

# 000 001 002 003 004 005 006 007 008 009 010 011 012 013 014 015 016 017 018 019 020 021 022 023 024 025 026 027 028 029 030 031 032 033 034 035 036 037 038 039 040 041 042 043 044 045 046 047 048 049 050 051 052 053 IMPROVING CLASSIFIER-FREE GUIDANCE IN MASKED DIFFUSION: LOW-DIM THEORETICAL INSIGHTS WITH HIGH-DIM IMPACT

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## ABSTRACT

Classifier-Free Guidance (CFG) is a widely used technique for conditional generation and improving sample quality in continuous diffusion models, and its extensions to discrete diffusion has recently started to be investigated. In order to improve the algorithms in a principled way, this paper starts by analyzing the exact effect of CFG in the context of a low-dimensional masked diffusion model, with a special emphasis on the guidance schedule. Our analysis shows that high guidance early in sampling (when inputs are heavily masked) harms generation quality, while late-stage guidance improves it. These findings provide a theoretical explanation for empirical observations in recent studies on guidance schedules. The analysis also reveals an imperfection of the current CFG implementations. These implementations can unintentionally cause imbalanced transitions, such as unmasking too rapidly during the early stages of generation, which degrades the quality of the resulting samples. To address this, we draw insight from the analysis and propose a novel classifier-free guidance mechanism. Intuitively, our method smooths the transport between the data distribution and the initial (masked) distribution, resulting in improved sample quality. Remarkably, our method is achievable via a simple one-line code change. Experiments on conditional image and text generation empirically confirm the efficacy of our method.

## 1 INTRODUCTION

Continuous-state diffusion models (Ho et al., 2020; Song et al.) have proven effective in both unconditional and conditional generation tasks, such as generating data from natural language prompts. Prominent examples include text-to-image and text-to-video models like Stable Diffusion, Sora, and others (Rombach et al., 2022; Esser et al., 2024; Brooks et al., 2024). More recently, progress in discrete diffusion modeling (Campbell et al., 2022; Lou et al., 2023; Huang et al., 2023; Gruver et al., 2023; Ou et al., 2024; Shi et al., 2024; Sahoo et al., 2024) has extended the applicability

images/Promo.png

Figure 1: We proposed an improved guidance mechanism through column normalization. Our method produces sharper images while being more stable to the guidance strength. Notably, it requires only a minor code modification.

of diffusion-based generation to new domains, including molecular design, protein synthesis, and languages.

Despite their success, these models often produce outputs that lack fine detail or strong alignment with conditioning inputs (e.g., text prompts). A widely adopted technique to address this issue is classifier-free guidance (CFG) (Ho and Salimans, 2021), which improves fidelity but typically at the cost of reduced sample diversity (Karras et al., 2024).

A growing body of work has sought to understand the theoretical foundations of CFG in diffusion models (Chidambaram et al., 2024; Pavasovic et al., 2025; Bradley and Nakkiran, 2024; Ye et al., 2025), while others have developed improved guidance algorithms (Karras et al., 2024; Li et al., 2024). Classifier-free guidance has also been adapted to discrete diffusion models (Nisonoff et al., 2024; Schiff et al., 2024), yielding promising empirical gains.

Among these improvements, dynamic guidance schedules—where guidance strength varies over the generation trajectory—have shown especially effective. Strategies such as guidance intervals (Kynkänniemi et al., 2024) and gradually increasing schedules (Xi et al., 2024) can significantly enhance sample quality and are increasingly adopted in practice (Hoogeboom et al., 2024; Yu et al., 2024; Karras et al., 2024). However, such scheduling techniques remain exclusive to the continuous setting.

While recent adaptations of CFG to discrete diffusion have improved empirical performance, defining and optimizing effective guidance strategies in discrete spaces remains a fundamentally challenging and open research problem.

In our work we aim to better understand the mechanisms by which guidance affects the sampling process in discrete diffusion. Specifically, we aim to answer the following questions:

- How does the guidance schedule affect the distribution of the generated samples?
- Is it possible to characterize properties of good guidance schedules?

To do so, we start by deriving explicit formulas for the sampled distribution under varying guidance schedules in 1 and 2 dimensions. Our analysis not only reveals flaws in current CFG implementations, but also leads to effective design principles for effective guidance schedules in masked diffusion. Our contributions can be summarized as:

- We identify a key flaw in existing discrete guidance mechanisms that complicates simulation, and provide a theoretical explanation of its cause.
- To address the flaw, we propose a novel classifier-free guidance mechanism based on a simple yet principled column normalization of the rate matrix. This change is theoretically justified, easy to implement (pseudocode in Sec.1), and compares favorably to existing approaches in practice.
- The first theoretical justifications to characterize guidance schedules and the mechanisms by which they improve sample generation

```
095 def normalized_guidance_euler_transition(
096     x, c, t, dt, w
097     ):
098     uncond = model(x, cond=None)
099     cond = model(x, cond=c)
100     logits = w * cond + (1 - w) * uncond
101     p_theta = logits.softmax(dim=-1)
102     s, s_bar = sigma(t), sigma_bar(t)
103     change = dt * s * (1 - exp(-s_bar))
104     return sample(delta(x) + change * p_theta)
```

Listing 1: Our guidance in the special case of masked diffusion using Euler transitions. Our method is a simple one line change *but clearly motivated by theory*

```
def other_guidance_euler_transition(
    x, c, t, dt, w
    ):
    uncond = model(x, cond=None)
    cond = model(x, cond=c)
    logits = w * cond + (1 - w) * uncond
    p_theta = logits.exp()
    s, s_bar = sigma(t), sigma_bar(t)
    change = dt * s * (1 - exp(-s_bar))
    return sample(delta(x) + change * p_theta)
```

Listing 2: Unlocking/Simple guidance for the special case of masked diffusion using Euler transitions.

108 

## 2 PRELIMINARIES

109

110 This paper considers a vocabulary of size  $M$  and state space  $S = \{1, 2, \dots, M\}^d$ , with each element  
111 being a sequence of tokens. The number of tokens  $d$  will also be referred to as the dimension. Each  
112 probability distribution on  $S$  is represented as a vector in  $\mathbb{R}^{M^d}$  whose entries sum to one.  
113

114 

### 2.1 INTRODUCTION TO DISCRETE DIFFUSION VIA CTMC

115

116 Given an initial distribution  $p \in \mathbb{R}^{M^d}$ , discrete diffusion is defined by considering a rate matrix  
117  $R_t \in \mathbb{R}^{M^d \times M^d}$  and defining a continuous time Markov chain (CTMC):  
118

119 
$$\frac{dp_t}{dt} = R_t p_t, \quad p_0 = p. \quad (1)$$
120

121 we pick  $R_t$  such that when  $t \rightarrow \infty$ ,  $p_t$  converges to a simple distribution. Additionally,  $R_t$  must  
122 satisfy that its non-diagonal entries are non-negative and each column must add up to zero. The time  
123 reversal of this process corresponds to a different CTMC given by:  
124

125 
$$\frac{dp_{T-t}}{dt} = \bar{R}_{T-t} p_{T-t}. \quad (2)$$
126

127 This process is considered as the time reversal since it has the same law as (1) for all values of  $t$  and  
128 the reverse transition matrix can be found through the following identities:  
129

130 
$$\bar{R}_t(y, x) = R_t(x, y) \cdot \frac{p_t(y)}{p_t(x)}, \quad \bar{R}_t(x, x) = - \sum_{y \neq x} \bar{R}_t(y, x). \quad (3)$$
131

132 The ratios  $\frac{p_t(y)}{p_t(x)}$  are called the concrete score and they enable sampling through Euler schemes,  
133  $\tau$ -leaping (Lou et al., 2023) or higher order methods (Ren et al., 2025).

134 **Masked Discrete Diffusion** is a special case of diffusion where a clean sequence  $x_0$  is gradually  
135 corrupted over time by randomly masking some of its entries. Typically, the forward process is  
136 chosen such that at time  $t = 0$ , the data is completely unmasked, and at  $t = T$  the data is completely  
137 masked. Formally, the distribution of each token can be written in a simple form:  
138

139 
$$p_t(x_t^i | x_0) = \begin{cases} x_0^i & \text{with probability } e^{-\bar{\sigma}_t} \\ M & \text{with probability } 1 - e^{-\bar{\sigma}_t} \end{cases}$$
140

141 Where  $\bar{\sigma}_t$  is an increasing function that defines the unmasking schedule. The forward dynamics  
142 are defined such that tokens transition only from a clean state to a masked state, remaining masked  
143 thereafter. Generation is achieved by starting from a fully masked state and iteratively unmasking  
144 tokens until a clean sequence is recovered by following Equation (2).

145 Masked diffusion enjoys a simple and structured design, which has enabled its successful scaling to  
146 large practical tasks (Nie et al.; Xie et al., 2025; Ou et al., 2024; Sahoo et al., 2024; Shi et al., 2024;  
147 Campbell et al., 2022). For this reason, we adopt it as the primary setting for our analysis.  
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149 

### 2.2 CLASSIFIER-FREE GUIDANCE

150

151 Classifier-free guidance (CFG) (Ho and Salimans, 2021) was introduced to improve conditional  
152 diffusion models, like generating images from class labels or text. Models often failed to capture  
153 fine details, which led to less accurate and misaligned samples (Karras et al., 2024).

154 CFG tackles this by comparing predictions with and without conditioning, and biasing generation  
155 toward the conditional signal. Formally, the method defines a reweighted distribution:  
156

157 
$$p^{(w)}(x|y) \propto p^w(x|y) p^{1-w}(x)$$
158

159 Where  $w$  is called the guidance strength. Setting  $w = 1$  recovers the usual conditional distribution  
160  $p(x|y)$  while  $w = 0$  corresponds to unconditional sampling. The crucial insight is that by setting  
161  $w > 1$  it is possible to emphasize the conditional part, effectively pulling the generation closer to  
162 satisfying the required condition. CFG is now a standard tool in conditional diffusion models, more  
163 controllable generations across tasks such as text-to-image synthesis.

162 While the original formulation contrasted the conditional model against its unconditional counter-  
 163 part, later works recognized that this can be extended by replacing the unconditional distribution  
 164 with other distributions. For example, [Karras et al. \(2024\)](#) used a weaker conditional model as the  
 165 guiding distribution. This view has led to the understanding that the essence of guidance lies in  
 166 balancing a **target distribution**  $p$  with a **guiding distribution**  $q$ .

$$p^{(w)}(x) \propto p^w(x)q^{1-w}(x) \quad (4)$$

169 This view highlights that the unconditional model is simply one possible choice of  $q$ . By carefully  
 170 selecting  $q$  recent works ([Karras et al., 2024](#); [Li et al., 2024](#); [Rojas et al., 2025](#)) have proposed novel  
 171 guidance strategies that further improve sample quality and control.

### 173 2.3 GUIDANCE FOR DISCRETE DIFFUSION MODELS

175 In parallel to advances in continuous domains, discrete diffusion models have emerged as powerful  
 176 generative models, enabling diffusion-based approaches on modalities that were previously out of  
 177 reach—most notably, text. Improving the fidelity and controllability of these models is crucial, and  
 178 guidance offers a natural path forward. Extending classifier-free guidance to the discrete setting has  
 179 therefore become an active line of research with two main approaches having been proposed, which  
 180 we describe below, followed by a discussion in Section 3.3 comparing them to our method.

181 **Unlocking Guidance** ([Nisonoff et al., 2024](#)) introduced the first classifier-free guidance mecha-  
 182 nisms for discrete diffusion models. Inspired by the continuous case, they constructed a guided  
 183 backwards transition by interpolating between two transition matrices in equation 2, yielding

$$\bar{R}_t^{(w)}(y, x) = R_t(x, y) \cdot \left( \frac{p_t(y)}{p_t(x)} \right)^w \left( \frac{q_t(y)}{q_t(x)} \right)^{1-w}, \quad \bar{R}_t^{(w)}(x, x) = - \sum_{y \neq x} \bar{R}_t^{(w)}(y, x), \quad (5)$$

187 where  $p_t, q_t$  follows the forward CTMC (1). Here  $p_0 = p$  is the distribution that we want to generate  
 188 from and  $q$  serves as the guiding distribution. <sup>1</sup>. Notice how the products mimic those present in  
 189 equation 4. A useful way to interpret this is by introducing the notion of the **tilted distribution**:

$$p^{(w)}(x) = Z_w^{-1} p^w(x) \cdot q^{1-w}(x), \quad Z_w = \sum_{y \in S} p^w(y) \cdot q^{1-w}(y).$$

193 The generation process follows the dynamics induced by the guided transition matrix substituted in  
 194 equation 2. [Nisonoff et al. \(2024\)](#) showed that guidance in the discrete setting serves a role analogous  
 195 to its continuous counterpart—steering the model toward more faithful conditional samples—thus  
 196 providing an important step toward improving the quality of discrete diffusion generations.

197 **Simple Guidance.** Concurrently, [Schiff et al. \(2024\)](#) proposed an alternative formulation of  
 198 classifier-free guidance for discrete diffusion. Rather than interpolating the rate matrices as in  
 199 [Nisonoff et al. \(2024\)](#), they directly interpolate the transition probabilities themselves. Specifically,  
 200 when transitioning from time  $t$  to time  $s < t$ , the following transition was proposed:

$$p_{\text{simple}}^{(w)}(z_s | z_t) \propto p^w(z_s | z_t) p^{1-w} q(z_s | z_t). \quad (6)$$

203 As before, increasing  $w$  biases towards the target distribution  $p$ . Although the construction appears  
 204 different, in the limit  $s \rightarrow t$  the transitions coincide with those of [Nisonoff et al. \(2024\)](#). In practice,  
 205 however, a finite number of steps is used, and the resulting methods are distinct. To implement these  
 206 transitions, one can use equation (2) together with a suitable numerical integration scheme.

### 208 2.4 DYNAMIC GUIDANCE SCHEDULES

210 In our work we will consider dynamic guidance schedules, i.e. making  $w$  a function of time. Such  
 211 schedules have become more popular in practice. For instance, guidance interval ([Kynkänniemi  
 212 et al., 2024](#)) only applies guidance on a segment of the generation process. Doing so produces  
 213 a boost in the performance of diffusion models. However, existing work on dynamic guidance

214 <sup>1</sup>In existing literature,  $p$  is usually a class-conditional distribution, and  $q$  is an unconditional distribution.  
 215 We adopt the general setup since recent works have shown that  $q$  can be chosen in different ways ([Karras et al.,  
 2024](#); [Li et al., 2024](#); [Rojas et al., 2025](#)).

216 schedules (Kynkäanniemi et al., 2024; Xi et al., 2024) has been limited to a continuous (state-space)  
 217 diffusion models. It remains unclear whether such schedules are also effective in discrete state  
 218 diffusion—a question that serves as the main focus of our investigation.

219 Specifically, this work will consider  $w : [0, T] \rightarrow \mathbb{R}$ , i.e. guidance strength as a function of time,  
 220 referred to as the guidance schedule. The schedule induces a generative process given by:  
 221

$$\frac{dp_{T-t}}{dt} = \bar{R}_{T-t}^{(w_{T-t})} p_{T-t} \quad (7)$$

224 Understanding which schedules result in the best generation is of crucial importance to further im-  
 225 prove the sample accuracy of discrete diffusion models.  
 226

### 227 3 METHODOLOGY

230 We begin by analyzing the guided process in the simplest case of a single token in Section 3.1,  
 231 which already reveals a key limitation of existing guidance. We then introduce our proposed remedy  
 232 in Section 3.2 via column normalization. Afterwards, we analyze the effect of guidance schedules  
 233 on two tokens in Section 3.4. Finally, we present experimental results of our methods in Section 4.

#### 235 3.1 IDENTIFYING AN ISSUE IN THE GUIDANCE OF DISCRETE DIFFUSION

236 We start by studying guidance in the case where  $d = 1$  where exact analysis is possi-  
 237 ble. The following result characterizes the distribution at time  $t$  under constant guidance:  
 238

239 **Theorem 3.1.** *(Informal) Along the dynamics of equa-  
 240 tion (7), starting from a fully masked state, the distri-  
 241 bution at time  $t$  is given by:*

$$243 p_t = \left( 1 - \left( \frac{1 - e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_T}} \right)^{Z_w} \right) \cdot p^{(w)} \quad (8)$$

246 We present a full proof, as well as a more general result  
 247 for varying guidance schedules in Theorem B.1. This  
 248 shows that for  $d = 1$  the guided process exactly recov-  
 249 ers the tilted distribution, with the unmasking speed  
 250 controlled by the factor in front of  $p^{(w)}$ . Although  
 251 low-dimensional, this result already reveals important  
 252 properties of the guided backwards process.

253 Crucially, the partition function  $Z_w$  appears in the  
 254 exponent of the rate term, meaning that even small  
 255 changes in  $w$  can result in fast changes in the sam-  
 256 pling rate. Figure 2 shows the percentage of tokens  
 257 that remain masked as a function of time  $p_t(M)$  for different values of  $Z_w$ . Applying guidance  
 258 can significantly accelerate unmasking rates. While this can lead to faster generation, it may also  
 259 introduce stiffness (Rathinam et al., 2003) and inefficiencies if not properly controlled.

#### 261 3.2 IMPROVED GUIDANCE MECHANISMS FOR DISCRETE DIFFUSION VIA COLUMN 262 NORMALIZATION

264 In order to alleviate the *unintentional* fast unmasking rates, we propose a simple yet effective change  
 265 to the guidance mechanism. To understand where this issue is coming from, we explicitly write the  
 266 transition rates between a masked state  $M$  a nonmasked state:

267 **Lemma 3.1.** *The transition rates between a masked state and an unnormalized state are given by:*

$$269 \bar{R}_t^{(w)}(y, M) = R_t(x, y) \frac{e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} Z_w p^{(w)}(y)$$



268 Figure 2: We plot the unmasking rates as  
 269 a function of time under guidance. Faster  
 270 unmasking ( $Z_w > 1$ ) leads to worse nu-  
 271 merical solvers, demonstrating an issue in  
 272 the existing guidance mechanism.

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 274 images/Tilted-distribution-w=1-comparison.pdf  
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280 Figure 3: Tilted distributions for varying values of  $w$ . A large  $w$  produces a large concentration on  
 281 one mode.

282  
 283  
 284 Notice how  $Z_w$  appears directly as a multiplying factor in the transition rate. However, when  $w =$   
 285 1, (i.e. the conditional setup) this constant would play no role! This elucidates the effect we observe  
 286 in Figure 2, the **rates are being increased disproportionately** due to the multiplication by the constant.  
 287 To fix this, we must normalize the columns of the transition rate matrix appropriately. In the case of  
 288 masked diffusion this can be achieved in a very simple fashion as follows:

$$289 \quad \bar{R}_{\text{nor},t}^{(w)}(\hat{\mathbf{x}}, \mathbf{x}) = \frac{R_t(\mathbf{x}, \hat{\mathbf{x}}) e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} \text{Softmax}(w \log p_0(\hat{\mathbf{x}}^i | \mathbf{x}^{\text{UM}}) + (1 - w) \log q_0(\hat{\mathbf{x}}^i | \mathbf{x}^{\text{UM}})). \quad (9)$$

290 The new rate matrix is normalized via the softmax function and fixes the issue introduced by the  
 291 guidance mechanism. For the case of other discrete diffusions we refer the reader to Appendix E  
 292 where we present a simple way of performing the normalization in general.

293 The normalization introduced in (9) has the effect of smoothing the transport between the starting  
 294 distribution and the data distribution. This simple change stabilizes the sampling process and allows  
 295 for a cleaner theory. Notably, this change can be done with a simple one line change to the code as  
 296 presented in the pseudocode in 1. We further elaborate on the experimental benefits on Section 4.

### 297 3.3 COMPARISON OF GUIDANCE MECHANISMS

300 We now clarify the distinctions between the various classifier-free guidance mechanisms. While  
 301 some differences between our method and that of Nisonoff et al. (2024) were already discussed,  
 302 we further highlight how our formulation also differs from the approach of Schiff et al. (2024). To  
 303 better understand these differences, we begin by comparing the unlocking guidance mechanism of  
 304 Nisonoff et al. (2024) with the simple guidance proposed by Schiff et al. (2024). For this analysis,  
 305 we keep the guidance strength fixed throughout. Notice that:  $p(x_s | x_t) = \exp\left(\int_s^t \bar{R}_\tau^{(w)} d\tau\right) p_t$ .  
 306 Therefore, if  $p_t$  denotes the law of  $x_t$ , we can write the transition probabilities for each method:

$$307 \quad p_{\text{unlocking}}(x_s | x_t) = \exp\left(\int_s^t \bar{R}_\tau^w(\cdot | c) \bar{R}_\tau^{1-w}(\cdot) d\tau\right) p_t,$$

$$311 \quad p_{\text{simple}}(x_s | x_t) = Z_{\text{simple}} \left( \exp\left(\int_s^t \bar{R}_\tau(\cdot | c) d\tau\right) p_t \right)^w \left( \exp\left(\int_s^t \bar{R}_\tau(\cdot) d\tau\right) p_t \right)^{1-w}.$$

313 where  $Z_{\text{simple}}$  is a normalizing constant. Now we look at the  $w$ -dependence inside the exponential.  
 314 For  $\log p_{\text{unlocking}}$ , the  $w$ -dependence is *exponential* as it appears in the exponent of the rate matrices,  
 315 while for  $\log p_{\text{simple}}$ , the  $w$ -dependence is *linear*. Therefore, the transitions induced by the unlocking  
 316 guidance method get much more aggressive when  $w$  increases. On the other hand, our normalization  
 317 (depending on  $w$ ) normalizes the columns so that it maintains the smoothness of the transition when  
 318  $w$  increases. Therefore, our method approximates the convergence rates of the original process.

### 319 3.4 ANALYSIS OF GUIDANCE SCHEDULES IN 2D

320 Having addressed the existing issue we switch our focus to the analysis of guidance schedules in the  
 321 case of two tokens. Although the analysis can be extended to higher-dimensions, the complexity of  
 322 the problem grows exponentially with the dimension, leading to increasingly intricate expressions

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Figure 5: Evolution of the coefficients in Corollary 3.1 for different values of  $t_2$ . Notice that we must have  $t_1 \leq t_2$ . We observe that for moderate  $t_2$  no coefficient dominates others, resulting in a balanced target distribution.

and reduced interpretability. This low-dimensional analysis already reveals the underlying mechanisms that define a good guidance schedule, and its impacts in high-dimensions are remarkable.

We start by stating our main theorem, in a simple to understand case that is used in practice. This simplification doesn't result in loss of generality, but significantly increases the interpretability of the results. We present a more general version in Theorem C.1.

**Corollary 3.1.** *Consider a time partition  $0 = t_0 < t_1 < t_2 < t_3 = T$  with guidance  $w_i$  in the interval  $[t_i, t_{i+1})$ . With  $\bar{\sigma} = -\log(1 - \delta t)$  and  $p_T(M, M) = 1$ . Then the sampled distribution follows the following formula:*

$$p_{t_0}(i, j) = \left(\frac{t_3 - t_2}{t_3}\right)^2 p^{(w_2)}(i, j) + \left(\frac{t_2 - t_1}{t_3}\right)^2 p^{(w_1)}(i, j) + \left(\frac{t_1 - t_0}{t_3}\right)^2 p^{(w_0)}(i, j) \\ + \frac{(t_3 - t_2)(t_2 - t_1)}{t_3^2} p^{(w_1, w_2)}(i, j) + \frac{(t_3 - t_2)(t_1 - t_0)}{t_3^2} p^{(w_0, w_2)}(i, j) + \frac{(t_2 - t_1)(t_1 - t_0)}{t_3^2} p^{(w_0, w_1)}(i, j),$$

where  $p^{(w, \gamma)}(i, j) = p^{(w)}(i, j | X_1 = i)p^{(\gamma)}(X_1 = i) + p^{(w)}(i, j | X_2 = j)p^{(\gamma)}(X_2 = j)$ , notice that this is not exactly a probability distribution as it is not normalized, but we will refer to it as one.

This theorem states that guidance schedules induce an interpolation of different distributions, which depend only on the guidance strengths and that the portion assigned to each one depends on the time parameters. We analyze the role of each component separately.

images/Combined-distribution-comparison

Figure 4: Notice that when  $\omega < \gamma$  the combined distribution doesn't bias the leftmost mode, making this setting less efficient for guidance.

effective schedules have higher guidance at the final and middle phases of the generation while keeping early guidance small.

**The role of the time parameters:** As observed in corollary 3.1, the time parameters set the proportion of each distribution that will contribute towards the final output. As observed in Figures 3, 4, biasing just one of the distributions usually results in oversampling from a certain area. A good schedule is one that appropriately balances the contribution of each distribution.

We fix several values of  $t_2$  and plot the coefficients as a function of  $t_1$  in Figure 5. When  $t_2 = 1$ , we only have two intervals, and the curves change quickly; this implies that finding the right balance requires more careful tuning. On the other hand when  $t_2 = .75$ , many values of  $t_1$  result in balanced combinations of all distributions, which ensures that we sample in a balanced way.

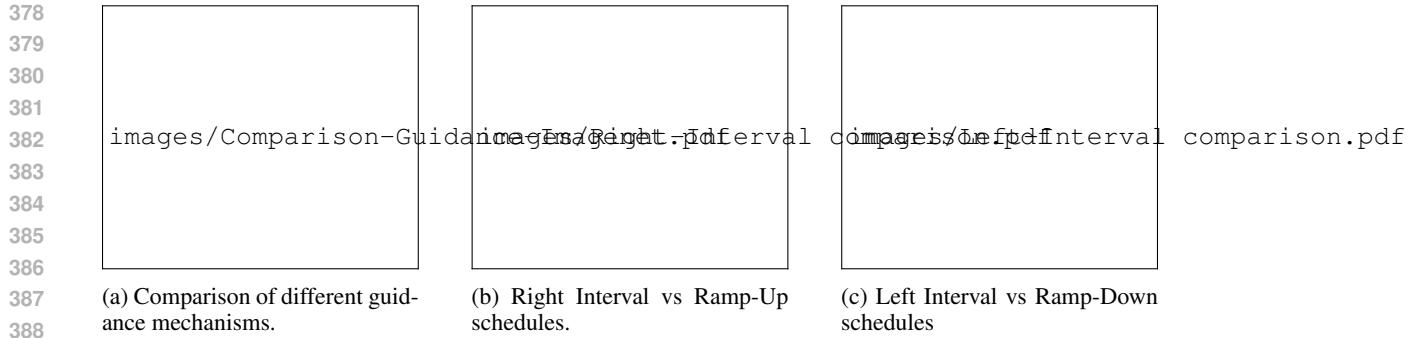


Figure 7: Evaluation of different guidance mechanisms and schedules on Imagenet

**Which schedules perform best?** Our theoretical analysis provides several insights into the design of effective guidance schedules. As discussed earlier, schedules that apply stronger guidance **during the middle and later stages** of the sampling process, while keeping early guidance small, tend to perform better. These selections seem to be the most critical, as they govern which distributions are mixed. Moreover, our theory predicts that using all **three intervals** (early, middle, and late) in the schedule facilitates **easier tuning** and yields more balanced output distributions. Based on these principles, we evaluate (according to our theory) various guidance schedules for discrete diffusion in Table 1, and we validate these predictions empirically in Section 4.2.

Table 1: Comparison of several guidance schedules.

	Low G. Beg	High G. Mid	High G. End	# Params	Tune	Difficulty to Tune
<b>Constant</b>	✗	✓	✓	1		High
<b>Interval</b>	✓	✓	✗	3		Low
<b>Increasing</b>	✓	✓	✓	1		Low
<b>Decreasing</b>	✗	✓	✗	1		Low

## 4 NUMERICAL RESULTS

In this section, we examine whether the theoretical insights from low dimensions extend to high-dimensional image and text domains. On Section 4.1 we study the effect of our normalization and in Section 4.2 the impact of different guidance schedules. We present more details and samples of different methods in Appendix H including experiments with Show-O Xie et al..

### 4.1 EFFECT OF NORMALIZATION

Recall that our theory predicted that failing to normalize complicates the simulation, so normalization should improve results in practice, which we confirm below.

**Testing on Imagenet:** We assess MaskGIT on the ImageNet dataset (Deng et al., 2009) and evaluate FID on ImageNet-256 using 50K samples, following standard practices. For our method and for the Unlocking Guidance baseline (Nisonoff et al., 2024), we use the  $\tau$ -leaping sampler. For Simple Guidance (Schiff et al., 2024), we interpolate Euler transitions. For all methods, we use 50 steps. Figure 7a shows FID as a function of guidance strength using a constant schedule. Our experiments demonstrate that *failing to normalize can substantially degrade sample quality* as suggested by our theory.

**Testing on text-to-image:** We evaluate our method on the GenEval benchmark (Ghosh et al., 2023) using the pre-trained Meissonic model (Bai et al., 2024). This benchmark provides a comprehensive measure of both prompt alignment and perceptual image quality.

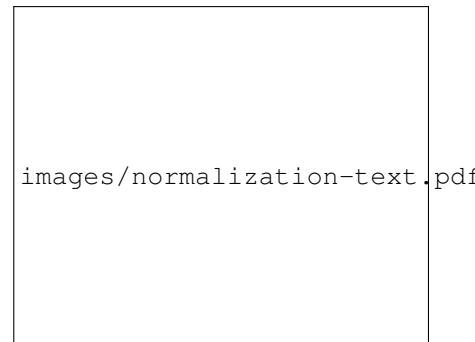


Figure 6: MATH-500 performance for LLada-8B-Instruct under a simple sampler without remasking to isolate the effect of the guidance mechanism. Normalization always yields better results.

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 440 Figure 8: GenEval with and without normalization. Red denotes an improved performance due to  
 441 normalization. Normalization leads to more faithful prompt adherence and image quality.

442 Figure 12 compares generations with and without normalization. Red regions indicate prompts  
 443 where normalization improved the score. Overall, we observe consistent gains: *normalization en-*  
 444 *hances prompt adherence* and yields images that better match the target distribution.

445 **Testing on text generation:** To assess the effectiveness of normalization in the text generation  
 446 domain, we evaluated using LLaDA-8B-Instruct (Nie et al.) on the MATH-500 dataset, generating  
 447 up to 256 tokens. We sample autoregressively in blocks of 32 tokens using a simple Euler sampler  
 448 with 32 denoising steps per block, resulting in a total of 256 steps for the full generation.

449 Figure 6 presents the results of such an experiment. The results clearly show that *normalization*  
 450 *consistently improves performance across all guidance strengths*. We note that the results are not  
 451 directly comparable to those reported in the LLaDA paper; we use a simple Euler sampler without  
 452 remasking to better isolate the effect of guidance and normalization in a simple setting.

453 **Empirical effect of normalization:** All our empirical findings demonstrate that including normal-  
 454 ization is a helpful step in improving the simulation of classifier-free guidance for discrete diffusion.  
 455 This aligns with our low-dimensional theoretical analysis in Section 3.1, demonstrating that *low-*  
 456 *dimensional studies can have a significant impact in high dimensions*.

## 4.2 STUDY OF GUIDANCE SCHEDULES

460 Previously, our theory predicted that increasing  
 461 schedules improve discrete diffusion while decreas-  
 462 ing ones degrade generation. We test this theory on  
 463 Imagenet-256 with 10K samples. For precise for-  
 464 mulas for the schedules, see Table 2. When testing  
 465 increasing schedules (Ramp-Up and Right Interval)  
 466 in 7b, we observe that both schedules can significantly improve the results. Furthermore, the Right  
 467 Interval schedule exhibits a convex trend with respect to  $r$ , while the Ramp-Up schedule is mono-  
 468 tone in  $r$ , and reaches a lower FID value, indicating that a gradual, linear increase in guidance  
 469 outperforms abrupt alternatives. When testing the decreasing schedules (Left interval and Ramp-  
 470 Down), we observe that they consistently damage the generation as seen in Figure 7c. Overall, *our*  
 471 *experiments confirm our theory* that increasing schedules are most effective for masked diffusion.

## 5 CONCLUSIONS

472 In this work, we introduced a framework for analyzing guidance schedules in masked diffusion.  
 473 Our analysis led to a novel approach for classifier-free guidance in the discrete setting. We validate  
 474 the effectiveness of our method through experiments and show that guidance applied near  $t = T$   
 475 is harmful to the generation quality while near  $t = 0$  can improve the it. This insights enabled  
 476 us to identify effective scheduling strategies. Our theoretical insights align closely with empirical  
 477 observations, bridging the gap between theory and practice.

478 **Limitations and Future work.** While our framework provides a principled and tractable approach  
 479 to CFG in discrete diffusion, our theoretical analysis is currently limited to masked diffusion in  
 480 low-dimensional settings. Although the method is applicable to more complex real-world settings,  
 481 our current theoretical study does not cover such regimes. Promising directions include extending  
 482 the framework to other forms of discrete diffusion, such as uniform diffusion, scaling to higher  
 483 dimensions, and analyzing the role of score estimation error in the guidance dynamics.

Table 2: Description of guidance schedules.

Schedule	Formula $w(t)$
Left Interval	$w \cdot \mathbf{1}_{[0,l]}(t)$
Right Interval	$w \cdot \mathbf{1}_{[r,1]}(t)$
Ramp-Up	$\min\left(w, w \cdot \frac{1-t}{1-r}\right)$
Ramp-Down	$\min\left(w, w \cdot \frac{t}{\ell}\right)$

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648 **A NOTATION AND GENERAL RESULTS**  
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650 **A.1 SPECIAL PROPERTIES OF MASKED DIFFUSION**  
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652 We will use the following notations specific to masked diffusion. Let  $\mathbf{x}_t = (\mathbf{x}_t^1, \dots, \mathbf{x}_t^d)$  denote  
 653 a random variable on  $S$ , and  $M$  be the masked token. We will write  $\mathbf{x}^{UM}$  for the set of elements  
 654 such that  $\mathbf{x}_t^i \neq M$ , meaning the entries that are not the masked token. Additionally, we will denote  
 655  $\bar{\sigma}_t = \int_0^t \sigma_s ds$ .

656 Masked diffusion has several appealing properties, one being the following shown by [Ou et al. \(2024\)](#):  
 657

658 **Lemma A.1.** *Along the dynamics (1) given by the masked rate matrix, if  $\mathbf{x}_t = (\mathbf{x}_t^1, \dots, \mathbf{x}_t^d)$  and  
 659  $\hat{\mathbf{x}}_t = (\mathbf{x}_t^1, \dots, \hat{\mathbf{x}}_t^i, \dots, \mathbf{x}_t^d)$  in such a way that  $\hat{\mathbf{x}}_t^i \neq M$  and  $\mathbf{x}_t^i = M$ , we have the following identity  
 660 for the score*

$$662 \frac{p_t(\hat{\mathbf{x}}_t)}{p_t(\mathbf{x}_t)} = \frac{e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} p_0(\hat{\mathbf{x}}_t^i | \mathbf{x}^{UM}).$$

665 This result is of great importance, as it tells us that it is possible to decompose the scores as a  
 666 probability distribution independent of time multiplied by a time-dependent term.

667 **B PROOFS IN 1D**  
 668

669 We first prove a small lemma:

670 **Lemma B.1.** *Given a matrix of the form*

$$673 \quad A = \begin{pmatrix} 0 & \dots & 0 & v_1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & v_n \end{pmatrix}$$

677 If  $v_n \neq 0$ , then its matrix exponential is given by  $e^A = I + A \cdot \frac{e^{v_n} - 1}{v_n}$ .  
 678

679 *Proof.* First notice that for  $k > 0$  it holds that  $A^k = v_n^{k-1} A$  then we can write:  
 680

$$681 \quad e^A = I + A + \frac{1}{2!} A^2 + \frac{1}{3!} A^3 + \dots \\ 682 \quad = I + A + \frac{1}{2!} A v_n + \frac{1}{3!} A v_n^2 + \dots \\ 683 \quad = I + A \left( 1 + \frac{1}{2!} v_n + \frac{1}{3!} v_n^2 + \dots \right) \\ 684 \quad = I + A \left( 1 + \frac{1}{v_n} \left( \frac{1}{2!} v_n^2 + \frac{1}{3!} v_n^3 + \dots \right) \right) \\ 685 \quad = I + A \left( 1 + \frac{1}{v_n} \left( -1 - v_n + 1 + v_n + \frac{1}{2!} v_n^2 + \frac{1}{3!} v_n^3 + \dots \right) \right) \\ 686 \quad = I + A \left( 1 + \frac{1}{v_n} \left( -1 - v_n + e^{v_n} \right) \right) \\ 687 \quad = I + A \frac{e^{v_n} - 1}{v_n}$$

688 As we wanted. □  
 689

690 We now state and prove the general version Theorem 3.1:  
 691

692 **Theorem B.1.** *Along the dynamics of equation (7). The distribution  $p_t$  is given by:*

$$693 \quad p_t = \left( A_1 \cdot \frac{1 - e^A}{A}, \dots, A_{M-1} \cdot \frac{1 - e^A}{A}, e^A \right)^\top.$$

702 Where, for  $i = 0, \dots, M - 1$ :

$$704 \quad A_i = \int_t^T \sigma_s \frac{e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_s}} Z_{w_s} \cdot p^{z, w_s}(i) ds, \quad A = - \sum_{i=0}^{M-1} A_i = \int_t^T \sigma_s \frac{e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_s}} Z_{w_s} ds.$$

707 *Proof.* Recall that the rate matrix in the one-dimensional case is:

$$709 \quad \bar{R}_t^{(w_t)} = \sigma_t \frac{e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} Z_{w_t} \begin{pmatrix} 0 & 0 & \cdots & 0 & p^{(w_t)}(1) \\ 0 & 0 & \cdots & 0 & p^{(w_t)}(2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & p^{(w_t)}(M-1) \\ 0 & 0 & \cdots & 0 & -1 \end{pmatrix} \quad (10)$$

715 By direct integration we know that:

$$717 \quad p_t = \exp \left( \int_t^T \bar{R}_\tau^{(w_\tau)} d\tau \right) p_T.$$

720 Therefore applying Lemma B.1 we get that (in vector notation):

$$722 \quad p_t = p_T + p_T(M) \left( \int_t^T \sigma_s \frac{e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_s}} Z_{w_s} \cdot p^{z, w_s} ds \cdot \frac{1 - e^A}{A} \right),$$

725 with

$$727 \quad A = - \sum_{i=0}^{M-1} A_i = \int_t^T \sigma_s \frac{e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_s}} Z_{w_s} ds.$$

730 The result is proved.  $\square$

731 We can now use the previous theorem to compute the distribution under constant guidance:

732 **Corollary B.1.** *If we start with a distribution  $p_t$  and keep guidance to be constant  $w$ . Then at time  $s$  the distribution is given by:*

$$736 \quad p_s(i) = p_t(i) + p_s(M) \left( \frac{1 - e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_s}} - 1 \right)^{Z_w} p^{(w)}(i)$$

$$739 \quad \text{for } i \neq M \text{ and } p_s(M) = \left( \frac{1 - e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_s}} - 1 \right)^{Z_w} p_t(M)$$

741 *Proof.* The proof follows by keeping  $w$  constant in the above theorem:

$$743 \quad p_s = p_t + p_t(M) \left( \int_s^t \sigma_s \frac{e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_s}} Z_{w_s} \cdot p^{z, w_s} ds \cdot \frac{1 - e^A}{A} \right)$$

$$744 \quad = p_t + p_t(M) \left( \int_t^T \sigma_s \frac{e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_s}} Z \cdot ds \cdot \frac{1 - e^A}{A} p^{(w)} \right)$$

$$745 \quad = p_t + p_t(M) \left( \frac{(1 - e^A) p^{(w)}}{e^A} \right)$$

752 Substituting  $A$  gives the desired result.  $\square$

754 We can now chain the above argument to obtain a result for general piece-wise constant guidance 755 schedules:

**Theorem B.2.** Let  $\delta = t_0 < t_1 < \dots < t_k = T$  be a time partition and let  $w_i$  the guidance strength on the interval  $(t_i, t_{i+1}]$ . Along the dynamics of equation (7), the sampled distribution  $p_\delta$  is given by:

$$p_\delta = p_T + \sum_{i=0}^{k-1} p_{t_{i+1}}(M) \cdot \left( 1 - \left( \frac{1 - e^{-\bar{\sigma}_{t_i}}}{1 - e^{-\bar{\sigma}_{t_{i+1}}}} \right)^{Z_{w_i}} \right) p^{(w_i)}. \quad (11)$$

Additionally, probability mass at  $M$  at different time satisfies  $p_{t_i}(M) = p_{t_{i+1}}(M) \left( \frac{1 - e^{-\bar{\sigma}_{t_i}}}{1 - e^{-\bar{\sigma}_{t_{i+1}}}} \right)^{Z_{w_i}}$  for all  $i = 0, 1, \dots, k-1$ .

**Lemma B.2.** The transition rates between a masked state and an unnormalized state are given by:

$$\bar{R}_t^{(w)}(y, M) = R_t(x, y) \frac{e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} Z_w p^{(w)}(y)$$

*Proof.* Using Lemma A.1 we can write:

$$\begin{aligned} \bar{R}_t^{(w)}(y, M) &= R_t(M, y) \cdot \left( \frac{p_t(x)}{p_t(M)} \right)^w \left( \frac{q_t(x)}{q_t(M)} \right)^{1-w} \\ &= R_t(M, y) \cdot \left( \frac{e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} p_0(y) \right)^w \left( \frac{e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} q_0(y) \right)^{1-w} \\ &= R_t(x, y) \frac{e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} p_0^w(y) q_0^{1-w}(y) \\ &= R_t(x, y) \frac{e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} Z_w p^{(w)}(y) \end{aligned}$$

□

The results for the normalized process are identical to the ones above, so we omit them for brevity.

## C PROOFS IN 2D

We begin by writing a simple lemma that will come in handy later.

**Lemma C.1.** Given a matrix of the form

$$A = \begin{pmatrix} 0 & a & b & 0 \\ 0 & -1 & 0 & c \\ 0 & 0 & -1 & d \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

Then for any  $\alpha \in \mathbb{R}$ , its matrix exponential is given by:

$$\exp(\alpha A) = \begin{pmatrix} 1 & a(1 - e^{-\alpha}) & b(1 - e^{-\alpha}) & \frac{(ac+bd)(e^\alpha - 1)^2 e^{-2\alpha}}{2} \\ 0 & e^{-\alpha} & 0 & c(e^\alpha - 1) e^{-2\alpha} \\ 0 & 0 & e^{-\alpha} & d(e^\alpha - 1) e^{-2\alpha} \\ 0 & 0 & 0 & e^{-2\alpha} \end{pmatrix}$$

*Proof.* The proof of the above statement is easy by noticing that  $A = PDP^{-1}$  with:

$$\begin{aligned} P &= \begin{pmatrix} \frac{ac}{2} + \frac{bd}{2} & -a & -b & 1 \\ -c & 1 & 0 & 0 \\ -d & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \\ D &= \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

Then  $\exp(\alpha A) = P \exp(\alpha D) P^{-1}$  and the result follows. □

Now for the main proof we start by explicitly writing down the rate matrix in the case of two tokens. In this case the rate matrix will have the following structure:

$$\bar{R}_{\text{nor},t}^{(w)} = \frac{\sigma_t e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} \begin{pmatrix} D_1 & \mathbf{0} & \dots & C_1 \\ \mathbf{0} & D_2 & \dots & C_2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & L \end{pmatrix} := \frac{\sigma_t e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_t}} \bar{R}_{\text{nor}}^{(w)},$$

where each block is an  $M \times M$  matrix given by the following formulas:

$$D_i = \begin{pmatrix} 0 & \dots & 0 & p^{(w)}(X_2 = 1 \mid X_1 = i) \\ 0 & \dots & 0 & p^{(w)}(X_2 = 2 \mid X_1 = i) \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & p^{(w)}(X_2 = M-1 \mid X_1 = i) \\ 0 & \dots & 0 & -1 \end{pmatrix}$$

$$C_i = \begin{pmatrix} p^{(w)}(X_1 = i \mid X_2 = 1) & 0 & & \dots & 0 \\ \vdots & \ddots & & \vdots & 0 \\ 0 & \dots & p^{(w)}(X_1 = i \mid X_2 = M-1) & 0 & 0 \\ 0 & \dots & \dots & 0 & p^{(w)}(X_1 = i) \end{pmatrix}$$

$$L = \begin{pmatrix} -1 & 0 & \dots & 0 & p^{(w)}(X_2 = 1) \\ 0 & -1 & \dots & 0 & p^{(w)}(X_2 = 2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & -1 & p^{(w)}(X_2 = M-1) \\ 0 & \dots & 0 & 0 & -2 \end{pmatrix}$$

We can now state the main theorem:

**Theorem C.1.** *Given a starting distribution  $p_t$  following the dynamics given by (7) the distribution at time  $s$  is given by:*

$$p_s(i, j) = \begin{cases} p_t(i, j) + \left(1 - \frac{1 - e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_t}}\right)^2 p^{(w)}(i, j) p_t(M, M) \\ \quad + \left(1 - \frac{1 - e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_t}}\right) \left[ p^{(w)}(X_2 = j \mid X_1 = i) p_t(i, M) \right. \\ \quad \left. + p^{(w)}(X_1 = i \mid X_2 = j) p_t(M, j) \right] & \text{if } i, j \neq M \\ \left( \frac{1 - e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_t}} \right) p_t(i, M) \\ \quad + \left( \frac{1 - e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_t}} \right)^2 \left( \frac{1 - e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_s}} - 1 \right) p^{(w)}(X_1 = i) p_t(M, M) & \text{if } i \neq M, j = M \\ \left( \frac{1 - e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_t}} \right) p_t(M, j) \\ \quad + \left( \frac{1 - e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_t}} \right)^2 \left( \frac{1 - e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_s}} - 1 \right) p^{(w)}(X_2 = j) p_t(M, M) & \text{if } i = M, j \neq M \\ \left( \frac{1 - e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_t}} \right)^2 p_t(M, M) & \text{if } i = j = M \end{cases}$$

*Proof.* By direct integration we know that:

$$p_s = \exp \left( \int_s^t \frac{\sigma_\tau e^{-\bar{\sigma}_\tau}}{1 - e^{-\bar{\sigma}_\tau}} d\tau \bar{R}_{\text{nor}}^{(w)} \right) = \exp \left( \ln \left( \frac{1 - e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_s}} \right) \bar{R}_{\text{nor}}^{(w)} \right).$$

864  
865Due to the block structure of  $\bar{R}_{\text{nor}}^{(w)}$ , it is enough to be able to compute the exponential of:

866

867

$$\begin{array}{c} 868 \\ 869 \\ 870 \\ 871 \\ 872 \\ 873 \\ 874 \\ 875 \\ 876 \\ 877 \end{array} \left( \begin{array}{cc} D_i & C_i \\ \mathbf{0} & L \end{array} \right) = \left[ \begin{array}{cc|ccccc} 0 & \dots & 0 & p^{(w)}(X_2 = 1 \mid X_1 = i) & p^{(w)}(X_1 = i \mid X_2 = 1) & 0 & \dots & 0 \\ 0 & \dots & 0 & p^{(w)}(X_2 = 2 \mid X_1 = i) & \vdots & \ddots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & -1 & 0 & \dots & 0 & p^{(w)}(X_1 = i) \\ \hline 0 & \dots & 0 & 0 & -1 & 0 & \dots & p^{(w)}(X_2 = 1) \\ 0 & \dots & 0 & 0 & 0 & -1 & \dots & p^{(w)}(X_2 = 2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & 0 & \dots & 0 & -2 \end{array} \right]$$

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879

where once again we can exploit the structured form of the matrix to simplify the calculation. It is clear that when computing products of this matrix, coordinates will only get affected by the smaller subblocks:

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884

$$\begin{array}{c} 885 \\ 886 \\ 887 \\ 888 \end{array} \left( \begin{array}{cc|cc} 0 & p^{(w)}(X_2 = j \mid X_1 = i) & p^{(w)}(X_1 = i \mid X_2 = j) & 0 \\ 0 & -1 & 0 & p^{(w)}(X_1 = i) \\ \hline 0 & 0 & -1 & p^{(w)}(X_2 = j) \\ 0 & 0 & 0 & -2 \end{array} \right)$$

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890

This is not only clear from the structure, but it also reveals a true intuitive understanding. The probability mass at a given position can only be affected by those states that are reachable from the current state by masking or unmasking the entries. We can now use Lemma C.1 to find the exponential:

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896

$$\begin{array}{c} 897 \\ 898 \\ 899 \\ 900 \end{array} \left( \begin{array}{cc|cc} 1 & p^{(w)}(X_2 = j \mid X_1 = i)(1 - e^{-\alpha}) & p^{(w)}(X_1 = i \mid X_2 = j)(1 - e^{-\alpha}) & p^{(w)}(i, j)(e^\alpha - 1)^2 e^{-2\alpha} \\ 0 & e^{-\alpha} & 0 & p^{(w)}(X_1 = i)(e^\alpha - 1) e^{-2\alpha} \\ 0 & 0 & e^{-\alpha} & p^{(w)}(X_2 = j)(e^\alpha - 1) e^{-2\alpha} \\ 0 & 0 & 0 & e^{-2\alpha} \end{array} \right)$$

901

902

903

where  $\alpha = \ln \left( \frac{1 - e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_s}} \right)$  and we used that  $2p^{(w)}(i, j) = p^{(w)}(X_2 = j \mid X_1 = i)p^{(w)}(X_1 = i) + p^{(w)}(X_1 = i \mid X_2 = j)p^{(w)}(X_2 = j)$ . Putting this together, we get that exponentiation each block we get:

907

908

909

$$\begin{array}{c} 910 \\ 911 \\ 912 \\ 913 \\ 914 \\ 915 \\ 916 \\ 917 \end{array} \left[ \begin{array}{cc|ccccc} 1 & \dots & 0 & p^{(w)}(X_2 = 1 \mid X_1 = i)(1 - e^{-\alpha}) & p^{(w)}(X_1 = i \mid X_2 = 1)(1 - e^{-\alpha}) & \dots & p^{(w)}(i, 1)(e^\alpha - 1)^2 e^{-2\alpha} \\ 0 & \dots & 0 & p^{(w)}(X_2 = 2 \mid X_1 = i)(1 - e^{-\alpha}) & \vdots & \ddots & p^{(w)}(i, 1)(e^\alpha - 1)^2 e^{-2\alpha} \\ \vdots & \vdots & \vdots & \vdots & 0 & & \vdots \\ 0 & \dots & 0 & e^{-\alpha} & 0 & \dots & p^{(w)}(X_1 = i)(1 - e^\alpha) e^{-2\alpha} \\ 0 & \dots & 0 & 0 & e^{-\alpha} & \dots & p^{(w)}(X_2 = 1)(1 - e^\alpha) e^{-2\alpha} \\ 0 & \dots & 0 & 0 & 0 & e^{-\alpha} & p^{(w)}(X_2 = 2)(1 - e^\alpha) e^{-2\alpha} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & 0 & \dots & e^{-2\alpha} \end{array} \right]$$

With this, we have a full characterization of the matrix exponential. Therefore, we can simply write down the probability distribution by multiplying by  $p_t$ :

$$p_s(i, j) = \begin{cases} p_t(i, j) + (1 - e^{-\alpha})^2 p^{(w)}(i, j) p_t(M, M) \\ \quad + (1 - e^{-\alpha}) [p^{(w)}(X_2 = j \mid X_1 = i) p_t(i, M) \\ \quad + p^{(w)}(X_1 = i \mid X_2 = j) p_t(M, j)] & \text{if } i, j \neq M \\ e^{-\alpha} p_t(i, M) \\ \quad + e^{-2\alpha} (e^\alpha - 1) p^{(w)}(X_1 = i) p_t(M, M) & \text{if } i \neq M, j = M \\ e^{-\alpha} p_t(M, j) \\ \quad + e^{-2\alpha} (e^\alpha - 1) p^{(w)}(X_2 = j) p_t(M, M) & \text{if } i = M, j \neq M \\ e^{-2\alpha} p_t(M, M) & \text{if } i = j = M \end{cases}$$

We can now replace  $\alpha = \ln \left( \frac{1 - e^{-\bar{\sigma}_t}}{1 - e^{-\bar{\sigma}_s}} \right)$  into the formula above to obtain the result.  $\square$

**Corollary C.1.** *Given a starting distribution  $p_t$  following the dynamics given by (7) with  $\bar{\sigma}_t = -\log(1 - \delta t)$  the distribution at time  $s$  is given by:*

$$p_s(i, j) = \begin{cases} p_t(i, j) + \left( \frac{t-s}{t} \right)^2 p^{(w)}(i, j) p_t(M, M) \\ \quad + \left( \frac{t-s}{t} \right) [p^{(w)}(X_2 = j \mid X_1 = i) p_t(i, M) \\ \quad + p^{(w)}(X_1 = i \mid X_2 = j) p_t(M, j)] & \text{if } i, j \neq M \\ \frac{s}{t} \cdot p_t(i, M) + \left( \frac{s}{t} \right)^2 \left( \frac{t-s}{s} \right) p^{(w)}(X_1 = i) p_t(M, M) & \text{if } i \neq M, j = M \\ \frac{s}{t} \cdot p_t(M, j) + \left( \frac{s}{t} \right)^2 \left( \frac{t-s}{s} \right) p^{(w)}(X_2 = j) p_t(M, M) & \text{if } i = M, j \neq M \\ \left( \frac{s}{t} \right)^2 p_t(M, M) & \text{if } i = j = M \end{cases}$$

*Proof.* Notice that under this schedule we have that:

$$\frac{1 - e^{-\bar{\sigma}_s}}{1 - e^{-\bar{\sigma}_t}} = \frac{\delta s}{\delta t} = \frac{s}{t}$$

Substituting this in gives the corollary above.  $\square$

*Proof of Corollary 3.1.* We track the changes in the distribution in every time interval. This can be found by plugging in the result of the corollary above. Firstly, on the interval  $T \rightarrow t_2$  we obtain:

$$p_{t_2}(M, M) = \left( \frac{t_2}{T} \right)^2$$

$$\begin{aligned}
972 \quad p_{t_2}(M, j) &= \left(\frac{t_2}{T}\right)^2 \left(\frac{T-t_2}{t_2}\right) p^{(w_2)}(X_2 = j) \\
973 \quad p_{t_2}(i, M) &= \left(\frac{t_2}{T}\right)^2 \left(\frac{T-t_2}{t_2}\right) p^{(w_2)}(X_1 = i) \\
974 \quad p_{t_2}(i, j) &= \left(\frac{T-t_2}{T}\right)^2 p^{(w_2)}(i, j)
\end{aligned}$$

975 Then on the interval from  $t_2 \rightarrow t_1$  we get:

$$\begin{aligned}
976 \quad p_{t_1}(M, M) &= \left(\frac{t_1}{T}\right)^2 \\
977 \quad p_{t_1}(M, j) &= \left(\frac{t_1}{t_2}\right) p_{t_2}(M, j) + \left(\frac{t_1}{t_2}\right)^2 \left(\frac{t_2-t_1}{t_1}\right) p^{(w_1)}(X_2 = j) p_{t_2}(M, M) \\
978 \quad p_{t_1}(i, M) &= \left(\frac{t_1}{t_2}\right) p_{t_2}(i, M) + \left(\frac{t_1}{t_2}\right)^2 \left(\frac{t_2-t_1}{t_1}\right) p^{(w_1)}(X_1 = i) p_{t_2}(M, M) \\
979 \quad p_{t_1}(i, j) &= p_{t_2}(i, j) + \left(\frac{t_2-t_1}{t_2}\right)^2 p^{(w_1)}(i, j) p_{t_2}(M, M) \\
980 \quad &+ \left(\frac{t_2-t_1}{t_2}\right) [p^{(w_1)}(X_2 = j | X_1 = i) p_{t_2}(i, M) + p^{(w_1)}(X_1 = i | X_2 = j) p_{t_2}(M, j)]
\end{aligned}$$

981 Replacing the values for  $p_{t_1}$  into this equation we get:

$$\begin{aligned}
982 \quad p_{t_1}(M, M) &= \left(\frac{t_1}{T}\right)^2 \\
983 \quad p_{t_1}(M, j) &= \left(\frac{t_1(T-t_2)}{T^2}\right) p^{(w_2)}(X_2 = j) + \left(\frac{t_1}{T}\right)^2 \left(\frac{t_2-t_1}{t_1}\right) p^{(w_2)}(X_2 = j) \\
984 \quad p_{t_1}(i, M) &= \left(\frac{t_1(T-t_2)}{T^2}\right) p^{(w_2)}(X_1 = i) + \left(\frac{t_1}{T}\right)^2 \left(\frac{t_2-t_1}{t_1}\right) p^{(w_2)}(X_1 = i) \\
985 \quad p_{t_1}(i, j) &= \left(\frac{T-t_2}{T}\right)^2 p^{(w_2)}(i, j) + \left(\frac{t_2-t_1}{t_2}\right)^2 \left(\frac{t_2}{T}\right)^2 p^{(w_1)}(i, j) \\
986 \quad &+ \left(\frac{t_2-t_1}{t_2}\right) \left(\frac{t_2}{T}\right)^2 \left(\frac{T-t_2}{t_2}\right) p^{(w_1)}(X_2 = j | X_1 = i) p^{(w_2)}(X_1 = i) \\
987 \quad &+ \left(\frac{t_2-t_1}{t_2}\right) \left(\frac{t_2}{T}\right)^2 \left(\frac{T-t_2}{t_2}\right) p^{(w_1)}(X_1 = i | X_2 = j) p^{(w_2)}(X_2 = j) \\
988 \quad &= \left(\frac{T-t_2}{T}\right)^2 p^{(w_2)}(i, j) + \left(\frac{t_2-t_1}{T}\right)^2 p^{(w_1)}(i, j) + \frac{(t_2-t_1)(T-t_2)}{T^2} p^{(w_1, w_2)}
\end{aligned}$$

989 Finally, we can proceed with the final step from  $t_1 \rightarrow t_0$ . In this case, we have:

$$\begin{aligned}
990 \quad p_{t_0}(M, M) &= 0 \\
991 \quad p_{t_0}(M, j) &= 0 \\
992 \quad p_{t_0}(i, M) &= 0 \\
993 \quad p_{t_0}(i, j) &= p_{t_1}(i, j) + \left(\frac{t_1-t_0}{t_1}\right)^2 p^{(w_0)}(i, j) p_{t_1}(M, M) \\
994 \quad &+ \left(\frac{t_1-t_0}{t_1}\right) [p^{(w_0)}(X_2 = j | X_1 = i) p_{t_1}(i, M) + p^{(w_0)}(X_1 = i | X_2 = j) p_{t_1}(M, j)]
\end{aligned}$$

995 Then substituting in the previous results:

$$996 \quad p_{t_0}(i, j) = p_{t_1}(i, j) + \left(\frac{t_1-t_0}{t_1}\right)^2 \left(\frac{t_1}{T}\right)^2 p^{(w_0)}(i, j)$$

$$\begin{aligned}
& + \left( \frac{t_1 - t_0}{t_1} \right) \left[ p^{(w_0)}(X_2 = j | X_1 = i) \right. \\
& \quad \left( \frac{t_1(T - t_2)}{T^2} p^{(w_2)}(X_1 = i) + \left( \frac{t_1}{T} \right)^2 \left( \frac{t_2 - t_1}{t_1} \right) p^{(w_2)}(X_1 = i) \right) \\
& \quad + p^{(w_0)}(X_1 = i | X_2 = j) \\
& \quad \left. \left( \frac{t_1(T - t_2)}{T^2} p^{(w_2)}(X_2 = j) + \left( \frac{t_1}{T} \right)^2 \left( \frac{t_2 - t_1}{t_1} \right) p^{(w_2)}(X_2 = j) \right) \right]
\end{aligned}$$

Grouping by coefficient we get:

$$\begin{aligned}
p_{t_0}(i, j) &= p_{t_1}(i, j) + \left( \frac{t_1 - t_0}{t_1} \right)^2 \left( \frac{t_1}{T} \right)^2 p^{(w_0)}(i, j) \\
&\quad + \left( \frac{t_1 - t_0}{t_1} \right) \cdot \\
&\quad \left[ \left( \frac{t_1(T - t_2)}{T^2} \right) [p^{(w_0)}(X_2 = j | X_1 = i) p^{(w_2)}(X_1 = i) + p^{(w_0)}(X_1 = i | X_2 = j) p^{(w_2)}(X_2 = j)] \right. \\
&\quad \left. + \left( \frac{t_1}{T} \right)^2 \left( \frac{t_2 - t_1}{t_1} \right) [p^{(w_0)}(X_2 = j | X_1 = i) p^{(w_1)}(X_1 = i) + p^{(w_0)}(X_1 = i | X_2 = j) p^{(w_1)}(X_2 = j)] \right] \\
&= p_{t_1}(i, j) + \left( \frac{t_1 - t_0}{t_1} \right)^2 \left( \frac{t_1}{T} \right)^2 p^{(w_0)}(i, j) \\
&\quad + \left( \frac{t_1 - t_0}{t_1} \right) \cdot \left[ \frac{t_1(T - t_2)}{T^2} p^{(w_0, w_2)} + \left( \frac{t_1}{T} \right)^2 \left( \frac{t_2 - t_1}{t_1} \right) p^{(w_0, w_1)} \right]
\end{aligned}$$

Simplifying and substituting the term of  $p_{t_1}$  this becomes:

$$\begin{aligned}
p_{t_0}(i, j) &= \left( \frac{t_3 - t_2}{t_3} \right)^2 p^{(w_2)}(i, j) + \left( \frac{t_2 - t_1}{t_3} \right)^2 p^{(w_1)}(i, j) + \left( \frac{t_1 - t_0}{t_3} \right)^2 p^{(w_0)}(i, j) \\
&\quad + \frac{(t_3 - t_2)(t_2 - t_1)}{t_3^2} p^{(w_1, w_2)}(i, j) + \frac{(t_3 - t_2)(t_1 - t_0)}{t_3^2} p^{(w_0, w_2)}(i, j) \\
&\quad + \frac{(t_2 - t_1)(t_1 - t_0)}{t_3^2} p^{(w_0, w_1)}(i, j).
\end{aligned}$$

□

## D DETAILS ON TOY EXAMPLE

We now present the details of the toy example that we used to demonstrate our theoretical results. In figure 9 we present plots of each class and the full data distribution. Each cluster is defined via the following matrix

$$\begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.4 & 0.6 & 0.7 & 0.8 & 0.7 & 0.6 & 0.4 & 0.2 \\ 0.3 & 0.6 & 0.8 & 0.9 & 1.0 & 0.9 & 0.8 & 0.6 & 0.3 \\ 0.4 & 0.7 & 0.9 & 1.0 & 1.0 & 1.0 & 0.9 & 0.7 & 0.4 \\ 0.5 & 0.8 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.8 & 0.5 \\ 0.4 & 0.7 & 0.9 & 1.0 & 1.0 & 1.0 & 0.9 & 0.7 & 0.4 \\ 0.3 & 0.6 & 0.8 & 0.9 & 1.0 & 0.9 & 0.8 & 0.6 & 0.3 \\ 0.2 & 0.4 & 0.6 & 0.7 & 0.8 & 0.7 & 0.6 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.4 & 0.3 & 0.2 & 0.1 \end{bmatrix}$$

Each class is equally weighted. A pseudocode for generating the above dataset is:

```

1080
1081
1082
1083
1084 images/toy_problem_schedules.png
1085
1086
1087
1088
1089

```

Figure 9: Definitions of the class and unconditional distributions for the toy problem.

```

1090
1091
1092
1093     height, width = 30, 30
1094
1095     matrix1 = torch.zeros((height, width))
1096     matrix1[1:10, 1:10] = torch.tensor(cluster)
1097     matrix1[9:18, 9:18] = torch.tensor(cluster)
1098
1099     matrix2 = torch.zeros((height, width))
1100     matrix2[11:20, 11:20] = torch.tensor(cluster)
1101     matrix2[19:28, 19:28] = torch.tensor(cluster)
1102
1103
1104
1105
1106
1107
1108
1109
1110
1111
1112
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1133

```

Listing 3: Code to generate our toy dataset

1134 E NORMALIZATION FOR GENERAL DIFFUSION PROCESSES  
1135

1136 In this section we demonstrate how it is possible to extend our normalization to the general class  
1137 of diffusion processes. We propose a simple but effective per-column normalization that applies to  
1138 different discrete diffusion models (masked/uniform).

$$\begin{aligned}
 & \left[ \begin{array}{c} \frac{p_t^w(1)q_t^{1-w}(1)}{p_t^w(x)q_t^{1-w}(x)} \\ \frac{p_t^w(2)q_t^{1-w}(2)}{p_t^w(x)q_t^{1-w}(x)} \\ \vdots \\ -\sum_{i \neq x} \frac{p_t^w(i)q_t^{1-w}(i)}{p_t^w(x)q_t^{1-w}(x)} \\ \vdots \\ \frac{p_t^w(M)p_t^{1-w}(M)}{p_t^w(x)p_t^{1-w}(x)} \end{array} \right] \xrightarrow{\text{Normalized vector}} \underbrace{\frac{\left( \sum_{i \neq x} p_t(i) \right)^w \left( \sum_{i \neq x} q_t(i) \right)^{1-w}}{\sum_{i \neq x} p_t(i)^w q_t(i)^{1-w}}}_{\text{Normalized vector}} \cdot \left[ \begin{array}{c} \frac{p_t^w(1)q_t^{1-w}(1)}{p_t^w(x)q_t^{1-w}(x)} \\ \frac{p_t^w(2)q_t^{1-w}(2)}{p_t^w(x)q_t^{1-w}(x)} \\ \vdots \\ -\sum_{i \neq x} \frac{p_t^w(i)q_t^{1-w}(i)}{p_t^w(x)q_t^{1-w}(x)} \\ \vdots \\ \frac{p_t^w(M)p_t^{1-w}(M)}{p_t^w(x)p_t^{1-w}(x)} \end{array} \right].
 \end{aligned} \tag{12}$$

1152 As seen in (12), our normalization applies to general diffusion models as long as we have access  
1153 to the scores for models of  $p, q$ . For the multiple-token case ( $d > 1$ ), due to the fact that we only use  
1154 one column vector that corresponds to a single-dimension jump every time, the normalization for  
1155 one-token in (12) can be applied to that column vector.

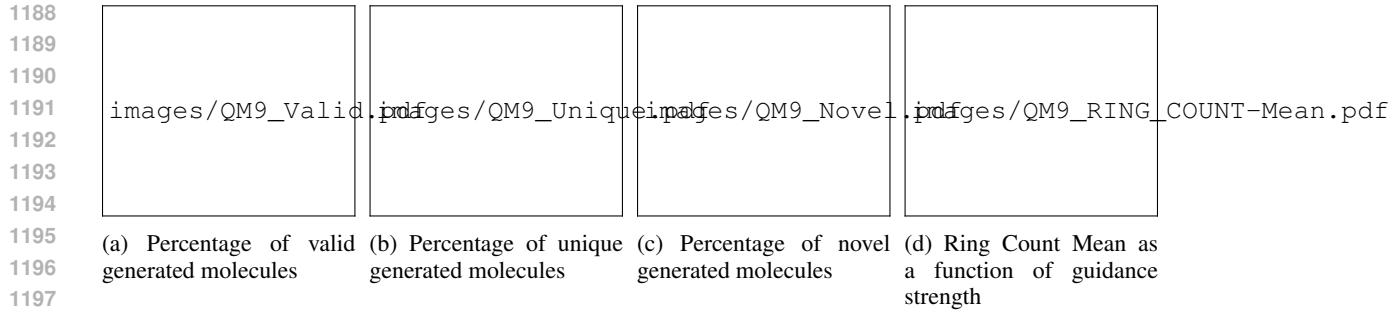
```

1157 def get_normalized_rate(
1158     x, c, t, dt, w
1159     ):
1160     # Get scores
1161     log_score_c = get_score(x, t, cond=c)
1162     log_score_u = get_score(x, t, cond=None)
1163     log_score_w = w * log_score_c + (1-w) * log_score_u
1164
1165     score_c = log_score_c.exp()
1166     score_u = log_score_u.exp()
1167     score_w = log_score_w.exp()
1168
1169     # Set diagonal terms
1170     score_c.scatter_(-1, x[..., None], torch.zeros_like(score_c))
1171     score_u.scatter_(-1, x[..., None], torch.zeros_like(score_u))
1172     score_w.scatter_(-1, x[..., None], torch.zeros_like(score_w))
1173
1174     normalized_rate = edge * score_w
1175     normalized_rate.scatter_(-1, x[..., None], -normalized_rate.sum(dim=-1, keepdim=True))
1176
1177     # Normalize appropriately
1178     sum_c = score_c.sum(-1, keepdim=True) ** w
1179     sum_u = score_u.sum(-1, keepdim=True)
1180     sum_u = torch.where(sum_u > 0, sum_u**(1-w), 0)
1181     sum_w = score_w.sum(-1, keepdim=True)
1182     normalized_rate = (sum_c * sum_u / sum_w) * normalized_rate
1183
1184     return sample(delta(x) + dt * sigma(t) * normalized_rate)

```

1177 Listing 4: Our guidance in the general case using Euler transitions  
11781179 E.1 RESULTS ON QM9  
1180

1181 We present results using our guidance mechanism in the context of uniform diffusion, applied to  
1182 the QM9 small molecule dataset (Ruddigkeit et al., 2012; Ramakrishnan et al., 2014). QM9 is a  
1183 dataset containing small organic molecules containing up to 9 heavy atoms. We train a conditional  
1184 model on QM9 using uniform diffusion, based on the official implementation of Schiff et al. (2024),  
1185 without modifying the architecture or hyperparameters. The model is conditioned on the number  
1186 of rings in each molecule (ring count). Unlike ImageNet, evaluation on QM9 is more nuanced: we  
1187 generate 1,024 samples and assess several metrics. First, generated molecules must satisfy chemical  
1188 constraints to be considered valid. Second, a key goal of generative models is to produce novel



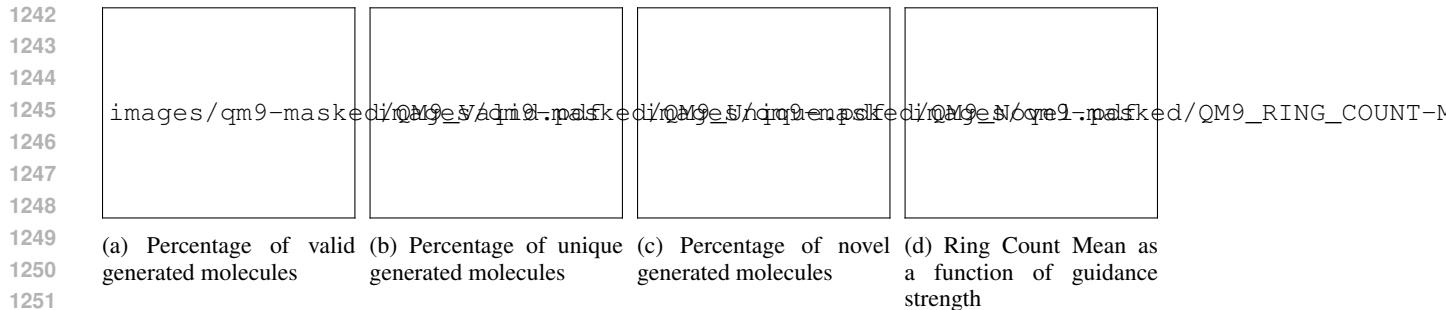


Figure 11: We display the percentage of valid, unique, and novel molecules. We find that our method is the most robust to an increase in guidance strength.

## F EXTRA EXPERIMENTS ON QM9 FOR MASKED DIFFUSION

We present similar results using our guidance mechanism but in the context of masked diffusion, applied to the QM9 small molecule dataset (Ruddigkeit et al., 2012; Ramakrishnan et al., 2014). We train a conditional model on QM9 using uniform diffusion, based on the official implementation of Schiff et al. (2024), without modifying the architecture or hyperparameters. The model is conditioned on the number of rings in each molecule (ring count). We report statistics under the same setting as in Appendix E.

We plot the results in Figure 11, we find that our method is the most robust to increases in guidance strength and generally achieves better results across various guidance strengths. Suggesting that normalization is also a helpful technique on this domain.

## G ANALYSIS OF DIVERSITY TRADEOFFS

We present a simple study of the diversity tradeoff when using different guidance mechanisms for discrete diffusion. We leverage the Imagenet dataset as in the main paper, using the same set of hyperparameters. We analyze the precision-recall Kynkänniemi et al. (2019) and report the results in Table 3. Precision measures the proportion of generated images that lie close to the real data manifold (fidelity), while recall quantifies the coverage of the real data distribution (diversity), allowing us to understand this correctly.

As guidance strength increases from  $w=1$  to  $w=5$ , all methods exhibit stable recall, ranging from 0.72 to 0.79. The precision statistic is more revealing: for Unlocking and Simple guidance, using  $w \geq 1$  always results in degraded precision (worse fidelity) while our method is capable of improving on it (better fidelity). Putting this together, all methods show similar diversity across guidance strengths. However, only our method can improve the fidelity while maintaining diverse samples.

## H GENERATED SAMPLES TEXT TO IMAGE

To generate these samples we made use of a single node with 8 NVIDIA A100 GPUs. We present samples to compare our method against other guidance methods as well as the detailed results of the GenEval benchmark in Table 4 and 5. The results demonstrate that normalization is key in order to improve the sample quality.

Table 3: Performance across guidance strengths  $w$ . Each cell shows *Precision/Recall*.

Method / Strength	$w=1$ (no guidance)	$w=2$	$w=3$	$w=4$	$w=5$
Our Method	0.48 / 0.73	0.51 / 0.75	0.51 / 0.75	0.52 / 0.76	0.18 / 0.72
Unlocking Guidance	0.48 / 0.73	0.43 / 0.79	0.29 / 0.77	0.19 / 0.77	0.12 / 0.76
Simple Guidance	0.46 / 0.72	0.45 / 0.79	0.34 / 0.76	0.24 / 0.74	0.19 / 0.72

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1299 images/clean\_horizontal\_difference\_heatmap\_showo.pdf

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Figure 12: GenEval with and without normalization using Show-o as a base model. Red denotes an improved performance due to normalization. Normalization leads to more faithful prompt adherence and image quality.

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Table 4: Performance comparison across different guidance weights using Meissonic as a base model

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Metric	Ours				Unlocking			
	$w = 1$	$w = 3$	$w = 6$	$w = 9$	$w = 1$	$w = 3$	$w = 6$	$w = 9$
Overall	8.2	40.8	45.9	44.7	8.2	43.1	28.5	19.9
Objects Single	23.8	89.4	91.9	91.2	23.8	88.4	80.0	64.1
Objects Two	3.0	36.9	48.2	48.7	3.0	47.5	23.2	18.2
Counting	0.6	27.2	33.8	28.4	0.6	33.1	11.9	3.1
Position	20.7	72.3	77.4	77.7	20.7	72.3	48.7	29.0
Color Attribution	0.2	8.5	7.8	7.8	0.2	6.2	7.8	3.8
Colors	1.0	10.5	16.5	14.5	1.0	10.8	2.8	1.0

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Table 5: Performance comparison across different guidance weights using Show-o as a base model

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Metric	Ours				Unlocking			
	$w = 2$	$w = 4$	$w = 6$	$w = 8$	$w = 2$	$w = 4$	$w = 6$	$w = 8$
Overall	56.42	62.46	63.13	63.39	53.73	52.84	52.89	43.96
Objects Single	96.88	98.75	99.06	98.75	95.94	98.44	97.19	86.88
Objects Two	65.66	75.76	78.28	80.05	64.14	61.36	60.61	60.10
Counting	41.56	50.00	50.94	50.94	39.69	35.00	35.00	31.25
Position	78.19	81.38	79.79	81.12	73.14	75.00	76.60	51.33
Color Attribution	22.75	28.5	30.25	27.25	26.25	22.75	22.00	20.25
Colors	33.5	41.5	40.5	42.25	23.25	24.50	26.00	14.00

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## H.1 GENERATED SAMPLES FROM MEISSONIC

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We now present some samples from our method:

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1358 images/meissonic/comparison_00000.jpg
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1375 images/meissonic/comparison_00001.jpg
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Figure 13: Comparison of samples generated by different guidance methods across various seeds and configurations. Using the prompts: A photo of a bench (top), A photo of a cow (bottom).

## H.2 GENERATED SAMPLES FROM SHOW-O

We now present some samples from our method using Show-o [Xie et al.](#):

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1412 images/meissonic/comparison_00002.jpg  
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1429 images/meissonic/comparison_00003.jpg  
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Figure 14: Comparison of samples generated by different guidance methods across various seeds and configurations. Using the prompts: A photo of a bike (top), A photo of a clock (bottom).

## I GENERATED SAMPLES IMAGENET

### I.1 GUIDANCE STRENGTH $w = 2$

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1466 images/meissonic/comparison_00004.jpg  
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1483 images/meissonic/comparison_00005.jpg  
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```

Figure 15: Comparison of samples generated by different guidance methods across various seeds and configurations. Using the prompts: A photo of a carrot (top), A photo of a suitcase (bottom).

## I.2 GUIDANCE STRENGTH $w = 3$

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1498  
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1520 images/meissonic/comparison_00006.jpg  
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1537 images/meissonic/comparison_00007.jpg  
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Figure 16: Comparison of samples generated by different guidance methods across various seeds and configurations. Using the prompts: A photo of a fork (top), A photo of a surfboard (bottom).

### I.3 GUIDANCE STRENGTH $w = 4$

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1574 images/meissonic/comparison_00008.jpg  
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1591 images/meissonic/comparison_00009.jpg  
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Figure 17: Comparison of samples generated by different guidance methods across various seeds and configurations. Using the prompt: A photo of a refrigerator (top), A photo of a cup (bottom)

#### 1604 I.4 GUIDANCE STRENGTH $w = 5$

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1628 images/showo/comparison_00027.jpg
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1645 images/showo/comparison_00028.jpg
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1655 Figure 18: Comparison of samples generated by different guidance methods across various seeds  
1656 and configurations. Using the prompts: A photo of a dog (top), A photo of a tie (bottom).  
1657

1658 **STATEMENT ON THE USE OF LARGE LANGUAGE MODELS**

1660  
1661 This work made use of large language models to assist with proofreading and improving the clarity  
1662 of the writing. All research ideas, theoretical development, and experiments were carried out solely  
1663 by the authors. When used for coding, it was solely used for plotting purposes.

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images/showo/comparison\_00029.jpg

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images/showo/comparison\_00030.jpg

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Figure 19: Comparison of samples generated by different guidance methods across various seeds and configurations. Using the prompts: A photo of a laptop (top), A photo of a computer mouse (bottom).

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1761 images/showo/comparison\_00032.jpg

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1771 Figure 20: Comparison of samples generated by different guidance methods across various seeds  
1772 and configurations. Using the prompts: A photo of a sandwich (top), A photo of a baseball bat  
1773 (bottom).

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1799 images/showo/comparison\_00033.jpg

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1816 images/showo/comparison\_00034.jpg

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1826 Figure 21: Comparison of samples generated by different guidance methods across various seeds  
1827 and configurations. Using the prompts: A photo of a train (top), A photo of a cell phone (bottom).

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1853 images/showo/comparison\_00035.jpg

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1870 images/showo/comparison\_00036.jpg

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1880 Figure 22: Comparison of samples generated by different guidance methods across various seeds  
1881 and configurations. Using the prompt: A photo of a chair (top), A photo of a tv (bottom)

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1899 images/guidance_1/comparison_1.png
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1910 images/guidance_1/comparison_20.png
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1921 images/guidance_1/comparison_2.png
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1932 images/guidance_1/comparison_42.png
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1939 Figure 23: Comparison of samples generated by different guidance methods across various seeds or
1940 configurations.
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images/guidance\_2/comparison\_31.png

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images/guidance\_2/comparison\_32.png

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images/guidance\_2/comparison\_33.png

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images/guidance\_2/comparison\_34.png

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Figure 24: Comparison of samples generated by different guidance methods across various seeds or configurations.

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images/guidance\_3/comparison\_0.png

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images/guidance\_3/comparison\_38.png

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images/guidance\_3/comparison\_39.png

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images/guidance\_3/comparison\_40.png

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Figure 25: Comparison of samples generated by different guidance methods across various seeds or configurations.

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2061 images/guidance\_4/comparison\_43.png

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2072 images/guidance\_4/comparison\_44.png

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2083 images/guidance\_4/comparison\_45.png

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2094 images/guidance\_4/comparison\_54.png

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2101 Figure 26: Comparison of samples generated by different guidance methods across various seeds or  
2102 configurations.

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