# Hyperbolic Image-Text Representations 

Karan Desai ${ }^{1}$ Maximilian Nickel ${ }^{2}$ Tanmay Rajpurohit ${ }^{3}$ Justin Johnson ${ }^{12}$ Ramakrishna Vedantam ${ }^{4}$


#### Abstract

Visual and linguistic concepts naturally organize themselves in a hierarchy, where a textual concept "dog" entails all images that contain dogs. Despite being intuitive, current large-scale vision and language models such as CLIP (Radford et al., 2021) do not explicitly capture such hierarchy. We propose MERU, a contrastive model that yields hyperbolic representations of images and text. Hyperbolic spaces have suitable geometric properties to embed tree-like data, so MERU can better capture the underlying hierarchy in image-text datasets. Our results show that MERU learns a highly interpretable and structured representation space while being competitive with CLIP's performance on standard multi-modal tasks like image classification and image-text retrieval.


## 1. Introduction

Visual-semantic hierarchy. It is commonly said that 'an image is worth a thousand words' - consequently, images contain a lot more information than the sentences which typically describe them. For example, given the middle image in Figure 1 one might describe it as 'a cat and a dog playing in the street' or with a less specific sentence like 'exhausted doggo' or 'so cute $<3$ '. These are not merely diverse descriptions but contain varying levels of detail about the underlying semantic contents of the image.

As humans, we can reason about the relative detail in each caption, and can organize such concepts into a meaningful visual-semantic hierarchy (Vendrov et al., 2016), namely, 'exhausted doggo' $\rightarrow$ 'a cat and a dog playing in the street' $\rightarrow$ (Figure 1 middle image). Providing multimodal models access to this inductive bias about vision and language has the potential to improve generalization (Radford et al., 2021), interpretability (Selvaraju et al., 2017) and enable

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Figure 1. Hyperbolic image-text representations. Left: Images and text depict concepts and can be jointly viewed in a visual-semantic hierarchy, wherein text 'exhausted doggo' is more generic than an image (which might have more details like a cat or snow). Our method MERU embeds images and text in a hyperbolic space that is well-suited to embed tree-like data. Right: Representation manifolds of CLIP (hypersphere) and MERU (hyperboloid) illustrated in 3D. MERU assumes the origin to represent the most generic concept, and embeds text closer to the origin than images.
better exploratory data analysis of large-scale datasets (Radford et al., 2021; Schuhmann et al., 2022).

Vision-language representation learning. Approaches such as CLIP (Radford et al., 2021) and ALIGN (Jia et al., 2021) have catalyzed a lot of recent progress in computer vision by showing that Transformer-based (Vaswani et al., 2017) models trained using large amounts of image-text data from the internet can yield transferable representations, and such models can perform zero-shot recognition and retrieval using natural language queries. All these models represent images and text as vectors in a high-dimensional Euclidean, affine space and normalize the embeddings to unit $L^{2}$ norm. However, such a choice of geometry can find it hard to capture the visual-semantic hierarchy.

An affine Euclidean space treats all embedded points in the same manner, with the same distance metric being applied to all points (Murphy, 2013). Conceptually, this can cause issues when modeling hierarchies - a generic concept (closer to the root node of the hierarchy) is close to many other concepts compared to a specific concept (which is only close to its immediate neighbors). Thus, a Euclidean space can find it hard to pack all the images that say a generic
concept 'curious kitty' should be close to while also respecting the embedding structure for ' $a$ cat and a dog playing on the street'. Such issues are handled naturally by hyperbolic spaces - the volume increases exponentially as we move away from the origin (Lee, 2019), making them a continuous relaxation of trees. This allows a generic concept ( 'cat') to have many neighbors by placing it close to the origin (Nickel \& Kiela, 2017), and more specific concepts further away. Thus, distinct specific concepts like images in Figure 1 can be far away from each other while being close to some generic concept ('animal').

Hyperbolic representations with MERU. In this work, we train the first large-scale contrastive image-text models that yield hyperbolic representations (Nickel \& Kiela, 2017) - MERU ${ }^{1}$ that captures the visual-semantic hierarchy (Figure 1). Our method conceptually resembles current state-of-the-art contrastive methods (Jia et al., 2021; Radford et al., 2021). Importantly the hierarchy emerges in the representation space, given access only to image-text pairs during training such models.
Practically, MERU confers multiple benefits such as (a) better performance on image retrieval and classification tasks, (b) more efficient usage of the embedding space, making it suited for resource-constrained, on-device scenarios, (c) an interpretable representation space that allows one to infer the relative semantic specificity of images and text. Overall, we summarize our contributions as follows:

- We introduce MERU, the first implementation of deep hyperbolic representations we are aware of, training ViTs (Dosovitskiy et al., 2021) with 12 M image-text pairs.
- We provide a strong CLIP baseline that outperforms previous re-implementations (Mu et al., 2022) at comparable data scale, and systematically demonstrate the benefits of hyperbolic representations over this baseline on zero-shot retrieval and classification, and effectiveness for small embedding dimensions (Kusupati et al., 2022).
- We perform thorough qualitative analysis with MERU to demonstrate its potential for exploratory data analysis of large-scale multimodal datasets.


## 2. Preliminaries

We briefly review Riemannian manifolds (Section 2.1) and essential concepts of hyperbolic geometry (Section 2.2). For a more thorough treatment of the topic, we refer the reader to textbooks by Ratcliffe (2006) and Lee (2019).

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### 2.1. Riemannian manifolds

A smooth surface is a two-dimensional sheet which is locally Euclidean - every point on the surface has a local neighborhood which can be mapped to $\mathbb{R}^{2}$ via a differentiable and invertible function. Smooth manifolds extend the notion of smooth surfaces to higher dimensions.
A Riemannian manifold $(\mathcal{M}, g)$ is a smooth manifold $\mathcal{M}$ equipped with a Riemannian metric $g$. The metric $g$ is a collection of inner product functions $g_{\mathbf{x}}$ for all points $\mathbf{x} \in \mathcal{M}$, and varies smoothly over the manifold. At any point $\mathbf{x}$, the inner product $g_{\mathbf{x}}$ is defined in the tangent space $\mathcal{T}_{\mathbf{x}} \mathcal{M}$, which is a Euclidean space that gives a linear approximation of $\mathcal{M}$ at $\mathbf{x}$. Euclidean space $\mathbb{R}^{n}$ is also a Riemannian manifold, where $g$ is the standard Euclidean inner product.
Our main topic of interest is hyperbolic spaces, which are Riemannian manifolds with constant negative curvature. They are fundamentally different from Euclidean spaces that are flat (zero curvature). A hyperbolic manifold of $n$ dimensions cannot be represented with $\mathbb{R}^{n}$ in a way that preserves both distances and angles. There are five popular models of hyperbolic geometry that either represent $n$-dimensional hyperbolic spaces either in $\mathbb{R}^{n}$ while distorting distances and/or angles (e.g. Poincaré ball model), or as a sub-manifold of $\mathbb{R}^{n+1}$ (e.g. the Lorentz model).

### 2.2. Lorentz model of hyperbolic geometry

We use the Lorentz model of hyperbolic geometry for developing MERU. This model represents a hyperbolic space of $n$ dimensions on the upper half of a two-sheeted hyperboloid in $\mathbb{R}^{n+1}$. See Figure 1 for an illustration of $\mathcal{L}^{2}$ in $\mathbb{R}^{3}$. Hyperbolic geometry has a direct connection to the study of special relativity theory (Einstein, 1905; Einstein et al., 2015). We borrow some of its terminology in our discussion - we refer to the hyperboloid's axis of symmetry as time dimension and all other axes as space dimensions (Minkowski, 1908). Every vector $\mathbf{x} \in \mathbb{R}^{n+1}$ can be written as $\left[\mathbf{x}_{\text {space }}, x_{\text {time }}\right]$, where $\mathbf{x}_{\text {space }} \in \mathbb{R}^{n}$ and $x_{\text {time }} \in \mathbb{R}$.

Definition. Let $\langle\cdot, \cdot\rangle$ is Euclidean inner product and $\langle\cdot, \cdot\rangle_{\mathcal{L}}$ denote the Lorentzian inner product that is induced by the Riemannian metric of the Lorentz model. For two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{n+1}$, it is computed as follows:

$$
\begin{equation*}
\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}=\left\langle\mathbf{x}_{\text {space }}, \mathbf{y}_{\text {space }}\right\rangle-x_{\text {time }} y_{\text {time }} \tag{1}
\end{equation*}
$$

The induced Lorentzian norm is $\|\mathbf{x}\|_{\mathcal{L}}=\sqrt{\left|\langle\mathbf{x}, \mathbf{x}\rangle_{\mathcal{L}}\right|}$. The Lorentz model possessing a constant curvature $-c$ is defined as a following set of vectors:

$$
\begin{equation*}
\mathcal{L}^{n}=\left\{\mathbf{x} \in \mathbb{R}^{n+1}:\langle\mathbf{x}, \mathbf{x}\rangle_{\mathcal{L}}=-1 / c, c>0\right\} \tag{2}
\end{equation*}
$$

All vectors in this set satisfy the following constraint:

$$
\begin{equation*}
x_{\text {time }}=\sqrt{1 / c+\left\|\mathbf{x}_{\text {space }}\right\|^{2}} \tag{3}
\end{equation*}
$$

Geodesics. A geodesic is the shortest path between two points on the manifold. Geodesics in the Lorentz model are curves traced by the intersection of the hyperboloid with hyperplanes passing through the origin of $\mathbb{R}^{n+1}$. The Lorentzian distance between two points $\mathbf{x}, \mathbf{y} \in \mathcal{L}^{n}$ is:

$$
\begin{equation*}
d_{\mathcal{L}}(\mathbf{x}, \mathbf{y})=\sqrt{1 / c} \cdot \cosh ^{-1}\left(-c\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}\right) \tag{4}
\end{equation*}
$$

Tangent space. The tangent space at some point $\mathbf{z} \in \mathcal{L}^{n}$ is a Euclidean space of vectors that are orthogonal to $\mathbf{z}$ according to the Lorentzian inner product:

$$
\begin{equation*}
\mathcal{T}_{\mathbf{z}} \mathcal{L}^{n}=\left\{\mathbf{v} \in \mathbb{R}^{n+1}:\langle\mathbf{z}, \mathbf{v}\rangle_{\mathcal{L}}=0\right\} \tag{5}
\end{equation*}
$$

Any vector in ambient space $\mathbf{u} \in \mathbb{R}^{n+1}$ can be projected to the tangent space $\mathcal{T}_{\mathbf{z}} \mathcal{L}^{n}$ via an orthogonal projection:

$$
\begin{equation*}
\mathbf{v}=\operatorname{proj}_{\mathbf{z}}(\mathbf{u})=\mathbf{u}+c \mathbf{z}\langle\mathbf{z}, \mathbf{u}\rangle_{\mathcal{L}} \tag{6}
\end{equation*}
$$

Exponential and logarithmic maps. The exponential map provides a way to map vectors from tangent spaces onto the manifold. For a point $\mathbf{z}$ on the hyperboloid, it is defined as $\operatorname{expm}_{\mathbf{z}}: \mathcal{T}_{\mathbf{z}} \mathcal{L}^{n} \rightarrow \mathcal{L}^{n}$ with the expression:
$\mathbf{x}=\operatorname{expm}_{\mathbf{z}}(\mathbf{v})=\cosh \left(\sqrt{c}\|\mathbf{v}\|_{\mathcal{L}}\right) \mathbf{z}+\frac{\sinh \left(\sqrt{c}\|\mathbf{v}\|_{\mathcal{L}}\right)}{\sqrt{c}\|\mathbf{v}\|_{\mathcal{L}}} \mathbf{v}$

Intuitively the exponential map shows how $\mathcal{T}_{x} \mathcal{L}^{n}$ folds on the manifold. Its inverse is the logarithmic map $\left(\operatorname{logm}_{\mathrm{z}}\right.$ : $\mathcal{L}^{n} \rightarrow \mathcal{T}_{\mathbf{z}} \mathcal{L}^{n}$ ), that maps $\mathbf{x}$ from the hyperboloid back to $\mathbf{v}$ in the tangent space:

$$
\begin{equation*}
\mathbf{v}=\operatorname{logm}_{\mathbf{z}}(\mathbf{x})=\frac{\cosh ^{-1}\left(-c\langle\mathbf{z}, \mathbf{x}\rangle_{\mathcal{L}}\right)}{\sqrt{\left(c\langle\mathbf{z}, \mathbf{x}\rangle_{\mathcal{L}}\right)^{2}-1}} \operatorname{proj}_{\mathbf{z}}(\mathbf{x}) \tag{8}
\end{equation*}
$$

For our approach, we will only consider these maps where $\mathbf{z}$ is the origin of the hyperboloid $(\mathbf{O}=[\mathbf{0}, \sqrt{1 / c}])$.

## 3. Approach

In this section, we discuss the modeling pipeline and learning objectives of MERU to learn hyperbolic representations of images and text. We use the tools of hyperbolic geometry introduced in Section 2 throughout our discussion.

Our model design is inspired by CLIP (Radford et al., 2021) due to its simplicity and scalability. As shown in Figure 2, we process images and text using two separate encoders, and obtain embedding vectors of a fixed dimension $n$. Beyond this, there are two crucial design choices: (1) transferring embeddings from Euclidean space to the Lorentz hyperboloid, and (2) designing suitable training objectives that induce semantics and structure in the representation space.


Figure 2. MERU model design: MERU comprises similar architectural components as standard image-text contrastive models like CLIP. While CLIP projects the embeddings to a unit hypersphere, MERU lifts them onto the Lorentz hyperboloid using the exponential map. The contrastive loss uses the negative of Lorentzian distance as a similarity metric, and a special entailment loss enforces 'text entails image' partial order in the representation space.

Lifting embeddings onto the hyperboloid. Let the embedding vector from the image encoder or text encoder, after linear projection be $\mathbf{v}_{\text {enc }} \in \mathbb{R}^{n}$. We need to apply a transformation such that the resulting vector x lies on the Lorentz hyperboloid $\mathcal{L}^{n}$ in $\mathbb{R}^{n+1}$. Let the vector $\mathbf{v}=\left[\mathbf{v}_{\text {enc }}, 0\right] \in \mathbb{R}^{n+1}$. We observe that $\mathbf{v}$ belongs to the tangent space at the hyperboloid origin $\mathbf{O}$, as Eqn. 5 is satisfied: $\langle\mathbf{O}, \mathbf{v}\rangle_{\mathcal{L}}=0$. Thus, we parameterize only the space components of the Lorentz model $\left(\mathbf{v}_{e n c}=\mathbf{v}_{\text {space }}\right)$. Due to such parameterization, we can simplify the exponential map from Eqn. 7 by writing only space components:

$$
\mathbf{x}_{\text {space }}=\cosh \left(\sqrt{c}\|\mathbf{v}\|_{\mathcal{L}}\right) \mathbf{0}+\frac{\sinh \left(\sqrt{c}\|\mathbf{v}\|_{\mathcal{L}}\right)}{\sqrt{c}\|\mathbf{v}\|_{\mathcal{L}}} \mathbf{v}_{\text {space }}
$$

The first term reduces to $\mathbf{0}$. Moreover, the Lorentzian norm of $\mathbf{v}$ simplifies to the Euclidean norm of space components: $\|\mathbf{v}\|_{\mathcal{L}}^{2}=\langle\mathbf{v}, \mathbf{v}\rangle_{\mathcal{L}}=\left\langle\mathbf{v}_{\text {space }}, \mathbf{v}_{\text {space }}\right\rangle-0=\left\|\mathbf{v}_{\text {space }}\right\|^{2}$. This substitution simplifies the above equation as follows:

$$
\begin{equation*}
\mathbf{x}_{\text {space }}=\frac{\sinh \left(\sqrt{c}\left\|\mathbf{v}_{\text {space }}\right\|\right)}{\sqrt{c}\left\|\mathbf{v}_{\text {space }}\right\|} \mathbf{v}_{\text {space }} \tag{9}
\end{equation*}
$$

The corresponding time component $x_{\text {time }}$ can be computed from $\mathbf{x}_{\text {space }}$ using Eqn. 3, the resulting $\mathbf{x}$ always lies on the hyperboloid. This eliminates the need for an orthogonal projection (Eqn. 6) and simplifies the exponential map. Our parameterization is simpler than previous work which parameterizes vectors in full ambient space $\mathbb{R}^{n+1}$ (Law et al., 2019; Le et al., 2019; Nickel \& Kiela, 2018).
Preventing numerical overflow. The exponential map scales $\mathbf{v}_{\text {space }}$ using an exponential operator. According to CLIP-style weight initialization, $\mathbf{v}_{\text {space }} \in \mathbb{R}^{n}$ would have an expected norm $=\sqrt{n}$. After exponential map, it becomes $e^{\sqrt{n}}$, which can be numerically large (e.g., $n=512$ and $c=1$ gives $\left\|\mathbf{x}_{\text {space }}\right\| \approx 6.7 \times 10^{10}$ ).

To fix this issue, we scale all vectors $\mathbf{v}_{\text {space }}$ in a batch before applying $\operatorname{expm}_{\mathbf{O}}$ using two learnable scalars $\alpha_{i m g}$ and $\alpha_{t x t}$. These are initialized to $\sqrt{1 / n}$ so that the Euclidean embeddings have an expected unit norm at initialization. We learn these scalars in logarithmic space to avoid collapsing all embeddings to zero. After training, they can be absorbed into the preceding projection layers.
Learning structured embeddings. Having lifted standard Euclidean embeddings onto the hyperboloid, we next discuss the losses used to enforce structure and semantics in representations learned by MERU. Recall that our motivation is to capture the visual-semantic hierarchy (Figure 1) to better inform the generalization capabilities of visionlanguage models. For this, an important desideratum is a meaningful notion of distance between semantically similar text and image pairs. We also want to induce a partial order between text and images as per the visual-semantic hierarchy to have better interpretability. We do this with a modified version of an entailment loss proposed by Le et al. (2019), that works for arbitrary hyperboloid curvatures $-c$.

### 3.1. Contrastive learning formulation

Given a batch of size $B$ of image-text pairs and any $j^{\text {th }}$ instance in batch, its image embedding $\mathbf{y}_{j}$ and text embedding $\mathbf{x}_{j}$ form a positive pair, whereas the remaining $B-1$ text embeddings in the batch $\mathbf{x}_{i}(i \neq j)$ form negative pairs.

In contrastive learning, we compute the negative Lorentzian distance as a similarity measure (Eqn. 4) for all $B$ pairs in the batch. These logits are divided by a temperature $\tau$ and apply a softmax operator. Similarly, we also consider a contrastive loss for text, that treats images as negatives. The total loss $\mathcal{L}_{\text {cont }}$ is the average of these two losses computed for every image-text pair in the batch. Our implementation of the contrastive loss is the same as the multi-class N-pair loss from (Sohn, 2016) used in CLIP (Radford et al., 2021) with the crucial difference being that we compute distances on the hyperboloid instead of cosine similarity.

### 3.2. Entailment loss

In addition to the contrastive loss, we adapt an entailment loss (Ganea et al., 2018; Le et al., 2019) to enforce partial order relationships between paired text and images. Ganea et al. (2018) is more different from ours since they parameterize their representations according to the Poincaré ball model. Le et al. (2019) use this loss with a fixed $c=1$, which we extend to handle arbitrary, learned curvatures.
Refer Figure 3 for an illustration in two dimensions. Let $\mathbf{x}$ and $\mathbf{y}$ denote the text and image embeddings of a single image-text pair. Note that the encoders only give $\mathbf{x}_{\text {space }}$ and $\mathbf{y}_{\text {space }}$ according to our parameterization. Corresponding $x_{\text {time }}$ and $y_{\text {time }}$ are calculated using Eqn. 3. We define an


Figure 3. Entailment loss (illustrated for $\mathcal{L}^{2}$ ): This loss pushes image embedding $y$ inside an imaginary cone projected by the paired text embedding $\mathbf{x}$, and is implemented as the difference of exterior angle $\angle O \mathbf{x y}$ and half aperture of the cone. Loss is zero if the image embedding is already inside the cone (left quadrant).
entailment cone for each $\mathbf{x}$, which narrows as we go farther from the origin. This cone is defined by the half-aperture:

$$
\begin{equation*}
\operatorname{aper}(\mathbf{x})=\sin ^{-1}\left(\frac{2 K}{\sqrt{c}\left\|\mathbf{x}_{\text {space }}\right\|}\right) \tag{10}
\end{equation*}
$$

where a constant $K=0.1$ is used for setting boundary conditions near the origin. We now aim to identify and penalize when the paired image embedding y lies outside the entailment cone. For this, we measure the exterior angle $\operatorname{ext}(\mathbf{x}, \mathbf{y})=\pi-\angle \mathbf{O x y}$ as shown in Figure 3:

$$
\begin{equation*}
\operatorname{ext}(\mathbf{x}, \mathbf{y})=\cos ^{-1}\left(\frac{y_{\text {time }}+x_{\text {time }} c\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}}{\left\|\mathbf{x}_{\text {space }}\right\| \sqrt{\left(c\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}\right)^{2}-1}}\right) \tag{11}
\end{equation*}
$$

If the exterior angle is smaller than the aperture, then the partial order relation between $\mathbf{x}$ and $\mathbf{y}$ is already satisfied and we need not penalize anything, while if the angle is greater, we need to reduce it. This is captured by the following loss function (written below for a single $\mathbf{x}, \mathbf{y}$ pair):

$$
\begin{equation*}
\mathcal{L}_{\text {entail }}(\mathbf{x}, \mathbf{y})=\max (0, \operatorname{ext}(\mathbf{x}, \mathbf{y})-\operatorname{aper}(\mathbf{x})) \tag{12}
\end{equation*}
$$

We provide exact derivations of the above equations for halfaperture and exterior angle in Appendix A. Overall, our total loss is $\mathcal{L}_{\text {cont }}+\lambda \mathcal{L}_{\text {entail }}$ averaged over each minibatch.

## 4. Experiments

Our main objective in the experiments is to establish the competitiveness of hyperbolic representations of MERU as compared to Euclidean representations obtained from CLIP-style models. To this end, we train models using large amounts of image-text pairs and transfer them to a variety of image classification and retrieval tasks.

### 4.1. Training details

Baselines. We primarily compare with CLIP (Radford et al., 2021), that embeds images and text on a unit hypersphere in a Euclidean space. CLIP was trained using a private dataset of 400M image-text pairs. Several followup works re-implement CLIP and use publicly accessible datasets like YFCC (Thomee et al., 2016), Conceptual Captions (Changpinyo et al., 2021; Sharma et al., 2018), and LAION (Schuhmann et al., 2021; 2022); notable examples are OpenCLIP (Ilharco et al., 2021), SLIP (Mu et al., 2022), DeCLIP (Li et al., 2022), and FILIP (Yao et al., 2022). We develop our CLIP baseline and train it using a single public dataset - RedCaps (Desai et al., 2021) - for easier reproducibility. Our smallest model trains using $8 \times$ V100 GPUs in less than one day and significantly outperforms recent CLIP re-implementations that use YFCC (Mu et al., 2022).

Refer Appendix B for details about our CLIP baseline. Our implementation is based on PyTorch (Paszke et al., 2019) and timm (Wightman, 2019) libraries.

Models. We use the Vision Transformer (Dosovitskiy et al., 2021) as image encoder, considering three models of varying capacity - ViT-S (Chen et al., 2021; Touvron et al., 2021), ViT-B, and ViT-L. All use a patch size of 16. The text encoder is same as CLIP - a 12-layer, 512 dimensions wide Transformer (Vaswani et al., 2017) language model. We use the same byte-pair encoding tokenizer (Sennrich et al., 2016) as CLIP, and truncate input text at maximum 77 tokens.

Data augmentation. We randomly crop 50-100\% area of images and resize them to $224 \times 224$, following (Mu et al., 2022). For text augmentation, we randomly prefix the subreddit names to captions as '\{subreddit \} : \{caption\}'.

Initialization. We initialize image/text encoders in the same style as CLIP, except for one change: we use a sinecosine position embedding in ViT, like (Chen et al., 2021; He et al., 2022), and keep it frozen while training. We initialize the softmax temperature as $\tau=0.07$ and clamp it to a minimum value of 0.01 . For MERU, we initialize the learnable projection scalars $\alpha_{i m g}=\alpha_{t x t}=1 / \sqrt{512}$, the curvature parameter $c=1.0$ and clamp it in $[0.1,10.0]$ to prevent training instability. All scalars are learned in $\operatorname{logarithmic~space~as~} \log (1 / \tau), \log (c)$, and $\log (\alpha)$.

Optimization. We use AdamW (Loshchilov \& Hutter, $2019)$ with weight decay 0.2 and $\left(\beta_{1}, \beta_{2}\right)=(0.9,0.98)$. We disable weight decay for all gains, biases, and learnable scalars. All models are trained for 120 K iterations with batch size 2048 ( $\approx 20$ epochs). The maximum learning rate is $5 \times 10^{-4}$, increased linearly for the first $4 K$ iterations, followed by cosine decay to zero (Loshchilov \& Hutter, 2016). We use mixed precision (Micikevicius et al., 2018) to accelerate training, except computing exponential map and losses for MERU in FP32 precision for numerical stability.

Table 1. Zero-shot image and text retrieval. Best performance in every column is highlighted in green. MERU performs better than CLIP for both datasets and across all model sizes.

|  |  | text $\rightarrow$ image |  |  |  | image $\rightarrow$ text |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | COCO |  | Flickr |  | COCO |  | Flickr |  |
|  |  | R5 | R10 | R5 | R10 | R5 | R10 | R5 | R10 |
| ViT | CLIP | 29.9 | 40.1 | 35.3 | 46.1 | 37.5 | 48.1 | 42.1 | 54.7 |
| S/16 | MERU | 30.5 | 40.9 | 37.1 | 47.4 | 39.0 | 50.5 | 43.5 | 55.2 |
| ViT | CLIP | 32.9 | 43.3 | 40.3 | 51.0 | 41.4 | 52.7 | 50.2 | 60.2 |
| B/16 | MERU | 33.2 | 44.0 | 41.1 | 51.6 | 41.8 | 52.9 | 48.1 | 58.9 |
| ViT | CLIP | 31.7 | 42.2 | 39.0 | 49.3 | 40.6 | 51.3 | 47.8 | 58.5 |
| L/16 | MERU | 32.6 | 43.0 | 39.6 | 50.3 | 41.9 | 53.3 | 50.3 | 60.6 |

Loss multiplier ( $\lambda$ ) for MERU. We set $\lambda=0.2$ by running a hyperparameter sweep with ViT-B/16 models for one epoch. Some $\lambda>0$ is necessary to induce partial order structure, however, quantitative performance is less sensitive to the choice of $\lambda \in[0.01,0.3]$; Higher values of $\lambda$ strongly regularize against the contrastive loss and hurt performance.

### 4.2. Image and text retrieval

CLIP-style contrastive models perform image and text retrieval within batch during training, making them ideal for retrieval-related downstream applications. We evaluate the retrieval capabilities of MERU as compared to CLIP on two established benchmarks: COCO and Flickr30K (Chen et al., 2015; Young et al., 2014), that comprise 5000 and 1000 images respectively and five captions per image. COCO evaluation uses the val2017 split while Flickr30K uses the test split defined by Karpathy \& Fei-Fei (2015). We perform zero-shot transfer, without any additional training using these datasets. We squeeze images to $224 \times 224$ pixels before processing them through the image encoder.

Inference with MERU. We rank a pool of candidate image/text embeddings for retrieval in decreasing order of their Lorentzian inner product (Eqn. 1) with a text/image query embedding. Some transfer tasks like open-vocabulary detection (Gu et al., 2022; Zareian et al., 2021) may require calibrated scores, for them we recommend using the training procedure - compute the negative of distance (Eqn. 4), divide by temperature and apply a softmax classifier.

Results. Table 1 reports recall@ $\{5,10\}$ of MERU and the reproduced CLIP baselines on these benchmarks. Hyperbolic representations of MERU mostly perform best for all tasks and models (except Flickr30K text retrieval with ViT$\mathrm{B} / 16$ ). This is encouraging evidence that hyperbolic spaces have suitable geometric properties to learn strong representations for retrieval applications. Surprisingly, increasing model size (ViT-B/16 $\rightarrow$ ViT-L/16) does not improve image retrieval for both, MERU and CLIP. We believe that better quality of text queries is important for image retrieval increasing the size of text encoder can alleviate this issue.

Table 2. Zero-shot image classification. We train MERU and CLIP models with varying parameter counts and transfer them zero-shot to 20 image classification datasets. Best performance in every column is highlighted in green. Hyperbolic representations from MERU match or outperform CLIP on 13 out of the first 16 datasets. On the last four datasets (gray columns), both MERU and CLIP have near-random performance, as concepts in these datasets are not adequately covered in the training data.

|  |  | $\begin{aligned} & \ddot{0}_{0}^{2} \\ & \text {. } \\ & \tilde{\Xi} \end{aligned}$ | $\begin{aligned} & \overrightarrow{0} \\ & \frac{1}{0} \\ & 0 \\ & \hline \end{aligned}$ |  |  | $\stackrel{e}{S}_{0}^{n}$ | $\begin{aligned} & \text { N} \\ & \underset{\sim}{n} \\ & \underset{\sim}{n} \end{aligned}$ | نَّ |  | $\stackrel{冃}{0}$ | $\stackrel{\square}{2}$ |  |  | $\begin{aligned} & \frac{0}{4} \\ & \stackrel{y}{4} \end{aligned}$ | $\begin{aligned} & \text { E } \\ & \text { N } \\ & \text { ob } \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \underset{U N}{N} \\ & \tilde{\sim} \\ & \text { N } \end{aligned}$ | $\begin{aligned} & \text { ت } \\ & \text { त् } \\ & \text { I } \\ & \text { U } \end{aligned}$ | $\frac{5}{2}$ | $\begin{aligned} & \text { N } \\ & \text { ud } \\ & \hline \end{aligned}$ | $\sum_{\substack{\text { d }}}$ | $\stackrel{\mathrm{N}}{\sim}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ViT | CLIP | 34.3 | 74.5 | 60.1 | 24.4 | 33.8 | 27.5 | 11.3 | 1.4 | 15.0 | 73.7 | 63.9 | 47.0 | 88.2 | 18.6 | 31.4 | 5.2 | 10.0 | 19.4 | 50.2 | 50.1 |
| S/16 | MERU | 34.4 | 75.6 | 52.0 | 24.7 | 33.7 | 28.0 | 11.1 | 1.3 | 16.2 | 72.3 | 64.1 | 49.2 | 91.1 | 30.4 | 32.0 | 4.8 | 7.5 | 14.5 | 51.0 | 50.0 |
| ViT | CLIP | 37.9 | 78.9 | 65.5 | 33.4 | 33.3 | 29.8 | 14.4 | 1.4 | 17.0 | 77.9 | 68.5 | 50.9 | 92.2 | 25.6 | 31.0 | 5.8 | 10.4 | 14.3 | 54.1 | 51.5 |
| B/16 | MERU | 37.5 | 78.8 | 67.7 | 32.7 | 34.8 | 30.9 | 14.0 | 1.7 | 17.2 | 79.3 | 68.5 | 52.1 | 92.5 | 30.2 | 34.5 | 5.6 | 13.0 | 13.5 | 49.8 | 49.9 |
| ViT | CLIP | 38.4 | 80.3 | 72.0 | 36.4 | 36.3 | 32.0 | 18.0 | 1.1 | 16.5 | 78.8 | 68.3 | 48.6 | 93.7 | 26.7 | 35.4 | 6.1 | 14.8 | 13.6 | 51.2 | 51.1 |
| L/16 | MERU | 38.8 | 80.6 | 68.7 | 35.5 | 37.2 | 33.0 | 16.6 | 2.2 | 17.2 | 80.0 | 67.5 | 52.1 | 93.7 | 28.1 | 36.5 | 6.2 | 11.8 | 13.1 | 52.7 | 49.3 |

### 4.3. Image classification

Learning from language supervision allows CLIP to perform zero-shot image classification, wherein one may specify label sets as text queries (Elhoseiny et al., 2013) instead of using pre-defined ontologies (Deng et al., 2009; Miller, 1992). Classifier weights are obtained by embedding labelbased queries (also called prompts) using the text encoder.
In this section, we evaluate MERU on 20 image classification benchmarks covering a wide variety of visual concepts. These are used by Radford et al. (2021) and several follow-up works (Li et al., 2022; Mu et al., 2022; Yao et al., 2022), and available with open-source libraries like tensorflow-datasets and torchvision ${ }^{2}$. We report top1 mean per-class accuracy for all datasets to account for any label imbalance. We use multiple prompts per dataset, most of which follow Radford et al. (2021). We ensemble these multiple prompts by averaging their embeddings before lifting them onto the hyperboloid (Eqn. 9). See Tables 6 and 8 in Appendix for details about datasets and prompts.
Results. Table 2 shows strong transfer performance of MERU, matching or outperforming CLIP on 13 out of 16 standard datasets. While MERU is effective on recall-based measures (Table 1), it does not come at the expense of precision (Murphy, 2013). Overall, hyperbolic representations from MERU are competitive with their Euclidean counterparts across varying model architectures (ViT-S/B/L).

All models have near-random performance on four benchmarks. Concepts in these datasets have low coverage in RedCaps, like PCAM (Veeling et al., 2018) containing medical scans, or SST2 (Socher et al., 2013) containing movie reviews rendered as images. Performance on these benchmarks does not indicate the efficacy of our RedCaps-trained models; using larger training datasets like LAION (Schuhmann et al., 2022) may yield meaningful trends.

[^2]|  |  | Embedding width |  |  |  |  |
| :---: | :--- | :--- | :---: | :---: | :---: | :---: |
|  |  | 512 | 256 | 128 | 96 | 64 |
| COCO | CLIP | 31.7 | 31.8 | 31.4 | 29.6 | 25.7 |
| text $\rightarrow$ image | MERU | $\mathbf{3 2 . 6}$ | $\mathbf{3 2 . 7}$ | $\mathbf{3 2 . 7}$ | $\mathbf{3 1 . 0}$ | $\mathbf{2 6 . 5}$ |
| COCO | CLIP | 40.6 | 41.0 | 40.4 | 37.9 | 33.3 |
| image $\rightarrow$ text | MERU | $\mathbf{4 1 . 9}$ | $\mathbf{4 2 . 5}$ | $\mathbf{4 2 . 6}$ | $\mathbf{4 0 . 5}$ | $\mathbf{3 4 . 2}$ |
| ImageNet | CLIP | 38.4 | 38.3 | 37.9 | 35.2 | 30.2 |
|  | MERU | $\mathbf{3 8 . 8}$ | $\mathbf{3 8 . 8}$ | $\mathbf{3 8 . 8}$ | $\mathbf{3 7 . 3}$ | $\mathbf{3 2 . 3}$ |

Table 3. MERU and CLIP with different embedding widths. We report zero-shot COCO recall@5 and ImageNet top-1 accuracy. MERU outperforms CLIP at lower embedding widths.

### 4.4. Resource-constrained deployment

We hypothesize that embeddings that capture a rich visualsemantic hierarchy can use the volume in the representation space more efficiently. This is useful for on-device deployments with runtime or memory constraints that necessitate low-dimensional embeddings (Kusupati et al., 2022).

To verify this hypothesis, we train MERU and CLIP models that output 64-512 dimensions wide embeddings. We initialize the encoders from ViT-L/16 models (Table 2, last two rows) to reduce compute requirements, keep them frozen, and re-initialize projection layers and learnable scalars. We train for $30 K$ iterations and evaluate on zero-shot COCO retrieval and ImageNet (Russakovsky et al., 2014) classification. Results in Table 3 show that MERU consistently performs better at low embedding widths. This indicates that hyperbolic embeddings may be an appealing solution for resource-constrained on-device applications.

### 4.5. Ablations

In this section, we ablate our MERU models to observe the impact of our design choices. We experiment with two image encoders, ViT-B/16 and ViT-L/16, and evaluate for zero-shot COCO retrieval and ImageNet classification.

Table 4. MERU ablations. We ablate three design choices of MERU and report zero-shot COCO recall@5 and ImageNet top-1 accuracy. Our design choices are crucial for training stability when using a larger model (ViT-L/16) with MERU.

|  | COCO <br> text $\rightarrow$ image | COCO <br> image $\rightarrow$ text | ImageNet |
| :--- | :---: | :---: | :---: |

Specifically, we train three ablations with the default hyperparameters (Section 4.1), except having one difference each. Results are shown in Table 4 above.

No entailment loss: We only use the contrastive loss for training this ablation. This effectively means setting $\lambda=0$. Note that this ablation is mathematically impossible for CLIP as there is no obvious notion of entailment that can be defined when all the embeddings have a unit norm. Disabling the entailment loss is mostly inconsequential to MERU's performance. This shows that choosing a hyperbolic space is sufficient to improve quantitative performance over CLIP. Entailment loss is crucial for better structure and interpretability, as will be discussed in Section 5.

Fixed curvature parameter: Recall that our models treat the hyperboloid curvature as a learnable parameter during training. Here we train an ablation using a fixed curvature $c=1$. This has negligible impact on MERU ViT-B/16, but learning curvature is crucial when scaling model size - MERU ViT-L/16 model with fixed $c=1$ is difficult to optimize and performs poorly on convergence. As far as we are aware, no prior work learns the curvature (Atigh et al., 2022; Khrulkov et al., 2020; Nickel \& Kiela, 2018).

Lorentzian inner product in contrastive loss: CLIP-style contrastive loss uses the inner product defined on the hypersphere (cosine similarity). Similarly, we consider the Lorentzian inner product (Eqn. 1) in the contrastive loss instead of negative Lorentzian distance. With this, MERU ViT-L/16 is difficult to train. Loss diverges due to numerical overflow, as Lorentzian inner product is numerically large and unbounded in $(-\infty,-1 / c]$, unlike cosine similarity $\in[-1,1]$. Lorentzian distance applies a logarithmic operator ( $\cosh ^{-1}$ ) on the Lorentzian inner product, slowing down its growth and hence improving numerical stability.

We hope these ablations serve as guidelines for work in other domains that study hyperbolic representation learning.

MERU (ViT-L/16) CLIP (ViT-L/16)


Figure 4. Distribution of embedding distances from [ROOT]: We embed all 12 M training images and text using trained MERU and CLIP. Note that precise distance is not necessary for this analysis, so we compute simple monotonic transformations of distances, $d(\mathbf{z})$. MERU embeds text closer to [ROOT] than images.

## 5. Qualitative analysis

In this section, we probe our trained models to infer the visual-semantic hierarchy captured by MERU and CLIP. Apriori we hypothesize that MERU is better equipped to capture this hierarchy due to the geometric properties of hyperbolic spaces and an entailment loss that enforces the partial-order relationship 'text entails image'. All our analysis in this section uses ViT-L/16 models.

Preliminary: [ROOT] embedding. Recall Figure 1 - if we think of the visual-semantic hierarchy as a tree, then its leaf nodes are images and the intermediate nodes are text descriptions with varying semantic specificity. Naturally, the root node should represent the most generic concept. We denote its embedding in the representation space as [ROOT].

For MERU, [ROOT] is the origin of the Lorentz hyperboloid as it entails the entire representation space. The location of [ROOT] for CLIP is not as intuitive - the notion of entailment is mathematically not defined, and the origin does not lie on the hypersphere. We empirically estimate CLIP's [ROOT] as an embedding vector that has the least distance from all embeddings of the training dataset. Hence, we average all $2 \times 12 \mathrm{M}$ embeddings of images and text in RedCaps, followed by $L^{2}$ normalization. [ROOT] will be different for different CLIP models, whereas it is fixed for MERU.

Embedding distances from [ROOT]. In a representation space that effectively captures the visual-semantic hierarchy, text embeddings should lie closer to [ROOT] than image embeddings, since text is more generic than images (Figure 1). Figure 4 shows the distribution of embedding distances from [ROOT] - these distributions overlap for CLIP but are separated for MERU. The range of distributions in Figure 4 (left) hints that MERU embeds text and images in two concentric, high-dimensional rings around [ROOT]. The ring of text is more spread out, whereas the ring of images is relatively thin. This resembles the structure of the visual-semantic hierarchy - images only occupy leaf nodes whereas text occupies many intermediate nodes.


Figure 5. Image traversals with MERU and CLIP. We perform text retrieval at multiple steps while traversing from an image embedding to [ROOT]. Overall, CLIP retrieves fewer textual concepts (top row), but in some cases it reveals a coarse hierarchy (bottom row). MERU captures hierarchy with significantly greater detail, we observe that: (1) Text becomes more generic we move towards [ROOT], e.g., white horse $\rightarrow$ equestrian and retro photo camera $\rightarrow$ vintage. (2) MERU has higher recall of concepts than CLIP, like words in bottom row: homemade, city, monument. (3) MERU also shows systematic text $\rightarrow$ image entailment, e.g., day entails many images captured in daylight.

Image traversals. In a discrete tree, one can discover the ancestors of any node by performing shortest-path traversal to the root node (Dijkstra, 1959). We perform such traversals for images with MERU and CLIP. If the representation space has captured the visual-semantic hierarchy, then a shortestpath traversal from an image to [ROOT] should let us infer textual concepts that describe the image with varying levels of abstraction. We briefly describe this analysis here, refer Appendix D for more details.

We traverse from an image and [ROOT] by interpolating 50 equally spaced steps along the geodesic connecting their embedding vectors. We use every interpolated step embedding as a query to perform retrieve the nearest neighbor from a set of text embeddings $\mathcal{X}$, that also include [ROOT].

We display results with 60 randomly selected images collected from pexels.com, a website that offers freely usable stock photos. We use two different sets $\mathcal{X}$ having text sourced from: (1) 750 captions obtained using the image metadata from pexels.com, and (2) 8.7 M captions from the YFCC dataset (Thomee et al., 2016).

Figure 5 shows results with 8 selected images and captions from pexels.com. Appendix D includes results with 52 other images and with YFCC captions without cherrypicking. CLIP seems to capture hierarchy to some extent, often retrieving very few (or zero) captions between image and [ROOT]. MERU captures it with much finer granularity, retrieving concepts that gradually become more generic as we move closer to [ROOT].

## 6. Related work

Visual-language representation learning. Soon after the initial success of deep learning on ImageNet (Krizhevsky et al., 2012), deep metric learning (Sohn, 2016; Song et al., 2015) was used to learn vision-language representations in a shared semantic space (Frome et al., 2013; Karpathy \& Fei-Fei, 2015). The motivations at the time included the possibility of improving vision models (Frome et al., 2013), enabling zero-shot learning by expressing novel categories as sentences (Elhoseiny et al., 2013; Frome et al., 2013), and better image-text retrieval (Karpathy \& Fei-Fei, 2015; Young et al., 2014). Another line of work proposed learning visual models from language supervision via objectives like textual n-gram prediction (Li et al., 2017), or generative objectives like masked language modeling (Bulent Sariyildiz et al., 2020) or image captioning (Desai \& Johnson, 2021).

More recent approaches like CLIP (Radford et al., 2021) and ALIGN (Jia et al., 2021) use contrastive metric learning to pre-train Vision Transformers (Dosovitskiy et al., 2021) and have helped to better realize the motivations of the earlier works in practice. While all prior works learn Euclidean embeddings, MERU explicitly works in the hyperbolic space that is conceptually better for embedding the visual-semantic hierarchy (Figure 1) underlying images and text. Our results (Section 4) demonstrate that MERU yields strong performance as prior works, and also offers better interpretability to the representation space.

Entailment embeddings. In a vision and language context, Order Embeddings (Vendrov et al., 2016) propose capturing the partial order between language and vision by enforcing that text embeddings $\mathbf{x}$ and image embeddings $\mathbf{y}$, should satisfy $\mathbf{y} \leq \mathbf{x}$ for all dimensions $i$. While enforcing order is useful for retrieval, in our initial experiments, we found that distance-based contrastive learning to be crucial for better performance on classification and retrieval. Thus, we focus on adapting the currently successful contrastive learning and add our entailment objective in conjunction, to obtain the desired structure in the representation space.

For NLP and knowledge graph embedding applications, several approaches embed partially ordered data (Bai et al., 2021; Dasgupta et al., 2020; Ganea et al., 2018; Nguyen et al., 2017; Vilnis et al., 2018) or discover ordering from pairwise similarities (Le et al., 2019; Nickel \& Kiela, 2017; Tifrea et al., 2018). Our work has a flavor of both these lines of work, since we impose structure across modalities, but order also emerges within modality (Figure 5).

Hyperbolic representations in computer vision. Khrulkov et al. (2020) learn hyperbolic image embeddings using image-label pairs, while Atigh et al. (2022) study image segmentation by utilizing hyperbolic geometry. More recently, Ermolov et al. (2022) and Ge et al. (2023) extend standard
contrastive self-supervised learning framework (He et al., 2020; Wu et al., 2018) in vision to learn hyperbolic representations. In contrast to all these works, MERU learns multimodal representations with an order of magnitude more data and shows strong zero-shot transfer abilities across generic artificial intelligence tasks (Radford et al., 2021).

## 7. Conclusion

In this paper, we learn large-scale image-text representations (MERU) to capture the visual-semantic hierarchy underlying images and text. Our key innovation is to bring advances in learning hyperbolic representations to practical, largescale deep learning applications. MERU is competitive or more performant than approaches that learn Euclidean representations (like CLIP). It does so along with capturing hierarchical knowledge which allows one to make powerful inferences such as reasoning about images at different levels of abstraction. Beyond this, our model also provides clear performance gains for small embedding dimensions (which are useful in resource-constrained settings). We hope this work catalyzes progress in learning useful representations from large amounts of unstructured data.

Future work. In this scaling era, we are seeing rapid progress with large multi-modal models trained using millions (or even billions) of image-text pairs. The quality and concept distribution of training data plays a vital role in the efficacy of these models. Such training data is becoming increasingly opaque and black-box due to its unprecedented scale. We believe that the time is ripe to revisit the unreasonable effectiveness of data in deep learning (Halevy et al., 2009; Sun et al., 2017). Modeling hierarchies can help uncover higher-order relationships beyond basic data statistics. As a concrete example, Figure 1 "so cute $<3$ " is an extremely generic caption and does not the precise details in images. Such captions add noisy supervision in contrastive loss by making false negative pairs with many images in the batch. Image traversals with MERU Figure 5 can discover such noisy captions. ML practitioners can filter or re-caption such training images to improve dataset quality and train subsequent models for improved performance.
Limitations. Our work is not without limitations. While MERU yields hyperbolic representations that excel at zeroshot retrieval and image classification tasks, the linear probe evaluations in Table 7 show that the underlying Euclidean representations from the image encoder of MERU underperform CLIP. Exploring MERU's transferability to other tasks that involve few-shot learning or full-model fine-tuning is also beyond scope of this paper. Finally, while we provide ample qualitative analysis of image traversals, future work can propose more systematic ways to evaluate the hierarchical knowledge captured by vision-language models.

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## Appendix

## A. Entailment loss derivations

We derive the entailment loss components (Eqn. 12) used in our approach. Note that for $c>0$, the curvature of the hyperboloid is $-c$.

Half-aperture. To derive the entailment loss for arbitrary curvatures $c>0$, we start with the expression of halfaperture for the Poincaré ball, introduced by Ganea et al. (2018). Let $\mathbf{x}_{b}$ be a point on the Poincaré ball, the cone half-aperture is defined as follows:

$$
\begin{equation*}
\operatorname{aper}_{b}\left(\mathbf{x}_{b}\right)=\sin ^{-1}\left(K \frac{1-c\left\|\mathbf{x}_{b}\right\|^{2}}{\sqrt{c}\left\|\mathbf{x}_{b}\right\|}\right) \tag{13}
\end{equation*}
$$

The Poincaré ball model and Lorentz hyperboloid model are isometric to each other - one can map any point $\mathbf{x}_{b}$ from the Poincaré ball to another point $\mathbf{x}_{h}$ on the hyperboloid using the following differentiable transformation:

$$
\begin{equation*}
\mathbf{x}_{h}=\frac{2 \mathbf{x}_{b}}{1-c\left\|\mathbf{x}_{b}\right\|^{2}} \tag{14}
\end{equation*}
$$

The half-aperture of a cone should be invariant to the exact hyperbolic model we use, hence $\operatorname{aper}_{h}\left(\mathrm{x}_{h}\right)=\operatorname{aper}_{b}\left(\mathrm{x}_{b}\right)$. Substituting Eqn. 14 in Eqn. 13, we get the expression:

$$
\operatorname{aper}_{h}\left(\mathbf{x}_{h}\right)=\sin ^{-1}\left(\frac{2 K}{\sqrt{c}\left\|\mathbf{x}_{h}\right\|}\right)
$$

Exterior angle. Consider three points $\mathbf{O}$ (the origin), $\mathbf{x}$ (text embedding) and $\mathbf{y}$ (image embedding). Then, a hyperbolic
triangle is a closed shape formed by the geodesics connecting each pair of points. Similar to the Euclidean plane, the hyperbolic plane also has its law of cosines that allows us to talk about the angles in the triangle (Lee, 2019). Let the Lorentzian distances (Eqn. 4) be $x=d(\mathbf{O}, \mathbf{y}), y=d(\mathbf{O}, \mathbf{x})$, and $z=d(\mathbf{x}, \mathbf{y})$. We can write the expression of exterior angle as follows:

$$
\begin{aligned}
& \operatorname{ext}(\mathbf{x}, \mathbf{y})=\pi-\angle \mathbf{O} \mathbf{x y} \\
& =\pi-\cos ^{-1}\left[\frac{\cosh (z \sqrt{c}) \cosh (y \sqrt{c})-\cosh (x \sqrt{c})}{\sinh (z \sqrt{c}) \sinh (y \sqrt{c})}\right]
\end{aligned}
$$

We use the relation $\pi-\cos ^{-1}(t)=\cos ^{-1}(-t)$ in the above equation. Then, let us define a function $g(t)=\cosh (t \sqrt{c})$ for brevity, and substitute in the above equation. We also substitute $\sinh (t)=\sqrt{\cosh ^{2}(t)-1}$ as per the hyperbolic trigonometric identity. Putting it all together, we get:

$$
\begin{equation*}
\operatorname{ext}(\mathbf{x}, \mathbf{y})=\cos ^{-1}\left[\frac{g(x)-g(z) g(y)}{\sqrt{g(z)^{2}-1} \sqrt{g(y)^{2}-1}}\right] \tag{15}
\end{equation*}
$$

Now all we need is to compute $g(x), g(y)$, and $g(z)$. We substitute the $z=d(\mathbf{x}, \mathbf{y})$ in $g(z)$ below:

$$
\begin{aligned}
g(z) & =\cosh (d(\mathbf{x}, \mathbf{y}) \sqrt{c}) \\
& =\cosh \left(\frac{1}{\sqrt{c}} \cosh ^{-1}\left(-c\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}\right) \cdot \sqrt{c}\right) \\
& =-c\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}
\end{aligned}
$$

Similarly, $g(x)=-c\langle\mathbf{O}, \mathbf{y}\rangle_{\mathcal{L}}$ and $g(y)=-c\langle\mathbf{O}, \mathbf{x}\rangle_{\mathcal{L}}$. The Lorentzian inner product (Eqn. 1) with origin $\mathbf{O}$ simplifies:

$$
\langle\mathbf{O}, \mathbf{x}\rangle_{\mathcal{L}}=-\frac{x_{\text {time }}}{\sqrt{c}} \quad \text { and } \quad\langle\mathbf{O}, \mathbf{y}\rangle_{\mathcal{L}}=-\frac{y_{t i m e}}{\sqrt{c}}
$$

Through this, we get $g(x)=x_{\text {time }} \sqrt{c}$ and $g(y)=y_{\text {time }} \sqrt{c}$. Finally, we can substitute $g(x), g(y)$, and $g(z)$ to re-write Eqn. 15 to give the final expression as follows:

$$
\operatorname{ext}(\mathbf{x}, \mathbf{y})=\cos ^{-1}\left(\frac{y_{\text {time }}+x_{\text {time }} c\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}}{\sqrt{x_{\text {time }}^{2} c-1} \sqrt{\left(c\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}\right)^{2}-1}}\right)
$$

Finally, we use the relation between $x_{\text {time }}$ and $\mathbf{x}_{\text {space }}$ (Eqn. 3) to simplify the denominator, giving the final expression of exterior angle as follows:

$$
\operatorname{ext}(\mathbf{x}, \mathbf{y})=\cos ^{-1}\left(\frac{y_{\text {time }}+x_{\text {time }} c\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}}{\left\|\mathbf{x}_{\text {space }}\right\| \sqrt{\left(c\langle\mathbf{x}, \mathbf{y}\rangle_{\mathcal{L}}\right)^{2}-1}}\right)
$$

Table 5. CLIP baseline. We develop a strong CLIP baseline that trains on an 8-GPU machine in less than one day (ViT-S image encoder), starting with SLIP (Mu et al., 2022) as a reference. We benchmark improvements on zero-shot image classification across 16 datasets. Our RedCaps-trained CLIP baseline (last row) is a significantly stronger baseline than its YFCC-trained counterparts.

|  |  | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \ddot{0}_{0} \\ & \text { ̈ㅔ } \end{aligned}$ |  |  | $\begin{aligned} & 8 \\ & \frac{8}{1} \\ & \stackrel{y}{4} \\ & \sqrt[3]{2} \end{aligned}$ | $\stackrel{N}{S}_{0}^{n}$ | $\underset{\sim}{\hat{N}}$ | نَّ |  | $\stackrel{0}{0}$ | $\stackrel{n}{2}$ |  | $\begin{aligned} & \tilde{0} \\ & 0 \\ & \frac{3}{I I} \\ & 0 \end{aligned}$ | $\frac{O}{4}$ |  | $\begin{aligned} & \text { む } \\ & \text { U } \\ & \text { N } \\ & \text { H } \end{aligned}$ | $\begin{aligned} & \text { ت } \\ & \text { In } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| YFCC15M-trained models |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| SLIP's CLIP (Mu et al., 2022) | 368M | 32.0 | 43.7 | 61.9 | 30.2 | 30.9 | 41.3 | 3.5 | 3.9 | 18.1 | 26.1 | 51.4 | 48.7 | 87.3 | 17.5 | 16.8 | 8.7 | 32.6 |
| Our implementation | 368M | 33.1 | 42.3 | 64.9 | 34.4 | 33.7 | 43.8 | 2.9 | 5.1 | 19.1 | 25.0 | 49.8 | 47.2 | 87.4 | 26.8 | 21.6 | 9.0 | 34.1 |
| + BS 4096 $\rightarrow 2048$ | 184M | 28.2 | 34.2 | 58.7 | 29.4 | 27.4 | 39.4 | 2.9 | 4.3 | 16.5 | 20.1 | 43.8 | 42.2 | 85.4 | 20.2 | 19.0 | 8.5 | 30.0 |
| + sin-cos pos embed | 184M | 28.7 | 34.2 | 67.3 | 33.6 | 25.4 | 41.1 | 3.1 | 4.2 | 17.8 | 21.0 | 44.3 | 43.6 | 86.4 | 18.6 | 19.6 | 8.3 | 31.1 |
| RedCaps-trained models |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| + YFCC $\rightarrow$ RedCaps | 184M | 32.6 | 71.5 | 61.4 | 25.6 | 29.9 | 27.5 | 10.1 | 1.5 | 14.3 | 72.7 | 62.8 | 42.2 | 88.0 | 18.1 | 30.5 | 4.9 | 37.1 |
| $+90 \mathrm{~K} \rightarrow 120 \mathrm{~K}$ iters. | 246M | 33.9 | 72.5 | 60.1 | 24.4 | 30.0 | 27.5 | 11.3 | 1.4 | 13.1 | 73.7 | 63.9 | 44.4 | 88.2 | 18.6 | 31.4 | 5.2 | 37.5 |
| + our zero-shot prompts | 246M | 34.3 | 74.5 | 60.1 | 24.4 | 33.8 | 27.5 | 11.3 | 1.4 | 15.0 | 73.7 | 63.9 | 47.0 | 88.2 | 18.6 | 31.4 | 5.2 | 38.1 |

## B. Developing a strong CLIP baseline

One of our contributions is to establish a lightweight, yet strong CLIP baseline. The original CLIP models (Radford et al., 2021) are trained using a private dataset of 400 M image-text pairs across 128 GPUs for more than 10 days. We aim to maximize accessibility for future works, hence we decide our hyperparameters such that our smallest model can train on a single 8-GPU machine in less than one day.
We start with a reference CLIP ViT-S/16 baseline from SLIP (Mu et al., 2022) and carefully introduce one modification at a time. We benchmark improvements on zero-shot image classification across 16 datasets used in our main experiments, using text prompts used by (Radford et al., 2021). Results are shown in Table 5.

CLIP baseline by SLIP. This re-implemented baseline was trained using a 15 M subset of the YFCC dataset (Thomee et al., 2016). We re-evaluate the publicly released ViT-S/16 checkpoint ${ }^{3}$ using our evaluation code; it obtains $32.6 \%$ average accuracy across all datasets.
Our re-implementation. We attempt a faithful replication of CLIP by following hyperparameters in SLIP. Our implementation obtains slightly higher average performance ( $34.1 \%$ ) with three minor changes:

- We use an undetached gather operation to collect all image/text features across all GPUs for contrastive loss. This ensures proper gradient flow across devices.
- The above change allows using weight decay $==0.2$ like OpenAI's CLIP, unlike 0.5 used by SLIP's CLIP.
- During training and inference, we resize input images using bicubic interpolation like original CLIP, instead of bilinear interpolation in SLIP's CLIP.

[^3]Fitting the model on 8-GPUs. This CLIP model requires $16 \times$ V100 32GB GPUs with a batch size of 4096 and automatic mixed precision (Micikevicius et al., 2018). Techniques like gradient checkpointing (Chen et al., 2016) can reduce memory requirements, but it comes at a cost of reduced training speed. Hence we avoid making it a requirement and simply reduce the batch size to 2048 . This incurs a performance drop as the effective images seen by the model are halved. We offset the effective shortening of the training schedule by using fixed sine-cosine position embeddings in ViT, so learning position-related inductive biases is not required. This change slightly improves average accuracy ( $30.0 \% \rightarrow 31.1 \%$ average accuracy).

Training with RedCaps dataset. RedCaps dataset (Desai et al., 2021) comprises 12M image-text pairs from Reddit, sourced from Pushshift (Baumgartner et al., 2020). Training with RedCaps significantly improves performance over YFCC-trained models ( $31.1 \% \rightarrow 37.1 \%$ average accuracy), especially on datasets whose concepts have high coverage in RedCaps, e.g., Food-101 (Bossard et al., 2014) and Pets (Parkhi et al., 2012).

To account for the smaller size of RedCaps, we increase the training iterations from 90 K up to 120 K . Finally, we modify zero-shot prompts for some datasets to match the linguistic style of RedCaps. For example, many captions in r/food simply mention the name of the dish in the corresponding image, hence we use the prompt 'food : $\}$ '. See Table 8 for the list of prompts for all datasets. We did not extensively tune these prompts, but we checked performance on the held-out validation sets to avoid cheating on the test splits.

Finally, our CLIP ViT-S/16 baseline trains on $8 \times$ V100 32 GB GPUs within $\approx 14$ hours and achieves $38.1 \%$ average performance across 16 datasets. We use these hyperparameters for all MERU and CLIP models in our experiments.

Table 6. Datasets used for image classification evaluation. Datasets in highlighted rows do not have an official validation split - we use a random held-out subset of the training split. EuroSAT and RESISC do not define any splits; we randomly sample non-overlapping splits. CLEVR Counts is derived from CLEVR (Johnson et al., 2017) and SST2 was introduced as an NLP dataset by (Socher et al., 2013).

| Dataset | Classes | Train | Val | Test |
| :--- | :---: | :---: | :---: | :---: |
| Food-101 (Bossard et al., 2014) | 101 | 68175 | 7575 | 25250 |
| CIFAR-10 (Krizhevsky, 2009) | 10 | 45000 | 5000 | 10000 |
| CIFAR-100 (Krizhevsky, 2009) | 100 | 45000 | 5000 | 10000 |
| CUB-2011 (Wah et al., 2011) | 200 | 4795 | 1199 | 5794 |
| SUN397 (Xiao et al., 2010) | 397 | 15880 | 3970 | 19849 |
| Stanford Cars (Krause et al., 2013) | 196 | 6515 | 1629 | 8041 |
| FGVC Aircraft (Maji et al., 2013) | 100 | 3334 | 3333 | 3333 |
| DTD (Cimpoi et al., 2014) | 47 | 1880 | 1880 | 1880 |
| Oxf-IIIT Pets (Parkhi et al., 2012) | 37 | 2944 | 736 | 3669 |
| Caltech-101 (Fei-Fei et al., 2004) | 102 | 2448 | 612 | 6084 |
| Flowers (Nilsback \& Zisserman, 2008) | 102 | 1020 | 1020 | 6149 |
| STL-10 (Coates et al., 2011) | 10 | 4000 | 1000 | 8000 |
| EuroSAT (Helber et al., 2019) | 10 | 5000 | 5000 | 5000 |
| RESISC (Cheng et al., 2017) | 45 | 3150 | 3150 | 25200 |
| Country211 (Radford et al., 2021) | 211 | 31650 | 10550 | 21100 |
| MNIST (LeCun et al., 2010) | 10 | 48000 | 12000 | 10000 |
| CLEVR Counts (Zhai et al., 2019) | 8 | 4500 | 500 | 5000 |
| PCAM (Veeling et al., 2018) | 2 | 262144 | 32768 | 32768 |
| SST2 (Radford et al., 2021) | 2 | 6920 | 872 | 1821 |

Table 7. Linear probe evaluation. We train a logistic regression classifier on embeddings extracted from the image encoders of CLIP and MERU (before projection layers). Note that embeddings from MERU are not lifted onto the hyperboloid.

|  |  | $\begin{aligned} & \text { oे } \\ & \frac{1}{8} \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & \frac{1}{x} \\ & \frac{1}{4} \\ & \end{aligned}$ | 8 $\stackrel{8}{2}$ 4 4 4 | $\stackrel{n}{U}$ | $\begin{aligned} & \hat{N} \\ & \underset{\sim}{\tilde{N}} \end{aligned}$ | ジ |  | $\stackrel{\theta}{0}$ | $\stackrel{n}{0}$ | $\begin{aligned} & \overline{0} \\ & \overline{1} \\ & \overline{0} \\ & \text { UU } \\ & \text { Ü } \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \frac{0}{0} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{y}{4} \\ & \stackrel{y}{5} \end{aligned}$ | $\begin{aligned} & \text { E } \\ & \text { ह } \\ & 0 \\ & \text { Hy } \end{aligned}$ |  | $\begin{aligned} & \text { E } \\ & \text { E } \\ & \text { E } \\ & 0 \end{aligned}$ | $\sum_{\Sigma}^{5}$ | $\underset{~ c}{\substack{y \\ u \\ \hline}}$ | $\sum_{U}^{\sum}$ | $\stackrel{N}{V}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ViT | CLIP | 85.3 | 89.6 | 72.3 | 68.8 | 61.1 | 60.5 | 42.2 | 71.2 | 87.9 | 88.4 | 96.2 | 95.5 | 95.7 | 88.1 | 15.0 | 98.5 | 57.5 | 84.6 | 54.9 |
| S/16 | MERU | 85.2 | 89.7 | 70.9 | 69.2 | 59.6 | 58.0 | 43.1 | 70.2 | 87.5 | 85.6 | 95.5 | 95.5 | 95.8 | 87.0 | 14.8 | 98.2 | 56.8 | 84.1 | 54.5 |
| ViT | CLIP | 88.4 | 92.2 | 76.5 | 73.2 | 64.7 | 71.1 | 50.4 | 72.6 | 90.2 | 89.6 | 97.3 | 97.1 | 96.9 | 90.0 | 16.7 | 98.9 | 52.7 | 84.4 | 57.6 |
| B/16 | MERU | 88.2 | 92.3 | 74.6 | 70.9 | 63.4 | 68.4 | 48.2 | 70.7 | 90.3 | 88.6 | 96.6 | 96.7 | 96.5 | 89.0 | 16.5 | 98.7 | 56.0 | 85.5 | 56.2 |
| ViT | CLIP | 89.6 | 95.3 | 80.5 | 75.7 | 66.0 | 75.7 | 54.5 | 75.7 | 92.0 | 92.0 | 97.4 | 97.6 | 96.9 | 90.5 | 17.8 | 99.2 | 55.6 | 87.5 | 56.1 |
| L/16 | MERU | 89.0 | 94.1 | 77.3 | 74.2 | 63.7 | 71.9 | 51.2 | 70.9 | 90.1 | 87.5 | 96.7 | 97.3 | 96.8 | 89.1 | 17.0 | 98.9 | 55.4 | 86.0 | 55.8 |

## C. Linear probe evaluation

Our experimental evaluations (Section 4) focus on zero-shot transfer (Elhoseiny et al., 2013; Radford et al., 2021). Another established protocol to evaluate visual representations is linear probe evaluation, which involves training linear models on frozen image embeddings. This protocol is popular in self-supervised representation learning literature, with Doersch et al. (2015), Zhang et al. (2016), and Noroozi \& Favaro (2016) being notable early works. We follow the implementation of Kornblith et al. (2019) as it is simple and less sensitive to choice of evaluation hyperparameters. This setup is also followed by CLIP (Radford et al., 2021) and many recent works on representation learning (El Banani et al., 2023; Fürst et al., 2022; Li et al., 2022).

We evaluate using datasets listed in Table 6. We train a logistic regression classifier on embeddings extracted from
the image encoder (before projection layer) of MERU and CLIP. For MERU, these underlying representations belong to a Euclidean space. We use the implementation from scikit-learn (Pedregosa et al., 2011) library, with LBFGS (Liu \& Nocedal, 1989) optimizer and search the regularization cost per dataset, $C \in\left[10^{-6}, 10^{6}\right]$, performing two-step search on val split like Radford et al. (2021). Then we train a final classifier on combined train and val splits for a maximum of 1000 iterations, then report top-1 mean per-class accuracy on the test split.

Results in Table 7 show that MERU mostly matches or underperforms CLIP. Our main focus is not on improving the underlying Euclidean representations from the encoders, but to demonstrate strong zero-shot transfer and interpretability benefits. Future work can focus on improving MERU's capabilities on other transfer applications.

Table 8. Prompts used for zero-shot classification (Section 4.3). Most of these prompts are same as (Radford et al., 2021). We modify prompts for some datasets, that significantly improved performance for both MERU and CLIP - We did not perform extensive prompt tuning, we simply checked the performance on val splits for our CLIP baseline (Appendix B). NOTE: Some prompts use the word 'porn' as it is included in the subreddit name. It does not indicate pornographic content but simply high-quality photographs.

| ImageNet (our prompts) |  |  |
| :---: | :---: | :---: |
| i took a picture : itap of a $\}$. | pics : a bad photo of the $\}$. | pics : a origami \{\}. |
| pics : a photo of the large $\}$. | pics : a $\}$ in a video game. | pics : art of the $\}$. |
| pics : a photo of the small $\}$. |  |  |
| Food-101 (our prompts) | DTD (our prompts) | Oxford Flowers (our prompts) |
| food : \{\}. | pics : \{\} texture. | flowers : \{\}. |
| food porn : $\}$. | pics : \{ \} pattern. | STL10 |
| CIFAR-10 and CIFAR-100 | pics : $\}$ thing. | a photo of a $\}$. |
| a photo of a $\}$. | pics : this $\}$ texture. | a photo of the $\}$. |
| a blurry photo of a $\}$. | pics : this $\}$ pattern. | EuroSAT |
| a black and white photo of a $\}$ | pics : this \{\} thing. | a centered satellite photo of \{\}. |
| a low contrast photo of a $\}$. | Oxford-IIIT Pets | a centered satellite photo of a |
| a high contrast photo of a $\}$. | a photo of a \{ , a type of pet. |  |
| a bad photo of a $\}$ | Caltech-101 | a centered satellite photo of the \{ |
| a good photo of a $\}$. | a photo of a $\}$. |  |
| a photo of a small $\}$. | a painting of a $\}$. | RESISC satellite imagery of \{\}. |
| a photo of a big $\}$. | a plastic $\}$. | satellite imagery of aerial imagery of $\{$. |
| a photo of the $\}$. | a sculpture of a $\}$. | satellite photo of |
| a blurry photo of the $\}$. | a sketch of a $\}$. | aerial photo of $\}$. |
| a black and white photo of the $\}$. | a tattoo of a $\}$. | satellite view of $\}$ |
| a low contrast photo of the $\}$. | a toy \{\}. | aerial view of $\}$. |
| a high contrast photo of the $\}$. | a rendition of a $\}$. | satellite imagery of a \{\}. |
| a bad photo of the $\}$. | a embroidered $\}$. | aerial imagery of a $\}$. |
| a good photo of the $\}$. | a cartoon $\}$. | satellite photo of a $\}$. |
| a photo of the small $\}$. | a $\}$ in a video game. | aerial photo of a $\}$. |
| a photo of the big $\}$. | a plushie $\}$. | satellite view of a \{\}. |
| CUB-2011 (our prompts) | a origami $\}$. | aerial view of a \{\}. |
| bird pics : \{\}. | art of a $\}$. | satellite imagery of the $\}$. |
| birding : \{\}. | graffiti of a $\}$. | aerial imagery of the $\}$. |
| birds : \{\}. | a drawing of a $\}$. | satellite photo of the $\}$. |
| bird photography : $\}$. | a doodle of a $\}$. | aerial photo of the $\}$. |
| SUN397 | a photo of the $\}$. | satellite view of the \{\}. |
| a photo of a $\}$. | a painting of the $\}$. | aerial view of the $\}$. |
| a photo of the $\}$. | the plastic $\}$. | Country211 |
| Stanford Cars | a sculpture of the $\}$. | a photo i took in $\}$. |
| a photo of a $\}$. | a sketch of the $\}$. | a photo i took while visiting \{ . |
| a photo of the $\}$. | a tattoo of the $\}$. | a photo from my home country of |
| a photo of my $\}$. | the toy $\}$. |  |
| i love my \{ ! | a rendition of the $\}$. | a photo from my visit to \{\}. |
| a photo of my dirty $\}$. | the embroidered $\}$. | a photo showing the country of $\}$. |
| a photo of my clean $\}$. | the cartoon $\}$. | MNIST |
| a photo of my new $\}$. | the $\}$ in a video game. | a photo of the number: "\{\}". |
| a photo of my old $\}$. | the plushie $\}$. | CLEVR |
| FGVC Aircraft | the origami $\}$. | a photo of \{\} objects. |
| a photo of a $\}$, a type of aircraft. | art of the $\}$. | Patch Camelyon |
|  | graffiti of the $\}$. | this is a photo of \{\}. |
| a photo of the $\}$, a type of aircraft. | a drawing of the $\}$. | Rendered SST2 |
|  | a doodle of the $\}$. | a \{\} review of a movie. |

## D. Image traversals: more details and results

Our qualitative analysis in Section 5 involves inferring the learned visual-semantic hierarchy in the representation space through image traversals. We performed shortest-path traversal from a given image embedding $\mathbf{y}$ to the [ROOT] embedding by interpolating 50 equally spaced steps. At each step, we retrieve text from a set $\mathcal{X}$ of text embeddings (including [ROOT]). Here we include the precise methodology details to perform image traversals.

MERU and CLIP have different methods for interpolation and nearest-neighbor retrieval due to the difference in geometric properties of Euclidean and hyperbolic spaces.

## Interpolating steps:

- CLIP: We linearly interpolate between $L^{2}$ normalized embeddings of $\mathbf{y}$ and [ROOT], and then $L^{2}$ normalize all step embeddings. In PyTorch (Paszke et al., 2019), torch. lerp can perform this linear interpolation.
- MERU: We linearly interpolate in the tangent space, between $\mathbf{v}=\operatorname{logm}_{\mathbf{O}}(\mathbf{y})($ Eqn. 8) and $\mathbf{O}$ (origin is [ROOT]), then lift all step embeddings onto the hyperboloid.


## Nearest-neighbor text retrieval:

- CLIP: We select $\mathbf{x} \in \mathcal{X}$ having the highest cosine similarity with the step embedding.
- MERU: First we create a subset $\mathcal{X}_{e} \subset \mathcal{X}$ of text embeddings that entail the given step embedding, i.e., Eqn. 12 evaluates to 0 (note that [ROOT] entails everything). Then we select $\mathbf{x} \in \mathcal{X}_{e}$ having the highest Lorentzian inner product with the step embedding.

At any given step, the caption associated with the retrieved texct embedding $\mathbf{x}$ (or [ROOT]) is the retrieved nearest neighbor. We observed that multiple consecutive steps retrieve the same caption, so our results only display unique captions encountered during the traversal.

Caption sources: We create the set of text embeddings $\mathcal{X}$ using captions collected from two different sources.

- pexels.com metadata: We manually collect metadata (Figure 6), then filter tags to only keep nouns and adjectives (total 750 captions and tags). We filter by performing parts-of-speech using the RoBERTa (Liu et al., 2019) model (en-core-web-trf) from SpaCy (Honnibal et al., 2020) library. Finally, we convert tags to captions by filling prompts - 'a photo of $\}$.' for nouns, and 'this photo is $\}$.' for adjectives.
- YFCC dataset: We use the text descriptions of the YFCC-15M subset (Radford et al., 2021). We perform minimal text processing of these captions according to RedCaps (Desai et al., 2021), to match the training data distribution. This involves converting to lowercase, using ftfy (Speer, 2019) to strips accents and non-latin charac-


Figure 6. pexels.com webpage of an image used in our results. We manually collect the closed caption (CC) and 'More like this' tags for all images to create the retrieval set for image traversals.
ters, removing all sub-strings enclosed in brackets ((.*), $[. \star]$ ), and replacing social media handles (words starting with '@') with a <usr>. We also remove captions having more than 20 tokens (for ease of visualization). Finally, we obtain $\approx 8.7 \mathrm{M}$ captions.

Results: Figure 5 shows selected qualitative examples with 8 out of 60 images. On the next pages, Figures 7 to 11 include results with other 52 images. After the image credits (Appendix E), we display results with YFCC captions.


Figure 7. Image traversals with MERU and CLIP (locations and landmarks). Retrieved captions are sourced from pexels.com metadata. MERU captures a more systematic and fine-grained visual-semantic hierarchy than CLIP - trends are same as Figure 5.


Figure 8. Image traversals with MERU and CLIP (flora and fauna). Retrieved captions are sourced from pexels.com metadata. MERU captures a more systematic and fine-grained visual-semantic hierarchy than CLIP - trends are same as Figure 5.


Figure 9. Image traversals with MERU and CLIP (food and drinks). Retrieved captions are sourced from pexels. com metadata. MERU captures a more systematic and fine-grained visual-semantic hierarchy than CLIP - trends are same as Figure 5.


Figure 10. Image traversals with MERU and CLIP (objects and scenes). Retrieved captions are sourced from pexels. com metadata. MERU captures a more systematic and fine-grained visual-semantic hierarchy than CLIP - trends are same as Figure 5.


Figure 11. Image traversals (objects and scenes). Retrieved captions are sourced from pexels. com metadata. MERU captures a more systematic and fine-grained visual-semantic hierarchy than CLIP - trends are same as Figure 5.

## E. Image credits

All images displayed in this paper are collected from pexels. com, a photography website that offers images with permissible usage licenses. Below is the list of the image source URLs listed in order of their appearance in the paper. We thank all the photographers for generously sharing these images.

## Illustration of the visual-semantic hierarchy (Figure 1).

- www.pexels.com/photo/adult-yellow-labrador-retriever-standing-on-snow-field-1696589
- www. pexels.com/photo/homeless-cat-fighting-with-dog-on-street-6601811
- www.pexels.com/photo/short-coated-gray-cat-20787

Image traversals - results in the main paper (Figure 5).
(1) www.pexels.com/photo/a-bengal-cat-sitting-beside-wheatgrass-on-a-white-surface-7123957
(2) www.pexels.com/photo/white-horse-running-on-green-field-1996337
(3) www.pexels.com/photo/photography-of-rainbow-during-cloudy-sky-757239
(4) www.pexels.com/photo/retro-photo-camera-on-table-7162551
(5) www.pexels.com/photo/avocado-toast-served-on-white-plate-10464867
(6) www.pexels.com/photo/photo-of-brooklyn-bridge-new-york-2260783
(7) www.pexels.com/photo/taj-mahal-through-an-arch-2413613
(8) www.pexels.com/photo/sydney-opera-house-7088958

## Image traversals - locations and landmarks (Figure 7).

(9) www.pexels.com/photo/golden-gate-bridge-san-francisco-california-1141853
(10) www.pexels.com/photo/white-cliffs-of-dover-in-england-9692909
(11) www.pexels.com/photo/the-famous-fountain-paint-pots-in-yellowstone-national-park-12767016
(12) www.pexels.com/photo/the-parthenon-temple-ruins-in-athens-greece-14446783
(13) www.pexels.com/photo/famous-big-ben-under-cloudy-sky-14434677
(14) www.pexels.com/photo/karlskirche-church-7018621
(15) www.pexels.com/photo/mt-fuji-3408353
(16) www.pexels.com/photo/horseshoe-bend-arizona-2563733
(17) www.pexels.com/photo/stars-at-night-1906667
(18) www.pexels.com/photo/volcano-erupting-at-night-under-starry-sky-4220967
(19) www.pexels.com/photo/northern-lights-1933319
(20) www.pexels.com/photo/attraction-building-city-hotel-415999

## Image traversals - flora and fauna (Figure 8).

(21) www.pexels.com/photo/squirrel-up-on-the-snow-covered-tree-15306429
(22) www.pexels.com/photo/a-seagull-flying-under-blue-sky-12509256
(23) www.pexels.com/photo/cute-pug-sitting-on-floor-in-white-kitchen-11199295
(24) www.pexels.com/photo/three-zebras-2118645
(25) www.pexels.com/photo/monarch-butterfly-perching-on-red-flower-1557208
(26) www.pexels.com/photo/red-hibiscus-in-bloom-5801054
(27) www.pexels.com/photo/white-chicken-on-green-grass-field-58902
(28) www.pexels.com/photo/yellow-blue-and-white-macaw-perched-on-brown-tree-branch-12715261
(29) www.pexels.com/photo/closeup-photo-of-red-and-white-mushroom-757292
(30) www.pexels.com/photo/photo-of-jellyfish-lot-underwater-3616240
(31) www.pexels.com/photo/yellow-labrador-retriever-wearing-red-cap-4588002
(32) www.pexels.com/photo/an-orca-whale-jumping-out-of-the-water-7767974

## Image traversals - food and drinks (Figure 9).

(33) www.pexels.com/photo/bread-and-coffee-for-breakfast-15891938
(34) www.pexels.com/photo/grilled-cheese-on-a-plate-14941252
(35) www.pexels.com/photo/bowl-of-ramen-12984979
(36) www.pexels.com/photo/green-chili-peppers-and-a-knife-5792428
(37) www.pexels.com/photo/spinach-caprese-salad-on-white-ceramic-plate-4768996
(38) www.pexels.com/photo/chocolate-cupcakes-635409
(39) www.pexels.com/photo/pav-bhaji-dish-on-a-bowl-5410400
(40) www.pexels.com/photo/clear-glass-bottle-filled-with-broccoli-shake-1346347
(41) www.pexels.com/photo/vada-pav-15017417
(42) www.pexels.com/photo/old-fashioned-cocktail-drink-4762719
(43) www.pexels.com/photo/coffee-in-white-ceramic-teacup-on-white-ceramic-suacer-894696
(44) www.pexels.com/photo/espresso-martini-in-close-up-photography-15082368

Image traversals - objects and scenes (Figure 10).
(45) www.pexels.com/photo/photograph-of-a-burning-fire-672636
(46) www.pexels.com/photo/white-clouds-in-blue-sky-8354530
(47) www.pexels.com/photo/raining-in-the-city-2448749
(48) www.pexels.com/photo/aerial-view-of-road-in-the-middle-of-trees-1173777
(49) www.pexels.com/photo/mountain-bike-on-the-beach-10542237
(50) www.pexels.com/photo/wax-candles-burning-on-ground-14184952
(51) www.pexels.com/photo/white-wooden-shelf-beside-bed-2062431
(52) www.pexels.com/photo/stainless-steel-faucet-on-white-ceramic-sink-3761560
(53) www.pexels.com/photo/jack-o-lantern-with-light-5659699
(54) www.pexels.com/photo/black-and-white-piano-keys-4077310
(55) www.pexels.com/photo/assorted-gift-boxes-on-floor-near-christmas-tree-3394779
(56) Www.pexels.com/photo/garden-table-and-chair-14831985

Image traversals - objects and scenes (Figure 11).
(57) www.pexels.com/photo/turned-on-floor-lamp-near-sofa-on-a-library-room-1907784
(58) www.pexels.com/photo/ripe-pineapple-on-gray-rock-beside-body-of-water-29555
(59) www.pexels.com/photo/close-up-shot-of-a-cockatiel-13511241
(60) www.pexels.com/photo/antique-bills-business-cash-210600

## Image traversals with YFCC captions.


(1)

| MERU | CLIP |
| :---: | :---: |
| leopard and stig have a beautiful piano at their home. | loki is a 1 year old bengal cat. |
| merlin wasn't impressed to leave the last house and <br> his precious cat grass | $\downarrow$ |
| my parents cat 'barry' loves being photographed! | $\downarrow$ |
| house cat posing | $\downarrow$ |
| mr . bo-majed | $\downarrow$ |
| our cat, our love our third member of our family. :) | $\downarrow$ |
| why are you taking pictures? it's dilo don't fill it up. :) | $\downarrow$ |


(2)

| MERU | CLIP |
| :---: | :---: |
| $\begin{array}{c}\text { caught my attention by the beautiful light cascading } \\ \text { on a grass behind this fellow. }\end{array}$ | pity about the camera shake in the evening light |
| $\begin{array}{c}\text { the focus is all wrong, but the white on the tail and } \\ \text { the tongue are pretty cool. }\end{array}$ | $\downarrow$ |
| just a goofy white guy. | $\downarrow$ |
| he was an active one, running to and fro. | $\downarrow$ |
| but then, she was happy to pose for me | $\downarrow$ |
| if she were a race horse her name would be |  |
| poopbiscuit. |  |$]$| dorky photo is dorky | $\downarrow$ |
| :---: | :---: |
| she looks so leery of the camera in this photo. | $\downarrow$ |
| this is only luky. | $\downarrow$ |
| [ROOT] | [ROOT] |
| MERU | CLIP |

going across brooklyn bridge on the way to brooklyn shot from the manhattan end of the brooklyn bridge
3 likes on instagram

(3)
manhattan depuis le brooklyn bridge park, a much more scenic to walk on than the brooklyn bridge brooklyn.

| bridge, manhattan skyline | new york new york! |
| :---: | :---: |
| shot from near the middle of the brooklyn bridge. | $\downarrow$ |
| this city goes on forever | $\downarrow$ |
| the city that never sleeps | $\downarrow$ |
| the city that never sleeps...it can't. | $\downarrow$ |
| it can be seen from most places in the city | $\downarrow$ |
| [ROOT] | [ROOT] |


(4)

| MERU | CLIP |
| :---: | :---: |
|  <br> raisins sourdough. | la tartine |
| with avocado slices - yum! | a toast to the new place |
| vaguely-healthy | $\downarrow$ |
| on bread, probably not what you should do with it, <br> but it was a good meal | $\downarrow$ |
| nice if you like this sort of thing. | $\downarrow$ |
| [ROOT] | [ROOT] |


(5)

(6)
\(\left.\begin{array}{cc}MERU \& CLIP <br>
\hline avenida paulista. fisheye 2 kodak elite chrome 100 \& leica m 7 with voigtlander zoom finder and dsptch <br>

camera strap\end{array}\right]\)| rolleiflex kodak portra 160 epson v500 scanner | zeiss ikon icarex, 02/2010 |
| :---: | :---: |
| ...of my brand new shiny 7.1mp camera. | faux lomo from www.dumpr.net - photo fun |
| camera de 5mp | zeiss-ikon |
| i'm a lumix camera fan now. | [ROOT] |
| [ROOT] | CLIP |
| MERU |  |


| double rainbows in our field were too good to pass up double rainbows in our field were too good to pass up <br> photographing |
| :---: | :---: |
| photographing |$|$| whoa... double rainbow | is that... a double rainbow? ;-) |
| :---: | :---: |
| is that... a double rainbow? ;-) | $\downarrow$ |
| what does it mean of this picture? | $\downarrow$ |
| only god could create something so beautiful. | $\downarrow$ |
| this is a good one to end with. reminds me of the |  |
| woman in this picture. |  |


| look out for that right one. | $\downarrow$ |
| :---: | :---: |
| [ROOT] | [ROOT] |
| MERU | CLIP |
| sydney opera house, october 2012. | gros plan sur l'opera de sydney |

sydney opera house see where this picture was taken. you can just make out the opera house in the far left.

| from the new opera house | my syd |
| :---: | ---: |
| $i$ think this is the last one $i$ have of the opera house. | $\downarrow$ |
| oh, and some opera house, too. | $\downarrow$ |
| just next to the famous opera house | $\downarrow$ |
| from horseshoe bay. | $\downarrow$ |
| taken from the donau. | $\downarrow$ |

[ROOT]
[ROOT]

(8)

MERU CLIP
captured this during my visit to taj mahal, seems like a weekend adventure to agra to see the taj mahal, see it still inspires young hearts.. also afternoon and night. luxury.

| the royal mausoleum on the grounds of 'iolani palace | this pre-dates the taj mahal |
| :---: | :---: | :---: |
| $\langle u s r>$ taj | you couldn't photograph inside the tombs, so this is | all i can show


| rotunda at nmai | the beauty of age, the mark of wisdom |
| :---: | :---: |
| outside of yet another palace | don't remember where. |
| taken from city palace. | $\downarrow$ |
| photography ii | $\downarrow$ |
| it can be seen from most places in the city | $\downarrow$ |
| kla photography | $\downarrow$ |

[ROOT]
[ROOT]

(9)

| MERU | CLIP |
| :---: | :---: |
| a high dynamic range shot of the golden gate bridge on a foggy afternoon | golden gate bridge thru photoshop lightroom |
| working on the perfect golden gate shot. | golden gates |
| golden gate iii | no...it's not the golden gate ;) |
| everyone who's been to sf has to take this photo at least once in their lives, right? | by gusf bit.ly/17hga6r |
| just got back from sf. will post more on my photoblog: ohad.me | the independent sf |
| ; 3 sf | thinking about painting this makes my shoulder hurt. |
| come out and play sf | still searching for the shot around here. |
| back from golden bay | $\downarrow$ |
| without fog | $\downarrow$ |
| [ROOT] | [ROOT] |
| MERU | CLIP |
| white cliffs of dover. august, maybe 2004? | coloured sand cliffs of alum bay, isle of wight, 1 may 2012. |
| calcite cumbria england | the cliffs are made of limestone. |
| poland rocks. | falkenberg from the south |
| white point natural area | at juta village |
| balderstone close | it's pretty rocky there. |
| nepomuk rocks... | $\downarrow$ |
| l'eglise de giverny | $\downarrow$ |
| one point if you can tell me where this was taken. | $\downarrow$ |
| also some kind of guenon, methinks. | $\downarrow$ |

[ROOT] [ROOT]

| MERU | CLIP |
| :--- | :---: |
| at yellowstone national park there are geyser pools <br> called painted pots because of the colors they exude. | yellowstone - noth entrance |
| yellowstone - noth entrance | ..like $i$ was, how yellowstone got its name |
| no trip to yellowstone is complete without it | i don't remember where this one was. it was striking. |
| people enjoy the hot spring, even at this time. | ain't he a beaut? |
| there are hot springs around here somewhere... | $\downarrow$ |
| wy'east |  |
| so many places that were stunning to look at. | $\downarrow$ |
| there are some special places in the earth. this is one ! | $\downarrow$ |
| with this photo... it's almost like taking a vacation | $\downarrow$ |
| just looking at this. | $\downarrow$ |

[ROOT]
[ROOT]

(12)

CLIP

| MERU | CLIP |
| :---: | :---: |
| the new parthenon museum, next to the acropolis. | athens archaeological site of the acropolis the <br> parthenon |
| this is the magnificent temple of zeus, located in the | temple of jupiter and ruins - selinunte |
| center of athens |  |

[^4]
(13)

| MERU | CLIP |
| :---: | :---: |
| obligatory big ben shot | big ben i el parlament |
| it's what they call a 'big clock' could be thought of as |  |
| 'big ben'' |  |$\quad$ or big ben, to his friends

[ROOT]

(14)


| MERU | CLIP |
| :---: | :---: |
| who needs mt.fuji | fuji provia 100 |
| fuji | fuji-q highlands |
| mt haba | fuji f30 |
| mount. | fuji-san in the background... |
| at quimbledon. | fairmount, in |

## [ROOT]

[ROOT]
(15)

(16)

| MERU | CLIP |
| :---: | :---: |
| a single exposure of horseshoe bend at sunrise. certainly one of my favorites from the trip. | yes, there is that backdrop of horseshoe bend :) |
| horseshoe bend | searching for the one ring |
| bend over | <usr>,usa |
| looking back at horseshoe canyon | $\downarrow$ |
| horseshoe bay... this is as close to paradise as you can get!!!! | $\downarrow$ |
| canyon country, specifically | $\downarrow$ |
| if you use my photo please post a link and let me know. | $\downarrow$ |
| [ROOT] | [ROOT] |
| MERU | CLIP |
| the milk way over bleriot ferry provincial park near drumheller, alberta. | the milky way as it appeared above the farmhouse in grey county - 30 sec exposure |
| the south-western part of the milky way | outside a house from the austmarka region |
| we were in quite a rural place, although there were still lights on the horizon. | this was just a couple of miles from the farmhouse we stayed in. |
| keeping the peace while bush was in town | $\downarrow$ |
| [ROOT] | [ROOT] |

(17)

[ROOT]
[ROOT]

Hyperbolic Image-Text Representations

(18)

| MERU | CLIP |
| :---: | :---: |
| lava as seen through night shot of volcan arenal | lava as seen through night shot of volcan arenal |
| the majestic villarica volcano | volcan osorno |
| nice photo of us in front of an active volcano. | volcanic origin |
| still an active volcano | the volcano! |
| around the khorgo volcano. | with volcan lanin in the background |
| mt stromlo | $\downarrow$ |
| i'm not sure where we were for this shot. | $\downarrow$ |
| for some reason, i think this photo is great! | $\downarrow$ |
| [ROOT] | [ROOT] |
| MERU | CLIP |
| arcs of the northen lights over the mountains neat |  |
| troms, norway, | northern lights on the otherside of patreksfjorur |
| northern lights, norway | aurora boreale a kangerlussuaq |
| in the night see where this picture was taken. | to a cinema the aurora |
| from the north. see where this picture was taken. | the village at the end of the world |
| sometimes something just looks out of place! | over a year's worth of photos here. |

[ROOT]
[ROOT]

| MERU | CLIP |
| :---: | :---: |
| cozy cone motel sign with tower of terror in background. california's adventure park at disneyland resort. | adam taylor ollie over sign kodak: iso 200 |
| my favorite tourist attraction in la. | enjoying jason scott's talk. |
| if this place didn't scream la, i don't know what does. | lost in las vegas- max ruckman |
| funny, this place was empty. | hollywood rip, ride, rockit |
| photo : l.g. | $\downarrow$ |
| [ROOT] | [ROOT] |
| MERU | CLIP |
| a squirrel enjoying the snow on a not-very cold day. | a squirrel enjoying the snow on a not-very cold day. |
| winter male still coming to food after the snow. | winter male still coming to food after the snow. |
| $i$ don't usually see these type of squirrels down here. loved this little guy. :) | $\downarrow$ |
| the last nut, my dear! | $\downarrow$ |
| it's kinda fuzzy. but i love this picture for some reason. | $\downarrow$ |
| $i$ had to take one picture, okay? | $\downarrow$ |

[ROOT]
[ROOT]

| MERU | CLIP |
| :---: | :---: |
| a gull in flight in stratford | a common or arctic tern flying above the scottish peninsula of kintyre. |
| gull on the wing | more bird shots at dyrholaey. |
| we also saw gull-billed but never close enough to photo. | wouldn't be a trip without at least one picture of a seagull |
| taken at little gull islands - b, little gull islands | seagull! |
| taken with the seagull | some bird thing. |
| $i$ was running after the seagull as i took this photo. | it took patience to get this shot since the stupid bird kept looking away |
| not a very nice bird, but still interesting to take pictures of... | $\downarrow$ |
| $i$ took one westward shot, just to see it | $\downarrow$ |
| $i$ keep taking this photograph | $\downarrow$ |

[ROOT]

Hyperbolic Image-Text Representations

(23)

(24)

(25)

(26)

| MERU | CLIP |
| :---: | :---: |
| margaret willie sanborn-sebo harvey henry sanborn pug dog framed | sacha my friend and companion patiently waiting for dad to finish taking photos !!! |
| everyday bear takes up this position as he waits for his mom to make his food. | eli begged mommy to take some photos |
| patiently waiting for the photographer to get his "shot". | $\downarrow$ |
| "this picture isn't going to work, and i'm going to show you why..." | $\downarrow$ |
| thinking to himself, "what's missing?" | $\downarrow$ |
| this is us...t trying to be sultry. | $\downarrow$ |
| to do nothing or to do something. | $\downarrow$ |
| [ROOT] | [ROOT] |
| MERU | CLIP |
| zebras, ruaha national park | zebras at kidepo national park, uganda. |
| zebra fouls with their mohicans, south africa | zebra fouls with their mohicans, south africa |
| the zebras | zebras at the watering hole |
| photographer: simone kuipers | three zebras |
| photographer: mark antos | good things come in threes. apparently, so do zebras. |
| from photo safari, take 2 . | the group comes around the bend. |
| at oudja | wild animals |
| at quimbledon. | more straglers |
| $\downarrow$ | of the group |
| $\downarrow$ | just a little to the left of the middle |
| [ROOT] | [ROOT] |
| MERU | CLIP |


| MERU | CLIP |
| :---: | :---: |
| monarch, danaus plexippus. shot in waimanalo, <br> hawaii. | a monarch pauses for a drink at a butterfly bush. |
| taken at the desert botanical garden in phoenix, <br> arizona, during its seasonal monarch butterfly exhibit. | monarch in a standard profile |
| i'm exercising to capture butterflies. | the monarch |
| visitor to the butterfly tree, | $\downarrow$ |
| monarch | $\downarrow$ |
| butterflies are always free, so enjoy as many of them |  |
| as you want. | $\downarrow$ |
| butterfly photography | $\downarrow$ |
| i tried to get more shots but it flew away. | $\downarrow$ |
| insect porn | $\downarrow$ |

[ROOT]
[ROOT]

| MERU | CLIP |
| :---: | :---: |
| i love the big blooms of the hibiscus with their bold <br> colour. | some hibiscus-like blossoms beside the visitor center <br> at moody gardens in galveston, $t x$ |
| these looked like hibiscus, but i think they are <br> something else. | after many years nurturing this back to life, the recent <br> heat and rainfall have produced more spectacular <br> blooms. |
| only a few blooms this time of year... | this beautiful species may be the hibiscus according <br> to my wife but $i$ am not so sure, |
| much better bloom this time... | these looked like hibiscus, but ithink they are |
| something else. |  |

[ROOT]
[ROOT]

Hyperbolic Image-Text Representations

(27)

| MERU | CLIP |
| :---: | :---: |
| crele old english game bantam cockerel | read "the beautiful yellow bird - a cautionary tale for <br> new mps" only at acid rabbi. |
| sabrina is running and twirling to the bachanalia <br> song from the samson and delila opera. | flaunt your assets! |
| really interesting gadgetar! posted by second life <br> resident ina centaur. visit kaneohe. | most of the girls had gone to college with mary, <br> whose husband was competing. |
| harvey henry sanborn marie elizabeth campbell | $\downarrow$ |
| sam is showing good form and follow through here. | $\downarrow$ |
| this is only luky. | $\downarrow$ |


| MERU | CLIP |
| :---: | :---: |
| blue-and-yellow macaw at the zooparque itatiba. | i like this portrait of this blue and yellow macaw, <br> because of the black background... |
| one of my fav birds to shoot at the zoo | from parrot island |
| i like these birds, especially when you can see the <br> yellow of their eyes! | $<$ usr $>$ bird sanctuary. |
| there are more colorful birds, but there are few birds |  |
| with as much character. |  |


(29)

| MERU | CLIP |
| :---: | :---: |
| fly agaric in the forest with a little spider. | freshly popped amanita muscaria in the forest. |
| all alone on the forest floor | a fly agaric on the rise. |
| from a recent new forest trip | or "fly agaric". "if your viking gets to choose." |
| i like this photo, so here it is too. | reminded me a bit of alice in wonderland |
| $\downarrow$ | reminded me of alice in wonderland |
| $\downarrow$ | this one goes out to forest love. |

## [ROOT]

[ROOT]

(30)
$\frac{\text { MERU }}{\text { taken at mystic aquarium, } c t}$
CLIP
taken by michael i love watching jellyfish. wish these pics had turned out better.

| i don't really like jellyfish, but they are beautiful. | jell. |
| :--- | :---: |
| something about this reminds me of every photo i've <br> ever taken of jellyfish | $\downarrow$ |
| it is really very cool to be able to see them... | $\downarrow$ |
| it did not feel like an aquarium when i took the picture | $\downarrow$ |
| really not much they let you see here. | $\downarrow$ |
| that i met. | $\downarrow$ |

[ROOT]

(31)

| MERU | CLIP |
| :---: | :---: |
| in new hat from adji | includes my twit hat- wink |
| oui, girl friend. | in new hat from adji |
| the hat actually belongs to honey :) | :) he's still in japan right now... i miss him. |
| fashion photo session | $\downarrow$ |
| with new hat | $\downarrow$ |
| a very fit lady. | $\downarrow$ |
| the boy has style! | $\downarrow$ |

[ROOT]
[ROOT]

(32)
(33)

(34)


| MERU | CLIP |
| :---: | :---: |
| humpback coming up for air near juneau, alaska | orca, craig, alaska, tongass nf. usfs francisco sanchez |
| here, this orca swims about before the show starts. | named for its shape. not for the occasional whales |
| found in its waters. |  |

[ROOT]

| MERU | CLIP |
| :---: | :---: |
| a pastry \& a coffee for breakfast yummy | today's breakfast: toast with honey, egg and black |
| coffee |  |


| MERU | CLIP |
| :---: | :---: |
| 'tooey's delight' gcl6aga | dominic samonte \& stephanie estabillo |
| 30 seconds into gina's five-minute toast. | chicken and cheese sandwich. |
| the plain toast was, um, plain. | $\downarrow$ |
| with grilled cheese sandwich. | $\downarrow$ |
| i don't normally like toasted sandwiches, but this one <br> was delicious! | $\downarrow$ |
| on bread, probably not what you should do with it, <br> but it was a good meal | $\downarrow$ |
| call center del club cantv | $\downarrow$ |
| so. good. | $\downarrow$ |
| [ROOT] | [ROOT] |
| MERU | CLIP |
| at momofuku | where the magic happens at momofuku noodle bar |
| the ramen got pwned. | @ terakawa ramen 4sq.com/ekseer |
| siawase ramen exploration - still looking |  |
| this i ate, and it was great! | with michael cotta |
| i ate some of this. | $\downarrow$ |
| rRer |  |

[ROOT]
[ROOT]
(35)

(36)

| MERU | CLIP |
| :---: | :---: |
| tiny hot peppers from the freezer | poblanos to be roasted |
| the ones with jalapenos are the ones that are ruling | they're best as peppers from the garden! |
| add the chillies and cook for another minute. | do nothing gardening in action! |
| getting ready for some chili-dippin' | $\downarrow$ |
| yo tengo el poder! | $\downarrow$ |
| toying around with a f/1.8 | $\downarrow$ |
| you canno tmake this stuff up | $\downarrow$ |

[ROOT]

Hyperbolic Image-Text Representations


| MERU | CLIP |
| :---: | :---: |
| last nights' salad, cropped | the first of many caprese salads while we traveled. |
| nice salad, summery, fresh. | for a homegrown salad |
| contemp salad | another salad :) |
| i made the salad | $\downarrow$ |
| it's $a$ verso/kveton thing. | $\downarrow$ |
| [ROOT] | [ROOT] |

(37)

(38)

| MERU | CLIP |
| :---: | :---: |
| awesome guinness \& chocolate cupcakes by kari <br> stewart our wonderful studio manager | i made cupcakes and took a million photos... |
| peanut butter cupcakes with whipped chocolate <br> ganache. | from the chocolate lady |
| vegan cupcakes. chocolate, coffee and cinnamon. | $\downarrow$ |
| coffee and chocolate: a "can't miss" cupcake... | $\downarrow$ |
| including a flourless chocolate cupcake! | $\downarrow$ |
| chocolate makes everything better. thanks $<$ usr $>$ | $\downarrow$ |
| chocolate-y goodness! | $\downarrow$ |
| this week's take, brought over by a friend. | $\downarrow$ |
| [ROOT] | [ROOT] |
| MERU | CLIP |
| new delhi's best cholle bhature | pav bhaji appetizer, \$5 dinner time only tirupathi |
| bhimas, milpitas |  |
| en tlaquepaque, jal. | dal monte la motta. |

west indian food. you can't help but smile when you the famous curry mile, taken on saturday 5th march eat it.

| $<$ usr $>$ buse dal lof | sloppy everything |
| :---: | :---: |
| paraje guer aike | the midas, great food |
| manoush | $\downarrow$ |
| with sambuca | $\downarrow$ |
| [ROOT] | [ROOT] |
| MERU | CLIP |
| matcha green tea ice blended | chimichurri sauce recipe |
| yes $\boldsymbol{i}$ had a pesto drink | trust me, itried to make this less green. |
| hot/fermented | make some good ones! |


| that's right, i've been experimenting. trying to keep <br> things fresh. | $\downarrow$ |
| :---: | :---: |
| ondel-ondel | $\downarrow$ |
| one love hi pawa | $\downarrow$ |

[ROOT]

(41)

CLIP
sliders served at lee roy selmon's restaurant.
biergarten sliders \& greens
bread with olive oil and vinegar
my welcome brunch to vienna.. a cheese party
they serve it with some sort of sauce . this is their version of "bread".
a good shot of how the bread should look.
the bread is real, i think. at least, not glass.
trying out some bread
[ROOT]


(47)

(48)

(49)

(50)

| MERU | CLIP |
| :---: | :---: |
| a drizzly february day in vancouver. | taken with a disposable camera on a rainy seattle day. |
| weather gloomy all the way to chicago | a cold rainy day in chicago |
| summer rain in the city. | junechicago |
| street $<$ usr $>$ | $\downarrow$ |
| $<$ usr $>$ street | $\downarrow$ |
| urban as usual | $\downarrow$ |
| street pics | $\downarrow$ |
| [ROOT] | [ROOT] |


| MERU | CLIP |
| :---: | :---: |
| this beautiful track goes through deep gorges of <br> nilgiris towards mangalore. | sepang international circuit malaysia |
| cipularang highway | very well finished road for indonesia!! ... looking |
|  | down the road |
| $<$ usr $>$ road | nature highway.. |

if you don't know where you are going, any road will take you there.

| crookhaven | look, there's a road and everything. |
| :---: | :---: |
| [ROOT] | [ROOT] |
| MERU | CLIP |

lebei is driving the quadricycle we rented on the my cruiser on 80 mile beach. messing with my 15 mm
toronto islands.
zeiss
this moto is still the best anything i've ever owned. biked to the nearest beach and took some pictures. you have no idea how much i love this thing. i was traveling alone, so instead of me i had to take pictures of my bike
...and she rides like a dream. initial impressions ride
report

| [ROOT] | [ROPort |
| :---: | :---: |
| MERU | CLIP |
| in a few of the branches of some trees were little <br> tea-light lanterns | in a few of the branches of some trees were little |
| tea-light lanterns |  |

if you use my photo please
[ROOT]


| MERU | CLIP |
| :---: | :---: |
| simple scandinavian style decor. clifton, bristol | ikea/helsinki design week party |
| rental apartment in berlin | with jeff from simple plan in denmark |
| my flat getting more and more comfortable with more <br> furniture coming in - and notice i now have fans! | $\downarrow$ |
| our first apartment | $\downarrow$ |
| ready made living space | $\downarrow$ |

[ROOT]
[ROOT]

Hyperbolic Image-Text Representations

(52)

| MERU | CLIP |
| :---: | :---: |
| studio di personaggio. character design. | model: yasemin snoek stylist: melanie vink |
| decorated with silver and nickle plated | we bought one of these for a friend's wedding. no, not |
|  | you julee. |
| new faucet set. | $\downarrow$ |
| elegant bath items, though | $\downarrow$ |
| oh to have that kind of luxury | $\downarrow$ |
| a little luxury | $\downarrow$ |
| rich sigfrit kicks things off. | $\downarrow$ |
| the things i'll do for a shot | $\downarrow$ |
| can you guess what club this is for?! | $\downarrow$ |


(53)

(54)

(55)

(56)

| MERU | CLIP |
| :---: | :---: |
| pentax *ist ds/ iso 1600__ happy halloween! | jack-o-lantern with other light up decorations. <br>  <br>  <br>  <br> jack-o-lantern. |
| we wish you a happy all hallows night! | i am so spoooooky |
| happy hallowe'en, everyone! | $\downarrow$ |
| happy halloween, yo. | $\downarrow$ |
| no be long now jack | $\downarrow$ |
| can you feel the spirit of e-xtrategy? | $\downarrow$ |

[ROOT]
[ROOT]

| MERU | CLIP |
| :---: | :---: |
| piano keyboard | vintage typewriter photo by rusty blazenhoff |
| musical ben | the typewriter is the best dead thing $i$ ever found |
| she was really good, great voice, excellent guitar <br> playing, and really nice to chat to. | musical harmony |
| the highly-touted prospect, not the guitar player | where some parts of me came of age |
| l.a.r.g.e | $\downarrow$ |
| [ROOT] | [ROOT] |


| MERU | CLIP |
| :---: | :---: |
| my christmas shopping is done, and what's more, my <br> presents are all wrapped! | my homemade christmas book - dec. 5th and 6th. |
| this year's wrapping job. | gift-wrapped for any occasion. |
| and gift wrapped! | a wrapped xmas present |
| 21 presents for my 21st for msh may | our christmas present put to good use. |
| christmas is coming. won't someone think of my |  |
| needs? |  |

the only good thing the guys did was dropped off the gifts.

| [ROOT] | [ROOT] |
| :--- | :---: |
| MERU | CLIP |

table in the backyard of the summer house in melby, chair detail denmark.

| kitch at airbnb at corte del correggio - note window <br> behind chairs | $\downarrow$ |
| :--- | :--- |
| photo by laura nawrocik some patio furniture that <br> needs a little cleaning. | $\downarrow$ |
| from a bench on the north side. | $\downarrow$ |
| the backyard of the $b$ and $b$ we stayed at | $\downarrow$ |
| another from this shoot in a more traditional style. | $\downarrow$ |
| traditional place to take a picture. | $\downarrow$ |
| a nice place to take pictures! | $\downarrow$ |

[ROOT]
[ROOT]

(57)

| MERU | CLIP |
| :---: | :---: |
| found at city lights book store, sf www.citylights.com | ikea catalog waiting for pickup, fairborn, ohio. |
| in powells rare book room | special collections - amsterdam, netherlands |
| in the rare book reading room | one needs to get there to read the it full. |
| book heaven. | $\downarrow$ |
| not a book in sight | $\downarrow$ |
| even more books | $\downarrow$ |
| sorry... i really like this space. | $\downarrow$ |
| with something like this, you would have to get a few | $\downarrow$ |

[ROOT]
[ROOT]

(58)

| MERU | CLIP |
| :---: | :---: |
| a pineapple grows in the wild between goa gajah and <br> yeh pulu, bali. | so much organic burden on its way to the sea. |
| the pineapple, dunmore park. $n$ | everything is so organic on lamu island. |

it started with more fruits but $i$ didn't take this picture it's been that long since $i$ took this that the next lot is till late already growing.

| shot $<$ usr $>$ beach | the one that didn't get picked yet |
| :---: | :---: |
| a regular sight from our coast | $\downarrow$ |
| la nature au carre. | $\downarrow$ |
| $i$ would be happy if all my photos turned out like this |  |
| one. | $\downarrow$ |
| this is one is good. | $\downarrow$ |

[ROOT] [ROOT]

(59)

| MERU | CLIP |
| :---: | :---: |
| sulphur crested cockatoos are great characters | soleil when she was a baby with her green feathers |
| au pied du ciel | lenny white $)$ |
| pale male's mate \#5 | white tee's - photos for everyone |
| i think this is $a$ nice photo of sean, though $i$ doubt he |  |
| will think so. | $\downarrow$ |
| photo: george struikelblok | $\downarrow$ |
| this is birdy. | $\downarrow$ |
| q's and a's | $\downarrow$ |

[ROOT]
[ROOT]

(60)

| MERU | CLIP |
| :---: | :---: |
| uzi usb drive by dan helmick. brass, wood, riveted. | $i$ swear, this is how the coins landed. so i had to take |
| a photo. |  |

[ROOT]


[^0]:    KD and Rama did this work while at Meta. ${ }^{1}$ University of Michigan ${ }^{2}$ Meta $\mathrm{AI}^{3}$ Independent Researcher ${ }^{4}$ New York University. Correspondence to: Karan Desai [kdexd@umich.edu](mailto:kdexd@umich.edu).

    Proceedings of the $40^{\text {th }}$ International Conference on Machine Learning, Honolulu, Hawaii, USA. PMLR 202, 2023. Copyright 2023 by the author(s).

[^1]:    ${ }^{1}$ Meru is a mountain that symbolizes the center of all physical, metaphysical, and spiritual universes in Eastern religions like Hinduism and Buddhism. Our method is named MERU because the origin of the hyperboloid entails everything and plays a more vital role than in Euclidean (or generally, affine) spaces. See also: Mount Semeru, Indonesia (Sources - wikipedia.org/wiki/ Mount_Meru and wikipedia.org/wiki/Semeru)

[^2]:    ${ }^{2}$ tensorflow.org/datasets and pytorch.org/vision

[^3]:    ${ }^{3}$ github.com/facebookresearch/slip

[^4]:    [ROOT]

