

000 001 002 003 004 005 MEAN FLOW POLICY WITH INSTANTANEOUS VELOC- 006 ITY CONSTRAINT FOR ONE-STEP ACTION GENERATION 007 008 009

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ABSTRACT

024 Learning expressive and efficient policy functions is a promising direction in reinforcement learning (RL). While flow-based policies have recently proven effective
025 in modeling complex action distributions with a fast deterministic sampling process,
026 they still face a trade-off between expressiveness and computational burden, which
027 is typically controlled by the number of flow steps. In this work, we propose mean
028 flow policy (MFP), a new generative policy function that models the mean velocity
029 field to achieve the fastest one-step action generation. To ensure its high expressiveness,
030 an instantaneous velocity constraint (IVC) is introduced on the mean velocity
031 field during training. We theoretically prove that this design explicitly serves as a
032 crucial boundary condition, thereby improving learning accuracy and enhancing
033 policy expressiveness. Empirically, our MFP achieves state-of-the-art success
034 rates across several challenging robotic manipulation tasks from Robomimic and
035 OGBench. It also delivers substantial improvements in training and inference speed
036 over existing flow-based policy baselines.
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1 INTRODUCTION

039 A promising topic in reinforcement learning (RL) community is to develop expressive and efficient
040 policies, particularly in complex control environments where action distributions can be multi-
041 modal (Zhu et al., 2023; Wang et al., 2023). Generative policies, such as diffusion model and
042 flow matching, have recently emerged as a powerful alternative to Gaussian or mixture policies by
043 transforming simple base distributions into flexible action distributions via learnable transfor-
044 mations (Song et al., 2020; Chi et al., 2023). However, a key limitation of existing generative policies
045 is their dependence on iterative multi-step refinement from noise to actions (Wang et al., 2024a;
046 2025; Ding et al., 2024). This computational dependency imposes a significant overhead that hinders
047 training speed, particularly for online RL where action sampling is a per-step requirement (Li, 2023;
048 Yang et al., 2023). Moreover, this overhead translates to considerable inference latency, which is a
049 major impediment to achieving high closed-loop performance in real-time control systems (Zhan
050 et al., 2024; 2025; Jiang et al., 2024).
051

052 A question naturally arises: *Can we unify the expressiveness of generative policies with the efficiency of
053 one-step action generation for online RL?*

054 In this paper, we propose the mean flow policy (MFP) as an affirmative answer. While existing flow policies
055 learn instantaneous velocities and require multi-step
056 iterative sampling (Lipman et al., 2023; Park et al.,
057 2025; Bharadhwaj et al., 2024), MFP instead learns
058 the mean velocity field (Geng et al., 2025a). This
059 design enables a direct, single-step mapping from a
060 base Gaussian noise to a multi-modal action distribution,
061 thereby preserving the expressive power of
062 flow-based models while drastically improving training
063 and inference efficiency (Kornilov et al., 2024).
064

065 Although the time-efficiency gains of MFP are very
066 promising, its learning difficulty is higher than that
067



068 **Figure 1: Performance-efficiency evalua-
069 tion on 9 robotic manipulation tasks.** Our
070 MFP achieved the highest success rate and
071 fastest training speed on a single A100 GPU.
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of a standard flow policy. One reason is that our MFP requires modeling the mean velocity for any time interval specified by two time points (Geng et al., 2025a). A more significant reason is that its learning process is governed by a first-order ordinary differential equation (ODE) derived from the definition of mean velocity. However, this ODE theoretically suffers from the problem of multiple solutions due to a lack of explicit boundary conditions, that is, the value at any boundary point is not enforced. This poses a non-trivial challenge to learning accuracy and consequently affects policy expressiveness (Birkhoff & Langer, 1923).

To address this, we introduce an instantaneous velocity constraint (IVC) to compensate for the lack of boundary conditions. Intuitively, IVC pairs the average velocity loss for each interval with an instantaneous velocity loss at the interval’s start point. In practice, IVC is implemented as a auxiliary policy loss, adding negligible computational overhead while materially improving accuracy. We evaluate our MFP on Robomimic (Mandlekar et al., 2021) and OGBench (Park et al., 2024), two demanding robot manipulation benchmarks. As shown in Figure 1 and Table 3, MFP achieves state-of-the-art success rates while delivering substantial speed-ups in training and per-step inference over strong flow-policy baselines, on average across both suites.

Our contributions are summarized threefold:

- We propose a new flow-based policy, namely mean flow policy (MFP), that enables fastest one-step action generation. By modeling the mean velocity field, MFP retains the expressiveness of generative policies while eliminating multi-step sampling overhead.
- We design a training enhancement technique, namely instantaneous velocity constraint (IVC), to improve the learning accuracy of mean velocity field. This technique explicitly serves as a boundary condition, thereby stabilizing learning and enhancing policy expressiveness.
- We empirically achieve state-of-the-art success rates on two challenging robotic manipulation benchmarks: Robomimic and OGBench. Moreover, our approach provides a substantial speedup in both training and inference over existing flow-policy baselines, highlighting its practicality for real-time application.

2 PRELIMINARIES

Reinforcement Learning. We consider an agent interacting with an environment modeled as a Markov Decision Process (MDP), defined by a tuple $\mathcal{M} = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}, r, \gamma \rangle$ (Li, 2023). The components are the state space $\mathcal{S} \subseteq \mathbb{R}^n$, the action space $\mathcal{A} \subseteq \mathbb{R}^m$, the state transition function $\mathcal{P}(s'|s, a)$, the reward function $r(s, a)$, and the discount factor $\gamma \in [0, 1]$. The primary goal in reinforcement learning (RL) is to learn a policy $\pi(a|s)$ that maximizes the expected cumulative discounted reward, namely return, given by

$$J_\pi = \mathbb{E}_{\pi, \mathcal{P}} \left[\sum_{k=0}^{\infty} \gamma^k r(s_k, a_k) \right]. \quad (1)$$

Grounded in the off-policy learning paradigm, our approach utilizes an action-value function (Q-function) to guide policy improvement, which denotes the expected cumulative return for taking an action a in a state s and thereafter following the policy π .

$$Q^\pi(s, a) = \mathbb{E}_{\pi, \mathcal{P}} \left[\sum_{i=0}^{\infty} \gamma^i r(s_i, a_i) | s_0 = s, a_0 = a \right]. \quad (2)$$

The optimal action-value function, $Q^*(s, a)$, represents the maximum expected return achievable from state s by taking action a . The optimal policy π^* can then be found by selecting the action that maximizes this function: $\pi^*(s) = \arg \max_{a \in \mathcal{A}} Q^*(s, a)$.

Flow Matching. Flow matching is a principled methodology for constructing continuous-time generative models (Lipman et al., 2023). In contrast to diffusion models, which employ stochastic differential equations (SDEs) (Song et al., 2020), flow matching is built upon deterministic dynamics governed by an ordinary differential equation (ODE). By directly learning a continuous-form instantaneous vector field, this approach simplifies the training objective and enables more efficient

108 sampling. Specifically, it trains a neural network $v_\theta : \mathbb{R}^d \times [0, 1] \rightarrow \mathbb{R}^d$ to parameterize a velocity
 109 field $v(x(t), t)$ that matches a predefined conditional target velocity $v(1)$.
 110

111 For a source distribution $q(x(0))$ and a target distribution $p(x(1))$, the velocity field is trained by
 112 minimizing the flow matching loss (Lipman et al., 2024):
 113

$$\mathcal{L}_{\text{FM}}(\theta) = \mathbb{E}_{\substack{t \sim \mathcal{U}([0, 1]) \\ x(1) \sim p, x(0) \sim q}} \|v_\theta(x(t), t) - v(x(t), t)\|_2^2, \quad (3)$$

115 where any intermediate point along the generating path $x(t) = tx(1) + (1 - t)x(0)$ is a linear
 116 interpolation between source and target points. The velocity for this path is assumed to be a constant
 117 vector $v(x(t), t) = x(1) - x(0)$. This formulation defines the target vector field along a straight path
 118 between the source and target samples. Once trained, the learned field v_θ defines a probability flow
 119 via the following ODE:
 120

$$\frac{dx(t)}{dt} = v_\theta(x(t), t), \quad x(0) \sim q, \quad (4)$$

121 which allows us to effectively generate samples of the target distribution p , starting from samples
 122 of the source distribution q . Although flow matching is conceptually designed to generate along
 123 a straight line, the paths still curve in practice when fitting between two distributions (Song et al.,
 124 2023). Therefore, multi-step discretization and numerical methods like the Euler method are often
 125 required to solve the ODE in Eq. (4) to obtain high-quality generated results (Park et al., 2025).
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3 METHOD

130 First, we introduce the mean flow policy (MFP), showing how its integration with a “generate-and-
 131 select” mechanism enables a direct mapping from noise to optimal actions. We then present the
 132 instantaneous velocity constraint (IVC) and theoretically justify its role in improving the learning
 133 accuracy. Finally, the complete pseudo-code for our mean flow RL algorithm is provided.
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3.1 MEAN FLOW POLICY

135 In RL, a policy $\pi(\cdot | s)$ defines a distribution over actions
 136 given a state s . For standard flow-based policies, this
 137 mapping is framed as a generative process: a velocity
 138 model, $v(a(t), t, s)$, transforms a standard Gaussian noise
 139 (source) into the optimal action (target), with the state
 140 serving as a conditioning input. The final output action
 141 $a(1)$ is calculated by
 142

$$a(1) = a(0) + \int_0^1 v(a(\tau), \tau, s) d\tau, \quad a(0) \sim \mathcal{N}(0, I). \quad (5)$$

143 Unlike standard flow policies that learn the instantaneous
 144 velocity field $v(a(t), t, s)$, we center on the mean velocity
 145 field $u(a(t), t, r, s)$, modeling the mean velocity over any
 146 given time interval $[t, r]$ as
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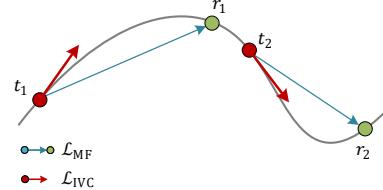
$$u(a(t), t, r, s) \triangleq \frac{1}{r - t} \int_t^r v(a(\tau), \tau, s) d\tau. \quad (6)$$

148 The relationship between these two types of velocities is shown in Figure 2. If the mean velocity
 149 model u^* is ideally learned, the policy inference is formulated as
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$$a(1) = a(0) + u^*(a(0), 0, 1, s), \quad a(0) \sim \mathcal{N}(0, I). \quad (7)$$

151 **(1) How to imitate a given target action.** To train the mean velocity model u , we first multiply
 152 both sides of Eq. (6) by $(r - t)$ and then differentiate with respect to t (treating r as independent of
 153 t), which yields the mean flow identity:
 154

$$-u(a(t), t, r, s) + (r - t) \frac{d}{dt} u(a(t), t, r, s) = -v(a(t), t, s). \quad (8)$$



155 Figure 2: Velocity field: blue arrows denote the mean velocity over a time interval,
 156 with red arrows representing the instantaneous velocity at a time point.
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We assume there is a target optimal action a^* , serving as $a(1)$ for imitation. For two randomly sampled time points, t and r (with $t < r$), the intermediate point $a(t)$ is defined by the linear interpolation: $a(t) = t \cdot a(1) + (1 - t) \cdot a(0)$, and $v = a^* - a(0)$. Let θ denote the learnable parameters, the training objective is to minimize the residual of the mean flow identity in Eq. (8) as

$$\mathcal{L}_{\text{MF}}(\theta) = \mathbb{E}_{t, r < t, a(t)} \left\| u_\theta(a(t), t, r, s) - \text{sg} \left(v - (t - r) \frac{d}{dt} u_\theta(a(t), t, r, s) \right) \right\|_2^2, \quad (9)$$

where “sg” denotes the stop-gradient operator to stabilize training. Calculating this training loss requires computing the total time derivative term, $\frac{d}{dt} u$. Using the chain rule, this term is expanded as shown in Eq. (10), and its computation can be performed efficiently using the Jacobian-vector product (JVP) in modern automatic differentiation libraries.

$$\frac{d}{dt} u_\theta(a(t), t, r, s) = v(a(t), t, s) \cdot \frac{\partial}{\partial a} u_\theta(a(t), r, t, s) + \frac{\partial}{\partial t} u_\theta(a(t), r, t, s), \quad (10)$$

By minimizing Eq. (9), the mean velocity model u can effectively transform a standard Gaussian noise into the desired target action distribution. In RL, however, there is no ground-truth dataset of optimal actions to imitate. Next, we will detail how to gradually find better actions and eventually approach the optimal target action.

(2) How to find the optimal target action to imitate. Finding the optimal action a^* directly is not realistic. However, we can use the Q-function to progressively find better actions as imitation targets, and eventually find the optimal action a^* in a bootstrap manner. This mechanism is called “generate-and-select” or “best-of- N ”.

In practice, at any given state s , the agent first generate N diverse candidate actions as

$$a^i = a_k^i(1) = \epsilon_i + u_\theta(\epsilon_i, 0, 1, s), \quad a^i(0) = \epsilon^i \sim \mathcal{N}(0, I), \quad i = 1, \dots, N. \quad (11)$$

Then the critic function Q_ϕ parameterized by ϕ is employed to evaluate all candidates, and the action yielding the highest Q-value is identified as the final output action for that state:

$$a^* = \arg \max_{a^i} Q_\phi(s, a^i). \quad (12)$$

We treat the combined mean flow generation process in Eq. (11) and the best-of- N mechanism in Eq. (12) as a unified policy $\pi_\theta^{\text{unified}}$, or simply denoted as π_θ . The resulting action, a^* , then serves three purposes: (1) interacting with the environment, (2) acting as the target action for policy training, and (3) calculating the target value for value training.

Although this bootstrap mechanism is intuitive, next we will formally prove that such a imitation-based update guarantees policy improvement by the following theorem.

Theorem 1 (Mean Flow Policy Improvement). *Let the new policy π_{new} be derived from an old policy π_{old} via the N -candidate generative update process described above. Under assumptions of a bounded Q-function error (ϵ_Q), L_Q -Lipschitz continuity of the Q-function, and a bounded mean flow matching error (ϵ_A), the performance difference between these two policies is lower-bounded by*

$$V^{\pi_{\text{new}}}(s) - V^{\pi_{\text{old}}}(s) \geq \underbrace{\mathbb{E}_{\tau \sim \pi_{\text{new}}} \left[\sum_{t=0}^{\infty} \gamma^t \Delta_N^{\pi_{\text{old}}}(s_t) \right]}_{\text{BFN improvement term } \Delta_1} - \underbrace{\frac{2\epsilon_Q + L_Q \epsilon_A}{1 - \gamma}}_{\text{fitting error term } \Delta_2}, \quad (13)$$

where $V(s)$ is the state-value function, and $\Delta_N^{\pi_{\text{old}}}(s)$ is the best-of- N advantage gain, satisfying

$$\Delta_N^{\pi_{\text{old}}}(s) := \mathbb{E}_{a_1, \dots, a_N \sim \pi_{\text{old}}(\cdot | s)} \left[\max_{i=1, \dots, N} Q^{\pi_{\text{old}}}(s, a_i) \right] - V^{\pi_{\text{old}}}(s) \geq 0. \quad (14)$$

Proof. See Appendix A. □

This theorem decomposes the performance difference into two distinct components: a *BFN improvement term* Δ_1 and a *fitting error term* Δ_2 . As we prove in Appendix A.4, the best-of- N advantage gain ($\Delta_N^{\pi_{\text{old}}}$) in Δ_1 is strictly non-negative, implying that the policy improvement can be driven by the benefit of sampling N diverse action candidates. It is also important to note that this improvement is partially counteracted by the fitting error term Δ_2 , which stems from the critic’s inaccuracy (ϵ_Q) and the mean flow matching error (ϵ_A). This highlights the importance of reducing ϵ_A to further enhance policy performance. In the next subsection, we will introduce the instantaneous velocity constraint (IVC) to help reduce this fitting error.

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3.2 THE INSTANTANEOUS VELOCITY CONSTRAINT AS A BOUNDARY CONDITION

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As we mentioned above, the underlying principle for training the mean velocity model is Eq. (8), which is a first-order ordinary differential equation (ODE) with respect to u . Mathematically speaking, we need to know both the dynamics and at least a boundary condition to accurately solve the unique solution of a ODE. Back to our practical setting, Eq. (8) only provides the dynamics given $t < r$. Although sampling pairs where r is very close to t could implicitly serve as the boundary condition, such events are too rare to ensure robust training.

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Inspired by this, we introduce the instantaneous velocity constraint (IVC), a training objective that explicitly enforces a boundary condition at t . The mean velocity from t to t is exactly the known instantaneous velocity $v = a^* - a(0)$, so the IVC objective is expressed as:

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$$\mathcal{L}_{\text{IVC}}(\theta) = \mathbb{E}_{t,a(t)} \|u_\theta(a(t), t, t) - v\|_2^2. \quad (15)$$

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To understand the effectiveness of this IVC objective, we first derive the following theorem, which demonstrates the multiplicity of solutions for Eq. (8) in the absence of a boundary condition.

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Theorem 2 (Multiplicity of Solutions for the Mean Flow Identity). *Let the **cumulative error** Δ_u be defined as the difference between the learned and the true mean velocity field, $\Delta_u(a(t), t, r) \triangleq u_\theta(a(t), t, r) - u^*(a(t), t, r)$. Assume u_θ perfectly satisfies the mean flow identity (Eq. (8)) for all $t < r$. Then, its cumulative error Δ_u satisfies*

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$$\Delta_u(a(t), t, r) = \frac{C(a, r)}{r - t} \quad (16)$$

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where $C(a, r)$ is an integration constant independent of time t .

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Proof. By assumption, both u_θ and u^* precisely satisfy the mean flow identity. Subtracting the identity for u^* from that of u_θ yields a homogeneous linear differential equation with respect to the cumulative error Δ_u as

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$$\Delta_u + (t - r) \frac{d}{dt} \Delta_u = 0. \quad (17)$$

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By the product rule for derivatives, this is equivalent to $\frac{d}{dt}[(r - t)\Delta_u] = 0$. Integrating with respect to t yields $(r - t)\Delta_u = C(a, r)$, which completes the proof. \square

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Theorem 2 reveals that any solution learned by perfectly minimizing the \mathcal{L}_{MF} loss belongs to a family of functions with a unknown constant $C(a, r)$. Because the \mathcal{L}_{MF} loss is blind to the boundary, it cannot provide a gradient to force $C(a, r)$ to zero. This allows an arbitrary, persistent bias to exist in the learned u_θ . Next, we derive the following theorem to prove that the explicit introduction of IVC eliminates this degree of freedom and restricts the solution space to the unique correct u^* .

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Theorem 3 (Uniqueness via the Instantaneous Velocity Constraint). *Let the **boundary error** Δ_v be defined as the error at the boundary t , $\Delta_v(a(t), t) \triangleq u_\theta(a(t), t, t) - u^*(a(t), t)$. Minimizing the IVC loss, $\mathcal{L}_{\text{IVC}} = \mathbb{E}[\|\Delta_v\|^2]$, forces the integration constant $C(a, r)$ in Theorem 2 to zero. This eliminates the boundary error and ensures the cumulative error Δ_u vanishes for all $t < r$.*

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Proof. The boundary error Δ_v is the limit of the cumulative error Δ_u as $r \rightarrow t^+$. Applying Eq. (16):

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$$\Delta_v(a(t), t) = \lim_{r \rightarrow t^+} \frac{C(a, r)}{r - t}. \quad (18)$$

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This limit diverges if $C(a, r) \neq 0$. To keep the IVC loss finite, the optimization must prevent this divergence, which requires $C(a, r) = 0$. With $C(a, r) = 0$, the boundary error Δ_v is zero. Consequently, by invoking Theorem 2, the cumulative error Δ_u also becomes zero. This completes the proof. \square

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In essence, Theorem 3 proves that the IVC provides the necessary boundary condition to make the learning problem well-posed. By forcing the error constant to zero at the boundary, the IVC eliminates the cumulative error inherent to the mean flow objective, thereby retaining the high expressive power of the generative model. Moreover, the resulted smaller mean flow matching error ϵ_A in Theorem 1, which helps enable a more effective policy improvement with each update.

270 3.3 MEAN FLOW REINFORCEMENT LEARNING
271272 This section systematically presents the complete picture of our mean flow RL. The policy training
273 loss $\mathcal{L}_{\text{policy}}$ combines the mean velocity model loss in Eq. (9) with the IVC loss in Eq. (15):
274

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$$\mathcal{L}_{\text{policy}}(\theta) = \mathcal{L}_{\text{MF}}(\theta) + \lambda \mathcal{L}_{\text{IVC}}(\theta), \quad (19)$$

276

277 where the balancing hyperparameter $\lambda > 0$ is called IVC coefficient, and the default value is 1.0.
278279 Concurrently, the critic Q_ϕ is trained to minimize a standard TD-error in Eq. (20) on transitions
280 (s_k, a_k, r_k, s_{k+1}) from the replay buffer, where k is the step index.
281

282
$$\mathcal{L}_Q(\phi) = \mathbb{E} \left[(Q_\phi(s_k, a_k) - (r_k + \gamma Q_\phi(s_{k+1}, a_{k+1}^*)))^2 \right]. \quad (20)$$

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284 Recall that we treat the combined mean flow generation process in Eq. (11) and the best-of- N
285 mechanism in Eq. (12) as a unified policy π_θ , so the calculation of a_{k+1}^* also involves a generative-
286 then-select process for ensuring an unbiased policy evaluation.
287288 The overall training scheme decouples the generative policy’s imitative training from the Q-value
289 gradient. Instead, the policy improvement is guaranteed by a best-of- N mechanism under the guidance
290 of Q-value selection. This design allows us to leverage the policy’s full expressive power while
291 ensuring stable and effective policy improvement. The complete algorithm is shown in Algorithm 1.
292293 **Algorithm 1** Mean Flow RL

294 **Input:** Mean Flow policy π_θ , where θ is the parameters of u_θ , Critic Q_ϕ , offline dataset $\mathcal{D}_{\text{offline}}$
295 Initialize replay buffer \mathcal{D} with $\mathcal{D}_{\text{offline}}$ ▷ Phase 1: Offline Pre-training
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297 **for** offline training step **do**
298 Replay a mini-batch from \mathcal{D}
299 Update policy π_θ by minimizing $\mathcal{L}_{\text{policy}}(\theta)$ with Eq. (19)
300 Update critic Q_ϕ by minimizing $\mathcal{L}_Q(\phi)$ with Eq. (20)
301 **end for** ▷ Phase 2: Online Interaction and Fine-tuning
302

303 **for** online training step $k = 1, 2, \dots$ **do**
304 Observe s_k , execute $a_k^* = \pi_\theta(s_k)$, receive s_{k+1}, r_k , and store $(s_k, a_k^*, r_k, s_{k+1})$ into \mathcal{D}
305 Replay a mini-batch from \mathcal{D}
306 Update policy π_θ by minimizing $\mathcal{L}_{\text{policy}}(\theta)$ with Eq. (19)
307 Update critic Q_ϕ by minimizing $\mathcal{L}_Q(\phi)$ with Eq. (20)
308 **end for**

309 4 EXPERIMENTS

310 4.1 MAIN EXPERIMENT

311 **Benchmark.** We consider a total of 9 sparse-reward robotic manipulation tasks with varying diffi-
312 culties. This includes 3 tasks from the Robomimic benchmark (Mandlekar et al., 2021), Lift, Can
313 and Square, and 6 tasks from OGBench (Park et al., 2024), cube-double-task 2/3/4 and
314 cube-triple-task 2/3/4. For Robomimic, we use the multi-human datasets. For OGBench,
315 we use the default play-style datasets. See more details of these tasks in Appendix C.

316 **Baselines and our method.** We compare with three latest strong offline-to-online RL baselines.
317 (1) **FQL (flow Q learning)** (Park et al., 2025) first uses behavioral cloning to train a multi-step
318 flow policy on offline data. It then trains a separate one-step policy that imitates the multi-step
319 policy and maximizes Q-values, enabling efficient and stable learning within the data distribution. (2)
320 **BFN (best-of- N)** (Ghasemipour et al., 2021) combines the best-of- N sampling with an expressive
321 multi-step flow policy. Specifically, BFN first generates N candidate actions and picks the action
322 (out of N) that maximizes the current Q-value. (3) **QC (Q-chunking)** (Li et al., 2025) applies action
323 chunking (Bharadhwaj et al., 2024) on the basis of BFN to improve exploration and sample efficiency.
324 (4) **Ours MFP** also adopts the chunking trick, but leverages a more efficient mean flow policy to
325 achieve the fastest one-step action generation with maintained or even enhanced expressiveness.

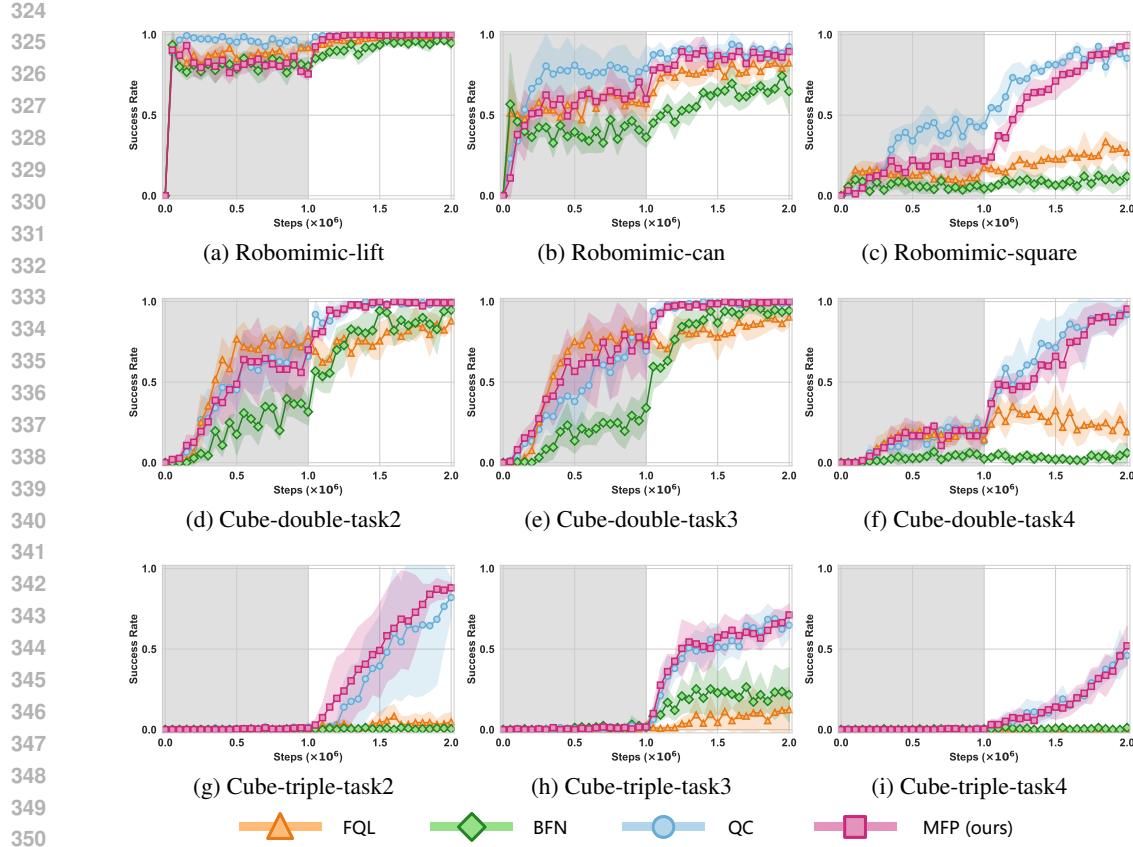


Figure 3: **Training curves on benchmarks.** The solid lines correspond to mean and shaded regions correspond to 95% confidence interval over five runs. The shadow background indicates the offline training phase, while the white background indicates the online training phase.

Table 1: Success rates. Mean \pm Std over 5 seeds. **Bold** = best, underlined = 2nd-best.

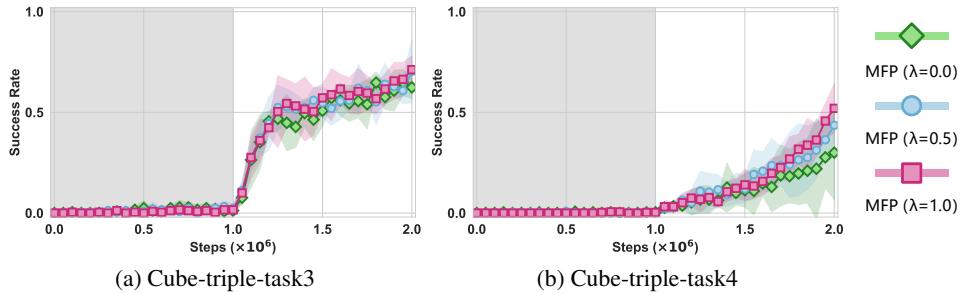
Task	FQL	BFN	QC	MFP (ours)
Robomimic-lift	0.96 ± 0.03	1.00 ± 0.01	1.00 ± 0.00	1.00 ± 0.00
Robomimic-can	0.74 ± 0.11	0.82 ± 0.03	0.94 ± 0.06	0.92 ± 0.07
Robomimic-square	0.12 ± 0.05	0.34 ± 0.06	0.92 ± 0.01	0.93 ± 0.01
Cube-double-task2	0.95 ± 0.04	0.88 ± 0.05	1.00 ± 0.00	1.00 ± 0.00
Cube-double-task3	0.97 ± 0.04	0.90 ± 0.06	1.00 ± 0.00	1.00 ± 0.00
Cube-double-task4	0.08 ± 0.04	0.35 ± 0.09	0.93 ± 0.08	0.95 ± 0.04
Cube-triple-task2	0.01 ± 0.02	0.08 ± 0.06	0.82 ± 0.10	0.88 ± 0.03
Cube-triple-task3	0.12 ± 0.13	0.26 ± 0.14	0.69 ± 0.05	0.71 ± 0.06
Cube-triple-task4	0.00 ± 0.00	0.02 ± 0.02	0.46 ± 0.13	0.52 ± 0.11
Average	0.44 ± 0.05	0.52 ± 0.06	0.86 ± 0.05	0.88 ± 0.05

Main results. As shown in Table 1, our MFP matches or exceeds state-of-the-art multi-step flow-matching baselines on eight of nine tasks. On the remaining task, MFP ranks second, with a performance of 0.92, which is just 0.02 points below the top-performing baseline’s score of 0.94. While all methods achieve near-perfect performance on simpler tasks like Robomimic-lift, Cube-double-task2, and Cube-double-task3, MFP demonstrates clear superiority on the more challenging tasks. Specifically, MFP consistently outperforms all baselines on Robomimic-square, Cube-double-task4, and all Cube-triple tasks, where it consistently achieves the highest success rates. For instance, on the most difficult task, Cube-triple-task4, MFP achieves a success rate of 0.52 ± 0.11 , which is

378 significantly higher than the next-best baseline, QC (0.46 ± 0.13), and substantially exceeds both
 379 FQL and BFN. Overall, our MFP secures the top position with an average success rate of 0.88 ± 0.05 .
 380 This result highlights its strong capability that is competitive with multi-step flow policies in solving
 381 long-horizon, sparse-reward tasks.
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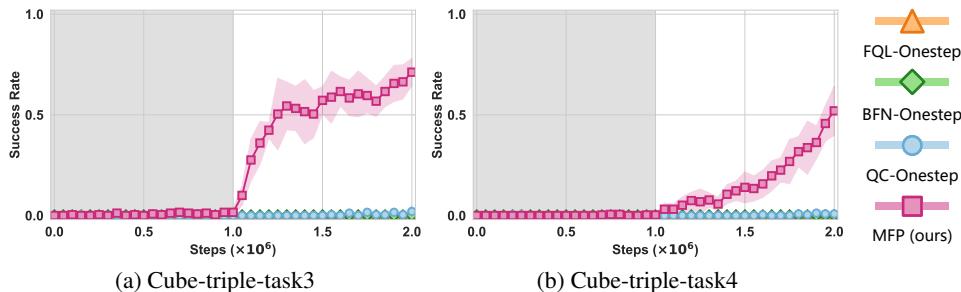
383 4.2 ABLATION STUDY AND SUPPLEMENTARY EXPERIMENTS

385 **(1) Ablation on the instantaneous velocity constraint (IVC).** We perform an ablation study on the
 386 IVC coefficient λ . Our full version ($\lambda = 1.0$) was compared against variants with a reduced constraint
 387 ($\lambda = 0.5$) and without the constraint ($\lambda = 0.0$). The results, as shown in Figure 4, indicate a positive
 388 correlation between the IVC weight and performance, while also demonstrating that the method is not
 389 overly sensitive to λ . For example, the success rate on the challenging Cube-triple-task4 significantly
 390 increases from 0.30 ± 0.21 (with no IVC) to 0.45 ± 0.15 (with a partial IVC), and further to $0.52 \pm$
 391 0.11 (with full IVC). Detailed numerical results are listed in Table 4 in Appendix B.1. These findings
 392 empirically validate our theoretical claims, confirming the IVC’s role as a crucial component for
 393 modeling an accurate mean velocity field and consequently achieving significant performance gains.
 394



403 (a) Cube-triple-task3 (b) Cube-triple-task4
 404
 405 Figure 4: Training curves of ablation on the IVC.
 406

407 **(2) Comparison with one-step variants of the aforementioned baselines.** We compared our
 408 MFP against one-step variants of the aforementioned baselines: FQL-Onestep, BFN-Onestep, and
 409 QC-Onestep. As shown in Figure 5, a naive one-step configuration is insufficient for solving these
 410 complex, long-horizon tasks, with baselines achieving success rates near zero on both Cube-triple-
 411 task3 and Cube-triple-task4. In stark contrast, our MFP achieves success rates of 0.71 ± 0.06 and 0.52
 412 ± 0.11 on these tasks, respectively. Detailed numerical results can be seen in Table 5 in Appendix B.1.
 413 This supplementary comparison highlights that simply using a one-step standard flow is not enough;
 414 the superior expressive capability and stable learning process of our mean flow policy are critical for
 415 tackling these challenging long-horizon manipulation tasks.
 416



425 (a) Cube-triple-task3 (b) Cube-triple-task4
 426
 427 Figure 5: Training curves of comparison with one-step flow.
 428

430 **(3) Training and inference time analysis.** Figure 1 presents a comparison of the average success
 431 rate (%) versus online training speed (iter/s) across the all 9 tasks. Our MFP achieves highest
 432 success rate and fastest training speed. This superior training efficiency stems directly from our

432 one-step action generation, which eliminates the expensive iterative sampling process required by
 433 prior multi-step flow policies.
 434

435 Table 2: Comparison of online training speed
 436

437 Task	FQL	BFN	QC	MFP (ours)
439 Average	108.5 ± 7.7 iter/s	68.0 ± 5.8 iter/s	92.6 ± 6.3 iter/s	153.6 ± 11.5 iter/s

440 Regarding inference time-efficiency, we conducted an evaluation with a focus on its suitability for
 441 real-world robotic deployment. Since robotic platforms often have limited computational resources,
 442 our experiments were conducted on a CPU-only environment, AMD Ryzen Threadripper 3960X
 443 24-Core Processor. To simulate a more realistic deployment scenario without hardware acceleration,
 444 we disabled JAX’s Just-In-Time (JIT) compilation during all evaluations.
 445

446 The results are listed in Table 3. our MFP and FQL exhibit very similar inference times, with both
 447 approaches being significantly faster than BFN and QC.
 448

449 **Why BFN and QC are slow.** The poor performance of BFN and QC is primarily because they rely
 450 on a 10-step flow policy, which requires iterative computation to transform noise into an action.
 451

452 **Why FQL is fast.** FQL simultaneously learns both a 10-step flow policy for high-accuracy imitation
 453 and a separate one-step flow policy. The one-step policy is obtained through a distillation process
 454 combined with a Q-function maximization loss, which allows FQL to achieve a high inference speed
 455 comparable to our MFP.
 456

457 **Why MFP is better than FQL comprehensively.** Despite FQL’s relatively fast inference, its
 458 training process is very slow due to the involvement of multiple policies, including a multi-step flow
 459 policy. Furthermore, benchmark results indicate that its success rate is very low, averaging only half
 460 of our MFP’s. When considering FQL’s overall low success rate and slow training speed, our MFP
 461 still maintains a significant advantage.
 462

463 Table 3: Comparison of inference time
 464

464 Task	FQL	BFN	QC	MFP (ours)
466 Average	10.76 ± 1.02 ms	117.3 ± 13.23 ms	113.22 ± 11.92 ms	10.93 ± 0.95 ms

468

4.3 SUPPLEMENTARY RESULTS

 469

470 We also extended our evaluation to high-dimensional D4RL tasks and visual manipulation tasks,
 471 and conduct a series of ablation results to demonstrate MFP’s superior performance across diverse
 472 situations. Please see Appendix E and F for full details.
 473

474

5 RELATED WORK

 475

476 **Offline-to-online RL.** The offline-to-online RL first uses a static, pre-collected dataset to offline train
 477 initial policy and Q-value functions (Levine et al., 2020; Kumar et al., 2020). This process provides
 478 a warm start, giving the agent a foundational understanding of the environment that significantly
 479 accelerates and improves the efficiency of subsequent online interactive fine-tuning (Lee et al.,
 480 2022). Numerous algorithmic designs have been proposed to improve offline-to-online RL, including
 481 behavioral regularization (Ashvin et al., 2020; Tarasov et al., 2023), conservatism (Kumar et al., 2020),
 482 in-sample maximization (Kostrikov et al., 2022; Garg et al., 2023), out-of-distribution detection (Yu
 483 et al., 2020; Kidambi et al., 2020; Nikulin et al., 2023) and dual RL (Lee et al., 2021; Sikchi et al.,
 484 2024). Beyond these algorithmic solutions, the choice of policy function also plays an important
 485 role (Wang et al., 2023). A policy network with high expressiveness can better capture the intricate
 distribution of the behavioral policy during offline training stage. This capability is also crucial for

486 adapting to target environments during online fine-tuning, as optimal actions in these settings often
 487 possess a naturally multi-modal nature (Park et al., 2025; Li et al., 2025). Since the policy must
 488 infer actions at every step during both the online fine-tuning and real-time deployment stages, the
 489 inference time efficiency of the policy function is also critical (Li, 2023; Park et al., 2025). Our work
 490 contributes a new policy function, MFP, which achieves the fastest single-step action generation and
 491 maintains a high expressive capacity to achieve high performance.

492 **Generative models as RL policies.** The expressive power of generative models, such as denoising
 493 diffusion models (Ho et al., 2020) and flow matching (Lipman et al., 2022), makes them promising
 494 for representing complex, multi-modal policies in both offline (Janner et al., 2022; Chi et al., 2023)
 495 and online (Yang et al., 2023; Wang et al., 2024a; Ding et al., 2024) RL. However, their iterative
 496 sampling process requires a high number of function evaluations (NFE), creating a prohibitive latency
 497 for the high-throughput nature of online RL. A common approach in RL to improving time-efficiency
 498 of generative policies is distillation, which compresses a trained iterative model into a one-step
 499 policy (Wang et al., 2024b; Park et al., 2025). Beyond the field of RL, recent studies have begun to
 500 explore training a single-step generative flow directly for tasks like image generation (Lu & Song,
 501 2025). These methods typically operate either by enforcing consistency constraints on the model’s
 502 outputs at different time steps (Song et al., 2023; Song & Dhariwal, 2024; Geng et al., 2025b) or
 503 by explicitly modeling the flow velocity over a specific time interval (Boffi et al., 2024; Frans et al.,
 504 2024). The latter class of methods generally requires more iterations to train, but ultimately performs
 505 better (Frans et al., 2024). A typical representative, mean flow, has achieved the best one-step fitting
 506 performance on Imagenet (Geng et al., 2025a). We propose a new policy function, MFP, which
 507 combines mean flow with RL to enable the fastest single-step action generation. In the context of RL,
 508 the dynamic shifts in data distribution during sampling place higher demands on imitation-based flow
 509 matching training. To address this, our proposed IVC serves as an explicit boundary condition during
 510 MFP training, which reduces fitting error and consequently ensures strong policy expressiveness.

511 6 CONCLUSION

512 We propose the mean flow policy (MFP), a new generative RL policy that enjoys both high time-
 513 efficiency and expressiveness. The former stems from the fastest one-step action generation, and the
 514 latter contributes to our designed instantaneous velocity constraint (IVC), which explicitly serves as
 515 a necessary boundary condition and reliably improves the learning accuracy. Empirical results on
 516 the Robomimic and OGBench benchmarks confirm that our MFP achieves state-of-the-art success
 517 rates and offers substantial improvements in training and inference speed. We believe that this work
 518 represents a significant step towards developing highly expressive and efficient policy functions for
 519 complex robotic control tasks. Regarding limitation, a primary one is the additional GPU memory
 520 consumption during training, which stems from the Jacobian-Vector Product (JVP) operation. In
 521 future work, we plan to validate our method on more tasks and real robotic platforms.

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702 A THEORETICAL ANALYSIS ON THE MEAN FLOW POLICY IMPROVEMENT

704 In this section, we provide the detailed theoretical analysis for the policy improvement guarantee of
 705 our training paradigm. It includes the implementation procedures of policy update detailed in A.1,
 706 core assumptions and useful lemma in A.2, the formal proof of the mean flow policy improvement
 707 theorem in A.3, and three key properties of the improvement gain in A.4.

709 A.1 IMPLEMENTATION PROCEDURES OF POLICY UPDATE

711 Let π_{old} denote the base mean flow policy prior to an update. At the same time, we have a learned
 712 critic, $Q_\phi(s, a)$, which is an approximation of the true action-value function of the base policy,
 713 $Q^{\pi_{\text{old}}}(s, a)$. Let π_{new} denote the updated policy, which is derived from π_{old} through the following
 714 three-step generative process for any given state s :

- 715 1. **Sample:** Generate N candidate actions $\{a_1, \dots, a_N\}$ by sampling from the base policy, i.e.,
 $a_i \sim \pi_{\text{old}}(\cdot|s)$.
- 717 2. **Select:** Use the learned critic Q_ϕ to select the best action from the candidates, which
 $a^*(s) = \arg \max_{a_i} Q_\phi(s, a_i).$

- 721 3. **Match:** The new policy π_{new} is obtained by training the parameters of π_{old} to match the
 $a^*(s)$. Note that an action $a_{\text{new}} \sim \pi_{\text{new}}$ is not guaranteed to perfectly match
 $a^*(s)$. We formally bound the expected distance between them in Assumption 3.

725 For analytical clarity, we introduce a conditional matching distribution, $M(\cdot|a^*)$, to model the
 726 outcome of the matching process (Step 3). Specifically, we assume the specific target a^* has been
 727 selected in the previous Steps 1 and 2, then generation of the final action a_{new} is expressed as

$$728 a_{\text{new}} \sim M(\cdot|a^*).$$

729 The difference between $M(\cdot|a^*)$ and π_{new} is that $M(\cdot|a^*)$ describes the action distribution conditioned
 730 on a specific target action a^* , whereas π_{new} represents the marginal distribution of the action, which
 731 results from averaging over all possible targets $a^*(s)$ generated in Steps 1 and 2.

733 A.2 CORE ASSUMPTIONS AND USEFUL LEMMA

735 Our analysis relies on the following assumptions and lemma.

736 **Assumption 1** (Bounded Q-Value Fitting Error). *The error between the learned critic Q_ϕ and the*
 737 *true Q-function of the base policy $Q^{\pi_{\text{old}}}$ is uniformly bounded by a constant $\epsilon_Q \geq 0$.*

$$738 \forall (s, a) \in \mathcal{S} \times \mathcal{A}, \quad |Q_\phi(s, a) - Q^{\pi_{\text{old}}}(s, a)| \leq \epsilon_Q.$$

740 **Assumption 2** (Q-Value Smoothness). *The true Q-value of the base policy, $Q^{\pi_{\text{old}}}(s, a)$, is L_Q -*
 741 *Lipschitz continuous with respect to the action a .*

$$743 \forall s \in \mathcal{S}, a_1, a_2 \in \mathcal{A}, \quad |Q^{\pi_{\text{old}}}(s, a_1) - Q^{\pi_{\text{old}}}(s, a_2)| \leq L_Q \cdot d(a_1, a_2).$$

745 **Assumption 3** (Bounded Mean Flow Matching Error). *The expected distance between the action*
 746 *a_{new} generated by the new policy and the target action a^* is bounded by a constant $\epsilon_A \geq 0$.*

$$748 \forall s \in \mathcal{S}, \quad \mathbb{E}_{a_{\text{new}} \sim M(\cdot|a^*)}[d(a_{\text{new}}, a^*(s))] \leq \epsilon_A.$$

750 **Lemma 1** (Performance Difference Lemma (Kakade & Langford, 2002)). *Let $\mathcal{M} = (\mathcal{S}, \mathcal{A}, P, R, \gamma)$*
 751 *be a Markov Decision Process, and let π and π' be two arbitrary policies. The difference in the*
 752 *state-value function at a starting state s , or denoted as s_0 , can be expressed in terms of the advantage*
 753 *function of the new policy π' with respect to the old policy π as:*

$$755 V^{\pi'}(s) - V^\pi(s) = \mathbb{E}_{s \sim d^{\pi'}, a \sim \pi'(\cdot|s)} \left[\sum_{t=0}^{\infty} \gamma^t A^\pi(s, a) \right].$$

756 where $d^{\pi'}(s)$ is the discounted state visitation distribution under policy π' , and $A^{\pi}(s, a) =$
 757 $Q^{\pi}(s, a) - V^{\pi}(s)$ is the advantage function of policy π .
 758

759 *Proof.* The proof relies on the key identity $V^{\pi}(s) = \sum_{a \in \mathcal{A}} \pi(a|s)Q^{\pi}(s, a)$. We start with the
 760 definition of the value difference for a state s :

$$\begin{aligned} 761 \quad V^{\pi'}(s) - V^{\pi}(s) &= V^{\pi'}(s) - \sum_{a \in \mathcal{A}} \pi'(a|s)Q^{\pi}(s, a) + \sum_{a \in \mathcal{A}} \pi'(a|s)Q^{\pi}(s, a) - V^{\pi}(s) \\ 762 &= V^{\pi'}(s) - \sum_{a \in \mathcal{A}} \pi'(a|s)Q^{\pi}(s, a) + \sum_{a \in \mathcal{A}} \pi'(a|s)(Q^{\pi}(s, a) - V^{\pi}(s)) \\ 764 &= V^{\pi'}(s) - \sum_{a \in \mathcal{A}} \pi'(a|s)Q^{\pi}(s, a) + \sum_{a \in \mathcal{A}} \pi'(a|s)A^{\pi}(s, a) \\ 766 &= V^{\pi'}(s) - \sum_{a \in \mathcal{A}} \pi'(a|s)Q^{\pi}(s, a) + \sum_{a \in \mathcal{A}} \pi'(a|s)A^{\pi}(s, a) \end{aligned}$$

768 Using the Bellman expectation equation for $V^{\pi'}(s)$:

$$771 \quad V^{\pi'}(s) = \sum_{a' \in \mathcal{A}} \pi'(a'|s) \left(R(s, a') + \gamma \sum_{s' \in \mathcal{S}} P(s'|s, a')V^{\pi'}(s') \right)$$

773 And the definition of $Q^{\pi}(s, a)$:

$$775 \quad Q^{\pi}(s, a) = R(s, a) + \gamma \sum_{s' \in \mathcal{S}} P(s'|s, a)V^{\pi}(s')$$

777 Substitute these into our first equation:

$$\begin{aligned} 779 \quad V^{\pi'}(s) - V^{\pi}(s) &= \sum_{a' \in \mathcal{A}} \pi'(a'|s) \left(R(s, a') + \gamma \sum_{s' \in \mathcal{S}} P(s'|s, a')V^{\pi'}(s') - Q^{\pi}(s, a') \right) + \sum_{a' \in \mathcal{A}} \pi'(a'|s)A^{\pi}(s, a') \\ 781 &= \sum_{a' \in \mathcal{A}} \pi'(a'|s) \left(\gamma \sum_{s' \in \mathcal{S}} P(s'|s, a')(V^{\pi'}(s') - V^{\pi}(s')) \right) + \mathbb{E}_{a \sim \pi'(\cdot|s)}[A^{\pi}(s, a)] \\ 784 &= \gamma \mathbb{E}_{a \sim \pi'(\cdot|s), s' \sim P(\cdot|s, a)}[V^{\pi'}(s') - V^{\pi}(s')] + \mathbb{E}_{a \sim \pi'(\cdot|s)}[A^{\pi}(s, a)] \end{aligned}$$

788 This recursive formula shows that the value difference at state s depends on the expected value
 789 difference at the next state s' . By recursively unrolling this expression over time and averaging over
 790 the state visitation distribution $d^{\pi'}(s)$, we arrive at the final result.

$$791 \quad V^{\pi'}(s) - V^{\pi}(s) = \mathbb{E}_{s \sim d^{\pi'}, a \sim \pi'(\cdot|s)} \left[\sum_{t=0}^{\infty} \gamma^t A^{\pi}(s, a) \right].$$

794 This completes the proof. □

796 A.3 PROOF OF THE MEAN FLOW POLICY IMPROVEMENT THEOREM

798 This subsection provides a formal proof of the mean flow policy improvement theorem, that is,
 799 Theorem 1 in the main text.

801 *Proof.* Our proof begins by invoking the Performance Difference Lemma, i.e., Lemma 1, which
 802 yields

$$803 \quad V^{\pi_{\text{new}}}(s) - V^{\pi_{\text{old}}}(s) = \mathbb{E}_{\tau \sim \pi_{\text{new}}} \left[\sum_{t=0}^{\infty} \gamma^t A^{\pi_{\text{old}}}(s_t, a_t) \right]. \quad (21)$$

805 This equation connects the improvement in value function to the expected advantage of the new
 806 policy's actions, measured with respect to the old policy.

808 The following proof proceeds in two parts: first, we derive the single-step advantage bound
 809 $\mathbb{E}_{a_t \sim \pi_{\text{new}}(\cdot|s_t)}[A^{\pi_{\text{old}}}(s_t, a_t)]$, and second, we aggregate them over time to obtain the required bound of
 810 value difference, which completes the proof.

810
Part 1: Lower bound the single-step expected advantage We begin by analyzing the Q-value
811 decay due to the imperfect matching for a *given* target action a^* . The final action a_{new} is distributed
812 according to $M(\cdot|a^*)$. We can bound the expected difference as follows:

$$\begin{aligned} 814 \quad & Q^{\pi_{\text{old}}}(s, a^*) - \mathbb{E}_{a_{\text{new}} \sim M(\cdot|a^*)}[Q^{\pi_{\text{old}}}(s, a_{\text{new}})] \\ 815 \quad &= \mathbb{E}_{a_{\text{new}}}[Q^{\pi_{\text{old}}}(s, a^*) - Q^{\pi_{\text{old}}}(s, a_{\text{new}})] \\ 816 \quad &\leq \mathbb{E}_{a_{\text{new}}}[|Q^{\pi_{\text{old}}}(s, a^*) - Q^{\pi_{\text{old}}}(s, a_{\text{new}})|] \quad (\text{by Jensen's inequality}) \\ 817 \quad &\leq \mathbb{E}_{a_{\text{new}}}[L_Q \cdot d(a^*, a_{\text{new}})] \quad (\text{by Q-Value Smoothness, Assumption. 2}) \\ 818 \quad &\leq L_Q \epsilon_A \quad (\text{by Bounded Matching Error, Assumption 3}) \\ 819 \end{aligned}$$

820 Rearranging gives us the following bound for a given a^* :

$$821 \quad \mathbb{E}_{a_{\text{new}} \sim M(\cdot|a^*)}[Q^{\pi_{\text{old}}}(s, a_{\text{new}})] \geq Q^{\pi_{\text{old}}}(s, a^*) - L_Q \epsilon_A. \\ 822$$

823 Now, we take the expectation over the distribution of target actions $a^*(s)$ to get the bound for the
824 marginal policy π_{new} :

$$\begin{aligned} 826 \quad & \mathbb{E}_{a \sim \pi_{\text{new}}}[Q^{\pi_{\text{old}}}(s, a)] = \mathbb{E}_{\{a_i\} \sim \pi_{\text{old}}}[\mathbb{E}_{a_{\text{new}} \sim M(\cdot|a^*(s))}[Q^{\pi_{\text{old}}}(s, a_{\text{new}})]] \\ 827 \quad & \geq \mathbb{E}_{\{a_i\} \sim \pi_{\text{old}}}[Q^{\pi_{\text{old}}}(s, a^*)] - L_Q \epsilon_A. \\ 828 \end{aligned}$$

829 Next, we relate the Q-value of the target action, $Q^{\pi_{\text{old}}}(s, a^*)$, back to the Q-values of the original
830 candidates $\{a_i\}$:

$$\begin{aligned} 831 \quad & Q^{\pi_{\text{old}}}(s, a^*) \geq Q_{\phi}(s, a^*) - \epsilon_Q \quad (\text{by Bounded Q-Value Fitting Error, Assumption 1}) \\ 832 \quad &= \max_{i=1, \dots, N} Q_{\phi}(s, a_i) - \epsilon_Q \quad (\text{by definition of } a^*) \\ 833 \quad &\geq \max_{i=1, \dots, N} Q^{\pi_{\text{old}}}(s, a_i) - 2\epsilon_Q. \quad (\text{by Assumption 1 on each candidate } a_i) \\ 835 \end{aligned}$$

836 Substituting this result back into our bound for $\mathbb{E}_{a \sim \pi_{\text{new}}}[Q^{\pi_{\text{old}}}(s, a)]$, we get:

$$837 \quad \mathbb{E}_{a \sim \pi_{\text{new}}}[Q^{\pi_{\text{old}}}(s, a)] \geq \mathbb{E}_{\{a_i\} \sim \pi_{\text{old}}} \left[\max_{i=1, \dots, N} Q^{\pi_{\text{old}}}(s, a_i) \right] - 2\epsilon_Q - L_Q \epsilon_A. \\ 838$$

840 Finally, subtracting $V^{\pi_{\text{old}}}(s) = \mathbb{E}_{a \sim \pi_{\text{old}}}[Q^{\pi_{\text{old}}}(s, a)]$ from both sides gives the desired single-step
841 advantage bound:

$$842 \quad \mathbb{E}_{a \sim \pi_{\text{new}}}[A^{\pi_{\text{old}}}(s, a)] \geq \Delta_N^{\pi_{\text{old}}}(s) - 2\epsilon_Q - L_Q \epsilon_A,$$

843 where $\Delta_N^{\pi_{\text{old}}}(s)$ is the **best-of- N advantage gain**, defined as

$$845 \quad \Delta_N^{\pi_{\text{old}}}(s) := \mathbb{E}_{a_1, \dots, a_N \sim \pi_{\text{old}}(\cdot|s)} \left[\max_{i=1, \dots, N} Q^{\pi_{\text{old}}}(s, a_i) \right] - V^{\pi_{\text{old}}}(s). \\ 846$$

848 **Part 2: Extension to the value function difference** With the single-step advantage bound estab-
849 lished, we substitute it into Eq. (21) and obtain:

$$\begin{aligned} 851 \quad & V^{\pi_{\text{new}}}(s) - V^{\pi_{\text{old}}}(s) \\ 852 \quad &= \mathbb{E}_{\tau \sim \pi_{\text{new}}} \left[\sum_{t=0}^{\infty} \gamma^t A^{\pi_{\text{old}}}(s_t, a_t) \right] \\ 853 \quad &= \sum_{t=0}^{\infty} \gamma^t \mathbb{E}_{s_t \sim d_{s, \pi_{\text{new}}^t}} [\mathbb{E}_{a_t \sim \pi_{\text{new}}(\cdot|s_t)}[A^{\pi_{\text{old}}}(s_t, a_t)]] \quad (\text{by law of total expectation}) \\ 854 \quad &\geq \sum_{t=0}^{\infty} \gamma^t \mathbb{E}_{s_t \sim d_{s, \pi_{\text{new}}^t}} [\Delta_N^{\pi_{\text{old}}}(s_t) - 2\epsilon_Q - L_Q \epsilon_A] \quad (\text{by substituting the bound from Part 1}) \\ 855 \quad &= \mathbb{E}_{\tau \sim \pi_{\text{new}}} \left[\sum_{t=0}^{\infty} \gamma^t \Delta_N^{\pi_{\text{old}}}(s_t) \right] - \frac{2\epsilon_Q + L_Q \epsilon_A}{1 - \gamma}. \quad (\text{by rearranging the geometric series}) \\ 856 \end{aligned}$$

857 This completes the proof. □

864 A.4 KEY PROPERTIES OF THE BEST-OF- N ADVANTAGE GAIN
865866 The term $\Delta_N^{\pi_{\text{old}}}(s)$ has several important properties:
867

- 868 1. **Non-negativity:** $\Delta_N^{\pi_{\text{old}}}(s) \geq 0$.
- 869 2. **Monotonicity with N :** $\Delta_{N+1}^{\pi_{\text{old}}}(s) \geq \Delta_N^{\pi_{\text{old}}}(s)$.
- 870 3. **Special case ($N = 1$):** $\Delta_1^{\pi_{\text{old}}}(s) = 0$.

872 *Proof.* Our proof sketch is to first rewrite $\Delta_N^{\pi_{\text{old}}}(s)$ as an integral involving the cumulative distribution
873 function (CDF) of the Q-values, and then demonstrate the claimed properties of the rewritten form.
874875 Let us define a random variable X representing the candidate Q-values. For a given state s , the
876 randomness of X is induced by the candidate action a independently sampled from the base policy:
877

878
$$X = Q^{\pi_{\text{old}}}(s, a), \quad \text{where } a \sim \pi_{\text{old}}(\cdot | s).$$

879 Recall that the definition of the **best-of- N advantage gain** is
880

881
$$\Delta_N^{\pi_{\text{old}}}(s) := \mathbb{E}_{a_1, \dots, a_N \sim \pi_{\text{old}}(\cdot | s)} \left[\max_{i=1, \dots, N} Q^{\pi_{\text{old}}}(s, a_i) \right] - V^{\pi_{\text{old}}}(s).$$

882

883 We can equivalently describe this definition using the CDF notation of the random variable X .
884 Let $F_X(x)$ be the CDF of X , and X_1, \dots, X_N be N independent and identically distributed (i.i.d.)
885 samples of X , and let $Y_N = \max(X_1, \dots, X_N)$. The CDF of Y_N is $F_{Y_N}(y) = [F_X(y)]^N$. Therefore,
886 we have $\Delta_N^{\pi_{\text{old}}}(s) = \mathbb{E}[Y_N] - \mathbb{E}[X]$.887 Moving forward, since the expectation of a random variable X in terms of its CDF satisfies $\mathbb{E}[X] =$
888 $\int_0^\infty (1 - F_X(x))dx - \int_{-\infty}^0 F_X(x)dx$, we can further rewrite $\Delta_N^{\pi_{\text{old}}}(s)$ as:
889

890
$$\Delta_N^{\pi_{\text{old}}}(s) = \left(\int_0^\infty (1 - F_{Y_N}(x))dx - \int_{-\infty}^0 F_{Y_N}(x)dx \right) - \left(\int_0^\infty (1 - F_X(x))dx - \int_{-\infty}^0 F_X(x)dx \right)$$

891
892
$$= \int_{-\infty}^\infty (F_X(x) - F_{Y_N}(x))dx.$$

893

894 Substituting $F_{Y_N}(x) = [F_X(x)]^N$, we arrive at the final rewritten form:
895

896
$$\Delta_N^{\pi_{\text{old}}}(s) = \int_{-\infty}^\infty (F_X(x) - [F_X(x)]^N)dx.$$

897

898 Next we use this form to prove the three key properties of $\Delta_N^{\pi_{\text{old}}}(s)$.
899900 **1. Proof of non-negativity** ($\Delta_N^{\pi_{\text{old}}}(s) \geq 0$)901 The CDF $F_X(x)$ takes values in the range $[0, 1]$. For any value $p \in [0, 1]$ and any integer $N \geq 1$, we
902 have $p^N \leq p$. Therefore, the integrand $F_X(x) - [F_X(x)]^N$ is always greater than or equal to zero
903 for all x . The integral of a non-negative function is non-negative. Thus, $\Delta_N^{\pi_{\text{old}}}(s) \geq 0$.
904905 **2. Proof of monotonicity with N** ($\Delta_{N+1}^{\pi_{\text{old}}}(s) \geq \Delta_N^{\pi_{\text{old}}}(s)$)906 We examine the difference between consecutive terms using the integral form:
907

908
$$\Delta_{N+1}^{\pi_{\text{old}}}(s) - \Delta_N^{\pi_{\text{old}}}(s) = \int_{-\infty}^\infty (F_X(x) - [F_X(x)]^{N+1})dx - \int_{-\infty}^\infty (F_X(x) - [F_X(x)]^N)dx$$

909
910
$$= \int_{-\infty}^\infty ([F_X(x)]^N - [F_X(x)]^{N+1})dx$$

911
912
$$= \int_{-\infty}^\infty [F_X(x)]^N (1 - F_X(x))dx.$$

913

914 Since $F_X(x) \in [0, 1]$, both terms in the integrand, $[F_X(x)]^N$ and $(1 - F_X(x))$, are non-negative.
915 Their product is therefore non-negative. The integral of this non-negative function is non-negative,
916 which implies $\Delta_{N+1}^{\pi_{\text{old}}}(s) - \Delta_N^{\pi_{\text{old}}}(s) \geq 0$.
917

918 **3. Proof of special case ($\Delta_1^{\pi_{\text{old}}}(s) = 0$)**919 Substituting $N = 1$ into the integral identity yields:

920
$$\Delta_1^{\pi_{\text{old}}}(s) = \int_{-\infty}^{\infty} (F_X(x) - [F_X(x)]^1) dx = \int_{-\infty}^{\infty} (F_X(x) - F_X(x)) dx = \int_{-\infty}^{\infty} 0 dx = 0.$$

921 This confirms the special case. \square 922 **B SUPPLEMENTARY RESULTS**923 **B.1 NUMERICAL RESULTS OF ABLATION STUDY**

924 Table 4: Ablation on the impact of IVC.

925

Task	MFP ($\lambda = 0.0$)	MFP ($\lambda = 0.5$)	MFP ($\lambda = 1.0$)
Cube-triple-task3	0.65 ± 0.05	0.70 ± 0.14	0.71 ± 0.06
Cube-triple-task4	0.30 ± 0.21	0.44 ± 0.08	0.52 ± 0.11

926 Table 5: Comparison with one-step variants of the aforementioned baselines.

927

Task	FQL-Onestep	BFN-Onestep	QC-Onestep	MFP (ours)
Cube-triple-task3	0.00 ± 0.01	0.00 ± 0.00	0.02 ± 0.03	0.71 ± 0.06
Cube-triple-task4	0.00 ± 0.00	0.00 ± 0.00	0.01 ± 0.01	0.52 ± 0.11

928 **C ENVIRONMENTS DESCRIPTION**929 **Robomimic benchmark.** We use three challenging tasks from the robomimic domain (Mandlekar
930 et al., 2021). We use the multi-human datasets that were collected by six human operators. Each
931 dataset consists of 50 trajectories provided by each operator, for a total of 300 successful trajectories.
932 The operators were varied in proficiency – there were 2 “worse” operators, 2 “okay” operators, and 2
933 “better” operators, resulting in diverse, mixed quality datasets. The three tasks are as described below.934 (1) **lift**: requires the robot arm to pick a small cube. This is the simplest task of the benchmark.935 (2) **can**: requires the robot arm to pick up a coke can and place in a smaller container bin.936 (3) **square**: requires the robot arm to pick a square nut and place it on a rod. The nut is slightly bigger
937 than the rod and requires the arm to move precisely to complete the task successfully.938 All of the three robomimic tasks use binary task completion rewards where the agent receives -1
939 reward when the task is not completed and 0 reward when the task is completed.940 **OGBench** cube-double/triple: These three domains contain 2/3 cubes respectively. The
941 tasks in the two domains all involve moving the cubes to their desired locations. The reward is
942 $-n_{\text{wrong}}$ where n_{wrong} is the number of the cubes that are at the wrong position. The episode
943 terminates when all cubes are at the correct position (reward is 0).

944 Below we highlight three representative tasks from each environment:

945 (4) **Cube-double-task2 (move)**: Two cubes in different colors are initialized at $(0.35, -0.1, 0.02)$
946 and $(0.50, -0.1, 0.02)$, and the goal is to move them to $(0.35, 0.1, 0.02)$ and $(0.50, 0.1, 0.02)$.947 (5) **Cube-double-task3 (move)**: The initial cube positions are $(0.35, 0.0, 0.02)$ and $(0.50, 0.0, 0.02)$,
948 and they must be placed at $(0.425, -0.2, 0.02)$ and $(0.425, 0.2, 0.02)$.949 (6) **Cube-double-task4 (swap)**: Two cubes start from $(0.425, -0.1, 0.02)$ and $(0.425, 0.1, 0.02)$,
950 and the objective is to swap their positions to $(0.425, 0.1, 0.02)$ and $(0.425, -0.1, 0.02)$.

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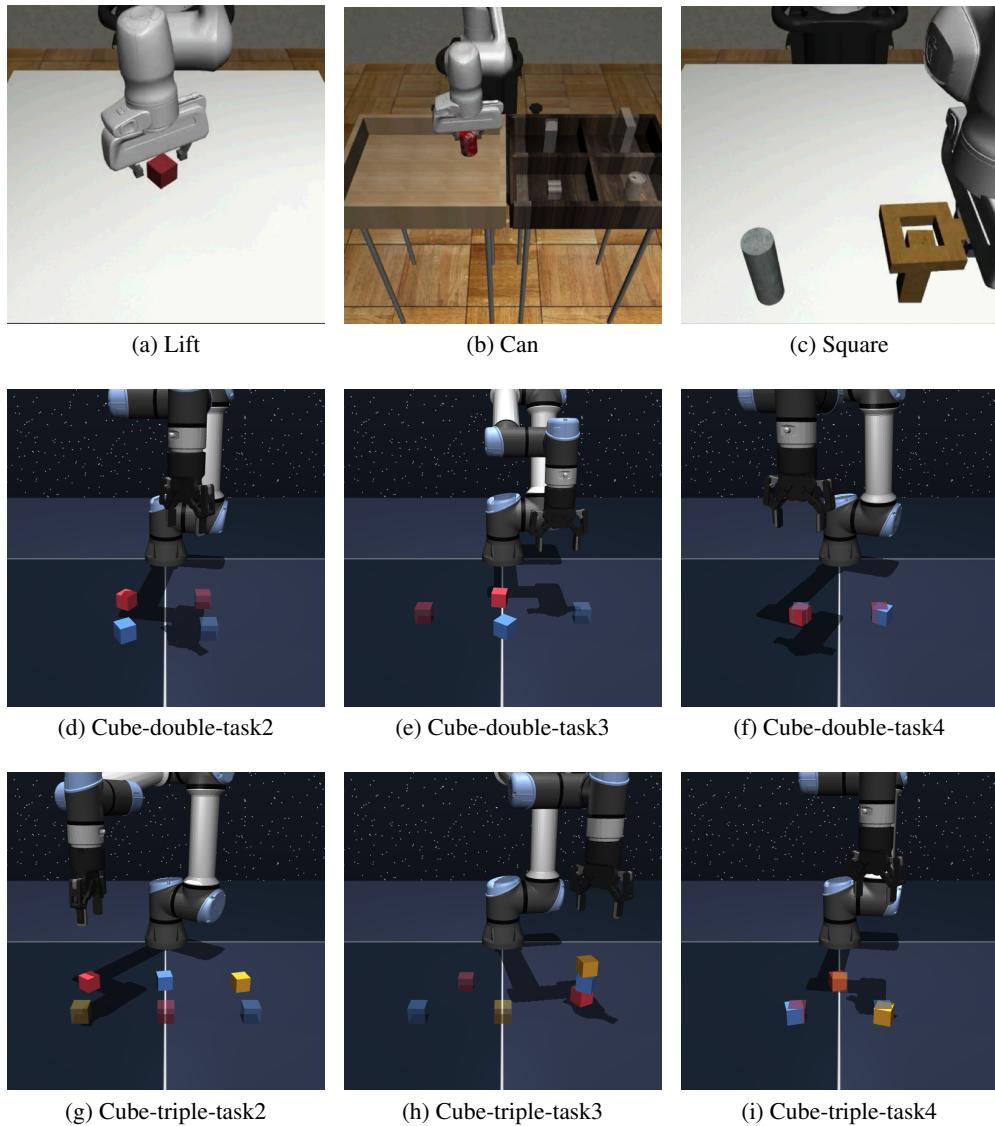


Figure 6: Snapshots of the 9 challenging long-horizon, sparse-reward manipulation tasks.

(7) Cube-triple-task2 (move): Three cubes are initialized at $(0.35, -0.2, 0.02)$, $(0.35, 0.0, 0.02)$, and $(0.35, 0.2, 0.02)$, with goals at $(0.50, 0.0, 0.02)$, $(0.50, 0.2, 0.02)$, and $(0.50, -0.2, 0.02)$.

(8) Cube-triple-task3 (stack): A stacked tower of three cubes is initialized at $(0.425, 0.2, 0.02)$, $(0.425, 0.2, 0.06)$, and $(0.425, 0.2, 0.10)$, and the robot must relocate them to $(0.35, -0.1, 0.02)$, $(0.50, -0.2, 0.02)$, and $(0.50, 0.0, 0.02)$, respectively.

1017 **(9) Cube-triple-task4 (swap):** Three cubes are initialized at $(0.35, 0.0, 0.02)$, $(0.50, -0.1, 0.02)$,
1018 and $(0.50, 0.1, 0.02)$, and they must be cyclically rearranged to $(0.50, -0.1, 0.02)$, $(0.50, 0.1, 0.02)$,
1019 and $(0.35, 0.0, 0.02)$.

These tasks highlight increasingly complex spatial reasoning requirements, ranging from coordinated multi-cube relocation to disassembly of stacked structures and cyclic rearrangement.

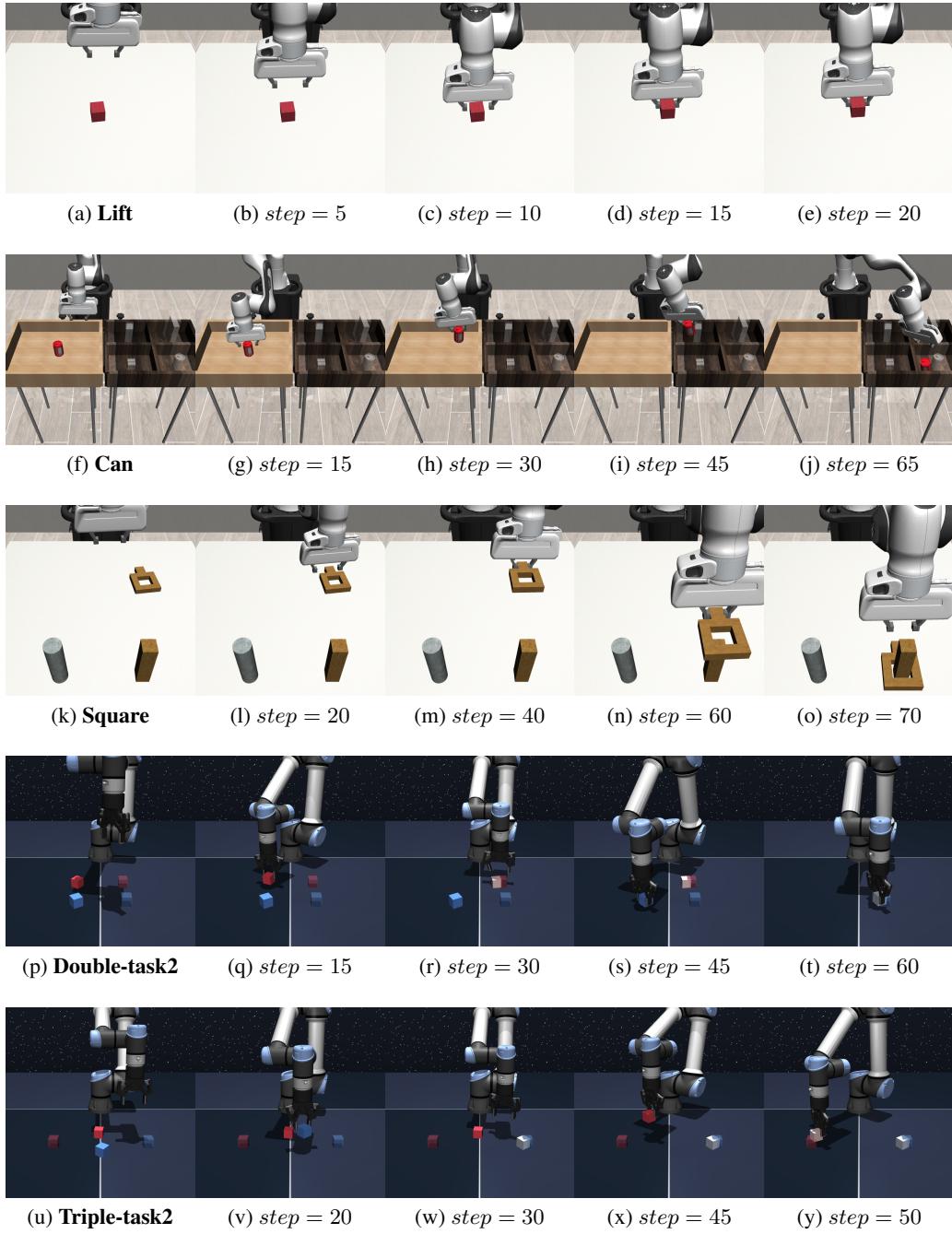
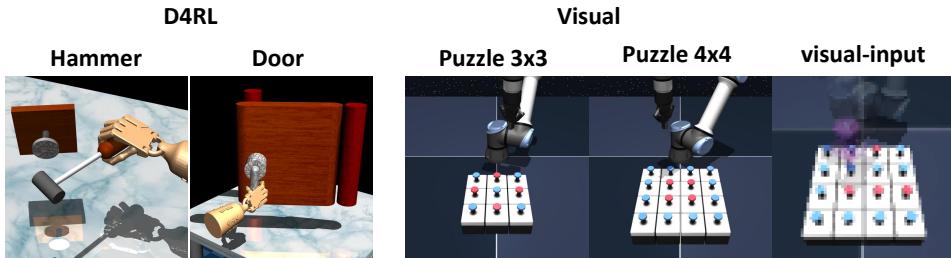
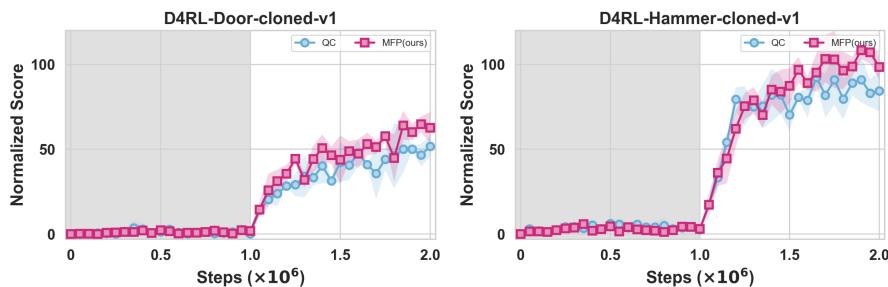
1026 **D VISUALIZATIONS**
10271028 This section provides supplementary visual material to demonstrate our model’s performance. The
1029 included visualizations of successful episodes highlight our policy’s ability to generate precise
1030 and robust trajectories, showcasing its effectiveness in handling the complexities of **long-horizon**
1031 **reasoning** and **sparse rewards** inherent in these robotic manipulation tasks.
10321068 **Figure 7: Visualizations of typical success episodes: Robomimic-lift, Robomimic-can, Robomimic-square,**
1069 **Cube-double-task2, and Cube-double-task3.**
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Figure 8: Visualizations of typical success episodes: Cube-double-task4, Cube-triple-task2, Cube-triple-task3, and Cube-triple-task4.

1134 **E ADDITIONAL EXPERIMENTS ON HIGH-DIMENSIONAL AND VISUAL TASKS**
11351136 To verify the scalability and robustness of MFP across diverse domains, we extended our evaluation
1137 to include two high-dimensional control tasks and two visual manipulation tasks, as shown in 9.
11381149 Figure 9: Snapshots of the additional tasks.
11501153 **E.1 HIGH-DIMENSIONAL CONTROL (D4RL ADROIT)**
11541155 We evaluated MFP on the D4RL Adroit suite (Hammer and Door), which involves high-dimensional
1156 dexterous manipulation. As shown in Table 6 and Figure 10, MFP significantly outperforms the
1157 strong baseline QC, achieving 14 points increase in normalized scores on average. This highlights
1158 MFP’s superior capability in modeling complex action distributions in high-dimensional spaces.
11591160 Table 6: Comparison of normalized scores on D4RL Adroit tasks (Mean \pm Std).
1161

Algorithm	D4RL-Door	D4RL-Hammer
QC	51.8 ± 6.0	92.2 ± 4.5
MFP (ours)	64.7 ± 4.7	108.3 ± 5.2

1178 Figure 10: Training curves on D4RL Adroit tasks.
11791182 **E.2 VISUAL-BASED MANIPULATION**
11831184 We further evaluated MFP on visual-based Puzzle-3x3 and Puzzle-4x4 tasks, which require spatial
1185 reasoning directly from pixel inputs. The results are summarized in Table 7. While both methods
1186 solve the 3x3 task, MFP achieves a higher success rate (0.61 vs. 0.47) and better peak performance
1187 on the significantly harder 4x4 task, demonstrating its effectiveness in learning visual policies with
1188 sparse rewards.
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Table 7: Success rates on Visual-based manipulation tasks.

Algorithm	Visual-puzzle-3x3	Visual-puzzle-4x4
QC	1.0 ± 0.0	0.47 ± 0.33
MFP (ours)	1.0 ± 0.0	0.61 ± 0.40

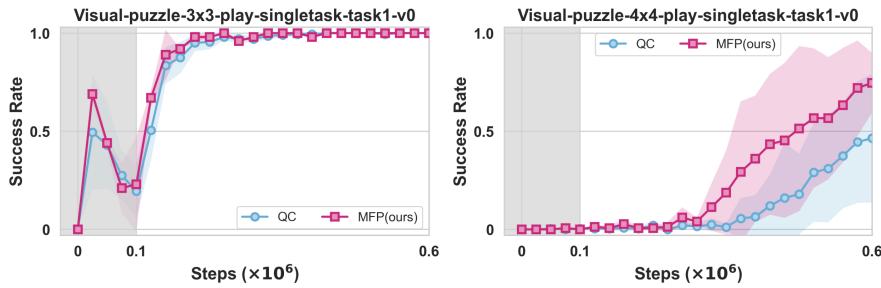


Figure 11: Training curves on visual tasks.

F ADDITIONAL ABLATION STUDIES

F.1 IMPACT OF INFERENCE STEPS

To demonstrate the trade-off between efficiency and performance, we evaluated the baseline QC with varying inference steps (1, 5, and 20). As shown in Figure 12 and Table 8, reducing the baseline steps leads to a catastrophic performance drop. In contrast, MFP is structurally designed for one-step generation (NFE=1) and effectively dominates the Pareto frontier, achieving SOTA success rates with minimal computational cost.

Table 8: Performance comparison with varying inference steps.

Algorithm	Cube-triple-task3	Cube-triple-task4
QC (1 step)	0.02 ± 0.02	0.01 ± 0.91
QC (5 steps)	0.33 ± 0.40	0.08 ± 0.04
QC (20 steps)	0.69 ± 0.07	0.30 ± 0.08
MFP (ours)	0.71 ± 0.08	0.52 ± 0.11

F.2 COMPARISON WITH PROBABILISTIC MIXING

We compared our proposed Instantaneous Velocity Constraint (IVC) against the heuristic probabilistic mixing strategy used in prior MeanFlow work (Geng et al., 2025a). As shown in Figure 13, our simultaneous IVC loss yields significantly higher success rates and stability compared to mixing strategies (25%, 50%, 75%), confirming that explicit boundary supervision is crucial for RL tasks.

G EVALUATION ON IMITATION LEARNING

To verify the generality of our method, we conducted a pure Behavioral Cloning (BC) experiment on the Robomimic Square task. Our one-step MFP outperforms the multi-step baseline in success rate while maintaining significantly faster inference. Additionally, visualization on the Push-T task (Figure 15) confirms that MFP successfully captures multi-modal behaviors.

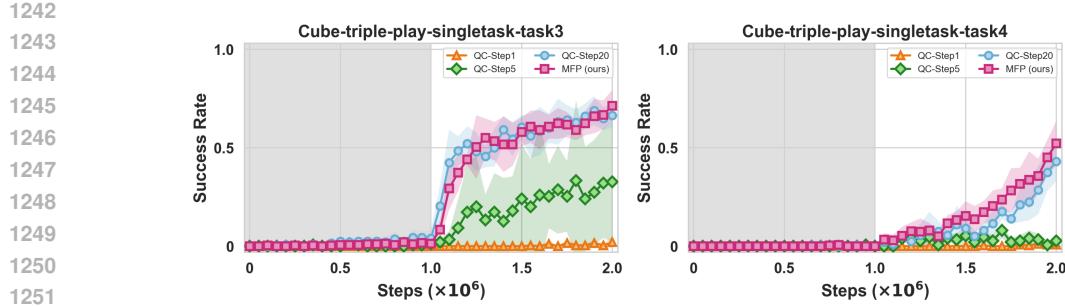


Figure 12: Ablation study on flow inference steps.

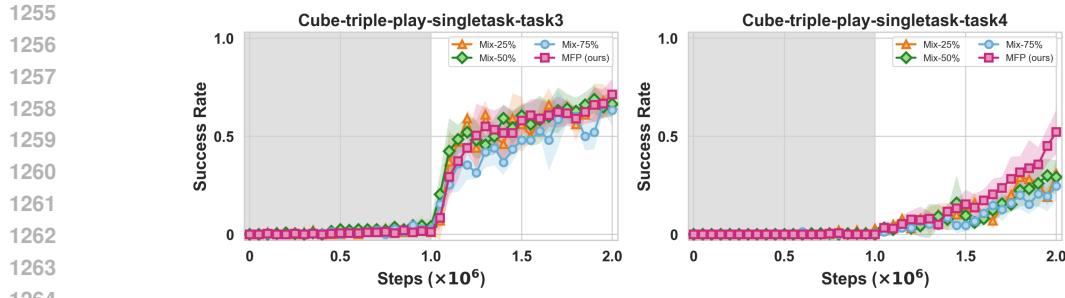


Figure 13: Ablation study comparing IVC with probabilistic mixing strategies.

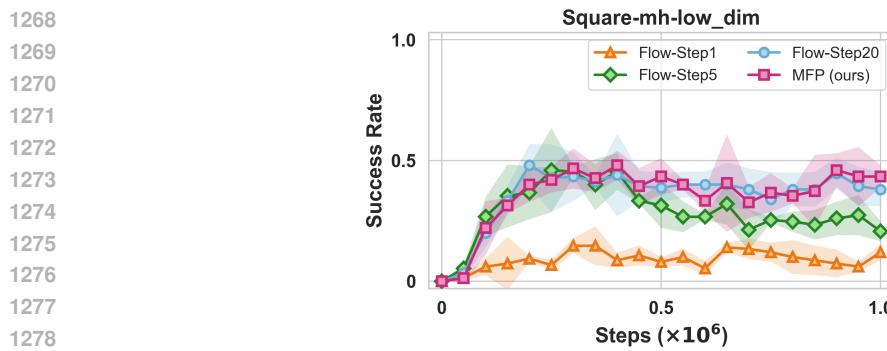


Figure 14: Behavior cloning experiment.

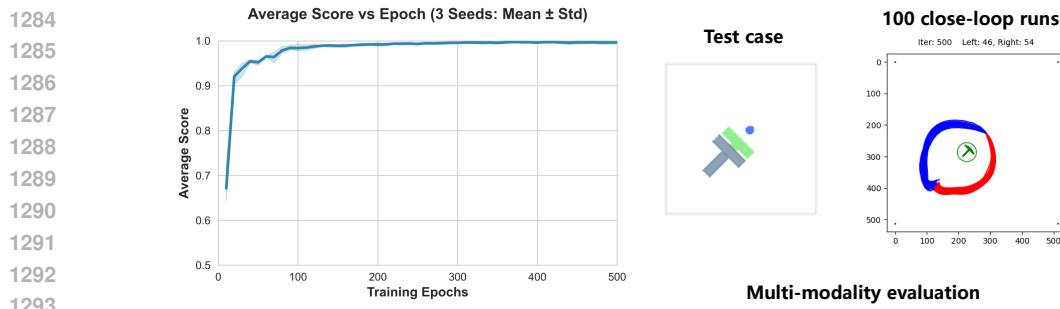


Figure 15: Visualization of multi-modal behaviors learned by MFP on the Push-T task.

1296 **H REPRODUCIBILITY STATEMENT**
12971298 The hyperparameters of all algorithms are shown in Table 9.
1299

1300 Parameter	1301 Value
Shared	
1303 Batch size	256
1304 Discount factor (γ)	0.99
1305 Optimizer	Adam
1306 Learning rate	3×10^{-4}
1307 Target network update rate (τ)	5×10^{-3}
1308 UTD Ratio	1
1309 Evaluation interval	5000
1310 Number of evaluation episodes	50
1311 Number of offline training steps	1×10^6 (1M)
1312 Number of online training steps	1×10^6 (1M)
1313 Number of flow steps (T)	10
1314 Policy network width	512
1315 Policy network depth	4 hidden layers
1316 Policy activation function	GELU
1317 Policy layer normalization	False
1318 Value network width	512
1319 Value network depth	4 hidden layers
1320 Value activation function	GELU
1321 Value layer normalization	True
1322 Value ensemble size (K)	2
1323 Value ensemble operator	MEAN
FQL	
1325 Flow step	10
1326 BC weight (α)	10000 for lift, can and square 300 for cube-double-* and cube-triple-*
QC	
1330 Chunking horizon length	5
1331 Flow step	10
1332 Number of best-of- N	16 for lift, can and square 32 for cube-double-* and cube-triple-*
MFP-IVC (ours)	
1336 IVC ratio (λ)	1.0
1337 Number of best-of- N	16 for lift, can and square 32 for cube-double-* and cube-triple-*

1340 **Table 9: Detailed hyperparameters.**
1341

1350 **I LLM USAGE DISCLOSURE**

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1352 We used ChatGPT to refine the grammar and improve the clarity of the text. All LLM-generated
1353 suggestions were reviewed and edited by the authors, who take full responsibility for the final content.
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