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ABSTRACT

Existing methods for evaluating graph generative models primarily rely on Maximum Mean Discrepancy (MMD) metrics based on graph descriptors. While these metrics can rank generative models, they do not provide an absolute measure of performance. Their values are also highly sensitive to extrinsic parameters, namely kernel and descriptor parametrization, making them incomparable across different graph descriptors. We introduce PolyGraphScore (PGS), a new evaluation framework that addresses these limitations. It approximates the Jensen-Shannon (JS) distance of graph distributions by fitting binary classifiers to distinguish between real and generated graphs, featurized by these descriptors. The data log-likelihood of these classifiers approximates a variational lower bound on the JS distance between the two distributions. Resulting scores are constrained to the unit interval $[0, 1]$ and are comparable across different graph descriptors. We further derive a theoretically grounded summary score that combines these individual metrics to provide a maximally tight lower bound on the distance for the given descriptors. Thorough experiments demonstrate that PGS provides a more robust and insightful evaluation compared to MMD metrics.

1 INTRODUCTION

Graph generative models (GGMs) are seeing wider adoption across scientific domains, from retrosynthesis (Somnath et al., 2021) and social network modeling (Bojchevski et al., 2018) to the discovery of novel drugs and materials (Liu et al., 2024; Kelvinius et al., 2025). However, progress in this field is increasingly bottlenecked by the lack of robust methods for evaluating generated graphs (Thompson et al., 2022; O’Bray et al., 2022).

This evaluation challenge is not unique to graphs. In image generation, the community has largely converged on pretrained embeddings paired with distribution distances, such as Inception-v3 coupled with Fréchet distance yielding the widely used Fréchet Inception distance (FID) (Heusel et al., 2017), or DinoV2 and density estimation producing the Feature Likelihood Divergence (FLD) (Jiralerspong et al., 2023). While these approaches provide standardized metrics adapted to other fields such as materials (Kelvinius et al., 2025), video (Unterthiner et al., 2019), and audio (Kilgour et al., 2018), limitations remain (Barratt & Sharma, 2018). As an alternative, classifier two-sample tests (C2STs) (Lopez-Paz & Oquab, 2017) recasts evaluation as a supervised classification task, turning classifier performance into evaluation metrics. To date, the applicability of these approaches to graph-structured data has not yet been explored.

The *de facto* standard for evaluating GGMs is to compute the Maximum Mean Discrepancy (MMD) (Gretton et al., 2012) between distributions of hand-crafted graph descriptors (e.g., degrees, Laplacian spectra, etc.) *on a small set of synthetic and real-world graphs* (You et al., 2018). *While convenient, this approach has critical inherent limitations:* (i) MMD estimates lack an *intrinsic scale*, meaning that a single reported value without context does not provide an absolute notion of the goodness of fit of the generative model; (ii) rankings are *sensitive to descriptor and kernel choice* (O’Bray et al., 2022), with no way of obtaining a single ranking across descriptors for consistent and systematic model comparison; and (iii) *in the small-sample regimes common to current GGM benchmarks, MMD estimates suffer from high bias and variance* (Krimmel et al., 2025)

We introduce PolyGraphScore (PGS), a novel evaluation framework that estimates the Jensen-Shannon distance (JSD) (Endres & Schindelin, 2003) between true and generated graph distributions

054 using *probabilistic classification* instead of kernel-based distances. A discriminator is trained to distinguish real from generated graphs using standard graph descriptors, where the classifier’s data
 055 log-likelihood provides a lower bound on the JSD. This yields scores in $[0, 1]$ that are directly *com-*
 056 *parable across descriptors*. Taking the maximum over descriptors gives the tightest available bound
 057 while identifying the most informative descriptor.
 058

059 Our formulation of PGS uses TabPFN (Hollmann et al.,
 060 2025), a fast, hyperparameter-free discriminator, making
 061 it robust and simple to use. Empirically, we show
 062 that PGS monotonically tracks synthetic data perturba-
 063 tions, strongly correlates with model training progress,
 064 and accurately captures generated graph quality. It also
 065 produces robust rankings across representative GGMs.
 066 Table 1 summarizes the advantages of PGS over MMD.

067 Our work makes four primary contributions:
 068

- **A rigorous reassessment of MMD for GGM evaluation.** We empirically show that standard MMD estimators are plagued by high bias and variance at typical benchmark sizes (20-40 graphs), leading to unreliable model rankings, and we provide actionable remedies.
- **PolyGraphScore (PGS): an estimate of the JSD distance between distributions.** We propose a method to derive interpretable evaluation scores by approximating variational lower bounds on the JSD via probabilistic discrimination on graph descriptors.
- **A comprehensive empirical validation.** We show that PGS tracks data perturbations monotonically and correlates strongly with training dynamics of state-of-the-art models. We also provide comprehensive PGS-based benchmark results across synthetic and real-world graphs, including molecules.
- **An open-source library to advance GGM evaluation.** We release the PolyGraph library, including implementations of PGS, MMD estimators, and new, larger benchmark datasets (SBM-L, LOBSTER-L, PLANAR-L), to facilitate more robust and reproducible future research.

082 2 RELATED WORK

084 We present here related work on the evaluation of graph generative models and classifier-based
 085 evaluation for general generative models.
 086

087 **Evaluation of Graph Generative Models.** The evaluation of GGMs has largely been shaped by
 088 methods based on the MMD (Gretton et al., 2012). You et al. (2018) first proposed computing the
 089 MMD between generated and real graph distributions using a Wasserstein Gaussian kernel on a set of
 090 graph descriptors, including degree histograms, clustering coefficients, and orbit counts. To reduce
 091 the computational cost of this method, Liao et al. (2019) introduced a simpler kernel formulation
 092 using a Gaussian kernel with the squared total variation distance, which gained widespread adoption
 093 (Martinkus et al., 2022; Vignac et al., 2023; Chen et al., 2025). However, this simplified kernel
 094 was shown to be indefinite and highly sensitive to hyperparameter choices (O’Bray et al., 2022).
 095 Subsequent work has focused on correcting these flaws, either by modifying the kernel to ensure
 096 it is positive definite (O’Bray et al., 2022) or by employing standard RBF kernels with automated
 097 hyperparameter tuning (Thompson et al., 2022; Sriperumbudur et al., 2009). A parallel research
 098 effort has concentrated on identifying more expressive graph descriptors for use within the MMD
 099 framework. The initial set of statistics was augmented with the graph Laplacian spectrum by Liao
 100 et al. (2019), and more recently, graph neural networks (GNNs) have been used as powerful graph
 101 featurizers (Thompson et al., 2022; Shirzad et al., 2022). Despite these advances, a key limitation
 102 remains: *since MMD has no inherent scale, it is difficult to assess whether newly proposed descrip-*
 103 *tors are suited for discriminating between real and generated graphs.* PGS, however, is comparable
 104 across descriptors and thus explicitly quantifies their discriminative power.

105 Departing from MMD, other evaluation paradigms have been proposed. Southern et al. (2023) used
 106 tools from topological data analysis, featurizing graphs via persistent homology and comparing dis-
 107 tributions based on their average persistence landscapes. In a different direction, Martinkus et al.
 108 (2022) introduced synthetic benchmark datasets (Planar and SBM) that allow for judging the struc-
 109 tural validity of individual graph samples—such as planarity. The small size of the synthetic datasets

Table 1: Comparison of Maximum Mean Discrepancy and PolyGraphScore.

Property	MMD	PGS
Range	$[0, \infty)$	$[0, 1]$
Intrinsic Scale	✗	✓
Descriptor Comparison	✗	✓
Single Ranking	✗	✓

108 and the resulting variance in MMD estimates were criticized by Krimmel et al. (2025). *We expand*
 109 *on these observations and propose concrete techniques for quantifying the uncertainty in GGM*
 110 *evaluation metrics, addressing a critical need for more reliable and reproducible evaluations.*

111 **Classifier-Based Evaluation.** One relevant family of metrics used for generative model evaluation
 112 is derived from the classifier two-sample test (C2ST) (Lopez-Paz & Oquab, 2017). This work pro-
 113 poses to discriminate generated from reference samples via binary classification and repurpose the
 114 resulting accuracy as a measure for the separability of the generated and reference distributions. By
 115 extension, it assesses the quality of the generative model.

116 The MMD can also be viewed through this lens, as it corresponds to the optimal linear risk of a
 117 kernel classifier (Sriperumbudur et al., 2009; Gretton & Jitkrittum, 2016). Generative adversarial
 118 networks (GANs) (Goodfellow et al., 2014; Li et al., 2015; Bińkowski et al., 2018; Arjovsky et al.,
 119 2017) also leverage a classifier’s output, not just for training but also for evaluation, where classifier-
 120 based divergences (including MMD) have been shown to correlate well with the perceptual quality
 121 of generated images (Im et al., 2018).

122 Despite the success of these methods in other domains, their application to graph generation has
 123 been limited. While some work has used fixed multi-class classifiers on generated graphs to mea-
 124 sure performance (Liu et al., 2019), classifiers that discriminate between real and generated graphs
 125 have not been explored beyond the MMD framework. *Our work addresses this gap, proposing a*
 126 *novel classifier-based evaluation framework for GGMs that provides scores that are (i) absolute,*
 127 *(ii) comparable across different graph descriptors, and (iii) capable of estimating lower bounds on*
 128 *certain probability metrics.*

130 3 PRELIMINARIES

131 In this section, we review two divergences, MMD and the Jensen-Shannon (JS) divergence, from a
 132 unified variational perspective: the optimal performance of a discriminator tasked with distin-
 133 guishing between two distributions. We first discuss MMD, interpreting it as the linear risk of a classifier
 134 in a reproducing kernel Hilbert space (RKHS) (Sriperumbudur et al., 2009). We highlight its lim-
 135 itations in the context of graph generation, primarily its lack of an absolute scale, which motivates
 136 our subsequent review of the JS distance as a foundation for more interpretable, classifier-based
 137 evaluation metrics such as the PolyGraphScore.

140 3.1 MMD AND ITS INTERPRETATION AS CLASSIFICATION RISK

141 Given two probability distributions P and Q over a space \mathcal{X} (in our case, the space of graphs) and a
 142 kernel $k : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}$, the squared MMD is defined as:

$$144 \text{MMD}^2(P, Q, k) := \mathbb{E}_{x, x' \sim P}[k(x, x')] - 2\mathbb{E}_{x \sim P, y \sim Q}[k(x, y)] + \mathbb{E}_{y, y' \sim Q}[k(y, y')]. \quad (1)$$

145 The MMD can be expressed as the distance between the mean embeddings of P and Q in the RKHS
 146 \mathcal{H} induced by k . This framing leads to a variational formulation where the MMD is precisely the
 147 optimal linear classification risk achievable by a discriminator in the unit ball of \mathcal{H} (Sriperumbudur
 148 et al., 2009). We refer to Appendix D for a detailed derivation.

149 **Limitations.** A fundamental limitation of MMD for model evaluation is its lack of an absolute scale
 150 (O’Bray et al., 2022). The MMD value is sensitive to the choice of kernel and the scaling of input
 151 features. For instance, using a linear kernel, simply scaling the input features by a scalar factor will
 152 scale the resulting MMD by the same factor. This makes it impossible to compare MMD scores
 153 across different graph descriptors. While MMD can rank models relative to a baseline for a fixed
 154 descriptor, it provides no absolute measure of performance.

155 To overcome this, we turn to metrics that possess a fixed intrinsic scale, making them compara-
 156 ble across different graph descriptors. This leads us to the Jensen-Shannon divergence and, more
 157 generally, to the family of f -divergences.

159 3.2 VARIATIONAL ESTIMATION OF THE JENSEN-SHANNON DISTANCE

160 The Jensen-Shannon (JS) divergence is a symmetrized version of the Kullback-Leibler (KL) diver-
 161 gence: $\frac{1}{2}(D_{\text{KL}}(P \parallel M) + D_{\text{KL}}(Q \parallel M))$ with $M := \frac{1}{2}(P + Q)$ being the mixture of P and Q . It is

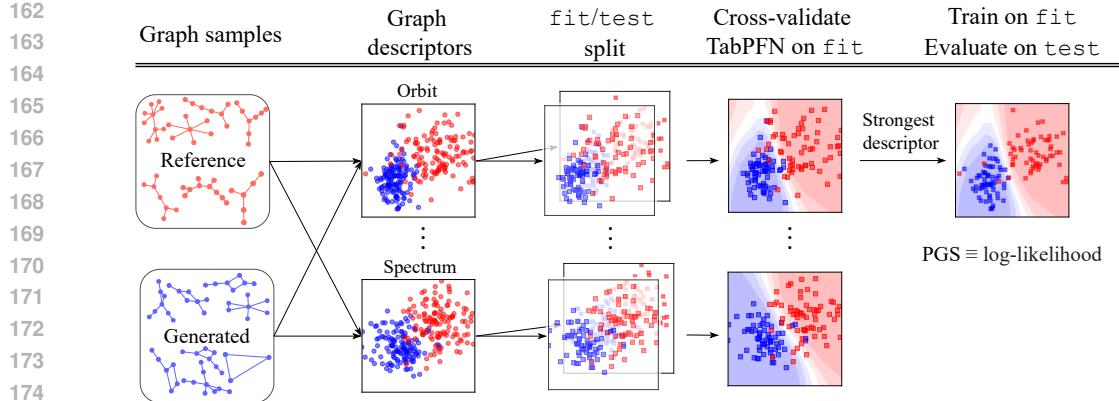


Figure 1: Computation of the PGS metric. TabPFN is trained to discriminate between generated and reference graphs based on different vectorial descriptions. The most expressive descriptor (here: orbit) is used to derive the final PGS, yielding a maximally tight lower bound on the Jensen-Shannon (JS) distance between the generated and reference graph distributions.

constrained to the unit interval $[0, 1]$ and, in contrast to MMD, is independent of extrinsic parameters such as kernel choice. As extensively leveraged in GANs (Goodfellow et al., 2014), the JS divergence admits (under mild conditions) a variational formulation as the maximal data log-likelihood (up to constants) achievable by a binary classifier D distinguishing between samples from P and Q :

$$D_{\text{JS}}(P \parallel Q) = \sup_{D: \mathcal{X} \rightarrow [0, 1]} \frac{1}{2} \mathbb{E}_{x \sim P} [\log_2 D(x)] + \frac{1}{2} \mathbb{E}_{x \sim Q} [\log_2 (1 - D(x))] + 1. \quad (2)$$

Importantly, the log-likelihood of *any* classifier provides a valid lower bound on the JS divergence and the bound is tightened by fitting a classifier via maximum likelihood methods. While the JS divergence is not a metric, its square root (termed the JS distance) is (Endres & Schindelin, 2003).

The JS divergence belongs to the larger family of f -divergences. As shown by Nguyen et al. (2010), any f -divergence admits a variational formulation similar to Eqn. (2). In Appendix E, we investigate the total variation (TV) distance as a possible alternative to the JS distance. Instead of log-likelihood, we show that the variational objective of the TV distance is given by the classifier’s *informedness*.

4 POLYGRAPHSCORE: VARIATIONAL ESTIMATES OF THE JS DISTANCE

Building on the variational view of divergences, we introduce PolyGraphScore (PGS), a framework for evaluating GGMs. PGS estimates the JS distance between a distribution of reference graphs and a distribution of generated graphs. The core idea is to reframe the divergence estimation as a classification task: we featurize graphs using a variety of established graph descriptors and measure how well a powerful, non-parametric classifier can distinguish between the two sets. The resulting classifier performance, measured in terms of log-likelihood, serves as a tight, empirical lower bound on the true JS divergence between the underlying graph distributions. Fig. 1 shows this procedure.

Our method proceeds in two main stages. First, we detail how to estimate a lower bound on the divergence using a *single* graph descriptor in Section 4.1. Second, we describe in Section 4.2 how to systematically combine *multiple* descriptors from a larger set to compute the final PGS, which represents the tightest lower bound from the given descriptors. We provide pseudocode in Appendix B.

4.1 ESTIMATING THE JS DISTANCE WITH A SINGLE DESCRIPTOR

Given a multiset of reference graphs P_{ref} and generated graphs Q_{gen} , along with a single graph descriptor $d: \mathcal{X} \rightarrow \mathbb{R}^n$, we estimate the divergence of P_{ref} and Q_{gen} via featurization by d .

To prevent overfitting, where a classifier might perfectly memorize the training data and thus overestimate the true divergence, we randomly partition both P_{ref} and Q_{gen} into disjoint `fit` and `test`

216 sets of equal size. Our goal is to approximate the supremum in Eqn. (2) by training a discriminator
 217 exclusively on the `fit` set, and computing the final divergence estimate on the held-out `test` set.
 218

219 **Discriminator Choice.** An appropriate discriminator for this task must satisfy three criteria:

220 1. **Probabilistic:** It must output class probabilities to estimate the JS divergence via its log-
 221 likelihood objective.
 222 2. **Efficient:** It must be fast to train, enabling rapid evaluation across many descriptors.
 223 3. **Hyperparameter-Free:** It should be robust and require no manual tuning to ensure fair and
 224 reproducible comparisons.

225 These requirements rule out the training of deep neural networks with gradient descent, because it
 226 is computationally expensive and requires hyperparameter tuning. It also rules out non-probabilistic
 227 models such as decision trees and SVMs. As a result, we choose TabPFN (Hollmann et al., 2025) in
 228 this work. TabPFN is a transformer-based model that approximates Bayesian inference over a large
 229 space of simple models consisting of Bayesian neural networks and structural causal models. It is
 230 fast (see Table 15), requires no hyperparameter tuning, and has proven to be a powerful classifier
 231 for tabular data, making it an ideal choice for our framework since our classifier operates on graph
 232 descriptors. In Appendix J, we investigate logistic regression as an alternative choice and show that
 233 TabPFN yields tighter bounds in practice. In Appendix R we show that kernel logistic regression
 234 also fits naturally into the PGS framework, allowing for the use of, e.g., graph kernels (Borgwardt
 235 & Kriegel, 2005; Shervashidze et al., 2011; Grauman & Darrell, 2007). However, similar to logistic
 236 regression, we found that those kernel logistic regression-based PGS scores yielded looser bounds
 237 in practice, and elected to proceed with TabPFN.

238 **Estimation Procedure.** With a discriminator selected, we first apply the descriptor $d : \mathcal{X} \rightarrow \mathbb{R}^n$
 239 to the graphs in the `fit` set to create vectorial features. We then train the binary classifier on
 240 these features using TabPFN. We apply the descriptor to the `test` set and use the trained classifier
 241 to evaluate the data log-likelihood, providing an approximate lower bound of the JS divergence.
 242 Finally, we take the square root to estimate the JS *distance*.

244 4.2 DESCRIPTOR SELECTION FOR THE TIGHTEST BOUND

245 A single graph descriptor captures only one specific aspect of graph structure. To obtain a more
 246 comprehensive evaluation, we consider a collection of K distinct descriptors $\{d_1, \dots, d_K\}$. The
 247 goal is to identify the single descriptor that most effectively distinguishes between the reference and
 248 generated graphs, as this descriptor will yield the tightest lower bound on the true JS distance. This
 249 descriptor selection process must be performed carefully to avoid data leakage from the `test` set,
 250 which would invalidate our final estimate. We therefore perform selection using only the `fit` data
 251 via cross-validation.

252 **Cross-Validation on the Fit Set.** For each descriptor $d_k : \mathcal{X} \rightarrow \mathbb{R}^n$, we estimate its ability to
 253 separate the distributions by performing 4-fold stratified cross-validation on the $(P_{\text{ref}}^{\text{fit}}, Q_{\text{gen}}^{\text{fit}})$ data.
 254 In each fold, three-quarters of the data are used for training a discriminator, and the remaining
 255 quarter is used for validation. The average validation score across the four folds provides a robust
 256 estimate of the lower bound achievable by that descriptor.

257 **The PolyGraphScore.** After performing cross-validation for all K descriptors, we select the
 258 descriptor d^* that yielded the highest average score. This is the descriptor that is empirically the
 259 most informative. Finally, we train a new discriminator for d^* on the *entire* `fit` set and evaluate
 260 it on the held-out `test` set. The resulting score is the PolyGraphScore (PGS). This procedure
 261 ensures that the descriptor selection and final evaluation are performed on separate data, yielding a
 262 principled and tight estimate of the divergence between the graph distributions.

264 5 EXPERIMENTS

265 We empirically validate PGS through a series of experiments designed to test its robustness, sen-
 266 sitivity, and practical utility against standard MMD-based metrics for evaluating graph generative
 267 models. Our investigation consists of four stages:

- 270 • First, Section 5.1 shows that MMD evaluations suffer from *substantial bias and variance on*
 271 *current datasets*, motivating the use of larger datasets, unbiased estimators, and subsampling to
 272 assess estimate stability.
- 273 • In Section 5.2, we show that *PGS correlates well with controlled perturbations* applied to syn-
 274 *thetic datasets, showing the power of JSD to distinguish samples from different distributions*.
- 275 • In a realistic use case for a state-of-the-art diffusion model (Section 5.3), we show that *PGS*
 276 *reliably tracks training progress and performance gains when increasing the number of denoising*
 277 *steps*. Our results indicate that PGS captures model quality more reliably than MMD metrics.
- 278 • Finally, in Section 5.4 we leverage PGS to conduct a *comprehensive benchmark* of several repre-
 279 *sentative GGMs*.

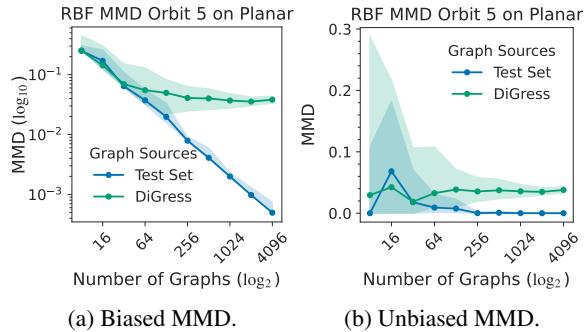
280 Unless otherwise stated, all PGS scores are based on the Jensen-Shannon (JS) distance estimated
 281 using TabPFN as the discriminator. Following previous works (You et al., 2018; Liao et al., 2019;
 282 Thompson et al., 2022), we use degree histograms (abbreviated as Degree/Deg. in our tables
 283 and figures), clustering coefficient histograms (Clustering/Clust.), the Laplacian spectrum (Spec-
 284 tral/Spec.), orbit counts (Orbit), and GIN embeddings (GIN) as descriptors. For molecular graphs,
 285 we use domain-specific descriptors based on topological properties, physico-chemical parameters,
 286 and learned representations. We refer to Appendix C for further details.

288 5.1 HIGH BIAS AND VARIANCE PLAGUE MMD-BASED GGM BENCHMARKS

290 The evaluation of GGMs is predominantly
 291 conducted on synthetic, procedurally gen-
 292 erated datasets, including lobster graphs,
 293 stochastic block models (SBMs), and planar
 294 graphs, which permit the generation
 295 of arbitrarily large numbers of samples.
 296 Krimmel et al. (2025) first raised the
 297 issue that MMD values computed on such
 298 datasets can exhibit considerable variance,
 299 thereby casting doubt on the robustness of
 300 model rankings derived from these met-
 301 rics. In order to more rigorously char-
 302 acterize this phenomenon, we exploited the
 303 procedural nature of these datasets to sys-
 304 tematically vary the subsample sizes used in MMD. The MMD shown here is obtained with the
 305 radial basis function (RBF) kernel; more details are given in Appendix G.

306 In the regime of commonly used synthetic graph benchmarks (between 20 and 40 test graphs, c.f.
 307 Appendix P), bias dominates the MMD values (Figure 2a, in log scale for clarity). Even when
 308 using the unbiased MMD estimator¹, the variance across subsamples remains large enough to make
 309 model comparisons at these sample sizes unreliable (Figure 2b). Figure 2 illustrates these issues for
 310 DiGress-generated samples for planar graphs described with orbit counts, but extensive experiments
 311 in Appendix G show that they persist across all combinations of models, descriptors, and datasets.

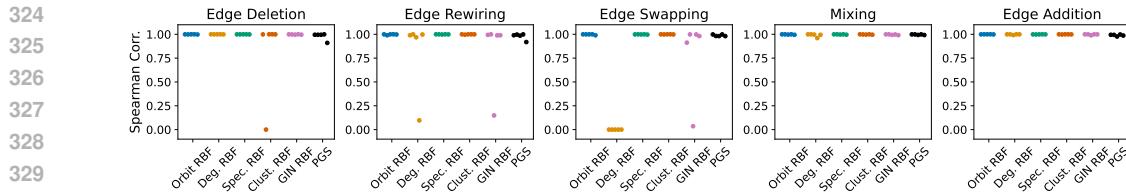
312 This finding yields three actionable insights. First, prefer *unbiased MMD* estimates, as bias depends
 313 heavily on sample size. Second, akin to Krimmel et al. (2025), use *larger sample sizes* to reduce
 314 estimator variance; we propose SBM-L, PLANAR-L, and LOBSTER-L for this purpose (with more
 315 details in Appendix M)². Third, report the *variance* of MMD across subsamples to quantify the
 316 stability of the estimates. To assess the effect of dataset size on PGS, we conducted analogous
 317 experiments in Appendix L, which show that its mean and variance stabilize beyond subsample sizes
 318 of about 256. This is particularly relevant because TabPFN’s discriminative power may depend on
 319 sample size.



321 Figure 2: Examples of MMD estimates that suffer from
 322 high bias (left) and variance (right).

323 ¹Our MMD estimates are not unbiased, as we take the maximum MMD value over a set of kernel band-
 324 widths, but we do use the unbiased MMD estimate without diagonals, see Eq. 3 in Gretton et al. (2012).

325 ²AutoGraph reaches similar VUN scores with markedly lower loss in SBM-L than on the original SBM
 326 dataset (see Appendix N), showing reduced overfitting, which is underexplored in GGMs (Vignac et al., 2023).

331 Figure 3: Spearman correlation of MMDs and PGS with magnitude of perturbation.
332
333334

5.2 POLYGRAPHSCORE TRACKS SYNTHETIC DATA PERTURBATIONS

335 To validate PGS as a reliable metric, we verify its ability to correlate with the magnitude of perturbations applied to graph datasets, a standard procedure for evaluating graph metrics (O’Bray
336 et al., 2022; Thompson et al., 2022). Our experiments demonstrate that PGS effectively tracks these
337 changes, performing on par with MMDs.
338

339 **Experimental Setup.** We conduct our experiments on five datasets: Protein contact graphs (Dobson
340 & Doig, 2003), ego nets extracted from Citeseer (Sen et al., 2008), and three procedural datasets
341 (Planar, SBM, Lobster). Each procedural dataset contains 4096 samples, while the proteins dataset
342 contains 918 samples, and the ego dataset contains 757 samples. Dataset details are in Appendix P.
343

344 To simulate data corruption, we apply five distinct perturbation types, four of which are adapted
345 from previous studies (O’Bray et al., 2022; Thompson et al., 2022). Each perturbation modifies
346 the graph structure (or dataset) in a controlled manner. Edge deletion/addition removes or adds a
347 specified number of edges selected at random. Edge rewiring replaces one of the incident vertices
348 of some edges with a randomly selected vertex. Mixing operates on the dataset level by replacing a
349 fraction of the graphs within a dataset with new samples from an Erdős–Rényi model. Finally, we
350 propose a novel perturbation type which we term “edge swapping”. Edge swapping selects pairs of
351 edges and swaps two of their incident vertices. This transformation preserves the vertex degrees,
352 making it a more challenging perturbation for some metrics to detect.
353

354 **Perturbation Experiments.** Our core experiment involves splitting each dataset into two equal
355 subsets: one serves as a fixed reference distribution, and the other is subjected to the perturbations.
356 We then measure the distance between the reference and the perturbed subset using PGS and MMD
357 metrics. Unlike MMD, PGS is a bounded metric in $[0, 1]$. This means it can saturate, or reach its
358 maximum value, when perturbations are too large and the distributions become non-overlapping. To
359 account for this, we first determine the perturbation magnitude at which PGS saturates (specifically,
360 exceeds 0.95). We then apply perturbations only within this non-saturating range and compute the
361 Spearman correlation between the metric scores and perturbation magnitudes. We visualize these
362 correlation coefficients in Fig. 3, where each data point represents a combination of dataset, pertur-
363 bation type, and metric. Our results show that PGS consistently exhibits a strong *rank* correlation
364 with perturbation magnitude, comparable to that of MMD metrics. We note that while the degree-
365 based and GIN-based MMD metrics struggle to detect the edge-swapping perturbation, PGS remains
366 robust by leveraging multiple descriptors that compensate for compromised ones.
367

368 We provide more details in Appendix H, where we illustrate the behavior of PGS as a function
369 of perturbation magnitude for all combinations of datasets and perturbations. From that analysis,
370 we conclude that no single descriptor dominates the others across all combinations of datasets and
371 perturbation types, underlining the necessity of considering a diverse set of graph descriptors. We
372 present additional experiments for a PGS estimating the Total Variation distance in Appendix F.2.
373 Appendix J provides similar results for a PGS variant using logistic regression instead of TabPFN.
374

375

5.3 POLYGRAPHSCORE CORRELATES WITH MODEL QUALITY

376 To demonstrate practical utility, we evaluate PGS on DiGress (Vignac et al., 2023), a state-of-the-art
377 GGM, using denoising iterations and training epochs as proxies for model quality. PGS strongly
378 correlates with both, capturing model improvement more faithfully than MMD metrics while main-
379 taining a strong linear correlation with the percentage of valid graphs generated. All metrics were
380 computed comparing 2048 reference graphs against 2048 generated graphs.
381

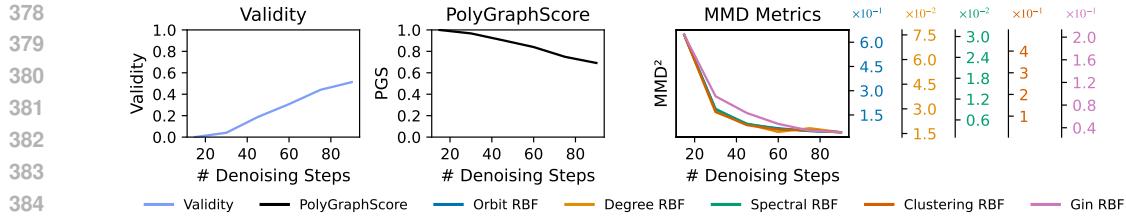


Figure 4: Trajectory of validity, PGS, and MMDs when increasing the number of denoising steps in DiGress on PLANAR-L.

Table 2: Negative Pearson correlation (\uparrow) of validity with other distance-based metrics. Denoising refers to the experiments in which we vary the number of denoising iterations. Training refers to the experiments in which we monitor performance metrics during the training of DiGress models.

	PGS	Orbit RBF	Deg. RBF	Spec. RBF	Clust. RBF	GIN RBF
Denoising	99.52	73.49	70.79	73.34	71.48	82.78
Training	99.05	84.33	76.52	79.05	81.61	81.07
PLANAR-L	88.07	51.05	15.77	36.76	83.97	14.12
SBM-L	89.32	-34.81	-33.40	-22.79	87.05	-30.31
LOBSTER-L						

Denoising Iterations. We first analyze the impact of the number of denoising steps on sample quality. Six DiGress models are trained on the large procedural planar dataset using a range of 15 to 90 denoising steps. As shown in Fig. 4, increasing the number of steps generally improves model performance across all metrics. We find that PGS has a much stronger *linear* relationship with validity than MMD metrics, as shown by the *Pearson correlation coefficients* in Table 2. This tight relationship is especially encouraging as validity, alongside uniqueness and novelty, is often considered a gold standard metric for assessing model quality. Yet, validity is not always defined. Uniqueness and novelty can be provided jointly with PGS to offer complementary insights.

Training Iterations. Similarly, we assess the ability of MMD and PGS to track model quality throughout the training process on LOBSTER-L, PLANAR-L, and SBM-L. The central hypothesis is that reliable metrics should improve monotonically with training duration. **We note that this relationship is non-linear, hence the use of Spearman’s correlation coefficient.** As illustrated for the SBM-L dataset in Fig. 5, PGS and validity align with this hypothesis, whereas MMD metrics exhibit erratic behavior. Analogous results for PLANAR-L and LOBSTER-L are provided in Appendix I. Spearman’s rank correlation in Table 3 confirms this quantitatively across all datasets: both PGS and validity are strongly correlated with training duration, while MMD metrics show weak or even negative correlations. The Pearson correlations in Table 2 further show that *PGS maintains its strong linear correlation with validity* during training, a property not consistently shared by MMD metrics.

5.4 BENCHMARKING REPRESENTATIVE MODELS

We next present concrete PGS values and their associated subscores on a set of well-established models spanning distinct generative paradigms, including autoregressive architectures such as GRAN (Liao et al., 2019) and AutoGraph (Chen et al., 2025) and diffusion models such as ESGG (Bergmeister et al., 2023) and DiGress (Vignac et al., 2023). We benchmark them on our proposed datasets, SBM-L, LOBSTER-L, and PLANAR-L (with 2048 samples each, see Ap-

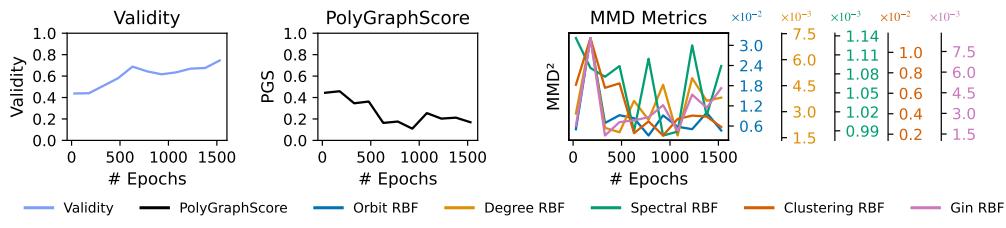


Figure 5: Trajectory of validity, PGS, and MMD metrics throughout training of DiGress on SBM-L.

Table 3: Sign-adjusted Spearman correlation (\uparrow) of validity, PGS, and MMDs with the number of training iterations for DiGress.

	Validity	PGS	Orbit RBF	Deg. RBF	Spec. RBF	Clust. RBF	GIN RBF
PLANAR-L	92.31	93.71	86.71	41.96	83.22	67.83	81.82
SBM-L	83.64	62.73	20.00	-19.09	18.18	58.18	-38.18
LOBSTER-L	85.47	78.19	-8.09	-4.66	13.73	68.14	-2.70

Table 4: Mean PGS \pm standard deviation across synthetic and real-world graphs. AutoGraph* denotes a model pretrained on the PubChem dataset. More details can be found in the original paper (Chen et al., 2025). Values are multiplied by 100 for readability. Subscores are computed on the training set to select the best descriptor, and the final PGS refers to the score computed on the test set with the best descriptor.

Dataset	Model	PGS subscores							
		VUN (↑)	PGS (↓)	Clust. (↓)	Deg. (↓)	GIN (↓)	Orb5. (↓)	Orb4. (↓)	Eig. (↓)
PLANAR-L	AutoGraph	85.1	34.0 ± 1.8	7.0 ± 2.9	7.8 ± 3.2	8.8 ± 3.0	34.0 ± 1.8	28.5 ± 1.5	26.9 ± 2.3
	DiGRESS	80.1	45.2 ± 1.8	24.8 ± 2.0	23.3 ± 1.2	29.0 ± 1.1	45.2 ± 1.8	40.3 ± 1.8	39.4 ± 2.0
	GRAN	1.6	99.7 ± 0.2	99.3 ± 0.2	98.3 ± 0.3	98.3 ± 0.3	99.7 ± 0.1	99.2 ± 0.2	98.5 ± 0.4
	ESGG	93.9	45.0 ± 1.4	10.9 ± 3.2	21.7 ± 3.0	32.9 ± 2.2	45.0 ± 1.4	42.8 ± 1.9	29.6 ± 1.6
LOBSTER-L	AutoGraph	83.1	18.0 ± 1.6	4.2 ± 1.9	12.1 ± 1.6	14.8 ± 1.5	18.0 ± 1.6	16.1 ± 1.6	13.0 ± 1.1
	DiGRESS	91.4	3.2 ± 2.6	2.0 ± 1.3	1.2 ± 1.5	2.3 ± 2.0	3.0 ± 3.1	4.5 ± 2.3	1.3 ± 1.1
	GRAN	41.3	85.4 ± 0.5	20.8 ± 1.1	77.1 ± 1.2	79.8 ± 0.6	85.4 ± 0.5	85.0 ± 0.6	69.8 ± 1.2
	ESGG	70.9	69.9 ± 0.6	0.0 ± 0.0	63.4 ± 1.1	66.8 ± 1.0	69.9 ± 0.6	66.0 ± 0.6	51.7 ± 1.8
SBM-L	AutoGraph	85.6	5.6 ± 1.5	0.3 ± 0.6	6.2 ± 1.4	6.3 ± 1.3	3.2 ± 2.2	4.4 ± 2.0	2.5 ± 2.2
	DiGRESS	73.0	17.4 ± 2.3	5.7 ± 2.8	8.2 ± 3.3	13.8 ± 1.7	17.4 ± 2.3	14.8 ± 2.5	8.7 ± 3.0
	GRAN	21.4	69.1 ± 1.4	50.2 ± 1.9	58.6 ± 1.4	69.1 ± 1.4	65.7 ± 1.3	62.8 ± 1.3	55.9 ± 1.5
	ESGG	10.4	99.4 ± 0.2	97.9 ± 0.5	97.5 ± 0.6	98.3 ± 0.4	96.8 ± 0.4	89.2 ± 0.7	99.4 ± 0.2
Proteins	AutoGraph	-	67.7 ± 7.4	47.7 ± 5.7	31.5 ± 8.5	45.3 ± 5.1	67.7 ± 7.4	47.4 ± 7.0	53.2 ± 6.9
	DiGRESS	-	88.1 ± 3.1	36.1 ± 4.3	29.2 ± 5.0	23.2 ± 5.3	88.1 ± 3.1	60.8 ± 3.6	23.4 ± 11.8
	GRAN	-	89.7 ± 2.7	86.0 ± 2.0	70.6 ± 3.1	71.5 ± 3.0	90.4 ± 2.4	84.4 ± 3.3	76.7 ± 4.7
	ESGG	-	79.2 ± 4.3	58.2 ± 3.6	54.0 ± 3.6	57.4 ± 4.1	80.2 ± 3.1	72.5 ± 3.0	24.3 ± 11.0
Dataset	Model	PGS subscores							
		Valid (↑)	PGS (↓)	Topo (↓)	Morgan (↓)	ChemNet (↓)	MolCLR (↓)	Lipinski (↓)	
GUACAMOL	AutoGraph	91.6	22.9 ± 0.5	8.2 ± 0.7	15.7 ± 0.8	22.9 ± 0.5	16.6 ± 0.4	19.4 ± 0.7	
	AutoGraph*	95.9	10.4 ± 1.2	4.3 ± 0.7	4.7 ± 1.4	4.6 ± 0.6	1.7 ± 1.0	10.4 ± 1.2	
	DiGRESS	85.2	32.7 ± 0.5	19.6 ± 0.6	20.4 ± 0.5	32.5 ± 0.7	22.9 ± 0.6	32.8 ± 0.5	
MOSES	AutoGraph	87.4	29.6 ± 0.4	22.4 ± 0.4	16.3 ± 1.3	25.8 ± 0.7	20.5 ± 0.5	29.6 ± 0.4	
	DiGRESS	85.7	33.4 ± 0.5	26.8 ± 0.4	24.8 ± 0.8	29.1 ± 0.6	24.3 ± 0.7	33.4 ± 0.5	

pendix M) as well as the Proteins dataset with 92 samples (Dobson & Doig, 2003). Additionally, we present PGS benchmarks of AutoGraph and DiGress on the molecular datasets GuacaMol (Brown et al., 2019) and MOSES (Polykovskiy et al., 2020) using 10,000 generated samples for benchmarking. For these datasets, we propose domain-specific descriptors which we describe in Appendix C.2. Appendix K contains further benchmarking methodological details.

As shown in Table 4, AutoGraph and DiGress achieve the best overall PGS scores across most datasets. PGS generally aligns with VUN or validity rankings, though some exceptions exist—ESGG ranks highest in VUN on PLANAR-L but performs worse in PGS. The Proteins dataset yields the highest scores, suggesting greater modeling difficulty. Max-reduction proves helpful in edge cases like LOBSTER-L, where clustering coefficients are uniformly zero, preventing a single uninformative subscore from masking other structural flaws. When interpreting the final PGS score, note it can differ from individual subscores since they use different datasets. Subscores are averaged over cross-validation splits on the *training set* to select the most informative descriptor, while the final PGS is computed on the *test set*, potentially yielding different results. Appendix K compares MMD and PGS values using Gaussian TV pseudo-kernels (Table 11) and optimized RBF kernels (Tables 12 and 13). Overall, PGS yields more interpretable model rankings than MMDs.

We also consider a feature concatenation variant of PGS as an alternative to max-reduction, where we concatenate all descriptors and apply PCA to fit TabPFN’s feature limits (500 for v2.0) in Appendix Q. While this yields tighter bounds (higher JSD estimates), it prevents identifying the most informative descriptor; therefore, we recommend max-reduction in practice.

486 **6 CONCLUSION**
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488 We introduce PGS, a classifier-based evaluation that yields unit-scale metrics by training a dis-
 489 criminator on standard graph descriptors and selecting the most informative one. Instantiated with
 490 TabPFN to estimate the JS distance, PGS is fast and tuning-free. Across perturbation and model-
 491 quality studies, PGS increases monotonically with synthetic noise and correlates strongly—and often
 492 linearly—with validity and training progress. It also produces robust rankings with descriptor-specific
 493 subscores. To standardize GGM evaluation and model selection, we release the PolyGraph library,
 494 PGS, and the larger datasets, which we show are necessary to avoid high bias and variance observed
 495 in evaluation metrics. We discuss potential limitations in Appendix A. We hope that our work cat-
 496 alyzes progress in graph generation and, more broadly, enables effective evaluations of generative
 497 models where multiple combinations of possibly complementary descriptors are required.
 498

499 **ETHICS STATEMENT**
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501 This work focuses on the development of evaluation methods for graph generative models. Our study
 502 does not involve human subjects, animals, or personal data. We do not foresee harm to individuals,
 503 groups, or the environment.

504 **REPRODUCIBILITY STATEMENT**
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506 To ensure reproducibility, we will publicly release the PolyGraph library that implements the
 507 PolyGraphScore, MMD metrics, and datasets discussed in this paper. This library is also pro-
 508 vided to the reviewers as anonymized supplementary material. Unit tests ensure consistency of
 509 the MMD metrics implemented in PolyGraph with the implementations of Liao et al. (2019);
 510 Thompson et al. (2022). We refer to Appendices B and C for a more detailed explanation of the
 511 PGS estimation procedure and the graph descriptors considered in this work. Appendix M details
 512 how to generate our improved procedural datasets. We are committed to storing all data generated in
 513 this work (including model checkpoints and computed metrics) in a long-term private archive with
 514 a minimum guaranteed access period of ten years.

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A LIMITATIONS

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758 Here, we touch upon some of the limitations of this work.
759760 **Descriptor dependence and information loss.** PGS operates on hand-crafted descriptors rather
761 than raw graphs. It therefore yields a lower bound of the divergence between *descriptor distributions*,
762 which itself is a lower bound of the divergence between the *graph distributions*. If the
763 divergence between descriptor distributions does not tightly approximate the divergence between
764 graph distributions, the PGS is also inherently a loose bound on the divergence between graph dis-
765 tributions. This highlights the importance of considering expressive descriptors.766 The final max-reduction can also under-utilize complementary signals across descriptors. This could
767 be addressed by combining features prior to TabPFN fitting, and using TabPFN extensions for auto-
768 matic feature selection³. If future TabPFN-like foundation models also support more input features,
769 this limitation will vanish.770 **Sample-size dependence** On the one hand, PGS requires several hundred samples to get an accu-
771 rate metric value, as indicated in Appendix L, which requires some computational burden, especially
772 for difficult-to-compute descriptors. On the other hand, the sample size used in our formulation of
773 PGS is constrained by TabPFN’s recommended 10k training limit, though this restriction is only an
774 implementation detail. This might be problematic in practice if a large number of samples is re-
775 quired to obtain a tight bound. We recommend that users assess the variance of PGS carefully when
776 considering new descriptors, graph types, and discriminators. The TabPFN extensions package also
777 implements some approaches to extend the training size via subsampling and ensembling⁴.
778779 **Limited feature dimensionality.** While MMD can operate on high-dimensional graph descrip-
780 tors, the classifier used in PGS may impose limits on the dimensionality of these features. The
781 TabPFN model that we use in our work has been shown to be effective on up to 500-dimensional
782 features. The graph descriptors proposed in previous works (c.f. Appendix C.1) are well within these
783 limits. In the context of evaluating molecule generative models, we employ random projections to
784 map 512-dimensional graph representations to a more compact feature space (c.f. Appendix C.2). A
785 more sophisticated feature selection process may yield tighter bounds on the JS distance. We leave
786 the exploration of optimal feature selection to future work.787 **Scopes of graph types, datasets, and models.** Our experiments focus on common procedural
788 datasets (with specific parameters), proteins, and molecules. We do not evaluate directed, temporal,
789 or heterogeneous graphs, and leave this to future work. While we benchmark four different GGMs,
790 covering autoregressive and denoising diffusion paradigms, we hope that future works adopt the
791 PGS framework to extend benchmarks to a wider variety of methods.792 **Application to other domains.** We focus on applying PGS to generative graph evaluation, where
793 the need for rigorous assessment is particularly acute. Nonetheless, the same approach could ex-
794 tend to other domains, though we leave this unexplored. One promising direction is improving
795 InceptionV3-style scoring: our multi-descriptor strategy could mitigate the sensitivity of FID to net-
796 work initialization by max-reducing across multiple InceptionV3 initializations, which was shown
797 to be problematic by Barratt & Sharma (2018).798

B PGS PSEUDOCODE

800 We provide pseudocode for the computation of PGS in Algorithm 1. We note that the pro-
801 cedure `estimate_divergence` corresponds to the algorithm we describe in Section 4.1 while
802 `polygraphscore` implements the combination of descriptors we outline in Section 4.2.
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809 ³<https://github.com/PriorLabs/tapfn-extensions>
see Footnote 3

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Algorithm 1 PGS computation

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824: **procedure** estimate_divergence(train, val, mode)
825: clf \leftarrow fit_tabpfn(train)
826: preds \leftarrow clf.predict(val.x)
827: **if** mode = "jsd" **then**
828: metric \leftarrow $\sqrt{\max(\text{log_likelihood}(\text{preds}, \text{val.y}), 0)}$
829: **else**
830: $\gamma \leftarrow \text{max_info_threshold}(\text{clf.predict}(\text{train.x}), \text{train.y})$
831: metric $\leftarrow \text{informedness}(\text{preds}, \text{val.y}, \gamma)$
832: **return** metric
833
834: **procedure** train_test_divergence(reference, generated, descriptor, mode, k)
835: ref_train, ref_test \leftarrow reference[0 :: 2], reference[1 :: 2] \triangleright Split reference graphs
836: gen_train, gen_test \leftarrow generated[0 :: 2], generated[1 :: 2] \triangleright Split generated graphs
837: $(X, Y) \leftarrow (\text{descriptor}(\text{ref_train} \parallel \text{gen_train}), [0 \dots 0, 1 \dots 1])$
838: folds $\leftarrow \text{stratified_folds}(X, Y, k)$
839: cv_metric $\leftarrow 0$
840: **for** train, val \in folds **do**
841: cv_metric \leftarrow cv_metric + estimate_divergence(train, val, mode)
842: cv_metric \leftarrow cv_metric/k
843: $(X_{\text{test}}, Y_{\text{test}}) \leftarrow (\text{descriptor}(\text{ref_test} \parallel \text{gen_test}), [0 \dots 0, 1 \dots 1])$
844: test_metric \leftarrow estimate_divergence((X, Y) , $(X_{\text{test}}, Y_{\text{test}})$, mode)
845: **return** cv_metric, test_metric
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864 **C GRAPH DESCRIPTORS**
865866 In this section, we discuss the vectorial graph descriptions used in our work. In Appendix C.1,
867 we provide details on the descriptors we apply to the synthetic datasets (PLANAR-L, SBM-L,
868 LOBSTER-L) and the Proteins dataset. These descriptors are, for the most part, identical to estab-
869 lished descriptors introduced for MMD evaluations (You et al., 2018; Liao et al., 2019; Thompson
870 et al., 2022). In Appendix C.2, we introduce novel descriptors for evaluating generative models for
871 molecules.872 We recommend that practitioners use domain-specific and expressive descriptors whenever possible,
873 similar to our procedure for molecules in Appendix C.2. As discussed previously, one should aim to
874 maximize the PGS metric when engineering graph descriptors.
875876 **C.1 GENERIC DESCRIPTORS**
877878 We use graph descriptors that have previously been proposed for evaluations via Maximum Mean
879 Discrepancy. Histograms of clustering coefficients and node degrees, as well as 4-node orbit counts,
880 have been proposed by You et al. (2018). These descriptors were extended by Liao et al. (2019) via
881 the spectrum of the graph Laplacian. Finally, Thompson et al. (2022) proposed to featurize graphs
882 via randomly initialized GIN models. We extend these descriptors with 5-node orbit counts, com-
883 puted with the ORCA algorithm (Hočvar, 2025). In our model benchmarks, we find that 5-node
884 orbit counts oftentimes yield the highest PGS, hence representing a strong descriptor (c.f. Table 4).
885 However, we find in the perturbation experiments (c.f. Appendix H) that no single descriptor con-
886 sistently dominates the others. This demonstrates the importance of considering a wide variety of
887 graph featurizers. We summarize our descriptors in Table 5.888
889 Table 5: Generic graph descriptors.890
891

Descriptor	Meaning	Reference
Clust.	Histogram of clustering coefficients, discretized to 100 bins in $[0, 1]$	You et al. (2018)
Deg.	Histogram of node degrees	You et al. (2018)
GIN	Activations of a randomly initialized GIN graph neural network	Thompson et al. (2022)
Eig.	Histogram of Laplacian spectrum, discretized to 200 bins in $[-10^{-5}, 2]$	Liao et al. (2019)
Orb. 4	4-node orbit counts	You et al. (2018); Hočvar (2025)
Orb. 5	5-node orbit counts	Hočvar (2025)

905
906 **C.2 MOLECULE-SPECIFIC DESCRIPTORS**
907908 We propose several novel descriptors for evaluating generative models for molecules
909 via the PolyGraphScore framework. Some of these descriptors are established in
910 chemoinformatics and are computed via RDKit (RDKit, 2024). Namely, topo-
911 logical quantities (`rdkit.Chem.GraphDescriptors`), physico-chemical pa-
912 rameters (`rdkit.Chem.Lipinski`) and classical Morgan molecule fingerprints
913 (`rdkit.Chem.AllChem.GetMorganGenerator`). Additionally, we use learned repre-
914 sentations extracted either from a SMILES-based LSTM model (Mayr et al., 2018) (termed
915 ChemNet), or from the contrastively trained MolCLR graph neural network (Wang et al., 2022).
916 The SMILES-based model has previously been used to formulate the Fréchet ChemNet dis-
917 tance (Preuer et al., 2018). To obtain more compact features, we map the learned representations
into a 128-dimensional space via sparse random projections with a fixed random seed.

918 These descriptors can only be computed for molecular graphs which can be converted into
 919 `rdkit.Chem.rdcchem.Mol` objects, i.e., for graphs which are chemically valid. Hence, we must
 920 filter generated graphs before computing a PGS score. A similar approach has been taken in the
 921 Fréchet ChemNet distance.

922 We summarize these descriptors in more detail in Table 6.
 923

924
 925
 926 Table 6: Descriptors used for molecular graphs.
 927

928 Descriptor	929 Meaning	930 Features	931 Reference
930 Morgan	931 128-D Morgan count fin- 932 gerprint	933 Substructure hash counts	934 RDKit (2024)
931 ChemNet	932 128-D projection of 933 ChemNet embedding 934 of canonical SMILES 935 string	936 Latent	937 Mayr et al. (2018)
935 MolCLR	936 128-D projection of 937 MolCLR embedding of 938 molecule graph	939 Latent	940 Wang et al. (2022)
938 Topo	939 Topological/topochemical 940 descriptors based on the 941 bond structure	942 1. AvgIpc 943 2. BertzCT 944 3. BalabanJ 945 4. HallKierAlpha 946 5. Kappa1 947 6. Kappa2 948 7. Kappa3 949 8. Chi0 950 9. Chi0n 951 10. Chi0v 952 11. Chi1 953 12. Chi1n 954 13. Chi1v 955 14. Chi2n 956 15. Chi2v 957 16. Chi3n 958 17. Chi3v 959 18. Chi4n 960 19. Chi4v	961 RDKit (2024)
961 Lipinski	962 Structural and physico- 963 chemical parameters	964 1. HeavyAtomCount 965 2. NHOHCount 966 3. NCount 967 4. NumHAcceptors 968 5. NumHDonors 969 6. NumHeteroatoms 970 7. NumRotatableBonds 971 8. RingCount 972 9. NumAliphaticCarbocycles 973 10. NumAliphaticHeterocycles 974 11. NumAliphaticRings 975 12. NumAromaticCarbocycles 976 13. NumAromaticHeterocycles 977 14. NumAromaticRings 978 15. NumHeterocycles 979 16. NumSaturatedCarbocycles 980 17. NumSaturatedHeterocycles 981 18. NumSaturatedRings 982 19. NumAmideBonds 983 20. NumAtomStereoCenters 984 21. NumUnspecifiedAtomStereoCenters 985 22. NumBridgeheadAtoms 986 23. NumSpiroAtoms 987 24. FractionCSP3 988 25. Phi	989 RDKit (2024)

972 **D MMD AS LINEAR CLASSIFICATION RISK**
973

974 In this section, we expand on the discussion in Section 3.1 and derive how MMD may be seen as
975 the optimal risk for distinguishing between P and Q of a binary classifier in the reproducing kernel
976 Hilbert space \mathcal{H} .
977

978 Using the notation $\mathbb{E}_x[k(x, \cdot)]$ for the Riesz representative of the (under mild conditions) bounded
979 linear form $f \mapsto \mathbb{E}_x[\langle f, k(x, \cdot) \rangle]$, one may show:
980

$$\begin{aligned} \text{MMD}(P, Q, k) &= \|\mathbb{E}_{x \sim P}[k(x, \cdot)] - \mathbb{E}_{y \sim Q}[k(y, \cdot)]\|_{\mathcal{H}} \\ &= \sup_{\|D\|_{\mathcal{H}} \leq 1} \langle D, \mathbb{E}_{x \sim P}[k(x, \cdot)] - \mathbb{E}_{y \sim Q}[k(y, \cdot)] \rangle \\ &= \sup_{\|D\|_{\mathcal{H}} \leq 1} \langle D, \mathbb{E}_{x \sim P}[k(x, \cdot)] \rangle - \langle D, \mathbb{E}_{y \sim Q}[k(y, \cdot)] \rangle \\ &= \sup_{\|D\|_{\mathcal{H}} \leq 1} \mathbb{E}_{x \sim P}[D(x)] - \mathbb{E}_{y \sim Q}[D(y)]. \end{aligned} \quad (3)$$

981 We use the Cauchy-Schwarz inequality in the second equality, the linearity of the inner product in
982 the third equality, and the definition of the Riesz representative in the last equality.
983

984 This framing reveals that MMD is precisely the optimal linear classification risk achievable by a
985 discriminator D in the unit ball of the function space induced by the kernel.
986

987 **E BACKGROUND ON f -DIVERGENCES AND TOTAL VARIATION DISTANCE**
988

989 Let P and Q be probability measures on \mathcal{X} that are assumed to be absolutely continuous with respect
990 to a base measure μ , having densities p and q . For now, also assume P to be absolutely continuous
991 w.r.t. Q . For a convex, lower-semicontinuous function $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ satisfying $f(0) = 1$, the
992 f -divergence of P from Q is defined as:
993

$$D_f(P \parallel Q) := \int_{\mathcal{X}} q(x) f\left(\frac{p(x)}{q(x)}\right) d\mu \quad (4)$$

994 As shown by Nguyen et al. (2010), f -divergence can be estimated via a variational objective similar
995 to that of MMD. Using the Fenchel conjugate $f^*(v) := \sup_{u \in \mathbb{R}_+} uv - f(u)$, the f -divergence is
996 lower-bounded by:
997

$$D_f(P \parallel Q) \geq \sup_{D \in \mathcal{F}} \mathbb{E}_{x \sim P}[D(x)] - \mathbb{E}_{y \sim Q}[f^*(D(x))], \quad (5)$$

998 for any family \mathcal{F} of measurable functions $D : \mathcal{X} \rightarrow \mathbb{R}$. The bound is tight if and only if the func-
999 tional class \mathcal{F} is sufficiently expressive to contain a subderivative of f at the density ratio $p(x)/q(x)$.
1000 Such a function then achieves the supremum. The variational formulation of the Jensen-Shannon
1001 divergence in Eqn. (2) is a special case of Eqn. (5)
1002

1003 **Total Variation Distance.** The total variation (TV) distance corresponds to $f(x) = \frac{1}{2}|1-x|$. One
1004 may easily verify that the integral in Eqn. (4) evaluates to half of the L^1 distance between p and q .
1005 As we show in Appendix F.1, its variational objective in Eqn. (5) can be reduced to:
1006

$$\sup_{\substack{D: \mathcal{X} \rightarrow [0,1] \\ \gamma \in [0,1]}} \mathbb{E}_{x \sim P} [[D(x) > \gamma]] - \mathbb{E}_{x \sim Q} [[D(x) > \gamma]], \quad (6)$$

1007 where we use the Iverson bracket $[D(x) > \gamma]$ to denote the binarization of D at the threshold γ .
1008 This objective is also known as the Informedness (or Youden’s J statistic) of the discriminator D . It
1009 has a clear geometric interpretation as the maximum vertical distance between the ROC curve of D
1010 and the chance diagonal, with a fixed scale of $[0, 1]$.
1011

1022 **F PGS-TV: ESTIMATING TOTAL VARIATION DISTANCES**
1023

1024 In this section, we propose an alternative variant of the PGS, using variational estimates of the total
1025 variation (TV) distance in place of the Jensen-Shannon distance. We term this variant PGS-TV.
1026

We recall from Appendix E that the variational objective for the TV distance is given by the informedness of a dichotomized classifier. We provide a proof of this fact in Appendix F.1. When computing PGS-TV, the choice of binarization threshold is considered part of the fitting process of the classifier. Hence, we choose γ to maximize the vertical distance of the ROC on the *fit* set. We refer to Appendix B for pseudocode. In Appendices F.2 and F.3, we present an empirical investigation of PGS-TV, analogous to the experiments presented in Sections 5.2 and 5.3. Finally, we discuss the advantages of PGS over PGS-TV in Appendix F.4

F.1 VARIATIONAL FORMULATION OF TV DISTANCE

One may easily verify that for $f(u) = \frac{1}{2}|1-u|$, we have the following Fenchel conjugate:

$$f^*(v) = \sup_{u \in \mathbb{R}_+} uv - \frac{1}{2}|1-u| = \begin{cases} -\frac{1}{2} & \text{if } v < -\frac{1}{2} \\ v & \text{if } v \in [-\frac{1}{2}, \frac{1}{2}] \\ \infty & \text{if } v > \frac{1}{2} \end{cases} \quad (7)$$

We recall the variational lower bound:

$$D_{TV}(P \parallel Q) \geq \sup_{D \in \mathcal{F}} \mathbb{E}_{x \sim P}[D(x)] - \mathbb{E}_{y \sim Q}[f^*(D(x))] \quad (8)$$

Without weakening the lower bound, we may restrict ourselves to families of functions which are upper-bounded by $\frac{1}{2}$ almost everywhere w.r.t. Q . Indeed, discriminators D that do not satisfy this have a variational bound of $-\infty$. Since we are assuming $P \ll Q$, the discriminators are then also upper-bounded almost everywhere w.r.t. P . Hence, w.l.o.g., we may assume that they are upper-bounded by $\frac{1}{2}$ everywhere. Under these assumptions, we obtain the simpler formulation:

$$\begin{aligned} D_{TV}(P \parallel Q) &\geq \sup_{D \in \mathcal{F}} \mathbb{E}_{x \sim P}[D(x)] - \mathbb{E}_{y \sim Q} \left[\max \left(D(x), -\frac{1}{2} \right) \right] \\ &= \sup_{D \in \mathcal{F}} \int_{\mathcal{X}} D(x)p(x) - \max \left(D(x), -\frac{1}{2} \right) q(x)d\mu \end{aligned} \quad (9)$$

Under the constraint that $D(x) \leq \frac{1}{2}$, we may maximize the expression above in a pointwise fashion by:

$$D(x) = \begin{cases} \frac{1}{2} & \text{if } p(x) > q(x) \\ -\frac{1}{2} & \text{if } p(x) \leq q(x) \end{cases} \quad (10)$$

We note that this is consistent with the finding of Nguyen et al. (2010) that $D(x)$ should attain a subderivative of f at the point $\frac{p(x)}{q(x)}$. Therefore, without weakening the lower bound, we may write:

$$\begin{aligned} D_{TV}(P \parallel Q) &\geq \sup_{D: \mathcal{X} \rightarrow \{-\frac{1}{2}, \frac{1}{2}\}} \mathbb{E}_{x \sim P}[D(x)] - \mathbb{E}_{x \sim Q} \left[\max \left(D(x), -\frac{1}{2} \right) \right] \\ &= \sup_{D: \mathcal{X} \rightarrow \{-\frac{1}{2}, \frac{1}{2}\}} \mathbb{E}_{x \sim P}[D(x)] - \mathbb{E}_{x \sim Q}[D(x)] \\ &= \sup_{D: \mathcal{X} \rightarrow \{0,1\}} \mathbb{E}_{x \sim P}[D(x)] - \mathbb{E}_{x \sim Q}[D(x)] \\ &= \sup_{\substack{D: \mathcal{X} \rightarrow [0,1] \\ \gamma \in [0,1]}} \mathbb{E}_{x \sim P}[[D(x) > \gamma]] - \mathbb{E}_{x \sim Q}[[D(x) > \gamma]] \end{aligned} \quad (11)$$

The first equality is derived from the observation that $D(x) \geq -\frac{1}{2}$ always holds and the maximum is therefore redundant. The second equality is obtained by noting that the expression is invariant under the addition of constants to D (in this case, we add $\frac{1}{2}$).

Without relying on the results of Nguyen et al. (2010), we now show that this bound is tight, even when $P \not\ll Q$. To work in this more general setting, we redefine the total variation distance as half the L^1 distance of p and q :

$$D_{TV}(P \parallel Q) := \frac{1}{2} \|p - q\|_{L^1(\mathcal{X}, \mu)} = \frac{1}{2} \int_{\mathcal{X}} |p(x) - q(x)|d\mu \quad (12)$$

1080 One may verify that this matches our original definition when $P \ll Q$. For any measurable set
 1081 $A \subset \mathcal{X}$, we note that:
 1082

$$1083 \quad 1 = \int_A p(x) d\mu + \int_{A^C} p(x) d\mu = \int_A q(x) d\mu + \int_{A^C} q(x) d\mu \quad (13)$$

1085 Hence, rearranging, we obtain:
 1086

$$1087 \quad \int_A p(x) - q(x) d\mu = \int_{A^C} q(x) - p(x) d\mu \quad (14)$$

1090 Defining $A := \{x \in \mathcal{X} : p(x) \geq q(x)\}$ and applying this identity, we get:
 1091

$$1092 \quad \frac{1}{2} \int_{\mathcal{X}} |p(x) - q(x)| d\mu = \frac{1}{2} \int_A p(x) - q(x) d\mu + \frac{1}{2} \int_{A^C} q(x) - p(x) d\mu \\ 1093 \quad = \int_A p(x) - q(x) d\mu \quad (15)$$

1096 Since A is exactly the set on which $p(x) - q(x)$ is non-negative, it is also clear that for any other
 1097 $B \subset \mathcal{X}$, we have:
 1098

$$1099 \quad \frac{1}{2} \int_{\mathcal{X}} |p(x) - q(x)| d\mu \geq \int_B p(x) - q(x) d\mu \quad (16)$$

1100 Thus, we may write:
 1101

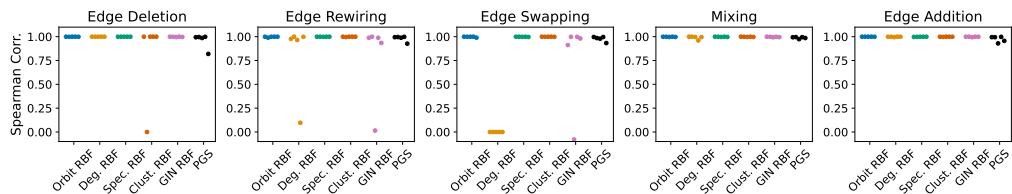
$$1103 \quad D_{TV}(P \parallel Q) = \sup_{B \subset \mathcal{X}} \int_B p(x) - q(x) d\mu \\ 1104 \quad = \sup_{D: \mathcal{X} \rightarrow \{0,1\}} \int_{\mathcal{X}} D(x)(p(x) - q(x)) d\mu \\ 1105 \quad = \sup_{D: \mathcal{X} \rightarrow \{0,1\}} \mathbb{E}_{x \sim P}[D(x)] - \mathbb{E}_{x \sim Q}[D(x)] \quad (17)$$

1109 This is exactly the variational lower bound which we have derived above. Hence, we have shown it
 1110 to be tight, even in the setting where $P \ll Q$.
 1111

1112 F.2 PGS-TV TRACKS SYNTHETIC DATA PERTURBATIONS

1115 We now present perturbation experiments for the PGS-TV variant that are analogous to those shown
 1116 in Section 5.2.

1117 We plot a summary of the Spearman correlation of the metrics with perturbation magnitude in Fig. 6.
 1118 Compared to Fig. 3, we find that PGS-TV exhibits slightly lower correlations. Figs. 7 and 8 show
 1119 the response of PGS-TV to perturbation over the entire and cropped magnitude range, respectively.
 1120 For a more detailed explanation of this type of plot, we refer to Appendix H. From the plots we
 1121 conclude that PGS-TV qualitatively exhibits the expected behavior of increasing with perturbation
 1122 magnitude and eventually saturating. However, in some cases (e.g., edge addition on proteins), the
 1123 PGS-TV flattens out, leading to lower correlations.



1131 Figure 6: Spearman correlation of MMD metrics and PGS-TV with the magnitude of perturbation
 1132 of datasets.
 1133

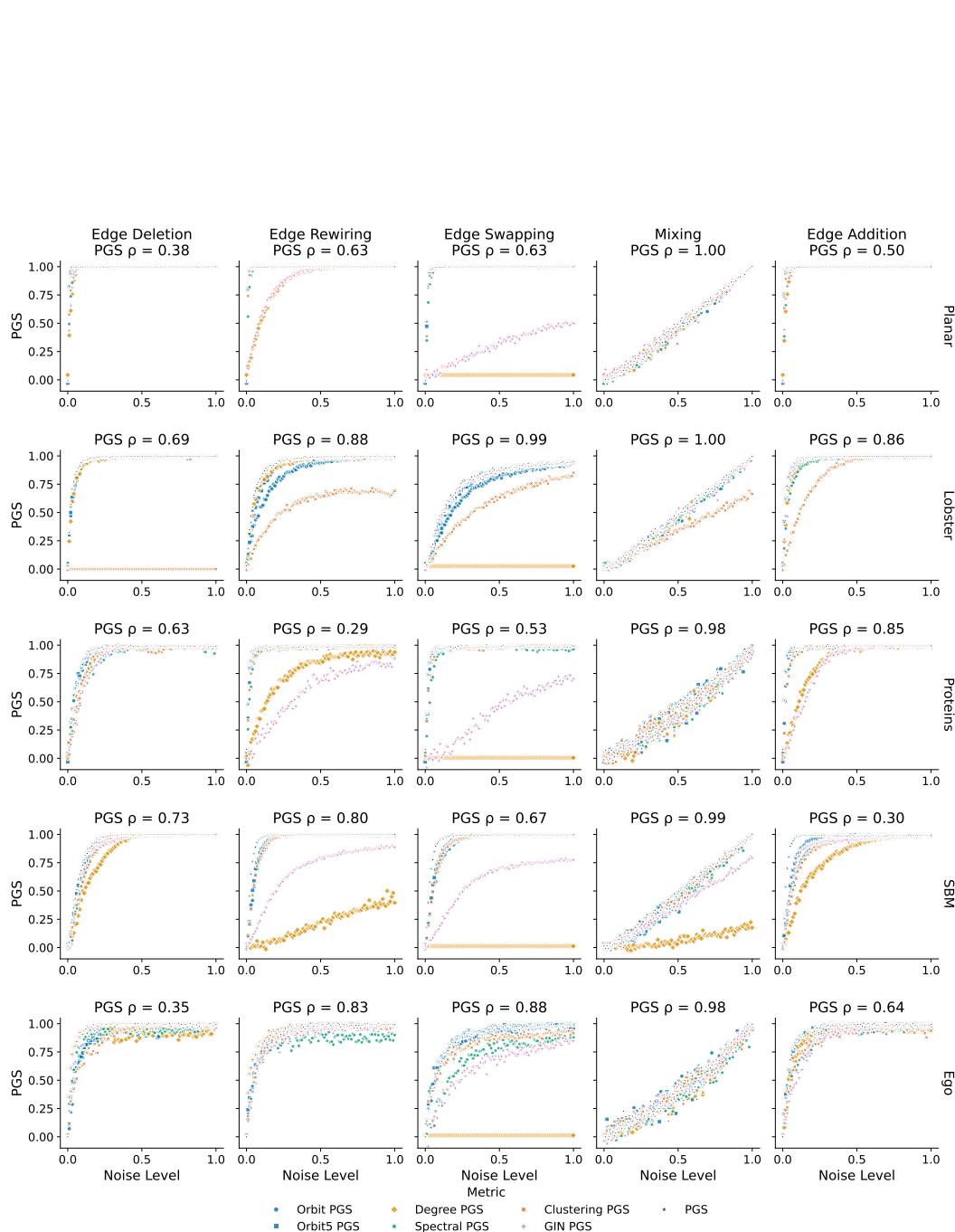


Figure 7: Behavior of descriptor-specific and aggregated PGS-TV as data distributions are perturbed. The perturbation type varies across rows while dataset varies across columns. The Spearman correlation of the aggregate PGS and the perturbation level is denoted by ρ .

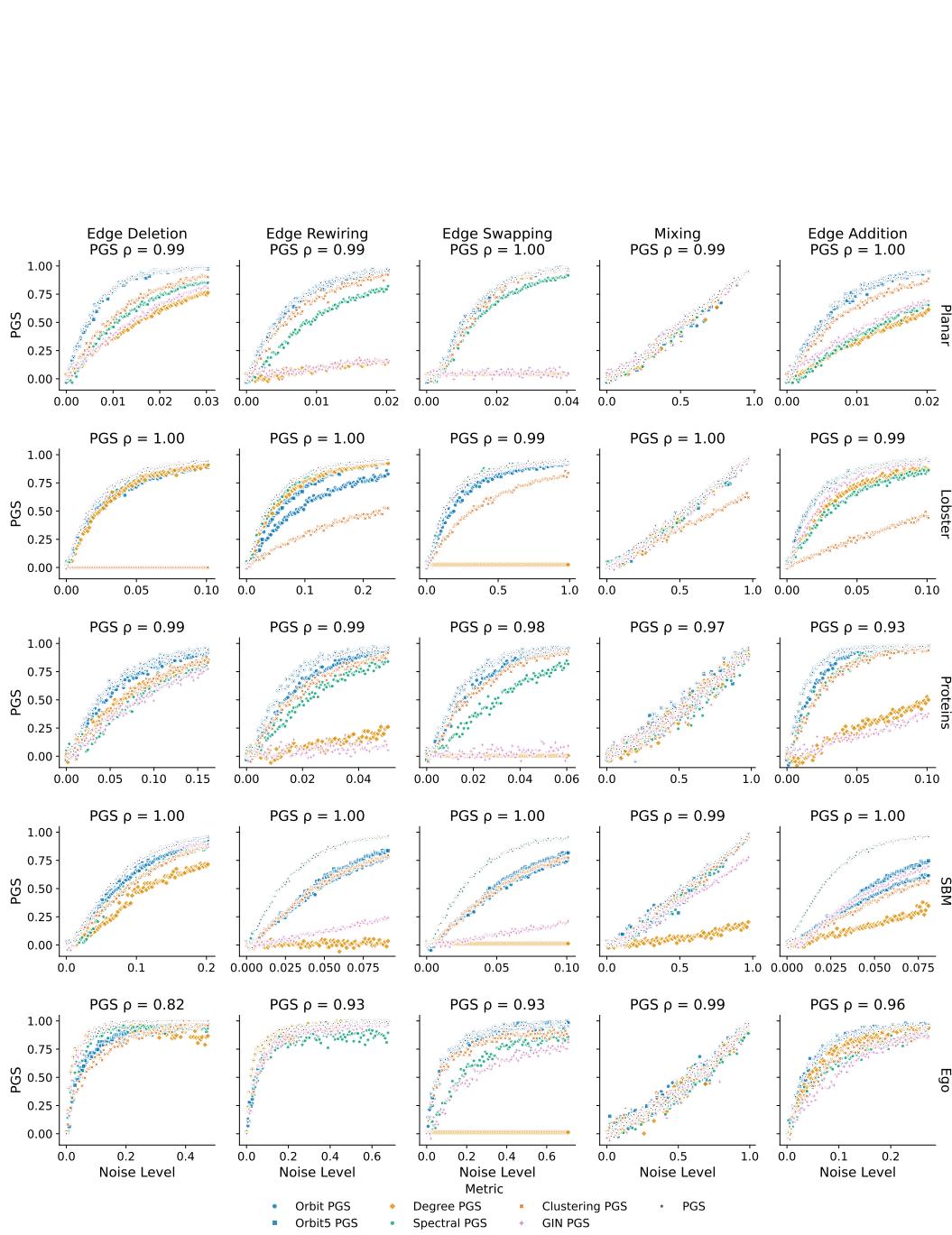
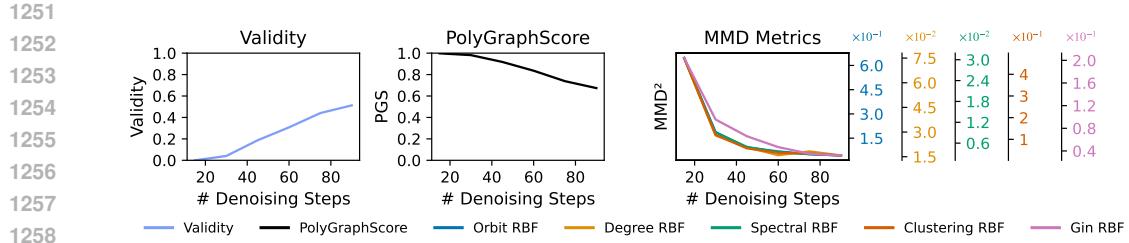


Figure 8: Behavior of descriptor-specific and aggregated PGS-TV as data distributions are perturbed. The perturbation type varies across rows, while the dataset varies across columns. The Spearman correlation of the aggregate PGS and the perturbation level is denoted by ρ .

1242 F.3 PGS-TV CORRELATES WITH MODEL QUALITY
1243

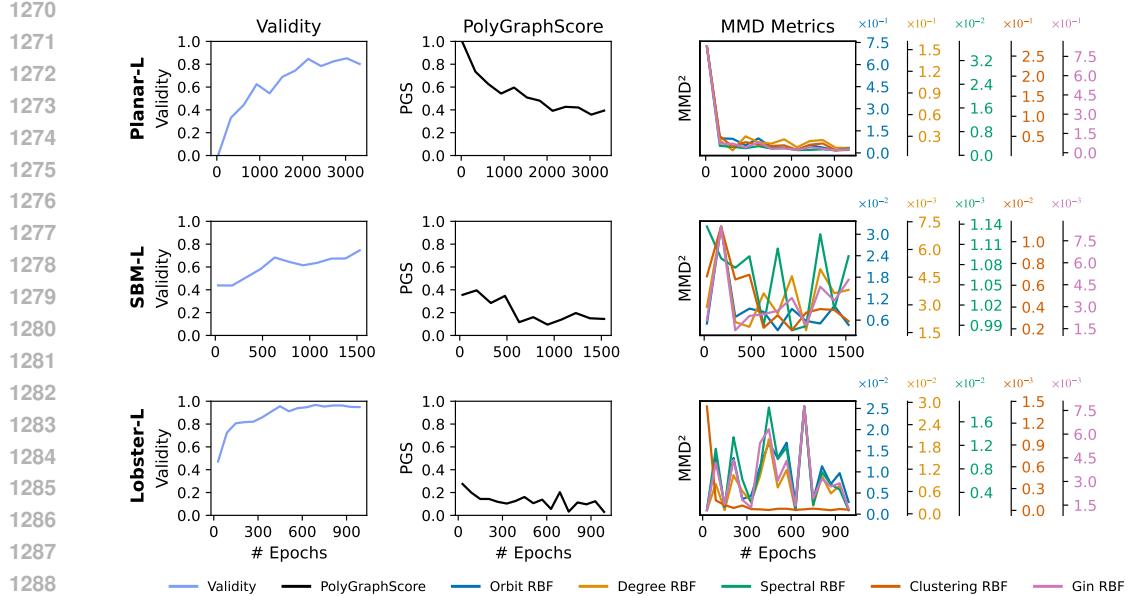
1244 Analogous to Section 5.3, we now investigate how the PGS-TV variant correlates with proxy vari-
1245 ables of model quality. In Fig. 9, we illustrate how PGS-TV behaves as the number of denoising
1246 steps in DiGress is varied. As in Fig. 4, we find that PGS-TV correlates with validity in a highly
1247 linear fashion.

1248 As in Section 5.3, we compute Pearson correlation coefficients between PGS-TV and validity. When
1249 varying the number of denoising steps, we find that PGS-TV exhibits a more linear relationship with
1250 validity than any of the MMD metrics.



1259 Figure 9: Behavior of validity, PGS-TV, and MMDs as the number of denoising steps in DiGress is
1260 varied on PLANAR-L.

1262 We examine the behavior of PGS-TV throughout training in Fig. 9. Qualitatively, a clear positive
1263 relationship emerges between training duration and PGS-TV. This trend is confirmed quantitatively
1264 in Table 8, where Spearman correlation coefficients show that most MMD metrics often exhibit weak
1265 or negative correlations, while PGS-TV consistently correlates positively with training duration.
1266 However, this correlation is weaker than that of PGS-JS (see Table 3) and the clustering-based MMD
1267 metric. A similar pattern appears in Table 7 (bottom three rows): PGS-TV correlates reliably with
1268 validity, whereas most MMD metrics show inconsistent behavior. Nevertheless, in two out of three
1269 cases, the clustering-based MMD metric achieves a stronger correlation with validity than PGS-TV.



1290 Figure 10: Behavior of validity, PGS-TV, and MMD metrics throughout training of DiGress on
1291 procedural graph datasets.

1292
1293
1294
1295

1296 Table 7: Negative Pearson correlation (\uparrow) of validity with other performance metrics. Denoising
 1297 refers to the experiments in which we vary the number of denoising iterations. Training refers to the
 1298 experiments in which we monitor performance metrics during the training of DiGress models.
 1299

		TV-PGS	Orbit RBF	Deg. RBF	Spec. RBF	Clust. RBF	GIN RBF
Denoising	PLANAR-L	99.24	73.49	70.79	73.34	71.48	82.78
	PLANAR-L	99.42	84.33	76.52	79.05	81.61	81.07
Training	SBM-L	84.07	52.07	16.60	35.32	83.82	14.64
	LOBSTER-L	69.18	-34.81	-33.40	-22.79	87.05	-30.31

1300
 1301 Table 8: Sign-adjusted Spearman correlation (\uparrow) of validity, PGS-TV, and MMDs with number of
 1302 training iterations of DiGress.
 1303

		Validity	PGS	Orbit RBF	Deg. RBF	Spec. RBF	Clust. RBF	GIN RBF
PLANAR-L		92.31	93.71	86.71	41.96	83.22	67.83	81.82
SBM-L		82.73	63.64	20.00	-19.09	18.18	58.18	-38.18
LOBSTER-L		85.47	62.91	-8.09	-4.66	13.73	68.14	-2.70

1314 F.4 COMPARISON OF PGS AND PGS-TV

1315 Overall, the experiments in Appendices F.2 and F.3 have demonstrated that PGS-TV is a viable
 1316 alternative to the PGS metric we presented in the main paper, correlating to a high degree with
 1317 synthetic data perturbations and proxy variables of model quality. Nevertheless, we found that PGS
 1318 exhibits stronger correlations and appears like a more robust choice.
 1319

1320 While we have no definite explanation for these observations, we hypothesize that the choice of binarization
 1321 threshold in PGS-TV may introduce some noise into the estimate. Additionally, maximum
 1322 likelihood classifiers (like logistic regression) inherently maximize the log-likelihood objective of
 1323 the JS divergence. Bayesian inference (approximated by TabPFN) may be expected to behave simi-
 1324 larly in the large sample size limit (van der Vaart, 1998). However, neither maximum likelihood
 1325 estimation nor Bayesian inference directly optimizes the variational objective of the TV distance,
 1326 i.e., informedness. This can lead to a misalignment when estimating the PGS-TV, potentially result-
 1327 ing in looser variational bounds.
 1328

1329 For these reasons, we recommend using the PGS variant presented in the main paper, estimating
 1330 lower bounds on the Jensen-Shannon distance.
 1331

1332 G SUPPLEMENTAL FOR: HIGH BIAS AND VARIANCE PLAGUE MMD-BASED 1333 GGM BENCHMARKS

1334 Here, we show that the conclusions of Section 5.1 expand to all combinations of models,
 1335 descriptors, and datasets, and provide additional experimental details. All MMD estimates
 1336 provided here and in Figs. 2a and 2b are RBF MMDs, as proposed by Thompson et al.
 1337 (2022). The kernel is selected by taking the maximum over the bandwidths $\{\sigma_i\}_{i=1}^1 =$
 1338 $\{0.01, 0.1, 0.25, 0.5, 0.75, 1.0, 2.5, 5.0, 7.5, 10.0\}$.
 1339

1340 Specifically, we subsampled 8 to 4096 graphs 100 times with replacement from a total of 8192 sam-
 1341 ples for the reference and generated graphs. We subsequently computed the median, 5th and 95th
 1342 quantiles to estimate the variation of MMD. We computed such experiments for all model-generated
 1343 samples we considered (ESGG, AutoGraph, DiGress and GRAN) and considered all descriptors (de-
 1344 gree histogram, clustering histogram, orbit count for graphlet sizes 4 and 5, and the graph Laplacian
 1345 eigenvalues) and all procedural datasets (SBM, Lobster and Planar).
 1346

1347 Based on those findings, we introduce PLANAR-L, SBM-L, and LOBSTER-L, larger versions of the
 1348 previously used datasets. Details for these new datasets are presented in Appendix M
 1349

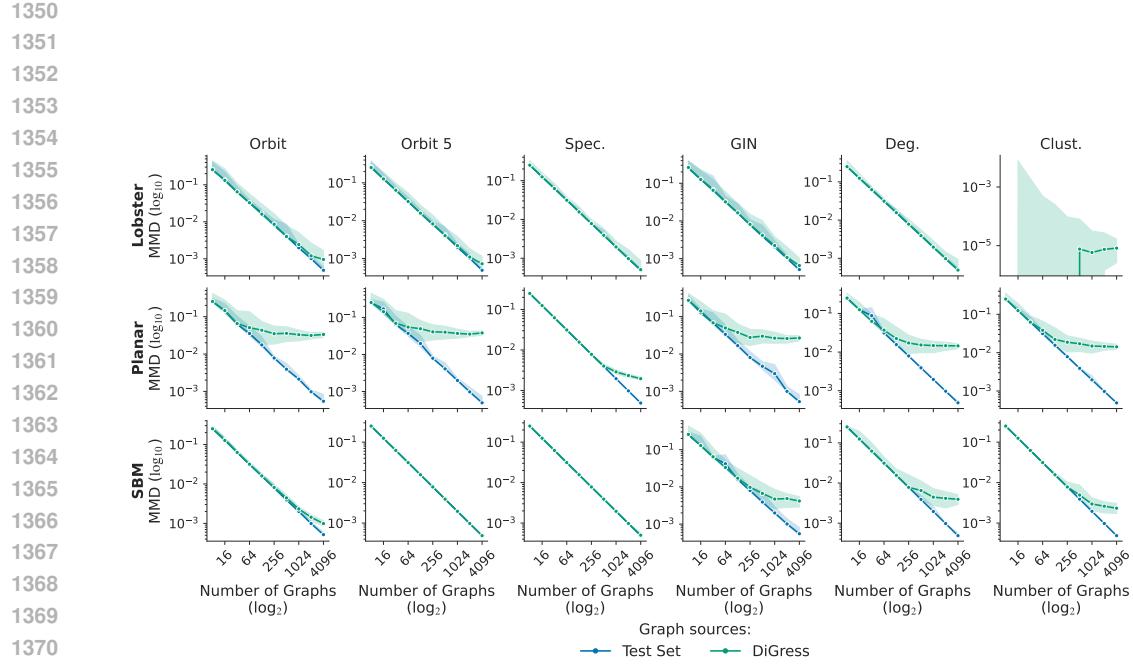


Figure 11: Behavior of biased MMD estimates as the number of samples is varied for DiGress.

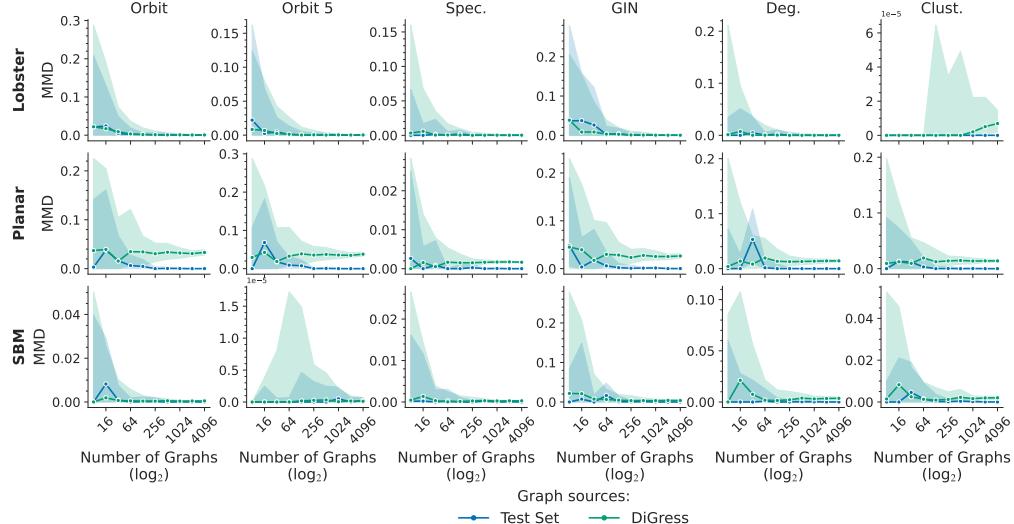
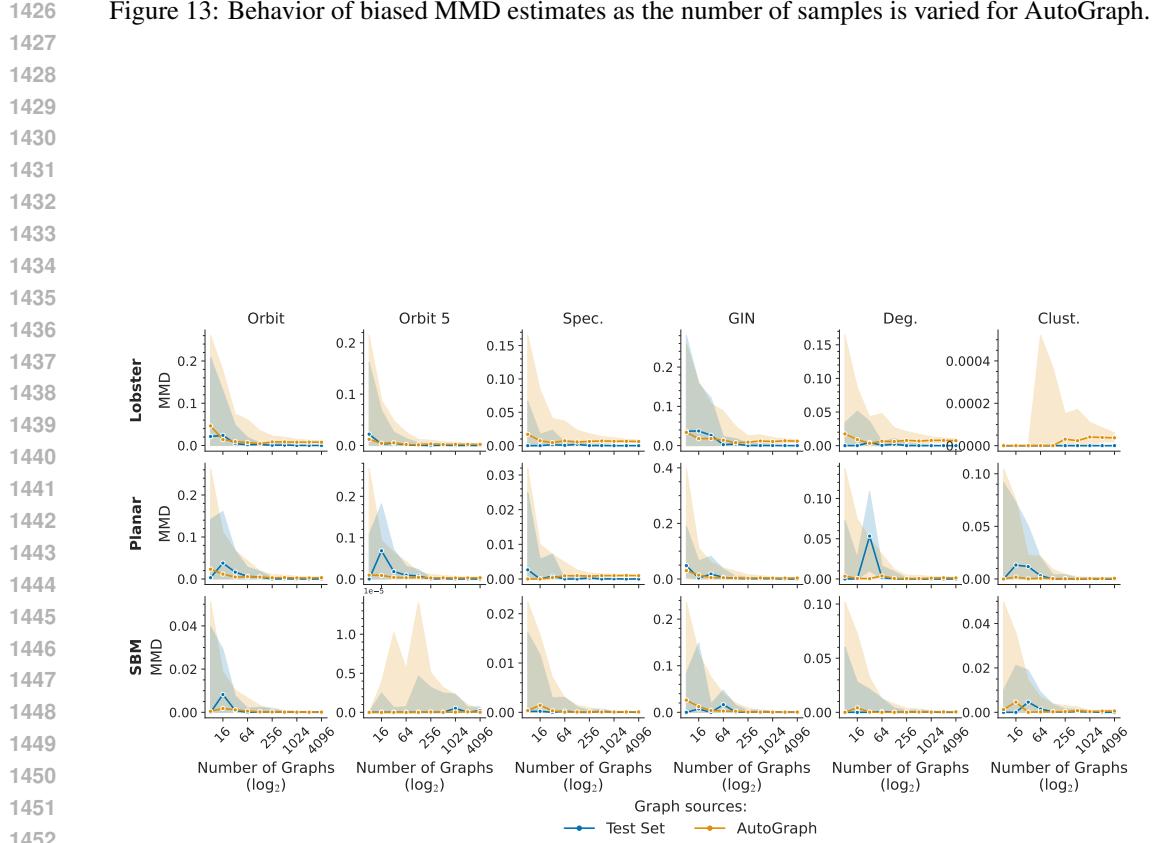
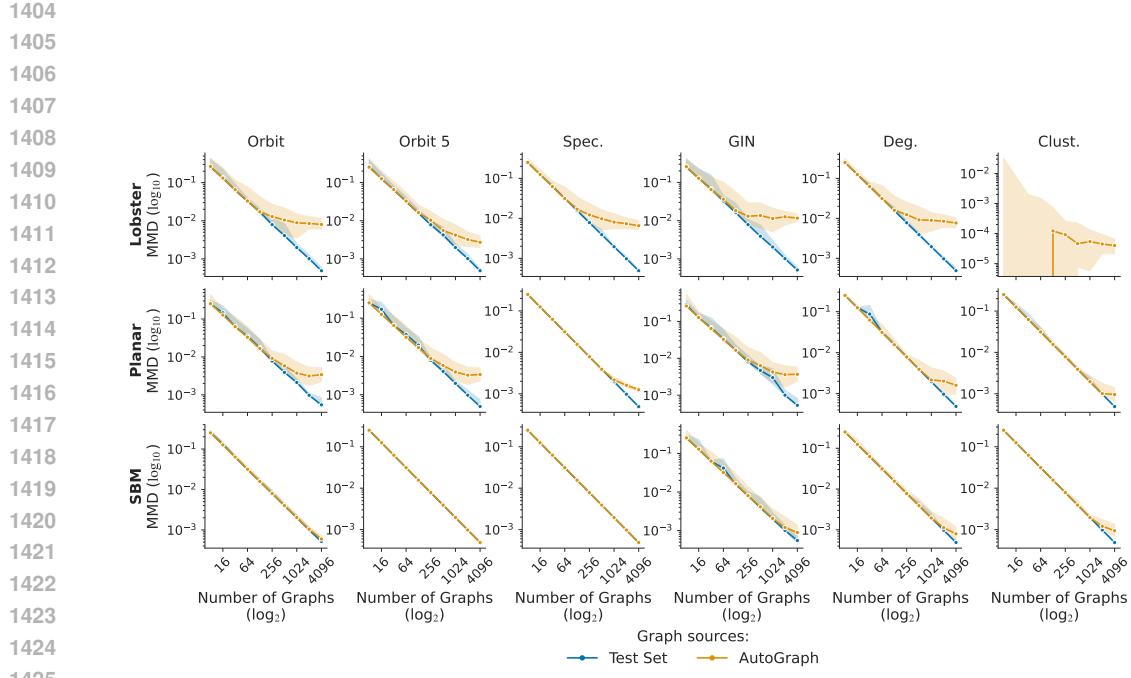


Figure 12: Behavior of unbiased MMD estimates as the number of samples is varied for DiGress.



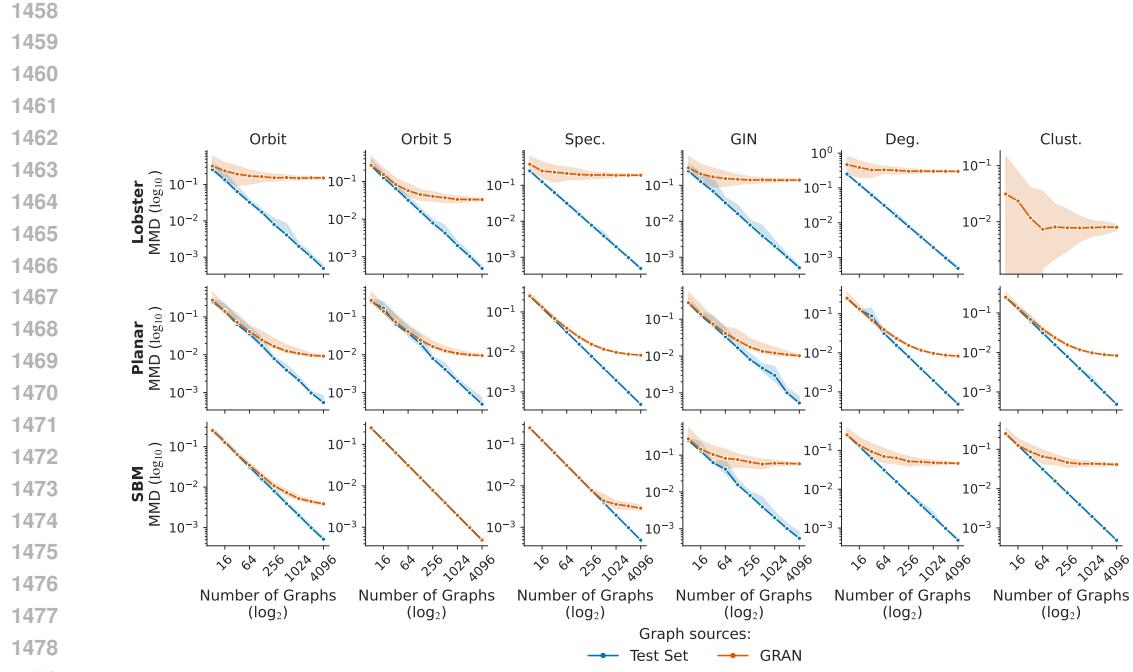


Figure 15: Behavior of biased MMD estimates as the number of samples is varied for GRAN.

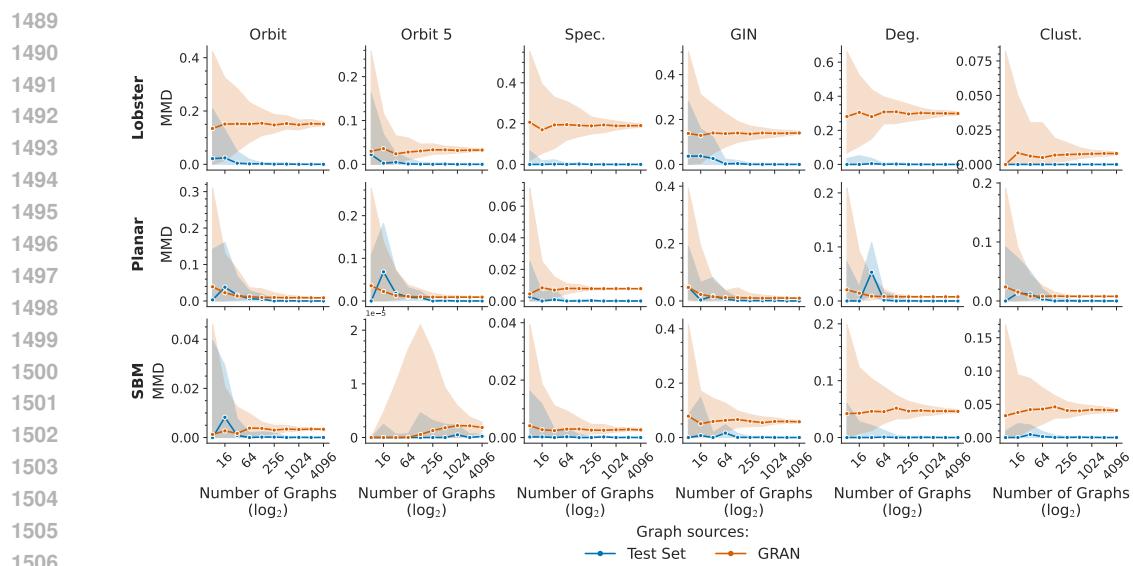


Figure 16: Behavior of unbiased MMD estimates as the number of samples is varied for GRAN.

1509
1510
1511

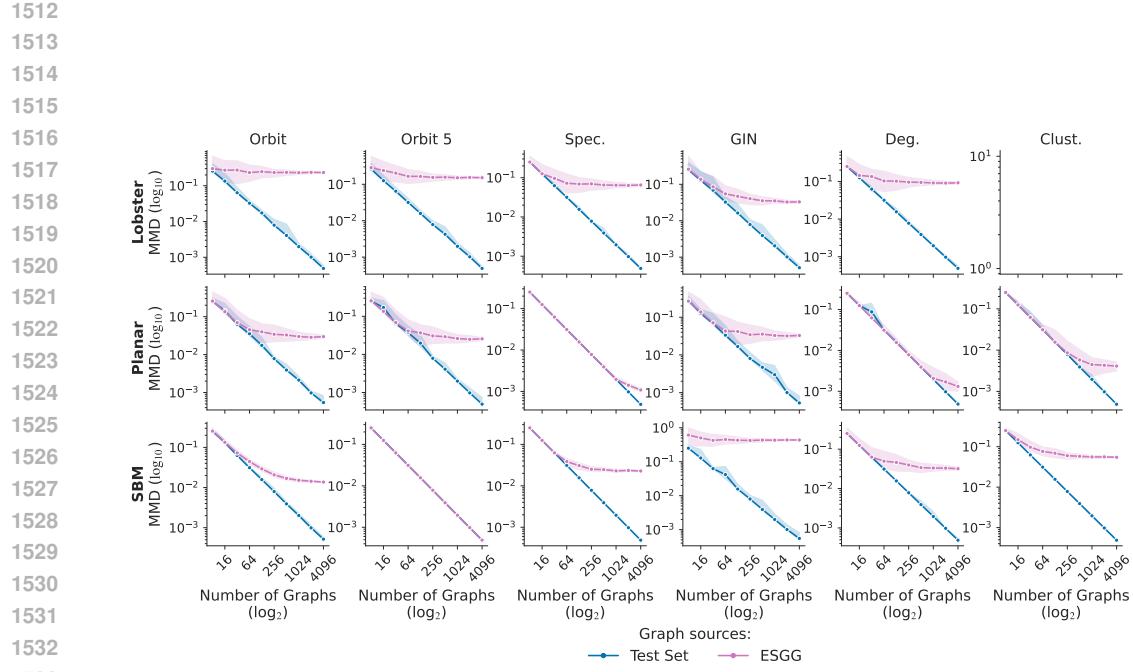


Figure 17: Behavior of biased MMD estimates as the number of samples is varied for ESGG.

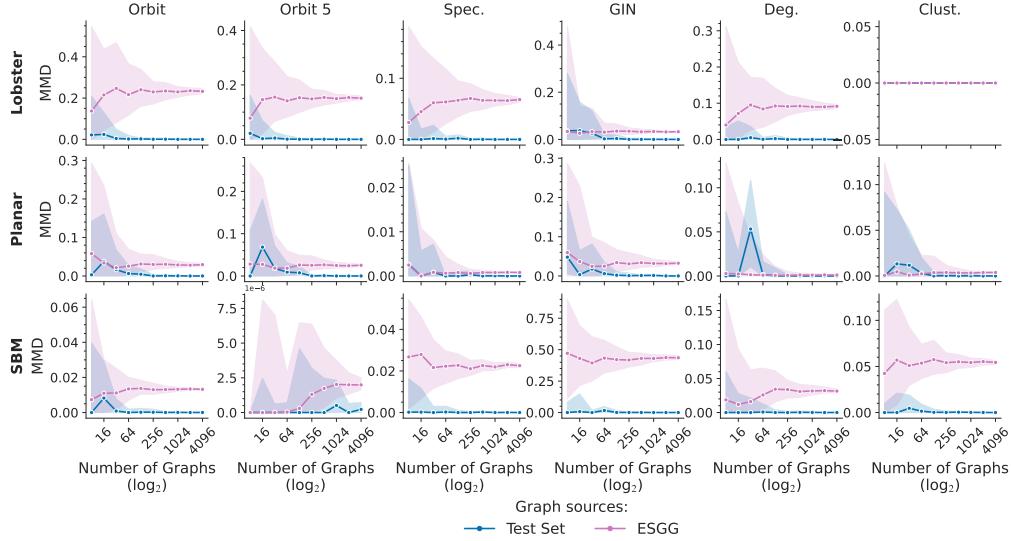
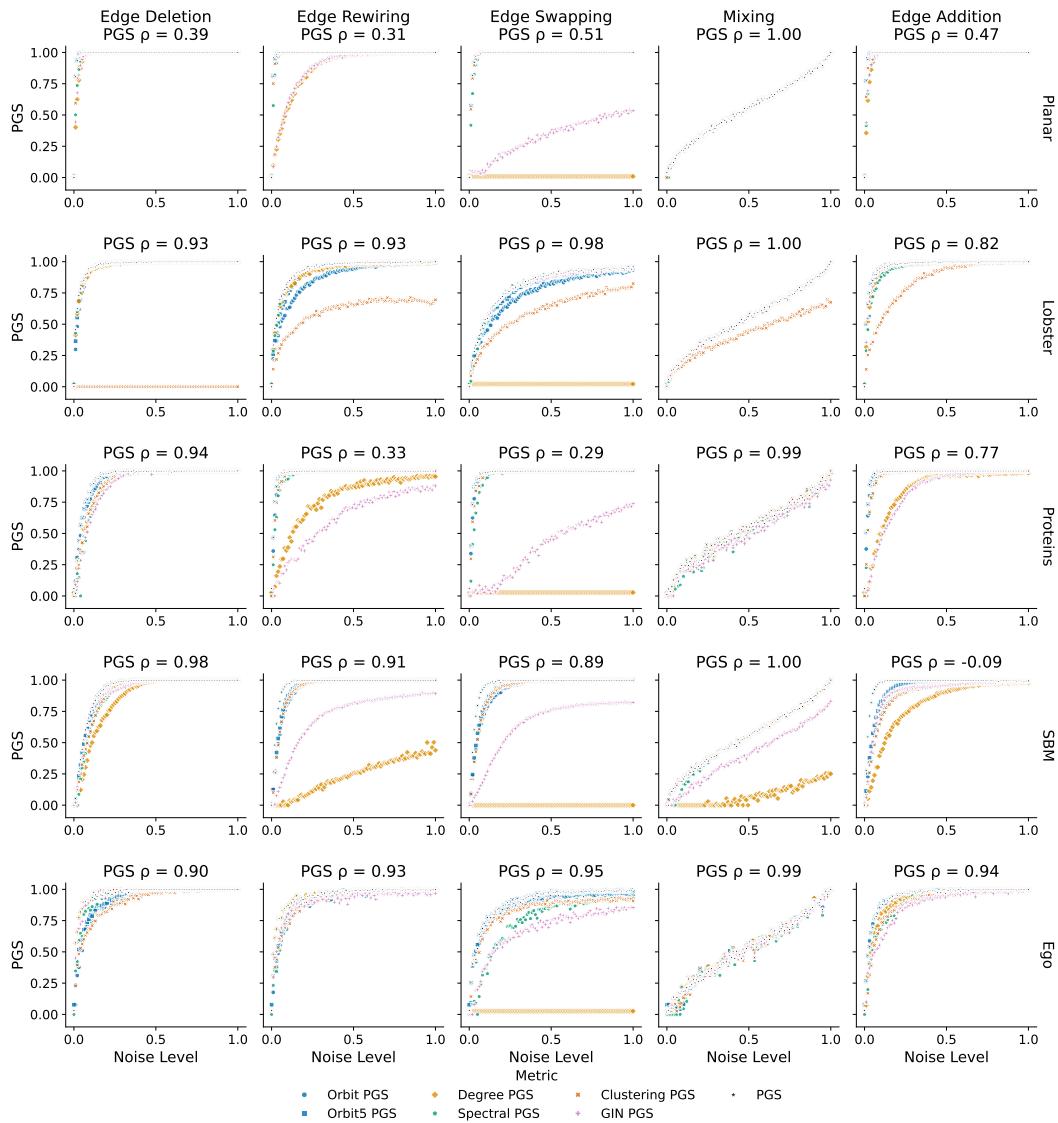


Figure 18: Behavior of unbiased MMD estimates as the number of samples is varied for ESGG.

1566 H SUPPLEMENTAL FOR: PGS TRACKS SYNTHETIC DATA PERTURBATIONS
15671568 In this section, we provide further details for the experiments presented in Section 5.2. In particular,
1569 we illustrate in more detail how PGS responds to perturbations and present results for the TV variant.
15701571 In Fig. 19, we illustrate how PGS (descriptor-specific scores and the summary PGS) responds to
1572 various perturbations on different datasets. In this figure, we illustrate the response over the whole
1573 range of magnitudes $[0, 1]$. As anticipated, the PGS saturates quickly as the support of the perturbed
1574 distribution becomes disjoint from the support of the true data distribution. We note that the PGS
1575 consistently responds in a monotonic fashion to the magnitude of perturbation.
15761612 Figure 19: Behavior of descriptor-specific and aggregated PGS (JS) as data distributions are per-
1613 turbed.
16141615 Based on the data from Fig. 19, we select a threshold for each combination of perturbation type and
1616 dataset at which the summary PGS saturates above 0.95. We illustrate the behavior of PGS-JS on
1617 these cropped ranges in Fig. 20.
16181619 We find that there is no single descriptor that consistently provides the tightest PGS estimate. This
highlights the importance of evaluating many different descriptors when computing a PGS.
1620

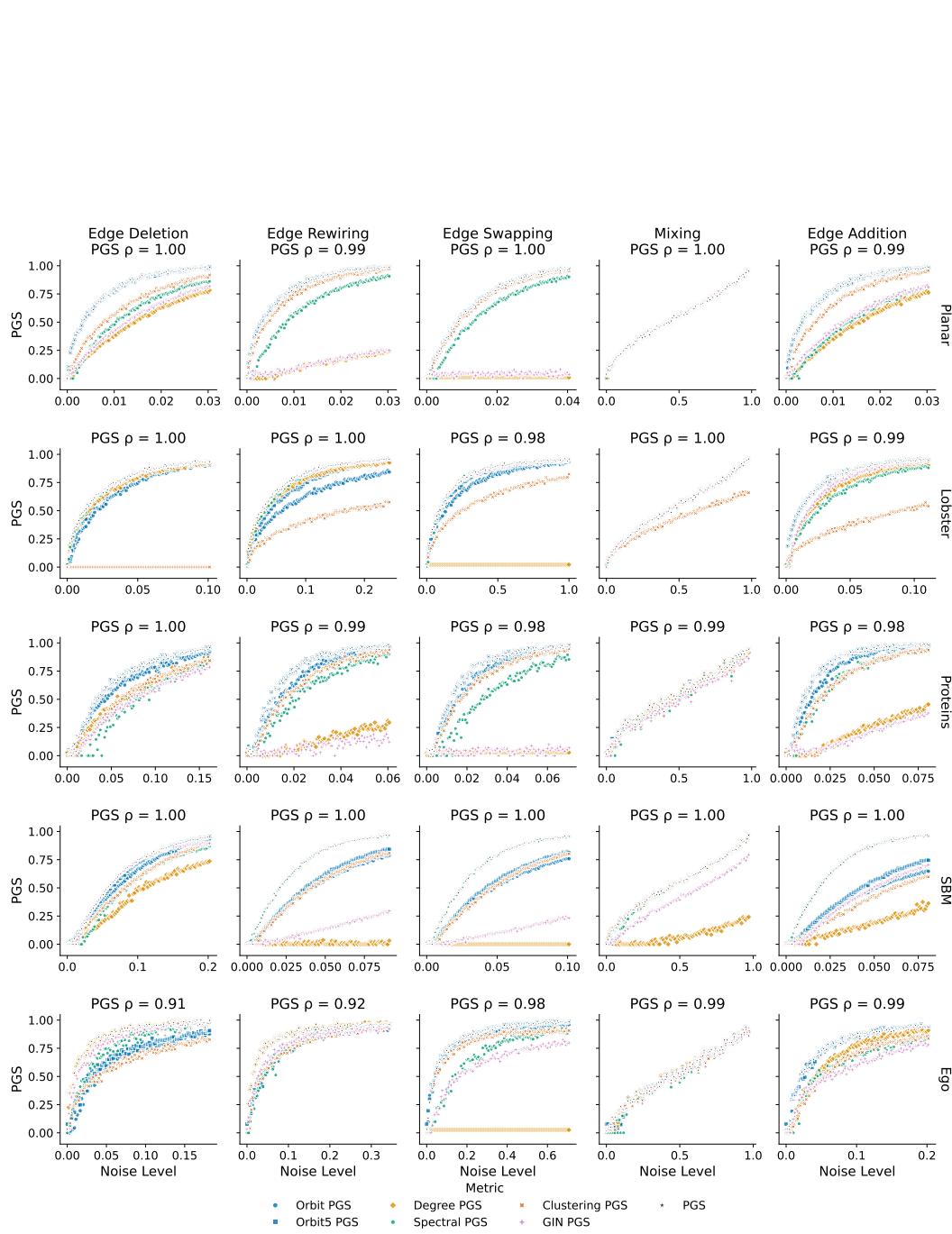


Figure 20: Behavior of descriptor-specific and aggregated PGS (JS) as data distributions are perturbed. The perturbation type varies across rows, while the dataset varies across columns. The Spearman correlation of the aggregate PGS and the perturbation level is denoted by ρ .

1674 I SUPPLEMENTAL FOR: PGS CORRELATES WITH MODEL QUALITY

1675
1676 In this section, we provide further details for the experiments presented in Section 5.3.
1677

1678 In Table 9 we provide the exact MMD metrics attained by DiGress as the number of denoising
1679 iterations is varied. Analogously, we provide the values of the PGS and descriptor-specific subscores
1680 in Table 10. We find that orbit counts appear to be the most discriminative descriptors, as they lead
1681 to the highest PGS values.

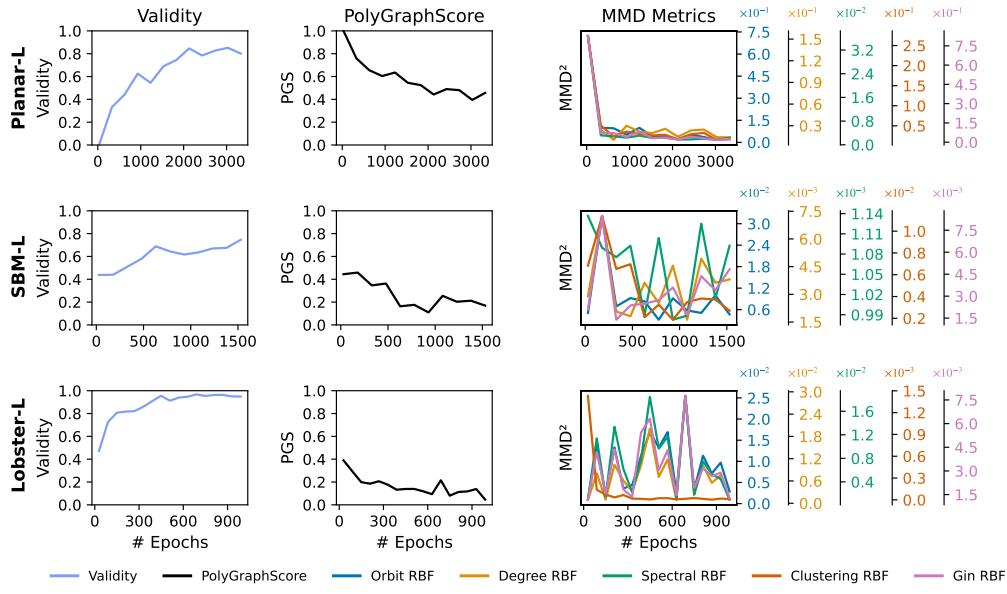
1682 Table 9: Behavior of RBF-based MMD metrics as the number of denoising steps in DiGress is
1683 varied. A separate model is trained for each row for 5k epochs on PLANAR-L.
1684

# Steps	Validity	Orbit RBF	Deg. RBF	Spec. RBF	Clust. RBF	GIN RBF
15	0.00	0.6460	0.0751	0.0305	0.4751	0.2041
30	4.05	0.1879	0.0280	0.0090	0.1206	0.0956
45	18.70	0.0921	0.0208	0.0049	0.0584	0.0660
60	30.76	0.0680	0.0159	0.0034	0.0377	0.0468
75	44.09	0.0506	0.0182	0.0028	0.0349	0.0350
90	51.27	0.0432	0.0158	0.0025	0.0258	0.0321

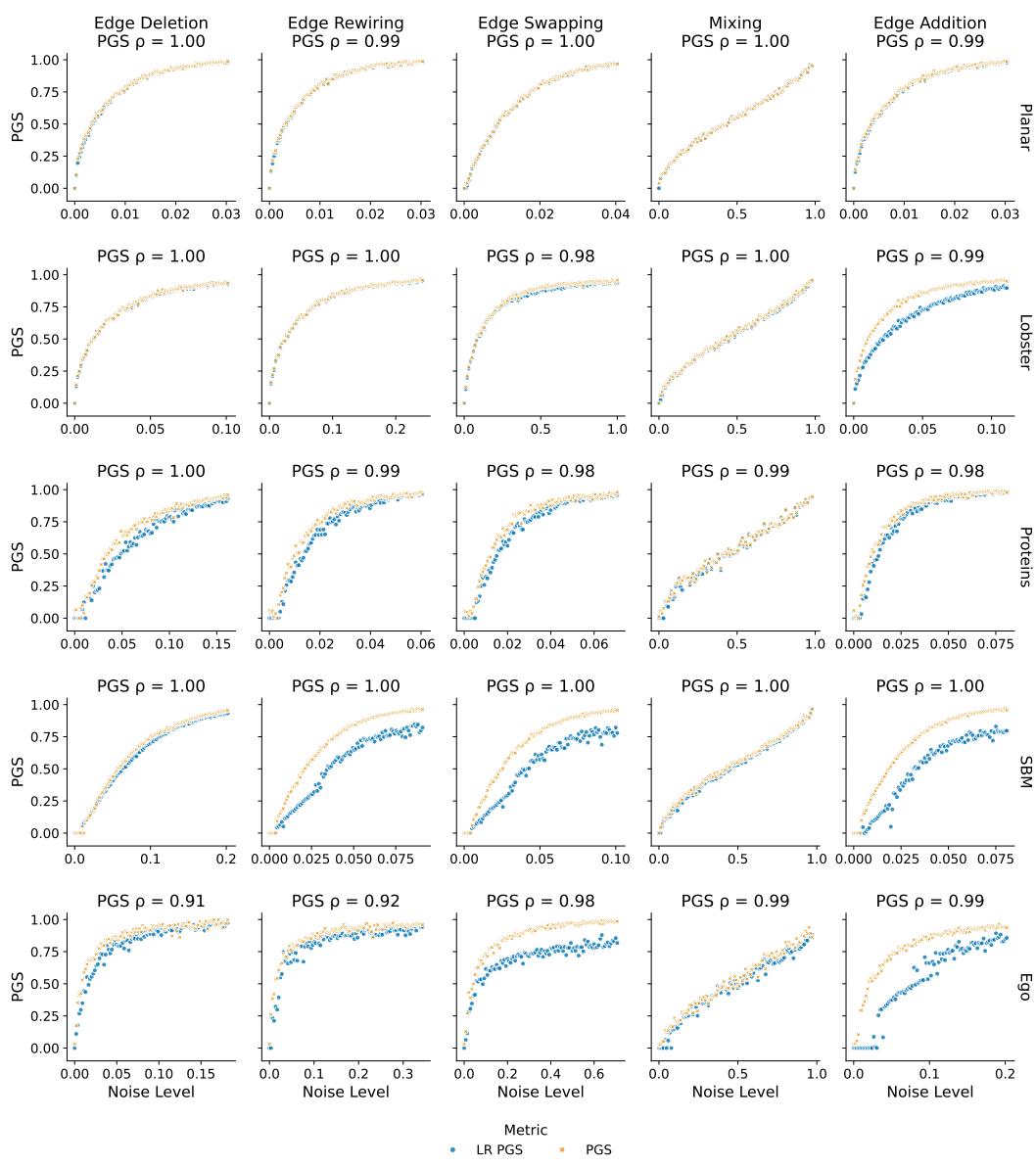
1693 Table 10: Behavior of PGS as the number of denoising steps in DiGress is varied. A separate model
1694 is trained for each row for 5k epochs on PLANAR-L.
1695

# Steps	Validity	PGS	Orbit PGS	Orbit5 PGS	Deg. PGS	Spec. PGS	Clust. PGS	GIN PGS
15	0.00	99.96	99.99	99.96	68.74	99.25	99.94	78.65
30	4.05	96.76	96.84	96.76	43.14	80.15	89.89	57.04
45	18.70	90.48	89.84	90.48	33.34	66.30	75.75	44.99
60	30.76	84.03	82.49	84.03	29.09	52.39	67.34	39.07
75	44.09	74.90	73.35	74.90	32.69	45.69	57.50	38.75
90	51.27	69.16	67.13	69.16	28.11	41.43	48.94	35.08

1704 In Fig. 21, we supplement the experiments presented previously in Fig. 5 with the corresponding
1705 results on PLANAR-L and LOBSTER-L.
1706



1726 Figure 21: Behavior of validity, PGS, and MMD metrics throughout training of DiGress on proce-
1727 dural datasets.

1728 **J ABLATION: TABPFN VS LOGISTIC REGRESSION**
17291730 In this section, we study logistic regression as an alternative to TabPFN as a discriminator. To
1731 this end, we repeat the perturbation experiments from Section 5.2 and Appendix H with logistic
1732 regression as a discriminator. We refer to the PGS variant using logistic regression LR PGS.
17331734 In Fig. 22 we plot the response of PGS and LR PGS to synthetic perturbations. We find that TabPFN
1735 consistently produces PGS estimates that are at least as high as those obtained by logistic regression.
1736 In some cases, TabPFN clearly outperforms logistic regression. This may be attributed to the fact
1737 that TabPFN can model non-linear decision boundaries and is thus more powerful than logistic
1738 regression. We also qualitatively observe that logistic regression leads to a noisier response to the
1739 variation of perturbation magnitude.
17401741 Hence, since TabPFN simultaneously produces tighter bounds and less noisy estimates, we prefer it
1742 to logistic regression.
17431778 Figure 22: Comparing the behavior of the aggregated PGS (JS) computed via logistic regression
1779 (LR PGS) to the aggregated PGS computed via a TabPFN classifier (PGS).
1780

1782 **K SUPPLEMENTAL FOR: BENCHMARKING REPRESENTATIVE MODELS**
1783

1784 In this Appendix, we do a thorough benchmark of PGS and MMD on LOBSTER-L, PLANAR-L,
1785 SBM-L and Proteins. To obtain the standard deviations for PGS scores and MMD values in Tables 4
1786 and 11 to 13, we subsample half of the dataset *without replacement* (2048 samples for procedural
1787 datasets, and 92 samples for proteins) 10 times. In all those tables, means and standard deviations
1788 are scaled by a factor of 100 for legibility purposes. The time taken to compute each of those metrics
1789 is reported in Table 15. Timing experiments were run on a compute node equipped with two AMD
1790 EPYC 9534 CPUs (using 10 vCPUs in total), an NVIDIA H100 GPU with 80 GB memory (CUDA
1791 12.2, driver 535.230.02), and 128 GB system RAM. Reference values (i.e. the score obtained by
1792 computing the metric between the train and test set) for all metrics discussed are in Tables 16 and 17.
1793 We note that the PGS discrepancy between the train and test set of MOSES is relatively high, as the
1794 test set consists of a separate scaffold split. Importantly, the PGS between the train and test set is
1795 very close to 0 (save for MOSES due to changes in the underlying distribution), further showing the
1796 absolute nature of PGS, making it much easier to interpret compared to MMD.
1797

1797 Table 11: Comparison of VUN and PGS with biased Gaussian TV-based MMD formulations from
1798 Liao et al. (2019). We computed the standard deviation from 10 subsamples of size 2048 except for
1799 Proteins, where the subsample size is 92 (50% of the size of the test set). All MMD hyperparameter
1800 choices are specified in table 14.

Dataset	Model	VUN (\uparrow)	PGS (\downarrow)	GTV MMD ² Deg. (\downarrow)	GTV MMD ² Clust. (\downarrow)	GTV MMD ² Orb. (\downarrow)	GTV MMD ² Eig. (\downarrow)
PLANAR-L	AutoGraph	0.851	33.965 \pm 1.786	7.814e-05 \pm 2.508e-05	1.630e-03 \pm 2.971e-04	1.088e-04 \pm 3.000e-05	8.229e-04 \pm 4.737e-05
	DIGRESS	0.801	45.189 \pm 1.770	6.317e-04 \pm 4.638e-05	1.438e-02 \pm 1.203e-03	3.675e-03 \pm 5.031e-04	1.284e-03 \pm 5.673e-05
	GRAN	0.016	99.663 \pm 0.171	6.272e-05 \pm 1.422e-05	4.658e-03 \pm 6.998e-04	6.620e-04 \pm 1.940e-04	1.198e-03 \pm 8.786e-05
	ESGG	0.939	45.010 \pm 1.395	2.288e-05 \pm 9.304e-06	4.196e-03 \pm 5.231e-04	1.466e-03 \pm 3.001e-04	6.797e-04 \pm 4.239e-05
LOBSTER-L	AutoGraph	0.831	18.022 \pm 1.608	4.453e-04 \pm 5.697e-05	3.336e-06 \pm 1.710e-06	6.333e-03 \pm 8.331e-04	9.893e-04 \pm 1.120e-04
	DIGRESS	0.914	3.167 \pm 2.607	2.816e-05 \pm 1.870e-05	1.067e-06 \pm 6.954e-07	4.166e-04 \pm 2.512e-04	1.571e-04 \pm 1.937e-05
	GRAN	0.413	85.370 \pm 0.501	1.158e-02 \pm 4.212e-04	3.344e-03 \pm 2.449e-04	1.955e-01 \pm 4.405e-03	2.303e-02 \pm 5.802e-04
	ESGG	0.709	69.886 \pm 0.557	6.252e-03 \pm 3.830e-04	0.000e+00 \pm 0.000e+00	6.359e-02 \pm 2.028e-03	1.030e-02 \pm 4.646e-04
SBM-L	AutoGraph	0.856	5.638 \pm 1.455	4.897e-05 \pm 1.537e-05	1.017e-03 \pm 2.634e-05	1.080e-03 \pm 2.241e-04	1.400e-04 \pm 1.859e-05
	DIGRESS	0.730	17.384 \pm 2.285	7.500e-04 \pm 1.785e-04	1.048e-03 \pm 2.796e-05	2.307e-03 \pm 3.480e-04	2.449e-04 \pm 4.943e-05
	GRAN	0.214	69.114 \pm 1.445	9.540e-03 \pm 3.929e-04	3.040e-03 \pm 7.289e-05	1.306e-02 \pm 7.980e-04	1.104e-03 \pm 7.706e-05
	ESGG	0.104	99.374 \pm 0.212	3.482e-03 \pm 2.877e-04	5.687e-03 \pm 1.007e-04	4.546e-02 \pm 1.449e-03	2.736e-02 \pm 3.318e-04
Proteins	AutoGraph	-	67.661 \pm 7.409	2.454e-03 \pm 4.456e-04	3.750e-02 \pm 4.022e-03	1.759e-02 \pm 3.502e-03	2.708e-03 \pm 1.730e-04
	DIGRESS	-	88.118 \pm 3.075	2.039e-04 \pm 8.748e-05	2.471e-02 \pm 3.015e-03	2.263e-02 \pm 7.034e-03	1.073e-03 \pm 5.723e-05
	GRAN	-	89.674 \pm 2.687	3.286e-02 \pm 1.852e-03	1.068e-01 \pm 4.791e-03	2.841e-01 \pm 1.214e-02	9.344e-03 \pm 5.235e-04
	ESGG	-	79.238 \pm 4.254	1.518e-03 \pm 2.904e-04	4.031e-02 \pm 1.987e-03	6.474e-03 \pm 1.315e-03	1.269e-03 \pm 1.318e-04

1814 Table 12: Unbiased RBF kernel-based MMD estimates. We computed the standard deviation from
1815 10 subsamples of size 2048 except for Proteins, where the subsample size is 92 (50% of the size of
1816 the test set). All MMD hyperparameter choices are specified in table 14.

Dataset	Model	VUN (\uparrow)	PGS (\downarrow)	RBF MMD ² Deg. (\downarrow)	RBF MMD ² Clust. (\downarrow)	RBF MMD ² Orb. (\downarrow)	RBF MMD ² Eig. (\downarrow)
PLANAR-L	AutoGraph	0.851	33.965 \pm 1.786	1.961e-03 \pm 6.688e-04	5.616e-04 \pm 1.687e-04	2.488e-03 \pm 3.305e-04	1.035e-03 \pm 7.394e-05
	DIGRESS	0.801	45.189 \pm 1.770	1.623e-02 \pm 1.130e-03	1.487e-02 \pm 1.508e-03	3.059e-02 \pm 3.484e-03	1.713e-03 \pm 9.201e-05
	GRAN	0.016	99.663 \pm 0.171	3.250e-03 \pm 6.760e-04	3.761e-03 \pm 8.004e-04	9.068e-03 \pm 7.194e-04	4.742e-03 \pm 1.555e-04
	ESGG	0.939	45.010 \pm 1.395	1.322e-03 \pm 2.961e-04	3.778e-03 \pm 6.875e-04	2.708e-02 \pm 1.779e-03	8.337e-04 \pm 7.146e-05
LOBSTER-L	AutoGraph	0.831	18.022 \pm 1.608	8.446e-03 \pm 1.241e-03	5.017e-06 \pm 2.677e-06	7.725e-03 \pm 1.340e-03	6.748e-03 \pm 1.198e-03
	DIGRESS	0.914	3.167 \pm 2.607	2.969e-04 \pm 5.087e-04	1.208e-06 \pm 9.237e-07	7.208e-04 \pm 5.402e-04	2.389e-04 \pm 2.738e-04
	GRAN	0.413	85.370 \pm 0.501	2.965e-01 \pm 8.501e-03	4.605e-03 \pm 3.158e-04	1.526e-01 \pm 4.294e-03	1.774e-01 \pm 7.080e-03
	ESGG	0.709	69.886 \pm 0.557	8.650e-02 \pm 3.577e-03	0.000e+00 \pm 0.000e+00	2.163e-01 \pm 7.297e-03	4.552e-02 \pm 1.243e-03
SBM-L	AutoGraph	0.856	5.638 \pm 1.455	2.085e-04 \pm 1.663e-04	3.275e-04 \pm 1.506e-04	9.928e-05 \pm 6.512e-05	7.888e-05 \pm 2.978e-05
	DIGRESS	0.730	17.384 \pm 2.285	3.385e-03 \pm 8.299e-04	1.738e-03 \pm 3.772e-04	4.252e-04 \pm 8.053e-05	2.832e-04 \pm 7.796e-05
	GRAN	0.214	69.114 \pm 1.445	4.543e-02 \pm 1.560e-03	4.111e-02 \pm 1.828e-03	3.194e-03 \pm 2.032e-04	2.671e-03 \pm 2.659e-04
	ESGG	0.104	99.374 \pm 0.212	3.255e-02 \pm 2.096e-03	5.523e-02 \pm 1.585e-03	1.334e-02 \pm 2.608e-04	2.262e-02 \pm 5.563e-04
Proteins	AutoGraph	-	67.661 \pm 7.409	4.025e-02 \pm 5.459e-03	5.165e-02 \pm 5.930e-03	1.715e-02 \pm 2.728e-03	3.967e-03 \pm 3.339e-04
	DIGRESS	-	88.118 \pm 3.075	2.389e-02 \pm 4.234e-03	2.230e-02 \pm 3.158e-03	5.588e-02 \pm 1.390e-02	1.239e-03 \pm 1.592e-04
	GRAN	-	89.674 \pm 2.687	2.853e-01 \pm 1.816e-02	2.495e-01 \pm 1.232e-02	3.731e-01 \pm 1.399e-02	2.967e-02 \pm 2.078e-03
	ESGG	-	79.238 \pm 4.254	5.391e-02 \pm 7.314e-03	5.968e-02 \pm 3.388e-03	3.669e-02 \pm 8.273e-03	1.431e-03 \pm 3.791e-04

1836 Table 13: Biased RBF kernel-based MMD estimates. We computed the standard deviation from 10
 1837 subsamples of size 2048 except for Proteins, where the subsample size is 92 (50% of the size of the
 1838 test set). All MMD hyperparameter choices are specified in table 14.

1840 Dataset	1841 Model	VUN (\uparrow)	PGS (\downarrow)	RBF MMD ² Deg. (\downarrow)	RBF MMD ² Clust. (\downarrow)	RBF MMD ² Orb. (\downarrow)	RBF MMD ² Eig. (\downarrow)
1841 PLANAR-L	AutoGraph	0.851	33.965 \pm 1.786	2.514e-03 \pm 6.689e-04	1.147e-03 \pm 1.681e-04	3.154e-03 \pm 3.387e-04	1.624e-03 \pm 7.366e-05
	DIGRESS	0.801	45.189 \pm 1.770	1.679e-02 \pm 1.129e-03	1.546e-02 \pm 1.507e-03	3.098e-02 \pm 3.389e-03	2.303e-03 \pm 9.193e-05
	GRAN	0.016	99.663 \pm 0.171	3.800e-03 \pm 6.768e-04	4.347e-03 \pm 8.007e-04	9.981e-03 \pm 6.747e-04	5.330e-03 \pm 1.557e-04
	ESGG	0.939	45.010 \pm 1.395	1.884e-03 \pm 2.951e-04	4.367e-03 \pm 6.874e-04	2.769e-02 \pm 1.831e-03	1.423e-03 \pm 7.155e-05
1843 LOBSTER-L	AutoGraph	0.831	18.022 \pm 1.608	9.012e-03 \pm 1.239e-03	6.324e-06 \pm 3.509e-06	8.229e-03 \pm 1.340e-03	7.486e-03 \pm 1.197e-03
	DIGRESS	0.914	3.167 \pm 2.607	8.316e-04 \pm 5.338e-04	1.760e-06 \pm 1.137e-06	1.509e-03 \pm 4.723e-04	9.372e-04 \pm 3.026e-04
	GRAN	0.413	85.370 \pm 0.501	2.972e-01 \pm 8.498e-03	4.795e-03 \pm 3.334e-04	1.533e-01 \pm 4.293e-03	1.782e-01 \pm 7.078e-03
	ESGG	0.709	69.886 \pm 0.557	8.710e-02 \pm 3.578e-03	0.000e+00 \pm 0.000e+00	2.167e-01 \pm 7.310e-03	4.627e-02 \pm 1.244e-03
1846 SBM-L	AutoGraph	0.856	5.638 \pm 1.455	9.239e-04 \pm 1.680e-04	7.998e-04 \pm 7.636e-05	1.068e-03 \pm 6.223e-05	5.036e-04 \pm 3.006e-05
	DIGRESS	0.730	17.384 \pm 2.285	4.100e-03 \pm 8.285e-04	2.140e-03 \pm 2.579e-04	1.392e-03 \pm 8.057e-05	7.082e-04 \pm 7.822e-05
	GRAN	0.214	69.114 \pm 1.445	4.617e-02 \pm 1.560e-03	3.392e-02 \pm 1.390e-03	4.163e-03 \pm 2.031e-04	3.115e-03 \pm 2.666e-04
	ESGG	0.104	99.374 \pm 0.212	3.329e-02 \pm 2.095e-03	3.564e-02 \pm 9.014e-04	1.430e-02 \pm 2.607e-04	2.313e-02 \pm 5.551e-04
1849 Proteins	AutoGraph	-	67.661 \pm 7.409	4.648e-02 \pm 5.412e-03	5.857e-02 \pm 5.924e-03	2.674e-02 \pm 2.481e-03	6.070e-03 \pm 3.329e-04
	DIGRESS	-	88.118 \pm 3.075	3.500e-02 \pm 4.196e-03	2.876e-02 \pm 3.076e-03	6.312e-02 \pm 1.386e-02	3.605e-03 \pm 1.646e-04
	GRAN	-	89.674 \pm 2.687	2.917e-01 \pm 1.812e-02	2.543e-01 \pm 1.237e-02	3.784e-01 \pm 1.398e-02	3.228e-02 \pm 2.108e-03
	ESGG	-	79.238 \pm 4.254	6.034e-02 \pm 7.284e-03	6.691e-02 \pm 3.333e-03	4.505e-02 \pm 8.287e-03	3.923e-03 \pm 4.190e-04

1852 Table 14: Mapping of display columns in results tables to MMD configurations. For all RBF MMDs,
 1853 the final MMD was computed as the maximum value over the following bandwidths $\{\sigma_i\}_{i=1}^6 =$
 1854 $\{0.1, 0.5, 1.0, 2.0, 5.0, 10.0\}$ as per Thompson et al. (2022). For the descriptor parameters, we used
 1855 100,000 for the width of the sparse degree histogram, 100 bins for the clustering histogram, and
 1856 4 for the orbit count. RBF: radial basis function; GTV: Gaussian total variation distance; UMVE:
 1857 unbiased minimum variance estimator, see Gretton et al. (2012).

1859 Name	1860 Variant	1861 Name	1862 Kernel Parameter	1863 Descriptor	
				1864 Biased	1865 GTV
1863 GTV MMD ² Deg.	1864 GTV MMD ² Clust.	1865 GTV MMD ² Orb.	1866 GTV MMD ² Eig.	1.0	Degree
				0.1	Clustering
				30	Orbit
				1.0	Eigenvalues
1867 RBF MMD ² Deg.	1868 RBF MMD ² Clust.	1869 RBF MMD ² Orb.	1870 RBF MMD ² Eig.	1871 UMVE	1872 RBF
				{ σ_i } _{i=1} ⁶	Degree
					Clustering
					Orbit
1873 RBF MMD ² Deg.	1874 RBF MMD ² Clust.	1875 RBF MMD ² Orb.	1876 RBF MMD ² Eig.		Eigenvalues
			1877 Biased	Degree	
				Clustering	
				Orbit	
1878 GTV MMD ² Deg.	1879 GTV MMD ² Clust.	1880 GTV MMD ² Orb.		1881 GTV MMD ² Eig.	

1871 Table 15: Compute time (s) per metric across datasets. Standard deviations are obtained from the
 1872 metrics computed on different model samples. Caching of intermediate or reused MMD values in
 1873 PolyGraph help make MMD computations substantially faster. Int. indicates whether the metric
 1874 yields an interval through subsampling. VUN scores were parallelized across 10 CPUs.

Metric	Int.	PLANAR-L	LOBSTER-L	SBM-L	Proteins	Overall
VUN	✗	425.60 \pm 17.72	253.32 \pm 8.95	1181.26 \pm 101.98	-	620.06 \pm 37.98
PGS	✗	73.64 \pm 3.01	338.82 \pm 190.27	125.02 \pm 17.77	140.35 \pm 73.67	169.46 \pm 52.81
PGS	✓	192.13 \pm 14.27	696.11 \pm 367.93	280.98 \pm 47.45	223.67 \pm 111.39	348.22 \pm 118.35
RBF MMD ² Deg.	✓	12.61 \pm 0.33	12.38 \pm 0.14	12.72 \pm 0.27	3.68 \pm 0.59	10.35 \pm 0.23
Biased RBF MMD ² Deg.	✓	10.50 \pm 0.21	10.32 \pm 0.24	10.46 \pm 0.21	1.49 \pm 0.65	8.19 \pm 0.23
GTV MMD ² Deg.	✓	7.74 \pm 1.22	8.04 \pm 0.21	8.46 \pm 2.54	3.26 \pm 0.40	6.88 \pm 0.78
GTV MMD ² Deg.	✗	3.53 \pm 0.25	3.54 \pm 0.32	3.83 \pm 0.39	3.51 \pm 0.70	3.60 \pm 0.21
RBF MMD ² Clust.	✓	16.23 \pm 0.46	13.69 \pm 0.38	22.63 \pm 1.61	16.48 \pm 8.16	17.26 \pm 1.86
Biased RBF MMD ² Clust.	✓	16.60 \pm 0.50	14.00 \pm 1.22	25.60 \pm 1.86	16.73 \pm 8.25	18.23 \pm 2.26
GTV MMD ² Clust.	✓	11.80 \pm 1.17	10.24 \pm 0.13	16.75 \pm 2.00	14.16 \pm 8.20	13.24 \pm 1.93
GTV MMD ² Clust.	✗	7.63 \pm 0.06	5.54 \pm 0.13	12.90 \pm 2.09	14.27 \pm 8.35	10.08 \pm 2.03
RBF MMD ² Orb.	✓	11.87 \pm 0.20	11.84 \pm 0.32	14.58 \pm 0.62	4.84 \pm 2.64	10.78 \pm 0.66
Biased RBF MMD ² Orb.	✓	11.82 \pm 0.07	11.95 \pm 0.36	14.64 \pm 0.52	4.75 \pm 2.69	10.79 \pm 0.70
GTV MMD ² Orb.	✓	5.75 \pm 1.08	5.85 \pm 0.08	6.71 \pm 1.31	3.73 \pm 2.13	5.51 \pm 0.56
GTV MMD ² Orb.	✗	1.64 \pm 0.02	1.22 \pm 0.02	2.73 \pm 0.41	3.71 \pm 2.12	2.32 \pm 0.50
RBF MMD ² Eig.	✓	21.56 \pm 0.83	19.13 \pm 0.71	25.83 \pm 1.47	31.99 \pm 16.42	24.63 \pm 4.14
Biased RBF MMD ² Eig.	✓	25.16 \pm 6.52	18.75 \pm 0.47	25.86 \pm 1.84	33.11 \pm 16.31	25.72 \pm 2.80
GTV MMD ² Eig.	✓	17.85 \pm 1.18	17.55 \pm 0.24	20.77 \pm 1.83	29.67 \pm 17.44	21.46 \pm 4.21
GTV MMD ² Eig.	✗	13.80 \pm 0.09	12.92 \pm 0.16	16.88 \pm 1.56	32.26 \pm 19.52	18.97 \pm 4.82

Table 16: Reference values between the test and training set for various metrics.

Metric	PLANAR-L	LOBSTER-L	SBM-L	Proteins
PGS (↓)	0.6 ± 1.2	0.8 ± 1.6	0.2 ± 0.6	2.1 ± 3.4
Clust. (↓)	0.1 ± 0.4	0.0 ± 0.0	0.1 ± 0.2	3.2 ± 3.6
Deg. (↓)	1.4 ± 1.4	0.7 ± 1.1	0.6 ± 1.1	5.2 ± 3.9
GIN (↓)	0.1 ± 0.4	0.4 ± 0.8	0.1 ± 0.4	3.0 ± 3.2
Orb5. (↓)	0.2 ± 0.5	0.5 ± 0.8	0.0 ± 0.1	1.1 ± 2.0
Orb4. (↓)	0.5 ± 0.7	0.6 ± 1.1	0.3 ± 0.6	2.0 ± 2.5
Eig. (↓)	0.0 ± 0.0	1.3 ± 1.5	0.2 ± 0.6	0.9 ± 2.7
GTV MMD² Clust. (↓)	2.91e-04	0.00e+00	4.87e-04	0.0068
GTV MMD² Clust. (↓)	5.87e-04 ± 1.3e-04	0.00e+00 ± 0.0e+00	9.69e-04 ± 9.4e-06	0.0104 ± 9.4e-04
RBF MMD² Clust. (↓)	3.44e-05 ± 5.1e-05	0.00e+00 ± 0.0e+00	1.62e-06 ± 3.7e-06	0.0014 ± 0.0016
RBF MMD² Clust. (↓)	5.34e-04 ± 1.5e-04	0.00e+00 ± 0.0e+00	6.10e-04 ± 2.6e-05	0.0077 ± 0.0020
GTV MMD² Deg. (↓)	1.51e-05	1.79e-05	1.69e-05	3.16e-04
GTV MMD² Deg. (↓)	2.14e-05 ± 1.1e-05	3.06e-05 ± 1.3e-05	3.86e-05 ± 2.4e-05	5.67e-04 ± 4.6e-04
RBF MMD² Deg. (↓)	1.69e-04 ± 1.7e-04	1.19e-04 ± 1.2e-04	1.48e-04 ± 1.2e-04	0.0052 ± 0.0038
RBF MMD² Deg. (↓)	6.38e-04 ± 2.7e-04	6.03e-04 ± 2.0e-04	8.54e-04 ± 1.3e-04	0.0117 ± 0.0039
GTV MMD² Orb. (↓)	3.43e-06	1.36e-05	3.26e-04	0.0032
GTV MMD² Orb. (↓)	2.18e-05 ± 2.1e-05	5.79e-05 ± 2.8e-05	8.79e-04 ± 2.1e-04	0.0065 ± 0.0042
RBF MMD² Orb. (↓)	1.05e-04 ± 9.8e-05	3.41e-04 ± 2.8e-04	2.98e-05 ± 3.7e-05	0.0044 ± 0.0055
RBF MMD² Orb. (↓)	0.0010 ± 3.3e-05	0.0012 ± 2.3e-04	9.99e-04 ± 3.4e-05	0.0132 ± 0.0038
GTV MMD² Eig. (↓)	7.39e-05	5.12e-05	4.93e-05	4.85e-04
GTV MMD² Eig. (↓)	1.27e-04 ± 2.5e-05	1.10e-04 ± 2.6e-05	9.75e-05 ± 1.9e-05	6.97e-04 ± 1.1e-04
RBF MMD² Eig. (↓)	1.69e-05 ± 2.9e-05	2.78e-05 ± 4.0e-05	5.21e-06 ± 9.7e-06	1.41e-04 ± 2.1e-04
RBF MMD² Eig. (↓)	5.80e-04 ± 5.0e-05	6.43e-04 ± 1.0e-04	4.02e-04 ± 3.1e-05	0.0024 ± 2.9e-04

Table 17: Reference PGS metrics between the molecule test and training sets. Note that MOSES uses a scaffold split, resulting in a high discrepancy between the train and test set.

Dataset	PGS subscores					
	PGS (↓)	Topo (↓)	Morgan (↓)	ChemNet (↓)	MolCLR (↓)	Lipinski (↓)
GUACAMOL	0.2 ± 0.4	0.2 ± 0.4	0.3 ± 0.5	0.3 ± 0.6	0.1 ± 0.2	0.0 ± 0.0
MOSES	21.0 ± 0.6	21.0 ± 0.6	17.8 ± 0.7	16.0 ± 1.2	18.0 ± 0.8	20.7 ± 0.7

1944 L STABILITY OF PGS UNDER VARYING SAMPLE SIZES.

1945
 1946 Figs. 23 to 26 show the relationship between the PGS score and the number of samples. The PGS
 1947 score of the reference graphs with respect to another set of reference graphs issued from the same
 1948 distribution is given as a comparison. For all experiments, we show the mean as well as the 5th and
 1949 95th quantile to give an estimate of the variance of PGS at different sample sizes.

1950 For most models, some separation from the test set occurs above 256 samples, with PGS scores, and
 1951 especially the upper bound is mostly stable beyond this range. This both showcases the stability of
 1952 the metric, the number of samples required to get a reliable PGS estimate, as well as the overall PGS
 1953 ranges for the various models we considered for this study.

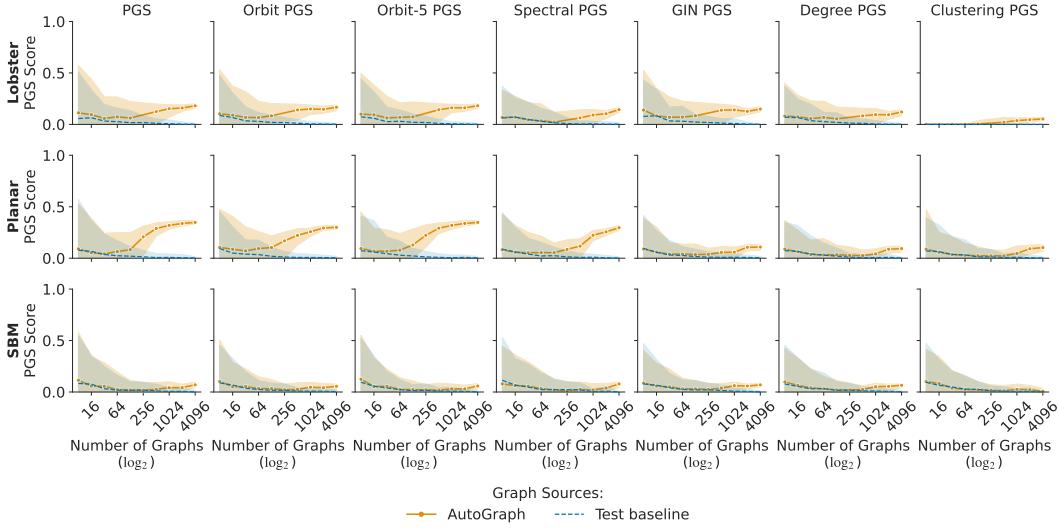


Figure 23: PGS obtained from varying sample sizes generated by AutoGraph.

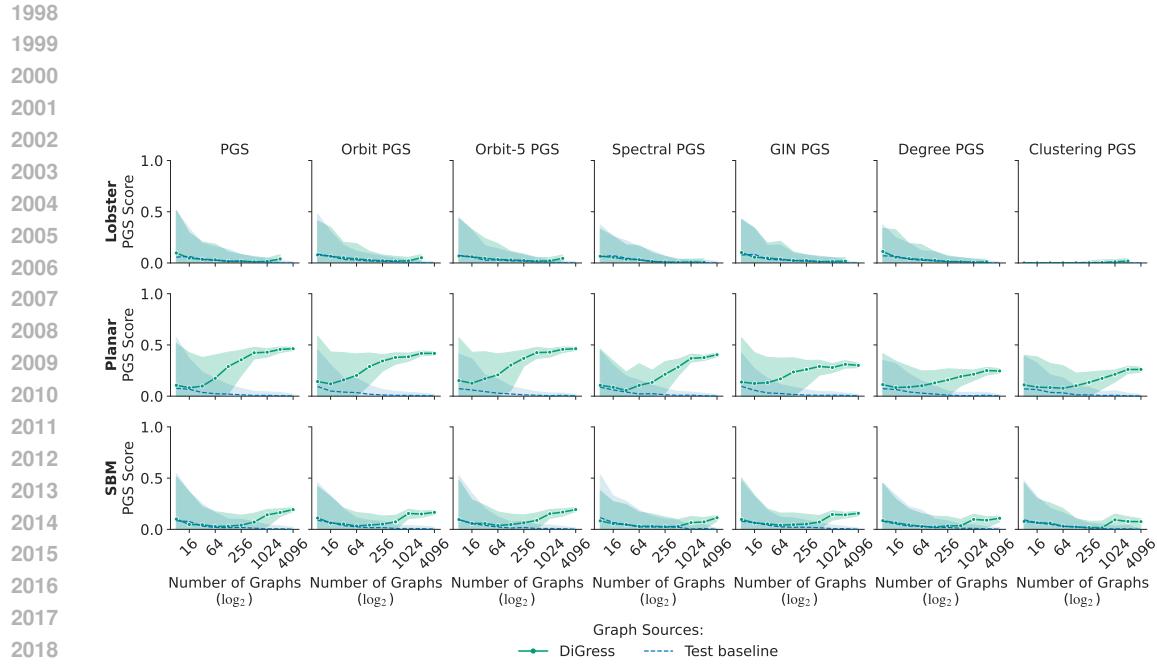


Figure 24: PGS obtained from varying sample sizes generated by DiGress.

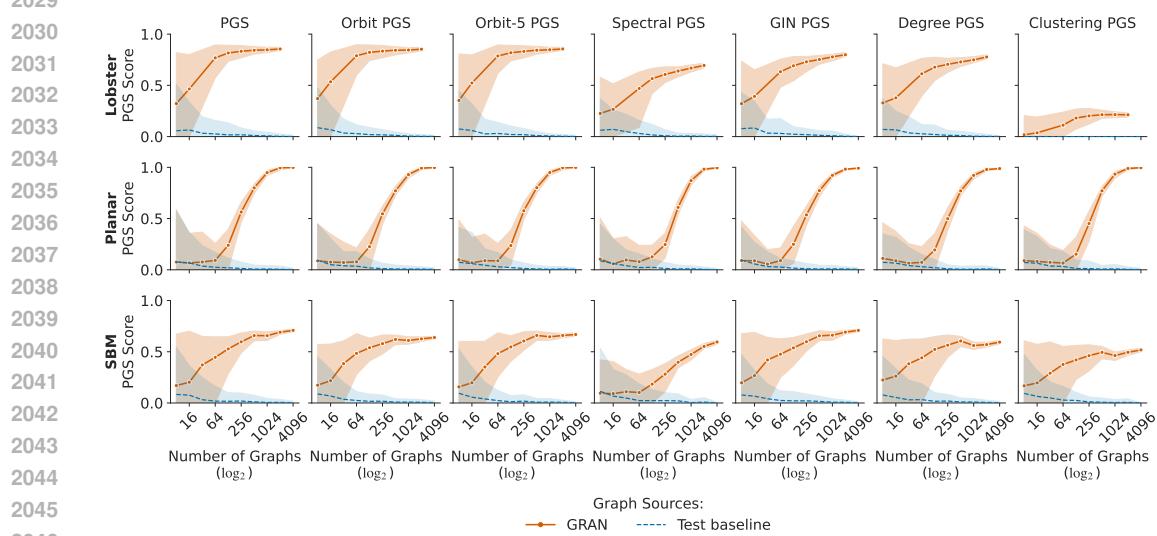


Figure 25: PGS obtained from varying sample sizes generated by GRAN.

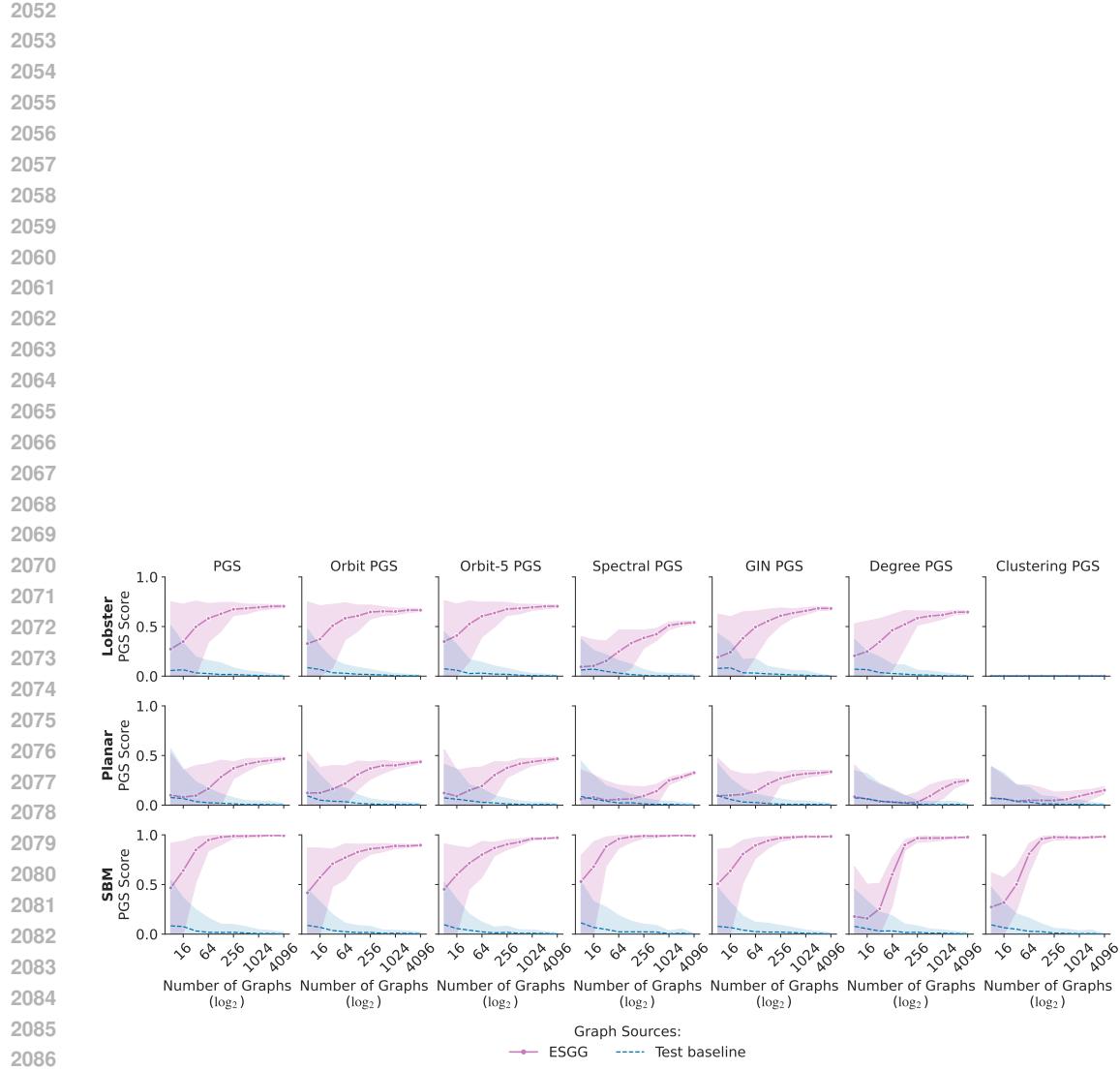


Figure 26: PGS obtained from varying sample sizes generated by ESGG.

2106 M LARGER PROCEDURAL REFERENCE DATASETS FOR BETTER GGM 2107 BENCHMARKING 2108

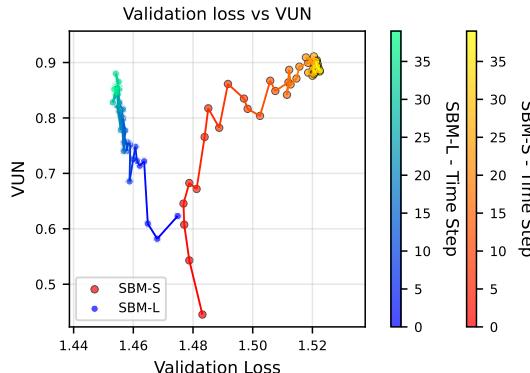
2109 Following our findings of Section 5.1 and Appendix G, we introduce larger procedurally-generated
2110 datasets for planar, lobster and SBM graphs, which we term PLANAR-L, LOBSTER-L and SBM-L.
2111 LOBSTER-L is a set of tree-shaped lobster graphs generated using `nx.random_lobster`, controlled
2112 by expected node count (80) and attachment probabilities to the backbone and its neighbors
2113 (set to 0.7 for both). PLANAR-L is a set of connected planar graphs generated by uniformly
2114 sampling 64 node positions in the unit square and forming the Delaunay triangulation via
2115 `scipy.spatial.Delaunay`, yielding planar edge sets from triangle simplices. SBM-L is a set
2116 of stochastic block model graphs with the number of communities sampled uniformly from 2 to 5
2117 and nodes per community from 20 to 40, where edges are drawn with intra-community probability
2118 0.3 and inter-community probability 0.005. SBM-L, PLANAR-L, and LOBSTER-L datasets follow
2119 `networkx`'s BSD-3 license.

2120 Table 18: Dataset sizes (number of graphs) per split.
2121

2122 Dataset	2123 Train	2124 Val	2125 Test
2126 SBM-L	2127 8192	2128 4096	2129 4096
2130 PLANAR-L	2131 8192	2132 4096	2133 4096
2134 LOBSTER-L	2135 8192	2136 4096	2137 4096

2138 N INFLUENCE OF TRAINING SET SIZE ON AUTOGRAPH

2139 As shown in Fig. 27, AutoGraph converges to similar VUN values across datasets, yet the loss is
2140 substantially lower for SBM-L after training than for SBM-S. This finding indicates that models
2141 may overfit on the existing small procedural datasets, further drawing into question the validity of
2142 previously reported evaluation results (Vignac et al., 2023).



2143 Figure 27: VUN vs. Loss for AutoGraph over the course of a training run.
2144

2145 O COMMON SHORTFALLS OF EXISTING SOLUTIONS

2146 To address the lack of inherent scale in MMD, some have proposed normalizing the MMD between
2147 generated and test graphs by the MMD between train and test graphs (Martinkus et al., 2022).
2148 However, this approach has several shortcomings:

2149 **Limited theoretical justification** MMD was originally introduced as a kernel two-sample test. Its
2150 manipulation beyond direct use as a performance metric or for p -value computation remains
2151 poorly understood.

2152 **Lack of composability** The MMD ratio does not enable combining information across multiple
2153 descriptors.

2160 **Sample-size sensitivity** As shown in Appendix G, MMD strongly depends on sample size. Divid-
 2161 ing MMDs computed on different sample sizes produces ratios with unclear or unreliable
 2162 interpretation.
 2163

2164 **P DATASET DETAILS**
 2165

2166 Here, we provide details about the datasets used in this study. Licenses for those datasets are sum-
 2167 marized in Table 19. Table 20 shows the dataset statistics of the Citeseer dataset (Sen et al., 2008).
 2168 The statistics for the small procedural datasets are presented in Table 21 (Planar), Table 22 (SBM),
 2169 and Table 23 (Lobster).
 2170

2171 Table 19: License and author information of the datasets used in our experiments.
 2172

2173 Dataset	2173 Author	2173 License
2174 Citeseer	2174 (Sen et al., 2008)	2174 CC BY-NC-SA 3.0
2175 Procedural (Planar, SBM, Lobster)	2175 (Martinkus et al., 2022; Hagberg et al., 2008)	2175 BSD-3
2176 Proteins	2176 (Dobson & Doig, 2003)	2176 CC0 1.0 Universal

2178 Table 20: Ego dataset statistics (extracted from Citeseer).
 2179

2180 Metric	2180 Train	2180 Val	2180 Test
2181 Number of Graphs	2181 454	2181 151	2181 152
2182 Minimum number of Nodes	2182 50	2182 50	2182 50
2183 Maximum number of Nodes	2183 399	2183 333	2183 364
2184 Average number of Nodes	2184 141.72	2184 139.29	2184 158.08
2185 Minimum number of Edges	2185 64	2185 56	2185 63
2186 Maximum number of Edges	2186 1066	2186 898	2186 1004
2187 Average number of Edges	2187 325.16	2187 321.87	2187 369.30
2188 Edge/Node Ratio	2188 2.29	2188 2.31	2188 2.34

2191 Table 21: Dataset statistics for the Planar dataset (train, validation, and test splits).
 2192

2193 Metric	2193 Train	2193 Validation	2193 Test
2194 Number of Graphs	2194 128	2194 32	2194 40
2195 Minimum Number of Nodes	2195 64	2195 64	2195 64
2196 Maximum Number of Nodes	2196 64	2196 64	2196 64
2197 Average Number of Nodes	2197 64.00	2197 64.00	2197 64.00
2198 Minimum Number of Edges	2198 173	2198 174	2198 174
2199 Maximum Number of Edges	2199 181	2199 181	2199 181
2200 Average Number of Edges	2200 177.83	2200 177.75	2200 177.93
2201 Edge-to-Node Ratio	2201 2.78	2201 2.78	2201 2.78

2204 **Q FEATURE CONCATENATION AS AN ALTERNATIVE TO MAX-REDUCTION**
 2205

2206 An alternative to taking the maximum JSD across descriptors consists of obtaining an overall PGS
 2207 score by concatenating all vectors arising from the different descriptors, and training a discrimina-
 2208 tor atop these concatenated features. However, because we are working with TabPFN and want to
 2209 keep the discriminator computation time reasonable, we need to apply a dimensionality reduction
 2210 technique (here, we choose PCA) for this concatenated vector to fit within the feature limits recom-
 2211 mended by TabPFN (for v2.0, this is 500). This makes attributing potentially high values to specific
 2212 descriptors impossible, but in practice still results in a tighter bound (i.e., higher scores) as can be
 2213 seen in Table 24.

2214 Table 22: Dataset statistics for the SBM dataset (train, validation, and test splits).
2215

Metric	Train	Validation	Test
Number of Graphs	128	32	40
Minimum Number of Nodes	44	49	54
Maximum Number of Nodes	187	162	174
Average Number of Nodes	105.99	91.28	107.85
Minimum Number of Edges	129	183	210
Maximum Number of Edges	1129	857	972
Average Number of Edges	512.51	425.19	521.88
Edge-to-Node Ratio	4.84	4.66	4.84

2227 Table 23: Dataset statistics for the Lobster dataset (train, validation, and test splits).
2228

Metric	Train	Validation	Test
Number of Graphs	60	20	20
Minimum Number of Nodes	10	11	14
Maximum Number of Nodes	98	98	84
Average Number of Nodes	53.67	56.30	50.80
Minimum Number of Edges	9	10	13
Maximum Number of Edges	97	97	83
Average Number of Edges	52.67	55.30	49.80
Edge-to-Node Ratio	0.98	0.98	0.98

2241

R KERNEL LOGISTIC REGRESSION WITH GRAPH KERNELS

2243 One can adopt the kernel logistic regression classifier and use graph kernels directly to evaluate
2244 GGMs, effectively showing that any (graph) kernel also suitable for MMD can also be used in PGS.
2245 We showcase this with the Weisfeiler-Lehman (Shervashidze et al., 2011), shortest path (Borgwardt
2246 & Kriegel, 2005), and PyramidMatch (Grauman & Darrell, 2007) graph kernels in Table 25. How-
2247 ever, they almost always show looser bounds compared to the standard PGS formulation, so we do
2248 not favor such kernels.

2250

S USE OF LARGE LANGUAGE MODELS

2252 The authors used large language models in the following ways:

2254 **Intelligent tab completion** During software development, tools for intelligent line-wise tab com-
2255 pletion were used.

2256 **Preparation of visualizations** LLMs were partly used to generate code for figure layouts. The
2257 correctness of all code and data was checked manually. The data shown in the figures was
2258 generated by manually written code.

2259 **Information retrieval** LLMs were queried for related work, but produced no relevant results. All
2260 related work presented in the manuscript was manually retrieved, save for Endres & Schin-
2261 delin (2003), which was manually checked to contain the required proof.

2262 **Polishing of manuscript** LLMs were occasionally used to refine or rephrase individual sentences.

2268
2269
2270 Table 24: Comparison of VUN, max-reduced PGS (the default we also use in Table 4) and PGS
2271 with concatenated descriptors. PGS-Concat. is obtained by concatenating all descriptor features,
2272 and subsequently applying a dimensionality reduction technique (PCA) for the feature vectors to fit
2273 within TabPFN’s recommended feature size limit (for v2.0, this is 500). The final score is obtained
2274 similarly to PGS.

Dataset	Model	VUN (\uparrow)	PGS (\downarrow)	PGS-Concat. (\downarrow)
PLANAR-L	AutoGraph	<u>85.1</u>	34.0 \pm 1.8	44.8 \pm 1.3
	DiGRESS	80.1	45.2 \pm 1.8	55.3 \pm 1.5
	GRAN	1.6	99.7 \pm 0.2	99.4 \pm 0.2
	ESGG	93.9	<u>45.0</u> \pm 1.4	<u>52.4</u> \pm 1.1
LOBSTER-L	AutoGraph	<u>83.1</u>	<u>18.0</u> \pm 1.6	29.0 \pm 2.1
	DiGRESS	91.4	3.2 \pm 2.6	<u>43.2</u> \pm 1.4
	GRAN	41.3	85.4 \pm 0.5	86.4 \pm 0.9
	ESGG	70.9	69.9 \pm 0.6	69.9 \pm 1.0
SBM-L	AutoGraph	85.6	5.6 \pm 1.5	27.2 \pm 3.0
	DiGRESS	<u>72.8</u>	<u>17.4</u> \pm 2.3	<u>32.0</u> \pm 2.0
	GRAN	21.4	69.1 \pm 1.4	78.0 \pm 0.8
	ESGG	10.6	99.4 \pm 0.2	98.1 \pm 0.4
Proteins	AutoGraph	-	67.7 \pm 7.4	94.8 \pm 2.6
	DiGRESS	-	88.1 \pm 3.1	99.6 \pm 0.3
	GRAN	-	89.7 \pm 2.7	99.8 \pm 0.1
	ESGG	-	<u>79.2</u> \pm 4.3	<u>99.4</u> \pm 0.3

2293
2294
2295
2296
2297 Table 25: Comparison of PGS (as shown in Table 4) with a PGS variant with a graph kernel logistic
2298 regression (GKLR) model as the classifier. The kernels used here are the PyramidMatch (PM)
2299 kernel, the shortest-path (SP) kernel, and the Weisfeiler-Lehman (WL) kernel.

Dataset	Model	Subscores			
		PGS (\downarrow)	PGS-GKLR (\downarrow)	PM (\downarrow)	SP (\downarrow)
PLANAR-L	AutoGraph	34.0 \pm 1.8	6.2 \pm 2.1	<u>5.3</u> \pm 1.4	5.2 \pm 0.9
	DiGRESS	45.2 \pm 1.8	22.7 \pm 0.9	<u>19.3</u> \pm 0.5	22.8 \pm 0.6
	GRAN	99.7 \pm 0.2	43.1 \pm 0.3	8.8 \pm 0.8	5.2 \pm 2.4
	ESGG	<u>45.0</u> \pm 1.4	<u>14.4</u> \pm 1.0	2.7 \pm 2.3	12.8 \pm 0.7
LOBSTER-L	AutoGraph	<u>18.0</u> \pm 1.6	<u>10.6</u> \pm 1.2	<u>10.3</u> \pm 0.9	8.4 \pm 1.4
	DiGRESS	3.2 \pm 2.6	2.4 \pm 2.5	2.6 \pm 1.7	2.5 \pm 2.2
	GRAN	85.4 \pm 0.5	72.7 \pm 0.8	52.3 \pm 0.8	57.9 \pm 1.2
	ESGG	69.9 \pm 0.6	56.1 \pm 0.6	42.0 \pm 0.6	41.8 \pm 1.0
SBM-L	AutoGraph	5.6 \pm 1.5	5.7 \pm 1.1	1.4 \pm 1.5	5.7 \pm 1.1
	DiGRESS	<u>17.4</u> \pm 2.3	<u>8.8</u> \pm 2.4	<u>7.8</u> \pm 2.4	4.0 \pm 2.2
	GRAN	69.1 \pm 1.4	47.4 \pm 1.0	46.8 \pm 1.0	32.7 \pm 1.3
	ESGG	99.4 \pm 0.2	93.5 \pm 0.3	23.8 \pm 1.8	93.5 \pm 0.3
Proteins	AutoGraph	67.7 \pm 7.4	<u>39.2</u> \pm 2.8	<u>14.0</u> \pm 2.5	<u>39.2</u> \pm 2.8
	DiGRESS	88.1 \pm 3.1	<u>44.8</u> \pm 1.3	3.6 \pm 3.0	44.8 \pm 1.3
	GRAN	89.7 \pm 2.7	<u>59.4</u> \pm 2.0	<u>55.0</u> \pm 1.8	45.7 \pm 1.9
	ESGG	<u>79.2</u> \pm 4.3	31.9 \pm 5.0	17.7 \pm 2.3	31.9 \pm 5.0