set to obtain the reported results for all models. We report the full variation of Selective Routing results on the investigated α value at Tab. 7.

[6] github.com/nelson-liu/lost-in-the-middle
[7] https://huggingface.co/datasets/cais/mmlu/viewer/abstract_algebra/validation

α =	no Selective Routing	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	**0.2**	0.3	0.4	0.5
Validation set (1531)		<— bias towards invariant model —— bias towards orig model —>										
Selective Routing (orig, pine)	64.9	65.3	65.8	66.4	67.3	67.4	67.6	67.5	**67.9**	67.8	68.0	67.9
Selective Routing (orig, ours-lexical)	63.9	64.1	64.5	65.4	65.8	67.0	67.4	68.1	**68.2**	68.1	67.9	67.9
Selective Routing (orig, ours-monot5)	66.0	66.1	66.3	66.7	67.0	67.6	68.0	68.1	**68.1**	68.1	67.7	67.9
Test set (14015)												
Selective Routing (orig, pine)	64.8	64.9	65.4	66.0	66.8	67.5	68.4	68.5	**68.5**	68.4	68.3	68.3
Selective Routing (orig, ours-lexical)	63.2	63.5	64.2	65.2	66.2	67.3	68.0	68.4	**68.5**	68.5	68.4	68.3
Selective Routing (orig, ours-monot5)	64.2	64.4	64.8	65.5	66.4	67.3	68.1	68.5	**68.4**	68.5	68.3	68.3

Table 7: Reporting full ablation results on application of Selective Routing. $\alpha = 0.2$ was the best for the validation set, which was then applied to obtain the reported results for all models.

A.6 Replication details on the runtime experiment

Apart from the theoretical runtime efficiency, we measured the actual end-to-end runtime in seconds, to better analyze the practical runtime efficiency between PINE and RoToR. The runtime of each experiment was measured on an ASUS ESC8000-E11 server featuring dual 4th Gen Intel Xeon Scalable processors, 64 CPU threads, 1.1 TB of RAM across 32 DIMM slots, and 8 NVIDIA A6000 GPUs with 48 GB of memory each. We Except for the experiments on Llama-3.1-70B-Instruct, we only use a single A6000 GPU for all of the experiments.

A.7 Input data examples

To illustrate the input and output formats used in our experiments, we provide example inputs for the Lost-in-the-Middle (LitM), Knowledge Graph Question Answering (KGQA), and MMLU datasets. For experiments using the Qwen-Chat model, special tokens were adjusted accordingly. While the example prompts are based on the Llama-3.1-8B-Instruct model, the specific differences in token usage for the Qwen-Chat variants can be observed by comparing the prompts in Fig. 8 and Fig. 9. This adjustment is consistently applied across all datasets. Note that no special tokens are added for the MMLU benchmark, which aligns with the `lm-evaluation harness` setup.

lost in the middle

Prefix:
<|begin_of_text|><|start_header_id|>system<|end_header_id|>

You are a helpful, respectful and honest assistant. Always answer as helpfully as possible, while being safe. Please ensure that your responses are socially unbiased and positive in nature. If a question does not make any sense, or is not factually coherent, explain why instead of answering something not correct. If you don't know the answer to a question, please don't share false information.<|eot_id|><|start_header_id|>user<|end_header_id|>

Write a high-quality answer for the given question using only the provided search results (some of which might be irrelevant).

Parallel texts:
Document [1](Title: List of Nobel laureates in Physics) The first ...
...
Document [10](Title: Nobel Prize in Chemistry) on December 10, the ...

Suffix:
Question: who got the first nobel prize in physics<|eot_id|><|start_header_id|>assistant
<|end_header_id|>

Figure 7: Example input for the lost in the middle dataset.

lost in the middle no indexing

Prefix:
<|begin_of_text|><|start_header_id|>system<|end_header_id|>

You are a helpful, respectful and honest assistant. Always answer as helpfully as possible, while being safe. Please ensure that your responses are socially unbiased and positive in nature. If a question does not make any sense, or is not factually coherent, explain why instead of answering something not correct. If you don't know the answer to a question, please don't share false information.<|eot_id|><|start_header_id|>user<|end_header_id|>

Write a high-quality answer for the given question using only the provided search results (some of which might be irrelevant).

Parallel texts:
[Document Title: List of Nobel laureates in Physics] The first ...
...
[Document Title: Nobel Prize in Chemistry] on December 10, the ...

Suffix:
Question: who got the first nobel prize in physics<|eot_id|><|start_header_id|>assistant
<|end_header_id|>

Figure 8: Example input for the lost in the middle dataset, without indexing by numbers. Prompt for the Llama-3.1-8B-Instruct model.

lost in the middle no indexing (Qwen variant)

Prefix:
<|im_start|>system
You are a helpful, respectful and honest assistant. Always answer as helpfully as possible, while being safe. Please ensure that your responses are socially unbiased and positive in nature. If a question does not make any sense, or is not factually coherent, explain why instead of answering something not correct. If you don't know the answer to a question, please don't share false information.<|im_end|><|im_start|>user

Write a high-quality answer for the given question using only the provided search results (some of which might be irrelevant).

Parallel texts:
[Document Title: Thorax] when deep breaths are attempted. Different people ...
...
[Document Title: Chest pain] present with chest pain, and carry a significantly higher ...

Suffix:
Question: for complaints of sudden chest pain patients should take a<|im_end|>
<|im_start|>assistant

Figure 9: Example input for the lost in the middle dataset, without indexing by numbers, prompt for the Qwen1.5-Chat model.

Mintaka

Prefix:
<|begin_of_text|><|start_header_id|>system<|end_header_id|>

Below are the facts in the form of the triple meaningful to answer the question. Answer the given question in a JSON format, such as "Answer": "xxx". Only output the JSON, do NOT say any word or explain.

<|eot_id|><|start_header_id|>user<|end_header_id|>

Parallel texts:
(Super Bowl XLII, winner, New York Giants)
(Super Bowl XLII, participating team, New York Giants)
(Super Bowl XLII, point in time, time: +2008-02-03)
(Super Bowl XLII, followed by, Super Bowl XLIII)
(Super Bowl XLII, location, State Farm Stadium)
...
(Super Bowl XLII, sport, American football)
(Super Bowl XLII, instance of, Super Bowl)

Suffix:
Question: which team did the super bowl xlii mvp play for?, Answer: <|eot_id|><|start_header_id|> assistant <|end_header_id|>

Gold Answer(s):
(‘NYG’, ‘Giants’, ‘NY Giants’, ‘New York Giants’)

Example generated output:
{"Answer": "New York Giants"} (Parsed to: New York Giants)

Figure 10: Example input for the Mintaka dataset.

MMLU

Prefix:
The following are multiple choice questions (with answers) about moral disputes.

Norcross agrees that if a being is incapable of moral reasoning, at even the most basic level, then it cannot be

Parallel texts:
A. a being of value.
B. an object of moral sympathy.
C. a moral agent.
D. a moral patient.

Suffix:
Answer:

Figure 11: Example input for the MMLU benchmark.

B Why is removing index bias an important detail for invariant models to be effective?

The alphabetic index (A/B/C/D) introduced in Fig. 1 associated with each segment, reportedly introduces token bias (Wei et al., 2024) of preferring the choice marked as 'A.' The same thing can be applied to listwise inputs with simple numeric indexing (1/2/3/4), which was the case for the lost-in-the middle benchmark. While a standard model with no modifications on positional encoding correctly places contexts indexed A before contexts indexed with D by positional encoding, an invariant model sees contexts in an order-agnostic way, meaning that the alphabetical indexing may not always be interpreted sequentially and thus can confuse the model from accurately interpreting the contexts. For example, even for cases where the index ordering of the input was in alphabetical order (A->B->C->D), the ordering-invariant model may interpret contexts with (C->A->B->D) at one point (e.g., when the query is D on self attention), which can cause unnatural, out-of-distribution representation, leading to decreased performance.

C Statisticial significance before and after shuffling segments

We conducted **paired two-tailed *t*-tests** (Table 9) for both the baseline ("original") model and our proposed method (RoToR), using the results in Table 8. Our goal was to determine whether the performance differences between the initial ordering and shuffled ordering are statistically significant. We excluded the Lost-in-the-Middle (LitM) dataset because it does not provide an initial ordering. Specifically, the tests evaluate whether the mean performance difference (Before Shuffle - After Shuffle) significantly deviates from zero.

For KGQA, we selected the F1 score as the representative metric among the three available, gathering data points from various task configurations and different models. For MMLU, the results are based on our RoToR variant with selective routing. As shown in Table 9, the original model shows a statistically significant drop in performance when the segments are shuffled, while RoToR does not, indicating increased robustness to segment-order perturbations. [8]

	Original Model			RoToR-lexical		
	Before Shuff.	After Shuff.	Diff.	Before Shuff.	After Shuff.	Diff.
Mintaka, Llama3.1-8B-Instruct, ndoc=30	51.9	51.0	0.9	54.1	53.8	0.3
Mintaka, Llama3.1-8B-Instruct, ndoc=50	51.7	51.0	0.7	53.6	53.5	0.1
Mintaka, Qwen1.5-4B-Chat, ndoc=30	34.9	34.7	0.2	35.7	35.5	0.2
Mintaka, Qwen1.5-4B-Chat, ndoc=50	35.8	35.0	0.8	36.2	36.1	0.1
Mintaka, Qwen1.5-7B-Chat, ndoc=30	35.4	35.0	0.4	37.7	37.7	0
Mintaka, Qwen1.5-7B-Chat, ndoc=50	35.7	35.5	0.2	38.0	38.0	0
MMLU, Llama3.1-8B-Instruct	68.3	65.5	2.8	68.5	65.7	2.8
MMLU, Qwen1.5-4B-Chat	53.6	52.6	1	53.7	52.6	1.1
MMLU, Qwen1.5-7B-Chat	60.1	58.6	1.5	60.1	58.8	1.3

Table 8: Performance of the **Original model** and **RoToR** before and after shuffling.

Derivation for the original model. Let the nine paired differences (Before $-$ After) be $\{d_1, d_2, d_3, ...d_8, d_9\}$. **Mean Difference:** $\bar{d} = \frac{1}{9}\sum_{i=1}^{9} d_i$. In this case, $\bar{d} \approx 0.9444\%$. **Sample Standard Deviation:** $s_d = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(d_i - \bar{d})^2} \approx 0.7632$. **Standard Error (SE):** $\mathrm{SE} = \frac{s_d}{\sqrt{n}} \approx 0.2544$. ***t*-Statistic:** $t = \frac{\bar{d}}{\mathrm{SE}} \approx 3.7124, \ (df = 8)$. Since the critical value at $df = 8$ and $\alpha = 0.05$ is 2.306, we have $3.71 > 2.306$. Therefore, the difference is statistically significant.

[8] All statistical calculations were validated using an online t-test calculator: `https://www.mathportal.org/calculators/statistics-calculator/t-test-calculator.php`

	Original	**RoToR**
Degrees of Freedom	8	
Mean Difference	0.94	0.66
t-Statistic	3.71	2.23
Critical Value	2.306	
Statistically Significant	**Yes**	**No**

Table 9: Paired two-tailed t-test results comparing the original model and ours.

Derivation for the RoToR model. Under the same procedure, $\bar{d} \approx 0.6556\%,\ s_d \approx 0.8833,\ \mathrm{SE} \approx 0.2944$. ***t*-Statistic:** $t \approx 2.2265$. Since $2.2265 < 2.306$, there is no significant difference in performance before and after shuffling for RoToR.

Conclusion. While the original model shows a statistically significant performance drop with shuffled inputs, RoToR remains unaffected, demonstrating greater robustness to segment-order perturbations.