This article was downloaded by: [171.66.13.55] On: 05 October 2023, At: 13:34
Publisher: Institute for Operations Research and the Management Sciences (INFORMS)
INFORMS is located in Maryland, USA

Management Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Simple and Approximately Optimal Contracts for Payment for Ecosystem Services

Wanyi Dai Li, Itai Ashlagi, Irene Lo

To cite this article:
Wanyi Dai Li, Itai Ashlagi, Irene Lo (2022) Simple and Approximately Optimal Contracts for Payment for Ecosystem Services. Management Science

Published online in Articles in Advance 10 Feb 2022

. https://doi.org/10.1287/mnsc.2021.4273

Full terms and conditions of use: https://pubsonline.informs.org/Publications/Librarians-Portal/PubsOnLine-Terms-and-Conditions

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2022 The Author(s)

Please scroll down for article—it is on subsequent pages

With 12,500 members from nearly 90 countries, INFORMS is the largest international association of operations research (O.R.) and analytics professionals and students. INFORMS provides unique networking and learning opportunities for individual professionals, and organizations of all types and sizes, to better understand and use O.R. and analytics tools and methods to transform strategic visions and achieve better outcomes.
For more information on INFORMS, its publications, membership, or meetings visit http://www.informs.org

MANAGEMENT SCIENCE
Articles in Advance, pp. 1–17
ISSN 0025-1909 (print), ISSN 1526-5501 (online)
http://pubsonline.informs.org/journal/mnsc

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

Simple and Approximately Optimal Contracts for Payment for Ecosystem Services

Wanyi Dai Li,[a,*] **Itai Ashlagi,**[a] **Irene Lo**[a]

[a] Department of Management Science and Engineering, Stanford University, Stanford, California 94305
*Corresponding author

Contact: wanyi.daili@gmail.com, https://orcid.org/0000-0002-4086-5047 (WDL); iashlagi@stanford.edu, https://orcid.org/0000-0003-2124-738X (IA); ilo@stanford.edu, https://orcid.org/0000-0002-0678-3494 (IL)

Received: December 19, 2020
Revised: May 19, 2021
Accepted: July 25, 2021
Published Online in Articles in Advance: February 10, 2022

https://doi.org/10.1287/mnsc.2021.4273

Copyright: © 2022 The Author(s)

Abstract. Many countries have adopted payment for ecosystem services (PES) programs to reduce deforestation. Empirical evaluations find such programs, which pay forest owners to conserve forest, can lead to anywhere from no impact to a 50% reduction in deforestation level. To better understand the potential effectiveness of PES contracts, we use a principal–agent model, in which the agent has an observable amount of initial forest land and a privately known baseline conservation level. Commonly used conditional contracts perform well when the environmental value of forest is sufficiently high or sufficiently low, but can do arbitrarily poorly compared with the optimal contract for intermediate values. We identify a linear contract with a distribution-free per-unit price that guarantees at least half of the optimal contract payoff. A numerical study using U.S. land use data supports our findings and illustrates when linear or conditional contracts are likely to be more effective.

History: Accepted by Beril Toktay, Special Issue of *Management Science*: business and climate change.
Open Access Statement: This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License. You are free to download this work and share with others for any purpose, except commercially, and you must attribute this work as "*Management Science*. Copyright © 2022 The Author(s). https://doi.org/10.1287/mnsc.2021.4273, used under a Creative Commons Attribution License: https://creativecommons.org/licenses/by-nc/4.0/."
Funding: W. Dai Li was supported by a Stanford Interdisciplinary Graduate Fellowship. I. Lo was supported by an Environmental Venture Project Grant from the Stanford Woods Institute for the Environment.
Supplemental Material: Data and the e-companion are available at https://doi.org/10.1287/mnsc.2021.4273.

Keywords: contract design • payment for ecosystem services • information asymmetry • additionality • conditionality

1. Introduction

The world has been losing tropical forests at an annual rate of 10 million hectares in the last two decades (Butler 2019). These alarming levels are driven largely by land use changes such as deforestation and agricultural expansion, which generate almost a quarter of the total greenhouse gas emissions worldwide (Hawken 2017). To mitigate climate change and to halt environmental degradation, there has been an emergence of programs that pay owners of natural assets to conserve their environmental resources. These payment for ecosystem services (PES) programs are widely implemented by governments and nongovernmental organizations (NGOs) seeking cost-effective methods to slow down deforestation and to combat climate change (Stern et al. 2006). Two prominent examples of forest PES programs are the Pago por Servicios Ambientales (PSA) program run by the national government of Costa Rica, which pays private forest owners $640 per hectare over 10 years in exchange for forest protection (Porras et al. 2017), and the Reducing Emissions from Deforestation and Forest Degradation (REDD+) program run by the United Nations. There are now more than 550 PES programs in the world, with combined annual payments over $36 billion (Salzman et al. 2018).

Governments and NGOs are typically interested in how much additional forest is conserved because of the payment scheme. However, such information is difficult to obtain, as forest owners have private information about their opportunity costs and the amount they plan to deforest in the absence of any incentives. Empirical studies have found mixed results regarding the effectiveness of PES programs, finding the payments led to anywhere from no impact to a 50% reduction in deforestation (for a survey, see Borner et al. 2017). This suggests PES can be a potentially promising tool to achieve environmental goals, but its

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

effectiveness is context dependent. Ferraro (2008) argues for the need to study asymmetric information in PES, yet theory is still limited in this space.

Most commonly used contracts in PES programs are *conditional contracts*, which pay forest owners in exchange for conserving their forests fully (Engel et al. 2008, Muñoz-Piña et al. 2008, Porras et al. 2017).[1] It is natural to ask when conditional contracts are effective, and whether other simple contracts could perform better.

We address these questions by studying a principal–agent model similar to that in Mason and Plantinga (2013). The model captures an important information asymmetry—the agent (forest owner) has private information about their *baseline conservation amount* absent any incentives. In addition, the agent cannot conserve more forest than their *initial forest area*. The principal (program designer) seeks to maximize the environmental value for the amount of conserved forest net payments. We use the optimal contract payoff (second best solution) as a benchmark to evaluate the performances of simple and practical contracts.

Conditional contracts work well when the principal has a relatively high value for forests. However, we show that for some parameter regimes they can perform arbitrarily poorly. In such regimes, conditional contracts may exclude a large population who find full conservation too costly even with the PES payments, leading to low levels of take-up.

In addition, we consider *linear contracts*, a class of simple yet intuitive contracts that pay the agent a fixed amount per unit of forest conserved. We identify a per-unit price independent of the baseline conservation distribution, with which a linear contract always yields at least half of the optimal contract payoff. This implies the best linear contract has the same approximation guarantee, regardless of the principal's environmental value or the agent's conservation behavior.

In a numerical exercise, we use empirically calibrated baseline conservation distributions and conservation cost functions from U.S. land use data to illustrate and discuss the performance of simple contracts. Three parameter regimes are of interest. First, when the environmental value is relatively low, neither simple contract can improve upon the baseline scenario; we find this regime applies to the majority of regions in the United States. Second, when the environmental value is relatively high, the best conditional contract achieves higher payoff than linear contracts. Finally, when the environmental value is in an intermediate regime, linear contracts can improve upon conditional contracts substantially if the baseline conservation level is low. We argue that this parameter regime is relevant for developing countries with high baseline deforestation levels and recommend PES designers consider linear contracts in this context.

Finally, we consider a generalization of both conditional and linear contracts. A *conditional linear* contract pays the agent a linear price in the conserved forest area but only if the conserved area exceeds a prespecified threshold. Such contracts allow the principal to adjust both the price and the stringency level. We identify a distribution-free conditional linear contract with an intermediate level of stringency that further improves upon the linear contract with the constant approximation guarantee.

1.1. Relation to the Literature

The first paper that uses a principal–agent framework to study optimal contracting in the PES context is Mason and Plantinga (2013). We extend their model to account for boundary solutions that naturally arise in this setting. In addition to characterizing the optimal contract completely, we analyze the performance of simple and practical contracts relative to the optimal one. Furthermore, Mason and Plantinga (2013) were the first to empirically calibrate the baseline conservation distributions and cost functions for the United States. We use their calibrations in our numerical exercise.

This paper contributes to the environmental economics and land use literature on PES by bringing in a new angle of contract design. Comprehensive reviews about PES programs implemented in practice are given by Pattanayak et al. (2010), Engel et al. (2016), and Alix-Garcia and Wolff (2014). Teytelboym (2019) reviews market design approaches used in the natural capital context and highlights the challenge of effectively designing and implementing PES constraints due to heterogeneity among landowners. Borner et al. (2017) reviews empirical results on evaluating PES programs. Using a field experiment, Jayachandran et al. (2017) found that paying landowners not to deforest led to a 50% reduction in deforestation level. Several papers address other frictions arising in PES programs: Jack and Jayachandran (2019) demonstrate the need for better targeting to alleviate inefficiencies due to self-selection, Peterson et al. (2015) study transaction costs of PES programs with uncertainty about the agent's willingness to enroll, and Harstad and Mideksa (2017) discuss theoretically how conservation policies can be affected by property rights.

The analysis of the principal–agent model is based on a well-established literature studying optimal mechanisms and contracts (Mirrlees 1971, Myerson 1979). Optimal contracts often require detailed knowledge of system parameters and distributions by the principal, and thus can be difficult to use in practice. Recent studies consider robust contract design and find simple contracts can provide approximation guarantees for the worst case outcome, when the

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

principal has limited distributional information (Carroll 2015, Dütting et al. 2019, Yu and Kong 2020). We take a similar approach in analyzing linear contracts and, specifically in the PES setting, provide guarantees on performance when the principal has no distributional information on the agent's type.

Our work broadly contributes to the study of contract design in operations management (OM) and sustainable OM. The OM community has long studied optimal contracts and their approximations in the context of supply chain contracts (Corbett et al. 2004, Perakis and Roels 2007, Kim and Netessine 2013, Bolandifar et al. 2018). The sustainable OM literature has flourished in recent years (see, e.g., Lee and Tang 2018, Atasu et al. 2020) and has studied information asymmetry in environmental disclosure policies (Kim 2015, Plambeck and Taylor 2016, Wang et al. 2016), as well as environmentally responsible sourcing strategies, which can alleviate deforestation pressure (Orsdemir et al. 2019, de Zegher et al. 2019). We bring together these strands of literature in designing approximately optimal contracts to combat deforestation and offer directions for future study at this intersection.

1.2. Organization of This Paper

Section 2 describes the model and the optimal contract. Conditional contracts are studied in Section 3. Linear contracts are studied in Section 4. A numerical analysis using U.S. data are given in Section 5. We discuss extensions in Section 6 and conclude with future directions in Section 7. Omitted proofs are in the appendices, and a more in-depth description of the numerical analysis is given in the e-companion to this paper.

2. Model

The agent has an initial forest area a_0, which is publicly observable.[2] The principal seeks to incentivize the agent to conserve their forest area. The agent has a privately known baseline conservation proportion θ, drawn independent and identically distributed from a publicly known distribution $F(\theta)$ with support $[\underline{\theta}, \overline{\theta}] \subseteq [0,1]$. That is, in the baseline scenario absent incentives, the agent will conserve θa_0 forest area. The agent can choose their conservation action, $a \in [0, a_0]$, and incurs a convex conservation cost $c(a-\theta a_0)$.[3] The conservation cost function $c(\cdot)$ captures the opportunity cost of not deforesting. It is costless for the agent to conserve up to the baseline amount θa_0, but it becomes increasingly costly for the agent to conserve beyond the baseline amount. We assume that, for $x := a - \theta a_0 \geq 0$, $c(x) = \frac{h}{2}x^2$, where $h > 0$, and for $x < 0$, $c(x) = 0$. Section 6 generalizes $c(x)$ to general convex functions.

The principal, who does not own the land, has environmental value \$$k$ per unit area of conserved forest. This can be interpreted as the carbon sequestration value or the biodiversity value of the forest.[4] Based on the principal's knowledge of a_0 and $F(\theta)$, the principal can offer a contract to the agent that specifies a payment for each conservation action, $P(a)$. We only consider contracts with limited liability; that is, payments are assumed to be nonnegative.[5] The principal's objective is to design a contract that maximizes the following expected payoff

$$\mathbb{E}_\theta[ka - P(a)],$$

which captures the environmental value from conserved forest minus the amount paid to the agent.

The agent with type θ is risk neutral and chooses a conservation action $a \in [0, a_0]$ to maximize their net utility

$$P(a) - c(a - \theta a_0).$$

There is no loss of generality to restrict attention to payment rules that are nondecreasing in the conservation amount. We assume that when indifferent, the agent chooses the conservation action that is preferred by the principal. The utility-maximizing action a^* is weakly greater than the baseline conservation level θa_0 for any such payment schemes. Observe that the principal's optimization problem is equivalent to maximizing the environmental value from the additional conservation amount net pay, $k\mathbb{E}[a-\theta a_0] - \mathbb{E}P(a)$, where $\mathbb{E}[a - \theta a_0]$ is the additional conservation amount induced by the contract.

2.1. The Optimal Contract

The optimal contract maximizes the principal's payoff given incentive compatibility (IC) and individual rationality (IR) constraints. We consider the space of direct revelation contracts, $\{(a(\theta), P(\theta))\}_{\theta \in [\underline{\theta}, \overline{\theta}]}$, where $a(\theta)$ and $P(\theta)$ are the conservation amount and the payment level of an agent with type θ.[6] The optimal contract offers the agent a continuum menu of choices. Formally, the principal's optimization problem is given by

$$\begin{aligned} Obj^{OPT} &= \max_{\{(a(\theta),P(\theta))\}_{\theta\in[\underline{\theta},\overline{\theta}]}} \mathbb{E}ka(\theta) - P(\theta) \\ \text{s.t.} &\quad P(\theta) - c(a(\theta) - \theta a_0) \geq 0,\ \forall \theta, \quad \text{(IR)} \\ &\quad \theta = \arg\max_{\theta' \in [\underline{\theta}, \overline{\theta}]} P(\theta') - c(a(\theta') - \theta a_0),\ \forall \theta. \quad \text{(IC)} \end{aligned}$$

Standard analysis (e.g., Mirrlees 1971) shows that the principal's expected payoff is equivalent to the following expression that integrates the IC constraint into the objective:

$$\int_{\theta=\underline{\theta}}^{\overline{\theta}} \left[ka(\theta) - c(a(\theta) - \theta a_0) - \frac{1-F(\theta)}{f(\theta)} a_0 c'(a(\theta) - \theta a_0) \right] f(\theta) d\theta. \quad (1)$$

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

In the optimal contract, the lowest type agent has a binding IR constraint; the optimal conservation quantity for each type, $a^{OPT}(\theta)$, is solved to maximize $J(\theta) \equiv ka(\theta) - c(a(\theta) - \theta a_0) - \frac{1-F(\theta)}{f(\theta)} a_0 c'(a(\theta) - \theta a_0)$. For ease of exposition, we state the results assuming $F(\theta)$ has a monotone hazard rate, that is, $\frac{1-F(\theta)}{f(\theta)}$ is nonincreasing in θ.[7] Taking the boundary condition of $a(\theta) \in [\theta a_0, a_0]$ into account, we have the following result.

Lemma 1. *The optimal direct revelation contract* $(a(\theta), P(\theta))$ *is given by*

$$a(\theta) = \begin{cases} \theta a_0, & \text{if } \theta \le \hat{\theta}_1, \\ \theta a_0 + \dfrac{k}{h} - \dfrac{1-F(\theta)}{f(\theta)} a_0, & \text{if } \theta \in (\hat{\theta}_1, \hat{\theta}_2), \\ a_0, & \text{if } \theta \ge \hat{\theta}_2, \end{cases}$$

where $\hat{\theta}_1$ *is defined by* $\frac{1-F(\hat{\theta}_1)}{f(\hat{\theta}_1)} = \frac{k}{ha_0}$ *if* $\frac{1}{f(\underline{\theta})} \ge \frac{k}{ha_0}$, *and otherwise is given by* $\hat{\theta}_1 = \underline{\theta}$, *and* $\hat{\theta}_2$ *is defined by* $1 - \hat{\theta}_2 + \frac{1-F(\hat{\theta}_2)}{f(\hat{\theta}_2)} = \frac{k}{ha_0}$ *if* $1 - \underline{\theta} + \frac{1}{f(\underline{\theta})} \ge \frac{k}{ha_0}$, *and otherwise is given by* $\hat{\theta}_2 = \underline{\theta}$. *In each* θ *interval, the payment level is given by*

$$P(\theta) = c(a(\theta) - \theta a_0) + \int_{\tau=\underline{\theta}}^{\theta} a_0 c'(a(\tau) - \tau a_0) d\tau.$$

Lemma 1 shows that in the optimal solution, the agent type space is divided into three regions. The top types in the interval $(\theta \in [\hat{\theta}_2, \overline{\theta}])$ are pooled and conserve the maximal amount. Moreover, it may be optimal to pool the bottom types in the interval $(\theta \in [\underline{\theta}, \hat{\theta}_1])$ and have them conserve at their baseline levels without payments. The optimal contract screens the middle types, where their conservation amounts are interior solutions. The quantity $\frac{k}{ha_0}$ measures how much the principal values the forest relative to the agent's conservation cost (and will be used throughout this paper). Both thresholds $\hat{\theta}_1$ and $\hat{\theta}_2$ decrease as $\frac{k}{ha_0}$ increases.

The optimal contract, as shown next, can also be implemented via a menu of affine contracts $\{p(\theta), T(\theta)\}_\theta$, where each option is composed of a linear price $p(\theta)$ as well as a lump sum transfer $T(\theta)$ that is independent of the agent's conservation quantity (see, e.g., Laffont and Tirole 1986). So an agent that chooses option $(p(\theta), T(\theta))$ and conserves an amount a of forest receives a total payment of $p(\theta)a + T(\theta)$. For this result, we assume $F(\theta)$ has a monotone hazard rate for ease of exposition.

Lemma 2. *If* $\frac{1-F(\theta)}{f(\theta)}$ *is nonincreasing in* θ, *the optimal allocation can be implemented via a menu of affine contracts* $\{p(\theta), T(\theta)\}_\theta$, *where*

$$p(\theta) = \begin{cases} 0, & \text{if } \theta \le \hat{\theta}_1, \\ k - ha_0 \dfrac{1-F(\theta)}{f(\theta)}, & \text{if } \theta \in (\hat{\theta}_1, \hat{\theta}_2), \\ h(1-\hat{\theta}_2)a_0, & \text{if } \theta \ge \hat{\theta}_2, \end{cases}$$

and

$$T(\theta) = \begin{cases} 0, & \text{if } \theta \le \hat{\theta}_1, \\ -\dfrac{1}{2h}\left(k - ha_0 \dfrac{1-F}{f}(\theta)\right)^2 \\ \quad + \displaystyle\int_{\theta_1}^{\theta} a_0\left(k - ha_0 \frac{1-F}{f}(\tau)\right) d\tau, & \text{if } \theta \in (\hat{\theta}_1, \hat{\theta}_2), \\ \dfrac{h}{2}(1-\theta_2)^2 a_0^2 - h(1-\theta_2)a_0^2 \\ \quad + \displaystyle\int_{\theta_1}^{\theta_2} a_0\left(k - ha_0 \frac{1-F}{f}(\tau)\right) d\tau, & \text{if } \theta \ge \hat{\theta}_2. \end{cases}$$

Here $\hat{\theta}_1$ *is defined implicitly by* $\frac{1-F(\hat{\theta}_1)}{f(\hat{\theta}_1)} = \frac{k}{ha_0}$ *if* $\frac{1}{f(\underline{\theta})} \ge \frac{k}{ha_0}$, *and otherwise is given by* $\hat{\theta}_1 = \underline{\theta}$, *and* $\hat{\theta}_2$ *is defined implicitly by* $1 - \hat{\theta}_2 + \frac{1-F(\hat{\theta}_2)}{f(\hat{\theta}_2)} = \frac{k}{ha_0}$ *if* $1 - \underline{\theta} + \frac{1}{f(\underline{\theta})} \ge \frac{k}{ha_0}$, *and otherwise is given by* $\hat{\theta}_2 = \underline{\theta}$. *Furthermore, the principal's payoff is identical to that in the optimal direct revelation contract.*

Observe that optimal contracts require an infinite number of options. In practice, PES contracts between landowners and NGOs/governments are typically very simple; contracts offering a menu with even a few options are considered complex and unlikely to be implemented, especially in developing countries. Furthermore, any optimal contract requires the principal to know the baseline conservation distribution $F(\theta)$, which is unrealistic. With this in mind, we turn to analyzing simpler contracts and let the optimal contract serve as a benchmark in our analysis of these contracts.

3. Conditional Contracts

A conditional contract is parametrized by a price p per unit area conserved. The principal pays a lump sum amount pa_0 if and only if the agent conserves the full amount $a = a_0$.

The agent with type θ can choose to either conserve all the forest they own, receiving utility $pa_0 - \frac{h}{2}(1-\theta)^2 a_0^2$, or continue with baseline activity, receiving utility zero. Define $\hat{\theta}(p) \equiv 1 - \sqrt{\frac{2p}{ha_0}}$ to be the threshold type indifferent between the two options. If $\theta \le \hat{\theta}(p)$, the agent conserves $a(\theta) = \theta a_0$; otherwise, $a(\theta) = a_0$.

The principal's objective can now be written as

$$\max_p Obj^C(p) = k\int_{\underline{\theta}}^{\hat{\theta}} \theta a_0 f(\theta) d\theta + (k-p)\int_{\hat{\theta}}^{\overline{\theta}} a_0 f(\theta) d\theta. \quad (2)$$

Denote by Obj^{C^*} the payoff of the *best* conditional contract with price p^{C^*}. In this section, we assume $F(\theta)$

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

has a monotone hazard rate in order to tractably characterize the outcome of the best conditional contract.

When the environmental value is sufficiently high relative to the conservation cost parameter h, the best conditional contract and the optimal contract both pay the agent for full conservation regardless of their type. The best conditional price is set to fully compensate the agent with the lowest baseline conservation level for their conservation cost.

Lemma 3. *If* $\frac{k}{ha_0} \geq (1-\underline{\theta}) + \frac{1}{f(\underline{\theta})}$, *the optimal contract and the best conditional contract coincide; thus,* $Obj^{C^*} = Obj^{OPT}$. *Every agent type* θ *conserves fully, that is,* $a(\theta) = a_0$, *and receives a payment of* $\frac{h}{2}(1-\underline{\theta})^2 a_0^2$.

When the relative environmental value $\frac{k}{ha_0}$ is sufficiently low, the best conditional contract is the baseline scenario. This is because the principal is not incentivized to set the price high enough to induce any additional conservation from the agent.

Lemma 4. *If* $\frac{k}{ha_0} \leq \frac{1}{2}(1-\overline{\theta})$, *the best conditional contract is the baseline scenario; that is,* $Obj^{C^*} = k\mathbb{E}[\theta]a_0$ *and* $p^{C^*} = 0$.

Next, we demonstrate that there exists baseline distributions such that the best conditional contract can perform arbitrarily poorly compared with the optimal contract.

Example 1. When there is only one agent type, $\theta = 0$, that is, $F(\theta) = \delta(0)$, the best conditional contract has objective value $Obj^{C^*} = \max\{0, ka_0 - \frac{h}{2}a_0^2\}$, because the principal will either ask the agent to conserve fully and compensate their conservation cost or not offer a conditional contract at all. The optimal contract is to maximize the socially efficient objective $ka - \frac{h}{2}a^2$, where a is the conservation amount of the agent type 0. Thus, $a(0) = \min\{\frac{k}{h}, a_0\}, P(0) = \min\left\{\frac{k^2}{2h}, \frac{h}{2}a_0^2\right\}$, resulting in positive payoff $Obj^{OPT} = \frac{k^2}{2h}$. When $\frac{k}{ha_0} \leq \frac{1}{2}$, $Obj^{C^*} = 0$, and the best conditional contract recovers 0% of the optimal contract payoff.

The poor performance of conditional contracts can occur for general $F(\theta)$ distributions with information asymmetry (i.e., more than one agent type) where the expectation of baseline conservation is small at intermediate relative environmental values, as demonstrated below.

Example 2. Let $F(\theta) = \frac{1-\exp(-\lambda\theta)}{1-\exp(-\lambda)}$ be a bounded exponential distribution with parameter λ with its support on $[0, 1]$. At an intermediate relative environmental value, $\frac{k}{ha_0} = \frac{1}{2}$, as λ tends to infinity, that is, the expected baseline conservation amount $\mathbb{E}[\theta]a_0$ tends to zero, the ratio between the best conditional contract and the optimal contract tends to zero. (See Appendix C for more details.)

The poor performance of the conditional contract manifests itself in low levels of take-up even with a seemingly attractive conditional price. This is because when $\frac{k}{ha_0}$ is intermediate, the optimal contract achieves higher payoff than the baseline scenario by capturing intermediate levels of additional conservation; in the conditional contract, for baseline distributions that are left skewed, the expected fraction of the population that commits to full conservation is still very low. As a result, the best conditional contract payoff is not that different from the baseline scenario.

In practice, the principal may not be aware of the conditional contract's potentially significant inefficiency due to lack of take-up. First, the conditional contract price may not be optimized; second, empirical studies often estimate the additional conservation from landowners who opt into the PES program relative to their estimated baseline conservation levels, but not the relative performance between the conditional contract and the optimal contract outcome.

Next we consider another simple class of contracts that overcome the drawbacks of conditional contracts.

4. Linear Contracts

A linear contract pays the agent a fixed price p per unit area conserved without any conditionality requirement and can be viewed as a uniform subsidy. Linear contracts are rare in practice, as conditionality is often required in PES schemes (Engel et al. 2016).

The agent's best response to a linear contract with price p is to choose a conservation amount a that maximizes $pa - \frac{h}{2}(a - \theta a_0)^2$. Solving this yields that the agent of type θ chooses to conserve an additional amount of $\frac{p}{h}$ beyond their baseline, or fully if their baseline conservation proportion θ is high enough that the boundary condition is met.

Denote the lowest agent type that will conserve the full amount by $\tilde{\theta}(p) \equiv 1 - \frac{p}{ha_0}$. The principal's objective can be written as

$$\max_p Obj^L(p) = \max_p (k-p)\left(\int_{\underline{\theta}}^{\min\{\tilde{\theta},\overline{\theta}\}} \left(\theta a_0 + \frac{p}{h}\right) f(\theta) d\theta + \int_{\min\{\tilde{\theta},\overline{\theta}\}}^{\overline{\theta}} a_0 f(\theta) d\theta\right). \quad (3)$$

We let Obj^{L^*} denote the objective of the *best* linear contract with price p^{L^*}.

Analogous to Lemma 4, when k is relatively small, a positive linear price is not sufficient to incentivize enough additional conservation:

Lemma 5. *When* $\frac{k}{ha_0} \leq \mathbb{E}[\theta]$, *the best linear contract is the baseline scenario; thus,* $Obj^{L^*} = k\mathbb{E}[\theta]a_0$ *and* $p^{L^*} = 0$.

Recall that when the baseline conservation level is at its worst case (i.e., $\mathbb{E}[\theta] = 0$), the best conditional

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

contract recovers 0% of the optimal contract payoff (Example 1). In contrast to conditional contracts where a large fraction of population may not participate, linear contracts always induce every agent to enroll and to conserve beyond their baseline level (except for the type $\theta = 1$, who already conserves fully). We study the linear contract for the same distribution.

Example 3. When there is only one agent type, $\theta = 0$, the ratio between the best linear contract and the optimal contract payoff is 0.5 regardless of the relative environmental value $\frac{k}{ha_0}$. The optimal contract is $a(0) = \min\{\frac{k}{h}, a_0\}, P(0) = \min\left\{\frac{k^2}{2h}, \frac{h}{2}a_0^2\right\}$ (equivalent to a per-unit price of $\frac{k}{2}$ when $k < ha_0$), leading to an objective value $Obj^{OPT} = \frac{k^2}{2h}$. The best linear contract is $a(0) = \frac{k}{2h}$ with a per-unit price $p^{L^*} = \frac{k}{2}$, leading to an objective value $Obj^{L^*} = \frac{k^2}{4h}$, which is half of the optimal payoff.

The next result shows that a linear contract with price $p = \frac{k}{2}$ always achieves at least half of the optimal contract payoff *for any environmental value k and any distribution* $F(\theta)$. Thus, the best linear contract always guarantees at least half the payoff of the optimal contract. The minimal ratio between the two contracts occurs when there is only one agent type, type 0. The intuition for why this is the worst case is because when there is only a single agent type, there is no information asymmetry, so the optimal contract is fully efficient, which maximizes the gap between the two contracts. (Because of the convexity of the cost function, linear contracts cannot recover full efficiency even with no information asymmetry.)

Theorem 1. *For all $k > 0$ and any $F(\theta)$, a linear contract with price $p^L = \frac{k}{2}$ achieves at least half of the optimal contract payoff. Therefore, the best linear contract always yields at least half of the optimal contract payoff.*

Proof Sketch. We provide a proof sketch assuming $F(\theta)$ has a monotone hazard rate; a proof without this assumption is given in Appendix D. First, we construct an upper bound for the optimal payoff, denoted by $\overline{Obj}^{OPT}$, which is the environmental outcome at the optimal contract minus the agent's conservation cost (i.e., the optimal payoff without the information rent). Using Equation (1), we have

$$Obj^{OPT} = \int_{\theta=\underline{\theta}}^{\overline{\theta}} \left[ka^{OPT}(\theta) - c(a^{OPT}(\theta) - \theta a_0) - \frac{1-F(\theta)}{f(\theta)} a_0 c'(a^{OPT}(\theta) - \theta a_0) \right] f(\theta) d\theta \tag{4}$$

$$\leq \int_{\theta=\underline{\theta}}^{\overline{\theta}} \left[ka^{OPT}(\theta) - c(a^{OPT}(\theta) - \theta a_0) \right] \times f(\theta) d\theta \equiv \overline{Obj}^{OPT}. \tag{5}$$

Explicitly substituting the optimal contract solution $a^{OPT}(\theta)$ from Lemma 1 (which holds when $F(\theta)$ satisfies the monotone hazard rate assumption) and simplifying the optimal payoff upper bound leads to

$$\overline{Obj}^{OPT} = \int_{\underline{\theta}}^{\hat{\theta}_1} k\theta a_0 f(\theta) d\theta + \int_{\hat{\theta}_1}^{\hat{\theta}_2} k\theta a_0 + \frac{k^2}{2h} - \frac{h}{2}\left(\frac{1-F(\theta)}{f(\theta)} a_0\right)^2 f(\theta) d\theta + \int_{\hat{\theta}_2}^{\overline{\theta}} ka_0 - \frac{h}{2}(1-\theta)^2 a_0^2 f(\theta) d\theta. \tag{6}$$

Next, applying Equation (3) with a price of $p = \frac{k}{2}$ to obtain the linear contract payoff and doubling both sides of the equation, we obtain

$$2Obj^L\left(p = \frac{k}{2}\right) = \int_{\underline{\theta}}^{\tilde{\theta}} \left(k\theta a_0 + \frac{k^2}{2h}\right) f(\theta) d\theta + \int_{\tilde{\theta}}^{\overline{\theta}} ka_0 f(\theta) d\theta. \tag{7}$$

Then, we show that, for *any* agent type θ, the integrand in Equation (7) is weakly larger than the integrand in Equation (6). When $F(\theta)$ does not have a monotone hazard rate, the optimal allocation $a^{OPT}(\theta)$ is still continuous in θ, allowing us to show that the integrand in (7) is weakly larger than the optimal payoff for any agent type. This completes the proof. □

In addition to guaranteeing a constant fraction of the optimal payoff, this simple linear contract with price $\frac{k}{2}$ is robust to misspecifications of the model parameters $\{F(\theta), h, a_0\}$, which are often difficult for the principal to know precisely. The linear price $\frac{k}{2}$ is independent of the agent type distribution $F(\theta)$, the agent's opportunity cost, and the initial land size a_0. Furthermore, Theorem 1 naturally generalizes to a model where the agents are endowed with different levels of initial forest areas.[8] This is particularly useful because the PES designer can use a single contract uniformly even when agents are heterogeneous in the amounts of forest land they own.

5. A Numerical Study Using U.S. Land Use Data

The theory suggests that there are three interesting regimes for comparing the performance of conditional and linear contracts. These regimes depend on the value of $\frac{k}{ha_0}$, which measures the value of conserving forest relative to its opportunity cost. Recall that in all these regimes, the linear contract provides a 0.5-approximation to the optimal contract (Theorem 1); however, the relative performance of the linear and conditional contracts varies. In this section, we discuss the implications of the theory and illustrate which contract is desirable via a numerical study, which uses

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

baseline conservation distributions and conservation cost functions calibrated using U.S. land use data.

The distributions and cost functions we consider are based on empirically calibrated data from Mason and Plantinga (2013) for 140 regions in the United States classified into four land qualities. Land quality class indicates the level of opportunity costs of forestation and is a rough measure of how profitable it may be to develop the land into nonforest uses. Higher (lower) land quality class regions have higher (lower) conservation costs and lower (higher) baseline conservation levels, and hence higher (lower) baseline deforestation levels. We directly apply the baseline conservation distributions in Mason and Plantinga (2013), and calibrate the parameter h in the conservation cost function $c(x) = \frac{h}{2}x^2$ using their cost functions. A detailed explanation of the baseline distributions and the cost functions is given in the e-companion (Section EC.1).

The environmental value of forests, k (per unit area), used in this numerical study is roughly twice the price of carbon per ton.[9] Although we can evaluate the performance of simple contracts in terms of any environmental value k, we center our discussion around the social cost of carbon, as most PES programs are implemented by national governments. The social cost of carbon instated by the Biden administration in 2021 is \$50 per ton of carbon, equivalent to $\$k = 100$ (Chemnick 2021). If the principal is a carbon buyer from the private sector, it is also reasonable to use the prevailing carbon price in a carbon market as the environmental value k. For example, in the California cap-and-trade auction, the carbon price is about \$20/ton which is equivalent to $k = \$40$.

5.1. Small Relative Value for Conservation

In the first regime, when $\frac{k}{ha_0}$ is very small, neither of the simple contracts can improve upon the baseline scenario, where the agent continues with their baseline conservation amount and no incentives are introduced (Lemmas 4 and 5). At an environmental value of $k = \$100$ per unit area, 130 (119) of the 140 regions in the United States fall in the small $\frac{k}{ha_0}$ regimes where conditional (linear) contracts cannot improve upon the baseline. We show the performance of simple contracts for such regions in the e-companion (Section EC.2) and verify that they do not improve upon the baseline scenario. The small $\frac{k}{ha_0}$ regime is likely to be less relevant in developing countries, where land development revenue is lower relative to the United States.

5.2. Large Relative Value for Conservation

In the second regime, when $\frac{k}{ha_0}$ is large, the best conditional contract is equivalent to the optimal contract (Lemma 3). Figure 1 plots the percentages of the optimal contract payoff that the two simple contracts, as well as the baseline scenario, are able to achieve, for high and low land quality classes in Montana. In the high land quality class (Figure 1(a)), the best conditional contract coincides with the optimal contract for sufficiently large k (larger than \$500 per unit area) and outperforms the best linear contract for k larger than \$350 per unit area. Similarly, in the low land quality class (Figure 1(b)), the best conditional contracts coincide with the optimal contracts when k is larger than \$280 per unit area, and outperform the best linear contracts when k is larger than \$150 per unit area.

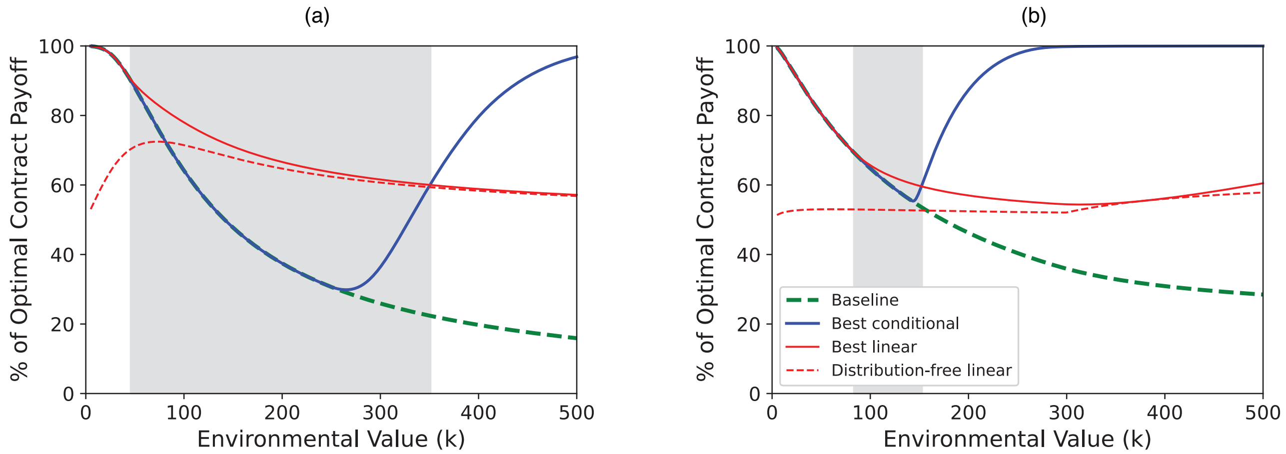
5.3. Intermediate Relative Value for Conservation

Finally, in the intermediate parameter region of $\frac{k}{ha_0}$ and when baseline conservation is low, we have shown that conditional contracts can perform arbitrarily poorly, whereas linear contracts are guaranteed to provide at least half of the optimal contract payoff (Theorem 1). This can be seen in the high land quality region in Montana: In Figure 1(a), for k between \$45 to \$350 per unit area (the grey region), the best linear contracts outperform the best conditional contracts. The best conditional contract achieves only about 30% of the optimal contract payoff, whereas the best linear contract always achieves more than 50%. More states that exhibit similar patterns are provided in the e-companion (Section EC.2).

Ideally, one would repeat the above exercise for many other countries. This requires empirical estimation of the value k of environmental conservation, as well as reliable estimates of the distribution of baseline conservation and other conservation cost parameters. The former is enabled by high-quality fine-grained data on the global carbon stock and biodiversity benefits, which are becoming increasingly available (Strassburg et al. 2020). However, the latter requires sophisticated econometric analysis using detailed parcel-level land use data, as done in Lubowski et al. (2006). As data on conservation costs and deforestation activity are often reported in aggregate on a regional level, aside from the data set for the United States provided in Mason and Plantinga (2013), similar data sets for other countries are not currently available.

Nevertheless, back-of-the-envelope calculations can still provide some helpful benchmarks. We take as an example Brazil, a developing country that continues to experience high levels of deforestation despite having forest conservation programs in place. By comparing the total agricultural land size and agricultural revenue of Brazil to those of the United States (public data from the Organisation for Economic Co-operation and Development land cover data set and the World Bank), we estimate that Brazil's conservation cost function parameter h is scaled down by a factor of more than two relative to the United States.[10]

Figure 1. (Color online) Ratios Between Simple Contracts and OPT at Different Values of k in Montana



Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

At an environmental value of $k \approx \$100$ per unit area, halving the conservation cost parameter h is equivalent to scaling k up to \$200 per unit area. If we assume that Brazil's baseline conservation distribution is similar to that of U.S. regions with high land quality, $\frac{k}{ha_0}$ will be in the intermediate regime (see Figure 1(a) at $k \approx \$200$).

There are reasons to believe that many regions, like Brazil, currently experience the intermediate regime of $\frac{k}{ha_0}$. On one hand, governments recognize the urgency of climate change and have joined international efforts to mitigate climate change (e.g., the REDD+ program), suggesting that $\frac{k}{ha_0}$ is large enough to induce some government action. On the other hand, we still observe high rates of deforestation in countries that already have PES programs set up (Hansen et al. 2013), suggesting that not every country is able to pay for full conservation. In this intermediate regime, conditional contracts can perform poorly and can be substantially improved upon by linear contracts.

6. Extensions

In this section, we consider two natural extensions to our theory, for which we provide robustness results.

6.1. Conditional Linear Contracts

Both the conditional contracts and the linear contracts belong to a class of simple contracts, which we term conditional linear contracts. Conditional linear contracts pay the agent a linear price p per unit area conserved, as long as the conserved area a is above a prespecified threshold wa_0, where $w \in [0,1]$. When $w = 1$, this contract is the conditional contract; when $w = 0$, this contract is the linear contract. This contract allows the principal to adjust both the price paid and the level of stringency of the contract.

Our running example with a single agent type illustrates the benefit from a conditional linear contract. Detailed analysis of the example and conditional linear contracts can be found in Appendix E.

Example 4. When there is only one agent type $\theta = 0$, if $\frac{k}{ha_0} \le 1$, the best conditional linear contract is $w = \frac{k}{ha_0}$ with price $p = \frac{k}{2}$; otherwise, the best conditional linear contract is $w = 1$ with price $p = \frac{ha_0}{2}$. Furthermore, this best conditional linear contract achieves 100% of the optimal contract payoff. However, that the best linear and the best conditional contracts guarantee only 50% and 0% of the optimal contract payoff, respectively.

We build on the guarantees provided by linear contracts (Theorem 1) by showing that a distribution-free conditional linear contract improves upon the distribution-free linear contract.

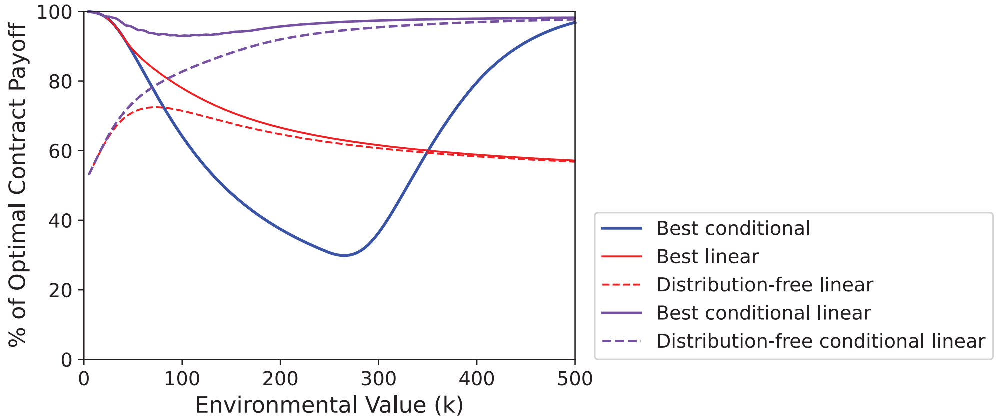
Theorem 2. *For all $k > 0$ and any $F(\theta)$, the conditional linear contract with $w = \min\{\frac{k}{ha_0}, 1\}$ and $p = \min\{\frac{k}{2}, \frac{ha_0}{2}\}$ always achieves higher payoff than the linear contract with price $\frac{k}{2}$, and guarantees at least half of the payoff of the optimal contract.*

When k is relatively small, this distribution-free conditional linear contract uses the same price as the distribution-free linear contract suggested in Theorem 1 but with an intermediate level of stringency; as k gets larger, this conditional linear contract becomes a conditional contract that pays enough so that any agent type conserves fully.

Figure 2 plots the simulated performances of the distribution-free conditional linear contracts identified in Theorem 2 and the best conditional linear contracts in the high land quality region of Montana. The best conditional linear contracts perform significantly better than both the best conditional and the best linear

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

Figure 2. (Color online) Ratios Between the Simple Contracts and OPT at Different Values of k in the High Land Quality Region of Montana



contracts. Both the conditional and linear contracts have only a single parameter (the price), but the conditional linear contracts have two parameters without sacrificing much simplicity. As the principal increases the number of parameters to be optimized in a contract, the principal can get closer to the optimal payoff. More importantly, the distribution-free conditional linear contracts significantly improve upon both the best conditional and the best linear contracts in the region $k \geq 100$.

6.2. General Convex Cost Function

The model assumes the cost function to be quadratic. We discuss here how the results change under a more general convex function $c(x)$ with a bounded second derivative, that is, $c''(x) \in [l, u]$, for all $x \in [0, a_0]$, where $u > l > 0$.

First, when $\frac{k}{ua_0}$ is large enough, the best conditional contract still coincides with the optimal contract.

Lemma 6. *When* $\frac{k}{ua_0} \geq (1 - \underline{\theta} + \frac{1}{f(\underline{\theta})})$, *the optimal contract and the best conditional contract coincide; thus,* $Obj^{C^*} = Obj^{OPT}$. *Every agent type* θ *conserves fully, that is,* $a(\theta) = a_0$, *and receives a payment of* $\frac{u}{2}(1-\underline{\theta})^2 a_0^2$.

Next, when $\frac{k}{la_0}$ is small enough, both kinds of simple contracts do not improve upon the baseline scenario.

Lemma 7. *When* $\frac{k}{la_0} \leq \frac{1-\overline{\theta}}{2}$, *the best conditional contract is the baseline scenario; thus,* $Obj^{C^*} = k\mathbb{E}[\theta]a_0$. *When* $\frac{k}{la_0} \leq \mathbb{E}[\theta]$, *the best linear contract is the baseline scenario; thus,* $Obj^{L^*} = k\mathbb{E}[\theta]a_0$.

The above two lemmas are simple extensions of Lemmas 3–5, where we bound the objectives using the appropriate u or l terms.

Finally, selecting the *best* contract out of the two kinds of simple contracts guarantees a constant-factor approximation to the optimal payoff.

Theorem 3. *For all* $k > 0$ *and* $F(\theta)$, *one of the simple contracts achieves at least* $\frac{l}{2u}$ *of the optimal contract payoff. Furthermore, there exists a threshold* $k^* \equiv \frac{2u^2}{u+l}a_0$ *such that*

i. *if* $k \leq k^*$, *the linear contract with price* $p^L = \frac{k}{2}$ *yields at least* $\frac{l}{2u}$ *of the optimal contract;*

ii. *if* $k > k^*$, *the conditional contract with price* $p^C = \frac{ua_0}{2}(1-\underline{\theta})^2$ *yields at least* $\frac{l}{2u}$ *of the optimal contract.*

In the general convex cost setting, in contrast with Theorem 1, a linear contract can no longer offer the approximation guarantee when k is large. Moreover, the approximation factor depends on the curvature of the cost function $c(\cdot)$.

The simple contracts in Theorem 3 are able to achieve the approximation guarantee with prices that are not optimized to the specific parameters in the model. The linear contract price $p^L = \frac{k}{2}$ is the same as that used in Theorem 1, which is independent of the baseline conservation distribution; the conditional contract price $p^C = \frac{ua_0}{2}(1-\underline{\theta})^2$ is the generalized version of $p^C = \frac{h}{2}(1-\underline{\theta})^2 a_0$ in the base model, which is the price used to induce full conservation regardless of the agent type.

7. Conclusion

This paper compares the performance of conditional and linear contracts for payments for ecosystem services. We find that the commonly used conditional contracts can have mixed outcomes: they can perform poorly when the relative environmental value is intermediate. Linear contracts are often overlooked, but we show that they have desirable robustness properties and can sometimes improve upon conditional contracts. In particular, a linear contract with an easily constructed per-unit price guarantees half the optimal contract payoff. Existing contracts with full conditionality should not be assumed to be a panacea for PES

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

programs; relaxing the conditionality requirement can actually improve conservation outcomes.

We believe that there is an abundance of research opportunity in using theoretical modeling to help inform program design for disincentivizing deforestation. Our model abstracts away from numerous features of potential interest. Contracts are typically offered over a long time period, over which the conservation costs can be stochastic. Lessons from the development economics literature can be helpful in better modeling agent behavior; for example, smallholder farmers in developing countries typically face complex and stochastic financial constraints (Jayachandran 2013, Lansing 2017). Monitoring conservation activities is still costly in practice at the moment, suggesting the use for robust contract design (Carroll 2015, Dütting et al. 2019, Yu and Kong 2020). Moreover, the value of natural assets are often heterogeneous and complementary; conserving one contiguous parcel of forest is better than conserving two noncontiguous parcels of half the size. Another important direction is to incorporate a richer action space to include sustainable land management practices, instead of a single dimensional deforestation quantity; such an approach will be especially valuable for achieving the UN Sustainable Development Goals. All of these directions merit future exploration. Finally, to identify the potential payoff of contracts, there is need for additional empirical research estimating conservation costs in high-deforestation regions across the world.

It is our hope that future interdisciplinary work can bring theoretical techniques from operations management together with empirical and practical insights from environmental and development economics to more holistically address problems at the heart of environmental sustainability.

Acknowledgments
The authors thank Andrew Plantinga for generously sharing data from Mason and Plantinga (2013) for the numerical simulation section. The authors also thank (in random order) Jim Leape, Lassi Ahlvik, Ben Roth, Alex Teytelboym, John Weyant, Seema Jayachandran, Eli Fenichel, Gretchen Daily, Ramesh Johari, Ilya Segal, two anonymous referees, the associate editor, the departmental editor, and seminar participants at the 2021 INFORMS Workshop on Market Design, the Surrey Business School Workshop on Public Policy and Deforestation, the Helsinki Graduate School of Economics, the 2020 Mechanism Design for Social Good workshop, the 2019 INFORMS annual conference, and the Stanford Natural Capital Project for helpful comments.

Appendix A. Analysis of the Optimal Contract
First, we provide the version of Lemma 1 with explicit expressions of payment levels when $F(\theta)$ has a monotone hazard rate (i.e., $\frac{1-F(\theta)}{f(\theta)}$ is weakly decreasing in θ). In the proof, we show how to apply the ironing technique on the region where $\frac{1-F(\theta)}{f(\theta)}$ is not weakly decreasing in θ.

Lemma A.1 (Full Version of Lemma 1). *The optimal contract (when $F(\theta)$ has a monotone hazard rate) is given by*

$$(a(\theta),P(\theta)) = \begin{cases} (\theta a_0, 0) & \text{if } \theta \le \hat{\theta}_1, \\ \Big(\theta a_0 + \frac{k}{h} - \frac{1-F(\theta)}{f(\theta)}a_0,\ \frac{h}{2}\Big(\frac{k}{h} - \frac{1-F(\theta)}{f(\theta)}a_0\Big)^2 \\ \quad + \int_{\hat{\theta}_1}^{\theta} ha_0\Big(\frac{k}{h} - \frac{1-F(\tau)}{f(\tau)}a_0\Big)d\tau\Big) & \text{if } \theta \in (\hat{\theta}_1, \hat{\theta}_2), \\ \Big(a_0,\ \min\Big\{\frac{h}{2}\Big(\frac{k}{h} - \frac{1-F(\hat{\theta}_2)}{f(\hat{\theta}_2)}a_0\Big)^2 \\ \quad + \int_{\hat{\theta}_1}^{\hat{\theta}_2} ha_0\Big(\frac{k}{h} - \frac{1-F(\tau)}{f(\tau)}a_0\Big)d\tau, h(1-\underline{\theta})^2 a_0^2\Big\}\Big) & \text{if } \theta \ge \hat{\theta}_2, \end{cases}$$

where $\hat{\theta}_1$ is defined by $\frac{1-F(\hat{\theta}_1)}{f(\hat{\theta}_1)} = \frac{k}{ha_0}$ if $\frac{1}{f(\underline{\theta})} \ge \frac{k}{ha_0}$ and otherwise is given by $\hat{\theta}_1 = \underline{\theta}$, and $\hat{\theta}_2$ is defined by $1-\hat{\theta}_2 + \frac{1-F(\hat{\theta}_2)}{f(\hat{\theta}_2)} = \frac{k}{ha_0}$ if $1-\underline{\theta} + \frac{1}{f(\underline{\theta})} \ge \frac{k}{ha_0}$, and otherwise is given by $\hat{\theta}_2 = \underline{\theta}$.

Proof of Lemma A.1. Standard analysis (e.g., Mirrlees 1971) shows the principal's problem is equivalent to solving the following:

$$a(\theta) = \underset{a(\theta)\in[\theta a_0, a_0]}{\arg\max} \int_{\theta=\underline{\theta}}^{\overline{\theta}} [ka(\theta) - c(a(\theta) - \theta a_0) - \frac{1-F(\theta)}{f(\theta)}a_0 c'(a(\theta)-\theta a_0)]f(\theta)d\theta, \tag{A.1}$$

$$P(\theta) = c(a(\theta) - \theta a_0) + \int_{\tau=\underline{\theta}}^{\theta} a_0 c'(a(\tau) - \tau a_0)d\tau, \tag{A.2}$$

along with the monotonicity constraint that $\frac{da(\theta)}{d\theta} \ge 0$. For each θ, $a(\theta)$ maximizes $J(\theta,a) \equiv ka - c(a-\theta a_0) - \frac{1-F(\theta)}{f(\theta)}a_0 c'(a-\theta a_0)$. When $\frac{1-F(\theta)}{f(\theta)}$ is nonincreasing in θ (monotone hazard rate), $a(\theta)$ can be determined by its first order condition because $\delta J(\theta,a)/\delta a$ is nondecreasing in θ. Explicitly,

$$\frac{\delta J(\theta)}{\delta a} = k - c'(a-\theta a_0) - \frac{1-F(\theta)}{f(\theta)}a_0 c''(a-\theta a_0) = k - h(a-\theta a_0) - ha_0\frac{1-F(\theta)}{f(\theta)}. \tag{A.3}$$

The boundary solution $a = \theta a_0$ appears when $\theta \le \hat{\theta}_1$, where $\hat{\theta}_1$ solves $\frac{\delta J(\theta)}{\delta a}|_{a=\hat{\theta}a_0} = k - ha_0\frac{1-F(\theta)}{f(\theta)} = 0$ if $k - ha_0\frac{1}{f(\underline{\theta})} \le 0$, and otherwise $\hat{\theta}_1 = \underline{\theta}$. The other boundary solution $a = a_0$ appears when $\theta \ge \hat{\theta}_2$, where $\hat{\theta}_2$ solves $\frac{\delta J(\theta)}{\delta a}|_{a=a_0} = k - h\,(1-\theta)a_0 - ha_0\frac{1-F(\theta)}{f(\theta)} = 0$ if $k - h(1-\underline{\theta})a_0 - ha_0\frac{1}{f(\underline{\theta})} \le 0$, and otherwise $\hat{\theta}_2 = \underline{\theta}$. The interior solution has the first order condition $\frac{\delta J(\theta)}{\delta a} = 0$, leading to $a(\theta) = \theta a_0 + \frac{k}{h} - \frac{1-F(\theta)}{f(\theta)}a_0$.

In the region of θ without a monotone hazard rate, we cannot set $\frac{\delta J(\theta)}{\delta a} = 0$ because otherwise $a(\theta)$ is decreasing in θ, thus violating the monotonicity constraint of $a(\theta)$. Instead, we apply standard ironing techniques, identifying an interval $[\theta_1, \theta_2]$ on which the monotonicity constraint

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

is binding, that is, a pooling solution $a(\theta)=\hat{a}$ for all $\theta\in[\theta_1,\theta_2]$. The interval (θ_1,θ_2) is determined via

$$\int_{\theta_1}^{\theta_2}\frac{\delta J(\theta,\hat{a})}{\delta\hat{a}}f(\theta)d\theta=0.$$

Explicitly, the optimal solution is pooling on the interval (θ_1,θ_2), that is, $a(\theta_1)=a(\theta_2)=\hat{a}$, for all $\theta\in(\theta_1,\theta_2)$. If $\theta_1<\hat{\theta}_1$ or $\theta_2>\hat{\theta}_2$, we can update the boundary solutions of $\hat{\theta}_1$ and $\hat{\theta}_2$ so that they are the largest θ such that $a(\theta)=\theta a_0$ and the smallest θ such that $a(\theta)=a_0$, respectively.

Finally, calculating the payment rules for each interval of $a(\theta)$ using Equation (A.2) gives the stated payment levels. □

Proof of Lemma 2. First, the principal's optimization problem when using a menu of affine contracts $\{p(\theta),T(\theta)\}_\theta$ is given by

$$Obj^{AFF}=\max_{\{p(\theta),T(\theta)\}}\mathbb{E}[ka(\theta)-(p(\theta)a(\theta)+T(\theta))]$$
$$\text{s.t.}\quad p(\theta)a(\theta)+T(\theta)-c(a(\theta)-\theta a_0)\geq 0, \qquad \text{(IR)}$$

$$\theta=\arg\max_{\theta'\in[0,1]}p(\theta')a(\theta'\mid\theta)+T(\theta')-c(a(\theta'\mid\theta)-\theta a_0), \qquad \text{(ICI)}$$
$$a(\theta'\mid\theta)=\arg\max_{a\in[\theta a,a_0]}p(\theta')a+T(\theta')-c(a-\theta a_0),\ \forall\theta,\ \forall\theta'. \qquad \text{(ICII)}$$

The first IC constraint states that the agent of type θ most prefers the affine contract $\{p(\theta),T(\theta)\}$. The second IC constraint states that the agent θ, having picked any contract $\{p(\theta'),T(\theta')\}$, maximizes their utility by choosing the best $a(\theta'\mid\theta)$. Together, both IC constraints ensure $a(\theta'\mid\theta)=a(\theta)$.

First, we find $a(\theta'\mid\theta)$ by solving (ICII). When an interior solution exists, we get

$$\frac{d}{da}(p(\theta')a+T(\theta')-c(a-\theta a_0))=p(\theta')-c'(a-\theta a_0)$$
$$=p(\theta')-h(a-\theta a_0)=0,$$
$$\Rightarrow a(\theta'\mid\theta)=\frac{1}{h}p(\theta')+\theta a_0.$$

Again, we integrate the constraint (ICI) into the objective to determine the exact pairs of $\{p(\theta),T(\theta)\}$ via information rent, that is, first using Equation (1) to rewrite the objective and then establishing the payment by

$$p(\theta)a(\theta)+T(\theta)=c(a(\theta)-\theta a_0)+\int_{\tau=\underline{\theta}}^{\theta}a_0c'(a(\tau)-\tau a_0)d\tau. \qquad \text{(A.4)}$$

Plugging in the solution of $a(\theta)$ into Equation (1),

$$Obj^{AFF}=\int_{\underline{\theta}}^{\theta_1}k\theta a_0dF(\theta)+\int_{\theta_1}^{\theta_2}k\left(\frac{1}{h}p(\theta)+\theta a_0\right)-\frac{h}{2}\left(\frac{1}{h}p(\theta')\right)^2$$
$$-\frac{1-F(\theta)}{f(\theta)}a_0p(\theta)dF(\theta)+\int_{\theta_2}^{\overline{\theta}}ka_0-\frac{h}{2}(1-\theta)^2a_0^2$$
$$-\frac{1-F(\theta)}{f(\theta)}h(1-\theta)a_0^2dF(\theta),$$

where θ_1,θ_2 are the boundaries where the agent type is not at an interior solution.

For each θ, the optimal linear price $p(\theta)$ is set by maximizing the integrand of Obj^{AFF} at θ. When an interior solution exists,

$$\frac{d}{dp}k\left(\frac{1}{h}p(\theta)+\theta a_0\right)-\frac{h}{2}\left(\frac{1}{h}p(\theta')\right)^2-\frac{1-F(\theta)}{f(\theta)}a_0p(\theta)$$
$$=\frac{k}{h}-\frac{p}{h}-a_0\frac{1-F(\theta)}{f(\theta)}=0,$$
$$\Rightarrow p(\theta)=k-ha_0\frac{1-F(\theta)}{f(\theta)}.$$

The boundary solutions are established by

$$\hat{\theta}_1: p(\hat{\theta}_1)=0\Rightarrow\frac{k}{h}-\frac{1-F(\hat{\theta}_1)}{f(\hat{\theta}_1)}a_0=0,$$

$$\hat{\theta}_2:\frac{1}{h}p(\hat{\theta}_2)+\hat{\theta}_2a_0=a_0\Rightarrow\frac{k}{h}-(1-\hat{\theta}_2)a_0-\frac{1-F(\hat{\theta}_2)}{f(\hat{\theta}_2)}a_0=0.$$

Note that $\hat{\theta}_1,\hat{\theta}_2$ are exactly the same boundaries that we established in the optimal allocation in Lemma 1. Furthermore, when $\theta\leq\hat{\theta}_1$, $p(\theta)=p(\hat{\theta}_1)=0$, and when $\theta\geq\hat{\theta}_2$, $p(\theta)=p(\hat{\theta}_2)$.

Now we can establish $T(\theta)$ for each θ using Equation (A.4):

- When $\theta\leq\hat{\theta}_1$, $p(\hat{\theta}_1)\theta a_0+T(\theta)=0$. Recall $p(\hat{\theta}_1)=0$, and thus $T(\theta)=0$. In this region, no one participates.
- When $\theta\in(\hat{\theta}_1,\hat{\theta}_2)$, $p(\theta)a(\theta)+T(\theta)=\frac{h}{2}\frac{p(\theta)^2}{h^2}+\int_{\hat{\theta}_1}^{\theta}a_0p(\tau)d\tau$. Thus,

$$T(\theta)=\frac{1}{2h}p^2(\theta)-p(\theta)a(\theta)+\int_{\hat{\theta}_1}^{\theta}a_0p(\tau)d\tau.$$

- When $\theta\geq\hat{\theta}_2$, similarly, we have

$$T(\theta)=\frac{h}{2}(1-\theta)^2a_0^2-p(\hat{\theta}_2)a_0+\int_{\hat{\theta}_1}^{\hat{\theta}_2}a_0p(\tau)d\tau+\int_{\hat{\theta}_2}^{\theta}h(1-\tau)a_0^2d\tau$$
$$=\frac{h}{2}(1-\theta)^2a_0^2-a_0^2h(1-\theta_2)+\int_{\hat{\theta}_1}^{\hat{\theta}_2}a_0\left(k-ha_0\frac{1-F}{f}(\tau)\right)d\tau$$
$$+\int_{\hat{\theta}_2}^{\theta}h(1-\tau)a_0^2d\tau$$
$$=\frac{1}{2}ha_0^2(1-\hat{\theta}_2)^2-(1-\hat{\theta}_2)ha_0^2+\int_{\hat{\theta}_1}^{\hat{\theta}_2}a_0\left(k-ha_0\frac{1-F}{f}(\tau)\right)d\tau.$$

Furthermore, this menu of affine contracts implements the optimal allocation because the conservation quantity $a(\theta)$ and the payment $P(\theta)=p(\theta)a(\theta)+T(\theta)$ are the same as those in Lemma 1. □

Appendix B. Structure of Simple Contracts

We show that both simple contracts have quasiconcave or concave objectives, which considerably simplifies our analysis of the best prices and best possible payoffs for each contract.

For the next lemma on the objective of conditional contracts, we assume $F(\theta)$ has a monotone hazard rate.

Lemma B.1. *When $\frac{1-F(\theta)}{f(\theta)}$ is nonincreasing in θ, the conditional contract objective Obj^C is quasiconcave in p. Hence, there exists a unique price $p\in[h(1-\overline{\theta})^2a_0,h(1-\underline{\theta})^2a_0]$ that maximizes the conditional contract objective.*

Proof of Lemma B.1. Recall that $\hat{\theta}(p)=1-\sqrt{\frac{2p}{ha_0}}$ is the threshold type at which the agent is indifferent between choosing full conservation and baseline conservation. We

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

can rewrite the conditional objective (Equation (2)) using Equation (1) by inserting the agent's best response to a conditional contract, $a(\theta) = \theta a_0$ for $\theta \le \hat{\theta}$ and $a(\theta) = a_0$ for $\theta > \hat{\theta}$, leading to

$$Obj^C(\hat{\theta}) = \int_{\underline{\theta}}^{\hat{\theta}} k\theta a_0 f(\theta)d\theta + \int_{\hat{\theta}}^{\overline{\theta}} \left(ka_0 - \frac{h}{2}(1-\theta)^2 a_0^2 - h(1-\theta)\frac{1-F(\theta)}{f(\theta)}a_0^2\right) f(\theta)d\theta$$
$$= ka_0\mathbb{E}\theta + \int_{\hat{\theta}}^{\overline{\theta}} a_0 X(\theta)(1-\theta)f(\theta)d\theta,$$

where $X(\theta) \equiv k - \frac{h}{2}(1-\theta)a_0 - h\frac{1-F(\theta)}{f(\theta)}a_0$. Observe that $X(\theta)$ is nondecreasing in θ when $\frac{1-F(\theta)}{f(\theta)}$ is nonincreasing in θ.

To show that $Obj^C(p)$ is quasiconcave on $p \in [\frac{h}{2}(1-\overline{\theta})^2 a_0, \frac{h}{2}(1-\underline{\theta})^2 a_0]$, it is equivalent to show that $Obj^C(\hat{\theta})$ is quasiconcave on $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$, that is, for any $\underline{\theta} < \tau_1 < \tau_2 \le \overline{\theta}$,

$$Obj^C(\hat{\theta} = \lambda\tau_1 + (1-\lambda)\tau_2) > \min\{Obj^C(\tau_1), Obj^C(\tau_2)\}.$$

The first order condition of the conditional contract objective is the following:

$$\frac{d}{d\hat{\theta}} Obj^C(\hat{\theta}) = -a_0 X(\hat{\theta})(1-\hat{\theta})f(\hat{\theta}) = 0.$$

The first order condition is satisfied at at most one point in $(\underline{\theta}, \overline{\theta})$ when $X(\hat{\theta}) = 0$, because $X(\hat{\theta})$ is nondecreasing. Set τ such that $X(\tau) = 0$. We know that when $\hat{\theta} < \tau$, the conditional contract objective increases because $\frac{d}{d\hat{\theta}} Obj^C(\hat{\theta}) > 0$; when $\hat{\theta} > \tau$, the conditional contract objective decreases because $\frac{d}{d\hat{\theta}} Obj^C(\hat{\theta}) < 0$. Thus, the conditional contract objective is quasiconcave. Furthermore, there exists a unique price that maximizes the conditional objective. The best conditional price has an interior solution $p^{C^*} \in (\frac{h}{2}(1-\overline{\theta})^2 a_0, \frac{h}{2}(1-\underline{\theta})^2 a_0)$ when $X(\hat{\theta}(p^{C^*})) = 0$; otherwise, it is a boundary solution. □

Lemma B.2. *The linear contract objective Obj^L is concave in $p \in [0, h(1-\underline{\theta})a_0]$.*

Proof of Lemma B.2. We show that the objective's second order derivative is negative. Recall that, depending on the price p, there may be agents whose conservation amount meets the boundary condition $a = a_0$ in the linear contract objective (Equation (3)). Thus, we need to consider two cases:

i. If $p \in [0, h(1-\overline{\theta})a_0]$, all agents have $a(\theta) = \theta a_0 + \frac{p}{h}$, $\frac{dObj^L}{dp} = \frac{1}{h}(k - 2p^{L^*}) - \mathbb{E}\theta a_0$, and $\frac{d^2 Obj^L}{dp^2} = -\frac{2}{h} < 0$.

ii. Otherwise, all agents with $\theta \le \tilde{\theta}(p)$ have conservation action $a(\theta) = \theta a_0 + \frac{p}{h}$, and all other agents with $\theta > \tilde{\theta}(p)$ have $a(\theta) = a_0$. Differentiating the objective, we have $\frac{dObj^L}{dp} = \frac{1}{h}(k-p) F(\tilde{\theta}) - a_0\left(\int_{\underline{\theta}}^{\tilde{\theta}} \theta dF(\theta) + 1 - \tilde{\theta}F(\tilde{\theta})\right)$; the second order derivative is

$$\frac{d^2 Obj^L}{dp^2} = \frac{1}{h}\left((k-p)f(\tilde{\theta})\frac{d\tilde{\theta}}{dp} - F(\tilde{\theta})\right) - a_0\left(\tilde{\theta}f(\tilde{\theta}) - \tilde{\theta}f(\tilde{\theta}) - F(\tilde{\theta})\right)\frac{d\tilde{\theta}}{dp}$$
$$= -\frac{1}{h}\left(\frac{1}{ha_0}(k-p)f(\tilde{\theta}) + 2F(\tilde{\theta})\right) \le 0.$$

Hence, the objective is concave in p. □

Lemmas B.1 and B.2 imply that to find the optimal price for each type of contract, it suffices to find the price that satisfies the first order condition (and to pick a boundary price if no such price exists). Moreover, the optimal price for each type of contract is unique.

Corollary B.1. *The best conditional and linear contracts are given by prices satisfying the following first order conditions if an interior solution exists, and a boundary solution otherwise. The best conditional price $p^{C^*} \in [\frac{h}{2}(1-\overline{\theta})^2 a_0, \frac{h}{2}(1-\underline{\theta})^2 a_0]$ has an interior solution that solves the following equation:*

$$\left.\frac{dObj^C}{dp}\right|_{p^{C^*}} = a_0 f(\hat{\theta})\left(\frac{k}{ha_0} - \frac{1}{2}\sqrt{\frac{2p^{C^*}}{ha_0}} - \frac{1-F(\hat{\theta})}{f(\hat{\theta})}\right) = 0, \quad \text{(B.1)}$$

where $\hat{\theta} = \hat{\theta}(p^{C^*}) = 1 - \sqrt{\frac{2p^{C^*}}{ha_0}}$. The best linear price $p^{L^*} \in [0, h(1-\underline{\theta})a_0]$ has an interior solution that solves the following equation:

$$\left.\frac{dObj^L}{dp}\right|_{p^{L^*}} = \begin{cases} \frac{1}{h}(k - p^{L^*})F(\tilde{\theta}) \\ \quad -a_0\left(\int_{\underline{\theta}}^{\tilde{\theta}} \theta\, dF(\theta) + 1 - \tilde{\theta}F(\tilde{\theta})\right) = 0, & \text{if } p^{L^*} \ge h(1-\overline{\theta})a_0, \\ \frac{1}{h}(k - 2p^{L^*}) - \mathbb{E}\theta a_0 = 0, & \text{if } p^{L^*} < h(1-\overline{\theta})a_0, \end{cases} \quad \text{(B.2)}$$

where $\tilde{\theta}(p) = 1 - \frac{p}{ha_0}$.

Appendix C. Proofs for Section 3 (Conditional Contracts)

Proof of Lemma 3. Recall that in Lemma 1, the optimal solution will pool agents with highest types $\theta \in [\hat{\theta}_2, \overline{\theta}]$ to fully conserve. When $\frac{k}{ha_0}$ is large enough so that $\hat{\theta}_2 = \underline{\theta}$, the optimal contract will pool the entire population to fully conserve. Explicitly, when $\frac{k}{ha_0} \ge (1-\underline{\theta}) + \frac{1}{f(\underline{\theta})}$, the optimal contract solution is $a(\theta) = a_0, p(\theta) = h(1-\underline{\theta})^2 a_0^2$ for all θ. Similarly, in the conditional contract, if $\frac{k}{ha_0}$ is large enough so that $\hat{\theta} = \underline{\theta}$, the best conditional price will be high enough such that the entire population will fully conserve. Using Corollary B.1, when $\frac{k}{ha_0} \ge \frac{1}{2}(1-\underline{\theta}) + \frac{1}{f(\underline{\theta})}$, the best conditional contract has $p^{C^*} = \frac{h}{2}(1-\underline{\theta})^2 a_0$, thus $\hat{\theta}(p^{C^*}) = \underline{\theta}$. Thus, the solutions to the optimal contract and the best conditional contract overlap when $\frac{k}{ha_0} \ge (1-\underline{\theta}) + \frac{1}{f(\underline{\theta})}$. □

Proof of Lemma 4. In a conditional contract, if the price is less than $\frac{h}{2}(1-\overline{\theta})^2 a_0$, then no agent will participate in the conditional contract, thus resulting in the baseline scenario. To identify the sufficient condition when the optimal conditional price is at most at $p^{C^*} = \frac{h}{2}(1-\overline{\theta})^2 a_0$, we use the first order condition of the conditional contracts in Corollary B.1: when $\frac{k}{ha_0} \le \frac{1}{2}(1-\overline{\theta})$, $\left.\frac{dObj^C}{dp}\right|_{p=\frac{h}{2}(1-\overline{\theta})^2 a_0} = f(\overline{\theta})(\frac{k}{ha_0} - \frac{1-\overline{\theta}}{2}) \le 0$. □

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

Proof of Example 2. Given an exponential distribution with support bounded by $[0,1]$, $F(\theta)=\frac{1-\exp(-\lambda\theta)}{1-\exp(-\lambda)}$, we show that $\lim_{\lambda\to\infty}\frac{Obj^{C^*}}{Obj^{OPT}}=0$ for $\frac{k}{ha_0}=\frac{1}{2}$.

First, we write out the optimal contract payoff by plugging the solution from Lemma 1 into Equation (A.1) and then simplifying it:

$$\begin{aligned}Obj^{OPT} &= \int_0^{\hat\theta_1} k\theta a_0 f(\theta)d\theta + \int_{\hat\theta_2}^1\left(ka_0-\frac{h}{2}(1-\theta)^2a_0^2\right.\\&\quad\left.-h(1-\theta)\frac{1-F(\theta)}{f(\theta)}a_0^2\right)f(\theta)d\theta\\&\quad+\int_{\hat\theta_1}^{\hat\theta_2}\left(k\left(\theta a_0+\frac{k}{h}-\frac{1-F(\theta)}{f(\theta)}a_0\right)-\frac{h}{2}\left(\frac{k}{h}-\frac{1-F(\theta)}{f(\theta)}a_0\right)^2\right.\\&\quad\left.-h\left(\frac{k}{h}-\frac{1-F(\theta)}{f(\theta)}a_0\right)\frac{1-F(\theta)}{f(\theta)}a_0\right)f(\theta)d\theta\\&=ka_0\mathbb{E}[\theta]+\int_{\hat\theta_1}^{\hat\theta_2}\frac{h}{2}\left(\frac{k}{h}-\frac{1-F(\theta)}{f(\theta)}a_0\right)^2f(\theta)d\theta\\&\quad+\int_{\hat\theta_2}^1(1-\theta)a_0\left(k-\frac{h}{2}(1-\theta)a_0-h\frac{1-F(\theta)}{f(\theta)}a_0\right)f(\theta)d\theta.\end{aligned}$$

The objective for the conditional contract (Equation (2)) can be also be written as a function of the threshold $\hat\theta(p)=1-\sqrt{\frac{2p}{ha_0}}$:

$$\begin{aligned}Obj^C(\hat\theta)&=ka_0\int_0^{\hat\theta}\theta f(\theta)d\theta+\left(k-\frac{h}{2}(1-\hat\theta)^2a_0\right)\int_{\hat\theta}^1 f(\theta)d\theta\\&=ka_0\mathbb{E}[\theta]+a_0\int_{\hat\theta}^1\left(k(1-\theta)-\frac{h}{2}(1-\hat\theta)^2a_0\right)f(\theta)d\theta.\end{aligned}$$

When $\lambda\to\infty$, $\mathbb{E}[\theta]$ tends to zero, and $\frac{1-F(\theta)}{f(\theta)}=\frac{1}{\lambda}\left(1-\frac{\exp(-\lambda)}{\exp(-\lambda\theta)}\right)$ approaches zero pointwise. Recall that the optimality condition of the conditional contract (Corollary B.1) in terms of $\hat\theta$ is $\frac{1-\hat\theta}{2}+\frac{1-F(\hat\theta)}{f(\hat\theta)}=\frac{k}{ha_0}=\frac{1}{2}$. This implies that when $\lambda\to\infty$, $\hat\theta$ approaches zero from the right. In the best conditional contract, $a_0\int_0^1(k(1-\theta)-\frac{h}{2}(1-\hat\theta)^2a_0)f(\theta)d\theta$ goes to zero.

Furthermore, recall that in the optimal solution (Lemma 1), $\hat\theta_2$ satisfies $1-\hat\theta_2+\frac{1-F(\hat\theta_2)}{f(\hat\theta_2)}=\frac{k}{ha_0}=\frac{1}{2}$. This implies that when $\lambda\to\infty$, $\hat\theta_2$ approaches $\frac{1}{2}$ from the left; because $\frac{1-F(0)}{f(0)}<\frac{k}{ha_0}$, $\hat\theta_1=0$ (Lemma 1). In the optimal contract, $\int_0^{\frac{1}{2}}\frac{h}{2}\left(\frac{k}{h}-\frac{1-F(\theta)}{f(\theta)}a_0\right)^2f(\theta)d\theta$ goes to $\frac{k^2}{2h}$ and the rest of the terms go to zero. Together we have that $\frac{Obj^{C^*}}{Obj^{OPT}}$ goes to zero. □

Appendix D. Proofs for Section 4 (Linear Contracts)

Proof of Lemma 5. We show that the optimal linear contract only beats the baseline when k is large enough. We need the sufficient condition when $\tilde\theta=1$ satisfies the first order condition of the linear contract and $p=0$ becomes the best linear price. According to Corollary B.1, when $\frac{k}{ha_0}\le\mathbb{E}\theta$, $\frac{dObj^L}{dp}\big|_{p=0}=\frac{k}{h}-\mathbb{E}\theta a_0\le 0$. □

Proof of Theorem 1. We show that $Obj^L(p=\frac{k}{2})\ge\frac{1}{2}Obj^{OPT}$ for all $k>0$ and any $F(\theta)$.

Let $\overline{Obj}^{OPT}$ denote an upper bound for the optimal payoff, given by considering the payoff from the optimal contract quantities specified in Lemma 1 with a reduced payment that covers only the agent's cost of conservation (i.e., the payment does not cover the agent's information rent). This upper bound $\overline{Obj}^{OPT}$ will allow us to simplify our analysis of the payoff for the second-best contract. In order to show the result is true regardless of $F(\theta)$, we show that, *pointwise at every* θ, the integrand without the $f(\theta)$ term in $2Obj^L(p=\frac{k}{2})$ from Equation (7), which we denote by $2L(\theta)$, is more than the integrand without the $f(\theta)$ term in $\overline{Obj}^{OPT}$ from Equation (5), which we denote by $\overline{OPT}(\theta)$. We can scale down both integrands by $f(\theta)$ because it appears in every term.

By explicitly substituting the optimal contract solution $a^{OPT}(\theta)$ from Lemma 1 into Equation (5), we have the following:

- When $\theta\in[\underline{\theta},\hat\theta_1]$, $a^{OPT}(\theta)=\theta a_0$ and $\overline{OPT}(\theta)=k\theta a_0$.
- When $\theta\in[\hat\theta_1,\hat\theta_2]$, there are two cases. When it is a separating solution without ironing ($\theta\notin[\theta_1,\theta_2]$), plug in $a^{OPT}(\theta)=\theta a_0+\frac{k}{h}-\frac{1-F(\theta)}{f(\theta)}a_0$ to get

$$\begin{aligned}\overline{OPT}(\theta)&=k\left(\theta a_0+\frac{k}{h}-\frac{1-F(\theta)}{f(\theta)}a_0\right)-\frac{h}{2}\left(\frac{k}{h}-\frac{1-F(\theta)}{f(\theta)}a_0\right)^2\\&=k\theta a_0+\frac{k^2}{2h}-\frac{h}{2}\left(\frac{1-F(\theta)}{f(\theta)}a_0\right)^2.\end{aligned}$$

When $F(\theta)$ satisfies the monotone hazard rate, this is the only case we need to consider. Otherwise, on the pooling interval of the ironing solution ($\theta\in[\theta_1,\theta_2]$), $a^{OPT}(\theta)=\hat a=a^{OPT}(\theta_1)=a^{OPT}(\theta_2)$ and $\overline{OPT}(\theta)=\overline{OPT}(\theta_1)=\overline{OPT}(\theta_2)$.

- When $\theta\in[\hat\theta_2,\overline{\theta}]$, $a^{OPT}(\theta)=a_0$ and $\overline{OPT}(\theta)=ka_0-\frac{h}{2}(1-\theta)^2a_0^2$.

With the price $p=\frac{k}{2}$, the linear contract payoff is given by Equation (3), then doubling:

- when $\theta\in[\underline{\theta},\tilde\theta]$, $2L(\theta)=2(k-\frac{k}{2})(\theta a_0+\frac{k}{2h})=k\theta a_0+\frac{k^2}{2h}$;
- when $\theta\in[\tilde\theta,\overline{\theta}]$, $2L(\theta)=2(k-\frac{k}{2})a_0=ka_0$.

All of the following six cases need to be considered because each θ can be at one of the three intervals in the optimal contract (i.e., $[\underline{\theta},\hat\theta_1],[\hat\theta_1,\hat\theta_2]$, and $[\hat\theta_2,\overline{\theta}]$) as well as one of the two intervals in the best linear contract (i.e., $[\underline{\theta},\tilde\theta]$ and $[\tilde\theta,\overline{\theta}]$):

i. If $\theta\in[\underline{\theta},\hat\theta_1]$ and $\theta\in[\underline{\theta},\tilde\theta]$, then $2L(\theta)-\overline{OPT}(\theta)=\left(k\theta_0+\frac{k^2}{2h}\right)-k\theta a_0=\frac{k^2}{2h}>0$.

ii. If $\theta\in[\hat\theta_1,\hat\theta_2]$ and $\theta\in[\underline{\theta},\tilde\theta]$, there are two cases: when θ is outside the pooling interval $[\theta_1,\theta_2]$, $2L(\theta)-\overline{OPT}(\theta)=\frac{h}{2}\left(\frac{1-F(\theta)}{f(\theta)}a_0\right)^2>0$; otherwise, $2L(\theta)-\overline{OPT}(\theta)=2L(\theta)-\overline{OPT}(\theta_1)=k\theta a_0+\frac{k^2}{2h}-k\theta_1a_0-\frac{k^2}{2h}+\frac{h}{2}\left(\frac{1-F(\theta_1)}{f(\theta_1)}a_0\right)^2=k(\theta-\theta_1)a_0+\frac{h}{2}\left(\frac{1-F(\theta_1)}{f(\theta_1)}a_0\right)^2>0$ because $\theta\ge\theta_1$.

iii. If $\theta\in[\hat\theta_2,\overline{\theta}]$ and $\theta\in[\underline{\theta},\tilde\theta]$, then $2L(\theta)-\overline{OPT}(\theta)=k\theta a_0+\frac{k^2}{2h}-ka_0+\frac{h}{2}(1-\theta)^2a_0^2=\frac{h}{2}\left(\frac{k}{h}-(1-\theta)a_0\right)^2>0$.

iv. If $\theta\in[\underline{\theta},\hat\theta_1]$ and $\theta\in[\tilde\theta,\overline{\theta}]$, then $2L(\theta)-\overline{OPT}(\theta)=ka_0-k\theta a_0>0$.

v. If $\theta \in [\hat{\theta}_1, \hat{\theta}_2]$ and $\theta \in [\tilde{\theta}, \overline{\theta}]$, again, there are two cases to consider. When θ is not in the pooling interval $[\theta_1, \theta_2]$,

$$\begin{aligned}2L(\theta) - \overline{OPT}(\theta) &= ka_0 - k\theta a_0 - \frac{k^2}{2h} + \frac{h}{2}\left(\frac{1-F(\theta)}{f(\theta)}a_0\right)^2 \\ &\geq k(1-\theta)a_0 - \frac{k^2}{2h} + \frac{h}{2}\left(\frac{k}{h} - (1-\hat{\theta}_2)a_0\right)^2 \\ &= k(1-\theta)a_0 - k(1-\hat{\theta}_2)a_0 + \frac{h}{2}(1-\hat{\theta}_2)^2 a_0^2 > 0.\end{aligned}$$

The first inequality uses the fact that $F(\theta)$ has the monotone hazard rate on this interval as well as the definition of $\hat{\theta}_2$ in Lemma 1. The last inequality is due to $\theta \leq \hat{\theta}_2$. On the pooling interval $[\theta_1, \theta_2]$, $2L(\theta) - \overline{OPT}(\theta) = 2L(\theta) - \overline{OPT}(\theta_2) \geq 0$ because $a(\theta)$ is continuous and we have shown that $2L(\theta_2) - \overline{OPT}(\theta_2) \geq 0$.

vi. If $\theta \in [\hat{\theta}_2, \overline{\theta}]$ and $\theta \in [\tilde{\theta}, \overline{\theta}]$, then $2L - \overline{OPT}(\theta) = \frac{h}{2}(1-\theta)^2 a_0^2 > 0$.

Together, these show that $2L(\theta) \geq \overline{OPT}(\theta)$ for all θ, and so twice the payoff from the linear contract with price $\frac{k}{2}$ is at least the optimal payoff; equivalently, $2Obj^L(p^L = \frac{k}{2}) \geq \overline{Obj}^{OPT} \geq Obj^{OPT}$. □

Appendix E. Extensions

E.1. Analysis for Section 6.1

Given a conditional linear contract (w, p), where the principal pays p per unit area as long as the conservation amount a is larger than w, the agent's best response is the following:

- When θ is very small, the agent does not conserve beyond the baseline: if $pwa_0 - c((w-\theta)a_0) < 0$, then $a(\theta) = \theta a_0$.
- As θ increases, the agent will conserve at the minimal requirement wa_0: if $pwa_0 - c((w-\theta)a_0) > 0$, then $a(\theta) = wa_0$.
- As θ increases further, the agent will conserve an additional amount beyond the minimal requirement by choosing a to maximize $pa - c(a - \theta a_0)^2$: if $\theta a_0 + \frac{p}{h} \geq wa_0$, then $a(\theta) = \theta a_0 + \frac{p}{h}$.
- The top types of θ conserves fully: if $\theta a_0 + \frac{p}{h} \geq a_0$, then $a(\theta) = a_0$.

The thresholds that divide the above four regions are as follows:

- $w - \sqrt{\frac{2wp}{ha_0}}$ is the type at which the agent indifferent between baseline conservation and conserving the minimal requirement wa_0;
- $w - \frac{p}{ha_0}$ is the largest θ type conserving the minimal requirement wa_0;
- $1 - \frac{p}{ha_0}$ is the smallest θ type conserving the full amount, which is the same with $\tilde{\theta}$ defined for a linear contract with price p.

Thus, the objective of the conditional linear contract (w, p) can be written as

$$\begin{aligned}Obj^{CL}(p) = k\int_{\underline{\theta}}^{w-\sqrt{\frac{2wp}{ha_0}}\theta} a_0 f(\theta)d\theta + (k-p)\Bigg(&\int_{w-\sqrt{\frac{2wp}{ha_0}}}^{w-\frac{p}{ha_0}} wa_0 f(\theta)d\theta \\ &+ \int_{w-\frac{p}{ha_0}}^{1-\frac{p}{ha_0}}\left(\theta a_0 + \frac{p}{h}\right)f(\theta)d\theta + \int_{1-\frac{p}{ha_0}\overline{\theta}a_0} f(\theta)d\theta\Bigg).\end{aligned}$$

In the example of $F(\theta) = \delta(0)$, when $\frac{k}{ha_0} \leq 1$, the principal can set $w = \frac{2p}{ha_0}$ and then maximize $(k-p)wa_0 = (k-p)\frac{2p}{h}$, arriving to $p = \frac{k}{2}$, $w = \frac{k}{ha_0}$, and recovering the optimal payoff $(k-p)wa_0 = \frac{k^2}{2h}$; otherwise, the principal can set $w = 1$ and pay $pa_0 = \frac{h}{2}a_0^2$.

Proof of Theorem 2. When $\frac{k}{ha_0} \leq 1$, simplify the objective of the conditional linear contract with $w = \frac{k}{h}, p = \frac{k}{2}$ to

$$\begin{aligned}&Obj^{CL}\left(w = \frac{k}{ha_0}, p = \frac{k}{2}\right) \\ &= \frac{k}{2}\left(\int_{\underline{\theta}}^{\frac{k}{2ha_0}} \frac{k}{h} f(\theta)d\theta + \int_{\frac{k}{2ha_0}}^{1-\frac{k}{2ha_0}}\left(\theta a_0 + \frac{k}{2h}\right)f(\theta)d\theta + \int_{1-\frac{k}{2ha_0}}^{\overline{\theta}} a_0 f(\theta)d\theta\right) \\ &= \int_{\underline{\theta}}^{\frac{k}{2ha_0}} \frac{k^2}{2h} f(\theta)d\theta + \int_{\frac{k}{2ha_0}}^{1-\frac{k}{2ha_0}}\left(\frac{k}{2}\theta a_0 + \frac{k^2}{4h}\right)f(\theta)d\theta + \int_{1-\frac{k}{2ha_0}}^{\overline{\theta}} \frac{k}{2}a_0 f(\theta)d\theta.\end{aligned}$$

Next we show that $Obj^{CL}(w = \frac{k}{ha_0}, p = \frac{k}{2}) > Obj^L(p = \frac{k}{2})$ at every integrand of θ, where

$$Obj^L\left(p = \frac{k}{2}\right) = \int_{\underline{\theta}}^{1-\frac{k}{2ha_0}}\left(\frac{k}{2}\theta a_0 + \frac{k^2}{4h}\right)f(\theta)d\theta + \int_{1-\frac{k}{2ha_0}}^{\overline{\theta}} \frac{k}{2}a_0 f(\theta)d\theta.$$

Observe that when $\theta > \frac{k}{2ha_0}$, the integrands of both Obj^{CL} and Obj^L are identical; this is because both types of contracts use the same linear price, inducing the same conservation actions from the agents with $\theta > \frac{k}{2ha_0}$. Thus, we only need to compare their integrands when $\theta \in [0, \frac{k}{2ha_0}]$. We have

$$\frac{k^2}{2h} - \frac{k\theta a_0}{2} - \frac{k^2}{4h} = \frac{k^2}{4h} - \frac{k\theta a_0}{2} \geq \frac{k^2}{4h} - \frac{k^2}{4h} = 0.$$

The inequality comes from $\theta \leq \frac{k}{2ha_0}$. When the agent's baseline conservation level is small, the conditional linear contract induces the agent to conserve more than the linear contract because of the required stringency level. At the same time, this conditional linear contract is not as stringent as a conditional contract so that any agent still conserves more than their baseline level.

When $\frac{k}{ha_0} > 1$, it is easy to observe that the conditional linear contract with $w = 1$ and $p = \frac{ha_0}{2}$ achieves a higher payoff than the linear contract with $p = \frac{k}{2}$. This is because the former has payoff $ka_0 - \frac{h}{2}a_0^2$, and the latter has payoff $\frac{k}{2}\mathbb{E}[\theta]a_0$. We have $ka_0 - \frac{h}{2}a_0^2 > \frac{k}{2}a_0 \geq \frac{k}{2}\mathbb{E}[\theta]a_0$, where the first inequality comes from $\frac{k}{ha_0} > 1$ and the second one comes from $\mathbb{E}[\theta] \leq 1$. □

E.2. Proofs for Section 6.2

Proof of Lemma 6. To get a sufficient condition such that the optimal solution will have $\hat{\theta}_2 = \underline{\theta}$, we can use the upper bound of the cost function and treat h in the base model as u; that is, when $\frac{k}{ua_0} \geq (1-\underline{\theta}) + \frac{1}{f(\underline{\theta})}$, the optimal contract solution is $a(\theta) = a_0, p(\theta) = \frac{u}{2}(1-\underline{\theta})^2 a_0^2$ for all θ. Applying the same upper bound will get us a sufficient condition such that the best conditional contract is to have all agents conserve fully; that is, when $\frac{k}{ua_0} \geq \frac{1}{2}(1-\underline{\theta}) + \frac{1}{f(\underline{\theta})}$,

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

the best conditional contract has $\hat{\theta} = \underline{\theta}$. Thus, the solutions to the optimal contract and the conditional contract overlap when $\frac{k}{ua_0} \geq (1-\underline{\theta}) + \frac{1}{f(\underline{\theta})}$. □

Proof of Lemma 7. This result is an extension of Lemma 4 and Lemma 5. It is sufficient to lower bound the cost function by treating l as h in the base model. □

Proof of Theorem 3. Theorem 3 follows from the following two lemmas, which show that for low values of k, the linear contract with per-unit price $\frac{k}{2}$ approximates the optimal payoff, and for high values of k, the conditional contract with per-unit price $\frac{ua_0}{2}$ approximates the optimal payoff.

Lemma E.1. *When* $\frac{k}{a_0} \leq \frac{2u^2}{u+l}$, $Obj^L(p^L = \frac{k}{2}) \geq \frac{l}{2u} Obj^{OPT}$.

Lemma E.2. *When* $\frac{k}{a_0} \geq \frac{2u^2}{u+l}$, $Obj^C(p^C = \frac{ua_0}{2}(1-\underline{\theta})^2) \geq \frac{l}{2u} Obj^{OPT}$.

Proof of Lemma E.1. This proof follows the same structure as the proof of Theorem 1. We first provide a lower bound on the objective of the linear contract with price $p^L = \frac{k}{2}$ by treating u equivalently to h in Equation (3):

$$Obj^L\left(p^L = \frac{k}{2}\right) \geq \int_{\underline{\theta}}^{\tilde{\theta}} \left(k - \frac{k}{2}\right)\left(\theta a_0 + \frac{k}{2u}\right) f(\theta)d\theta + \int_{\tilde{\theta}}^{\overline{\theta}} \left(k - \frac{k}{2}\right) a_0 f(\theta)d\theta.$$

We then upper bound the payoff of the optimal contract by removing the information rent terms and treating l as the h in Lemma 1. We write the proof assuming that $F(\theta)$ has a monotone hazard rate, but as in the proof of Theorem 1, this assumption is not necessary. We have

$$\overline{Obj}^{OPT} \leq \int_{\underline{\theta}}^{\hat{\theta}_1} k\theta a_0 dF + \int_{\hat{\theta}_1}^{\hat{\theta}_2} k\theta a_0 + \frac{k^2}{2l} - \frac{l}{2}\left(\frac{1-F(\theta)}{f(\theta)} a_0\right)^2 f(\theta)d\theta + \int_{\hat{\theta}_2}^{\overline{\theta}} ka_0 - \frac{l}{2}(1-\theta)^2 a_0^2 f(\theta)d\theta. \tag{E.1}$$

We again compare the payoff from the linear contract and the upper bound on the optimal payoff by directly comparing the integrands pointwise for each θ without $f(\theta)$, denoted by $L(\theta)$ and $\overline{OPT}(\theta)$, respectively. We show that $\frac{2u}{l}L(\theta) - \overline{OPT}(\theta) \geq 0$ in each of the following six cases:

i. If $\theta \in [\underline{\theta}, \hat{\theta}_1]$ and $\theta \in [\underline{\theta}, \tilde{\theta}]$, then $\frac{2u}{l}L(\theta) - \overline{OPT}(\theta) \geq \frac{k^2}{2l} + (\frac{u}{l} - 1)\theta k a_0 > 0$.

ii. If $\theta \in [\hat{\theta}_1, \hat{\theta}_2]$ and $\theta \in [\underline{\theta}, \tilde{\theta}]$, then $\frac{2u}{l}L(\theta) - \overline{OPT}(\theta) \geq (\frac{u}{l} - 1)\theta k a_0 + \frac{l}{2}(\frac{1-F(\theta)}{f(\theta)} a_0)^2 > 0$.

iii. If $\theta \in [\hat{\theta}_2, \overline{\theta}]$ and $\theta \in [\underline{\theta}, \tilde{\theta}]$, then $\frac{2u}{l}L(\theta) - \overline{OPT}(\theta) \geq \frac{k^2}{2l} - (1 - \frac{u}{l}\theta)ka_0 + \frac{l}{2}(1-\theta)^2 a_0^2 = \frac{1}{2l}[(k - l(1-\theta)a_0)^2 + 2ka_0\theta(u-l)] > 0$.

iv. If $\theta \in [\underline{\theta}, \hat{\theta}_1]$ and $\theta \in [\tilde{\theta}, \overline{\theta}]$, then $\frac{2u}{l}L(\theta) - \overline{OPT}(\theta) = (\frac{u}{l} - \theta)ka_0 > 0$.

v. If $\theta \in [\hat{\theta}_1, \hat{\theta}_2]$ and $\theta \in [\tilde{\theta}, \overline{\theta}]$, then

$$\begin{aligned}
&\frac{2u}{l}L(\theta) - \overline{OPT}(\theta) \\
&\geq \frac{u}{l}ka_0 - k\theta a_0 - \frac{k^2}{2l} + \frac{l}{2}\left(\frac{1-F(\theta)}{f(\theta)} a_0\right)^2 \\
&\geq \left(\frac{u}{l} - \theta\right)ka_0 - \frac{k^2}{2l} + \frac{l}{2}\left(\frac{k}{c''((1-\hat{\theta}_2)a_0)} - \frac{c'((1-\hat{\theta}_2)a_0)}{c''((1-\hat{\theta}_2)a_0)}\right)^2 \\
&\geq k\left(\frac{u}{l} - \theta\right)a_0 - \frac{k^2}{2l} + \frac{lk^2}{2u^2} - \frac{l}{2}\frac{k(1-\hat{\theta}_2)a_0}{u} + \frac{l}{2}(1-\hat{\theta}_2)^2 a_0^2 \\
&\geq \left[\frac{u}{l} - 1 + \left(1 - \frac{l}{2u}\right)(1-\hat{\theta}_2)\right]ka_0 + \frac{k^2}{2l}\left(\frac{l^2}{u^2} - 1\right) + \frac{l}{1}(1-\hat{\theta}_2)^2 a_0^2 \\
&\geq \left(\frac{u}{l} - 1\right)ka_0 + \frac{k^2}{2l}\left(\frac{l^2}{u^2} - 1\right) \geq 0.
\end{aligned}$$

The second inequality uses the monotone hazard rate assumption as well as the definition of $\hat{\theta}_2$ in Lemma 1. The third inequality uses the bounds of $c''(x)$. The fourth inequality uses $\theta \leq \hat{\theta}_2$. The fifth inequality uses $\hat{\theta}_2 \leq \overline{\theta}$. The last inequality comes from the assumption that $\frac{k}{a_0} \leq \frac{2u^2}{u+l}$.

vi. If $\theta \in [\hat{\theta}_2, \overline{\theta}]$ and $\theta \in [\tilde{\theta}, \overline{\theta}]$, then $\frac{2u}{l}L(\theta) - \overline{OPT}(\theta) = (\frac{u}{l} - 1)\, ka_0 + \frac{l}{2}(1-\theta)^2 a_0^2 > 0$. □

Proof of Lemma E.2. When $p^C = \frac{ua_0}{2}(1-\underline{\theta})^2$, the best response to this conditional contract is $a(\theta) = a_0$ for all θ. Thus, the payoff of this conditional contract is $Obj^C(p^C = \frac{ua_0}{2}(1-\underline{\theta})^2) = ka_0 - \frac{ua_0^2}{2}$. We can lower bound the conditional contract using the assumption that $\frac{k}{a_0} \geq \frac{2u^2}{u+l}$ as follows:

$$\begin{aligned}
\frac{2u}{l}Obj^C\left(p^C = \frac{ua_0}{2}\right) &= \frac{ua_0}{l}(2k - ua_0) \geq \frac{ua_0}{l}\left(2k - \frac{u+l}{2u}k\right) \\
&= ka_0\left(\frac{3u}{2l} - \frac{1}{2}\right).
\end{aligned}$$

Again we upper bound the optimal contract the same way we did in Lemma E.2 and Equation (E.1). Denote the integrands of the conditional and the optimal contract without the $f(\theta)$ term by $C(\theta)$ and $\overline{OPT}(\theta)$ for every θ. We show $\frac{2u}{l}C(\theta) - \overline{OPT}(\theta) \geq 0$ pointwise by considering the following three cases:

i. If $\theta \in [\underline{\theta}, \hat{\theta}_1]$, $\frac{2u}{l}C(\theta) - \overline{OPT}(\theta) \geq ka_0(\frac{3u}{2l} - \frac{1}{2} - \theta) > 0$.

ii. If $\theta \in [\hat{\theta}_1, \hat{\theta}_2]$,

$$\begin{aligned}
&\frac{2u}{l}C(\theta) - \overline{OPT}(\theta) \\
&\geq ka_0\left(\frac{3u}{2l} - \frac{1}{2} - \theta\right) - \frac{k^2}{2l} + \frac{l}{2}\left(\frac{1-F(\theta)}{f(\theta)} a_0\right)^2 \\
&\geq ka_0\left(\frac{3u}{2l} - \frac{1}{2} - \theta\right) + \frac{k^2}{2l}\left(\frac{l^2}{u^2} - 1\right) + \frac{l}{2}(1-\hat{\theta}_2)^2 a_0^2 - \frac{l}{2u}(1-\hat{\theta}_2)ka_0 \\
&\geq ka_0\left(\frac{3}{2}\left(\frac{u}{l} - 1\right) + \left(1 - \frac{l}{2u}\right)(1-\hat{\theta}_2)\right) + \frac{k^2}{2l}\left(\frac{l^2}{u^2} - 1\right) + \frac{l}{2}(1-\hat{\theta}_2)^2 a_0^2 \\
&\geq \frac{3}{2}ka_0\left(\frac{u}{l} - 1\right) + \frac{k^2}{2l}\left(\frac{l^2}{u^2} - 1\right) \\
&\geq ka_0\left(\frac{3}{2}\left(\frac{u}{l} - 1\right) + \frac{2u^2}{2l(u+l)}\left(\frac{l^2}{u^2} - 1\right)\right) = \frac{1}{2}ka_0\left(\frac{u}{l} - 1\right) > 0.
\end{aligned}$$

The second inequality comes from the definition of $\hat{\theta}_2$. The third inequality comes from $\theta < \hat{\theta}_2$. The fourth inequality comes from $\hat{\theta}_2 \leq \overline{\theta}$. The last inequality comes from the assumption that $\frac{k}{a_0} \geq \frac{2u^2}{u+l}$.

iii. If $\theta \in [\hat{\theta}_2, \overline{\theta}]$, $\frac{2u}{l}C(\theta) - \overline{OPT}(\theta) \geq ka_0(\frac{3u}{2l} - \frac{1}{2} - 1) + \frac{l}{2}(1-\theta)^2 a_0^2 > 0$. □

Endnotes

[1] The requirement of full conservation in order to receive payments is termed *conditionality* in the PES literature (Engel et al. 2008).

[2] Assuming that the forest area is observable captures the recent advancement in satellite imaging technology, which allows for monitoring

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

forests and tree coverage at the level of individual landowners (Hansen et al. 2013, Jean et al. 2019, Lütjens et al. 2019).

[3] We can equivalently normalize a_0 but do not do so in order to emphasize the dependence of contracts on a_0.

[4] We assume the environmental value of forest is linear in the forest area; this is a nontrivial ecological assumption. The linear assumption is reasonable if we consider only the carbon storage value of the forest and abstract away its biodiversity value. The biodiversity value of the forest is often heterogeneous and complementary, and thus nonlinear in the forest area. The k in our model can be interpreted as the social cost of carbon, or as the carbon price transacted on carbon credit markets.

[5] Fines for noncompliance are rarely used, although the PROFAFOR program in Ecuador asks forest owners to pay back past payments if they do not comply (Wunder and Albán 2008).

[6] The revelation principle (Dasgupta et al. 1979, Myerson 1979) states that without loss of implementability, we only need to consider direct revelation contracts where the agent truthfully reports their type (baseline conservation proportion θ) and the contract specifies an action-and-payment pair based on the agent's type.

[7] When $F(\theta)$ does not satisfy the monotone hazard rate assumption, standard "ironing" techniques can be applied (Mussa and Rosen 1978). Additional details are provided in the appendix.

[8] Because $\frac{k}{2}$ is independent of a_0, the same contract can be used for agents with different initial land sizes a_0.

[9] Mason and Plantinga (2013) used an environmental value $k = \$100$ per unit area, which is based on the estimate in Lubowski et al. (2006) of a value of \$50 per ton of carbon.

[10] Specifically, Brazil's crop land size is about 2 million square kilometers, and annual agricultural revenue is about \$81 billion; the U.S. crop land size is about 1.9 million square kilometers, and its annual agricultural revenue (2017) is about \$178 billion. This means the average revenues per square kilometer in Brazil and the United States are approximately \$40,000 and \$94,000, respectively, yielding a factor of $2.35 > 2$.

References

Alix-Garcia J, Wolff H (2014) Payment for ecosystem services from forests. *Annual Rev. Resource Econom.* 6(1):361–380.

Atasu A, Corbett CJ, Huang X, Toktay LB (2020) Sustainable operations management through the perspective of *Manufacturing & Service Operations Management*. *Manufacturing Service Oper. Management* 22(1):146–157.

Bolandifar E, Feng T, Zhang F (2018) Simple contracts to assure supply under noncontractible capacity and asymmetric cost information. *Manufacturing Service Oper. Management* 20(2):217–231.

Borner J, Baylis K, Corbera E, Ezzine-de Blas D, Honey-Roses J, Persson UM, Wunder S (2017) The effectiveness of payments for environmental services. *World Development* 96:359–374.

Butler RA (2019) Tropical forests' lost decade: The 2010s. Accessed September 30, 2020, https://news.mongabay.com/2019/12/tropical-forests-lost-decade-the-2010s/.

Carroll G (2015) Robustness and linear contracts. *Amer. Econom. Rev.* 105(2):536–563.

Chemnick J (2021) Cost of carbon pollution pegged at \$51 a ton. *Sci. Amer.* (March 1), https://www.scientificamerican.com/article/cost-of-carbon-pollution-pegged-at-51-a-ton/.

Corbett CJ, Zhou D, Tang CS (2004) Designing supply contracts: Contract type and information asymmetry. *Management Sci.* 50(4):550–559.

Dasgupta P, Hammond P, Maskin E (1979) The implementation of social choice rules: Some general results on incentive compatibility. *Rev. Econom. Stud.* 46(2):185–216.

de Zegher JF, Iancu DA, Lee HL (2019) Designing contracts and sourcing channels to create shared value. *Manufacturing Service Oper. Management* 21(2):271–289.

Dütting P, Roughgarden T, Talgam-Cohen I (2019) Simple vs. optimal contracts. *Proc. 2019 ACM Conf. Econom. Comput.* (Association for Computing Machinery, New York), 369–387.

Engel S (2016) The devil in the detail: A practical guide on designing payments for environmental services. *Internat. Rev. Environ. Resource Econom.* 9(1–2):131–177.

Engel S, Pagiola S, Wunder S (2008) Designing payments for environmental services in theory and practice: An overview of the issues. *Ecological Econom.* 65(4):663–674.

Ferraro PJ (2008) Asymmetric information and contract design for payments for environmental services. *Ecological Econom.* 65(4): 810–821.

Hansen MC, Potapov PV, Moore R, Hancher M, Turubanova SA, Tyukavina A, Thau D, et al. (2013) High-resolution global maps of 21st-century forest cover change. *Science* 342(6160):850–853.

Harstad B, Mideksa TK (2017) Conservation contracts and political regimes. *Rev. Econom. Stud.* 84(4):1708–1734.

Hawken P (2017) *Drawdown: The Most Comprehensive Plan Ever Proposed to Reverse Global Warming* (Penguin, New York).

Jack BK, Jayachandran S (2019) Self-selection into payments for ecosystem services programs. *Proc. Natl. Acad. Sci. USA* 116(12): 5326–5333.

Jayachandran S (2013) Liquidity constraints and deforestation: The limitations of payments for ecosystem services. *Amer. Econom. Rev.* 103(3):309–313.

Jayachandran S, De Laat J, Lambin EF, Stanton CY, Audy R, Thomas NE (2017) Cash for carbon: A randomized trial of payments for ecosystem services to reduce deforestation. *Science* 357(6348):267–273.

Jean N, Wang S, Samar A, Azzari G, Lobell D, Ermon S (2019) Tile2-vec: Unsupervised representation learning for spatially distributed data. *Proc. 33rd* AAAI *Conf. Artificial Intelligence* (AAAI Press, Palo Alto, CA), 3967–3974.

Kim SH (2015) Time to come clean? Disclosure and inspection policies for green production. *Oper. Res.* 63(1):1–20.

Kim SH, Netessine S (2013) Collaborative cost reduction and component procurement under information asymmetry. *Management Sci.* 59(1):189–206.

Laffont JJ, Tirole J (1986) Using cost observation to regulate firms. *J. Political Econom.* 94(3, Part 1):614–641.

Lansing DM (2017) Understanding smallholder participation in payments for ecosystem services: The case of Costa Rica. *Human Ecology* 45(1):77–87.

Lee HL, Tang CS (2018) Socially and environmentally responsible value chain innovations: New operations management research opportunities. *Management Sci.* 64(3):983–996.

Lubowski RN, Plantinga AJ, Stavins RN (2006) Land-use change and carbon sinks: Econometric estimation of the carbon sequestration supply function. *J. Environ. Econom. Management* 51(2):135–152.

Lütjens B, Liebenwein L, Kramer K (2019) Machine learning-based estimation of forest carbon stocks to increase transparency of forest preservation efforts. *Proc. NeurIPS Workshop Tackling Climate Change AI*. https://www.climatechange.ai/events/neurips2019#about-the-workshop.

Mason CF, Plantinga AJ (2013) The additionality problem with offsets: Optimal contracts for carbon sequestration in forests. *J. Environ. Econom. Management* 66(1):1–14.

Mirrlees JA (1971) An exploration in the theory of optimum income taxation. *Rev. Econom. Stud.* 38(2):175–208.

Muñoz-Piña C, Guevara A, Torres JM, Braña J (2008) Paying for the hydrological services of Mexico's forests: Analysis, negotiations and results. *Ecological Econom.* 65(4):725–736.

Mussa M, Rosen S (1978) Monopoly and product quality. *J. Econom. Theory* 18(2):301–317.

Downloaded from informs.org by [171.66.13.55] on 05 October 2023, at 13:34 . For personal use only, all rights reserved.

Myerson RB (1979) Incentive compatibility and the bargaining problem. *Econometrica* 47(1):61–73.

Orsdemir A, Hu B, Deshpande V (2019) Ensuring corporate social and environmental responsibility through vertical integration and horizontal sourcing. *Manufacturing Service Oper. Management* 21(2):417–434.

Pattanayak SK, Wunder S, Ferraro PJ (2010) Show me the money: Do payments supply environmental services in developing countries? *Rev. Environ. Econom. Policy* 4(2):254–274.

Perakis G, Roels G (2007) The price of anarchy in supply chains: Quantifying the efficiency of price-only contracts. *Management Sci.* 53(8):1249–1268.

Peterson JM, Smith CM, Leatherman JC, Hendricks NP, Fox JA (2015) Transaction costs in payment for environmental service contracts. *Amer. J. Agricultural Econom.* 97(1):219–238.

Plambeck EL, Taylor TA (2016) Supplier evasion of a buyer's audit: Implications for motivating supplier social and environmental responsibility. *Manufacturing Service Oper. Management* 18(2): 184–197.

Porras I, Barton DN, Miranda M, Chacón-Cascante A (2017) *Learning from 20 Years of Payments for Ecosystem Services in Costa Rica* (International Institute for Environment and Development, London).

Salzman J, Bennett G, Carroll N, Goldstein A, Jenkins M (2018) The global status and trends of payments for ecosystem services. *Nature Sustainability* 1(3):136–144.

Stern NH, Peters S, Bakhshi V, Bowen A, Cameron C, Catovsky S, Crane D, et al. (2006) *The Economics of Climate Change: The Stern Review* (Cambridge University Press, Cambridge, UK).

Strassburg BB, Iribarrem A, Beyer HL, Cordeiro CL, Crouzeilles R, Jakovac CC, Junqueira AB, et al. (2020) Global priority areas for ecosystem restoration. *Nature* 586(7831):724–729.

Teytelboym A (2019) Natural capital market design. *Oxford Rev. Econom. Policy* 35(1):138–161.

Wang S, Sun P, de Véricourt F (2016) Inducing environmental disclosures: A dynamic mechanism design approach. *Oper. Res.* 64(2):371–389.

Wunder S, Albán M (2008) Decentralized payments for environmental services: The cases of Pimampiro and PROFAFOR in Ecuador. *Ecological Econom.* 65(4):685–698.

Yu Y, Kong X (2020) Robust contract designs: Linear contracts and moral hazard. *Oper. Res.* 68(5):1457–1473.