

000 001 002 003 004 005 EDEL: ERROR-DRIVEN ENSEMBLE LEARNING FOR 006 IMBALANCED DATA CLASSIFICATION 007 008 009

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ABSTRACT

011 The class imbalance problem poses a critical challenge in high-stakes applica-
012 tions such as fraud detection, where the minority class often represents rare but
013 consequential cases. In such settings, misclassifying minority instances can lead
014 to substantial financial loss, underscoring the need for learning algorithms that re-
015 main reliable under severe imbalance. While deep learning methods have achieved
016 remarkable success across various domains, their effectiveness often depends on
017 large-scale datasets, and their black-box nature limits their interpretability, which
018 is a critical requirement in high-stakes scenarios. To address this gap, we propose
019 **Error-Driven Ensemble Learning (EDEL)**, an adaptive machine learning algorithm
020 that dynamically introduces misclassified instances during training, thereby plac-
021 ing greater emphasis on hard-to-classify samples. Through theoretical analysis
022 and extensive experiments on multiple real-world datasets, EDEL demonstrates
023 strong effectiveness, particularly under challenging imbalanced conditions.
024

1 INTRODUCTION

027 Real-world classification tasks are often affected by class imbalance, where the distribution of
028 classes is highly skewed, as in fraud detection, credit risk assessment, and medical diagnosis. In
029 these scenarios, the minority class typically corresponds to rare but critical instances (Gabidolla
030 et al., 2024; Shahana et al., 2023). The inherent imbalance in such tasks biases machine learning
031 models toward the majority class, leading them to overlook minority instances (Loffredo et al.,
032 2024). This bias significantly degrades the model’s performance on the minority class and under-
033 mines the reliability of decision-making in these systems (Sun et al., 2006). For instance, in fraud
034 detection, fraudulent transactions occur infrequently but may cause substantial financial losses if
035 overlooked, while in credit risk assessment and medical diagnosis, minority errors can result in
036 severe financial or health-related consequences.

037 Traditional learning algorithms are inherently biased toward the majority class, yielding superficially
038 high accuracy while neglecting minority samples. This results in degraded recall, a critical indicator
039 of the reliability and trustworthiness of deployed models. To address this, researchers have proposed
040 various techniques, including resampling methods (Maldonado et al., 2022; Sağlam & Cengiz, 2022;
041 Abedin et al., 2022; Dixit & Mani, 2023; Yan et al., 2023) cost-sensitive learning (Elkan, 2001;
042 Zhou & Liu, 2006; Ling & Sheng, 2010; Zhang & Hu, 2013; Cao et al., 2021), and advanced
043 ensemble approaches (Freund & Schapire, 1997b; Chawla et al., 2003; Sağlam & Cengiz, 2022;
044 Abedin et al., 2022; Zhao et al., 2025). Despite these advancements, accurately identifying minority
045 instances remains a central challenge in imbalanced learning. In this work, we emphasize two
046 key considerations: the need to effectively address *hard-to-classify* samples and the importance of
047 ensuring interpretability in model decisions.

048 **C1. Requirement of attention on *hard-to-classify* samples.** A key concern in imbalanced learn-
049 ing is the presence of *hard-to-classify* samples that remain difficult to predict accurately, whether
050 they belong to the minority or majority class. Such cases often arise from feature overlap, noise, or
051 atypical patterns. Neglecting these samples undermines model robustness, as errors tend to concen-
052 trate on precisely those instances that are most informative for improving generalization. Figure 1
053 illustrates how class imbalance skews classification performance and error distribution. While the
054 majority class is generally well recognized, a nontrivial portion of samples are misclassified as mi-
055 nority, placing erroneous signals into the minority space. These misclassified majorities increase

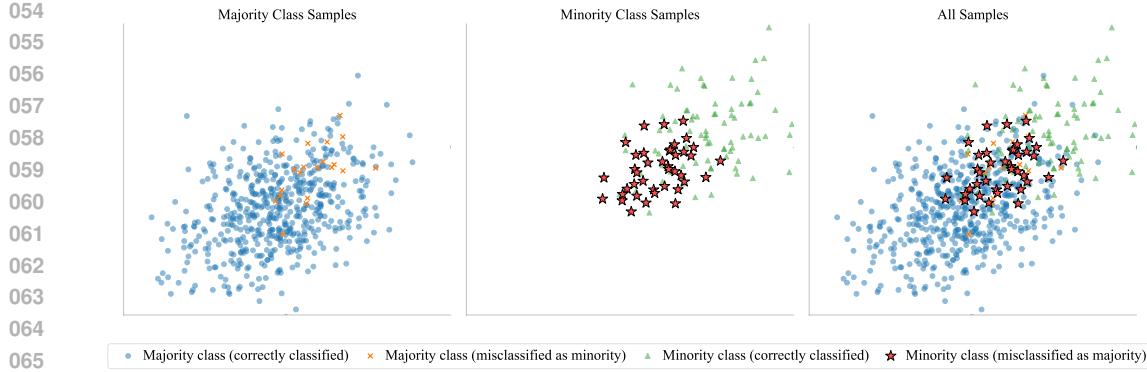


Figure 1: Impact of class imbalance on classification accuracy and error distribution.

overlap between classes, making the minority region appear less distinct, and further skew the decision boundary against the true minority. As a result, the classifier not only struggles with minority recognition but also inherits bias from misleading majority instances. We conjecture that these misclassifications occur because minority samples can closely resemble majority patterns, while some majority outliers deviate from typical distributions. Such concentration of errors is problematic: both minority instances misclassified as majority and majority outliers misclassified as minority contribute to the degradation of performance (especially the recall) and the erosion of applications' trustworthiness.

C2. Requirement of attention to interpretability. Another concern that should receive attention in imbalanced learning is interpretability, which is particularly critical in high-stakes real-world applications. Complex models, such as models based on Deep Learning algorithms, may achieve strong predictive performance but often operate as black boxes, making it difficult to understand why certain minority instances are misclassified or how decision boundaries are shaped by skewed data distributions. The lack of transparency hinders trust, complicates regulatory compliance, and prevents practitioners from diagnosing whether errors stem from data imbalance, noise, or just model bias. Therefore, beyond improving the predictive accuracy, methods for imbalanced classification should provide interpretable mechanisms for explaining decisions, especially for minority predictions where mistakes carry disproportionate costs, particularly in these real-world applications.

With these challenges in mind, we propose **Error-Driven Ensemble Learning** (EDEL), a novel machine learning approach that progressively focuses on misclassified samples, a.k.a., *hard-to-classify* samples, into the learning process. By iteratively emphasizing them, EDEL refines its understanding and decision boundaries, improving recognition of the minority class while correcting misleading majority errors. Moreover, EDEL operates by partitioning the training data into multiple subsets, training weak classifiers, reinjecting misclassified samples, and aggregating predictions through probability averaging to ensure robust performance. This error-driven design naturally enhances interpretability by revealing where classifiers fail and how corrections are applied. Overall, the main contributions of EDEL are as follows:

- We highlight two critical concerns in imbalanced learning: the prevalence of *hard-to-classify* samples and the lack of interpretability.
- We introduce EDEL, a novel machine learning approach that progressively focuses on misclassified samples during training, significantly improving the performance on *hard-to-classify* samples. Experimental results on seven real-world datasets within imbalance ratios ranging from 1.54 to 577.88 demonstrate EDEL's superiority over baselines.
- We provide a theoretical foundation for EDEL, showing via McDiarmid's inequality and Bayes' theorem that dynamically incorporating *hard-to-classify* instances reduces empirical error, refines decision boundaries, and ensures asymptotic consistency with robust generalization under extreme imbalance. This theoretical guarantee contributes to the broader task of imbalanced classification by establishing a principled basis for designing reliable and interpretable learning algorithms.

108 **2 PRELIMINARY**
 109

110 **Problem Definition.** For a K -class classification task, we have a training dataset with imbalanced
 111 distribution, consisting of m samples: $\mathcal{D} = \{(\mathcal{X}, \mathcal{Y})\} = \{(\mathbf{x}_1, \mathbf{y}_1), \dots, (\mathbf{x}_m, \mathbf{y}_m)\}$, where $\mathbf{x}_i \in \mathbb{R}^d$
 112 represents the d -dimensional features and \mathbf{y}_i is the corresponding ground truth. Without loss of
 113 generality, we assume the class sample sizes are sorted in descending order, i.e., $n_1 \geq n_2 \geq \dots \geq$
 114 n_K , where n_k denotes the number of samples in class k . Our objective is to conduct a scheme
 115 $\pi(\mathcal{X}; \theta)$ that accurately infers the label for \mathcal{X} under the class-balanced setting, i.e., $\pi(\mathcal{X}; \theta) \mapsto \mathcal{Y}$.

116 Within this setup, we further introduce the *hard-to-classify* and *easy-to-classify* samples in this work.
 117 Let (\mathbf{x}, \mathbf{y}) be an instance, let $\mathcal{F} = \{f_1, \dots, f_{|\mathcal{F}|}\}$ denote a set of classifiers.
 118

119 **DEFINITION 1:** *Easy-to-classify* samples correspond to data points that are well-aligned with their
 120 class distribution and consistently captured by different classifiers. That means,

121
$$f_i(\mathbf{x}) = \mathbf{y}, \quad \forall f_i \in \mathcal{F}. \quad (1)$$

 122

123 **DEFINITION 2:** *Hard-to-classify* samples lie in regions of feature overlap, noise, or atypical struc-
 124 ture, where even a subset of classifiers fail. In this work, an instance (\mathbf{x}, \mathbf{y}) is *Hard-to-classify* if it
 125 is misclassified by at least one classifier in \mathcal{F} , namely,

126
$$\exists f_i \in \mathcal{F} \quad \text{such that } f_i(\mathbf{x}) \neq \mathbf{y}. \quad (2)$$

 127

128 **3 METHODOLOGIES**
 129

130 **3.1 PIPELINE**
 131

132 Initially, we randomly form N -subsets from original dataset, i.e., $\mathcal{D} \rightarrow \{\mathcal{D}_i\}_{i=1}^N$, where
 133 $\sum_{i=1}^N |\mathcal{D}_i| = |\mathcal{D}|$. Herein, subset \mathcal{D}_i can be viewed as a partial observation drawn from the true
 134 data distribution. By partitioning the data into multiple subsets, EDEL generates diverse local views
 135 of the distribution, while each subset has its opportunity to emphasize certain characteristics or pat-
 136 terns. Whereafter, weak classifiers $\hat{\pi}$ are conducted on corresponding subset as,
 137

138
$$\hat{\mathbf{y}} = \hat{\pi}_i(\mathbf{x}), \quad \mathbf{x} \in \mathcal{D}_i, \quad i \in [1, N], \quad (3)$$

139 and *hard-to-classify* samples can be formed via these weak classifiers $\hat{\pi}$ by,
 140

141
$$\mathcal{D}_i^h \leftarrow \{(\mathbf{x}, \mathbf{y}) \mid \hat{\pi}_i(\mathbf{x}) \neq \mathbf{y}, (\mathbf{x}, \mathbf{y}) \in \mathcal{D} \setminus \mathcal{D}_i\}. \quad (4)$$

142 Therefore, we update \mathcal{D}_i by incorporating \mathcal{D}_i^h as,
 143

144
$$\mathcal{D}_i \leftarrow \mathcal{D}_i \cup \mathcal{D}_i^h. \quad (5)$$

 145

146 Notably, we have $\sum_{i=1}^N |\mathcal{D}_i| > |\mathcal{D}|$ after this operation.

147 Once the weak classifiers are well-trained, EDEL integrates parameters from each $\hat{\pi}(\cdot)$ to form the
 148 final classifier $\pi(\cdot)$ by employing a probability-averaging scheme, namely,
 149

150
$$\Theta = \frac{1}{N} \sum_{i=1}^N \hat{\Theta}_i, \quad \text{where} \quad \hat{\Theta}_i \leftarrow \hat{\pi}_i(\mathcal{D}_i). \quad (6)$$

 151

153 Algorithm 1 outlines the training process of the proposed EDEL. To ensure generality, we denote
 154 the training cost of a weak classifier as T_{π} , and divide EDEL's overall complexity into 5 stages:
 155

156 (1) **Data Partitioning.** Using stratified sampling to preserve class distribution on a dataset \mathcal{D}
 157 of size m , this step has time complexity $\mathcal{O}(m)$.
 158 (2) **Weak Classifier Training.** Training N weak classifiers $\hat{\pi}$ on each subset \mathcal{D}_i cost, giving a
 159 total complexity of $\mathcal{O}(N \times T_{\pi})$.
 160 (3) **Subset Update.** Each π_i conducts prediction on remaining $N - 1$ subsets, for a total of
 161 $\mathcal{O}((N - 1) \times m)$.

162
163 **Algorithm 1:** Training Process of EDEL.
164 **Input:** $\mathcal{D}, N, \hat{\pi}$
165 1 Initially partition $\mathcal{D} \rightarrow \{\mathcal{D}_i\}_{i=1}^N$
166 2 **while** not done **do**
167 3 **for** $i = 1$ to N **do**
168 4 **for** $(x, y) \in \mathcal{D}_i$ **do**
169 5 | $\hat{y} = \hat{\pi}_i(x)$
170 6 | **end**
171 7 | $\Theta_i \leftarrow \hat{\pi}_i(\mathcal{D}_i)$
172 8 | $\mathcal{D}_i^h \leftarrow \{\emptyset\}$
173 9 | **for** $(x, y) \in \mathcal{D} \setminus \mathcal{D}_i$ **do**
174 10 | | $\hat{y} = \hat{\pi}_i(x)$
175 11 | | **if** $\hat{y} \neq y$ **then**
176 12 | | | $\mathcal{D}_i^h \leftarrow \mathcal{D}_i^h \cup \{(x, y)\}$
177 13 | | | $(\mathcal{D} \setminus \mathcal{D}_i) \setminus \{(x, y)\}$
178 14 | | **end**
179 15 | | **end**
180 16 | | $\mathcal{D}_i \leftarrow \mathcal{D}_i \cup \mathcal{D}_i^h$
181 17 **end**
182 18 **end**
183 19 $\Theta \leftarrow \frac{1}{N} \sum_{i=1}^N \hat{\Theta}_i$
184 **Output:** π_Θ

186 (4) **Training Cycle.** Retraining each $\hat{\pi}_i$ on updated subset \mathcal{D}_i costs T_π , totaling $\mathcal{O}(N \times T_\pi)$.
187 (5) **Ensemble Strategy:** Forming the final classifier by averaging outputs costs $\mathcal{O}(N)$ per
188 sample, totaling $\mathcal{O}(N \times m)$.

190 Thus, the overall time complexity can be summed up by the 5 stages as:
191

$$192 \quad \mathcal{O}(m) + \mathcal{O}(N \times T_\pi) + \mathcal{O}((N-1) \times m) + \mathcal{O}(N \times T_\pi) + \mathcal{O}(N \times m) = \mathcal{O}(N \times (T_\pi + m)).$$

194 In summary, the time complexity of the EDEL algorithm is linear with respect to both the weak
195 classifier training time T_π and the dataset size m , and it scales proportionally with the number of
196 weak classifiers N . Through data partitioning and parallel computation, EDEL effectively scales
197 to large datasets. In highly imbalanced scenarios, adjusting the number of classifiers allows EDEL
198 to balance high performance with computational efficiency, making it a versatile solution for large-
199 scale and imbalanced data.

200 3.2 THEORETICAL FOUNDATION

202 **Data Aspect.** EDEL employs stratified sampling to split training dataset \mathcal{D} into N non-overlapping
203 subsets $\{\mathcal{D}_i\}_{i=1}^N$ initially. Following the definition on Sec. 2, the class proportion for class k is given
204 by $p_k = \frac{n_k}{m}$. Stratified sampling allocates $\frac{n_k}{N}$ samples of class k to each subset \mathcal{D}_i , resulting in a
205 subset size of $\frac{m}{N}$ (assuming m is divisible by N for simplicity). The proportion of class k in \mathcal{D}_i can
206 be described as,

$$207 \quad p_{i,k} = \frac{\frac{n_k}{N}}{\frac{m}{N}} = \frac{n_k}{m} = p_k. \quad (7)$$

210 Ideally, $p_{i,k} = p_k$, ensuring that each subset \mathcal{D}_i mirrors the overall class distribution of \mathcal{D} . However,
211 due to finite sampling, random allocation may introduce deviations. The number of class k samples
212 $n_{i,k}$ in \mathcal{D}_i follows a hypergeometric distribution, with expectation $\mathbb{E}[n_{i,k}] = \frac{n_k}{N}$ and variance:
213

$$214 \quad \text{Var}(n_{i,k}) = \frac{n_k}{N} \left(1 - \frac{1}{N}\right) \frac{m - n_k}{m - 1}. \quad (8)$$

216 The deviation of the proportion is defined as $\Delta p_{i,k} = |p_{i,k} - p_k|$. Using the Chebyshev inequality,
 217 the probability that this deviation exceeds δ is bounded by:
 218

$$219 \quad P(|\Delta p_{i,k}| \geq \delta) \leq \frac{\text{Var}(n_{i,k})}{n^2 \delta^2}, \quad (9)$$

221 where $n = \frac{m}{N}$. As the total sample size $m \rightarrow \infty$, the variance term $\frac{\text{Var}(n_{i,k})}{n^2} \rightarrow 0$, implying that
 222 $\Delta p_{i,k} \rightarrow 0$. This confirms that $p_{i,k} \approx p_k$ with high probability for large m , thus establishing the rep-
 223 resentativeness of \mathcal{D}_i with respect to \mathcal{D} . This representativeness supports the dynamic enhancement
 224 of minority class samples, as detailed in [Appendix B](#).

225 **Classifier Aspect.** Weak classifiers $\hat{\pi}_i^{(t+1)}$ are trained on the dynamically enhanced subset $\mathcal{D}_i^{(t)} =$
 226 $\mathcal{D}_i^{(t-1)} \cup \mathcal{D}_i^{(t-1),h}$, with its training error rate:
 227

$$228 \quad \hat{\epsilon}_i^{(t+1)} = \frac{1}{|\mathcal{D}_i^{(t)}|} \sum_{(\mathbf{x}, y) \in \mathcal{D}_i^{(t)}} \mathbb{I}(\hat{\pi}_i^{(t+1)}(\mathbf{x}) \neq y), \quad (10)$$

231 where $\mathbb{I}(\cdot)$ is the indicator function.

232 Assume $\mathcal{D}_i^{(t)}$ contains $n_i^{(t)}$ samples, with $n_{i,0}^{(t)}$ samples from the majority class (class 0) and $n_{i,1}^{(t)}$
 233 samples from the minority class (class 1), such that $n_i^{(t)} = n_{i,0}^{(t)} + n_{i,1}^{(t)}$. For a simple weak clas-
 234 sifier, such as a decision tree stump (which performs a single split), the optimal classification rule
 235 minimizes the training error rate by selecting a feature and threshold that partitions the data into two
 236 regions, assigning the majority class label to each region. In the degenerate case without any split
 237 (equivalent to a constant classifier predicting the majority class), the minimum training error rate is
 238 the minority class proportion:

$$239 \quad \hat{\epsilon}_i^{(t+1)} = \frac{n_{i,1}^{(t)}}{n_i^{(t)}}. \quad (11)$$

242 However, with splitting enabled, the actual training error rate is strictly lower than this baseline, as
 243 the split allows the classifier to capture more nuanced patterns in the data. Dynamic enhancement
 244 increases the number of minority class samples $n_{i,1}^{(t)}$ (due to the higher proportion of minority class
 245 instances in \mathcal{D}_i^h), making the subset $\mathcal{D}_i^{(t)}$ more balanced. This balance enables the classifier to
 246 identify better decision boundaries, reducing the actual $\hat{\epsilon}_i^{(t+1)}$ compared to the previous iteration
 247 or to random guessing (where $\epsilon = 0.5$ for a balanced dataset). Since $\rho' > \rho$ (where ρ and ρ'
 248 are the minority class proportions in $\mathcal{D}_i^{(t-1)}$ and $\mathcal{D}_i^{(t)}$, respectively), $n_{i,1}^{(t)}$ grows relative to $n_{i,0}^{(t)}$,
 249 ensuring the actual $\epsilon < 0.5$ and improving performance on minority class instances. This minority
 250 class enrichment is rigorously derived using Bayes' theorem in [Appendix B](#), with further analysis
 251 of extreme imbalance scenarios.

253 To assess the generalization performance of weak classifiers within EDEL, we employ Rademacher
 254 complexity (Bartlett & Mendelson, 2003) $\mathcal{R}_{n_i^{(t)}}(\mathcal{H})$, which measures the capacity of the hypothesis
 255 space \mathcal{H} (e.g., decision stumps) over the training set $\mathcal{D}_i^{(t)}$. It is defined as:

$$257 \quad \hat{\mathcal{R}}_{n_i^{(t)}}(\mathcal{H}) = \mathbb{E}_\sigma \left[\sup_{h \in \mathcal{H}} \left| \frac{2}{n_i^{(t)}} \sum_{j=1}^{n_i^{(t)}} \sigma_j h(\mathbf{x}_j) \right| \middle| \mathcal{D}_i^{(t)} \right], \quad (12)$$

261 where $\sigma_j \in \{-1, +1\}$ are independent Rademacher random variables, and the expectation is
 262 taken over their distribution conditional on the $\mathcal{D}_i^{(t)}$. The unconditional Rademacher complexity is
 263 $\mathcal{R}_{n_i^{(t)}}(\mathcal{H}) = \mathbb{E}[\hat{\mathcal{R}}_{n_i^{(t)}}(\mathcal{H})]$. For a decision tree with d features, $\mathcal{R}_{n_i^{(t)}}(\mathcal{H}) = \mathcal{O}(\sqrt{d/n_i^{(t)}})$, based on
 264 the empirical VC-dimension bound (where the VC-dimension of decision stumps is $O(d)$). Thereby,
 265 the true generalization error $\epsilon_i^{(t+1)}$ (i.e., the expected error over the data distribution) is bounded us-
 266 ing Rademacher complexity, with probability at least $1 - \delta$:

$$268 \quad \epsilon_i^{(t+1)} \leq \hat{\epsilon}_i^{(t+1)} + \frac{\mathcal{R}_{n_i^{(t)}}(\mathcal{H})}{2} + \sqrt{\frac{\ln(1/\delta)}{2n_i^{(t)}}}, \quad (13)$$

270 where $\delta > 0$ is a confidence parameter. As $n_i^{(t)}$ increases, $\mathcal{R}_{n_i^{(t)}}(\mathcal{H}) = \mathcal{O}(\sqrt{d/n_i^{(t)}})$ and $\sqrt{\frac{\ln(1/\delta)}{2n_i^{(t)}}}$
 271 both decrease, ensuring that the empirical error $\hat{\epsilon}_i^{(t+1)}$ is a good approximation of the true error
 272 $\epsilon_i^{(t+1)}$. This confirms that the weak classifier generalizes well from the enhanced training subset
 273 $\mathcal{D}_i^{(t)}$.
 274

275 **Convergence Aspect.** Building on the stratified sampling and dynamic enhancement mechanisms
 276 outlined in the **Data Aspect** and **Classifier Aspect**, we analyze the convergence of the empirical
 277 error $\hat{\epsilon}_i^{(t+1)}$ to the true error $\epsilon_i^{(t+1)}$ for the proposed EDEL. The empirical error is defined in Equa-
 278 tion equation 10 as:
 279

$$280 \hat{\epsilon}_i^{(t+1)} = \frac{1}{|\mathcal{D}_i^{(t)}|} \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}_i^{(t)}} \mathcal{Z}_i(\mathbf{x}, \mathbf{y}), \quad (14)$$

283 where $\mathcal{Z}_i(\mathbf{x}, \mathbf{y}) = \mathbb{I}(\hat{\pi}_i^{(t+1)}(\mathbf{x}) \neq \mathbf{y})$ is the indicator function for misclassification, and $\epsilon_i^{(t+1)} =$
 284 $\mathbb{E}_{(\mathbf{x}, \mathbf{y}) \sim \mathcal{D}}[\mathcal{Z}_i(\mathbf{x}, \mathbf{y})]$ is the true error under the data distribution \mathcal{D} . The initial stratified sam-
 285 pling ensures approximate i.i.d. conditions (Equation equation 7), with class proportion deviations
 286 $\Delta p_{i,k} \rightarrow 0$ as $m \rightarrow \infty$ (Sec. 3.2). However, the dynamic incorporation of misclassified samples \mathcal{D}_i^h
 287 into $\mathcal{D}_i^{(t)}$ introduces dependencies and shifts the subset distribution toward minority class instances
 288 (Appendix B).
 289

290 Under approximate i.i.d. conditions, the expected empirical error is approximately unbiased:
 291

$$292 \mathbb{E}[\hat{\epsilon}_i^{(t+1)}] = \mathbb{E} \left[\frac{1}{|\mathcal{D}_i^{(t)}|} \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}_i^{(t)}} \mathcal{Z}_i(\mathbf{x}, \mathbf{y}) \right] \approx \frac{1}{|\mathcal{D}_i^{(t)}|} \sum_{(\mathbf{x}, \mathbf{y}) \in \mathcal{D}_i^{(t)}} \mathbb{E}[\mathcal{Z}_i(\mathbf{x}, \mathbf{y})] \approx \epsilon_i^{(t+1)}, \quad (15)$$

294 The linearity of expectation ensures the first equality holds regardless of independence, but potential
 295 distribution shifts from dynamic enhancement may cause $\mathbb{E}[\mathcal{Z}_i(\mathbf{x}, \mathbf{y})]$ to vary across samples, intro-
 296 ducing a bias $|\mathbb{E}[\hat{\epsilon}_i^{(t+1)}] - \epsilon_i^{(t+1)}| \leq \mathcal{O}(\sqrt{d/|\mathcal{D}_i^{(t)}|})$, as controlled by the Rademacher complexity
 297 of the loss function (Bartlett & Mendelson, 2003).
 298

299 To address dependencies introduced by the enhancement process, we employ the McDiarmid's in-
 300 equality (McDiarmid, 1989), which requires only bounded differences rather than strict indepen-
 301 dence. Consider $\hat{\epsilon}_i^{(t+1)}$ as a function of the training samples in $\mathcal{D}_i^{(t)}$. Changing one sample $(\mathbf{x}_j, \mathbf{y}_j)$
 302 to $(\mathbf{x}'_j, \mathbf{y}'_j)$ affects \mathcal{Z}_j , altering $\hat{\epsilon}_i^{(t+1)}$ by at most $\frac{1}{|\mathcal{D}_i^{(t)}|}$, as $\mathcal{Z}_j \in [0, 1]$. Thus, the bounded difference
 303 constant is $c_j = \frac{1}{|\mathcal{D}_i^{(t)}|}$, and the sum of squared differences is $\sum_{j=1}^{|\mathcal{D}_i^{(t)}|} c_j^2 = \frac{1}{|\mathcal{D}_i^{(t)}|}$. The McDiarmid's
 304 inequality yields:
 305

$$306 P \left(\left| \hat{\epsilon}_i^{(t+1)} - \mathbb{E}[\hat{\epsilon}_i^{(t+1)}] \right| \geq \delta \right) \leq 2 \exp \left(-2\delta^2 |\mathcal{D}_i^{(t)}| \right). \quad (16)$$

307 This ensures that $\hat{\epsilon}_i^{(t+1)}$ concentrates around its expectation with high probability as the subset size
 308 $|\mathcal{D}_i^{(t)}|$ increases.
 309

310 To bridge $\mathbb{E}[\hat{\epsilon}_i^{(t+1)}]$ to $\epsilon_i^{(t+1)}$, we leverage the Rademacher complexity bound (Equation equation 13) (Bartlett & Mendelson, 2003). For the hypothesis space \mathcal{H} of weak classifiers (e.g., decision
 311 stumps), the true error is bounded with probability at least $1 - \delta$:
 312

$$313 \epsilon_i^{(t+1)} \leq \hat{\epsilon}_i^{(t+1)} + \frac{\mathcal{R}_{|\mathcal{D}_i^{(t)}|}(\mathcal{H})}{2} + \sqrt{\frac{\ln(1/\delta)}{2|\mathcal{D}_i^{(t)}|}}, \quad (17)$$

314 where $\mathcal{R}_{|\mathcal{D}_i^{(t)}|}(\mathcal{H}) = \mathcal{O}(\sqrt{d/|\mathcal{D}_i^{(t)}|})$ for decision stumps with d features. The bias
 315 $|\mathbb{E}[\hat{\epsilon}_i^{(t+1)}] - \epsilon_i^{(t+1)}|$ is thus bounded by $\mathcal{O}(\sqrt{d/|\mathcal{D}_i^{(t)}|})$, mitigating the impact of non-identical dis-
 316 tributions caused by dynamic enhancement.
 317

318 Combining the McDiarmid's inequality and Rademacher bounds, we establish that $\hat{\epsilon}_i^{(t+1)}$ converges
 319 to $\epsilon_i^{(t+1)}$ with high probability, at least $1 - 2 \exp(-2\delta^2 |\mathcal{D}_i^{(t)}|) - \delta$, where $|\mathcal{D}_i^{(t)}| = |\mathcal{D}_i^{(t-1)}| + |\mathcal{D}_i^h|$
 320

reflects the growth from dynamic enhancement. This convergence is robust to the dependencies and distribution shifts introduced by (\mathcal{D}_i^h) and the ensemble averaging further mitigates subset-specific biases, ensuring robust performance. Further analysis of minority class enrichment and integrated classifier performance is provided in [Appendix B](#) and [Appendix B.3](#).

4 EXPERIMENTS

Dataset. We conduct experiments on seven publicly available, real-world datasets that span diverse domains, detailed in Table 1. They exhibit a wide range of imbalance ratios (IR), from moderate (e.g., SBD with IR=1.54) to extreme (e.g., CCFD with IR=577.88), making them ideal for evaluating the robustness of our method across varying degrees of class imbalance. Each dataset has been preprocessed to handle missing values and eliminate redundant features. More details of datasets and metrics are described in [Appendix C.1](#).

Table 1: Summary of the datasets used in the experiments.

Dataset	Samples	Pos	Neg	Features	IR
SBD	4,601	1,813	2,788	57	1.54
AID	48,842	11,687	37,155	13	3.18
TCD	30,000	6,636	23,364	23	3.52
CDH	253,680	35,346	218,334	21	6.18
BMD	41,188	4,640	36,548	20	7.88
GMSC	150,000	10,026	139,974	10	13.96
CCFD	284,807	492	284,315	29	577.88

Baseline. We evaluate EDEL against five baselines, including (1) basic method, the Original instances with no imbalance handling applied; (2) data-level methods, SMOTE (Chawla et al., 2002), RandomUnderSampler (RUS) (He & Garcia, 2009) and SMOTE-TLNN-DEPSO (S-T-D) (Dixit & Mani, 2023); (3) ensemble learning methods, MESA (Liu et al., 2020) and CHRE (Zhao et al., 2025). All methods, including EDEL and the baselines, are evaluated using four classifiers: Decision Tree (DT), Random Forest (RF), XGBoost (XGB), and LightGBM (LGBM). Experimental results are reported by 5-fold stratified cross-validation, with each fold allocating 80% for training and 20% for testing. More details of classifier parameters are described in [Appendix C.2](#).

Table 2: Results in terms of AUC.

Clf	Mth	SBD	AID	TCD	CDH	BMD	GMSC	CCFD
DT	Orig	0.9030 \pm 0.0075	0.7717 \pm 0.0041	0.6143 \pm 0.0074	0.5979 \pm 0.0031	0.7294 \pm 0.0066	0.6120 \pm 0.0047	0.8921 \pm 0.0041
	RUS	0.9062 \pm 0.0139	0.7824 \pm 0.0036	0.6214 \pm 0.0041	0.6514 \pm 0.0031	0.8367 \pm 0.0044	0.7017 \pm 0.0033	0.9012 \pm 0.0137
	SMOTE	0.9126 \pm 0.0112	0.7780 \pm 0.0060	0.6131 \pm 0.0073	0.5979 \pm 0.0054	0.7451 \pm 0.0085	0.6358 \pm 0.0055	0.8953 \pm 0.0164
	MESA	0.9655 \pm 0.0134	0.8721 \pm 0.0052	0.7366 \pm 0.0074	0.7734 \pm 0.0056	0.9322 \pm 0.0036	0.8145 \pm 0.0128	0.9609 \pm 0.0177
	S-T-D	0.8893 \pm 0.0085	0.7765 \pm 0.0030	0.6165 \pm 0.0073	0.6210 \pm 0.0046	0.7935 \pm 0.0098	0.6648 \pm 0.0096	0.9085 \pm 0.0180
	CHRE	0.8993 \pm 0.0128	0.7840 \pm 0.0025	0.6383 \pm 0.0104	0.6633 \pm 0.0052	0.8226 \pm 0.0038	0.6932 \pm 0.0038	0.9123 \pm 0.0054
RF	EDEL	0.9911 \pm 0.0128	0.9519 \pm 0.0724	0.9187 \pm 0.1536	0.9241 \pm 0.1381	0.9745 \pm 0.0475	0.9457 \pm 0.1107	0.9827 \pm 0.0387
	Orig	0.9860 \pm 0.0041	0.8954 \pm 0.0036	0.7639 \pm 0.0046	0.7974 \pm 0.0025	0.9444 \pm 0.0024	0.8387 \pm 0.0055	0.9497 \pm 0.0108
	RUS	0.9859 \pm 0.0044	0.8973 \pm 0.0020	0.7684 \pm 0.0063	0.8077 \pm 0.0023	0.9430 \pm 0.0027	0.8519 \pm 0.0038	0.9777 \pm 0.0077
	SMOTE	0.9857 \pm 0.0040	0.8902 \pm 0.0027	0.7506 \pm 0.0039	0.7942 \pm 0.0032	0.9415 \pm 0.0027	0.8207 \pm 0.0055	0.9691 \pm 0.0103
	MESA	0.9858 \pm 0.0040	0.9015 \pm 0.0044	0.7729 \pm 0.0048	0.8168 \pm 0.0024	0.9464 \pm 0.0022	0.8450 \pm 0.0052	0.9829 \pm 0.0074
	S-T-D	0.9750 \pm 0.0072	0.8942 \pm 0.0025	0.7530 \pm 0.0072	0.7970 \pm 0.0029	0.9428 \pm 0.0017	0.8381 \pm 0.0041	0.9793 \pm 0.0108
XGB	CHRE	0.9428 \pm 0.0074	0.8207 \pm 0.0021	0.7112 \pm 0.0089	0.7139 \pm 0.0055	0.8782 \pm 0.0045	0.7283 \pm 0.0066	0.9104 \pm 0.0107
	EDEL	0.9972 \pm 0.0058	0.9656 \pm 0.0448	0.9461 \pm 0.1117	0.9506 \pm 0.0841	0.9795 \pm 0.0306	0.9583 \pm 0.0641	0.9923 \pm 0.0172
	Orig	0.9872 \pm 0.0031	0.9285 \pm 0.0025	0.7661 \pm 0.0059	0.8269 \pm 0.0025	0.9458 \pm 0.0018	0.8573 \pm 0.0043	0.9789 \pm 0.0062
	RUS	0.9869 \pm 0.0030	0.9258 \pm 0.0030	0.7579 \pm 0.0073	0.8223 \pm 0.0024	0.9411 \pm 0.0029	0.8515 \pm 0.0034	0.9780 \pm 0.0086
	SMOTE	0.9871 \pm 0.0027	0.9239 \pm 0.0030	0.7378 \pm 0.0084	0.8262 \pm 0.0029	0.9398 \pm 0.0025	0.8174 \pm 0.0034	0.9763 \pm 0.0110
	MESA	0.9872 \pm 0.0024	0.9273 \pm 0.0031	0.7715 \pm 0.0043	0.8259 \pm 0.0032	0.9463 \pm 0.0018	0.8566 \pm 0.0053	0.9727 \pm 0.0132
LGBM	S-T-D	0.9782 \pm 0.0047	0.9213 \pm 0.0030	0.7370 \pm 0.0084	0.8196 \pm 0.0032	0.9431 \pm 0.0027	0.8360 \pm 0.0048	0.9784 \pm 0.0077
	CHRE	0.9454 \pm 0.0064	0.8235 \pm 0.0037	0.6829 \pm 0.0094	0.7402 \pm 0.0034	0.8873 \pm 0.0042	0.7704 \pm 0.0075	0.9196 \pm 0.0153
	EDEL	0.9968 \pm 0.0065	0.9424 \pm 0.0127	0.9039 \pm 0.0897	0.8365 \pm 0.0125	0.9708 \pm 0.0244	0.8963 \pm 0.0270	0.9968 \pm 0.0071
	Orig	0.9882 \pm 0.0035	0.9287 \pm 0.0029	0.7805 \pm 0.0052	0.8302 \pm 0.0030	0.9508 \pm 0.0019	0.8649 \pm 0.0039	0.7549 \pm 0.0368
	RUS	0.9880 \pm 0.0029	0.9278 \pm 0.0029	0.7765 \pm 0.0051	0.8290 \pm 0.0030	0.9456 \pm 0.0029	0.8623 \pm 0.0035	0.9812 \pm 0.0104
	SMOTE	0.9881 \pm 0.0032	0.9250 \pm 0.0030	0.7544 \pm 0.0083	0.8266 \pm 0.0031	0.9437 \pm 0.0031	0.8313 \pm 0.0038	0.9682 \pm 0.0166

Performance. Tables 2 and 3 report the results in terms of AUC and F1-measure. The best gain is highlighted in **bold** while the second best is underlined. Generally, across all settings, EDEL demon-

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Table 3: Results in terms of F1-measure.

Clf	Mth	SBD	AID	TCD	CDH	BMD	GMSC	CCFD
DT	Orig	0.8819 \pm 0.0088	0.6171 \pm 0.0043	0.3999 \pm 0.0104	0.3072 \pm 0.0047	0.5155 \pm 0.0119	0.2679 \pm 0.0082	0.7741 \pm 0.0272
	RUS	0.8834 \pm 0.0171	0.6173 \pm 0.0056	0.4208 \pm 0.0043	0.3436 \pm 0.0023	0.5402 \pm 0.0071	0.2386 \pm 0.0028	0.0312 \pm 0.0043
	SMOTE	0.8914 \pm 0.0140	0.6242 \pm 0.0061	0.4048 \pm 0.0097	0.3074 \pm 0.0088	0.5233 \pm 0.0151	0.2482 \pm 0.0062	0.5405 \pm 0.0289
	MESA	0.9047 \pm 0.0262	0.6516 \pm 0.0099	0.4949 \pm 0.0099	0.4029 \pm 0.0034	0.6109 \pm 0.0078	0.3237 \pm 0.0111	0.5625 \pm 0.0375
	S-T-D	0.8673 \pm 0.0102	0.6451 \pm 0.0030	0.4070 \pm 0.0100	0.3373 \pm 0.0065	0.5686 \pm 0.0125	0.2780 \pm 0.0095	0.3996 \pm 0.0295
	CHRE	0.8712 \pm 0.0152	0.6292 \pm 0.0028	0.4384 \pm 0.0110	0.3650 \pm 0.0043	0.5761 \pm 0.0075	0.3140 \pm 0.0051	0.7419 \pm 0.0210
RF	EDEL	0.9709 \pm 0.0412	0.8215 \pm 0.1339	0.8096 \pm 0.2554	0.7584 \pm 0.2778	0.8375 \pm 0.2023	0.7690 \pm 0.2987	0.9182 \pm 0.1289
	Orig	0.9406 \pm 0.0095	0.6669 \pm 0.0045	0.4711 \pm 0.0086	0.2561 \pm 0.0046	0.5781 \pm 0.0086	0.2796 \pm 0.0085	0.8588 \pm 0.0186
	RUS	0.9425 \pm 0.0078	0.6725 \pm 0.0028	0.5212 \pm 0.0073	0.4249 \pm 0.0013	0.5934 \pm 0.0065	0.3241 \pm 0.0026	0.1157 \pm 0.0190
	SMOTE	0.9415 \pm 0.0055	0.6736 \pm 0.0040	0.4959 \pm 0.0092	0.2733 \pm 0.0050	0.6120 \pm 0.0139	0.3524 \pm 0.0059	0.8565 \pm 0.0272
	MESA	0.9380 \pm 0.0060	0.6843 \pm 0.0095	0.5353 \pm 0.0118	0.4156 \pm 0.0115	0.6440 \pm 0.0074	0.3629 \pm 0.0160	0.8243 \pm 0.0311
	S-T-D	0.9189 \pm 0.0101	0.6864 \pm 0.0025	0.4886 \pm 0.0116	0.3775 \pm 0.0035	0.6381 \pm 0.0076	0.3883 \pm 0.0072	0.8405 \pm 0.0273
XGB	CHRE	0.9244 \pm 0.0084	0.6720 \pm 0.0029	0.5361 \pm 0.0130	0.4501 \pm 0.0032	0.6379 \pm 0.0116	0.4296 \pm 0.0112	0.8414 \pm 0.0344
	EDEL	0.9824 \pm 0.0209	0.8192 \pm 0.1143	0.7448 \pm 0.2229	0.6829 \pm 0.1925	0.7145 \pm 0.1194	0.5929 \pm 0.1588	0.9370 \pm 0.0780
	Orig	0.9379 \pm 0.0094	0.7122 \pm 0.0066	0.4673 \pm 0.0157	0.2610 \pm 0.0030	0.5904 \pm 0.0159	0.2884 \pm 0.0128	0.8749 \pm 0.0298
	RUS	0.9396 \pm 0.0080	0.7056 \pm 0.0043	0.5047 \pm 0.0096	0.4362 \pm 0.0025	0.5944 \pm 0.0051	0.3217 \pm 0.0030	0.0798 \pm 0.0090
	SMOTE	0.9388 \pm 0.0112	0.7158 \pm 0.0052	0.4747 \pm 0.0090	0.2832 \pm 0.0041	0.6112 \pm 0.0123	0.3448 \pm 0.0026	0.8364 \pm 0.0267
	MESA	0.9428 \pm 0.0063	0.7201 \pm 0.0082	0.5194 \pm 0.0118	0.4738 \pm 0.0150	0.6420 \pm 0.0078	0.3273 \pm 0.0293	0.8145 \pm 0.0488
LGBM	S-T-D	0.9197 \pm 0.0089	0.7169 \pm 0.0037	0.4605 \pm 0.0117	0.3686 \pm 0.0053	0.6323 \pm 0.0097	0.3874 \pm 0.0045	0.7803 \pm 0.0286
	CHRE	0.9293 \pm 0.0103	0.6524 \pm 0.0047	0.4819 \pm 0.0093	0.4079 \pm 0.0039	0.6020 \pm 0.0073	0.3661 \pm 0.0053	0.8295 \pm 0.0275
	EDEL	0.9796 \pm 0.0282	0.7378 \pm 0.0316	0.6855 \pm 0.1595	0.4508 \pm 0.0296	0.7333 \pm 0.1409	0.4821 \pm 0.0827	0.9446 \pm 0.0628
	Orig	0.9447 \pm 0.0081	0.7124 \pm 0.0049	0.4770 \pm 0.0088	0.2506 \pm 0.0029	0.6074 \pm 0.0096	0.2895 \pm 0.0138	0.3834 \pm 0.1507
	RUS	0.9417 \pm 0.0062	0.7110 \pm 0.0049	0.5265 \pm 0.0117	0.4412 \pm 0.0026	0.5990 \pm 0.0058	0.3323 \pm 0.0060	0.0897 \pm 0.0116
	SMOTE	0.9441 \pm 0.0064	0.7162 \pm 0.0053	0.5031 \pm 0.0077	0.2977 \pm 0.0035	0.6322 \pm 0.0131	0.3613 \pm 0.0015	0.7310 \pm 0.0321
400	MESA	0.9427 \pm 0.0057	0.7211 \pm 0.0086	0.5236 \pm 0.0116	0.4371 \pm 0.0096	0.6420 \pm 0.0290	0.3515 \pm 0.0214	0.8360 \pm 0.0396
	S-T-D	0.9212 \pm 0.0113	0.7206 \pm 0.0052	0.4789 \pm 0.0091	0.3820 \pm 0.0051	0.6391 \pm 0.0038	0.4019 \pm 0.0048	0.7117 \pm 0.0342
	CHRE	0.9390 \pm 0.0055	0.6477 \pm 0.0102	0.4720 \pm 0.0064	0.4340 \pm 0.0076	0.6028 \pm 0.0132	0.3646 \pm 0.0071	0.1877 \pm 0.0964
	EDEL	0.9799 \pm 0.0280	0.7282 \pm 0.0201	0.5627 \pm 0.0769	0.4039 \pm 0.0181	0.6904 \pm 0.1018	0.3988 \pm 0.0228	0.9366 \pm 0.0753

strates consistent and often substantial improvements in both AUC and F1-measure, highlighting its robustness in imbalanced scenarios. Specifically, we have the following observations.

(O1). On low to moderate imbalance datasets such as SBD (IR=1.54) and AID (IR=3.18), EDEL achieves near-perfect performance with RF that AUC up to 0.9972, F1=0.9824 on SBD, and consistently outperforms all baselines across both RF and DT classifiers.

(O2). For medium imbalance datasets including TCD (IR=3.52) and CDH (IR=6.18), EDEL delivers significant gains. Notably, on CDH, EDEL achieves an AUC of 0.9506 and 0.6829 F1 score with RF, substantially higher than Orig (AUC=0.7974, F1=0.2561).

(O3). On high imbalance datasets, BMD (IR=7.88) and GMSC (IR=13.96), EDEL exhibits strong resilience. For instance, on GMSC, DT with EDEL achieves 0.7690 F1 score compared to 0.2679 with the basic method (orig), evidencing its ability to recover minority information effectively.

(O4). The CCFD, with IR=577.88, is an extremely imbalanced scenario. EDEL remains robust and yields the superior performance on both AUC and F1 score across all classifier-dataset combinations.

Overall, the above observations confirm that EDEL consistently performs superiorly across varying imbalance ratios and classifier backbones, with notable improvements in terms of F1, validating its effectiveness in capturing minority-class signals without sacrificing overall discrimination power.

Theoretical Consistency Validation. The experimental results further validate the theoretical foundation of EDEL in addressing imbalanced classification tasks. On the extremely imbalanced CCFD dataset (IR=577.88), EDEL’s error-driven update mechanism exhibits strong adaptability by dynamically re-injecting misclassified instances, which enables the model to continuously refine its decision boundary and improve minority class recognition as the number of weak classifiers increases. This progressive refinement directly reflects the convergence properties established in Section 3, where McDiarmid’s inequality guarantees iterative error reduction with high probability. In practice, we observe that EDEL steadily improves performance up to a certain ensemble size, after which the gains diminish. For example, the marginal improvements between 4 and 5 weak classifiers confirm the theoretical prediction of diminishing returns, as the empirical error approaches its asymptotic bound and convergence stabilizes (details in Appendix B.3 and Appendix D.1). Importantly, these observations not only highlight EDEL’s robustness on highly skewed data distributions but also demonstrate its alignment with the theoretical analysis, reinforcing confidence in its broad applicability to real-world imbalanced learning problems.

Feature Distribution Analysis. We present Table 4 to analyze the distribution shifts of features V1–V10 in GMSC, i.e., features change in $\mathcal{D}_i^{(t-1)}$ and $(\mathcal{D}_i^{(t)})$. The stratified sampling in Section 3.2

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Table 4: Feature distribution shifts in EDEL’s training subsets by class (GMSC, 5-fold CV).

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Feature	$\mathcal{D}_i^{(t-1),1}$ Mean	$\mathcal{D}_i^{t,1}$ Mean	% Change (minority)	$\mathcal{D}_i^{(t-1),0}$ Mean	$\mathcal{D}_i^{t,0}$ Mean	% Change (majority)	KS Statistic (p-value)
V1	4.37	3.82	-12.4	6.17	7.42	20.2	0.106 ($< 10^{-236}$)
V2	45.93	46.34	0.9	52.75	51.65	-2.1	0.049 ($< 10^{-46}$)
V3	2.39	1.81	-24.3	0.28	0.48	70.8	0.075 ($< 10^{-120}$)
V4	295.12	296.13	0.3	357.15	359.26	0.6	0.017 ($< 10^{-5}$)
V5	5525.81	5621.75	1.7	6397.43	6362.33	-0.5	0.028 ($< 10^{-13}$)
V6	7.88	8.11	2.9	8.49	8.49	0.0	0.020 ($< 10^{-8}$)
V7	2.09	1.48	-29.4	0.14	0.29	116.8	0.052 ($< 10^{-53}$)
V8	0.99	1.01	2.6	1.02	1.03	0.8	0.020 ($< 10^{-8}$)
V9	1.83	1.30	-29.1	0.13	0.26	107.0	0.041 ($< 10^{-36}$)
V10	0.93	0.91	-2.1	0.72	0.77	6.1	0.022 ($< 10^{-7}$)

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Table 5: Analysis of five representative samples from the GMSC dataset.

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ID	label	DT	RF	XGB	LGB	EDEL	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	Type
12057	0	0	0	0	0	0	0.0018	77	0	0.0000	5000	4	0	0	0	0	Easy
26462	1	1	1	1	1	1	0.9797	43	2	0.5431	2700	4	1	2	3	0	Easy
29011	1	0	0	0	0	1	0.3842	49	1	0.7734	1760	8	0	1	0	4	Hard
68219	1	0	0	0	0	1	0.9910	23	0	0.2045	1500	2	0	0	0	0	Hard
35849	0	1	1	1	1	0	1.0180	46	4	0.4021	5500	6	0	1	1	0	Hard

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ensures representative subsets ($p_{i,k} \approx p_k$). For minority, dynamic enhancement reduces V1 (-12.4%, from high to moderate utilization), V3 (-24.3%, fewer 30–59 day overdue), V7 (-29.4%, fewer 90+ day overdue), and V9 (-29.1%, fewer 60–89 day overdue), while increasing V6 (+2.9%, more open loans), enhancing minority class signals. For majority, V1 increases (+20.2%, higher utilization), V3 (+70.8%), and V7 (+116.8%) increase, reflecting inclusion of hard-to-classify majority samples. The average KS statistic (0.043, all $p < 10^{-5}$) confirms significant distribution shifts, validating the Bayes’ theorem prediction of minority class enrichment in \mathcal{D}_i^h . (See Appendix B.1).

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Case Study. To investigate the impact of class imbalance on classification performance and to further validate the interpretability of EDEL, we provide case-level analyses in Table 5, covering both *easy-to-classify* and *hard-to-classify* instances from GMSC (IR=13.96). The corresponding feature details (v1-v10) are shown in Table 6, which lists each variable, its description, and its preferences with respect to delinquency risk. A downward arrow (\downarrow) indicates that lower values are preferable, an upward arrow (\uparrow) indicates that higher values are preferable, and a dash (–) denotes neutral influence. These financial indicators, such as delinquency counts (V3, V7, V9), credit utilization (V1), and income level (V5), are critical for understanding classification challenges in imbalanced settings. The results in Table 5 illustrate the capacity of EDEL on addressing *hard-to-classify* samples. For ambiguous minority cases (29011, 68219), baselines misclassify them as majority due to weak or absent minority-indicative features (lower v3, v7, v9). In contrast, EDEL leverages the reinjection of misclassified instances (a.k.a \mathcal{D}^h), allowing minority cases (35849) that other models misclassify as minority. Overall, EDEL corrects almost 75.39% of baseline minority hard errors, yielding a $2.87 \times$ improvement in F1 (0.7690 vs. 0.2679, cf. Table 3), aligning with our theoretical foundations.

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Table 6: Feature descriptions for the GMSC dataset.

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Feature ID	Feature	Description	Preference
–	SeriousDlqin2yrs (label)	Borrower experienced 90+ days past due delinquency	–
V1	RevolvingUtilizationOfUnsecuredLines	Balance on credit cards and personal lines of credit (excluding real estate and installment debt) divided by credit limits	\downarrow
V2	Age	Borrower’s age in years	–
V3	NumberOfTime30-59DaysPastDueNotWorse	Times borrower was 30–59 days past due in the last 2 years	\downarrow
V4	DebtRatio	Monthly debt payments, alimony, and living costs divided by monthly gross income	\downarrow
V5	MonthlyIncome	Monthly income	\uparrow
V6	NumberOfOpenCreditLinesAndLoans	Number of open loans and lines of credit	–
V7	NumberOfTimes90DaysLate	Times borrower was 90+ days past due	\downarrow
V8	NumberOfRealEstateLoansOrLines	Number of mortgage and real estate loans	–
V9	NumberOfTime60-89DaysPastDueNotWorse	Times borrower was 60–89 days past due in the last 2 years	\downarrow
V10	NumberOfDependents	Number of dependents, excluding the borrower	\downarrow

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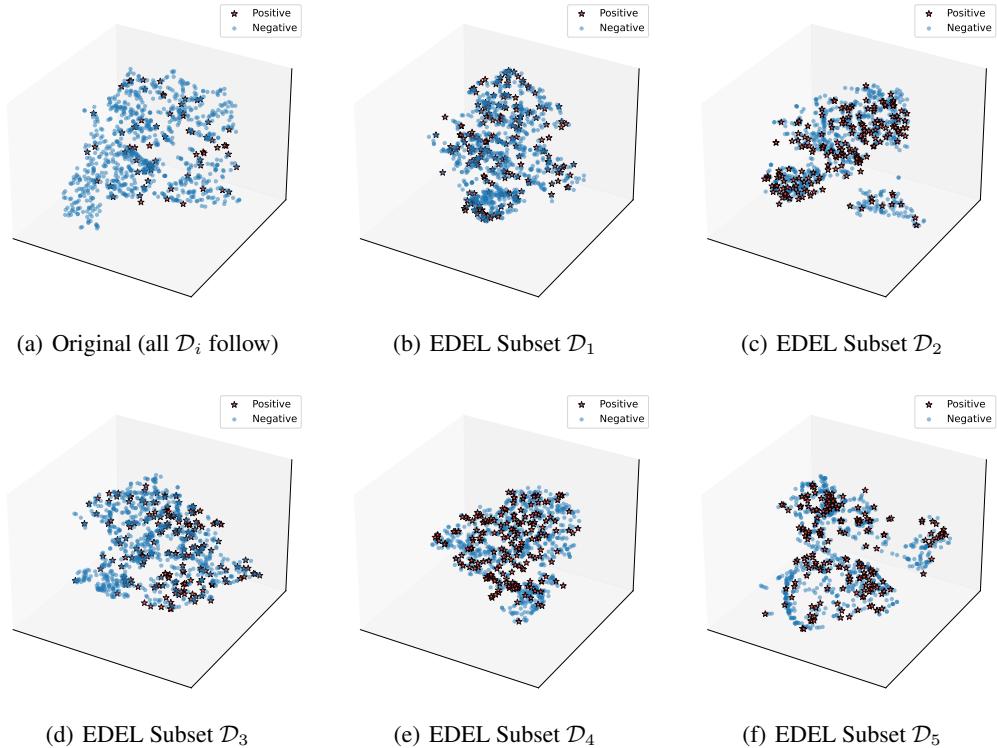
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Visualization Figure 2 visualizes the original training set \mathcal{D} with corresponding five subsets produced after one EDEL update under stratified sampling (size=1000) on GMSC. In subfigure (a), positive samples are sparse and dispersed and heavily intermixed with dense negative clusters, echoing the imbalance-induced boundary bias we discussed in Section 1. In contrast, subfigures (b)–(f)

486 show clear minority enrichment: positives become denser and more coherent, and boundary overlap
 487 is reduced, which matches our theoretical analysis in Section 3 and Appendix B.1. Moreover, the
 488 five subsets illustrate subtle distributional differences, reflecting the multi-view partitioning stra-
 489 t-
 490 tegy and contributing to ensemble diversity, further enabling inspection of iterative refinements on
 491 *hard-to-classify* samples to foster trust in high-stakes applications.



518 Figure 2: Visualization of data reinjection in EDEL.
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520 Due to space limitation, additional discussions and related works are listed in [Appendix D](#) and [E](#).
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524 5 CONCLUSION

526 This work introduces EDEL, a novel algorithm developed to address the challenge of class imbal-
 527 ance in machine learning. By dynamically adjusting the learning process using misclassified sam-
 528 ples and incorporating a multi-perspective learning approach, EDEL enhances the model’s ability
 529 to recognize minority class instances. Extensive experiments on seven real-world datasets, includ-
 530 ing domains such as financial fraud detection and credit risk assessment, demonstrate that EDEL
 531 significantly improves key metrics, such as AUC and F1-measure, particularly under extreme class
 532 imbalance. These findings validate the robustness and adaptability of EDEL in managing both mod-
 533 erate and severe imbalances. Moreover, EDEL’s adaptive nature makes it effective across a range of
 534 classifiers, consistently improving performance.

535 However, while EDEL significantly improves minority class recognition, particularly in high-risk
 536 domains, it introduces higher computational complexity, posing challenges for large-scale datasets.
 537 To address this, parallel and distributed computing methods could be considered. Overall, EDEL
 538 provides a robust and adaptable solution to the class imbalance problem, with considerable potential
 539 for future enhancements and broader real-world applications, particularly in fields where minority
 class recognition is critical.

540 REPRODUCIBILITY STATEMENT
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542 To ensure the reproducibility of our results, we provide the complete implementation of the
543 EDEL algorithm and all experimental details. The source code is publicly available at <https://anonymous.4open.science/r/EDEL-1A3E/Readme.md>, including data preprocessing
544 pipelines, algorithm codes, and experimental scripts. All experiments are conducted on seven
545 publicly available benchmark datasets, with detailed URLs provided in [Appendix C](#). We use 5-fold
546 stratified cross-validation with fixed random seeds 42 and report detailed hyperparameters for all
547 baseline methods and classifiers.

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APPENDIX

A THE USE OF LARGE LANGUAGE MODELS (LLMs)

In the preparation of this manuscript, large language models (LLMs) were utilized to assist with language polishing and grammar checking. Specifically, an LLM was employed to refine the text for improved clarity, coherence, and grammatical accuracy. This ensured that the final version of the article maintains high linguistic standards while preserving the original ideas and content generated by the authors. The use of LLMs was limited to these supportive tasks, and no substantial content was generated by the models themselves.

B THEORETICAL ANALYSIS

This section extends the data representativeness and classifier performance analysis in Section 3.2, providing detailed derivations for minority class enrichment using Bayes’ theorem.

Misclassified Sample Aspect. Weak classifiers trained on imbalanced data are prone to exhibiting biases toward the majority class, resulting in a higher misclassification rate for minority class samples. To quantify the proportion of minority class samples within \mathcal{D}_i^h (cf Eq.(4)), we derive the conditional probability using Bayes’ theorem:

$$P(\mathbf{y} = 1 \mid (\mathbf{x}, \mathbf{y}) \in \mathcal{D}_i^h) = \frac{P((\mathbf{x}, \mathbf{y}) \in \mathcal{D}_i^h \mid \mathbf{y} = 1)P(\mathbf{y} = 1)}{P((\mathbf{x}, \mathbf{y}) \in \mathcal{D}_i^h)}. \quad (18)$$

Herein, the numerator represents misclassification samples, i.e., $\epsilon_i = P(\pi_i(\mathbf{x}) \neq \mathbf{y})$. Using the law of total probability, ϵ_i can be expressed as:

$$\epsilon_i = P(\pi_i(\mathbf{x}) \neq \mathbf{y} \mid \mathbf{y} = 0)P(\mathbf{y} = 0) + P(\pi_i(\mathbf{x}) \neq \mathbf{y} \mid \mathbf{y} = 1)P(\mathbf{y} = 1), \quad (19)$$

where $P(\mathbf{y} = 0) = 1 - \rho$ and $P(\mathbf{y} = 1) = \rho$ (with $\rho \ll 0.5$ indicating class imbalance). Substituting the misclassification rates $\epsilon_{i,0} = P(\pi_i(\mathbf{x}) \neq 0 \mid \mathbf{y} = 0)$ and $\epsilon_{i,1}$, we obtain:

$$\epsilon_i = \epsilon_{i,0}(1 - \rho) + \epsilon_{i,1}\rho. \quad (20)$$

Combining the numerator and denominator, the proportion of minority class samples in \mathcal{D}_i^h is:

$$P(\mathbf{y} = 1 \mid (\mathbf{x}, \mathbf{y}) \in \mathcal{D}_i^h) = \frac{\epsilon_{i,1}\rho}{\epsilon_{i,0}(1 - \rho) + \epsilon_{i,1}\rho}. \quad (21)$$

To establish that the minority class is overrepresented in the misclassified set compared to the original dataset (i.e., $P(\mathbf{y} = 1 \mid (\mathbf{x}, \mathbf{y}) \in \mathcal{D}_i^h) > \rho$), we proceed with the following inequality:

$$\frac{\epsilon_{i,1}\rho}{\epsilon_{i,0}(1 - \rho) + \epsilon_{i,1}\rho} > \rho. \quad (22)$$

Inequality Transformation. Multiply both sides by the positive denominator $\epsilon_{i,0}(1 - \rho) + \epsilon_{i,1}\rho$:

$$\epsilon_{i,1}\rho > \rho [\epsilon_{i,0}(1 - \rho) + \epsilon_{i,1}\rho]. \quad (23)$$

Division by ρ : Since $\rho > 0$ (as it is a probability), divide both sides by ρ :

$$\epsilon_{i,1} > \epsilon_{i,0}(1 - \rho) + \epsilon_{i,1}\rho. \quad (24)$$

Rearrangement: Move terms involving $\epsilon_{i,1}$ to one side:

$$\epsilon_{i,1} - \epsilon_{i,1}\rho > \epsilon_{i,0}(1 - \rho). \quad (25)$$

Factorization: Factor out $\epsilon_{i,1}$ on the left-hand side:

$$\epsilon_{i,1}(1 - \rho) > \epsilon_{i,0}(1 - \rho). \quad (26)$$

Division by $1 - \rho$: Since $1 - \rho > 0$ (as $\rho < 0.5$), divide both sides by $1 - \rho$:

$$\epsilon_{i,1} > \epsilon_{i,0}. \quad (27)$$

This inequality holds because, in imbalanced datasets where $\rho \ll 0.5$, weak classifiers $\pi_i^{(t+1)}$ trained on $\mathcal{D}_i^{(t)}$ are biased toward the majority class due to the scarcity of minority class samples. This bias results in a higher misclassification rate for the minority class $\epsilon_{i,1}$ compared to the majority class $\epsilon_{i,0}$. Thus, $P(y = 1 \mid (\mathbf{x}, y) \in \mathcal{D}_i^h) > \rho$, confirming that the misclassified sample set \mathcal{D}_i^h contains a disproportionately high number of minority class instances compared to the original dataset.

B.1 ENHANCED MECHANISM EFFICACY

The dynamic enhancement mechanism augments the training subset $\mathcal{D}_i^{(t)}$ with the misclassified sample set \mathcal{D}_i^h , forming the updated training subset $\mathcal{D}_i^{(t)} = \mathcal{D}_i^{(t-1)} \cup \mathcal{D}_i^h$. To rigorously demonstrate the efficacy of this mechanism, we analyze the change in the minority class proportion within the training subset.

Initial Setup: Let the size of the original training subset be $|\mathcal{D}_i^{(t-1)}| = n$, with the number of minority class samples given by $n \cdot \rho$, where $\rho = P(y = 1)$ is the minority class proportion in $\mathcal{D}_i^{(t-1)}$. The size of the misclassified sample set is $|\mathcal{D}_i^h| = n_h$, and the number of minority class samples in \mathcal{D}_i^h is $n_h \cdot P(y = 1 \mid (\mathbf{x}, y) \in \mathcal{D}_i^h)$.

New Minority Class Proportion: The updated minority class proportion ρ' in $\mathcal{D}_i^{(t)}$ is:

$$\rho' = \frac{n \cdot \rho + n_h \cdot P(y = 1 \mid (\mathbf{x}, y) \in \mathcal{D}_i^h)}{n + n_h}. \quad (28)$$

Inequality Derivation: From the previous subsection, we established that $P(y = 1 \mid (\mathbf{x}, y) \in \mathcal{D}_i^h) > \rho$. Let $P(y = 1 \mid (\mathbf{x}, y) \in \mathcal{D}_i^h) = \rho + \delta$, where $\delta > 0$ represents the excess proportion of minority class samples in \mathcal{D}_i^h . Substitute this into the expression for ρ' :

$$\rho' = \frac{n \cdot \rho + n_h \cdot (\rho + \delta)}{n + n_h}. \quad (29)$$

Numerator Expansion: Expand the numerator:

$$n \cdot \rho + n_h \cdot (\rho + \delta) = n \cdot \rho + n_h \cdot \rho + n_h \cdot \delta = (n + n_h) \cdot \rho + n_h \cdot \delta. \quad (30)$$

Division by Denominator: Divide each term by the denominator $n + n_h$:

$$\rho' = \frac{(n + n_h) \cdot \rho}{n + n_h} + \frac{n_h \cdot \delta}{n + n_h} = \rho + \frac{n_h \cdot \delta}{n + n_h}. \quad (31)$$

Conclusion of Inequality: Since $n_h > 0$ (as \mathcal{D}_i^h contains misclassified samples) and $\delta > 0$ (due to the higher minority class proportion in \mathcal{D}_i^h), the term $\frac{n_h \cdot \delta}{n + n_h} > 0$. Therefore:

$$\rho' > \rho. \quad (32)$$

This increase in the minority class proportion ρ' indicates that the updated training subset $\mathcal{D}_i^{(t)}$ is more balanced than the original $\mathcal{D}_i^{(t-1)}$.

Error Rate Improvement: The retrained classifier h_i' on $\mathcal{D}_i^{(t)}$ benefits from the increased representation of minority class samples. The training error rate for the minority class, $\epsilon_{i,1}' = P(\pi_i^{(t+1)}(\mathbf{x}) \neq 1 \mid y = 1)$, is influenced by the loss function, which now assigns greater weight to minority class misclassifications due to their higher proportion. According to statistical learning theory, an increase in the number of training samples for a given class reduces the variance of the classifier's predictions for that class. Let the empirical risk for h_i' be:

$$\hat{R}(h_i') = \frac{1}{|\mathcal{D}_i^{(t)}|} \sum_{(\mathbf{x}, y) \in \mathcal{D}_i^{(t)}} \mathbb{I}(\pi_i^{t+1}(\mathbf{x}) \neq y). \quad (33)$$

810 The generalization error bound, based on the increased sample size and balanced distribution, is:
 811

$$812 \quad 813 \quad 814 \quad \epsilon'_{i,1} \leq \hat{\epsilon}'_{i,1} + \mathcal{O} \left(\sqrt{\frac{\ln(1/\delta)}{|\mathcal{D}_i^{(t)}|}} \right), \quad (34)$$

815 where $\hat{\epsilon}'_{i,1}$ is the empirical error rate for the minority class. As $|\mathcal{D}_i^{(t)}|$ increases and ρ' approaches a
 816 more balanced value, $\hat{\epsilon}'_{i,1}$ decreases, leading to $\epsilon'_{i,1} < \epsilon_{i,1}$.
 817

818 B.2 SPECIAL SCENARIO SUPERIORITY

820 In scenarios of extreme class imbalance (e.g., $\rho \ll 0.01$), the decision boundary of a classifier
 821 trained on $\mathcal{D}_i^{(t-1)}$ tends to shift toward the minority class to minimize the overall error rate, which is
 822 dominated by the majority class. This shift causes even "easy" minority class samples—those with
 823 distinct features and far from the decision boundary in a balanced setting—to be misclassified as
 824 majority class samples. These misclassified samples are identified as "hard-to-classify" due to the
 825 imbalance-induced bias.

826 The dynamic enhancement mechanism addresses this issue by incorporating \mathcal{D}_i^h into $\mathcal{D}_i^{(t)}$. Since \mathcal{D}_i^h
 827 is enriched with minority class samples (as $P(y = 1 | (\mathbf{x}, y) \in \mathcal{D}_i^h) > \rho$), the retrained classifier h'_i
 828 on $\mathcal{D}_i^{(t)}$ experiences an improved minority class representation. This adjustment allows h'_i to better
 829 capture the minority class's feature distribution. Mathematically, the decision boundary shift can be
 830 modeled in a linear classifier context, where the decision function is $f(\mathbf{x}) = \mathbf{w}^T \mathbf{x} + b$. In extreme
 831 imbalance, \mathbf{w} and b are adjusted to minimize:
 832

$$833 \quad 834 \quad 835 \quad \frac{1}{M} \sum_{i=1}^M \mathbb{I}(f(\mathbf{x}_i) y_i < 0), \quad (35)$$

836 favoring the majority class. The inclusion of \mathcal{D}_i^h increases the penalty for misclassifying minority
 837 samples, shifting the boundary back. Consequently, the minority class error rate $\epsilon'_{i,1}$ decreases,
 838 and the misclassification of "easy" minority samples is mitigated, demonstrating the superiority of
 839 EDEL in extreme imbalance scenarios.
 840

841 B.3 INTEGRATED LEARNING AND OVERALL PERFORMANCE ENHANCEMENT

843 Building on the convergence analysis of individual weak classifiers in Section 3.2, this section ana-
 844 lyzes the integrated classifier's performance and overall error reduction.

845 This section demonstrates how the EDEL algorithm integrates multiple weak classifiers to enhance
 846 overall performance, leveraging classical ensemble learning theories. The integration process com-
 847 bines the outputs of dynamically enhanced weak classifiers to form a robust final classifier, while
 848 the iterative refinement of average empirical error ensures convergence to an optimal performance
 849 level.
 850

851 B.3.1 INTEGRATED MECHANISM FOR PERFORMANCE BOOST

853 The final classifier in the EDEL algorithm is defined as $H(\mathbf{x}) = \text{sign} \left(\sum_{i=1}^N \alpha_i h'_i(\mathbf{x}) \right)$, where each
 854 h'_i is a weak classifier trained on the dynamically enhanced subset $\mathcal{D}_i^{(t)} = \mathcal{D}_i^{(t-1)} \cup \mathcal{D}_i^h$ (as specified
 855 in the pipeline), and α_i represents the weight assigned to the i -th weak classifier. The weight α_i
 856 is determined based on the classifier's performance, specifically using the formula $\alpha_i = \frac{1}{2} \ln \frac{1-\epsilon_i}{\epsilon_i}$,
 857 where $\epsilon_i = P(h'_i(\mathbf{x}) \neq y)$ is the error rate of h'_i over the true distribution \mathcal{D} . The error rate ϵ_i
 858 is ensured to be less than 0.5 due to the dynamic enhancement mechanism, which increases the
 859 minority class proportion in $\mathcal{D}_i^{(t)}$, thereby improving the classifier's ability to handle imbalanced
 860 data.
 861

862 To quantify the performance boost, we derive the upper bound on the final classifier's error rate
 863 $\epsilon_{\text{final}} = P(H(\mathbf{x}) \neq y)$ using classical AdaBoost theory (Freund & Schapire, 1997a). The error rate
 864 of the integrated classifier can be bounded as follows:

864
 865
 866

$$\epsilon_{\text{final}} = P \left(\text{sign} \left(\sum_{i=1}^N \alpha_i h'_i(\mathbf{x}) \right) \neq y \right). \quad (36)$$

 867
 868

869 Consider the weighted margin $S(\mathbf{x}) = \sum_{i=1}^N \alpha_i h'_i(\mathbf{x})$. The classification error occurs when $y \cdot S(\mathbf{x}) \leq 0$. The expected error can be analyzed using the exponential loss framework, where the
 870 upper bound is given by:
 871

872
 873

$$\epsilon_{\text{final}} \leq \prod_{i=1}^N \sqrt{4\epsilon_i(1-\epsilon_i)}. \quad (37)$$

 874
 875

876 To derive this bound, note that for each weak classifier h'_i , the weighted contribution to the margin
 877 is influenced by its error rate ϵ_i . The probability of correct classification by h'_i is $1 - \epsilon_i$, and the
 878 AdaBoost weighting scheme ensures that the margin grows with the number of classifiers. The
 879 term $\sqrt{4\epsilon_i(1-\epsilon_i)}$ arises from the analysis of the exponential loss $\exp(-y \cdot S(\mathbf{x}))$, where the error
 880 probability is bounded by the product of individual classifier contributions. Since $\epsilon_i < 0.5$, the
 881 maximum value of $4\epsilon_i(1-\epsilon_i)$ occurs at $\epsilon_i = 0.5$, yielding 1, and for $\epsilon_i < 0.5$, $4\epsilon_i(1-\epsilon_i) < 1$.
 882 Thus:

883
 884
$$\sqrt{4\epsilon_i(1-\epsilon_i)} < 1. \quad (38)$$

 885

886 As N increases, the product $\prod_{i=1}^N \sqrt{4\epsilon_i(1-\epsilon_i)}$ decreases exponentially because each factor is less
 887 than 1. This exponential decay demonstrates that the error rate ϵ_{final} diminishes with the number of
 888 weak classifiers, proving a significant performance boost through integration.

889 The diversity among weak classifiers h'_i further enhances this improvement. This diversity stems
 890 from the stratified sampling of $\mathcal{D}_i^{(t-1)}$ and the dynamic enhancement with \mathcal{D}_i^h , which introduces
 891 variations in the training data across different classifiers. The reduced correlation among h'_i ensures
 892 that their errors are not perfectly aligned, leading to a more robust ensemble. This diversity effect
 893 lowers the overall ϵ_{final} beyond the theoretical bound, reinforcing the efficacy of the integrated
 894 mechanism.

895
 896 **B.3.2 CONVERGENCE OF AVERAGE EMPIRICAL ERROR VIA McDIARMID INEQUALITY**

897 To ensure the iterative optimization process of EDEL converges to an optimal performance level, we
 898 analyze the convergence of the average empirical error using the McDiarmid inequality (McDiarmid,
 899 1989). The average empirical error at iteration t is defined as:

900
 901
$$\hat{E}_t = \frac{1}{n} \sum_{i=1}^n \hat{\epsilon}_i^{(t)}, \quad (39)$$

 902
 903

904 where n is the number of weak classifiers, and the empirical error for the i -th weak classifier is:

905
 906
$$\hat{\epsilon}_i^{(t+1)} = \frac{1}{|\mathcal{D}_i^{(t)}|} \sum_{(\mathbf{x}, y) \in \mathcal{D}_i^{(t)}} \mathbb{I}(\pi_i^{(t+1)}(\mathbf{x}) \neq y), \quad (40)$$

 907
 908

909 with $|\mathcal{D}_i^{(t)}|$ denoting the size of the training set $\mathcal{D}_i^{(t)}$ at iteration t . Our goal is to prove that the
 910 average empirical error in the next iteration \hat{E}_{t+1} is likely to decrease by at least $\Delta E > 0$, with a
 911 probability bound given by:
 912

913
 914
$$P(\hat{E}_{t+1} \leq \hat{E}_t - \Delta E) \geq 1 - 2 \exp(-2\Delta E^2 n). \quad (41)$$

 915

916 *Bounded Difference Condition:* We treat \hat{E}_{t+1} as a function of the training sets for all weak clas-
 917 sifiers, defined as $f(\mathcal{D}_1^{(t+1)}, \dots, \mathcal{D}_n^{(t+1)}) = \hat{E}_{t+1}$. To apply the McDiarmid inequality, we need to

918 verify the bounded difference condition. Consider changing the training set $\mathcal{D}_i^{(t+1)}$ to $\mathcal{D}'^{(t+1)}$ for a
 919 fixed i . This change affects only the empirical error $\hat{\epsilon}_i^{(t+1)}$ of the i -th weak classifier, while $\hat{\epsilon}_j^{(t+1)}$
 920 (for $j \neq i$) remains unchanged. The difference in the function value is:
 921

$$923 |f(\mathcal{D}_1^{(t+1)}, \dots, \mathcal{D}_i^{(t+1)}, \dots, \mathcal{D}_n^{(t+1)}) - f(\mathcal{D}_1^{(t+1)}, \dots, \mathcal{D}'_i^{(t+1)}, \dots, \mathcal{D}_n^{(t+1)})| = \frac{1}{n} |\hat{\epsilon}_i^{(t+1)} - \hat{\epsilon}'_i^{(t+1)}|. \quad (42)$$

926 Since $\hat{\epsilon}_i^{(t+1)}$ is a misclassification rate ranging from 0 to 1, the maximum possible change when
 927 altering the entire training set $\mathcal{D}_i^{(t+1)}$ is from 0 to 1 (or vice versa). Thus:
 928

$$930 |\hat{\epsilon}_i^{(t+1)} - \hat{\epsilon}'_i^{(t+1)}| \leq 1, \quad (43)$$

932 This establishes the bounded difference constant $c_i = \frac{1}{n}$ for each i . The sum of squared bounded
 933 differences is:
 934

$$935 \sum_{i=1}^n c_i^2 = \sum_{i=1}^n \left(\frac{1}{n}\right)^2 = n \cdot \frac{1}{n^2} = \frac{1}{n}. \quad (44)$$

938 *McDiarmid Inequality Application:* The McDiarmid inequality (McDiarmid, 1989) states that for
 939 a function $Y = f(X_1, \dots, X_n)$ satisfying the bounded difference condition, the probability of
 940 deviating from its expectation is bounded by:
 941

$$942 P(|Y - \mathbb{E}[Y]| \geq t) \leq 2 \exp\left(-\frac{2t^2}{\sum_{i=1}^n c_i^2}\right). \quad (45)$$

945 Substituting the sum of squared bounded differences:
 946

$$948 P(|\hat{E}_{t+1} - \mathbb{E}[\hat{E}_{t+1}]| \geq t) \leq 2 \exp\left(-\frac{2t^2}{\frac{1}{n}}\right) = 2 \exp(-2t^2 n). \quad (46)$$

950 *Error Reduction Analysis:* To prove $\hat{E}_{t+1} \leq \hat{E}_t - \Delta E$, we assume that the expected average em-
 951 pirical error after the update satisfies $\mathbb{E}[\hat{E}_{t+1}] \leq \hat{E}_t - \Delta E$, where $\Delta E > 0$ represents the expected
 952 reduction in error due to the inclusion of misclassified samples \mathcal{D}_i^h . This assumption is based on the
 953 intuition that focusing on hard-to-classify samples improves the classifier's performance. We need
 954 to bound the probability that \hat{E}_{t+1} exceeds $\hat{E}_t - \Delta E$.
 955

956 Set $t = \Delta E$ in the inequality:
 957

$$958 P(\hat{E}_{t+1} - \mathbb{E}[\hat{E}_{t+1}] \geq \Delta E) \leq \exp(-2\Delta E^2 n). \quad (47)$$

960 This represents the probability that \hat{E}_{t+1} is ΔE above its expectation. Since $\mathbb{E}[\hat{E}_{t+1}] \leq \hat{E}_t - \Delta E$,
 961 the event $\hat{E}_{t+1} > \hat{E}_t - \Delta E$ occurs if $\hat{E}_{t+1} - \mathbb{E}[\hat{E}_{t+1}] > \hat{E}_t - \Delta E - \mathbb{E}[\hat{E}_{t+1}]$. Given $\hat{E}_t - \Delta E -$
 962 $\mathbb{E}[\hat{E}_{t+1}] \geq 0$, the maximum deviation is bounded by ΔE . Thus:
 963

$$964 P(\hat{E}_{t+1} > \hat{E}_t - \Delta E) \leq \exp(-2\Delta E^2 n). \quad (48)$$

966 For a conservative estimate, considering both tails of the distribution (as \hat{E}_{t+1} could deviate in either
 967 direction), the double-sided bound is:
 968

$$970 P(|\hat{E}_{t+1} - \mathbb{E}[\hat{E}_{t+1}]| \geq \Delta E) \leq 2 \exp(-2\Delta E^2 n). \quad (49)$$

971 Therefore, the probability that \hat{E}_{t+1} does not exceed $\hat{E}_t - \Delta E$ is:

972
973
974

$$P(\hat{E}_{t+1} \leq \hat{E}_t - \Delta E) \geq 1 - 2 \exp(-2\Delta E^2 n). \quad (50)$$

975 *Convergence Property:* As the number of weak classifiers n increases, the exponent $-2\Delta E^2 n$ becomes more negative, causing the probability bound $2 \exp(-2\Delta E^2 n)$ to approach 0. This implies
976 that $\hat{E}_{t+1} \leq \hat{E}_t - \Delta E$ holds with probability approaching 1. Over multiple iterations, the average
977 empirical error \hat{E}_t converges to a limiting value E_∞ , which represents the optimal average error
978 achievable given the data distribution and the capacity of the weak classifiers.
979

980 This convergence is driven by the iterative process of EDEL, where the inclusion of \mathcal{D}_i^h refines the
981 training sets $\mathcal{D}_i^{(t+1)}$, allowing the weak classifiers to better capture the underlying patterns, partic-
982 ularly for minority class instances. The McDiarmid inequality thus provides a statistical guarantee
983 that the algorithm's performance improves iteratively, enhancing the overall efficacy of the ensem-
984 ble.
985

986 C EXPERIMENTAL DETAILS

987 C.1 DATASET AND METRIC

990 **Dataset.** These datasets were chosen because they represent real-world scenarios where class im-
991 balance is prevalent and poses significant challenges for traditional machine learning models. By
992 evaluating our method on these diverse datasets, we ensure its generalizability and effectiveness in
993 handling various imbalance scenarios. Below are the details of each dataset:
994

- 995 • **Spambase Dataset (SBD)**¹: This dataset contains 4,601 samples used to classify emails as spam
996 (positive class) or non-spam. With an IR of 1.54, it represents a moderately imbalanced sce-
997 nario. Spam detection is critical in email filtering systems to reduce user inconvenience caused by
998 unwanted messages.
- 999 • **Adult Income Dataset (AID)**²: This dataset includes 48,842 samples to predict whether an in-
1000 dividual's income exceeds 50K per year (positive class). The IR is 3.18, indicating a moderate
1001 imbalance. This dataset is often used to study socioeconomic factors influencing income levels,
1002 where accurately identifying high-income individuals is key.
- 1003 • **Taiwan Credit Card Default (TCD)**³: This dataset has 30,000 samples to predict whether a
1004 credit card holder will default on payments (positive class). With an IR of 3.52, it reflects a
1005 typical credit risk assessment scenario where identifying potential defaulters is crucial for financial
1006 institutions.
- 1007 • **CDC Diabetes Health Indicators (CDH)**⁴: This dataset consists of 253,680 samples used to
1008 predict whether an individual has diabetes (positive class) based on health indicators. The IR is
1009 6.18, highlighting the challenge of diagnosing rare diseases in large populations.
- 1010 • **Bank Marketing Dataset (BMD)**⁵: This dataset includes 41,188 samples for predicting whether
1011 a client will subscribe to a term deposit (positive class). With an IR of 7.88, it represents a
1012 marketing campaign scenario where successful subscriptions are relatively rare.
- 1013 • **Give Me Some Credit Dataset (GMSC)**⁶: This dataset consists of 150,000 samples to predict
1014 whether an individual will experience financial distress in the next two years (positive class). The
1015 IR is 13.96, emphasizing the difficulty of predicting rare but high-impact events in credit risk
1016 modeling.
- 1017 • **Credit Card Fraud Detection (CCFD)**⁷: This dataset includes 284,807 samples for detecting
1018 whether a transaction is fraudulent (positive class). With an extreme IR of 577.88, it poses signif-

1¹<https://archive.ics.uci.edu/ml/datasets/spambase>

2²<https://archive.ics.uci.edu/ml/datasets/adult>

3³<https://archive.ics.uci.edu/ml/datasets/default+of+credit+card+clients>

4⁴<https://www.kaggle.com/datasets/alexteboul/diabetes-health-indicators-dataset/data>

5⁵<https://archive.ics.uci.edu/ml/datasets/Bank+Marketing>

6⁶<https://www.kaggle.com/c/GiveMeSomeCredit>

7⁷<https://www.kaggle.com/mlg-ulb/creditcardfraud>

1026 significant challenges for fraud detection systems, where missing a fraudulent transaction can lead to
1027 substantial losses.

1029 **Evaluation Metrics.** We employ AUC and F-measure as evaluation metrics to assess the performance
1030 of the EDEL algorithm in imbalanced classification tasks. These metrics are particularly
1031 suitable for high-stakes applications such as fraud detection and credit risk assessment, where class
1032 imbalance poses significant challenges.

1033 AUC is the area under the Receiver Operating Characteristic (ROC) curve, which effectively reflects
1034 the overall accuracy of the classifier in distinguishing between positive and negative classes across all
1035 possible thresholds. It ranges from 0 to 1, with 1 indicating perfect classification and 0.5 representing
1036 random guessing. The AUC is calculated based on the True Positive Rate (TPR) and False Positive
1037 Rate (FPR), defined as:

$$\text{TPR} = \frac{\text{TP}}{\text{TP} + \text{FN}}, \quad \text{FPR} = \frac{\text{FP}}{\text{FP} + \text{TN}},$$

1042 where TP (True Positives) is the number of correctly predicted positive instances, FN (False Neg-
1043 atives) is the number of positive instances incorrectly predicted as negative, FP (False Positives) is
1044 the number of negative instances incorrectly predicted as positive, and TN (True Negatives) is the
1045 number of correctly predicted negative instances.

1046 **F-measure** is obtained using the formula:

$$F\text{-measure} = \frac{(1 + \beta^2) \cdot \text{Recall} \cdot \text{Precision}}{\beta^2 \cdot \text{Precision} + \text{Recall}},$$

1051 where $\text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}}$, $\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}$, and β is a factor used to balance and weight the
 1052 importance of Recall and Precision. When $\beta > 1$, Recall has a greater impact; when $\beta < 1$,
 1053 Precision has a greater impact. In this study, we use the F1-measure, where β is set to 1, giving equal
 1054 weight to Precision and Recall. The F1-measure ranges from 0 to 1, with higher values indicating
 1055 better balance between identifying positive instances and avoiding false positives.

C.2 EXPERIMENTAL SETTING

Baselines. We evaluate EDEL against five baselines, including:

(1) **basic method**, the Original instances with no imbalance handling applied;

(2) data-level methods:

- SMOTE (Chawla et al., 2002), which generates synthetic minority samples via interpolation between existing minority instances.
- RandomUnderSampler (RUS) (He & Garcia, 2009), which randomly selects majority class instances to match the minority class size.
- SMOTE-TLNN-DEPSO (Dixit & Mani, 2023), which integrates Two-Layer Nearest Neighbor and Differential Evolution Particle Swarm Optimization to handle noisy data.

(3) ensemble learning methods:

- MESA (Liu et al., 2020), which employs a meta-sampler trained via soft actor-critic reinforcement learning to learn an adaptive under-sampling strategy; it iteratively computes error distributions on training and validation sets to form a meta-state, uses the meta-sampler to output a parameter μ for a Gaussian function that assigns sampling weights based on classification errors, and builds a cascade ensemble of classifiers on balanced subsets, optimizing for generalization performance; configured in our experiments with `random_state = 42`, while keeping hyperparameters (e.g., `metric = aucprc`, `max_estimators = 10`, `num_bins = 5`, $\sigma = 0.2$, `train_ir = 1`, `update_steps = 1000`, `start_steps = 500`, `hidden_size = 50`) at their default values.

1080
 1081
 1082
 1083
 1084
 1085 • CHRE (Zhao et al., 2025), a hybrid resampling method, synthesizes new minority samples
 1086 based on sample contribution using Euclidean distance to balance information and noise
 1087 levels, configured in our experiments with hyperparameters $\lambda_1 = 0.8$, $\lambda_2 = 0.5$, $K = 5$,
 1088 and a maximum of 10 iterations.

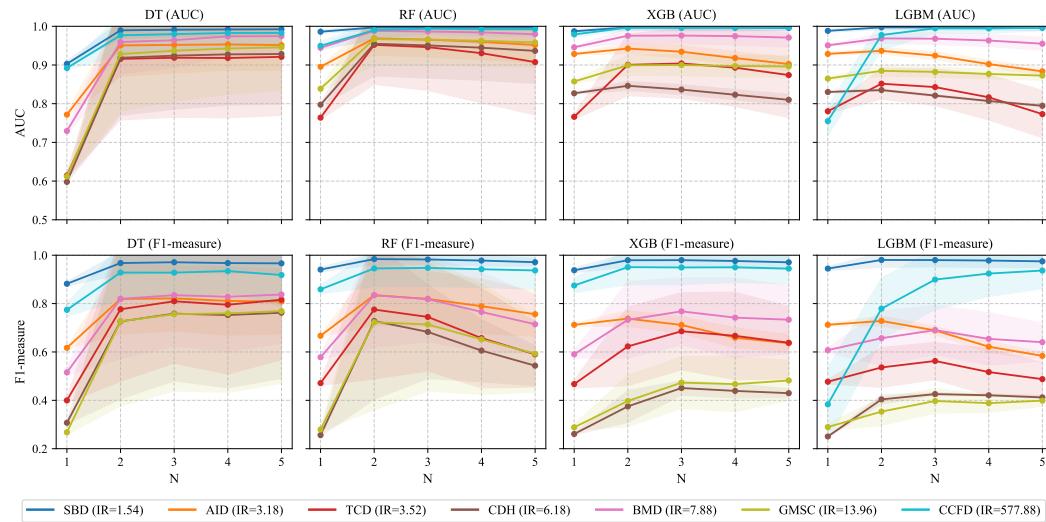
1085 All methods, including EDEL and the baselines, are evaluated using four classifiers: Decision Tree,
 1086 Random Forest, XGBoost, and LightGBM. Experimental results are reported by 5-fold stratified
 1087 cross-validation, with each fold allocating 80% for training and 20% for testing.

1088 **Classifier parameters Detailed.** Classifier parameters are set as follows: Decision-
 1089 TreeClassifier with criterion='gini', max_depth=None, min_samples_split=2, min_samples_leaf=1,
 1090 and max_features=None; RandomForestClassifier with n_estimators=100, criterion='gini',
 1091 max_depth=None, min_samples_split=2, min_samples_leaf=1, and max_features='sqrt'; XGBClassifier
 1092 with n_estimators=100, max_depth=6, learning_rate=0.3, and objective='binary : logistic';
 1093 and LightGBM with boosting_type='gbdt', num_leaves=31, max_depth=-1, learning_rate=0.1, and
 1094 n_estimators=100. These settings ensure reproducibility of experimental results.

1095 D DISCUSSION

1096 D.1 SENSITIVITY ANALYSIS

1100 To evaluate the impact of the number of weak classifiers N on the performance of Error-Driven En-
 1101 semble Learning (EDEL), we conduct a sensitivity analysis across seven real-world datasets: Spam-
 1102 base (SBD, IR=1.54), Adult Income (AID, IR=3.18), Taiwan Credit Card Default (TCD, IR=3.52),
 1103 CDC Diabetes Health Indicators (CDH, IR=6.18), Bank Marketing (BMD, IR=7.88), Give Me Some
 1104 Credit (GMSC, IR=13.96), and Credit Card Fraud Detection (CCFD, IR=577.88). We vary N from
 1105 1 to 5, where EDEL (1) represents the baseline without EDEL’s dynamic augmentation. Four clas-
 1106 sifiers are evaluated: Decision Tree (DT), Random Forest (RF), XGBoost (XGB), and LightGBM
 1107 (LGBM), using AUC and F1-measure as metrics (Section C.1).



1126 Figure 3: Sensitivity analysis of EDEL performance with varying number of weak classifiers (N)
 1127 across seven datasets. Top row: AUC for DT, RF, XGB, and LGBM. Bottom row: F1-measure.

1128
 1129 Figure 3 illustrates EDEL’s performance trends across datasets and classifiers(detailed data in Table 7
 1130 and Table 8). On CCFD (IR=577.88), EDEL with LGBM achieves an AUC of 0.9948 and F1-
 1131 measure of 0.8993 at $N = 3$, compared to 0.7549 and 0.3834 at $N = 1$, reflecting a 31.7% AUC
 1132 increase and 135% F1 improvement. Similarly, on GMSC (IR=13.96) with DT, F1 rises from 0.2679
 1133 ($N = 1$) to 0.7690 ($N = 5$), a 187% gain. For moderate imbalance, such as SBD (IR=1.54) with
 RF, AUC improves from 0.9860 ($N = 1$) to 0.9972 ($N = 3$). Performance typically peaks at

N = 3, with diminishing returns beyond this point, likely due to overlapping misclassified instances reducing subset diversity (Section B.3).

Table 7: AUC sensitivity analysis results for EDEL across datasets with varying number of weak classifiers (*N*). EDEL (1) represents the baseline (*N* = 1, no EDEL).

Clf	<i>N</i>	SBD	AID	TCD	CDH	BMD	GMSC	CCFD
DT	EDEL (1)	0.9030 ± 0.0075	0.7717 ± 0.0041	0.6143 ± 0.0074	0.5979 ± 0.0031	0.7294 ± 0.0066	0.6120 ± 0.0047	0.8921 ± 0.0041
	EDEL (2)	0.9896 ± 0.0211	0.9506 ± 0.0796	0.9158 ± 0.1578	0.9186 ± 0.1488	0.9594 ± 0.0822	0.9283 ± 0.1476	0.9767 ± 0.0521
	EDEL (3)	0.9911 ± 0.0182	0.9519 ± 0.0724	0.9187 ± 0.1536	0.9241 ± 0.1381	0.9641 ± 0.0705	0.9369 ± 0.1300	0.9797 ± 0.0454
	EDEL (4)	0.9918 ± 0.0162	0.9531 ± 0.0661	0.9181 ± 0.1546	0.9275 ± 0.1312	0.9740 ± 0.0500	0.9425 ± 0.1184	0.9827 ± 0.0386
	EDEL (5)	0.9923 ± 0.0148	0.9514 ± 0.0644	0.9210 ± 0.1509	0.9284 ± 0.1292	0.9745 ± 0.0475	0.9457 ± 0.1107	0.9827 ± 0.0387
RF	EDEL (1)	0.9860 ± 0.0041	0.8954 ± 0.0036	0.7639 ± 0.0046	0.7974 ± 0.0025	0.9444 ± 0.0024	0.8387 ± 0.0055	0.9497 ± 0.0108
	EDEL (2)	0.9971 ± 0.0062	0.9688 ± 0.0431	0.9521 ± 0.1020	0.9549 ± 0.0833	0.9881 ± 0.0246	0.9669 ± 0.0679	0.9900 ± 0.0224
	EDEL (3)	0.9972 ± 0.0058	0.9656 ± 0.0448	0.9461 ± 0.1117	0.9506 ± 0.0841	0.9865 ± 0.0271	0.9656 ± 0.0657	0.9939 ± 0.0137
	EDEL (4)	0.9971 ± 0.0060	0.9598 ± 0.0454	0.9305 ± 0.1280	0.9447 ± 0.0840	0.9842 ± 0.0287	0.9625 ± 0.0653	0.9925 ± 0.0167
	EDEL (5)	0.9970 ± 0.0057	0.9516 ± 0.0463	0.9075 ± 0.1364	0.9365 ± 0.0839	0.9795 ± 0.0306	0.9583 ± 0.0641	0.9923 ± 0.0172
XGB	EDEL (1)	0.9872 ± 0.0031	0.9285 ± 0.0025	0.7661 ± 0.0059	0.8269 ± 0.0025	0.9458 ± 0.0018	0.8573 ± 0.0043	0.9789 ± 0.0062
	EDEL (2)	0.9971 ± 0.0060	0.9424 ± 0.0127	0.9004 ± 0.0795	0.8462 ± 0.0113	0.9756 ± 0.0193	0.8992 ± 0.0258	0.9966 ± 0.0076
	EDEL (3)	0.9968 ± 0.0065	0.9343 ± 0.0160	0.9039 ± 0.0897	0.8365 ± 0.0125	0.9761 ± 0.0206	0.8999 ± 0.0270	0.9965 ± 0.0077
	EDEL (4)	0.9968 ± 0.0063	0.9179 ± 0.0164	0.8931 ± 0.1001	0.8231 ± 0.0131	0.9745 ± 0.0227	0.8973 ± 0.0276	0.9964 ± 0.0079
	EDEL (5)	0.9960 ± 0.0072	0.9025 ± 0.0166	0.8739 ± 0.1096	0.8100 ± 0.0141	0.9708 ± 0.0244	0.8963 ± 0.0270	0.9968 ± 0.0071
LGBM	EDEL (1)	0.9882 ± 0.0035	0.9287 ± 0.0029	0.7805 ± 0.0052	0.8302 ± 0.0030	0.9508 ± 0.0019	0.8649 ± 0.0039	0.7549 ± 0.0368
	EDEL (2)	0.9973 ± 0.0056	0.9366 ± 0.0079	0.8516 ± 0.0399	0.8352 ± 0.0024	0.9687 ± 0.0123	0.8849 ± 0.0138	0.9774 ± 0.0388
	EDEL (3)	0.9969 ± 0.0065	0.9243 ± 0.0100	0.8429 ± 0.0474	0.8209 ± 0.0021	0.9679 ± 0.0152	0.8821 ± 0.0130	0.9948 ± 0.0116
	EDEL (4)	0.9969 ± 0.0062	0.9023 ± 0.0098	0.8165 ± 0.0584	0.8070 ± 0.0021	0.9632 ± 0.0155	0.8768 ± 0.0125	0.9944 ± 0.0126
	EDEL (5)	0.9965 ± 0.0069	0.8836 ± 0.0090	0.7731 ± 0.0609	0.7946 ± 0.0027	0.9550 ± 0.0166	0.8726 ± 0.0121	0.9965 ± 0.0079

Table 8: F1-measure sensitivity analysis results for EDEL across datasets with varying number of weak classifiers (*N*). EDEL (1) represents the baseline (*N* = 1, no EDEL).

Clf	<i>N</i>	SBD	AID	TCD	CDH	BMD	GMSC	CCFD
DT	EDEL (1)	0.8819 ± 0.0088	0.6171 ± 0.0043	0.3999 ± 0.0104	0.3072 ± 0.0047	0.5155 ± 0.0119	0.2679 ± 0.0082	0.7741 ± 0.0272
	EDEL (2)	0.9676 ± 0.0465	0.8194 ± 0.1449	0.7765 ± 0.2973	0.7262 ± 0.3202	0.8194 ± 0.2467	0.7267 ± 0.3502	0.9283 ± 0.1116
	EDEL (3)	0.9709 ± 0.0412	0.8215 ± 0.1339	0.8096 ± 0.2554	0.7584 ± 0.2778	0.8349 ± 0.2103	0.7562 ± 0.3172	0.9279 ± 0.1106
	EDEL (4)	0.9675 ± 0.0403	0.8110 ± 0.1411	0.7952 ± 0.2864	0.7526 ± 0.3005	0.8282 ± 0.2249	0.7594 ± 0.3263	0.9345 ± 0.0882
	EDEL (5)	0.9663 ± 0.0425	0.8087 ± 0.1333	0.8161 ± 0.2472	0.7622 ± 0.2714	0.8375 ± 0.2023	0.7690 ± 0.2987	0.9182 ± 0.1289
RF	EDEL (1)	0.9406 ± 0.0095	0.6669 ± 0.0045	0.4711 ± 0.0086	0.2561 ± 0.0046	0.5781 ± 0.0086	0.2796 ± 0.0085	0.8588 ± 0.0186
	EDEL (2)	0.9840 ± 0.0232	0.8349 ± 0.1250	0.7754 ± 0.2866	0.7280 ± 0.3236	0.8344 ± 0.2153	0.7229 ± 0.3618	0.9453 ± 0.0744
	EDEL (3)	0.9824 ± 0.0209	0.8192 ± 0.1143	0.7448 ± 0.2229	0.6829 ± 0.1925	0.8185 ± 0.1752	0.7135 ± 0.2748	0.9475 ± 0.0768
	EDEL (4)	0.9779 ± 0.0222	0.7893 ± 0.1118	0.6574 ± 0.2060	0.6057 ± 0.1285	0.7656 ± 0.1626	0.6520 ± 0.2271	0.9420 ± 0.0746
	EDEL (5)	0.9711 ± 0.0154	0.7562 ± 0.0917	0.5901 ± 0.1350	0.5433 ± 0.0827	0.7145 ± 0.1194	0.5929 ± 0.1588	0.9370 ± 0.0780
XGB	EDEL (1)	0.9379 ± 0.0094	0.7122 ± 0.0066	0.4673 ± 0.0157	0.2610 ± 0.0030	0.5904 ± 0.0159	0.2884 ± 0.0128	0.8749 ± 0.0298
	EDEL (2)	0.9792 ± 0.0280	0.7378 ± 0.0316	0.6230 ± 0.1620	0.3749 ± 0.0658	0.7330 ± 0.1578	0.3972 ± 0.1052	0.9506 ± 0.0722
	EDEL (3)	0.9796 ± 0.0282	0.7116 ± 0.0345	0.6855 ± 0.1595	0.4508 ± 0.0296	0.7679 ± 0.1386	0.4732 ± 0.1067	0.9493 ± 0.0708
	EDEL (4)	0.9762 ± 0.0301	0.6587 ± 0.0407	0.6667 ± 0.1806	0.4390 ± 0.0290	0.7419 ± 0.1638	0.4670 ± 0.1136	0.9502 ± 0.0621
	EDEL (5)	0.9707 ± 0.0291	0.6361 ± 0.0356	0.6380 ± 0.1492	0.4296 ± 0.0176	0.7333 ± 0.1409	0.4821 ± 0.0827	0.9446 ± 0.0628
LGBM	EDEL (1)	0.9447 ± 0.0081	0.7124 ± 0.0049	0.4770 ± 0.0088	0.2506 ± 0.0029	0.6074 ± 0.0096	0.2895 ± 0.0138	0.3834 ± 0.1507
	EDEL (2)	0.9804 ± 0.0300	0.7282 ± 0.0201	0.5360 ± 0.0821	0.4039 ± 0.0181	0.6569 ± 0.1117	0.3534 ± 0.0572	0.7791 ± 0.1265
	EDEL (3)	0.9799 ± 0.0280	0.6891 ± 0.0180	0.5627 ± 0.0769	0.4258 ± 0.0058	0.6904 ± 0.1018	0.3972 ± 0.0495	0.8993 ± 0.1237
	EDEL (4)	0.9780 ± 0.0275	0.6217 ± 0.0234	0.5169 ± 0.0961	0.4209 ± 0.0037	0.6541 ± 0.1079	0.3884 ± 0.0386	0.9241 ± 0.0944
	EDEL (5)	0.9747 ± 0.0266	0.5840 ± 0.0146	0.4876 ± 0.0688	0.4121 ± 0.0047	0.6403 ± 0.0782	0.3988 ± 0.0228	0.9366 ± 0.0753

These results highlight EDEL’s effectiveness in enhancing minority class detection, particularly in extreme imbalance scenarios like CCFD, where dynamic incorporation of hard-to-classify instances (Section 2) refines decision boundaries. The performance saturation at *N* = 3 aligns with the theoretical convergence of average empirical error (Section B.3), balancing computational complexity and classification accuracy, making EDEL practical for real-world applications.

D.2 DIFFERENCES BETWEEN EDEL AND ADABoost

Although EDEL draws inspiration from AdaBoost (Freund & Schapire, 1997b) in its iterative, error-driven refinement, the two algorithms differ fundamentally in their design, mechanisms, and handling of class imbalance. These distinctions arise from EDEL’s focus on explicit data augmentation for imbalanced learning, contrasted with AdaBoost’s weight-based boosting for general classification. Below, we outline key differences, incorporating perspectives on ensemble integration and local iterative processes.

1. Core Mechanism: Sample Handling and Update Strategy.

- AdaBoost operates on the entire dataset per iteration, multiplicatively updating sample weights to emphasize misclassified instances. For a sample (\mathbf{x}_i, y_i) , the update is

1188
 1189 $w_i^{(t+1)} = w_i^{(t)} \cdot \exp(-\alpha_t \cdot y_i \cdot h_t(\mathbf{x}_i))$, where $\alpha_t = \frac{1}{2} \ln \frac{1-\epsilon_t}{\epsilon_t}$ and ϵ_t is the weighted error.
 1190 This forms a sequential process where each weak learner h_t is trained on a reweighted
 1191 distribution, implicitly focusing on hard samples via probabilistic resampling.
 1192

- 1193 • EDEL, however, partitions the dataset into N subsets $\{\mathcal{D}_i\}_{i=1}^N$ and explicitly augments each
 1194 with misclassified samples \mathcal{D}_i^h from the complement $(\mathcal{D} \setminus \mathcal{D}_i)$: $\mathcal{D}_i^{(t)} = \mathcal{D}_i^{(t-1)} \cup \mathcal{D}_i^h$. This
 1195 additive augmentation creates parallel, diverse views without global reweighting. From an
 1196 additional perspective, within each subset’s iteration, EDEL can be viewed as a 0/1 selection
 1197 process over the remaining samples—selecting (1) misclassified ones for inclusion and
 1198 discarding (0) correctly classified ones—forming a localized, binary decision mini-process
 1199 akin to a simplified expert allocation, but tailored for imbalance correction.
 1200

1201 2. Focus on Class Imbalance and Sample Difficulty.

- 1202 • AdaBoost is geared toward general boosting, converting weak learners (accuracy slightly
 1203 > 0.5) into strong ones without explicit imbalance handling. It amplifies misclassified sam-
 1204 ples globally, which may indirectly benefit minorities if they are frequently erred, but lacks
 1205 guarantees for disproportionate minority focus in extreme imbalance (e.g., IR > 100).
 1206 • EDEL explicitly addresses imbalance by proving (via Bayes’ theorem) that \mathcal{D}_i^h enriches
 1207 minorities ($P(y=1 \mid (\mathbf{x}, y) \in \mathcal{D}_i^h) > \rho$), leading to iterative balancing ($\rho' > \rho$). It targets
 1208 “hard-to-classify” samples (Definition 2), which are often minorities, making it superior
 1209 for high-IR scenarios (e.g., CCFD with IR=577.88), as validated empirically.
 1210

1211 3. Ensemble Integration and Extensibility.

- 1212 • AdaBoost integrates sequentially via weighted voting: $H(\mathbf{x}) = \text{sign}(\sum_t \alpha_t h_t(\mathbf{x}))$, with
 1213 α_t reflecting weak learner strength. This sequential nature limits parallelism.
 1214 • EDEL employs uniform probability averaging over parallel weak classifiers: $\Theta = \frac{1}{N} \sum_i \hat{\Theta}_i$, supporting efficient computation. From an extended perspective, this uniform
 1215 averaging can be generalized to weighted averaging (e.g., $\Theta = \sum_i \alpha_i \hat{\Theta}_i$, where α_i could
 1216 be derived from subset-specific errors), allowing flexibility for future adaptations while
 1217 maintaining focus on imbalance.
 1218

1219 4. Theoretical Foundations and Convergence.

- 1220 • AdaBoost’s bounds emphasize exponential error decay: training error $\leq \exp(-2 \sum_t \gamma_t^2)$,
 1221 with $\gamma_t = \frac{1}{2} - \epsilon_t$, rooted in exponential loss and VC-dimension for generalization.
 1222 • EDEL uses stratified sampling (Chebyshev for representativeness), Rademacher complexity
 1223 for generalization, and McDiarmid’s inequality for convergence ($P(|\hat{\epsilon} - \mathbb{E}[\hat{\epsilon}]| \geq \delta) \leq 2 \exp(-2\delta^2 n)$). Its proofs target imbalance-specific properties, like minority enrichment,
 1224 differing from AdaBoost’s margin-based analysis.
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1226 In essence, AdaBoost is a general booster via sequential weighting, while EDEL is an imbalance-
 1227 specialized ensemble through parallel augmentation and selective inclusion. These differences en-
 1228 able EDEL’s superior performance in imbalanced domains, as shown in experiments, without requir-
 1229 ing prior knowledge of weak learner accuracies.
 1230

1232 E RELATED WORKS

1233 Existing approaches to address class imbalance are categorized into three groups: data-level meth-
 1234 ods, algorithm-level methods, and ensemble learning methods, each offering distinct strategies to
 1235 address this issue.
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1237 **Data-Level Methods** Data-level methods seek to rebalance class distributions through dataset mod-
 1238 ification, providing classifier-agnostic solutions that enhance generalizability across models. The
 1239 Synthetic Minority Over-sampling Technique (SMOTE) (Chawla et al., 2002) generates synthetic
 1240 minority samples via interpolation, inspiring variants that address its over-generalization ten-
 1241 dencies. For instance, Borderline-SMOTE (Han et al., 2005) targets samples near decision boundaries,
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1242 while ADASYN (He et al., 2008) adaptively generates samples based on local density. Safe-Level-
 1243 SMOTE (Bunkhumpornpat et al., 2009) ensures synthetic samples remain within safe minority class
 1244 regions, and FW-SMOTE (Maldonado et al., 2022) incorporates feature weighting to improve sam-
 1245 ple relevance. RCSMOTE (Soltanzadeh & Hashemzadeh, 2021) refines SMOTE by constraining
 1246 synthetic sample ranges using distances to majority class samples, reducing noise and class overlap,
 1247 particularly effective for moderately imbalanced datasets. Similarly, MLBOITE (Teng et al., 2024)
 1248 extends SMOTE to multi-label datasets with a three-phase framework (seed set construction, border-
 1249 line sample resampling, and internal sample resampling), optimizing label assignment for complex
 1250 multi-label scenarios. ConvGeN (Schultz et al., 2024) leverages a deep-generative model to learn
 1251 convex combinations of minority class samples, producing high-quality samples for small tabular
 1252 datasets while reducing over-generalization compared to SMOTE and GAN-based methods (Good-
 1253 fellow et al., 2014; Xu et al., 2019). SMOTE-TLNN-DEPSO (Dixit & Mani, 2023) integrates
 1254 SMOTE with Tomek Links, neural networks, and differential evolution optimization to generate
 1255 and filter high-quality synthetic samples, enhancing robustness against noisy data. In contrast, un-
 1256 dersampling methods reduce majority class samples to balance datasets. Spatial Distribution-based
 1257 Undersampling (SDUS) (Yan et al., 2023) exploits spatial relationships, while Metaheuristic-based
 1258 Under-Sampling (MHUS) (Soltanzadeh et al., 2023) employs a genetic algorithm to select represen-
 1259 tative majority samples, minimizing information loss compared to random undersampling (Kubat
 1260 & Matwin, 1997). Despite their versatility, data-level methods face significant challenges. Over-
 1261 sampling techniques, such as SMOTE and its variants, may introduce synthetic noise, especially in
 1262 high-dimensional or overlapping datasets, potentially degrading classifier performance. GAN-based
 1263 methods often produce inconsistent sample quality, particularly for non-tabular data like images.
 1264 Undersampling risks discarding critical information, which can be detrimental in datasets with com-
 1265 plex distributions or extreme imbalance ratios.

1265 **Algorithm-Level Methods** Algorithm-level methods enhance minority class performance by mod-
 1266 ifying the learning process, offering fine-grained control over model optimization. Cost-sensitive
 1267 learning assigns higher penalties to misclassifications of minority class samples, prioritizing their
 1268 correct classification (Elkan, 2001; Zhou & Liu, 2006; Ling & Sheng, 2010). Recent advan-
 1269 tages address the challenge of specifying fixed cost matrices. Cost-Free Learning (CFL) (Zhang
 1270 & Hu, 2013) eliminates the need for explicit cost definitions, while Adaptive Threshold Error Costs
 1271 (ATEC) (Cao et al., 2021) dynamically adjusts error thresholds to adapt to class distributions. Cu-
 1272 mulative Cost-Sensitive Boosting (AdaCC) (Iosifidis et al., 2022) further advances this paradigm
 1273 by dynamically adjusting sample weights based on cumulative misclassification costs (false positive
 1274 and false negative rates), using two variants (AdaCC1 and AdaCC2) to incorporate costs into the
 1275 weight update formula, achieving superior performance without requiring a predefined cost matrix.
 1276 Modified loss functions provide an alternative approach. Focal Loss (Lin et al., 2017) reduces the
 1277 influence of easily classified samples, emphasizing hard-to-classify instances, while Class-Balanced
 1278 Loss (Cui et al., 2019) reweights losses based on class frequency to balance contributions. Tech-
 1279 niques such as adaptive learning rates and distribution-aware optimization further refine training
 1280 parameters to enhance minority class representation (He & Garcia, 2009; Kingma & Ba, 2014; Cao
 1281 et al., 2019). Algorithm-level methods excel in tailoring model behavior to specific class imbalance
 1282 scenarios, particularly when domain knowledge is limited. However, their reliance on hyperparam-
 1283 eter tuning, as seen in Focal Loss and ATEC, can complicate deployment across diverse datasets.
 1284 Additionally, these methods may struggle to generalize across datasets with extreme imbalance or
 1285 noisy boundaries, necessitating ensemble strategies to improve robustness.

1285 **Ensemble Learning Methods** Ensemble learning methods combine multiple classifiers to enhance
 1286 robustness, leveraging diverse models to mitigate class imbalance. Bagging (Breiman, 1996) gener-
 1287 ates varied subsets via bootstrap sampling, but often underrepresents minority classes in imbalanced
 1288 datasets, leading to biased models (Sun et al., 2007). Balanced Random Forests (Chen & Breiman,
 1289 2004) address this by undersampling the majority class in each subset, improving minority class rep-
 1290 resentation. Boosting methods, such as AdaBoost (Freund & Schapire, 1997b), iteratively increase
 1291 weights of hard-to-classify samples, enhancing their classification accuracy. SMOTEBoost (Chawla
 1292 et al., 2003) integrates SMOTE with Boosting, generating synthetic minority samples in each it-
 1293 eration to enrich minority class representation. RUSBoost (Seiffert et al., 2010) pairs random un-
 1294 dersampling with Boosting, offering a computationally efficient alternative. SMOTEWB (Sağlam
 1295 & Cengiz, 2022) enhances SMOTE with noise detection, dynamically adjusting parameters during
 1296 Boosting to reduce synthetic noise. MESA (Liu et al., 2020) enhances ensemble learning by em-
 1297 ploying a meta-sampler to adaptively learn sampling strategies, improving performance on extreme

1296 imbalance scenarios. WSMOTE-ensemble (Abedin et al., 2022) employs Weighted SMOTE with
1297 Bagging to generate diverse synthetic samples, improving model diversity. The Contribution-Based
1298 Hybrid Resampling Ensemble (CHRE) (Zhao et al., 2025) uses a Globally Unified Data Evaluation
1299 (GUDE) algorithm to assess sample contributions based on information and noise levels, guiding
1300 oversampling and undersampling within a serial ensemble framework to balance data distribution.
1301 Ensemble methods are particularly effective in handling moderately imbalanced datasets by leverag-
1302 ing multiple classifiers to capture diverse patterns. However, they face challenges with extreme class
1303 imbalances, where minority class underrepresentation persists, as seen in Bagging and AdaBoost.
1304 Methods like SMOTEBoost and CHRE mitigate this by integrating resampling, but their reliance on
1305 synthetic samples can introduce noise, particularly in complex or high-dimensional datasets. Fur-
1306 thermore, ensemble methods often incur significant computational costs due to multiple classifier
1307 training and hyperparameter tuning, limiting their scalability in large-scale applications.
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