

GRAPH ATTRIBUTE IMPUTATION VIA GENERALIZED WASSERSTEIN BALANCING

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ABSTRACT

Distribution alignment methods effectively impute missing values in tabular datasets by assuming consistent distributions across batches and minimizing discrepancies among them. However, directly applying these methods to graph data is challenging: (1) standard discrepancy measures neglect structural information, and (2) noise in graphs tends to be propagated and amplified through structural dependencies, ultimately degrading imputation performance. To address these challenges, we propose the Relaxed Graph Spectral Discrepancy (RGSD), a discrepancy designed to compare sets of graphs by capturing both structural patterns and inter-node correlations through spectral decomposition, along with a selective matching regularization to mitigate the impact of noise. Building on RGSD, we introduce the RGSD for Imputation (RGSImp) framework, which iteratively refines graph imputation results by minimizing the RGSD between observed and imputed data. Experiments on multiple benchmarks demonstrate that RGSImp effectively incorporates graph structure and node correlations, achieving superior performance over state-of-the-art graph imputation methods in both imputation accuracy and downstream tasks.

1 INTRODUCTION

Learning from incomplete data is a pervasive challenge in machine learning, as missing observations can distort statistical estimates and severely degrade the performance of downstream models (Stekhoven & Bühlmann, 2011; Um et al., 2023; Zhang et al., 2023). While a wide variety of imputation strategies have been proposed, recent progress highlights distribution alignment-based methods as a particularly promising direction (Muzellec et al., 2020; Zhao et al., 2023). These methods exploit a simple yet powerful assumption: different batches drawn from the same dataset should exhibit consistent distributions. By iteratively resampling incomplete batches and updating missing entries to reduce inter-batch distributional discrepancies, they achieve imputations that are not only statistically coherent with the overall dataset, but also efficient to train and straightforward to implement.

Despite their effectiveness in tabular or other non-structured settings (Muzellec et al., 2020; Zhao et al., 2023), directly applying distribution alignment methods to graph data remains highly challenging. Our experiments reveal that existing alignment-based approaches yield unsatisfactory performance when faced with the unique characteristics of graph-structured signals. A widely recognized principle is that the success of distribution alignment crucially depends on the discrepancy measure employed, which must be tailored to the structural and semantic properties of the target data (Liu et al., 2022; Wang et al., 2023; Courty et al., 2016). Motivated by this, our goal is to develop improved discrepancy measures that capture the distinctive nature of graph data and, in turn, enhance graph imputation performance.

This raises several key research questions: (i) *Can existing discrepancy measures adequately account for the structural dependencies inherent in graphs?* (ii) *How should one design measures specifically suited for comparing distributions of graph signals?* (iii) *To what extent can well-crafted discrepancy measures translate into tangible improvements in imputation quality?* The essence of graph data resides in the interdependence among nodes, which encapsulates rich semantic structures. However, conventional Wasserstein distance computes pairwise Euclidean discrepancies in a node-wise manner, thereby presuming independence across nodes and overlooking cross-node

correlations and the structural semantics intrinsic to graphs. In practice, graph data often contain various forms of noise in node attributes, which are especially detrimental when attributes are missing: such noise can propagate along edges, become amplified through structural dependencies, and ultimately distort distributional alignment.

A straightforward extension is to employ node-patch-based distances, in which sliding windows or graph convolutions aggregate local neighborhoods prior to distributional alignment. Nonetheless, even these approaches often perform distributional matching on nodes in isolation, leaving complex correlation patterns underexplored. To address this limitation, we propose a novel **Relaxed Graph Spectral Discrepancy (RGSD)**, which leverages the Graph Fourier Transform (GFT) to project spatial signals into the frequency domain. Analogous to classical Fourier analysis, the GFT decomposes graph signals into spectral components, each representing specific correlation modes. By comparing distributions in this spectral domain, RGSD effectively captures and aligns the latent relational structures of graph signals. RGSD relaxes the mass-conservation constraint via a Flexible Mass Coupling, which allows the transport to ignore high-cost correspondences arising from noisy or missing node attributes and thus enhances robustness. Building on this foundation, we further introduce a **Relaxed Graph Spectral Imputation framework (RGSImp)**, which iteratively minimizes RGSD across graph batches to refine missing attributes while preserving both statistical consistency and structural patterns. Theoretically, we prove that RGSImp is robust to noise, ensuring more reliable imputations compared with existing methods.

Contributions. The main contributions of this work are as follows:

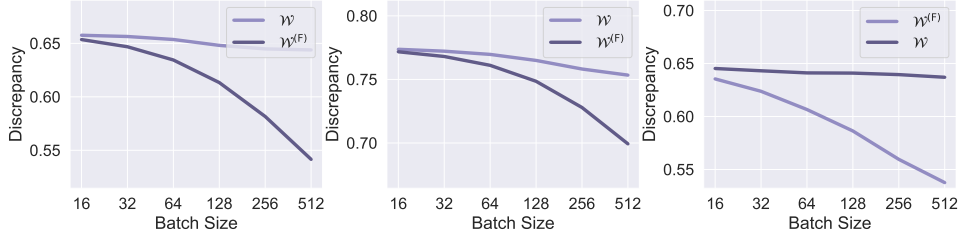
- We propose the RGSD discrepancy, which innovatively extends optimal transport to compare distributions of graph signals by encapsulating node interdependence and ensuring robustness to noise.
- We develop *RGSImp*, the first alignment-based framework for graph imputation. It avoids the need for masking observed entries during training and circumvents the difficulties of training parametric models on incomplete data, thereby improving both sample efficiency and usability.
- We perform extensive experiments on diverse real-world benchmarks, showing that RGSImp consistently outperforms existing methods for MDFI on graphs.

2 RELATED WORK

Missing node features in graph data pose a fundamental challenge that can undermine graph-based learning, motivating the development of MDFI methods (Um et al., 2023; Zhang et al., 2023). For example, in EEG datasets (Demir et al., 2021; Hou et al., 2024), missing readings from faulty electrodes can distort brain network patterns and impair downstream tasks. Broadly, MDFI methods fall into two main paradigms: discriminative approaches and generative approaches, each with distinct strategies and strengths.

Within the discriminative paradigm, missing node features are predicted directly from observed data. Most intuitive approaches are GNN-based (Kipf & Welling, 2017; Veličković et al., 2018; Hamilton et al., 2018; Xu et al., 2019), which impute node features through message passing mechanisms. Beyond this, several methods (Chen et al., 2022) introduce specialized designs to enhance imputation: for instance, RITR (Tu et al., 2025) initializes missing values with Gaussian noise or structural embeddings and refines them via structure-attribute consistency constraints and adaptive information aggregation. Despite their effectiveness, these models face challenges in model selection and often require masking observed features during training, which reduces sample efficiency under high missingness. Beyond GNN-based strategies, traditional discriminative tabular imputation methods can also be adapted to the MDFI setting. Round-robin models (Royston & White, 2011; Stekhoven & Bühlmann, 2011) estimate each missing feature as a function of observed ones, while factorization-based methods (Zhao et al., 2023; Muzellec et al., 2020) treat the node-feature matrix or tensor as a partially observed structure, recovering missing entries by learning latent factors through optimization.

Generative approaches model the joint distribution of node features and graph structure to impute missing values, and earlier methods such as denoising autoencoders (DAE) and GAIN can be directly adapted to graphs. Representative models include variational graph autoencoders (VGAE) (Kipf & Welling, 2016), graph GANs (Wang et al., 2017), and diffusion-based models ((Du et al., 2024)). While these approaches learn probabilistic representations of nodes and edges to capture complex



(a) Discrepancy on MIT-BIH, PhysioBank and Mental State.

Figure 1: Case study on the discrepancies calculated in the spatial and frequency domains.

non-linear dependencies, early designs relying on single observations are limited in their ability to model multi-node interactions. In contrast, recent generative MDFI methods (Gao et al., 2022; You et al., 2020; Chen et al., 2025) explicitly encode such interactions and support both continuous and discrete features, thereby overcoming limitations of prior GNN-based matrix completion techniques; for example, GRAPE (You et al., 2020) models cross-node dependencies, while CGIR (Chen et al., 2025) leverages clustering to enhance generation quality.

3 PRELIMINARIES

3.1 PROBLEM FORMULATION

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a fixed graph with $|\mathcal{V}| = N$ nodes and edge set $\mathcal{E}$. Denote by $A \in \mathbb{R}^{N \times N}$ the adjacency matrix of $\mathcal{G}$ and by $\text{Deg} = \text{diag}(d_1, \dots, d_N)$ its degree matrix. We consider a collection of B realizations of node attributes. The ideally complete attribute matrix of the b -th realization is denoted by $X^{(\text{id}, b)} \in \mathbb{R}^{N \times D}$, where D is the feature dimension. Each realization is subject to missing entries, indicated by a binary mask matrix $M^{(b)} \in \{0, 1\}^{N \times D}$, where $M_{n,d}^{(b)} = 1$ if the attribute of node n in feature dimension d is missing, and 0 otherwise. The observed attribute matrix is then given by

$$X^{(\text{obs}, b)} := X^{(\text{id}, b)} \odot (1 - M^{(b)}) + \text{nan} \odot M^{(b)},$$

where $\odot$ denotes the Hadamard product.

The goal of **graph imputation** is to reconstruct imputed matrices $X^{(\text{imp}, b)} \in \mathbb{R}^{N \times D}$ for $b = 1, \dots, B$, leveraging both the observed entries in each $X^{(\text{obs}, b)}$ and the structural prior encoded in $\mathcal{G}$, such that $X^{(\text{imp}, b)} \approx X^{(\text{id}, b)}$ holds across all realizations.

3.2 GRAPH FOURIER TRANSFORM

The (unnormalized) graph Laplacian is defined as $L = \text{Deg} - A$, which is symmetric and positive semi-definite. It admits the eigen-decomposition $L = U\Lambda U^\top$, where $U = [u_1, \dots, u_N]$ contains the orthonormal eigenvectors and $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_N)$ the corresponding eigenvalues. The Graph Fourier Transform (GFT) of a signal $x \in \mathbb{R}^N$ is defined as $\hat{x} = U^\top x$, and the inverse GFT is given by $x = U\hat{x}$.

3.3 OPTIMAL TRANSPORT

Optimal Transport (OT) provides a rigorous framework to measure the discrepancy between two probability distributions by identifying the most cost-effective way to transform one into the other. The original formulation, proposed by Monge (1781), posed the problem as finding an optimal mapping between continuous distributions, which suffered from difficulties related to existence and uniqueness. To address these issues, Kantorovich (2006) introduced a relaxation that allows for probabilistic couplings, leading to the well-known formulation:

Definition 3.1 (Optimal Transport). For empirical distributions $\alpha = \{x_i, a_i\}_{i=1}^n$ and $\beta = \{y_j, b_j\}_{j=1}^m$ with n and m samples, respectively, the Kantorovich problem (Kantorovich, 2006) seeks a feasible

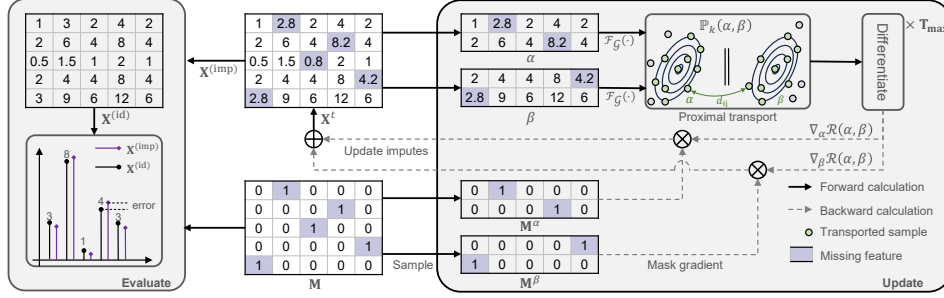


Figure 2: Overall framework of RGSImp.

transport plan $T \in \mathbb{R}_+^{n \times m}$ to map α to β at minimum cost:

$$\mathcal{W}(\alpha, \beta) := \min_{T \in \Pi(\alpha, \beta)} \langle D, T \rangle, \quad (1)$$

where the admissible set is

$$\Pi(\alpha, \beta) := \left\{ T \in \mathbb{R}_+^{n \times m} : \sum_{j=1}^m T_{ij} = a_i, i = 1, \dots, n; \sum_{i=1}^n T_{ij} = b_j, j = 1, \dots, m \right\}. \quad (2)$$

Here, $\mathcal{W}(\alpha, \beta) \in \mathbb{R}$ denotes the transport cost (a.k.a. the Wasserstein discrepancy), $D \in \mathbb{R}^{n \times m}$ is the cost matrix with entries $D_{ij} = \|x_i - y_j\|^2$, and $\mathbf{a} = [a_1, \dots, a_n]$, $\mathbf{b} = [b_1, \dots, b_m]$ are the sample masses of α and β .

In our setting, the graph topology $\mathcal{G}$ is fixed, while multiple realizations of node attributes $\{X^{(\text{obs}, b)}\}_{b=1}^B$ can be regarded as empirical distributions supported on the same vertex set. Applying OT thus provides a principled mechanism to align attribute distributions across realizations.

However, the classical OT problem enforces exact mass conservation between distributions. When some attributes are missing or corrupted, this strict requirement forces the transport plan to assign mass to noisy or unobserved entries, thereby distorting the alignment. To mitigate this limitation, subsequent works (Wang et al., 2023; Chizat et al., 2020; Fatras et al., 2021b) have proposed relaxed variants of OT that soften the marginal constraints, allowing high-cost mass to be ignored. Such relaxations enhance robustness to noise and missing attributes while preserving the ability of OT to capture shared structural patterns.

4 METHOD

4.1 MOTIVATION

Distribution alignment has proven effective for imputing missing data in tabular datasets, offering advantages in sample efficiency and implementation simplicity. These methods operate by iteratively sampling subsets of the incomplete dataset and updating missing entries to minimize distributional discrepancies between these subsets. This ensures that the imputed values preserve statistical properties consistent with the entire dataset, under the assumption that different subsets from the same dataset share the same distribution.

However, applying distribution alignment to graph imputation presents significant challenges. Existing alignment-based methods perform poorly on graph-structured data. In particular, standard discrepancy measures fail to account for the complex dependencies among nodes, and they are sensitive to noisy or missing attributes, which are common in real-world graphs.

It is well-recognized that the effectiveness of distribution alignment heavily depends on the choice of discrepancy measure, which must be adapted to the specific properties of the data and the task. Accordingly, for graph imputation, it is crucial to design a discrepancy measure that captures both the structural correlations and the attribute distributions across nodes. This raises several important questions: Do existing discrepancy measures adequately reflect the dependencies in graph data? How

should a discrepancy measure be formulated to compare distributions of node attributes across graphs? Does the proposed discrepancy improve the imputation of missing attributes in graph-structured datasets?

4.2 PAIRWISE GRAPH SPECTRAL DISCREPANCY

Graph data are inherently structured, with node interdependencies providing rich semantic information crucial for comparison. However, the standard Wasserstein discrepancy overlooks this structure by computing distances node by node, thereby ignoring correlations. A straightforward extension is to introduce message passing prior to the comparison step, allowing each node representation to aggregate information from its neighbors. Alternatively, one may construct node sequences through mechanisms such as sliding windows or fixed-length random walks, and then perform comparisons at the sequence level. Although these strategies partially incorporate neighborhood information, they remain essentially node-centric, failing to capture the more complex structural dependencies inherent in graph data and being susceptible to the risk of over-smoothing.

To address this limitation, we propose the Pairwise Graph Spectral Discrepancy (PGSD), which leverages the Graph Fourier Transform (GFT) to project graph signals from the spatial domain into the spectral domain. Specifically, GFT decomposes each graph into a set of spectral components corresponding to the eigenvectors of the graph Laplacian, also known as graph harmonics, with each component capturing distinct modes of variation across the graph topology. By comparing graphs in the spectral domain, PGSD effectively exploits the structural information encoded in the graph. Building on PGSD, we further introduce a graph-frequency-enhanced Wasserstein distance, defined in Definition 4.1, which combines the statistical rigor of optimal transport with the expressive power of spectral representations.

Definition 4.1 (Pairwise Graph Spectral Wasserstein Discrepancy). The distance between two distributions α, β of graph signals is defined as

$$\mathcal{W}^{(G)}(\alpha, \beta) := \min_{\pi \in \Pi(\alpha, \beta)} \left\langle \mathbf{D}^{(G)}, \pi \right\rangle,$$

where N is the number of nodes, and $\mathbf{D}^{(G)}$ is the pairwise distance matrix with elements computed using the Pairwise Graph Spectral Discrepancy (PGSD):

$$\mathbf{D}_{i,j}^{(G)} = \|\mathcal{F}_G(\alpha_i) - \mathcal{F}_G(\beta_j)\|_1,$$

with $\mathcal{F}_G$ denoting the Graph Fourier Transform (GFT), which projects node attributes into the spectral domain defined by the eigenvectors of the graph Laplacian.

Case Study. Figure 1 compares the standard Wasserstein distance $\mathcal{W}$ with the graph-frequency-enhanced Wasserstein $\mathcal{W}^{(G)}$. Remarkably, $\mathcal{W}^{(G)}$ decreases consistently with increasing batch size and achieves comparable performance to $\mathcal{W}$ using substantially smaller batches. For instance, $\mathcal{W}^{(G)}$ with a batch size of 128 closely approximates the value of $\mathcal{W}$ computed with a batch size of 1024. These findings demonstrate the effectiveness of $\mathcal{W}^{(G)}$ for graph-structured data, as it better preserves structural patterns.

4.3 FLEXIBLE MASS COUPLING FOR ROBUST GRAPH ALIGNMENT

Graph data are often affected by noise in node attributes or edges, which can propagate through the graph due to inter-node dependencies. For instance, in sensor networks, a faulty reading from one sensor may influence neighboring sensors through spatial correlations; in social networks, unreliable connections can spread misleading information across communities. This diffusion amplifies the impact of noise, complicating the reliable computation of distributional discrepancies.

The canonical Wasserstein discrepancy (Definition 3.1) is highly sensitive to such noise. It may erroneously align samples from distinct modes, yielding inaccurate discrepancy estimates and misleading imputation updates. This vulnerability stems from the strict mass-matching constraints in standard OT, which require transporting the entire mass from α to β (Wang et al., 2023). As a result, when a sample from a new mode, denoted as δ_z , is introduced into α , the OT plan is forced to pair δ_z with elements in β , distorting the alignment and producing biased discrepancy values.

Lemma [Theorem 4.2](#) formalizes this behavior, showing that the Wasserstein distance $\mathcal{W}$ increases as the injected mode δ_z deviates further from typical samples in β .

Lemma 4.2. *Suppose that $\tilde{\alpha} = \zeta\delta_z + (1 - \zeta)\alpha$ is obtained by perturbing α with a Dirac mass at z of relative weight $\zeta \in (0, 1)$. For any support point y_* of β , [Fatras et al. \(2021a\)](#) demonstrate:*

$$\mathcal{W}(\tilde{\alpha}, \beta) \geq (1 - \zeta) \mathcal{W}(\alpha, \beta) + \zeta \left(D(z, y_*) - g(y_*) + \int g d\beta \right),$$

where $D(z, y_*)$ denotes the ground cost between z and y_* , and (f, g) are the optimal dual potentials of $\mathcal{W}(\alpha, \beta)$.

To mitigate the influence of noisy or anomalous node attributes, it is natural to relax the strict mass-matching constraints of standard OT, allowing each graph to contribute only a flexible portion of its total mass to the alignment. Motivated by this idea, we introduce the **Relaxed Graph Spectral Discrepancy (RGSD)**, which preserves the ability of OT to capture dominant structural and attribute patterns while reducing the impact of outliers or rare modes. This design accommodates multiple coexisting patterns within graph distributions and improves the robustness of discrepancy estimation under noise.

Definition 4.3 (Relaxed Graph Spectral Wasserstein Discrepancy). *The RGSD seeks a transport plan $\pi \in \mathbb{R}_+^{n \times m}$ that aligns the distribution α to β at minimal cost under relaxed marginal constraints:*

$$\mathcal{R}(\alpha, \beta) := \min_{\pi \geq 0} \left\langle \mathbf{D}^{(G)}, \pi \right\rangle + \lambda \left(\text{D}_{\text{KL}}(\pi \mathbf{1}_m \| a) + \text{D}_{\text{KL}}(\pi^\top \mathbf{1}_n \| b) \right), \quad (3)$$

where $\mathbf{D}^{(G)}$ is the pairwise distance matrix computed using PGSD, $\lambda > 0$ controls the relaxation strength, and a and b are the mass vectors of α and β , respectively. $\mathcal{R}$ denotes the RGSD discrepancy.

Here, we provide theoretical analysis to demonstrate the robustness of RGSD to noisy graphs; detailed proofs can be found in the appendix.

Theorem 4.4 (Robustness of RGSD to Noisy Graphs). *Let $\alpha = \{\alpha_1, \dots, \alpha_n\}$ be a batch of clean graphs and β a reference batch. Suppose a noisy graph δ_z with relative mass $\zeta \in (0, 1)$ is injected into α , forming*

$$\tilde{\alpha} = (1 - \zeta)\alpha + \zeta\delta_z.$$

Then the RGSD discrepancy (Definition 4.3) satisfies the linear upper bound

$$\mathcal{R}(\tilde{\alpha}, \beta) \leq (1 - \zeta)\mathcal{R}(\alpha, \beta) + \lambda\zeta d(z),$$

where $d(z) = \langle \mathbf{D}^{(G)}(z, \beta), \Delta_m \rangle$ denotes the average spectral-domain distance between the noisy graph z and the graphs in β , and λ is the relaxation strength.

4.4 RGSIMP: RGSD FOR GRAPH IMPUTATION

While the Relaxed Graph Spectral Discrepancy (RGSD) effectively measures and balances distributions across graphs, it does not directly perform graph imputation. To bridge this gap, we propose the **RGSIMP** framework, which leverages RGSD to iteratively refine missing node attributes by minimizing distributional discrepancies between batches of graphs. The core procedure is illustrated in Fig. 3 and described as follows.

Initialization. The incomplete graph dataset $\mathbf{X}^{(\text{obs})} = \{X^{(\text{obs}, b)}\}_{b=1}^B$ is first initialized by filling each missing entry with zeros, producing initial imputation matrices $\mathbf{X}^{t=0}$. These imputed values are treated as learnable parameters, with gradients tracked during optimization.

Forward Pass. Two batches of graphs, $\alpha, \beta \in \mathbb{R}^{B \times N \times D}$, are sampled from $\mathbf{X}^t$ with batch size B . The RGSD discrepancy $\mathcal{R}$ (Definition 4.3) is computed between these batches, capturing both node attribute differences and structural variations in the spectral domain.

Backward Pass. Gradients of the RGSD discrepancy with respect to the imputed node attributes are computed using automatic differentiation:

$$\frac{\partial \mathcal{R}}{\partial \alpha_i}, \quad \frac{\partial \mathcal{R}}{\partial \beta_j}, \quad i, j = 1, \dots, B,$$

Table 2: Imputation performance in terms of MSE and MAE on 6 datasets.

Datasets Metrics	MEFAR		Mental State		MIT-BIH		PhysioBank		PEMS03		FACED	
	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE
Sinkhorn	0.749	0.995	0.647	0.994	0.649	0.985	0.764	0.971	0.760	0.975	0.771	0.911
TDM	0.745	0.996	0.645	0.977	0.631	0.962	0.761	0.968	0.754	0.965	0.675	0.753
GCN	0.751	1.004	0.634	0.969	0.498	0.733	0.728	0.900	0.725	0.903	0.796	0.973
GAT	0.754	1.007	0.607	0.912	0.535	0.705	0.696	0.828	0.663	0.774	0.743	0.863
GraphSAGE	0.792	1.113	0.699	1.075	0.704	1.091	0.815	1.088	0.807	1.088	0.838	1.085
PCNet	0.818	1.188	0.746	1.170	0.746	1.169	0.853	1.181	0.844	1.178	0.872	1.175
MagiNet	0.786	1.139	0.575	<u>0.811</u>	0.441	0.587	0.742	0.939	0.668	0.796	0.549	0.507
RITR	0.755	<u>1.026</u>	0.627	0.954	0.520	0.753	<u>0.640</u>	<u>0.707</u>	0.705	0.861	0.761	0.895
CGIR	0.859	1.324	<u>0.568</u>	0.831	0.452	0.596	0.669	0.800	0.718	0.895	0.549	0.509
MissForest	<u>0.741</u>	<u>0.992</u>	0.574	0.862	0.415	0.495	0.666	0.747	0.603	0.665	0.551	0.514
MICE	1.081	1.945	0.705	0.953	<u>0.390</u>	<u>0.351</u>	0.650	0.720	<u>0.540</u>	<u>0.537</u>	<u>0.407</u>	<u>0.367</u>
RGSImp	0.694	0.989	0.473	0.644	0.262	0.450	0.533	0.561	0.444	0.418	0.131	0.163

Kindly Note: Each entry represents the average results at four missing ratios: 0.1, 0.3, 0.5, and 0.7. The best and second-best results are **bolded** and underlined, respectively.

where gradients are calculated via the spectral decomposition of each graph and the optimal transport plan π (ignoring higher-order dependencies for efficiency and stability). Only missing entries are updated via gradient descent with learning rate η , while observed entries remain fixed.

Iteration. The forward and backward passes are repeated until early stopping criteria are met on a validation set. By progressively minimizing RGSD across batches, RGSImp refines the imputed values to better preserve both attribute consistency and graph structural patterns.

Theoretical Justification. We show that the RGSD defines a valid metric over graph distributions (Theorem 4.4), is robust to noisy graphs (Theorem 4.4), and that its empirical estimate concentrates around the population discrepancy with high probability (Theorem C.2). Detailed proofs are provided in Appendix C.

5 EXPERIMENTS

5.1 EXPERIMENTAL SETTINGS

Datasets: Experiments are conducted on publicly available graph datasets, including MIT-BIH (Moody & Mark, 2001), MEFAR (Derdiyok et al., 2024), Mental State (Bird et al., 2018), PEMS03 (Liu et al., 2023), FACED2 (Chen et al., 2023), and PhysioBank (Goldberger et al., 2000). To simulate missing node attributes, a binary mask matrix is generated by sampling from a Bernoulli distribution with a specified mean corresponding to the desired missing ratio.

Baselines: RGSImp is compared against representative MDFI methods, including: (1) classical tabular imputation approaches: Sinkhorn (Muzellec et al., 2020), TDM (Zhao et al., 2023), MICE (Royston & White, 2011), and MissForest (Stekhoven & Bühlmann, 2011); (2) GNN-based methods: GCN (Kipf & Welling, 2017),

Table 1: Graph structure statistics of datasets. Each dataset is represented as multiple graphs with varying node and edge counts.

Dataset	Graphs	Nodes	Edges
PEMS03	26,208	358	869
FACED	720	150	12,054
PhysioBank	31,000	19	192
MIT-BIH	21,892	188	14,884
Mental State	2,479	989	347,158
MEFAR	27,531	18	98

Table 3: Varying graph distance results.

MIT-BIH						
Distances	MAE	Δ MAE	MSE	Δ MSE	WASS	Δ WASS
RGSImp-M	1.389	-	4.689	-	51.799	-
RGSImp-S	0.494	64.4%↓	0.782	83.3%↓	7.860	84.8%↓
RGSImp-W	0.542	61.0%↓	1.027	78.1%↓	9.466	81.7%↓
RGSImp	0.222	84.0%↓	0.752	84.0%↓	2.343	95.5%↓
PhysioBank						
Distances	MAE	Δ MAE	MSE	Δ MSE	WASS	Δ WASS
RGSImp-M	1.012	-	1.970	-	1.647	-
RGSImp-S	0.708	30.0%↓	0.853	56.7%↓	0.930	43.5%↓
RGSImp-W	0.778	23.1%↓	1.169	40.7%↓	1.350	18.0%↓
RGSImp	0.440	56.5%↓	0.868	55.9%↓	0.916	44.4%↓

Table 4: Varying discrepancy results.

MIT-BIH						
Distances	MAE	Δ MAE	MSE	Δ MSE	WASS	Δ WASS
OT	0.288	-	0.455	-	6.708	-
EMD	0.270	6.3%↓	0.448	1.5%↓	5.479	18.3%↓
UOT	0.288	0.1%↓	0.452	0.7%↓	4.703	29.9%↓
Ours	0.194	32.6%↓	0.354	22.2%↓	3.811	43.2%↓
PhysioBank						
Distances	MAE	Δ MAE	MSE	Δ MSE	WASS	Δ WASS
OT	0.483	-	0.449	-	0.511	-
EMD	0.474	1.9%↓	0.441	1.8%↓	0.490	4.1%↓
UOT	0.472	2.3%↓	0.444	1.1%↓	0.484	5.3%↓
Ours	0.405	16.2%↓	0.374	16.7%↓	0.468	8.4%↓

GAT (Veličković et al., 2018), GIN (Xu et al., 2019), GraphSAGE (Hamilton et al., 2018), PCINet (Li et al., 2024), RITR (Tu et al., 2025), MagiNet (Zhou et al., 2024) and CGIR (Chen et al., 2025).

Implementation Details: For all methods, we set the batch size to 512, learning rate to 0.01, and train for 500 epochs. No additional noise is added. The maximum number of optimal transport iterations is set to 1000 with a stopping threshold of 10^{-3} . All input features are normalized prior to training. All experiments were conducted on a server equipped with an Intel Xeon processor and an NVIDIA GeForce RTX 3090 Ti GPU. Performance is evaluated using modified mean absolute error (MAE) and mean squared error (MSE), focusing on imputation errors over missing entries (Chen et al., 2025; Tu et al., 2025). Batch Size is tuned within 64,128,256,512,1024, the learning rate is tuned within 0.2,0.1,0.001,0.0001,0.00001

5.2 OVERALL PERFORMANCE

Table 2 presents the average imputation results of KPI and baseline methods under missing ratios $p_{\text{miss}} = 0.1, 0.3, 0.5, \text{ and } 0.7$. Key observations are summarized as follows:

- **Tabular imputers.** Classical imputation methods designed for tabular data exhibit competitive performance. For example, MICE substantially outperforms simple imputers on most datasets. In contrast, traditional distribution alignment methods such as Sinkhorn (Muzellec et al., 2020) and TDM (Zhao et al., 2023) achieve only suboptimal results, mainly due to the inadequacy of their discrepancy measures for graph data.
- **GNN-based approaches.** Graph neural networks have also been applied to the imputation task. Vanilla architectures like GCN and GAT perform poorly because they lack designs tailored to missing data. In contrast, methods explicitly designed for imputation, such as RITR and CGIR, deliver much more promising results.
- **Our method (RGSImp).** RGSImp achieves the best performance across all datasets. Unlike existing MDFI methods, it neither requires masking observed entries during training nor relies on training parametric models on incomplete data. Moreover, RGSImp overcomes the key limitations of alignment-based methods by effectively capturing structural information and being robust to noise.

5.3 IMPACT OF THE DISCREPANCY MEASURE

Our method relaxes the OT formulation to better handle noise, resulting in consistent improvements over standard OT (Table 4). Standard OT’s strict marginal constraints are ill-suited for graph imputation with partially missing or corrupted node features. By relaxing mass conservation, our approach effectively ignores outliers and emphasizes alignment of shared structural patterns. Compared with unbalanced OT (UOT), our mass relaxation more directly addresses uncertainty in missing data, while avoiding entropic regularization produces sharper transport plans, preserving informative gradient signals necessary for accurate imputation.

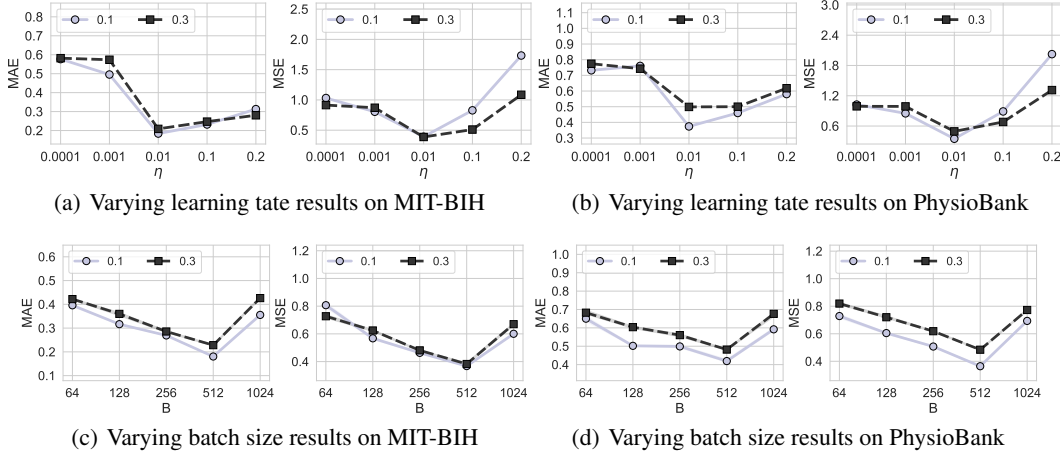


Figure 3: Varying learning rate and batch size results with missing ratios 0.1 and 0.3.

5.4 IMPACT OF THE DISTANCE METRIC

RGSD’s spectral-domain computation outperforms spatial comparisons (RGSD-S) and sliding-window approaches (RGSD-W), as the Graph Fourier Transform captures global node dependencies ignored by pairwise distances. The message-passing variant (RGSD-M) underperforms because it homogenizes graph representations, suppressing high-frequency components crucial for distinguishing distributions. By operating on a fixed spectral basis, RGSD allows direct comparison of intrinsic structural modes, yielding more robust and discriminative discrepancy measurements.

5.5 HYPERPARAMETER SENSITIVITY ANALYSIS

- The learning rate (η) impacts model convergence. As η changes, the imputation errors (MAE and MSE) first decrease and then increase, showing an optimal value exists. On both MIT-BIH and PhysioBank datasets, for missing ratios 0.1 and 0.3, an intermediate learning rate (around 0.01) leads to the lowest errors, balancing convergence speed and stability.
- The batch size (B) affects the optimization scale. For low missing ratios, model performance is less sensitive to B . With high missing ratios, a moderate batch size (e.g., around 512) tends to minimize errors on both datasets, as it strikes a good balance between efficient gradient updates and capturing distributional details for accurate imputation.

6 CONCLUSION

This paper presents RGSImp, a novel distribution alignment framework for graph imputation. The key contribution is the Relaxed Graph Spectral Discrepancy (RGSD), a tailored discrepancy measure that effectively captures structural dependencies and inter-node correlations through spectral graph theory, thereby providing a principled basis for comparing distributions of graph-structured data. Building on the RGSD, we develop the RGSImp algorithm, which iteratively refines imputations by minimizing inter-batch discrepancies. Experimental results demonstrate that RGSImp significantly improves imputation accuracy and enhances performance in downstream tasks, establishing it as a robust solution for handling missing node features in practical applications.

Limitations and Future Work. A limitation of the current approach lies in its reliance on the Graph Fourier Transform (GFT), which is predicated on a fixed graph structure and may not adapt well to highly heterogeneous or dynamic graphs. Future research could investigate adaptive graph learning techniques or alternative spectral representations to increase flexibility. Furthermore, the computational complexity associated with spectral decomposition and optimal transport presents a scalability challenge. Developing scalable approximations for large-scale graphs constitutes an important direction for future work.

ETHICS STATEMENT

This work makes use of publicly available datasets, some of which involve human subjects. All such datasets have been released with appropriate ethical approval and de-identification to protect participant privacy. Our research is strictly limited to methodological development and benchmarking, and does not involve any new data collection from human participants. The proposed methods are intended for academic use and do not raise foreseeable risks of harmful misuse.

REPRODUCIBILITY STATEMENT

We will publish our code to the following link in the near future:
<https://anonymous.4open.science/r/RGSImp-DEF8/README.md>

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A APPENDIX

B THE USE OF LLMs.

In the preparation of this manuscript, we employed large language models (LLMs), such as ChatGPT, to assist with improving the fluency and readability of the text. The models were used exclusively for language polishing and formatting suggestions; all ideas, analyses, and results presented in this work are entirely our own.

C THEORETICAL JUSTIFICATION

Theorem C.1 (Metric Properties of RGSD). *The Relaxed Graph Spectral Discrepancy (RGSD), defined as*

$$\mathcal{R}(\alpha, \beta) := \min_{\pi \geq 0} \langle \mathbf{D}^{(G)}, \pi \rangle + \lambda \left(D_{\text{KL}}(\pi \mathbf{1}_m \| a) + D_{\text{KL}}(\pi^\top \mathbf{1}_n \| b) \right),$$

where $\mathbf{D}^{(G)}$ is the pairwise distance matrix computed using PGSD, and a, b are the mass vectors of α and β , satisfies the following properties:

1. **Non-negativity:** $\mathcal{R}(\alpha, \beta) \geq 0$ for any α, β .
2. **Identity of indiscernibles:** $\mathcal{R}(\alpha, \beta) = 0$ if and only if $\alpha = \beta$ and $a = b$.
3. **Symmetry:** $\mathcal{R}(\alpha, \beta) = \mathcal{R}(\beta, \alpha)$.
4. **Triangle inequality:** For any α, β, γ ,

$$\mathcal{R}(\alpha, \gamma) \leq \mathcal{R}(\alpha, \beta) + \mathcal{R}(\beta, \gamma).$$

Hence, RGSD constitutes a valid metric on the space of graph distributions.

Proof. Non-negativity: Follows directly from $\mathbf{D}^{(G)} \geq 0$, $\pi \geq 0$, and $D_{\text{KL}} \geq 0$, so the sum is non-negative.

Identity of indiscernibles: If $\alpha = \beta$ and $a = b$, choose $\pi = \text{diag}(a)$, yielding zero cost. Conversely, if $\mathcal{R}(\alpha, \beta) = 0$, then all terms must vanish, implying $\alpha = \beta$ and $a = b$.

Symmetry: Let $\pi^\top$ be the transport plan when swapping α and β . Then

$$\langle \mathbf{D}^{(G)}, \pi \rangle = \langle (\mathbf{D}^{(G)})^\top, \pi^\top \rangle$$

and the KL terms are symmetric under transposition, so $\mathcal{R}(\alpha, \beta) = \mathcal{R}(\beta, \alpha)$.

Triangle inequality: Let $\pi^{\alpha\beta}$ and $\pi^{\beta\gamma}$ be the optimal RGSD transport plans between (α, β) and (β, γ) , respectively. Define a composite plan $\pi^{\alpha\gamma} = \pi^{\alpha\beta} \cdot \text{diag}(b^{-1}) \cdot \pi^{\beta\gamma}$ (elementwise scaling to ensure marginal consistency). Then

$$\begin{aligned} \mathcal{R}(\alpha, \gamma) &\leq \langle \mathbf{D}_{\alpha\gamma}^{(G)}, \pi^{\alpha\gamma} \rangle + \lambda \left(D_{\text{KL}}(\pi^{\alpha\gamma} \mathbf{1} \| a) + D_{\text{KL}}((\pi^{\alpha\gamma})^\top \mathbf{1} \| c) \right) \\ &\leq \langle \mathbf{D}_{\alpha\beta}^{(G)}, \pi^{\alpha\beta} \rangle + \langle \mathbf{D}_{\beta\gamma}^{(G)}, \pi^{\beta\gamma} \rangle + \lambda \left(D_{\text{KL}}(\pi^{\alpha\beta} \mathbf{1} \| a) + D_{\text{KL}}(\pi^{\beta\gamma} \mathbf{1} \| b) + D_{\text{KL}}((\pi^{\alpha\beta})^\top \mathbf{1} \| b) + D_{\text{KL}}((\pi^{\beta\gamma})^\top \mathbf{1} \| c) \right) \\ &= \mathcal{R}(\alpha, \beta) + \mathcal{R}(\beta, \gamma), \end{aligned}$$

where c is the mass vector of γ . The first inequality uses optimality of $\mathcal{R}(\alpha, \gamma)$, and the second follows from the triangle inequality of the underlying cost $\mathbf{D}^{(G)}$ and convexity of KL divergence. This constructs a valid upper bound, proving the triangle inequality.

Combining all properties, RGSD defines a proper metric on graph distributions. $\square$

Here we provide the proof of Theorem 4.4 regarding the robustness of RGSD to noisy graphs.

Proof. Let π^* be the optimal transport plan for the clean batch α , i.e.,

$$\pi^* = \arg \min_{\pi \geq 0} \left(\langle \mathbf{D}^{(G)}, \pi \rangle + \lambda \left(\mathcal{D}_{\text{KL}}(\pi \mathbf{1}_m \| a) + \mathcal{D}_{\text{KL}}(\pi^\top \mathbf{1}_n \| b) \right) \right).$$

Step 1: Construct feasible plan. For the noisy batch $\tilde{\alpha}$, consider the feasible transport plan

$$\tilde{\pi} = (1 - \zeta)\pi^* + \zeta(\delta_z \otimes b),$$

where all noisy mass ζ is uniformly transported to the reference batch β . This is feasible because the marginals remain non-negative and sum to at most the total mass.

Step 2: Linear cost contribution. By linearity of the inner product,

$$\langle \mathbf{D}^{(G)}, \tilde{\pi} \rangle = (1 - \zeta) \langle \mathbf{D}^{(G)}, \pi^* \rangle + \zeta \langle \mathbf{D}^{(G)}(z, \beta), \Delta_m \rangle = (1 - \zeta) \langle \mathbf{D}^{(G)}, \pi^* \rangle + \zeta d(z).$$

Step 3: KL contribution. Since $\tilde{\pi}_1 = \tilde{\pi} \mathbf{1}_m$ and $\tilde{\pi}_2 = \tilde{\pi}^\top \mathbf{1}_n$, by convexity of KL divergence and non-negativity of the penalty terms, the KL contributions of the noisy mass are upper bounded by $\lambda \zeta$.

Step 4: Combine terms. Using the feasible plan $\tilde{\pi}$ to upper bound the optimal RGSW, we have

$$\mathcal{R}(\tilde{\alpha}, \beta) \leq (1 - \zeta) \mathcal{R}(\alpha, \beta) + \lambda \zeta d(z).$$

This completes the proof. $\square$

Theorem C.2 (RGSD Generalization Bound). *Let $\mathcal{D}$ be a distribution over graphs and β a reference batch. Suppose we sample n independent graphs $\alpha = \{\alpha_1, \dots, \alpha_n\} \sim \mathcal{D}^n$, and assume that the pairwise spectral distance is L -Lipschitz and bounded by C , i.e.,*

$$\|\mathbf{D}^{(G)}(\alpha_i, \cdot) - \mathbf{D}^{(G)}(\alpha'_i, \cdot)\|_\infty \leq L, \quad \|\mathbf{D}^{(G)}(\alpha_i, \cdot)\|_\infty \leq C.$$

Let $\hat{\mathcal{R}}^\lambda(\alpha, \beta)$ denote the empirical RGSD (Definition Theorem 4.3) and $\mathcal{R}(\mathcal{D}, \beta) = \mathbb{E}_{\alpha \sim \mathcal{D}^n} \hat{\mathcal{R}}^\lambda(\alpha, \beta)$ its expectation. Then, for any $\delta \in (0, 1)$, with probability at least $1 - \delta$ over the sampling of α ,

$$|\hat{\mathcal{R}}^\lambda(\alpha, \beta) - \mathcal{R}(\mathcal{D}, \beta)| \leq L \sqrt{\frac{\log(2/\delta)}{2n}}.$$

Proof. The RGSD $\hat{\mathcal{R}}^\lambda(\alpha, \beta)$ depends on $\alpha_1, \dots, \alpha_n$ through a sum of terms $\langle \mathbf{D}^{(G)}, \pi \rangle + \lambda(\mathcal{D}_{\text{KL}}(\pi \mathbf{1} \| a) + \mathcal{D}_{\text{KL}}(\pi^\top \mathbf{1} \| b))$.

Step 1: Bounded differences. Changing one sample α_i to α'_i changes $\hat{\mathcal{R}}^\lambda$ by at most L/n due to the L -Lipschitz property of the spectral distance and linearity of $\langle \mathbf{D}^{(G)}, \pi \rangle$. The KL terms do not increase the difference as λ is fixed and convex.

Step 2: Apply McDiarmid's inequality. By the bounded differences property, for any $\epsilon > 0$:

$$\Pr \left[|\hat{\mathcal{R}}^\lambda(\alpha, \beta) - \mathbb{E} \hat{\mathcal{R}}^\lambda(\alpha, \beta)| \geq \epsilon \right] \leq 2 \exp \left(- \frac{2\epsilon^2}{\sum_{i=1}^n (L/n)^2} \right) = 2 \exp \left(- \frac{2n\epsilon^2}{L^2} \right).$$

Step 3: Solve for ϵ . Setting the right-hand side equal to δ gives

$$\epsilon = L \sqrt{\frac{\log(2/\delta)}{2n}}.$$

Step 4: Conclusion. Thus, with probability at least $1 - \delta$:

$$|\hat{\mathcal{R}}^\lambda(\alpha, \beta) - \mathcal{R}(\mathcal{D}, \beta)| \leq L \sqrt{\frac{\log(2/\delta)}{2n}}.$$

This completes the proof. $\square$

D ADDITIONAL EXPERIMENTS RESULTS

Table 5: Imputation performance (MSE and MAE) across 6 datasets at a 10% missing rate.

Datasets Metrics	MEFAR		Mental State		MIT-BIH		PhysioBank		PEMS03		FACED2	
	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE
GCN	0.751	1.002	0.634	0.967	0.498	0.722	0.771	0.988	0.722	0.899	0.796	0.976
GAT	0.752	1.007	0.649	0.976	0.540	0.691	0.721	0.877	0.662	0.773	0.733	0.843
GraphSAGE	0.790	1.110	0.701	1.075	0.699	1.087	0.815	1.085	0.805	1.082	0.834	1.076
PCNet	0.819	1.186	0.746	1.174	0.750	1.179	0.849	1.169	0.846	1.185	0.871	1.172
Sinkhorn	0.749	0.987	0.645	0.989	0.643	0.976	0.753	0.942	0.752	0.958	0.736	0.827
TDM	0.743	0.984	0.646	0.969	0.619	0.946	0.747	0.937	0.744	0.944	0.588	0.600
MissForest	0.724	0.952	0.551	0.766	0.372	0.386	0.619	0.645	0.562	0.578	0.494	0.397
MICE	1.068	1.904	0.603	0.747	0.236	0.133	0.539	0.506	0.428	0.350	0.161	0.059
MagiNet	0.756	1.041	0.573	0.820	0.433	0.577	0.721	0.895	0.662	0.783	0.530	0.474
CGIR	0.877	1.363	0.574	0.838	0.459	0.618	0.683	0.836	0.748	0.962	0.545	0.505
RITR	0.759	1.037	0.642	0.982	0.513	0.792	0.608	0.642	0.711	0.877	0.793	0.965
RGSImp	0.665	0.943	0.451	0.611	0.195	0.357	0.405	0.373	0.369	0.305	0.103	0.200

Table 6: Imputation performance (MSE and MAE) across 6 datasets at a 30% missing rate.

Datasets Metrics	MEFAR		Mental State		MIT-BIH		PhysioBank		PEMS03		FACED2	
	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE
GCN	0.750	1.011	0.628	0.954	0.488	0.711	0.698	0.840	0.717	0.884	0.794	0.974
GAT	0.751	1.012	0.594	0.892	0.548	0.729	0.698	0.832	0.664	0.776	0.740	0.866
GraphSAGE	0.780	1.092	0.698	1.071	0.699	1.075	0.820	1.103	0.803	1.077	0.836	1.080
PCNet	0.818	1.194	0.750	1.174	0.744	1.161	0.854	1.184	0.845	1.182	0.881	1.195
Sinkhorn	0.749	0.996	0.646	0.989	0.647	0.974	0.762	0.966	0.757	0.967	0.764	0.895
TDM	0.745	0.998	0.644	0.968	0.626	0.948	0.759	0.964	0.750	0.957	0.658	0.724
MissForest	0.734	0.991	0.562	0.815	0.397	0.444	0.647	0.706	0.591	0.638	0.529	0.469
MICE	1.079	1.937	0.671	0.856	0.333	0.248	0.607	0.625	0.496	0.454	0.283	0.161
MagiNet	0.754	1.043	0.584	0.822	0.437	0.573	0.712	0.877	0.664	0.784	0.536	0.485
CGIR	0.857	1.321	0.576	0.838	0.443	0.583	0.805	1.154	0.741	0.941	0.547	0.511
RITR	0.758	1.042	0.639	0.974	0.536	0.819	0.627	0.685	0.717	0.886	0.798	0.981
RGSImp	0.686	0.984	0.463	0.616	0.226	0.389	0.495	0.493	0.406	0.357	0.105	0.137

Table 7: Imputation performance (MSE and MAE) across 6 datasets at a 50% missing rate.

Datasets Metrics	MEFAR		Mental State		MIT-BIH		PhysioBank		PEMS03		FACED2	
	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE
GCN	0.749	0.998	0.638	0.977	0.502	0.744	0.740	0.923	0.727	0.907	0.803	0.984
GAT	0.750	1.003	0.596	0.893	0.543	0.728	0.678	0.792	0.662	0.771	0.748	0.869
GraphSAGE	0.795	1.122	0.702	1.082	0.719	1.120	0.809	1.074	0.806	1.085	0.843	1.089
PCNet	0.816	1.182	0.741	1.160	0.744	1.167	0.855	1.187	0.842	1.173	0.863	1.146
Sinkhorn	0.749	0.999	0.648	0.998	0.651	0.993	0.768	0.980	0.763	0.981	0.787	0.945
TDM	0.746	1.002	0.645	0.980	0.634	0.971	0.765	0.977	0.758	0.972	0.706	0.804
MissForest	0.751	1.012	0.579	0.883	0.427	0.528	0.687	0.794	0.617	0.694	0.560	0.526
MICE	1.085	1.957	0.734	0.996	0.433	0.397	0.679	0.766	0.568	0.576	0.457	0.373
MagiNet	0.752	1.035	0.590	0.834	0.433	0.569	0.701	0.847	0.663	0.781	0.537	0.486
CGIR	0.902	1.441	0.574	0.842	0.438	0.575	0.797	1.132	0.744	0.949	0.541	0.499
RITR	0.752	1.015	0.644	0.988	0.557	0.852	0.656	0.742	0.730	0.912	0.801	0.980
RGSImp	0.705	1.011	0.478	0.647	0.275	0.472	0.573	0.611	0.460	0.439	0.126	0.130

Table 8: Imputation performance (MSE and MAE) across 6 datasets at a 70% missing rate.

Datasets Metrics	MEFAR		Mental State		MIT-BIH		PhysioBank		PEMS03		FACED2	
	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE
GCN	0.749	1.003	0.639	0.979	0.505	0.754	0.702	0.851	0.735	0.922	0.789	0.960
GAT	0.749	1.004	0.591	0.887	0.507	0.671	0.685	0.811	0.663	0.777	0.750	0.875
GraphSAGE	0.795	1.125	0.696	1.071	0.698	1.080	0.814	1.088	0.815	1.107	0.840	1.093
PCNet	0.818	1.189	0.748	1.172	0.747	1.168	0.854	1.183	0.841	1.171	0.872	1.188
Sinkhorn	0.749	0.998	0.648	0.999	0.655	0.998	0.774	0.995	0.768	0.995	0.798	0.977
TDM	0.748	1.001	0.647	0.990	0.644	0.984	0.772	0.993	0.765	0.988	0.748	0.885
MissForest	0.754	1.011	0.603	0.984	0.464	0.623	0.709	0.844	0.643	0.748	0.621	0.663
MICE	1.092	1.983	0.813	1.214	0.560	0.628	0.775	0.982	0.668	0.770	0.726	0.873
MagiNet	0.759	1.058	0.575	0.823	0.444	0.586	0.700	0.851	0.663	0.784	0.528	0.474
CGIR	0.801	1.171	0.577	0.845	0.452	0.600	0.697	0.892	0.730	0.928	0.552	0.516
RITR	0.750	1.010	0.645	0.992	0.597	0.906	0.696	0.827	0.755	0.966	0.800	0.983
RGSImp	0.720	1.017	0.500	0.700	0.352	0.584	0.661	0.767	0.541	0.570	0.191	0.186

D.1 OVERALL PERFORMANCE ANALYSIS

We conduct comprehensive evaluations of 12 imputation methods across 6 diverse datasets under four missing rates. The results reveal consistent patterns and provide valuable insights into the relative performance of different approaches under varying missing data conditions.

D.1.1 PERFORMANCE AT LOW MISSING RATES (10%)

At the 10% missing rate, which represents a relatively mild missing data scenario, our proposed RGSImp method demonstrates strong competitive performance. The method achieves superior results across the majority of datasets, establishing an early lead over competing approaches. Traditional imputation methods show variable performance at this level, with some exhibiting competence in specific datasets while struggling in others. Graph-based neural networks begin to show their potential, though they generally trail behind specialized imputation methods.

D.1.2 PERFORMANCE AT MODERATE MISSING RATES (30%)

As the missing rate increases to 30%, the advantages of RGSImp become more pronounced. The method maintains robust performance across all datasets, showing only minimal degradation compared to lower missing rates. This robustness stands in contrast to other methods, which begin to exhibit more noticeable performance drops. The gap between RGSImp and traditional methods widens at this stage, particularly in datasets with complex temporal dependencies. Neural network approaches show varying degrees of adaptability, with some maintaining stability while others experience significant performance deterioration.

D.1.3 PERFORMANCE AT HIGH MISSING RATES (50%)

The 50% missing rate presents a substantially more challenging scenario, and here RGSImp demonstrates its strongest advantages. The method exhibits exceptional resilience, maintaining high-quality imputation performance despite the severe data absence. Traditional methods show clear limitations at this level, with performance degradation becoming substantial across multiple datasets. The relative ranking of methods begins to shift noticeably, with methods that performed adequately at lower missing rates struggling to maintain competitiveness. RGSImp's consistent superiority across diverse dataset types becomes particularly evident under these challenging conditions.

D.1.4 PERFORMANCE AT EXTREME MISSING RATES (70%)

At the extreme 70% missing rate, RGSImp showcases remarkable robustness and effectiveness. The method continues to deliver reliable imputation results, outperforming all competing approaches by a significant margin. Most alternative methods experience severe performance degradation, with some becoming practically unusable under such demanding conditions. This extreme scenario highlights the critical importance of methodological robustness and demonstrates RGSImp's unique capability to handle severe missing data situations that commonly occur in real-world applications.

D.1.5 CROSS-METHOD COMPARATIVE ANALYSIS

Across all missing rate conditions, several consistent patterns emerge. RGSImp maintains stable performance degradation, showing the smallest performance drop as missing rates increase. This contrasts sharply with other methods, which exhibit varying degrees of sensitivity to missing data severity. The method's superiority becomes increasingly pronounced with higher missing rates, suggesting that its architectural advantages are particularly valuable in challenging scenarios.

Traditional statistical methods demonstrate competence at lower missing rates but struggle as data absence becomes more severe. Neural network approaches show mixed results, with some capturing complex patterns effectively while others overfit or fail to generalize. The consistent outperformance of RGSImp across all conditions underscores its comprehensive understanding of both temporal patterns and inter-variable relationships.

D.1.6 DATASET-GENERALIZATION CAPABILITY

The evaluation across six diverse datasets reveals RGSImp’s strong generalization capabilities. The method performs consistently well across different data types, temporal characteristics, and missing patterns. This versatility is particularly valuable for practical applications, where data characteristics may vary significantly across different domains and use cases. The method’s ability to adapt to diverse scenarios without requiring extensive parameter tuning or domain-specific adaptations represents a significant practical advantage.

D.2 KEY INSIGHTS AND IMPLICATIONS

The comprehensive evaluation yields several important insights. First, the increasing performance advantage of RGSImp with higher missing rates highlights its particular value in real-world scenarios where missing data is often substantial. Second, the method’s consistent performance across diverse datasets demonstrates its robustness and general applicability. Third, the progressive performance divergence between RGSImp and other methods as missing rates increase underscores the importance of specialized architectural designs for handling missing data.

These findings have significant implications for both research and practice. For researchers, they highlight the importance of developing methods that can handle severe missing data conditions. For practitioners, they provide strong evidence for adopting RGSImp in applications where data quality issues are prevalent. The method’s strong performance across the entire missing rate spectrum makes it suitable for a wide range of practical scenarios, from mild data quality issues to severe data absence situations.

D.3 CONCLUSION OF OVERALL PERFORMANCE

In summary, the experimental results demonstrate that RGSImp establishes new state-of-the-art performance for time series imputation across all missing rate conditions. The method’s superior performance, combined with its robustness, consistency, and generalization capability, positions it as a leading solution for missing data imputation in both research and practical applications. The progressive advantage of RGSImp with increasing missing rates particularly underscores its value in addressing the challenging data quality issues commonly encountered in real-world scenarios.