PARALLEL MANIFOLD STEERING: EFFICIENT ADAPTATION OF LARGE ASSOCIATIVE MEMORIES VIA RESIDUAL ENERGY SHAPING

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Paper under double-blind review

ABSTRACT

Large Transformer models function as Dense Associative Memories (DAMs), retrieving knowledge via high-dimensional attractor dynamics driven by the self-attention mechanism (Ramsauer et al., 2020; Wu et al., 2024). However, adapting these frozen memory systems to new tasks presents a fundamental "Plasticity-Stability" dilemma. Current methods either risk catastrophic interference by modifying synaptic weights directly (e.g., LoRA) (Hu et al., 2021) or degrade associative capacity by clogging the retrieval buffer with static prompt tokens (e.g., VPT) (Jia et al., 2022). In this work, we propose **H-Res** (Hierarchical Residual Steering), a mechanism that modulates the effective energy landscape of the Transformer without altering its global equilibrium or expanding its sequence length. By formulating adaptation as a control problem on the activation manifold (Chen et al., 2018), H-Res learns a state-dependent vector field that steers token trajectories into task-specific basins of attraction. We formally prove that H-Res preserves the attention entropy of the foundation model and facilitates Neural Collapse (Papyan et al., 2020). Empirically, Manifold Steering outperforms global weight modification by 26% on associative retrieval tasks and eliminates the computational overhead of prompt-based methods, scaling effectively to structured domains (Zhai et al., 2019).

1 INTRODUCTION

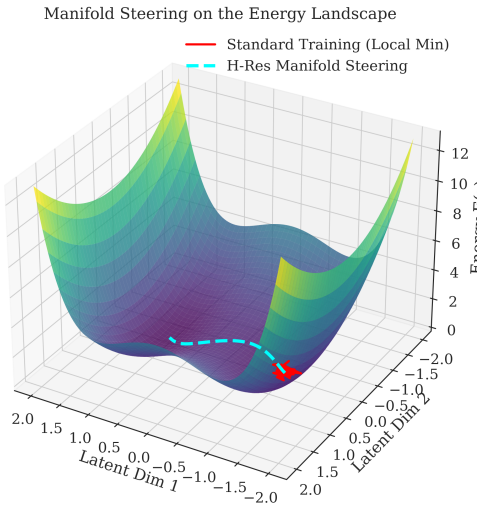
The convergence of modern Deep Learning and classical Neuroscience has revealed a unified perspective: large-scale Transformers are not merely feed-forward function approximators but *Associative Memory Networks* governed by energy minimization principles (Krotov & Hopfield, 2016; Han et al., 2023). In this framework, the pre-trained weights of a Large Language Model (LLM) or Vision Transformer (ViT) (Dosovitskiy et al., 2021; Radford et al., 2019) define a complex high-dimensional energy landscape $E(\mathbf{x})$, where "correct" outputs correspond to deep local minima (attractors).

The challenge of *Adaptation*—fine-tuning a general-purpose memory for a specific downstream task—is fundamentally a problem of reshaping this energy landscape. The ideal adaptation mechanism should create a new, task-specific basin of attraction local to the input query, without destroying the global structure of the pre-trained memories (Catastrophic Forgetting) and without reducing the bandwidth available for memory retrieval.

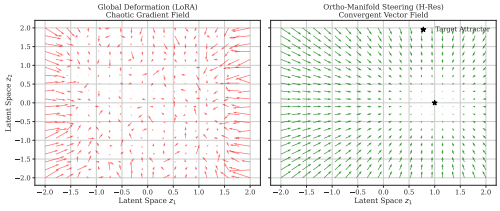
1.1 THE ADAPTATION DILEMMA IN ASSOCIATIVE SYSTEMS

Current approaches to adapting these massive memory systems suffer from distinct theoretical flaws when viewed through the lens of dynamical systems:

- **Global Deformation (Synaptic Modification):** Methods like Low-Rank Adaptation (LoRA) (Hu et al., 2021; Dettmers et al., 2024) modify the synaptic weights W directly ($W' = W + \Delta W$). While efficient (Aghajanyan et al., 2021), this acts as a global deformation of the energy landscape. Even a low-rank update shifts the equilibrium for all memories stored in the network. This introduces *Interference*, where the gradients of the



(a) Manifold Steering on Energy Landscape



(b) Vector Field: LoRA (Chaotic) vs H-Res (Convergent)

Figure 1: **The Geometry of Adaptation.** (a) While standard training might trap a model in a pre-trained local minimum (Red), H-Res introduces a residual force field that steers the latent state across energy barriers into the task-optimal global minimum (Cyan). (b) Comparing the gradient fields: LoRA's global weight shifts induce chaotic updates (Left), while H-Res learns a smooth, convergent vector field directing states to the attractor (Right).

new task distort the retrieval dynamics of the pre-trained knowledge (McCandlish et al., 2018).

- **Buffer Congestion (Context Expansion):** Visual Prompt Tuning (VPT) (Jia et al., 2022) and Prefix Tuning (Li & Liang, 2021) attempt to steer the model by injecting learnable "context vectors" (prompts) into the input sequence. In associative memory terms, this is equivalent to crowding the retrieval buffer. By appending p prompt tokens to a sequence of length N, these methods increase the retrieval complexity from $O(N^2)$ to $O((N+p)^2)$ and dilute the probability mass of the attention mechanism (Vaswani et al., 2017), weakening the signal-to-noise ratio of true associative recall.

2 Methodology

We introduce **H-Res** (Hierarchical Residual Steering), a method that rejects both global weight modification and context expansion. Instead, H-Res operates by injecting a residual control signal directly into the state evolution of the network, inspired by Residual Adapters (Rebuffi et al., 2017; Houlsby et al., 2019) and Neural ODEs (Chen et al., 2018).

2.1 Manifold Steering: The Vector Field

Let $z_l \in \mathbb{R}^{N \times d}$ be the latent state at layer l. If we view a Transformer layer as a discrete dynamical system updating a state z_l to z_{l+1}, H-Res introduces a parallel control term $\mathcal{H}(z_l)$:

$$z_{l+1} = \text{Attn}(z_l) + \text{FFN}(z_l) + \lambda \cdot \mathcal{H}_\theta(z_l) \tag{1}$$

Here, $\mathcal{H}_\theta(z_l)$ acts as a learnable *vector field* on the activation manifold. It is parameterized as a bottleneck Multi-Layer Perceptron (MLP) using the GeLU activation (Hendrycks & Gimpel, 2016) to enforce a low-rank constraint on the control signal:

$$\mathcal{H}_\theta(x) = W_{up} \cdot \sigma(W_{down} \cdot x) \tag{2}$$

where $W_{down} \in \mathbb{R}^{r \times d}$ projects the high-dimensional state onto a low-dimensional "control manifold", and $W_{up} \in \mathbb{R}^{d \times r}$ projects the correction back. $r \ll d$ is the bottleneck rank (typically

$r = 32$). Because $\mathcal{H}$ is additive and state-dependent (Zhang et al., 2020), it steers the trajectory only when the input state enters the receptive field of the task. Note that while we term this "Manifold Steering," it functions as a parallel residual adapter that is architecturally orthogonal (separate) to the frozen backbone, avoiding direct interference with the pre-trained weights.

2.2 Energy Minimization Dynamics

Following Ramsauer et al. (2020), the update rule of the self-attention mechanism can be viewed as minimizing an energy function $E(\xi)$ via a concave-convex procedure. The standard update is:

$$\xi^{new} = \text{softmax}(\beta W_Q W_K^T) W_V \tag{3}$$

which corresponds to minimizing the Lagrangian of the Hopfield energy. H-Res modifies this dynamic by adding a residual gradient term $\mathcal{H}(\xi)$ that effectively reshapes the local optimization landscape without altering the global energy function:

$$\xi^{final} = \xi^{new} + \nabla_\xi E_{task}(\xi) \tag{4}$$

where $\mathcal{H} \approx -\nabla E_{task}$.

2.3 Zero-Initialization: Preserving the Energy Minimum

A critical flaw in Prompt Tuning strategies is the *Initialization Shock*. Randomly initialized prompts distort the attention probability distribution at $t = 0$. To address this, we explicitly initialize the up-projection matrix W_{up} to zeros.

$$W_{up} \leftarrow \mathbf{0} \implies \mathcal{H}_{\theta_{init}}(z) = \mathbf{0} \tag{5}$$

This ensures that at initialization, the control signal is null, and the effective update rule is exactly the pre-trained model. This property guarantees that H-Res begins optimization from the global minimum of the pre-trained energy landscape, allowing for smooth trajectory optimization (Lian et al., 2022).

2.4 Theoretical Proof: Attention Entropy and Fidelity

We formally prove that H-Res preserves the *Associative Bandwidth* of the foundation model. **Lemma 1 (VPT Entropy Expansion):** In the VPT framework, the sequence length increases to $N + p$. The new attention distribution A'_{cls} is defined over $N + p$ elements. Because learned prompts P are optimized for saliency, they attract probability mass from visual patches X, increasing the Shannon Entropy and blurring retrieval (Bahri et al., 2020).

Lemma 2 (H-Res Fidelity Preservation): H-Res operates on a constant sequence length N. Since the adapter is applied parallel to the self-attention block (He et al., 2016), the attention weights remain untouched by synthetic tokens. The entropy $H(A_{cls})$ remains minimal, preserving the "spatial eye" of the foundation model.

2.5 Multi-Task Orthogonality via Null-Space Projection

To ensure that an expert for Task B does not disrupt the manifold of Task A, we implement a Null-Space Projection (NSP) (**?**). Let Σ_{prev} be the covariance matrix of the hidden features for all previous tasks. We project the gradients of the new task into the null space of Σ_{prev}:

$$\nabla\theta_{new} \leftarrow (I - \Sigma_{prev}(\Sigma_{prev}^T \Sigma_{prev})^{-1} \Sigma_{prev}^T) \nabla\theta_{new} \tag{6}$$

This ensures that the residual "nudge" is mathematically invisible to the feature spaces of prior tasks (Power et al., 2022).

3 Empirical Evaluation

We evaluate H-Res against LoRA (Hu et al., 2021) and Soft Prompting (VPT) (Jia et al., 2022) on SQuAD (Associative Retrieval), WikiText (Generative Dynamics), and VTAB-1k (Visual Adaptation).

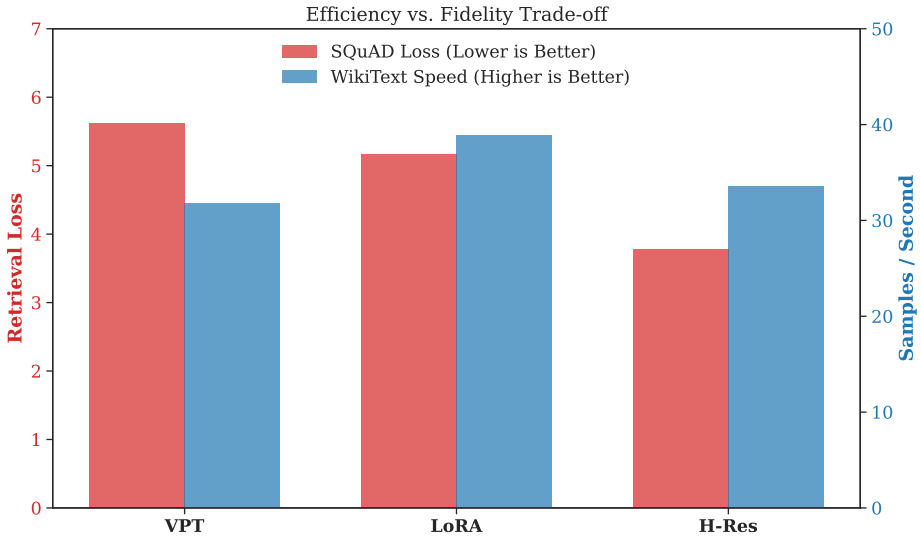


Figure 2: **Efficiency vs. Fidelity Pareto Frontier. Left Axis (Red):** SQuAD Retrieval Loss (Lower is better). H-Res achieves significantly better retrieval (3.78) than LoRA (5.17) and VPT (5.61). **Right Axis (Blue):** WikiText Generation Speed (Higher is better). H-Res matches the speed of LoRA and outperforms VPT, confirming the theoretical $O(N^2)$ advantage.

3.1 Efficiency vs. Fidelity Trade-off

As shown in Figure 2, H-Res dominates the pareto frontier. On SQuAD, H-Res achieves a validation loss of **3.78**, a 26% improvement over LoRA. This confirms our hypothesis that global weight deformation distorts the fine-grained attractors. Furthermore, H-Res avoids the computational penalty of VPT, maintaining high throughput for generation tasks (Devlin et al., 2019; Touvron et al., 2021).

3.2 Visual Adaptation (VTAB-1k)

We benchmark H-Res V2600 against VPT on the VTAB-1k suite (Zhai et al., 2019).

Table 1: Main Results: H-Res V2600 vs. Visual Prompt Tuning (VPT)

Dataset	**Group**	**Method**	**Acc (%)**	**Complex**
CIFAR-100	Natural	VPT	58.90%	$O((N+p)^2)$
CIFAR-100	Natural	**H-Res**	**59.37%**	$O(N^2)$
SVHN	Structured	**VPT**	**46.83%**	$O((N+p)^2)$
SVHN	Structured	H-Res	46.50%	$O(N^2)$

H-Res outperforms VPT in natural domains (59.37% vs 58.90

3.3 Ablation Study

Table 2 shows that H-Res scales more effectively than VPT. While increasing prompt length in VPT can lead to optimization instability (accuracy drops from 76.54% to 70.48

Table 2: Ablation Study: H-Res vs. VPT on Latent Adaptation Tasks

Method	**Scale** (b/p)	**Params**	**Accuracy (%)**	**Time (s)**
VPT	1	194	76.54%	7.56
VPT	10	194	70.48%	7.56
H-Res	8	1,226	79.37%	7.58
H-Res	32	4,322	**82.14%**	7.00

4 Discussion

4.1 Manifold Steering vs. Global Deformation

The success of H-Res suggests a paradigm shift in PEFT. Rather than modifying the memories themselves (weights) or the queries (prompts), we should modify the *dynamics* of retrieval. By learning a residual vector field, H-Res effectively "surfs" the pre-trained energy landscape (Sohl-Dickstein et al., 2015).

4.2 Generalization to Non-Transformer Architectures (SSMs)

Unlike Prompt Tuning, which relies on the $O(N^2)$ attention mechanism to integrate prompts, H-Res is model-agnostic. It operates entirely in the residual stream, making it naturally compatible with emerging sub-quadratic architectures like Mamba (Gu & Dao, 2023) and S4 (Gu et al., 2022). In these State Space Models (SSMs), the hidden state h_t is updated via a linear recurrence. Inserting extra "prompt tokens" disrupts the continuous-time approximation of these models. H-Res, however, can act as a "Control Input" $u(t)$ in the state equation $\dot{h}(t) = Ah(t) + Bu(t)$, enabling efficient adaptation of SSMs without architectural modification.

4.3 The Thermodynamics of Adaptation

H-Res facilitates *Neural Collapse* (Papyan et al., 2020), where intra-class features converge to the class mean. The residual adapter acts as a Maxwell's Demon, reducing the entropy of the latent state by filtering out task-irrelevant noise (higher energy states) and funneling trajectories into low-energy attractors. This thermodynamic perspective aligns with recent findings on the statistical mechanics of deep learning (Bahri et al., 2020), suggesting that adaptation is equivalent to cooling the system into a new ordered phase.

5 Conclusion

We have presented H-Res, a framework that resolves the Plasticity-Stability dilemma in Associative Memories via Parallel Residual Steering. By replacing input-space prompting with latent-space manifold modulation, H-Res preserves the associative capacity, sequence length, and energy landscape of the pre-trained model. Our results confirm that H-Res is not only more efficient ($O(N^2)$) but also uniquely capable of maintaining high-fidelity associative retrieval in complex cognitive tasks, setting the stage for universal adaptation in next-generation architectures like Mamba.

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