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Anonymous authors

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ABSTRACT

Multi-modal entity alignment (MMEA) aims to identify equivalent entities across heterogeneous multi-modal knowledge graphs (MMKGs), where each entity is described by attributes from various modalities. Existing methods typically assume that both intra-entity and inter-graph correspondences are faultless, which is often violated in real-world MMKGs due to the reliance on expert annotations. In this paper, we reveal and study a highly practical yet under-explored problem in MMEA, termed Dual-level Noisy Correspondence (DNC). DNC refers to misalignments in both intra-entity (entity-attribute) and inter-graph (entity-entity and attribute-attribute) correspondences. To address the DNC problem, we propose a robust MMEA framework termed RULE. RULE first estimates the reliability of both intra-entity and inter-graph correspondences via a dedicated two-fold principle. Leveraging the estimated reliabilities, RULE mitigates the negative impact of intra-entity noise during attribute fusion and prevents overfitting to noisy inter-graph correspondences during inter-graph discrepancy elimination. Beyond the training-time designs, RULE further incorporates a correspondence reasoning module that uncovers the underlying attribute-attribute connection across graphs, guaranteeing more accurate equivalent entity identification. Extensive experiments on five benchmarks verify the effectiveness of our method against the DNC compared with seven state-of-the-art methods. The code will be released upon acceptance.

1 INTRODUCTION

Multi-Modal Entity Alignment (Liu et al., 2021; Li et al., 2023) (MMEA) aims to identify equivalent entities across different Multi-modal Knowledge Graphs (MMKGs) (Liu et al., 2019; Zhu et al., 2022), where each entity is associated with attributes of various modalities (*e.g.*, structural triples and images). Due to the heterogeneity of attributes from different modalities and graphs from different sources (*e.g.*, Wikidata (Vrandečić & Krötzsch, 2014) and YAGO (Suchanek et al., 2007)), the key challenge of MMEA is to learn a comprehensive representation for each entity with its respective attributes while eliminating the cross-graph discrepancy. To this end, existing methods usually conduct multi-modal fusion for attributes within the same entity based on the intra-entity correspondences (*i.e.*, entity-attribute pairs), while performing cross-graph alignment by resorting to the inter-graph correspondences (*i.e.*, entity-entity pairs and attribute-attribute pairs).

Despite significant efforts in intra-entity attribute fusion (Chen et al., 2023a; Huang et al., 2024a) and inter-graph discrepancy elimination (Xu et al., 2023; Guo et al., 2021), existing MMEA methods heavily rely on the assumption of faultless intra-entity and inter-graph correspondences. However, as shown in Fig. 1(a), the assumption is daunting and even impossible to satisfy, leading to the Noisy Correspondence (NC) problem at dual levels. On the one hand, as the MMKG construction requires expert knowledge, it is inevitable to wrongly associate some entities with irrelevant attributes, resulting in intra-entity NC. For instance, image of “Elvis Tsui” is incorrectly associated with entity “Jason Momoa” because of the visual resemblance. On the other hand, due to the inherent complexities in attribute and entity association, accurately associating all the inter-graph entities and their corresponding attributes is impractical, leading to inter-graph NC. For example, movie entity “Mr. & Mrs. Smith” is mistakenly labeled with real-life couple “Will Smith and Mrs. Smith”. According to the statistics in Appendix B, real-world benchmarks always contain numerous NC (*e.g.*, over 50% in ICEWS benchmarks). As shown in Fig. 1(b), NC would not only undermine the fusion of within-entity attributes but also misleading the inter-graph alignment, both of which significantly degrade the performance.

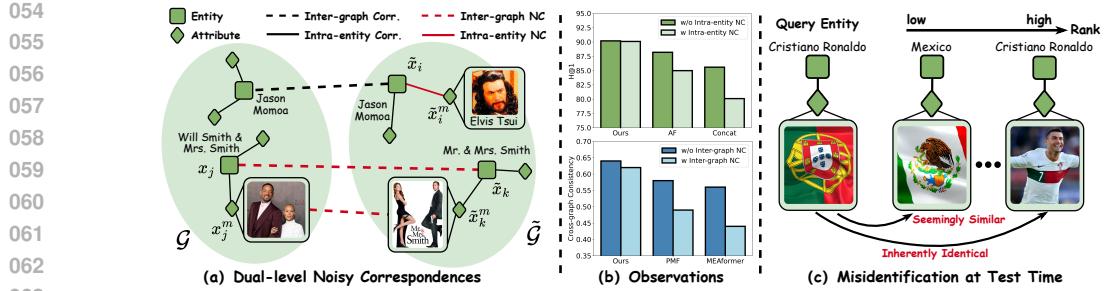


Figure 1: (a) **Dual-level Noisy Correspondence** occurs in both intra-entity level (*i.e.*, entity-attribute pairs such as $(\tilde{x}_i, \tilde{x}_i^m)$) and the inter-graph level (*i.e.*, entity-entity (x_j, \tilde{x}_k) or attribute-attribute pairs (x_j^m, \tilde{x}_k^m)). (b) **Observations**: On the one hand, both vanilla adaptive fusion (AF) and concatenation (Concat) tend to integrate erroneous attributes and thus degrade performance, while our method achieves reliable fusion against inter-graph NC. On the other hand, existing methods suffer in cross-graph alignment when encountering inter-graph NC, whereas our method boosts performance by mitigating the negative impact of intra-entity NC. (c) **Misidentification at Test Time**: Seemingly similar attribute pairs may prevent the query entity from being associated with its equivalent entity. For instance, the implicit connection between the football player “Cristiano Ronaldo” and his home country are often overlooked, resulting in misidentification during entity alignment.

Based on the above observations, we reveal a new problem for MMEA, termed Dual-level Noisy Correspondence (DNC). To conquer the DNC problem, we propose a novel method, dubbed dually Robust LEarning (RULE), for achieving robust MMEA against DNC. Specifically, RULE first estimates the reliability of both the intra-entity and inter-graph correspondences by resorting to a dedicatedly-designed two-fold principle and then divides the entity-attribute, entity-entity, and attribute-attribute pairs into different groups. Based on the estimated reliabilities and division results, RULE alleviates the negative impact of intra-entity NC during intra-entity attribute fusion, while preventing the model from overfitting the inter-graph NC during inter-graph discrepancy elimination. Beyond the training-time designs, RULE further incorporates a novel correspondence reasoning module to enhance the test-time robustness. In brief, this module performs deep reasoning to uncover the underlying attribute-attribute connections across graphs, thus preventing seemingly dissimilar but inherently identical attributes from being neglected (as shown in Fig. 1(c)) and guaranteeing more accurate equivalent entity identification during inference.

In summary, the major contributions and novelties of this work are given as follows.

- We reveal and study a novel and practical problem in MMEA, termed Dual-level Noisy Correspondence (DNC). In brief, DNC refers to the noisy correspondence rooted in the intra-entity (entity-attribute) pairs and inter-graph (entity-entity, attribute-attribute) pairs. We empirically demonstrate that DNC not only undermines multi-modal attribute fusion but also misleads the inter-graph alignment, leading to significant performance degradation for existing MMEA methods.
- To achieve robust MMEA against the DNC problem, we propose a novel method termed RULE, which estimates the reliability of both the intra-entity and inter-graph correspondences with a dedicatedly-designed two-fold principle and accordingly mitigates the negative impact of DNC during the multi-modal attribute fusion and inter-graph alignment processes.
- During inference, RULE employs a novel correspondence reasoning module to uncover inherently-identical attributes and accordingly achieve more precise cross-graph equivalent entity identification. To the best of our knowledge, this could be one of the first methods to enhance test-time robustness for the MMEA task.

2 METHOD

In this section, we introduce the proposed RULE for tackling the DNC problem. In Section 2.1, we present the formal definition of the MMEA task and the DNC problem. In Section 2.2, we elaborate on the two-fold principle for the reliability estimation and pair division. In Section 2.3-2.4, we introduce the robust attribute fusion and robust discrepancy elimination modules. In Section 2.5, we

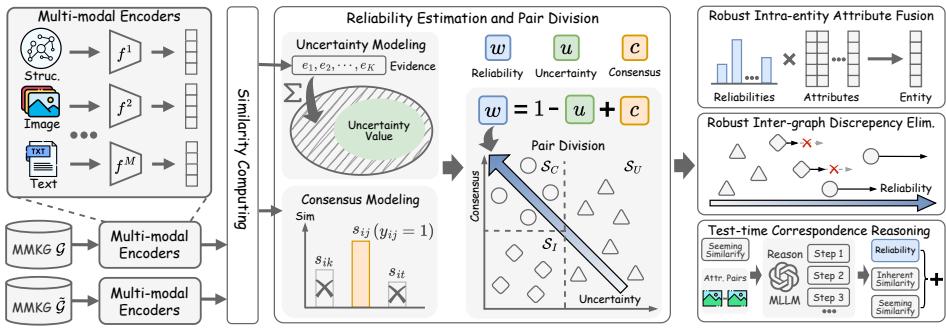


Figure 2: Overview of our method RULE. Given two MMKGs \mathcal{G} and $\tilde{\mathcal{G}}$, RULE first projects the entity attributes into a shared latent space and computes cross-graph attribute similarities. These similarities are used to estimate the reliability of inter-graph correspondences and categorize cross-graph pairs into three subsets: \mathcal{S}_C , \mathcal{S}_I , and \mathcal{S}_U . Subsequently, the robust intra-entity attribute fusion module and robust inter-graph discrepancy elimination are employed to mitigate the impact of both intra-entity and inter-graph noisy correspondences. Beyond training-time robustness, a test-time correspondence reasoning module uncovers latent attribute-attribute connections across graphs, enabling more accurate equivalent entity identification during inference.

design a test-time correspondence reasoning module to uncover underlying connections between inter-graph attributes, facilitating the equivalent entity identification.

2.1 PROBLEM FORMULATION

Given two heterogeneous multi-modal knowledge graphs (MMKGs), denoted as $\mathcal{G} = \{x_i, \{x_i^m\}_{m=1}^M\}_{i=1}^N$ and $\tilde{\mathcal{G}} = \{\tilde{x}_j, \{\tilde{x}_j^m\}_{m=1}^{\tilde{M}}\}_{j=1}^{\tilde{N}}$, where x_i and \tilde{x}_j are entities in \mathcal{G} and $\tilde{\mathcal{G}}$, respectively. Each entity $x_i \in \mathcal{G}$ is associated with M attribute-specific attributes $\{x_i^m\}_{m=1}^M$, such as structured triples, textual descriptions, and images.

Within a single graph, the association between an entity and its attributes is captured by entity-attribute pairs (x_i, x_i^m, h_i^m) , where $h_i^m \in \{0, 1\}$ is a binary indicator, $h_i^m = 1$ indicates the valid *intra-entity correspondence*, and $h_i^m = 0$ denotes no correspondence between x_i and x_i^m . Across graphs, *inter-graph correspondences* govern the alignment of both the entity-entity pairs and attribute-attribute pairs. To be specific, the entity-entity pair is represented by $(x_i, \tilde{x}_j, y_{ij})$, where the correspondence $y_{ij} = 1$ if x_i and \tilde{x}_j refer to the same real-world concepts, and $y_{ij} = 0$ otherwise. Similarly, the attribute-attribute pair is denoted by $(x_i^m, \tilde{x}_j^m, y_{ij}^m)$, where the correspondence $y_{ij}^m = 1$ iff. both attributes are linked to correct entities (i.e., $h_i^m = 1 \& \tilde{h}_j^m = 1$) and the corresponding entities x_i and \tilde{x}_j are aligned (i.e., $y_{ij} = 1$). In other words, once the inter-graph entities are associated, their corresponding attributes could be treated as matched.

Given a query entity $x_i \in \mathcal{G}$, the goal of multi-modal entity alignment is to identify its equivalent entity \tilde{x}_j from the other $\tilde{\mathcal{G}}$ such that $y_{ij} = 1$. To this end, existing approaches typically follow a two-stage pipeline: i) intra-entity attribute fusion: for each entity x_i , attribute representations are first extracted using attribute-specific encoders $z_i^m = f^m(x_i^m)$, and then aggregated to form a unified entity representation z_i ; ii) inter-graph discrepancy elimination: based on the fused entity representations z_i and \tilde{z}_j , contrastive learning (Chen et al., 2020) is employed to mitigate the inter-graph discrepancy. However, in practice, this pipeline assumes that both the intra-entity correspondences (i.e., entity-attribute h_i^m) and inter-graph correspondences (i.e., entity-entity y_{ij} and attribute-attribute y_{ij}^m) are perfectly labeled. However, due to annotation errors, such an assumption is often violated, leading to the DNC challenge. As discussed in Introduction, the DNC problem would undermine the inter-graph and intra-entity learning, leading to remarkable performance degradation.

2.2 RELIABILITY ESTIMATION AND PAIR DIVISION

To facilitate robust inter-graph discrepancy elimination and intra-entity attribute fusion, we first estimate the reliability of both the intra-entity and inter-graph correspondences by resorting to a

162 two-fold principle, *i.e.*, uncertainty and consensus. Without loss of generality, in the following,
 163 we take the inter-graph entity-entity correspondence as a showcase to elaborate on the process of
 164 correspondence reliability estimation. For a given entity x_i , the reliability w_i between x_i and its
 165 associated counterpart \tilde{x}_j ($y_{ij} = 1$) is estimated using the following principle:

$$166 \quad 167 \quad w_i = (1 - u_i) \alpha + c_i(1 - \alpha), \quad (1)$$

168 where α is the balanced hyper-parameters (fixed as 0.5 for simplicity, see Appendix G.10 for more
 169 choices), u_i and c_i denote the uncertainty and consensus for the correspondence and will be detailed
 170 in the following sections.

171 2.2.1 UNCERTAINTY MODELING

173 For a given entity, uncertainty in this work refers to whether its correspondence is trustworthy or not,
 174 which could serve as the principle to identify NC. According to the Dempster-Shafer Theory (Shafer,
 175 1992), uncertainty could be quantified by evidence, which measures how the data support the
 176 association between a query and a candidate. Specifically, the more evidences the entity accumulates,
 177 the lower uncertainty it embraces. Formally, evidence of the entity pairs (x_i, \tilde{x}_j) is defined as

$$178 \quad 179 \quad e_{ij} = \exp(\tanh(s_{ij}/\tau)), \quad (2)$$

180 where $s_{ij} = z_i \cdot \tilde{z}_j$ denotes the dot product between the entity representation z_i and \tilde{z}_j , τ is
 181 the temperature, and the evidence vector for x_i is $\mathbf{e}_i = [e_{i1}; e_{i2}; \dots; e_{i\tilde{N}}]$. Following Subjective
 182 Logic (Sensoy et al., 2018), we associates the evidence vector \mathbf{e}_i with the parameters of the Dirichlet
 183 distribution $\boldsymbol{\alpha}_i = [\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{i\tilde{N}}]$, where $\alpha_{ij} = e_{ij} + 1$.

184 **Definition 1.** *Uncertainty.* For a given entity x_i , the uncertainty and the corresponding belief mass
 185 are defined as

$$186 \quad 187 \quad u_i = \frac{\tilde{N}}{Q_i} \text{ and } b_{ij} = \frac{e_{ij}}{Q_i} = \frac{\alpha_{ij} - 1}{Q_i}, \quad (3)$$

188 where $Q_i = \sum_j^{\tilde{N}} (e_{ij} + 1) = \sum_j^{\tilde{N}} \alpha_{ij}$ and $u_i + \sum_j^{\tilde{N}} b_{ij} = 1$.

190 The Q_i denotes the Dirichlet distribution strength, and the belief mass assignment $\mathbf{b}_i =$
 191 $[b_{i1}; b_{i2}; \dots; b_{i\tilde{N}}]$, *i.e.*, subjective opinion, corresponds to the Dirichlet distribution with param-
 192 eters $\boldsymbol{\alpha}_i$. Such a formulation encourages the mismatched entity-entity pairs to yield limited evidence,
 193 as the given entity fails to associate with any entity in the other MMKG, resulting in high uncertainty.

194 2.2.2 CONSENSUS MODELING

196 Although the formulated uncertainty would help to identify noisy correspondence, we observe that a
 197 low uncertainty does not necessarily indicate a correct correspondence. Formally,

199 **Theorem 1.** A low uncertainty u_i does not necessarily imply that the highest belief is assigned to the
 200 annotated correspondence \mathbf{y}_i , *i.e.*,

$$201 \quad z_i \text{ with low } u_i \not\Rightarrow \arg \max \mathbf{b}_i = \arg \max \mathbf{y}_i. \quad (4)$$

202 The Proof is placed in Appendix E. Here, $\mathbf{y}_i = [y_{i1}; y_{i2}; \dots; y_{i\tilde{N}}]$ is a one-hot vector indicating the
 203 inter-graph entity-entity correspondence of entity x_i . Such a theorem highlights that uncertainty
 204 is insufficient to determine whether the belief is concentrated on the annotated correspondence.
 205 Therefore, we propose the consensus principle as follows.

206 **Definition 2.** *Consensus.* For a given entity x_i , the consensus is defined as

$$208 \quad c_i = \max(0, \mathbf{s}_i \cdot \mathbf{y}_i), \quad (5)$$

209 where $\mathbf{s}_i = [s_{i1}, s_{i2}, \dots, s_{iN}]$ denotes the similarity vector, $\max(0, \cdot)$ ensures the consensus is
 210 non-negative.

212 Intuitively, a low consensus c_i indicates that the given correspondence is unreliable, thus serving
 213 as another principle to identify noisy correspondence. However, during inference, the annotated
 214 correspondence \mathbf{y}_i in Eq. 5 is unavailable. To remedy this, we propose to estimate the correct
 215 correspondence through a greedy strategy based on marginal contribution. Here, we begin with a
 definition of marginal contribution.

216 **Definition 3.** For a given entity x_i , the marginal contribution of its m -th attribute is defined as
 217

$$\Delta = v(\pi \cup \{m\}) - v(\pi), \quad (6)$$

219 where $v(\cdot)$ indicates the value function, $\pi \subseteq \Pi \setminus \{m\}$ denotes a subset π of attributes excluding the
 220 m -th one, Π is the complete set of available attributes.

222 In the implementation, we define the value function as $v(\pi) = \max\left(\frac{1}{|\pi|} \sum_{j \in \pi} s_i^j\right)$ and $v(\pi \cup$
 223 $\{m\}) = \max\left(\frac{1}{|\pi|+1} \sum_{j \in \pi \cup \{m\}} s_i^j\right)$, where $|\cdot|$ denotes the number of attributes. Inspired by
 224 Shannon's principle that "the essence of information is to eliminate uncertainty", we expect that the
 225 informative attributes would contribute to establishing reliable correspondence for the entity-attribute
 226 pairs. Thus,

227 **Assumption 1.** For a given entity x_i , if x_i^m is correctly associated with x_i , then $\Delta \geq 0$. Conversely,
 228 if x_i^m is irrelevant to x_i , then $\Delta < 0$.

230 Assumption 1 provides a feasible way to estimate the correct correspondence. Specifically, incor-
 231 porating attributes until the marginal contribution no longer improves, and the established subset π
 232 would help to indicate a reliable correspondence. To implement this, we adopt the following greedy
 233 strategy,

$$\pi^* = \pi_0 \cup \{m \in (\Pi \setminus \pi_0) \mid v(\pi_0 \cup \{m\}) - v(\pi_0) > 0\}, \quad (7)$$

235 where π_0 denotes the initial subset with $|\pi_0| = \lfloor \frac{M}{2} + 1 \rfloor$ when $M \geq 3$. See more details in
 236 Appendix F.3. With the selected subset π^* , the estimated correspondence is finally given as $\mathbf{y}_i =$
 237 $\text{one-hot}(\arg \max(\frac{1}{|\pi^*|} \sum_{m \in \pi^*} s_i^m))$, where one-hot denotes the vector conversion.

239 2.2.3 PAIR DIVISION

240 With the formulated uncertainty and consensus, we could further identify the inter-graph NC.
 241 Specifically, we propose to divide the inter-graph pairs with $y_{ij} = 1$ into three portions: noisy
 242 portion with high uncertainty $\mathcal{S}_U = \{x_i, \tilde{x}_j \mid u_i > \beta_u\}$, noisy portion with low consensus
 243 $\mathcal{S}_I = \{x_i, \tilde{x}_j \mid u_i \leq \beta_u \text{ and } c_i < \beta_c\}$ and clean portion $\mathcal{S}_C = \{x_i, \tilde{x}_j \mid u_i \leq \beta_u \text{ and } c_i \geq \beta_c\}$. The
 244 thresholds β_u and β_c are determined in a self-adaptive manner via

$$\beta_u = \min(u^{TP}, 1 - \beta), \quad \beta_c = \max(\beta, c^{TP}), \quad (8)$$

245 where $u^{TP} = \max_{i \in \mathcal{S}^{TP}} u_i$, $c^{TP} = \min_{i \in \mathcal{S}^{TP}} c_i$, and β indicates the threshold hyperparameter. Here,
 246 $\mathcal{S}^{TP} = \{i \mid \arg \max(s_i) = \arg \max(y_i)\}$ denotes the set of true positive pairs. With the above pair
 247 division, the inter-graph pairs could be divided into \mathcal{S}_U , \mathcal{S}_I , and \mathcal{S}_C , which are further used for
 248 inter-graph discrepancy elimination.

252 2.3 ROBUST INTER-GRAPH DISCREPANCY ELIMINATION

253 With the established reliability and pair division results, we could obtain three subsets: \mathcal{S}_U , \mathcal{S}_I , and
 254 \mathcal{S}_C . Since the pairs in \mathcal{S}_U exhibit high uncertainty, they are considered unreliable and be excluded
 255 from the discrepancy elimination. As discussed in Section 2.2.2, inter-graph pairs with low consensus
 256 do not necessarily indicate correct matches, thus the pairs in \mathcal{S}_I cannot be regarded as reliable.
 257 Accordingly, we propose a novel Dually Robust Learning (DRL) that employs tailored strategies for
 258 the three subsets, thereby achieving robustness against inter-graph noisy correspondence. Formally,
 259 the overall objective is defined as

$$\mathcal{L} = \mathcal{L}_{DR} + \lambda \mathcal{L}_{Reg}, \quad (9)$$

260 where \mathcal{L}_{DR} and \mathcal{L}_{Reg} denotes the dually robust loss and regularization loss, λ denotes the trade-off
 261 parameter. Specifically, the dually robust loss and regularization loss are given by,

$$\mathcal{L}_{DR} = \mathcal{L}_{DR}(\boldsymbol{\alpha}_i, \hat{\mathbf{y}}_i) + \sum_{m=1}^M \mathcal{L}_{DR}(\boldsymbol{\alpha}_i^m, \hat{\mathbf{y}}_i^m), \quad \mathcal{L}_{Reg} = \mathcal{L}_{Reg}(\boldsymbol{\alpha}_i, \hat{\mathbf{y}}_i) + \sum_{m=1}^M \mathcal{L}_{Reg}(\boldsymbol{\alpha}_i^m, \hat{\mathbf{y}}_i^m), \quad (10)$$

262 where $\boldsymbol{\alpha}_i^m$ and $\hat{\mathbf{y}}_i^m$ are the Dirichlet parameter and refined correspondence for x_i^m . More specifically,
 263 for the given entity x_i , the dually robust loss is defined as

$$\mathcal{L}_{DR}(\boldsymbol{\alpha}_i, \hat{\mathbf{y}}_i) = \mathbb{I}(i \notin \mathcal{S}_U) \int \|\hat{\mathbf{y}}_i - \mathbf{p}_i\|_2^2 D(\mathbf{p}_i \mid \boldsymbol{\alpha}_i) d\mathbf{p}_i, \quad (11)$$

270 where $D(\mathbf{p}_i \mid \boldsymbol{\alpha}_i)$ denotes the density function of the Dirichlet distribution over the query probability
 271 $\mathbf{p}_i = [p_{i1}, p_{i2}, \dots, p_{i\tilde{N}}]$, and $\mathbb{I}(\cdot)$ indicates an indicator function evaluating to 1 *i.f.f* the condition is
 272 satisfied. The refined correspondence $\hat{\mathbf{y}}_i$ is defined as follows,

$$\hat{\mathbf{y}}_i = \begin{cases} \mathbf{y}_i, & \text{if } i \in \mathcal{S}_C \\ c_i \mathbf{y}_i + (1 - c_i) \text{Softmax}(\mathbf{s}_i), & \text{if } i \in \mathcal{S}_I \end{cases} \quad (12)$$

276 Such behavior enhances robustness against inter-graph noisy correspondences for the following
 277 reasons. On the one hand, the upper bound of query probability is proportional to Q_i (Theorem 2),
 278 thus preventing over-optimization when the accumulate Q_i is limited. On the other hand, excluding
 279 high-uncertainty correspondences in \mathcal{S}_U and refining the low-consensus correspondences in \mathcal{S}_I would
 280 prevent erroneous optimization caused by NC.

281 Although the proposed dually robust loss in Eq. 11 could encourage higher evidence for inter-graph
 282 pairs with reliable correspondence, it is unable to guarantee that unassociated inter-graph pairs
 283 generate limited evidence. To achieve this, a Kullback-Leibler (KL) divergence term is adopted to
 284 penalize the evidence of the unassociated inter-graph pairs, *i.e.*,

$$\mathcal{L}_{\text{Reg}}(\boldsymbol{\alpha}_i, \hat{\mathbf{y}}_i) = \text{KL} [D(\mathbf{p}_i \mid \tilde{\boldsymbol{\alpha}}_i) \parallel D(\mathbf{p}_i \mid \mathbf{1})] \quad (13)$$

286 where $\mathbf{1} \in \mathbb{R}^{\tilde{N}}$ is a \tilde{N} -dimensional vector of ones, $\tilde{\boldsymbol{\alpha}}_i = \hat{\mathbf{y}}_i + (1 - \hat{\mathbf{y}}_i) \odot \boldsymbol{\alpha}_i$ denotes the Dirichlet
 287 parameters which help to penalize the evidence of unassociated correspondence, $\Gamma(\cdot)$ and $\psi(\cdot)$ are
 288 the gamma and digamma function, respectively.

290 2.4 ROBUST INTRA-ENTITY ATTRIBUTE FUSION

292 As discussed in Section 2.1, inter-graph attribute associations emerge as the by-product of establishing
 293 entity-attribute and entity-entity correspondences. Therefore, for correctly paired entities, the attribute-
 294 attribute correspondence is incorrect, *i.f.f*, the corresponding entity-attribute correspondence is
 295 wrongly established. Thus, the inter-graph reliability w_i^m could be employed to identify unreliable
 296 intra-entity attributes and weaken the emphasis on them during attribute fusion. Specifically, for a
 297 given entity x_i , we employ the following Dually Robust Fusion (DRF) module to obtain the integrated
 298 representation,

$$z_i = \bigoplus_{m \in M} (w_i^m \cdot z_i^m), \quad (14)$$

299 where \bigoplus indicates the concatenation operator. Such behavior achieves robustness against noisy
 300 entity-attribute pairs by fusing the multi-modal attributes with adaptive weights. In other words,
 301 attributes with higher reliability are emphasized, while those with lower reliability are weakened.

303 2.5 TEST-TIME CORRESPONDENCE REASONING

305 As discussed in the Introduction, the seemingly similar attributes might hinder the identification of
 306 equivalent entities. To solve the problem, we propose Test-time correspondence Reasoning (TTR)
 307 module, which uncovers the underlying attribute-attribute connections across graphs, thus improving
 308 the equivalent entity identification during inference. Specifically, the refined entity-entity similarity
 309 scores are given by,

$$\hat{s}_i = \sum_{m \in M} \hat{w}_i^m \cdot \hat{s}_i^m, \quad (15)$$

312 where \hat{s}_i^m represents the similarity scores of the m -th attribute output by the MLLM and \hat{w}_i^m denotes
 313 the corresponding reliability weight. Such behaviour could mitigate the negative impact of intra-
 314 entity NC, which might undermine attribute fusion during test time. More specifically, we employ
 315 Chain-of-Thought (CoT) to guide the MLLM toward step-by-step reasoning. Mathematically,

$$\hat{s}_i^m = \text{Softmax} \left(\bigoplus_{j \in \mathcal{T}_i^m} (\text{CoT} [x_i^m, \tilde{x}_j^m, s_i^m]) \right), \quad (16)$$

317 where \mathcal{T}_i^m denotes the set of correspondences with the highest similarity in prior results s_i^m , CoT
 318 indicates the reasoning process. Although a feasible solution is to prompt the MLLM with simple
 319 instructions such as “Identify the similarities between these attributes.”, such vanilla prompts fail to
 320 fully activate the deep reasoning capabilities of MLLM. In contrast, the proposed CoT-based reasoning
 321 would enable the MLLM to leverage prior results and detailed steps for reasoning, preventing
 322 deviations from the prior knowledge while facilitating the mining of underlying connections. See
 323 Appendix F.5 and Appendix I for more details. Finally, the joint similarity score could be derived as
 $s_i^{\text{joint}} = s_i + \hat{s}_i$ and the identified equivalent entity is given by $\arg \max s_i^{\text{joint}}$.

324 Table 1: Comparisons with state-of-the-art methods on Non-name benchmarks under DNC setting.
325 “Inherent DNC” refers to the setting without any additional injected noise. $H@k$ indicates the top- k
326 retrieval accuracy while MRR denotes the mean reciprocal rank. The best and second best results are
327 marked in **bold** and underline.

Setting	Method	ICEWS-WIKI			ICEWS-YAGO			DBP15K ZH-EN			DBP15K JA-EN			DBP15K FR-EN			Avg. $H@1$
		$H@1$	$H@5$	MRR	$H@1$	$H@5$	MRR	$H@1$	$H@5$	MRR	$H@1$	$H@5$	MRR	$H@1$	$H@5$	MRR	
Inherent DNC	EVA	29.6	40.7	35.1	8.0	13.7	11.1	70.7	86.8	77.9	73.6	89.5	80.6	74.3	90.5	81.4	51.2
	MCLEA	43.2	63.1	52.4	30.1	47.7	38.8	76.6	90.8	83.0	77.8	92.0	84.1	78.7	92.7	84.9	61.3
	XGEA	49.8	61.5	55.5	35.5	46.7	41.2	81.1	93.0	86.3	82.6	94.3	87.8	83.1	94.7	88.3	66.4
	MEAformer	<u>53.5</u>	<u>70.1</u>	<u>61.3</u>	35.0	51.2	42.8	82.4	93.5	87.3	81.9	94.2	87.3	82.1	94.4	87.5	67.0
	UMAEA	51.2	70.0	59.9	32.4	49.4	40.6	79.1	93.2	85.3	79.6	93.9	85.8	81.2	95.0	87.3	64.7
	PMF	52.6	67.9	59.9	<u>38.3</u>	<u>53.2</u>	<u>45.4</u>	83.9	94.6	88.9	<u>83.9</u>	<u>94.9</u>	<u>89.0</u>	84.4	<u>95.3</u>	<u>89.6</u>	<u>68.6</u>
	HHEA	49.0	64.6	56.4	37.5	50.4	43.8	48.7	62.5	55.5	49.9	60.6	55.4	52.8	63.6	58.2	47.6
20% DNC	Ours	64.2	76.7	70.0	48.8	60.5	54.6	85.6	94.8	89.7	85.2	95.4	89.6	85.1	95.4	89.6	73.8
	EVA	15.2	21.6	18.4	0.2	0.4	0.4	51.0	70.2	59.7	54.5	73.4	63.1	53.4	73.8	62.6	34.9
	MCLEA	34.5	53.6	43.5	24.6	40.4	32.5	69.9	85.7	77.0	70.1	85.6	77.2	70.7	87.3	78.1	54.0
	XGEA	40.4	48.4	44.6	22.6	27.6	25.7	76.3	90.7	82.7	76.6	<u>91.1</u>	83.0	76.9	91.2	83.7	58.6
	MEAformer	<u>50.8</u>	<u>67.5</u>	<u>58.4</u>	35.9	<u>50.7</u>	43.0	<u>77.7</u>	90.6	<u>83.4</u>	<u>77.8</u>	90.9	<u>83.6</u>	<u>78.0</u>	<u>91.5</u>	<u>84.0</u>	<u>64.0</u>
	UMAEA	48.4	64.6	56.1	31.1	46.5	38.6	74.5	89.6	81.3	73.6	89.4	80.7	74.3	89.9	81.1	60.4
	PMF	45.4	60.6	52.6	36.2	49.9	42.7	76.7	90.2	82.7	76.5	89.9	82.5	77.1	90.7	83.2	62.4
50% DNC	HHEA	47.8	61.8	54.4	<u>37.4</u>	<u>49.5</u>	<u>43.3</u>	48.7	58.8	53.8	49.0	58.7	54.0	52.5	61.7	57.1	47.1
	Ours	62.4	75.1	68.5	48.3	59.5	53.9	81.1	92.0	86.0	80.5	92.2	85.6	80.5	92.2	85.8	70.6

3 EXPERIMENTS

In this section, we conduct extensive experiments on five widely-used MMEA datasets to validate the effectiveness of the proposed RULE. Due to space limitation, we present more experiments in Appendix G.

3.1 IMPLEMENTATION DETAILS AND EXPERIMENTAL SETTINGS

Our method contains two networks, the attribute-specific encoders f^m and the test-time correspondence reasoning module. Specifically, we first utilize a pre-trained CLIP model (Radford et al., 2021) to extract features from visual and textual attributes. After that, we employ the attribute-specific encoders to obtain the latent embeddings following (Huang et al., 2024a; Xu et al., 2023). For the test-time correspondence module, we use Qwen2.5-VL-72B-Instruct (Bai et al., 2025) as default to facilitate the test-time correspondence reasoning module (Section 2.5). Regarding hyperparameters, we set the trade-off parameter λ in Eq. 9, the threshold β in Eq. 8 are fixed as $1e^{-4}$, 0.3 for all the experiments, respectively. The temperature τ in Eq. 2 is set to 0.07 following (Chen et al., 2020).

We evaluate our method on five benchmark datasets: ICEWS-WIKI (Jiang et al., 2024), ICEWS-YAGO, DBP15K_{ZH-EN} (Liu et al., 2021), DBP15K_{JA-EN}, and DBP15K_{FR-EN}. Details of the dataset and evaluation metric are provided in Appendix F.1 and F.4. As discussed in Introduction, the MMEA benchmarks including ICEWS always contaminated by DNC which denoted as “Inherent DNC” in the paper. To further evaluate the robustness toward DNC, we manually inject noise to conduct more comprehensive evaluations by following the widely-adopted strategies in the noisy correspondence/label learning community (Natarajan et al., 2013; Huang et al., 2021). Specifically, the artificial noise are injected in the following three aspects: i) *entity-entity NC*: one entity in an aligned entity pair is randomly replaced with a different entity; ii) *entity-attribute NC*: a visual or textual attribute is randomly reassigned to a different entity; iii) *attribute-attribute NC*: visual attributes are perturbed with Gaussian noise, while textual attributes are corrupted via random character replacements. The artificial noise levels are set as 20% and 50% in our experiments, which represents the proportion of corrupted E-E/E-A/A-A pairs.

3.2 COMPARISONS WITH STATE-OF-THE-ARTS

In this section, we compare our method RULE with seven state-of-the-art MMEA methods under the Dual-level Noisy Correspondence setting, including EVA (Liu et al., 2021), MCLEA (Lin et al.,

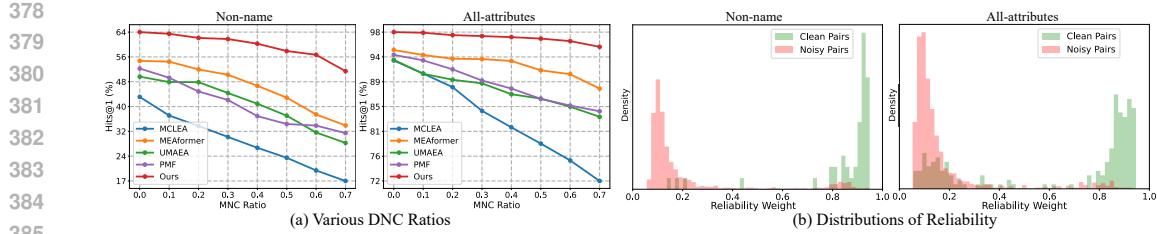


Figure 3: Analytic studies of various DNC ratios and reliability in Eq. 1.

Table 2: Comparisons with state-of-the-art methods on All-attributes benchmarks under DNC setting.

Setting	Method	ICEWS-WIKI			ICEWS-YAGO			DBP15K _{ZH-EN}			DBP15K _{JA-EN}			DBP15K _{FR-EN}			Avg. H@1
		H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	
Inherent DNC	EVA	90.7	95.7	93.0	86.5	94.0	89.8	89.8	96.6	92.8	94.8	98.8	96.5	98.7	99.8	99.2	92.1
	MCLEA	93.8	98.3	95.9	92.1	97.7	94.6	94.5	98.6	96.4	97.8	99.7	98.7	99.2	99.9	99.5	95.5
	XGEA	83.5	94.4	88.6	93.9	97.3	95.8	91.4	97.4	94.1	94.3	98.0	96.0	97.3	99.3	98.2	92.1
	MEAformer	95.9	98.8	97.2	93.8	97.9	95.7	96.7	99.0	97.7	98.8	99.8	99.3	99.6	100.0	99.8	97.0
	UMAEA	94.8	98.7	96.6	92.8	97.9	95.1	95.4	98.9	97.0	98.2	99.7	98.9	99.4	99.9	99.6	96.1
	PMF	94.9	98.4	96.5	92.8	97.7	95.0	96.3	99.1	97.6	98.5	99.7	99.1	99.5	100.0	99.7	96.4
	HHEA	89.9	95.5	92.5	89.7	95.2	92.2	68.1	78.8	73.2	77.0	86.0	81.1	85.8	92.2	88.7	82.1
20% DNC	Ours	98.9	99.2	99.1	97.6	98.8	98.2	98.3	99.5	99.3	99.9	99.6	99.8	100.0	99.9	98.8	
	EVA	67.4	76.2	71.6	17.9	21.4	19.7	64.2	78.9	70.8	72.6	85.7	78.5	88.0	95.2	91.3	62.0
	MCLEA	89.0	95.2	91.8	88.8	95.8	92.0	91.5	97.0	94.0	95.6	98.8	97.0	97.8	99.6	98.6	92.5
	XGEA	56.1	67.3	61.7	60.1	71.4	65.5	89.5	96.4	92.6	92.4	98.2	95.0	96.6	98.9	97.6	78.9
	MEAformer	93.8	97.6	95.6	91.8	97.2	94.3	95.5	98.5	96.8	98.3	99.6	98.9	99.4	99.9	99.7	95.7
	UMAEA	90.3	96.5	93.1	86.8	95.1	90.5	94.1	98.2	95.9	97.2	99.4	98.2	98.8	99.9	99.3	93.5
	PMF	92.2	96.9	94.3	90.9	96.3	93.4	94.8	98.1	96.3	97.6	99.3	98.3	99.2	99.9	99.5	94.9
50% DNC	HHEA	87.6	93.8	90.5	89.3	94.6	92.1	66.1	75.9	70.8	72.6	81.9	77.0	83.5	90.2	86.6	79.8
	Ours	98.3	98.9	98.6	97.5	98.7	98.1	97.6	99.1	99.3	99.5	99.8	100.0	99.9	98.5		
	EVA	2.7	3.8	3.4	0.0	0.1	0.2	17.5	31.7	24.2	18.4	33.2	25.2	15.3	30.5	22.4	10.8
	MCLEA	78.9	88.3	83.2	75.9	88.1	81.5	84.5	91.7	87.8	88.7	94.7	91.4	93.5	97.5	95.4	84.3
	XGEA	50.3	60.3	55.3	34.8	44.5	39.8	71.3	86.4	78.0	70.1	85.5	77.0	88.7	95.9	91.9	63.0
	MEAformer	91.9	96.7	94.1	91.9	96.8	94.1	93.4	97.3	95.2	97.3	99.1	98.1	99.1	99.9	99.5	94.7
	UMAEA	87.0	94.4	90.4	85.7	93.9	89.4	91.4	96.7	93.8	95.9	98.8	97.2	98.1	99.6	98.8	91.6
	PMF	86.9	93.9	90.0	87.6	94.4	90.7	92.2	96.5	94.2	96.1	98.9	97.3	98.6	99.6	99.1	92.3
	HHEA	86.2	92.8	89.2	84.2	92.1	87.8	56.8	71.3	63.7	70.5	82.2	75.9	76.9	86.1	81.1	74.9
	Ours	97.7	98.3	98.0	97.0	98.2	97.6	96.3	98.1	97.2	98.7	99.7	99.1	99.7	100.0	99.8	97.9

2022b), XGEA (Xu et al., 2023), MEAformer (Chen et al., 2023a), UMAEA (Chen et al., 2023b), PMF (Huang et al., 2024a), and HHEA (Jiang et al., 2024). Following (Chen et al., 2023a; Huang et al., 2024a; Xu et al., 2023), we conduct experiments under two widely-adopted evaluation protocols: *Non-name setting* denotes all attributes except for the entity name are used, while *All-attributes setting* includes all available modalities. For fair comparisons, we adopt the same backbone (*i.e.*, CLIP) for all baselines and our method. For more results on different backbones, please refer to Appendix G.11.

As shown in Tables 1-2, we could have the following conclusions: i) existing methods face substantial performance degradation as noise increases, highlighting their vulnerability to noisy correspondences. In contrast, RULE outperforms all baselines across different datasets and noise settings, demonstrating superior robustness against DNC; ii) even without any manually-injected noise, RULE still achieves performance gains compared to existing methods, as the real-world MMEA datasets contain a considerable number of DNC. To further verify the effectiveness of RULE, we conduct experiments under the manually-injected noise ratio from 0.0 to 0.7. As shown in Fig. 3 (a), RULE not only achieves higher performance across all noise levels but also exhibits significantly slower performance degradation, which further confirms the robustness of RULE against DNC.

3.3 ANALYSIS AND ABLATION STUDY

In this section, we conduct analysis and ablation studies on the ICEWS-WIKI dataset.

Analysis Studies on Uncertainty and Consensus. As discussed in Section 2.2, the estimated reliability plays a key role in identifying DNC. To better understand its behavior, we visualize the reliability distribution of all training entity pairs. As shown in Fig. 3(b), clean pairs are concentrated on the right side of the plot (indicating high reliability), while noisy pairs are predominantly on the left (indicating low reliability). This confirms that the proposed reliability serves an effective indicator for distinguishing clean and noisy pairs. To further explore how the proposed uncertainty

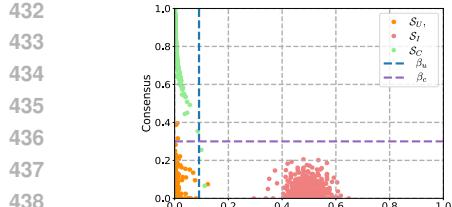


Figure 4: Quantitative analysis of the uncertainty and consensus on the name attribute.

Table 3: Ablation study of the various modules in the train and test stages.

Stage	Setting	Non-name			All-attributes		
		H@1	H@5	MRR	H@1	H@5	MRR
Train	w/o DRL	31.6	45.9	38.6	82.3	90.4	86.0
	w/o DRF	50.4	66.2	57.6	93.4	97.4	95.2
	Only Unc.	53.5	67.8	60.2	93.6	97.4	95.4
	Only Cons.	48.3	60.3	54.3	87.7	93.2	90.4
Test	w/o DRF	52.4	66.2	59.0	95.1	97.9	96.3
	w/o TTR	56.5	68.6	62.3	94.0	97.7	95.7
	MLLM Enhance	56.6	69.0	62.4	97.6	98.2	97.9
Both	Default	58.2	69.7	63.6	97.7	98.3	98.0



Figure 5: Visualization of the reliability for clean entity pairs and those with manually injected E-A NC in the image and name attributes. From top to bottom, the three rows denote the structure, image, and name attributes, respectively.

and consensus behave under noise, we construct subsets S_U and S_I by injecting synthetic noise and randomly shuffling the name attributes of the raw set S_C . As illustrated in Fig. 4, uncertainty and consensus principles successfully separate the three subsets, which supports the design of our tailored loss strategies in Eq. 11.

Effectiveness of Robust Fusion. To qualitatively study the effectiveness of RULE in handling entity-attribute noise, we visualize the reliability in Eq. 1 during the fusion process. As shown in Fig. 5, correctly associated attributes are assigned high reliability scores, while noisy or irrelevant attributes receive significantly lower scores. This behavior confirms that RULE effectively suppresses the influence of unreliable attributes during fusion, thereby enhancing robustness against entity-attribute noise.

Ablation studies. To verify the effectiveness of each component in our framework, we conduct ablation experiments on the modules involved in both training and test-time phases. According to the results in Table 3, one could have the following conclusions. First, during training phase, both the ‘‘Only Unc.’’ variant (which applies the uncertainty-guided loss in Eq. 18) and the ‘‘Only Cons.’’ variant (which uses a consensus-based MSE loss) outperform the baseline ‘‘w/o DRL’’ (which uses only a standard MSE loss). This demonstrates the effectiveness of our proposed Dually Robust Learning mechanism in handling noisy correspondence. Second, during the test phase, the TTR module significantly improves alignment performance by uncovering latent semantic connections. In particular, the comparison between the ‘‘MLLM Enhance’’ (which only uses rethinking scores in Eq. 16) and ‘‘w/o TTR’’ settings shows that combining rethinking scores with prior similarity scores leads to complementary effects, resulting in improved robustness and accuracy. Third, the Dually Robust Fusion (DRF) module effectively mitigates the influence of intra-entity NC. Its inclusion enhances performance in both the training and testing stages.

4 CONCLUSION

In this paper, we study a new problem in MMEA, *i.e.*, Dual-level Noisy Correspondence, which refers to the wrongly annotated intra-entity and inter-graph correspondences. To solve this problem, the proposed methods estimate the reliability of both the intra-entity and inter-graph correspondences and alleviate the negative impact of NC during the inter-graph discrepancy elimination and intra-entity attribution fusion. Beyond the training-time design, we employ a novel correspondence reasoning module to guarantee more accurate equivalent entity identification during inference. We believe this work might remarkably enrich the learning paradigm with noisy correspondence by simultaneously considering the noise across both training-time and test-time.

486 REFERENCES
487

488 Shuai Bai, Keqin Chen, Xuejing Liu, Jialin Wang, Wenbin Ge, Sibo Song, Kai Dang, Peng Wang,
489 Shijie Wang, Jun Tang, et al. Qwen2.5-vl technical report. *arXiv preprint arXiv:2502.13923*, 2025.

490 Cynthia Brame. Active learning. *Vanderbilt University Center for Teaching*, 2016.

491

492 Liyi Chen, Ying Sun, Shengzhe Zhang, Yuyang Ye, Wei Wu, and Hui Xiong. Tackling uncertain
493 correspondences for multi-modal entity alignment. *NeurIS*, 2024.

494 Ting Chen, Simon Kornblith, Mohammad Norouzi, and Geoffrey Hinton. A simple framework for
495 contrastive learning of visual representations. In *ICML*, 2020.

496

497 Zhuo Chen, Jiaoyan Chen, Wen Zhang, Lingbing Guo, Yin Fang, Yufeng Huang, Yichi Zhang, Yuxia
498 Geng, Jeff Z Pan, Wenting Song, et al. Meaformer: Multi-modal entity alignment transformer for
499 meta modality hybrid. In *ACM Multimedia*, 2023a.

500 Zhuo Chen, Lingbing Guo, Yin Fang, Yichi Zhang, Jiaoyan Chen, Jeff Z Pan, Yangning Li, Huajun
501 Chen, and Wen Zhang. Rethinking uncertainly missing and ambiguous visual modality in multi-
502 modal entity alignment. In *ISWC*, 2023b.

503

504 Yunpeng Gong, Liqing Huang, and Lifei Chen. Eliminate deviation with deviation for data aug-
505mentation and a general multi-modal data learning method. *arXiv preprint arXiv:2101.08533*,
506 2021.

507 Yunpeng Gong, Liqing Huang, and Lifei Chen. Person re-identification method based on color attack
508 and joint defence. In *CVPR*, 2022, pp. 4313–4322, 2022.

509

510 Yunpeng Gong, Zhun Zhong, Yansong Qu, Zhiming Luo, Rongrong Ji, and Min Jiang. Cross-
511 modality perturbation synergy attack for person re-identification. *Advances in Neural Information
512 Processing Systems*, 37:23352–23377, 2024.

513 Hao Guo, Jiuyang Tang, Weixin Zeng, Xiang Zhao, and Li Liu. Multi-modal entity alignment in
514 hyperbolic space. *Neurocomputing*, 2021.

515 Zongbo Han, Changqing Zhang, Huazhu Fu, and Joey Tianyi Zhou. Trusted multi-view classification
516 with dynamic evidential fusion. *IEEE Transactions on Pattern Analysis and Machine Intelligence*,
517 2022.

518 Sida Huang, Hongyuan Zhang, and Xuelong Li. Enhance vision-language alignment with noise. In
519 *AAAI*, 2025.

520

521 Yani Huang, Xuefeng Zhang, Richong Zhang, Junfan Chen, and Jaein Kim. Progressively modality
522 freezing for multi-modal entity alignment. *arXiv preprint arXiv:2407.16168*, 2024a.

523

524 Zhenyu Huang, Guocheng Niu, Xiao Liu, Wenbiao Ding, Xinyan Xiao, Hua Wu, and Xi Peng.
525 Learning with noisy correspondence for cross-modal matching. In *NeurIPS*, 2021.

526 Zhenyu Huang, Mouxing Yang, Xinyan Xiao, Peng Hu, and Xi Peng. Noise-robust vision-language
527 pre-training with positive-negative learning. *IEEE Transactions on Pattern Analysis and Machine
528 Intelligence*, 2024b.

529

530 Xuhui Jiang, Chengjin Xu, Yinghan Shen, Yuanzhuo Wang, Fenglong Su, Zhichao Shi, Fei Sun,
531 Zixuan Li, Jian Guo, and Huawei Shen. Toward practical entity alignment method design: Insights
532 from new highly heterogeneous knowledge graph datasets. In *WWW*, 2024.

533 Junnan Li, Dongxu Li, Caiming Xiong, and Steven Hoi. Blip: Bootstrapping language-image
534 pre-training for unified vision-language understanding and generation. In *ICML*, 2022.

535

536 Qian Li, Shu Guo, Yangifei Luo, Cheng Ji, Lihong Wang, Jiawei Sheng, and Jianxin Li. Attribute-
537 consistent knowledge graph representation learning for multi-modal entity alignment. In *WWW*,
538 2023.

539 Jia-Qi Lin, Man-Sheng Chen, Chang-Dong Wang, and Haizhang Zhang. A tensor approach for
uncoupled multiview clustering. *IEEE Transactions on Cybernetics*, 2022a.

540 Jia-Qi Lin, Man-Sheng Chen, Xi-Ran Zhu, Chang-Dong Wang, and Haizhang Zhang. Dual information
 541 enhanced multiview attributed graph clustering. *IEEE Transactions on Neural Networks and*
 542 *Learning Systems*, 2024.

543 Yijie Lin, Mouxing Yang, Jun Yu, Peng Hu, Changqing Zhang, and Xi Peng. Graph matching with
 544 bi-level noisy correspondence. In *ICCV*, 2023.

545 Zhenxi Lin, Ziheng Zhang, Meng Wang, Yinghui Shi, Xian Wu, and Yefeng Zheng. Multi-modal
 546 contrastive representation learning for entity alignment. *arXiv preprint arXiv:2209.00891*, 2022b.

547 Fangyu Liu, Muham Chen, Dan Roth, and Nigel Collier. Visual pivoting for (unsupervised) entity
 548 alignment. In *AAAI*, 2021.

549 Haotian Liu, Chunyuan Li, Qingsheng Wu, and Yong Jae Lee. Visual instruction tuning. In *NeurIPS*,
 550 2023.

551 Ye Liu, Hui Li, Alberto Garcia-Duran, Mathias Niepert, Daniel Onoro-Rubio, and David S Rosenblum.
 552 Mmkg: Multi-modal knowledge graphs. In *ESWC*, 2019.

553 Nagarajan Natarajan, Inderjit S Dhillon, Pradeep K Ravikumar, and Ambuj Tewari. Learning with
 554 noisy labels. In *NeurIPS*, 2013.

555 Heiko Paulheim. Knowledge graph refinement: A survey of approaches and evaluation methods.
 556 *Semantic web*, 2016.

557 Shichao Pei, Lu Yu, Guoxian Yu, and Xiangliang Zhang. Rea: Robust cross-lingual entity alignment
 558 between knowledge graphs. In *KDD*, 2020.

559 Alessandro Piscopo and Elena Simperl. Who models the world? collaborative ontology creation and
 560 user roles in wikidata. *Proceedings of the ACM on Human-Computer Interaction*, 2(CSCW):1–18,
 561 2018.

562 Alec Radford, Jong Wook Kim, Chris Hallacy, Aditya Ramesh, Gabriel Goh, Sandhini Agarwal,
 563 Girish Sastry, Amanda Askell, Pamela Mishkin, Jack Clark, et al. Learning transferable visual
 564 models from natural language supervision. In *ICML*, 2021.

565 Murat Sensoy, Lance Kaplan, and Melih Kandemir. Evidential deep learning to quantify classification
 566 uncertainty. In *NeurIPS*, 2018.

567 Glenn Shafer. Dempster-shafer theory. *Encyclopedia of Artificial Intelligence*, 1992.

568 Fabian M Suchanek, Gjergji Kasneci, and Gerhard Weikum. Yago: A core of semantic knowledge.
 569 In *WWW*, 2007.

570 Denny Vrandečić and Markus Krötzsch. Wikidata: A free collaborative knowledgebase. *Communications of the ACM*, 2014.

571 Yuanyi Wang, Haifeng Sun, Jiabo Wang, Jingyu Wang, Wei Tang, Qi Qi, Shaoling Sun, and Jianxin
 572 Liao. Towards semantic consistency: Dirichlet energy driven robust multi-modal entity alignment.
 573 In *IEEE ICDE*. IEEE, 2024.

574 Baogui Xu, Chengjin Xu, and Bing Su. Cross-modal graph attention network for entity alignment. In
 575 *ACM Multimedia*, 2023.

576 Baogui Xu, Yafei Lu, Bing Su, and Xiaoran Yan. Position-aware active learning for multi-modal
 577 entity alignment. In *ICASSP*. IEEE, 2024.

578 Mouxing Yang, Yunfan Li, Zhenyu Huang, Zitao Liu, Peng Hu, and Xi Peng. Partially view-aligned
 579 representation learning with noise-robust contrastive loss. In *CVPR*, 2021.

580 Mouxing Yang, Zhenyu Huang, Peng Hu, Taihao Li, Jiancheng Lv, and Xi Peng. Learning with twin
 581 noisy labels for visible-infrared person re-identification. In *CVPR*, 2022.

582 Xiaohua Zhai, Basil Mustafa, Alexander Kolesnikov, and Lucas Beyer. Sigmoid loss for language
 583 image pre-training. In *ICCV*, 2023.

594 Xiangru Zhu, Zhixu Li, Xiaodan Wang, Xueyao Jiang, Panglei Sun, Xuwu Wang, Yanghua Xiao, and
595 Nicholas Jing Yuan. Multi-modal knowledge graph construction and application: A survey. *IEEE*
596 *Transactions on Knowledge and Data Engineering*, 2022.
597
598
599
600
601
602
603
604
605
606
607
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609
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APPENDIX

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702 A THE USE OF LARGE LANGUAGE MODELS (LLMs)

704
705 Yes, we employ LLMs to aid and polish writing. Specifically, LLMs are used for language refinement
706 in the Introduction, Method, Experiments sections, and so on. Moreover, the multi-modal large
707 language model (MLLM) acts as one of the key parts for the test-time correspondence reasoning
708 module (see Section 2.5) in our method.

709 B NOISE STATISTICS IN REAL-WORLD BENCHMARKS

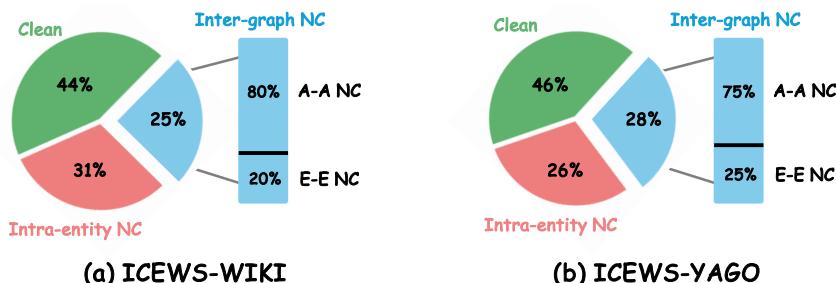
711 In this section, we elaborate on the necessity of addressing the DNC challenge in real-world scenarios
712 and discuss the underlying reasons behind the formation of the DNC.

714 B.1 STATISTICS ANALYSIS

716 As discussed in the manuscript, real-world benchmarks implicitly contain both intra-entity and
717 inter-graph noisy correspondences. To support this claim, we conducted an additional observational
718 study to analyze the types and distributions of noise present in the datasets. Specifically, we randomly
719 sample 1,000 entity pairs from the training set in the ICEWS-WIKI and ICEWS-YAGO benchmarks
720 and conduct manual statistical analysis. After that, we categorize each entity pair into one of four
721 types, *i.e.*, “Clean”, “E-E NC”, “E-A NC” or “A-A NC”. As shown in Fig. 6, one could have the
722 following observation and conclusions:

- 724 • Even after careful manual annotation, a considerable amount of DNC still exists in real-world datasets, *i.e.*, more than 50% of entity pairs in ICEWS benchmarks suffer from the
725 DNC challenge.
- 727 • Among the three types of NC, entity-attribute NC and attribute-attribute NC account for a
728 large proportion (*e.g.*, over 40% in ICEWS benchmarks). Compared with the entity-entity
729 NC, these two types of NC are more fine-grained, which makes it difficult to uncover
730 manually. It is worth emphasizing that although several NC-oriented methods have been
731 proposed, nearly all of them overlook such non-negligible fine-grained noise in MMEA,
732 further highlighting the necessity of addressing the DNC challenge.

733 As mentioned above, although the two commonly-used MMEA datasets are small in scale (*i.e.*,
734 containing up to 30K entity pairs) and carefully annotated by humans, they still exhibit non-negligible
735 DNC cases. Such a DNC challenge would become even more severe in large-scale knowledge graphs
736 such as Wikidata, DBpedia, and YAGO datasets (*i.e.*, containing over 10M-100M entities). We
737 believe that the DNC challenge is one of the key reasons why large-scale datasets are currently
738 lacking in the MMEA field. The proposed RULE framework offers a promising solution toward
739 achieving entity alignment on larger-scale datasets with the DNC challenge.



750 Figure 6: Distribution analysis of DNC noise in ICEWS-WIKI and ICEWS-YAGO benchmarks.
751

753 B.2 THE UNDERLYING REASON BEHIND DNC

755 To investigate the underlying causes of DNC, we analyze the formation of intra-entity NC and
756 inter-graph NC from two perspectives, *i.e.*, knowledge graph construction and cross-graph annotation.

756 **Knowledge Graph Construction:** The construction of existing knowledge graphs (Piscopo & Sim-
 757 perl, 2018) (*e.g.*, Wikidata, Freebase, and YAGO) heavily relies on crowdsourcing (Paulheim, 2016),
 758 *i.e.*, user editing on collaborative platforms. However, as most users lack expert knowledge, their edits
 759 are prone to errors such as incorrect data entry and conceptual misunderstandings, thereby inevitably
 760 introducing incorrect attributes within an entity. As a result, the editing errors in crowdsourcing
 761 would lead to intra-entity NC.

762 **Cross-graph Annotation:** In real-world scenarios, most MMEA datasets are annotated using semi-
 763 automated methods, which would inevitably introduce the inter-graph NC. Specifically, to construct
 764 entity pairs between the ICEWS and WIKI knowledge graphs, Jiang et al. (2024) employs the name
 765 attributes from the ICEWS graph as queries and uses the Wikidata API to retrieve the most relevant
 766 matches. As a result, entity-entity pairs are established between the queries and the retrieved results.
 767 However, such a process inevitably introduces inter-graph E-E NC due to the following reasons:
 768 i) some entities exist in ICEWS but do not appear in Wikidata, making it impossible to establish
 769 correct correspondences; ii) even some entities have highly similar name attributes, they could be very
 770 obvious mismatches. For instance, as shown in Fig. 1(a) in the manuscript, the movie entity “Mr. &
 771 Mrs. Smith” may be mistakenly aligned with the real-life actor couple “Will Smith and Mrs. Smith.”
 772 In this case, despite the similarity in names, the two refer to totally different entities with distinct
 773 multi-modal attributes. In other words, entities considered “neighbors” during cross-graph annotation
 774 (*e.g.*, retrieved via name-based lookups), may not actually be relevant in real-world scenarios. Such
 775 fine-grained errors require expert knowledge to distinguish, and may be undetected even during
 776 manual filtering. Both of the above reasons would lead to obvious mistakes, where the aligned entity
 777 pairs are totally unrelated. It is worth noting that such obvious mistakes prove the reasonableness of
 778 constructing incorrect correspondences to simulate real-world noise in our experiments.

779 Moreover, once the inter-graph entity pairs are associated, their corresponding attributes could be
 780 treated as matched. Therefore, as the by-product of the entity-attribute and entity-entity correspon-
 781 dences establishment, the association between inter-graph attributes might be mislead due to the
 782 aforementioned two kinds of annotation errors. In other words, E-A NC and E-E NC would further
 783 propagate to inter-graph attribute pairs, leading to noisy A-A correspondences. Beyond explicit
 784 annotation errors in A-A pairs, some attributes may contain textual typos or visual ambiguity (*e.g.*,
 785 noise under low-light conditions) in real-world scenarios, which would also lead to inter-graph A-A
 786 NC.

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810 C RELATED WORK
811812 In this section, we briefly review two topics related to this work, *i.e.*, multi-modal entity alignment
813 and learning with noisy correspondence.
814815 C.1 MULTI-MODAL ENTITY ALIGNMENT
816817 Multi-modal entity alignment aims to eliminate the discrepancy between heterogeneous MMKGs, so
818 that the equivalent entities from various MMKGs could be identified. Towards achieving this goal,
819 numerous MMEA approaches have been proposed, which typically involve the following two-stage
820 pipeline, namely, fuse the intra-entity attributes to form the representation for entities and perform
821 cross-graph alignment on the paired attributes and entities. According to their primary focus, most
822 existing approaches could be broadly grouped into two categories: i) fusion-centric methods (Chen
823 et al., 2023a; Huang et al., 2024a), which assign different weights according to the importance of
824 each attribute-specific attribute during the fusion stage. ii) alignment-centric methods (Li et al., 2023;
825 Lin et al., 2022b; Chen et al., 2024), which mitigate inter-graph discrepancy by maximizing the
826 similarity between associated pairs while minimizing that of mismatched ones across MMKGs during
827 the alignment stage.
828829 Among the existing approaches, REA (Pei et al., 2020) is the most relevant to our work, while
830 having the following remarkable differences. First, REA focuses on inter-graph uni-modality entity
831 alignment, whereas our work tackles multi-modal entity alignment, which involves mitigating
832 discrepancies not only between heterogeneous graphs but also across multi-modal attributes. Second,
833 REA only considers misalignment in entity-entity pairs, while our work comprehensively reveals and
834 studies the noisy correspondence at dual levels, namely, both the intra-entity and inter-graph. Third,
835 unlike REA solely concentrates on training-time robustness, our method further improves test-time
836 robustness, facilitating more precise cross-graph equivalent entity identification.
837838 C.2 LEARNING WITH NOISY CORRESPONDENCE
839840 Noisy correspondence refers to inherently irrelevant or relevant samples that are wrongly regarded
841 as associated (*a.k.a.*, false positive) or unassociated (*a.k.a.*, false negative), which is first revealed
842 and studied by (Huang et al., 2021; Yang et al., 2021). Considering that numerous applications
843 require paired data as input, including but not limited to visual instruction tuning (Huang et al., 2025),
844 vision-language pre-training (Huang et al., 2024b), object re-identification (Yang et al., 2022), and
845 graph matching (Lin et al., 2023), how to learn with noisy correspondence rooted in data pairs has
846 emerged as a new research direction, drawing increasing attention from both academia and industry.
847848 In this paper, we focus on mitigating the negative impact of the noisy positive correspondence
849 issue. Unlike most existing noisy correspondence studies that tackle the errors in correspondence
850 of a specific level (e.g., image-to-sample or pixel-to-pixel), this work delves into the multi-modal
851 entity alignment task and reveals the specific dual-level noisy correspondence (DNC) problem for
852 the first time. In brief, DNC refers to the noisy correspondence involving in the entity-attribute,
853 entity-entity, and attribute-attribute pairs, which misleads the multi-modal fusion and cross-graph
854 alignment processes. Therefore, it is desirable to customize a specific approach for MMEA with the
855 DNC problem.
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864 **D EVIDENTIAL LEARNING**
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866 In this section, we provide more details about the dually robust objective function in Eq. 9.
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868 Following (Sensoy et al., 2018; Han et al., 2022), the Dirichlet distribution is parameterized by
 869 $\alpha_i = [\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{iK}]$ and the probability density function in Eq. 11 is given by,

$$870 \quad D(\mathbf{p}_i \mid \alpha_i) = \begin{cases} \frac{1}{B(\alpha_i)} \prod_{j=1}^{\tilde{N}} p_{ij}^{\alpha_{ij}-1} & \text{for } \mathbf{p}_i \in \mathcal{S}^{\tilde{N}}, \\ 871 \quad 0 & \text{otherwise,} \end{cases} \quad (17)$$

873 where $B(\alpha_i)$ is the multinomial beta function, and $\mathcal{S}^{\tilde{N}}$ is the K -dimensional unit simplex.
 874

875 With the above derivation, MMEA aims to guide the query probability \mathbf{p}_i to approach the annotated
 876 correspondence \mathbf{y}_i . To achieve this, the uncertainty-based loss could be formulated as follows,

$$877 \quad \mathcal{L}_U(\alpha_i, \mathbf{y}_i) = \int \|\mathbf{y}_i - \mathbf{p}_i\|_2^2 \frac{1}{B(\alpha_i)} \prod_{j=1}^{\tilde{N}} p_{ij}^{\alpha_{ij}-1} d\mathbf{p}_i \\ 878 \\ 879 \\ 880 \\ 881 \\ 882 \\ 883 \\ 884 \\ 885 \\ 886$$

$$= \sum_{j=1}^{\tilde{N}} \left[(y_{ij} - \mathbb{E}[p_{ij}])^2 + \text{Var}(p_{ij}) \right] \\ = \sum_{j=1}^{\tilde{N}} \left(y_{ij} - \frac{\alpha_{ij}}{Q_i} \right)^2 + \frac{\alpha_{ij}(Q_i - \alpha_{ij})}{Q_i^2(Q_i + 1)}. \quad (18)$$

887 where $\mathbb{E}[p_{ij}]$ and $\text{Var}(p_{ij})$ are the expected value and the variance of p_{ij} , respectively. Following
 888 (Sensoy et al., 2018), the expected probability $\mathbb{E}[p_{ij}]$ could be estimated by $\frac{\alpha_{ij}}{Q_i}$. Intuitively,
 889 Eq. 18 encourages higher evidence for correct correspondence compared to mismatched ones, but also
 890 prevents excessive optimization when the overall evidence is limited. In other words, the probability
 891 $\frac{\alpha_{ij}}{Q_i}$ is proportional to the total evidence Q_i , thus limiting overconfidence in noisy correspondences.
 892 Specifically,

893 **Theorem 2.** *The uncertainty-aware probability $\frac{\alpha_{ij}}{Q_i}$ is upper bounded by $\frac{Q_i - K + 1}{Q_i}$, which is propor-
 894 tional to Q_i , i.e.,*

$$895 \quad \frac{\alpha_{ij}}{Q_i} \leq \frac{Q_i - K + 1}{Q_i} \propto Q_i. \quad (19)$$

896 *Proof.* According to the definition of evidence in Eq. 2, each correspondence satisfies $\alpha_{il} \geq 1$ for all
 897 $l \neq j$. Therefore, Q_i is lower-bounded by

$$901 \quad Q_i = \alpha_{ij} + \sum_{l \neq j} \alpha_{il} \geq \alpha_{ij} + (K - 1). \quad (20)$$

902 Rearranging the inequality yields,

$$903 \quad \alpha_{ij} \leq Q_i - (K - 1). \quad (21)$$

904 Then, dividing both sides by Q_i , we obtain the desired upper bound,

$$905 \quad \frac{\alpha_{ij}}{Q_i} \leq \frac{Q_i - K + 1}{Q_i}, \quad (22)$$

906 where the upper bound is proportional to Q_i , i.e., $\frac{Q_i - K + 1}{Q_i} \propto Q_i$. \square

918 E PROOF OF THEOREM 1
919920 In this section, we present detailed proofs for Theorem 1 in the main paper.
921922 **Theorem 3.** *A low uncertainty u_i does not necessarily imply that the highest belief is assigned to the
923 annotated correspondence, i.e., $y_{ij} = 1$,*

924
$$z_i \text{ with low } u_i \not\Rightarrow \arg \max \mathbf{b}_i = \arg \max \mathbf{y}_i. \quad (23)$$

925

926 *Proof.* We will prove the theorem by contradiction. Suppose that for *all* evidence vectors with low
927 uncertainty u_i , the highest belief is assigned to the annotated correspondence, i.e.,

928
$$\forall \mathbf{e}_i \text{ with low } u_i, \arg \max \mathbf{b}_i = \arg \max \mathbf{y}_i. \quad (24)$$

929

930 Let us now consider the two evidence vectors $\mathbf{e}_i^{(1)}$ and $\mathbf{e}_i^{(2)}$ defined as:
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$$e_{ij}^{(1)} = \begin{cases} Q_i, & \text{if } j = j_1, \\ 0, & \text{otherwise,} \end{cases} \quad e_{ij}^{(2)} = \begin{cases} Q_i, & \text{if } j = j_2, \\ 0, & \text{otherwise,} \end{cases} \quad (25)$$

933

934 where $j_1 \neq j_2$, and $Q_i > 0$ is fixed. According to Eq. 3, both evidence vectors yield the same total
935 evidence Q_i and hence the same uncertainty u_i . However, $\mathbf{e}_i^{(1)}$ suggests (i, j_1) is the most probable
936 correspondence, while $\mathbf{e}_i^{(2)}$ suggests (i, j_2) as the most plausible option. However, only one of them
937 might indicate the correct correspondence. Such an example contradicts the assumption that low
938 uncertainty invariably leads to the highest belief being assigned to the annotated correspondence j .
939 Consequently, the initial assumption is invalid. \square
940941 F MORE IMPLEMENTATION DETAILS
942943 F.1 DATASETS
944945 We place the detailed statistics of the five MMEA datasets used in our experiments in Table 4. Note
946 that, not every entity is paired with image-type attributes.
947948 Table 4: Details of MMEA datasets. “E-E pairs” indicates the number of cross-graph entity-entity
949 correspondences.
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Dataset	KG	Entity	Triples	Numeric	Image	E-E pairs
DBP15K _{ZH-EN}	ZH (Chinese)	19388	70414	8111	15912	15000
	EN (English)	19572	95142	7173	14125	15000
DBP15K _{JA-EN}	JA (Japanese)	19814	77214	5882	12739	15000
	EN (English)	19600	93484	6066	13741	15000
DBP15K _{FR-EN}	FR (French)	19661	105998	4547	10599	15000
	EN (English)	19993	115722	6422	13858	15000
ICEWS-WIKI	ICEWS	11047	3527881	-	33341	5058
	WIKI	15896	198257	-	47688	5058
ICEWS-YAGO	ICEWS	26863	4192555	-	80589	18824
	YAGO	22734	107118	-	68202	18824

962 F.2 NETWORKS
963964 In this section, we describe the network architectures used for encoding multi-modal attributes. For
965 the name and image attributes, we first employ the pre-trained CLIP ViT-L/14 (Radford et al., 2021)
966 model to obtain the features. Note that, all the experiments are conducted on the same visual and
967 textual features for fair comparison. Following (Chen et al., 2023a), we employ trainable fully
968 connected layers to project the image and name attributes into embedding vectors. Missing image
969 attributes are handled by assigning zero vectors as their embeddings.
970971 For the DBP15K benchmarks, the multi-modal attributes include structure, relation, numerical value,
972 image, name, and character attributes. Following (Xu et al., 2023), we adopt cross-modal graph

972 networks to encode the structure and relation attributes. For the numerical attribute, we employ the
 973 Bag-of-Words (BoW) model to encode the relations as fixed-length vectors. Besides, the character
 974 attributes are derived from character bigrams of the translated entity names.

975 For the ICEWS benchmarks, the multi-modal attributes include structure, image, and name attributes.
 976 Following (Chen et al., 2023a), we utilize Graph Attention Networks GAT to reduce the computational
 977 cost of encoding structural information, as the number of relation triplets in the ICEWS benchmark is
 978 significantly larger than that of the DBP15K benchmark.

980 F.3 DETAILS OF DUALLY ROBUST FUSION MODULE

982 To achieve robustness against intra-entity E-A NC, we propose a dually robust fusion that assigns
 983 low weights to unreliable attributes. Here, we provide more details on how to construct the initial
 984 subset for the greedy strategy in Eq. 7. Although some attributes are often weakly associated or even
 985 irrelevant to the entity, we assume that most attributes should be correctly associated, otherwise,
 986 the cross-graph correspondence would not hold. Based on this assumption, we adopt the following
 987 strategy for constructing the initial subset π_0 ,

$$988 \quad 989 \quad 990 \quad 991 \quad 992 \quad 993 \quad 994 \quad 995 \quad 996 \quad 997 \quad 998 \quad 999 \quad 1000 \quad 1001 \quad 1002 \quad 1003 \quad 1004 \quad 1005 \quad 1006 \quad 1007 \quad 1008 \quad 1009 \quad 1010 \quad 1011 \quad 1012 \quad 1013 \quad 1014 \quad 1015 \quad 1016 \quad 1017 \quad 1018 \quad 1019 \quad 1020 \quad 1021 \quad 1022 \quad 1023 \quad 1024 \quad 1025$$

$$|\pi_0| \geq \begin{cases} \left\lfloor \frac{M}{2} \right\rfloor + 1 & \text{if } M \geq 3 \\ 1 & \text{if } M \leq 2. \end{cases} \quad (26)$$

Such behavior ensures that the initial subset contains at least half or more of the multi-modal attributes.
 Notably, the formulation of the attribute uncertainty u_i^m and consensus c_i^m relies on correct cross-
 graph entity-entity correspondence. However, the cross-graph correspondences are mismatched when
 E-E NC occurs, leading to unreliable u_i^m and c_i^m . Thus, we employ DRF only to entities z_i that
 satisfy

$$(1 - u_i) + c_i \geq 1. \quad (27)$$

In other words, DRF is adopted to entities with reliable correspondences, which could prevent
 erroneous fusion.

1001 F.4 EVALUATION METRIC

1003 We evaluate multi-modal entity alignment using Hit@1 and Mean Reciprocal Rank (MRR) (Huang
 1004 et al., 2024a):

- 1005 • Hit@ k measures the percentage of cases where the ground-truth candidate appears in the
 1006 top- k retrieved entity.
- 1007 • Mean Reciprocal Rank (MRR) assesses the average inverse rank of the correct entity.

1009 F.5 DETAILS OF TEST-TIME REASONING MODULE

1011 In the manuscript, we propose a test-time reasoning module to capture the underlying connections
 1012 between attribute pairs. Here, we provide more details on the MLLM rethinking presented in Eq. 16.

1014 To reduce the computational cost of MLLM reasoning, we avoid rethinking attribute pairs that satisfy
 1015 the conditions as follows,

$$1016 \quad 1017 \quad 1018 \quad 1019 \quad 1020 \quad 1021 \quad 1022 \quad 1023 \quad 1024 \quad 1025 \quad \max(\mathbf{s}_i^m) \geq 0.2 \quad \vee \quad \max(\mathbf{s}_i^m) - \mathbf{s}_{ij}^m \geq 0.2, \quad \forall \mathbf{s}_{ij}^m \neq \max(\mathbf{s}_i^m) \quad (28)$$

Such behavior not only reduces the overhead of certain attribute pairs, but also encourages reasoning
 on unreliable pairs and thus improves MMEA performance. Notably, for the missing image attributes
 in the DBP15K benchmark, the corresponding rethinking scores are set to zero.

Taking the entity pair 7125 – 26134 as an example, we employ the following Chain-of-Thought for
 the Non-name and All-attributes setting.

For a given query entity z_i , the output results of the MLLM denotes as $\mathbf{o}_i = [o_{i1}; o_{i2}; \dots, o_{i10}]$,
 where each similarity satisfies $0 \leq o_{ij} \leq 10$. After that, \mathbf{o}_i is normalized as follow,

$$\hat{\mathbf{o}}_i = (\mathbf{o}_i - 5)/5. \quad (29)$$

1026 Such normalization ensures that the range of the MLLM output is consistent with that of the similarity
 1027 vector s_i in Eq. 2. The \hat{o}_i could then be employed to derive the rethinking scores via Eq. 16, *i.e.*,
 1028 $\oplus_{j \in \mathcal{T}_i^m} (\text{CoT} [x_i^m, (x'_j)^m, s_i^m]) = \hat{o}_i$.
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1030 **Chain-of-Thought for Non-name Setting**

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1032 **• Base Prompt:**

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1034 – Help me align or match entities of different knowledge graphs according to the given images
 and prior retrieval results.



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ID:7125



ID:26134

• Prior Results:

- Below are prior retrieval results focusing on visual similarity of the given images.
- Candidate Entities List which may be aligned with QUERY Entity (ID:7125) are shown in the following list [Format: ID Similarity]:
 - * 22646 0.40,
 - * 22946 0.36,
 - * 23688 0.30,
 - * 26364 0.26,
 - * 22619 0.20,
 - * 26052 0.14,
 - * 26134 0.10,
 - * 26518 0.02.

• Rethinking Image Similarity:

- The two provided images represent the query (ID:7125) and the candidate (ID:26134).
- Please evaluate the probability that the QUERY and the CANDIDATE belong to the same entity **STEP BY STEP**:
 1. Rethink the visual similarities based on the prior retrieval results and the given images.
 2. Analyze the similarities of detailed visual contents between the provided images.
 3. Consider the underlying connections between the given images.
- [Output Format]: [IMAGE SIMILARITY]= A out of 10, where A is in range [0,1,2,3,4,5,6,7,8,9,10], which represents the levels from VERY LOW to VERY HIGH.

NOTICE: You MUST output strictly in this format: [IMAGE SIMILARITY]= A out of 10.

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Chain-of-Thought for All-attributes Setting**• Base Prompt:**

- Help me align or match entities of different knowledge graphs according to the given names, images and prior retrieval results.



ID:7125 Name:African Development Fund



ID:26134 Name:African Development Bank

• Prior Results:

- Below are prior retrieval results focusing on visual and textual similarity of the given images and names, respectively.
- Candidate Entities List which may be aligned with QUERY Entity (ID:7125 Name:African Development Fund) are shown in the following list [Format: ID Name Similarity]:
 - * 26364 African Development Bank 0.42,
 - * 22619 Southern African Development Community 0.32,
 - * 26052 Joseph Ki-Zerbo 0.24,
 - * 22946 United Nations African Union Mission in Darfur 0.15,
 - * 23688 OHADA 0.07,
 - * 22646 Food and Agriculture Organization 0.05,
 - * 26134 Jorg Asmussen -0.02,
 - * 26518 Joyce Banda -0.08.

• Rethinking Image Similarity:

- The two provided images represent the query (ID:7125 Name:African Development Fund) and the candidate (ID:26134 Name:African Development Bank).
- Please evaluate the probability that the QUERY and the CANDIDATE belong to the same entity **STEP BY STEP**:
 1. Rethink the visual similarities based on the prior retrieval results and the given images.
 2. Analyze the similarities of detailed visual contents between the provided images.
 3. Consider the underlying connections between the given images.
- [Output Format]: [IMAGE SIMILARITY]= A out of 10, where A is in range [0,1,2,3,4,5,6,7,8,9,10], which represents the levels from VERY LOW to VERY HIGH.
NOTICE: You MUST output strictly in this format: [IMAGE SIMILARITY]= A out of 10.

• Rethinking Name Similarity:

- The two provided names represent the query (ID:7125 Name:African Development Fund) and the candidate (ID:26134 Name:African Development Bank).
- Based on the prior retrieval results and the given names, identify the similarities between the query entity and candidate entity.
- [Output Format]: [NAME SIMILARITY]= B out of 10, where B is in range [0,1,2,3,4,5,6,7,8,9,10], which represents the levels from VERY LOW to VERY HIGH.
NOTICE: You MUST output strictly in this format: [NAME SIMILARITY]= B out of 10.

1134 G MORE EXPERIMENT RESULTS

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 1136 In this section, we provide more experimental results of the proposed RULE. Unless otherwise
 1137 specified, all experiments are conducted on the ICEWS-WIKI dataset under the 50% DNC setting.
 1138

1139 G.1 RESULTS UNDER THE E-E, E-A AND A-A NOISY CORRESPONDENCE

1140
 1141 In the manuscript, we have carried out experiments under DNC settings. Here, we further provide
 1142 more results under single-type NC scenarios, *i.e.*, E-E, E-A, and A-A NC. From the results in
 1143 Table 5-6, Rule significantly outperforms all baselines across various settings and datasets.

1144
 1145 Table 5: Comparisons with state-of-the-art methods on Non-name benchmarks under NC setting
 1146 regarding the Hits and MRR metrics.

Setting	Method	ICEWS-WIKI			ICEWS-YAGO			DBP15K _{ZH-EN}			DBP15K _{JA-EN}			DBP15K _{FR-EN}			Avg.
		H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	
50%	EVA	4.4	6.9	5.7	0.1	0.1	0.2	17.4	30.6	23.6	21.9	34.9	28.1	18.9	31.1	24.9	12.5
	MCLEA	28.6	43.9	36.2	20.3	34.7	27.4	58.2	75.6	66.1	58.2	74.6	65.8	59.1	75.3	66.6	44.9
	XGEA	40.0	47.4	44.0	21.8	26.5	24.7	70.1	85.2	75.9	68.9	82.1	74.2	68.8	81.9	72.0	53.9
	MEAformer	46.0	62.6	53.6	33.6	48.3	40.7	68.9	83.1	75.3	65.0	80.5	72.1	66.8	81.4	73.5	56.1
	E-E	44.0	60.4	51.9	28.1	43.5	35.5	66.8	82.8	74.0	62.2	79.4	70.1	63.7	80.4	71.2	53.0
	UMAEA	44.0	60.4	51.9	28.1	43.5	35.5	66.8	82.8	74.0	62.2	79.4	70.1	63.7	80.4	71.2	53.0
	PMF	38.6	53.9	46.0	32.6	45.4	38.8	68.4	83.3	75.2	67.1	81.8	73.8	67.5	82.1	74.2	54.8
NC	HHEA	45.6	59.1	52.1	37.0	48.5	42.6	47.7	56.2	52.0	49.3	56.8	53.3	52.2	59.4	56.0	46.4
	Ours	61.0	72.5	66.6	48.7	59.5	54.0	73.6	86.1	79.3	71.6	84.6	77.6	71.5	84.7	77.6	65.3
	EVA	5.5	9.4	7.7	0.2	0.9	0.9	64.6	83.4	73.1	67.8	86.1	76.0	69.1	87.8	77.2	41.4
	MCLEA	34.4	51.9	42.9	24.0	39.1	31.5	75.3	89.6	81.7	75.4	90.4	82.0	76.2	90.9	82.7	57.1
	XGEA	40.0	47.9	44.3	22.4	27.9	25.7	76.1	91.2	82.6	75.8	91.2	82.5	76.0	92.1	83.3	58.1
	MEAformer	53.1	68.6	60.4	37.4	52.8	44.8	75.8	91.1	82.5	75.8	91.8	82.9	76.6	92.6	83.7	63.7
	E-A	50.7	67.5	58.5	34.3	49.0	41.6	72.3	89.8	80.1	72.2	90.2	80.2	73.8	91.6	81.7	60.7
50%	UMAEA	44.8	59.4	51.9	34.9	48.5	41.5	74.3	89.7	81.1	74.4	90.4	81.5	76.0	92.3	83.2	60.9
	PMF	49.9	63.7	56.4	35.8	48.9	42.1	46.4	60.8	53.6	48.1	61.2	54.6	52.4	64.0	58.3	46.0
	HHEA	62.6	75.5	68.7	47.7	59.1	53.5	80.3	92.4	85.7	79.1	92.5	84.9	80.1	93.2	85.9	70.0
	Ours	63.6	76.7	69.7	48.6	60.3	54.4	85.6	94.7	89.7	85.1	95.4	89.5	85.0	95.5	89.6	73.6
	EVA	29.3	41.1	35.1	5.5	10.3	8.1	70.2	86.4	77.5	72.3	89.1	79.8	73.6	90.4	80.9	50.2
	MCLEA	43.2	62.8	52.4	29.7	47.1	38.3	74.8	89.5	81.4	75.4	90.2	82.0	76.1	90.7	82.6	59.8
	XGEA	40.8	48.8	45.1	24.2	29.7	27.6	78.2	92.1	84.4	78.8	92.9	84.9	81.0	93.9	86.6	60.6
A-A	MEAformer	53.4	69.1	60.9	34.7	51.1	42.6	81.9	93.5	87.0	81.4	94.2	87.0	82.1	94.4	87.5	66.7
	UMAEA	49.6	68.0	58.0	29.5	45.8	37.6	77.8	92.1	84.3	78.3	92.3	84.5	79.3	93.5	85.6	62.9
	PMF	51.9	67.3	59.4	37.1	52.5	44.8	80.7	93.1	86.2	81.1	93.4	86.6	82.7	94.5	88.0	66.7
	HHEA	48.6	63.3	55.8	37.0	50.2	43.4	46.6	61.1	53.8	47.3	59.7	53.5	50.1	62.4	56.3	45.9
	Ours	63.6	76.7	69.7	48.6	60.3	54.4	85.6	94.7	89.7	85.1	95.4	89.5	85.0	95.5	89.6	73.6
	EVA	29.3	41.1	35.1	5.5	10.3	8.1	70.2	86.4	77.5	72.3	89.1	79.8	73.6	90.4	80.9	50.2
	MCLEA	43.2	62.8	52.4	29.7	47.1	38.3	74.8	89.5	81.4	75.4	90.2	82.0	76.1	90.7	82.6	59.8
NC	XGEA	40.8	48.8	45.1	24.2	29.7	27.6	78.2	92.1	84.4	78.8	92.9	84.9	81.0	93.9	86.6	60.6
	MEAformer	53.4	69.1	60.9	34.7	51.1	42.6	81.9	93.5	87.0	81.4	94.2	87.0	82.1	94.4	87.5	66.7
	UMAEA	49.6	68.0	58.0	29.5	45.8	37.6	77.8	92.1	84.3	78.3	92.3	84.5	79.3	93.5	85.6	62.9
	PMF	51.9	67.3	59.4	37.1	52.5	44.8	80.7	93.1	86.2	81.1	93.4	86.6	82.7	94.5	88.0	66.7
	HHEA	48.6	63.3	55.8	37.0	50.2	43.4	46.6	61.1	53.8	47.3	59.7	53.5	50.1	62.4	56.3	45.9
	Ours	63.6	76.7	69.7	48.6	60.3	54.4	85.6	94.7	89.7	85.1	95.4	89.5	85.0	95.5	89.6	73.6
	EVA	29.3	41.1	35.1	5.5	10.3	8.1	70.2	86.4	77.5	72.3	89.1	79.8	73.6	90.4	80.9	50.2
50%	MCLEA	43.2	62.8	52.4	29.7	47.1	38.3	74.8	89.5	81.4	75.4	90.2	82.0	76.1	90.7	82.6	59.8
	XGEA	40.8	48.8	45.1	24.2	29.7	27.6	78.2	92.1	84.4	78.8	92.9	84.9	81.0	93.9	86.6	60.6
	MEAformer	53.4	69.1	60.9	34.7	51.1	42.6	81.9	93.5	87.0	81.4	94.2	87.0	82.1	94.4	87.5	66.7
	UMAEA	49.6	68.0	58.0	29.5	45.8	37.6	77.8	92.1	84.3	78.3	92.3	84.5	79.3	93.5	85.6	62.9
	PMF	51.9	67.3	59.4	37.1	52.5	44.8	80.7	93.1	86.2	81.1	93.4	86.6	82.7	94.5	88.0	66.7
	HHEA	48.6	63.3	55.8	37.0	50.2	43.4	46.6	61.1	53.8	47.3	59.7	53.5	50.1	62.4	56.3	45.9
	Ours	63.6	76.7	69.7	48.6	60.3	54.4	85.6	94.7	89.7	85.1	95.4	89.5	85.0	95.5	89.6	73.6
A-A	EVA	29.3	41.1	35.1	5.5	10.3	8.1	70.2	86.4	77.5	72.3	89.1	79.8	73.6	90.4	80.9	50.2
	MCLEA	43.2	62.8	52.4	29.7	47.1	38.3	74.8	89.5	81.4	75.4	90.2	82.0	76.1	90.7	82.6	59.8
	XGEA	40.8	48.8	45.1	24.2	29.7	27.6	78.2	92.1	84.4	78.8	92.9	84.9	81.0	93.9	86.6	60.6
	MEAformer	53.4	69.1	60.9	34.7	51.1	42.6	81.9	93.5	87.0	81.4	94.2	87.0	82.1	94.4	87.5	66.7
	UMAEA	49.6	68.0	58.0	29.5	45.8	37.6	77.8	92.1	84.3	78.3	92.3	84.5	79.3	93.5	85.6	62.9
	PMF	51.9	67.3	59.4	37.1	52.5	44.8	80.7	93.1	86.2	81.1	93.4	86.6	82.7	94.5	88.0	66.7
	HHEA	48.6	63.3	55.8	37.0	50.2	43.4	46.6	61.1	53.8	47.3	59.7	53.5	50.1	62.4	56.3	45.9
NC	Ours	63.6	76.7	69.7	48.6	60.3	54.4	85.6	94.7	89.7	85.1	95.4	89.5	85.0	95.5	89.6	73.6
	EVA	29.3	41.1	35.1	5.5	10.3	8.1	70.2	86.4	77.5	72.3	89.1	79.8	73.6	90.4	80.9	50.2
	MCLEA	43.2	62.8	52.4	29.7	47.1	38.3	74.8	89.5	81.4	75.4	90.2	82.0	76.1	90.7	82.6	59.8
	XGEA	40.8	48.8	45.1	24.2	29.7	27.6	78.2	92.1	84.4	78.8	92.9	84.9	81.0	93.9	86.6	60.6
	MEAformer	53.4	69.1	60.9	34.7	51.1	42.6	81.9	93.5	87.0	81.4	94.2	87.0	82.1	94.4	87.5	66.7
	UMAEA	49.6	68.0	58.0	29.5	45.8	37.6	77.8	92.1	84.3	78.3	92.3	84.5	79.3	93.5	85.6	62.9
	PMF	51.9	67.3	59.4	37.1	52.5	44.8	80.7	93.1	86.2	81.1	93.4	86.6	82.7	94.5	88.0	66.7
	HHEA	48.6	63.3	55.8	37.0	50											

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G.2 PARAMETER ANALYSIS

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As mentioned in the manuscript, the hyperparameters in RULE include the trade-off parameter λ in Eq. 9, the temperature τ in Eq. 2, and the threshold β in Eq. 8. Here, we conduct a detailed parameter analysis to study their individual impacts. c in Fig. 7, one could have the following conclusions: i) RULE exhibits stable performance when λ is within the range of $[2e-5, 5e-4]$, τ within $[0.05, 0.2]$, and β within $[0.2, 0.4]$. ii) when λ is excessively large, the model would overemphasize the regularization term in Eq. 13, resulting in performance drop.

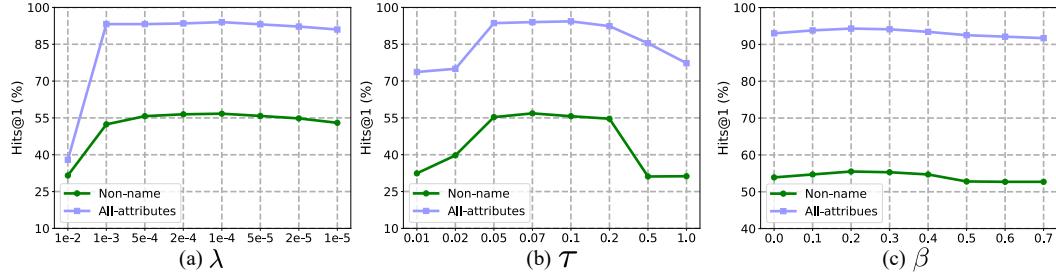
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Figure 7: The parameter analysis of the trade-off parameter λ in Eq. 9, the temperature τ in Eq. 2, and the threshold β in Eq. 8.

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G.3 MORE ABLATION STUDIES

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To further verify the effectiveness of the proposed RULE, we conduct additional ablation studies of the loss terms and the test-time rectification module. Specifically, we investigate the loss terms and the pairs division module in Table 7, resulting in the following conclusions. First, the regularization loss penalizes the evidence of negative pairs, thereby enhancing the performance. Second, employing tailored strategies to either \mathcal{S}_I or \mathcal{S}_U would contribute to the performance improvement.

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Table 7: Ablation study of the loss terms and the correspondence division module on the ICEWS-WIKI benchmark.

Setting	L_{DR}	L_{Reg}	Non-name			All-attributes		
			Hits@1	Hits@5	MRR	Hits@1	Hits@5	MRR
✓			55.8	68.3	61.8	93.0	97.7	96.1
✓	✓		56.5	68.6	62.3	94.0	97.7	95.7
w/o \mathcal{S}_I			55.6	67.8	61.5	93.5	97.4	95.4
w/o \mathcal{S}_U			54.8	67.9	61.1	93.5	97.2	95.2

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As discussed in the manuscript, the test-time reasoning module would help to capture the underlying connection and thus boost the performance. Here, we carry out more ablation studies about the test-time reasoning module. From the results in Table 8-9, the proposed TTR module significantly improves performance on both the Non-name and All-attribute benchmarks. It is worth noting that, despite the missing image attributes in the DBP15K benchmark, Rule still achieves stable performance gains.

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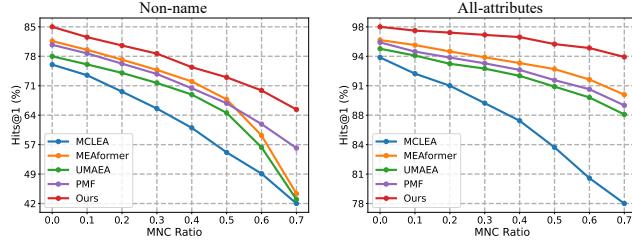
Table 8: Ablation study of TTR module on Non-name benchmarks under different DNC settings.

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Setting	Method	ICEWS-WIKI			ICEWS-YAGO			DBP15K _{ZH-EN}			DBP15K _{JA-EN}			DBP15K _{FR-EN}		
		H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR
Inherent DNC	w/o TTR	62.1	76.6	68.8	46.4	59.6	53.0	85.5	94.8	89.7	85.0	95.3	89.4	85.2	95.4	89.7
	w TTR	64.2	76.7	70.0	48.8	60.5	54.6	85.6	94.8	89.7	85.2	95.4	89.6	85.1	95.4	89.6
20% DNC	w/o TTR	60.3	74.7	67.0	46.2	58.6	52.2	80.9	92.0	85.9	80.4	92.2	85.6	80.6	92.2	85.8
	w TTR	62.4	75.1	68.5	48.3	59.5	53.9	81.1	92.0	86.0	80.5	92.2	85.6	80.5	92.2	85.8
50% DNC	w/o TTR	56.5	68.6	62.3	44.5	56.0	50.3	73.2	86.0	79.0	71.5	84.7	77.6	71.3	84.7	77.5
	w TTR	58.2	69.7	63.6	46.9	57.4	52.0	73.4	85.9	79.2	71.8	84.9	77.8	71.4	84.8	77.5

1242 Table 9: Ablation study of TTR module on All-attributes benchmarks under different DNC settings.
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Setting	Method	ICEWS-WIKI			ICEWS-YAGO			DBP15K _{ZH-EN}			DBP15K _{JA-EN}			DBP15K _{FR-EN}		
		H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR	H@1	H@5	MRR
Inherent DNC	w/o TTR	96.7	98.9	97.8	95.2	98.5	96.7	97.4	99.4	98.3	99.2	99.9	99.5	99.7	100.0	99.8
	w TTR	98.9	99.2	99.1	97.6	98.8	98.2	98.3	99.5	98.8	99.3	99.9	99.6	99.8	100.0	99.9
20% DNC	w/o TTR	95.8	98.4	97.1	95.1	98.4	96.6	96.4	98.8	97.5	98.8	99.8	99.3	99.6	100.0	99.8
	w TTR	98.3	98.9	98.6	97.5	98.7	98.1	97.6	99.1	98.3	99.1	99.9	99.5	99.8	100.0	99.9
50% DNC	w/o TTR	94.0	97.7	95.7	94.3	97.8	95.9	94.9	97.9	96.2	98.0	99.6	98.7	99.3	99.9	99.6
	w TTR	97.7	98.3	98.0	97.0	98.2	97.6	96.3	98.1	97.2	98.7	99.7	99.1	99.7	100.0	99.8

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1251 G.4 MORE EXPERIMENTS UNDER VARIOUS DNC RATIOS1253 In the manuscript, we have carried out the experiments under various DNC ratios on the ICEWS-
1254 WIKI dataset. Here, we provide more experiments under various DNC ratios on the DBP15K_{ZH-EN}
1255 dataset. As shown in Fig 8, the proposed RULE significantly outperforms all the baselines.
12561266 Figure 8: Performance with various DNC ratios on the DBP15K_{ZH-EN} dataset.
12671268 G.5 MORE ANALYTIC STUDY OF DUALLY ROBUST FUSION
12691270 We conduct more analytic studies on the ICEWS-WIKI dataset to verify that the DRF achieves
1271 better performance by estimating the correct correspondence during the test time. From the results
1272 in Table 10, the estimated correspondence achieves higher accuracy compared to the vanilla fusion
1273 method, *i.e.*, simple concatenation. Consequently, Rule benefits from the estimated correspondence
1274 and the DRF module, resulting in better performance. Note that, different from previous fusion
1275 methods (Lin et al., 2024; 2022a), the proposed DRF further explores robust fusion on the reasoning
1276 scores output by the MLLMs.1277 Table 10: The analytic study of the estimated correspondence on the ICEWS-WIKI dataset.
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Setting	Non-name			All-attributes		
	Hits@1	Hits@5	MRR	Hits@1	Hits@5	MRR
w/o DRF	51.4	66.7	58.6	93.0	97.3	94.9
Estimated	55.3	65.0	60.2	94.1	97.4	95.7
w DRF	56.5	68.6	62.3	94.0	97.7	95.7

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G.6 MORE ANALYTIC STUDY OF INTER-GRAPH LEARNING

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As discussed in 3.3 and G.3, the proposed RULE improves the performance by adopting the tailored strategies to the divided subsets, *i.e.*, \mathcal{S}_U , \mathcal{S}_I and \mathcal{S}_C . Here, we carry out more analytic studies about the statistics of the three kinds of pairs. From the results in Fig. 9 and Table 11, one could observe that the proposed correspondence division module effectively distinguishes the three subsets and thus boosts the performance. Different from previous discrepancy elimination methods (Gong et al., 2021; 2024; 2022), the proposed DRL not only divides correspondences into positive and negative, but also further distinguishes different types of negatives and adapts the tailored strategy to mitigate the negative impacts of them.

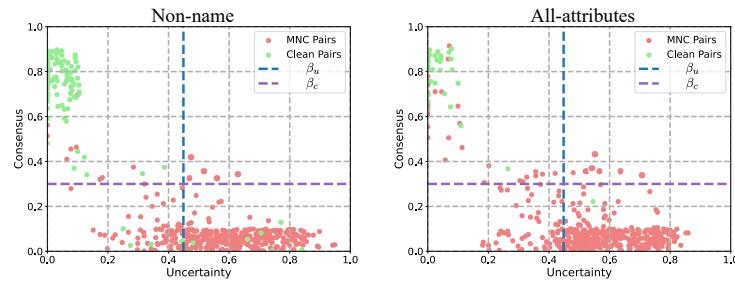
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Figure 9: Quantitative analysis of the uncertainty and consensus on the integrated entity.

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Table 11: Statistics of three kinds of pairs.

Entity	\mathcal{S}_C	\mathcal{S}_I	\mathcal{S}_U
Non-name	14.8%	33.9%	51.3%
All-attributes	14.1%	37.0%	48.9%

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G.7 RESULTS OF VARIOUS MLLM

To verify the generalizability of the TTR module, we conduct additional experiments on the MLLM with various architectures and parameter scales. Specifically, we evaluate the performance of the TTR module on Qwen2.5-VL models with 3B, 7B, and 72B parameters, as well as on LLaVA-1.6 (Liu et al., 2023) with 34B parameters. As shown in Table 12, Rule achieves consistent performance improvements across various model architectures and parameter scales, demonstrating the effectiveness of the proposed TTR module.

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Table 12: Performance comparison of different MLLMs on Non-name and All-attributes settings.

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Methods	Non-name			All-attributes		
	Hits@1	Hits@5	MRR	Hits@1	Hits@5	MRR
w/o TTR	56.5	68.6	62.3	94.0	97.7	95.7
Qwen2.5-VL 3B	57.2	68.8	62.7	96.5	98.2	97.3
Qwen2.5-VL 7B	57.4	69.0	62.9	97.1	98.2	97.6
Qwen2.5-VL 72B	58.2	69.7	63.6	97.7	98.3	98.0
LLaVA-1.6 34B	57.0	68.8	62.6	95.5	97.9	96.6

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G.8 COMPLEXITY ANALYSIS OF TEST-TIME CORRESPONDENCE REASONING

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To assess the efficiency of the TTR module, we report the time cost and memory cost across various parameter scales of Qwen2.5-VL. Note that all experiments were conducted on the NVIDIA RTX 3090 GPUs. From the results in Table 13, one can observe that employing Qwen2.5-VL 3B or 7B for TTR leads to a considerable performance boost, with up to a 5 \times speedup. Furthermore, when resources are limited, RULE can be deployed without MLLMs. Even without TTR, RULE still significantly outperforms the strongest baselines, achieving 56.5% vs. 43.9% (best-performing

baseline) in the “Non-name” setting and 94.0% vs. 91.9% (best-performing baseline) in the “All-attributes” setting. Moreover, the lightweight solutions substantially reduce the memory cost as well.

Table 13: Experiment results for the complexity analysis on the ICEWS-WIKI dataset.

Methods	Non-name		All-attributes		Memory Consumption
	H@1	Time Cost	H@1	Time Cost	
w/o TTR	56.5	103	94.0	109	1GPU \times ~16GB
Qwen2.5-VL 3B	57.2	2122	96.5	1008	1GPU \times ~8GB
Qwen2.5-VL 7B	57.4	2690	97.1	1329	1GPU \times ~8GB
Qwen2.5-VL 72B	58.2	10043	97.7	4373	8GPU \times ~20GB

G.9 RESULTS OF CROSS-ENTROPY IMPLEMENTATION

As discussed in (Sensoy et al., 2018), the uncertainty-based loss could be implemented using either the Mean Squared Error (MSE) loss or the Cross-Entropy (CE) loss. In Appendix D, we derive the uncertainty-based formulation based on MSE loss, which also serves as the foundation of the proposed dually robust loss in Eq. 11. Here, we derive the dually robust loss using the cross-entropy formulation as follows,

$$\begin{aligned} \mathcal{L}_{DR}(\alpha_i, \hat{y}_i) &= \mathbb{I}(u_i \leq \beta_u) \int \left[\sum_{j=1}^{\tilde{N}} -\hat{y}_{ij} \log(p_{ij}) \right] D(\mathbf{p}_i \mid \alpha_i) d\mathbf{p}_i \\ &= \mathbb{I}(u_i \leq \beta_u) \sum_{j=1}^{\tilde{N}} \hat{y}_{ij} (\psi(Q_i) - \psi(\alpha_{ij})), \end{aligned} \quad (30)$$

where $\psi(\cdot)$ is the digamma function. To further investigate the effectiveness of the proposed DRL, we conduct additional experiments of the cross-entropy implementation based on Eq. 30. From the results in Table 14, both implementations based on MSE and CE losses achieve competitive performance compared to other baselines shown in Tables 1 and 2.

Table 14: Comparisons of RULE with MAE and CE objectives under the DNC setting.

Objective	Setting	Non-name			All-attributes		
		Hits@1	Hits@5	MRR	Hits@1	Hits@5	MRR
MAE	Inherent DNC	62.1	76.6	68.8	96.7	98.9	97.8
	20% DNC	60.3	74.7	67.0	95.8	98.4	97.1
	50% DNC	56.5	68.6	62.3	94.0	97.7	95.7
CE	Inherent DNC	61.7	76.2	68.6	96.3	98.8	97.5
	20% DNC	59.6	74.0	66.3	95.9	98.6	97.2
	50% DNC	54.4	68.5	61.1	92.5	97.0	94.5

1404 G.10 MORE ANALYTIC STUDY OF THE RELIABILITY
14051406 Here, we explore alternative strategies that dynamically manipulate the weights assigned to uncertainty
1407 u_i and consensus c_I in Eq. 1, *i.e.*,

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$$w_i = (1 - u_i) \alpha + c_i(1 - \alpha) \quad (31)$$

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1410 where α is the balance parameter. From the results in Table 15, one could have the following
1411 conclusions: i) employ either uncertainty ($\alpha = 1$) or consensus ($\alpha = 0$) is inadequate in identifying
1412 DNC, leading to degraded performance; ii) thanks to the normalization in Eq. 2 and Eq. 5, a simple
1413 linear combination of uncertainty and consensus could yield considerable performance improvements.
1414 The results above indicate that our design for combining uncertainty and consensus is reasonable and
1415 effective.1416 Table 15: Analytic Experiments of the weighting strategy on the ICEWS-WIKI dataset under the
1417 “Non-name” setting.

α	H@1	H@5	MRR
0	51.8	62.4	57.0
0.2	53.4	64.4	58.7
0.4	54.9	66.5	60.4
0.5	56.5	68.6	62.3
0.6	55.1	67.1	60.8
0.8	53.9	66.6	60.0
1.0	52.5	65.7	58.8

1428 G.11 MORE EXPERIMENTS ON VARIOUS BACKBONES
14291430 The proposed RULE is a model-agnostic method, which could be employed in any mainstream vision-
1431 language backbones. To verify this, we further verify the effectiveness of RULE in various backbones.
1432 Specifically, we adopt SigLIP (Zhai et al., 2023) and BLIP (Li et al., 2022) as the backbones for
1433 extracting image and text features. Notably, we do not employ MLLM-based reasoning in the
1434 experiments for fairness and select the best-performing baselines from Tables 1-2 for comparisons.
1435 The results in Table 16 demonstrate that RULE outperforms all baselines across various backbones,
1436 which confirms the generality and effectiveness of RULE.1437 Table 16: Experiment results on the ICEWS-WIKI dataset with SigLIP and BLIP as backbone.
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Setting	Methods	SigLIP		BLIP	
		Non-name	All-attributes	Non-name	All-attributes
Inherent DNC	MEAformer	45.3	93.8	33.4	95.1
	UMAEA	41.5	92.0	31.5	93.6
	PMF	45.2	92.3	30.8	95.3
	Ours	55.2	95.2	39.1	96.8
20% DNC	MEAformer	42.2	89.0	28.9	95.0
	UMAEA	39.4	89.3	26.8	91.3
	PMF	37.8	89.0	26.6	89.7
	Ours	52.3	94.5	34.9	96.7
50% DNC	MEAformer	32.2	88.5	19.6	93.2
	UMAEA	27.4	85.5	15.3	89.5
	PMF	28.1	83.9	16.8	80.9
	Ours	45.4	93.2	24.4	97.1

1458 G.12 MORE EXPERIMENTS ON FB15K-DB15K AND FB15K-YAGO15K BENCHMARKS
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1460 Here, we conduct additional experiments on the FB15K-DB15K and FB15K-YAGO15K benchmarks
 1461 with their inherent handcrafted features. Notably, the raw data for these two datasets is no longer
 1462 available, so we don't employ MLLM for reasoning at test-time. Here, we select the best-performing
 1463 baselines from Tables 1-2 for comparisons. The results in Table 17-18 demonstrate that RULE
 1464 outperforms all baselines on FB15K-DB15K and FB15K-YAGO15K datasets under the settings with
 1465 various DNC ratios.

1466 Table 17: Experiment results on the FB15K-DB15K dataset under the “Non-name” setting. “Inherent
 1467 DNC” refers to the setting without any additional injected noise.

Setting	Methods	H@1	H@5	MRR
Inherent DNC	MEAformer	40.8	62.7	51.0
	UMAEA	40.8	62.0	50.7
	PMF	43.0	65.3	53.2
	Ours	44.4	63.6	53.8
20% DNC	MEAformer	28.1	49.7	39.6
	UMAEA	29.5	51.4	41.1
	PMF	29.6	51.4	41.2
	Ours	36.0	56.0	45.9
50% DNC	MEAformer	12.0	26.6	20.7
	UMAEA	15.0	32.1	24.7
	PMF	15.1	31.7	24.6
	Ours	20.3	36.9	28.9

1481 Table 18: Experiment results on the FB15K-YAGO15K dataset under the “Non-name” setting.

Setting	Methods	H@1	H@5	MRR
Inherent DNC	MEAformer	31.8	50.5	40.7
	UMAEA	31.2	50.9	40.3
	PMF	34.6	55.0	44.3
	Ours	38.9	55.7	47.0
20% DNC	MEAformer	20.5	38.1	30.1
	UMAEA	22.6	40.0	31.8
	PMF	23.7	42.0	33.2
	Ours	31.7	49.0	40.4
50% DNC	MEAformer	10.3	20.7	16.2
	UMAEA	11.9	24.0	18.6
	PMF	11.3	22.8	17.9
	Ours	17.9	31.9	25.1

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G.13 DISCUSSIONS WITH ACTIVE LEARNING PARADIGM

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One intuitive approach to address DNC is to introduce additional expert annotations and the active learning paradigm (Brame, 2016) might be the most representative paradigm. However, we argue that active learning is inadequate for addressing DNC for the following reasons:

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- Existing MMEA-oriented active learning methods (Xu et al., 2024) focus on annotating E-E correspondences, which have not explored the establishment of E-A and A-A associations. As a result, even with expert involvement, active learning is unable to solve E-A or A-A NC. It is important to emphasize that these two types of NC are highly practical in real-world scenarios, with their total proportion exceeding 40% in commonly-used ICEWS benchmarks.
- Active learning requires expert involvement, which is time-consuming and labor-intensive. In contrast, RULE provides a robust learning paradigm in an automated manner. Moreover, even after careful manual annotation, over 50% of the data suffer from DNC challenge in real-world datasets, which indicates that expert annotation remains prone to errors.
- Active learning selects the most valuable samples for human annotation to improve model performance. However, most active learning methods require an initial set of labeled data for value model training before conducting active learning. As a result, the DNC challenge is still inevitable in the initial set, thus degrading the performance of the value model and then undermining the active learning.

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G.14 DISCUSSIONS WITH RELATED WORKS

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Recently, several related studies have explored to address noise issues in MMEA task. Among them, DESAlign (Wang et al., 2024) might be the most relevant to our work. However, DESAlign and the proposed RULE differ in the following aspects: i) Different Motivations. DESAlign focuses on addressing disparities in attribute counts or the absence of certain modalities, while RULE aims to mitigate the negative impacts of E-E NC, E-A NC, and A-A NC on intra-entity attribute fusion and inter-graph discrepancy elimination; ii) Different Key Ideas. DESAlign employs a robust semantic learning to mitigate semantic inconsistency and interpolates missing modality through a propagation strategy. In contrast, RULE not only adopts an intra-entity attribute fusion and an inter-graph loss to achieve robustness against DNC, but also leverages the test-time reasoning module to mine underlying associations between attributes and thus guarantees more accurate equivalent entity identification.

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G.15 DISCUSSIONS ABOUT THE RELIABILITY ESTIMATION

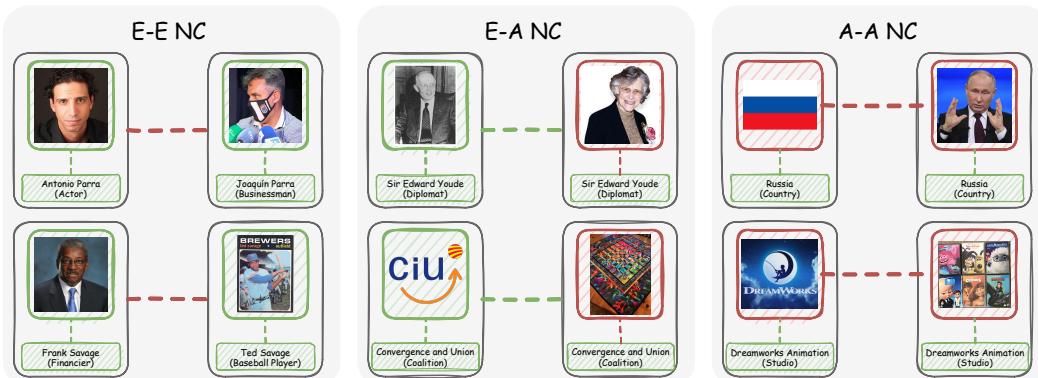
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To alleviate the negative impact of DNC, it is desirable to establish a unified reliability principle so that both intra-entity and inter-graph NC could be identified. An intuitive approach is to estimate the reliability of inter-graph pairs by leveraging intra-entity pairs, as intra-entity NC may further contaminate inter-graph attribute pairs. However, such a vanilla internal-to-external approach is limited, since some mismatched entity pairs may result from annotation errors (as shown in Fig. 1 (a)) rather than intra-entity NC. To remedy this, we explore a novel external-to-internal estimation paradigm. On the one hand, the reliability of inter-graph correspondences could be directly assessed. On the other hand, since intra-entity NC would inevitably lead to mismatched inter-graph attribute pairs, these attribute pairs could be employed to estimate the reliability of intra-entity associations.

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1566 H CASE STUDY OF DUAL-LEVEL NOISY CORRESPONDENCE

1568 In this section, we showcase some DNC examples from the ICEWS benchmark in Fig. 10.



1583 Figure 10: Examples of DNC in the ICEWS benchmark.

1586 I CASE STUDY OF TEST-TIME CORRESPONDENCE REASONING

1589 In this section, we visualize the test-time reasoning process in TTR module. Specifically, we present
1590 several representative successful and failure cases, and further provide visualizations of performing
1591 TTR on ambiguous and missing attributes.

1592 **Successful Cases:** As illustrated in Fig. 11-12, the TTR module uncovers the underlying connections
1593 between image and name attribute pairs, thus boosting performance.

1594 **Failure Cases:** Although the TTR module could rectify MMEA results by mining implicit associations,
1595 we still observe several failure cases as illustrated in Fig. 13. According to the visualization
1596 results, we attribute these failure cases to the following two reasons. On the one hand, the MLLM
1597 might lack enough knowledge to perform correct reasoning. For example, when the query refers
1598 to the abbreviation of Toronto Metropolitan University, the MLLM does not possess the necessary
1599 background knowledge and thus cannot find the corresponding university logo. On the other hand,
1600 the MLLM might sometimes resort to shortcut cues rather than performing deep reasoning, leading
1601 to undiscovered associations even after reasoning. For instance, the MLLM may still rely on visual
1602 similarity when identifying coastal cities, while ignoring underlying semantic cues that indicate the
1603 corresponding country.

1604 **Ambiguous Cases:** TTR could perform correct test-time reasoning even when the attributes are
1605 ambiguous. As illustrated in Fig. 14, the image attributes of the candidates are highly similar.
1606 Nevertheless, the TTR module is able to uncover meaningful associations, such as identifying the
1607 same individual across different scenes or recognizing variations in clothing and appearance.

1608 **Missing Cases:** Multimodal knowledge graphs often contain missing attributes, particularly missing
1609 images (Chen et al., 2023b). We present several visualizations in Fig. 15 to illustrate how TTR
1610 module behaves under missing attributes scenario. From the results, we observe that when the image
1611 attribute of the correct candidate is missing, the TTR module assigns low similarity scores to other
1612 images and instead relies on the available and more reliable textual attributes for more accurate
1613 reasoning. As a result, the TTR module could still achieve correct alignment even in the presence of
1614 missing attributes, demonstrating the effectiveness and robustness of RULE.

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1622	Query	Candidate	Prior Rank: 7 Final Rank: 1
1623	Charles Leslie Falconer	Charles Leslie Falconer Baron Falconer of Thoroton	MLLM Analysis: The names "Charles Leslie" and "Charles Leslie, Baron Falconer of Thoroton" share the same first and middle names, indicating a high level of similarity, but the additional title in the candidate's name suggests they may not be the exact same person. [Name SIMILARITY] = 8 out of 10.
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1627	Query	Candidate	Prior Rank: 3 Final Rank: 1
1628	Andrey Vladimirovich Kozyrev	Andrei Kozyrev	MLLM Analysis: The names "Andrey Vladimirovich Kozyrev" and "Andrei Kozyrev" share the same surname and a similar first name, with "Andrey" and "Andrei" being variations of the same name. The middle name is the only difference, which is not present in the candidate's name. [Name SIMILARITY] = 9 out of 10.
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1632	Query	Candidate	Prior Rank: 5 Final Rank: 1
1633	Peronist Youth	Peronism	MLLM Analysis: The names "Peronist Youth" and "Peronism" share a common root "Peron," indicating a strong thematic connection related to the political movement associated with Juan Perón, despite the specific focus of "Peronist Youth" being on the younger members of the movement. [Name SIMILARITY] = 7 out of 10.
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1637	Query	Candidate	Prior Rank: 8 Final Rank: 1
1638	Brick City	Newark New Jersey	MLLM Analysis: "Brick City" is a widely recognized nickname for "Newark, New Jersey", indicating a strong cultural and contextual link between the two names. While they differ lexically, they often refer to the same place, suggesting a high degree of name similarity. [Name SIMILARITY] = 8 out of 10.
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1644	Figure 11: Reasoning process for the name attributes in test-time reasoning.		
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1649	Query	Candidate	Prior Rank: 2 Final Rank: 1
1650			MLLM Analysis: The first image is Qatar's emblem, featuring a sailboat, palm trees, and desert, while the second highlights its location on a map. Both images effectively represent Qatar, with the emblem reflecting cultural identity and the map showing its geographical position. [IMAGE SIMILARITY] = 9 out of 10.
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1654	Query	Candidate	Prior Rank: 5 Final Rank: 1
1655			MLLM Analysis: The individuals in both images appear to be the same person at different stages of life, as indicated by the consistent facial features and posture. The attire in both images suggests a formal military or royal role, further supporting the likelihood that they represent the same entity. [IMAGE SIMILARITY] = 8 out of 10.
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1660	Query	Candidate	Prior Rank: 6 Final Rank: 1
1661			MLLM Analysis: The emblem in the first image matches the logo displayed in the bottom left corner of the second image, indicating they belong to the same institution. Additionally, the text Pontificia Universidad Católica de Chile in both images confirms their connection to the same entity. [IMAGE SIMILARITY] = 9 out of 10.
1662			
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1666	Query	Candidate	Prior Rank: 5 Final Rank: 1
1667			MLLM Analysis: The individual in both images appears to be the same person, as they share similar facial features, hair color, and style. The slight differences in expression and context do not detract significantly from the strong visual resemblance. [IMAGE SIMILARITY] = 9 out of 10.
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1672	Figure 12: Reasoning process for the image attributes in test-time reasoning.		
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Figure 12: Reasoning process for the image attributes in test-time reasoning.

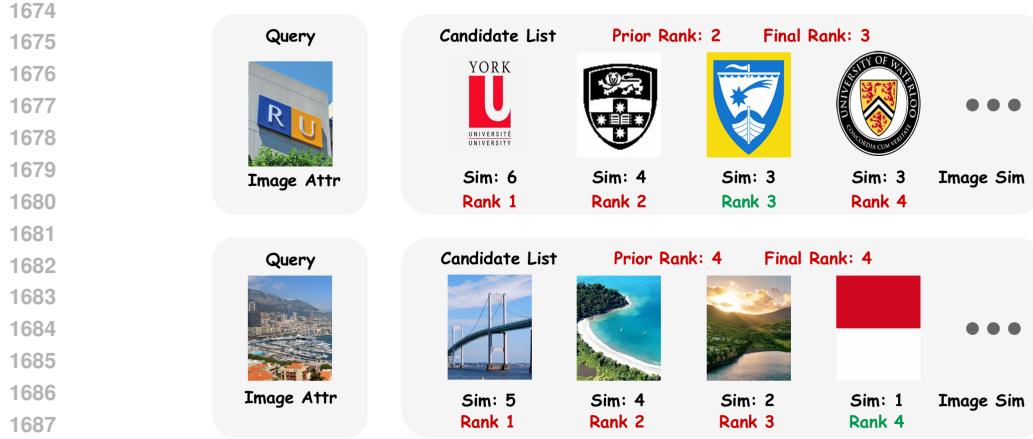


Figure 13: Failure cases in test-time reasoning.

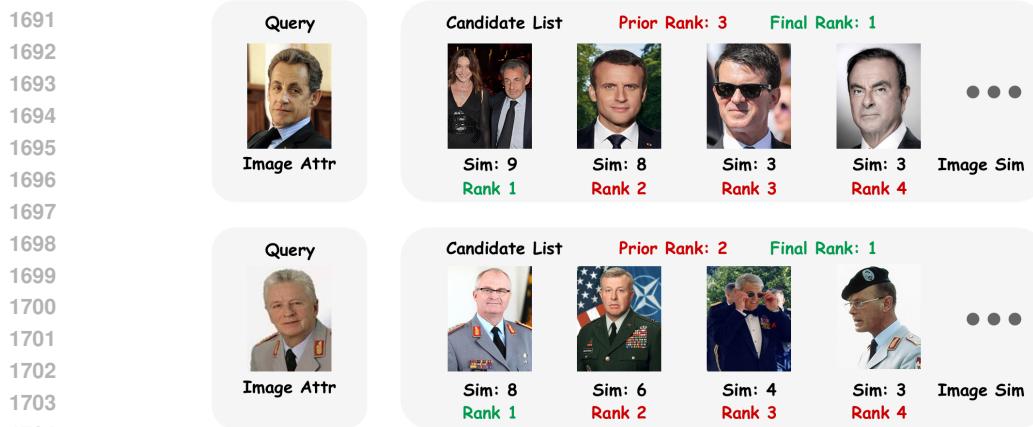


Figure 14: Ambiguous attributes in test-time reasoning.

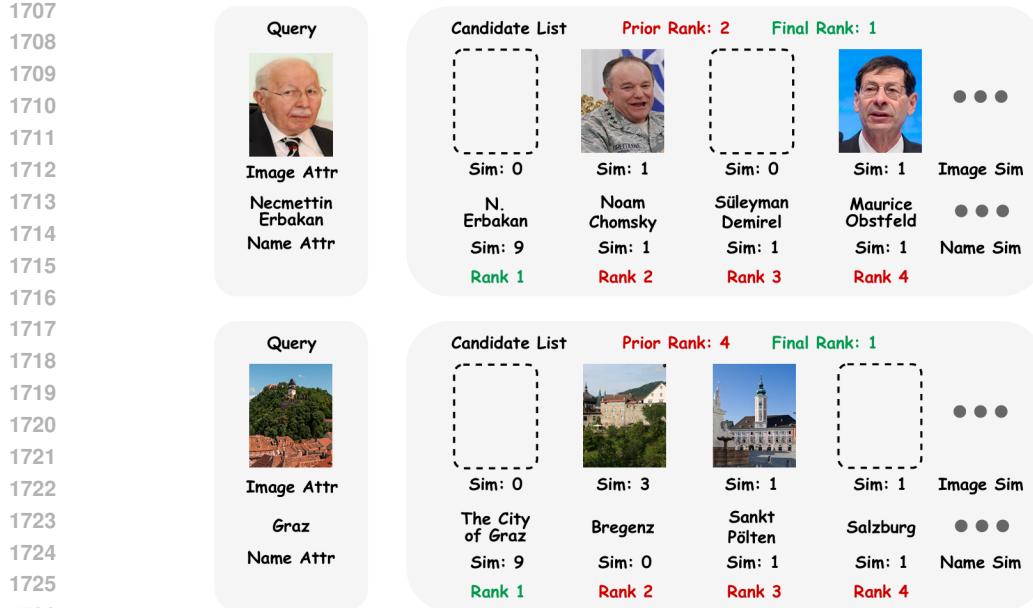


Figure 15: Missing attributes in test-time reasoning.