ALLEVIATING FORGETFULNESS OF LINEAR ATTENTION BY HYBRID SPARSE ATTENTION AND CONTEXTUALIZED LEARNABLE TOKEN EVICTION

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ABSTRACT

Linear-attention models that compress the entire past memory into a fixed-size recurrent state offer an efficient alternative to Transformers, but the finite memory induces forgetfulness that harms retrieval-intensive scenarios. To mitigate the issue, we explore a series of hybrid memory architectures that restore direct access to the past. We interleave layers with intermediate time and space complexity between linear and full attention, including sparse attention with token eviction, and the query-aware native sparse attention. Particularly, we propose a novel learnable eviction policy of past memory. Combined with sliding-window attention, an end-to-end trainable lightweight CNN aggregates information from both past and future adjacent context to adaptively retain a limited set of critical KV-pairs per head, maintaining linear attention's constant time and space complexity. Efficient Triton kernels for the sparse attention are provided. Empirical evaluations on retrieval-intensive benchmarks support the effectiveness of our approaches.

1 INTRODUCTION

The recent emergence of large language models (LLMs) highlights the strong capability and efficiency of the transformer architecture (Vaswani et al., 2017). By explicitly scoring pairwise token relevance, transformers excel in capturing long-term dependence and retrieving memory from the distant past. Furthermore, transformers can be efficiently trained in parallel on an unprecedented scale of model and data capacity on modern GPUs. All these advantages enable transformers to supplant recurrent neural networks (RNNs). However, the blessing of pairwise attention is also a curse: compute time scales linearly with past context length per query (i.e. quadratically for the whole sequence), and efficient decoding requires a key-value (KV) cache grows linearly as the working memory; both drive computational costs and hardware demands. This is a particular bottleneck in scenarios like AI agents with ultra-long accumulated context. In response, novel hardware-efficient recurrent or linear attention token mixers such as Mamba (Gu & Dao, 2024) and DeltaNet (Yang et al., 2024d) have gained traction. Like early RNNs, these models compress the entire history into a fixed-size memory recurrently updated for each new token (illustrated in Figure 1 (a)), yielding $O(1)$ time and space per step, while remaining competitive with transformers on many language tasks. However, this also comes with a cost: a variable-length history can never be losslessly compressed into a fixed-size state. As time elapses, information from distant tokens inevitably decays, resulting in diminishing performance on retrieval-intensive tasks compared to standard transformers, even at modest context lengths (Wen et al., 2025; Jelassi et al., 2024; Arora et al., 2024a;b).

Various methods have been proposed to mitigate this forgetfulness. A prominent direction is to improve the update rule itself to increase memory expressiveness and update selectivity (Gu & Dao, 2024; Yang et al., 2024b; Du et al., 2025), without altering the recurrent nature. Others interleave full and linear attention layers to create hybrid models (Lenz et al., 2025; Waleffe et al., 2024; MiniMax et al., 2025). While this combination or compromise of two worlds allows more effective direct retrieval of past memory, complexity issues resurface. This motivates a natural question: Can we directly access past tokens for better retrieval, without much sacrifice in computational complexity?

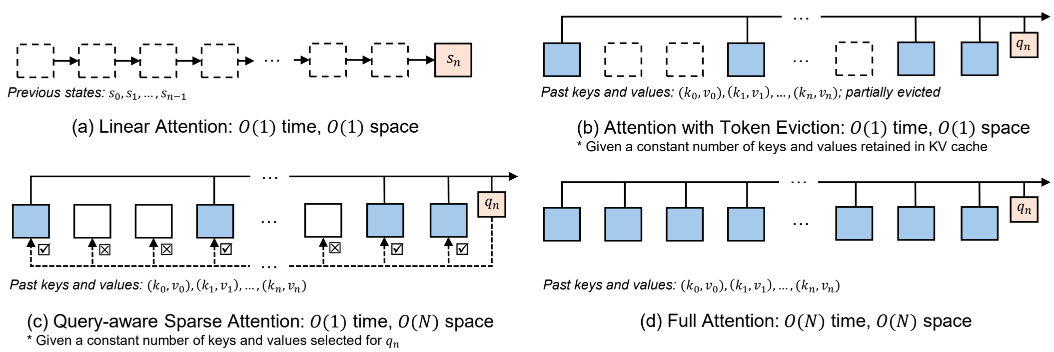

Figure 1: Hierarchy of token mixers across different time and space complexity per step. More complete and direct access to past tokens entails higher time and space costs.

Other than a complete fall back from linear to standard attention, there is a series of token mixers with time and space complexity between the two extremes, as exemplified in Figure 1. Among them recent query-aware sparse attention mechanisms such as Native Sparse Attention (NSA) (Yuan et al., 2025) are promising. These models introduce a lightweight "probing" step that first compares the current query with blocks of previous KVs, and select only a small set of possible most relevant past memory blocks for actual attention computation. This motivates our proposal of a hybrid **laNSA** model that interleaves **l**inear **a**ttention and **NSA** layers, aiming at directly accessing past information while largely preserving the favorable time complexity.

However, they still require accessing all past tokens and thus an $O(N)$-size memory, undermining linear attention's constant-space appeal; block-wise probing also limits granularity and adds overheads. Ideally, one could determine or *forecast* if the current token will be relevant for future queries using certain relevant criteria, and proactively discard less relevant tokens in advance, retaining only $O(1)$ tokens in the working memory (Figure 1 (b)), as if people underline key words when reading a long book. This has inspired a thread of research from the earlier sparse and local attention (Beltagy et al., 2020) to recent token eviction rules driven by accumulated attention scores (Zhang et al., 2023; Chen et al., 2024b). This yet leads back to forgetfulness: despite the direct access to the distant past, once a token is dropped, it is no longer available to any future query. An accurate eviction rule is henceforth of utmost importance. Most prior work relies on fixed and handcrafted heuristics. By contrast, we introduce learnable token eviction (LTE), which leverages a novel lightweight convolution neural network (CNN) that combines sliding-window attention (SWA) to predict the individual KV importance per head from both the past and future adjacent context. Trained end-to-end under sparsity regularization, LTE allows each sparse attention layer to retain the most relevant KVs under a learned per-head cache budget. We then build our **laLTE** model with interleaved linear and LTE-sparse attention layers. We further approach actual speedup with a dedicated KV cache layout, fused Triton kernels for SWA+KV-sparse attention computation and caching, and a lazy scoring scheme. In this way, laLTE retains both the constant time and space complexity of linear attention.

We pretrain language models at 0.4B and 1.4B scale, and evaluate them on several short-context and long-context/retrieval-intensive benchmarks. Both laLTE and laNSA outperform baselines including hybrids with SWA or heuristic eviction, and approach full transformer performance. To summarize, our contributions are: 1) propose a novel contextualized learnable token eviction mechanism for working memory management under strict budget; 2) build hybrid laNSA and laLTE architectures with efficient implementation; 3) examine the performance of hybrid linear attention models across the token mixer hierarchy. Our source code and models are publicly available at `https://mutiann.github.io/papers/laLTE`.

2 Background and related work

2.1 Sparse attention

Sparsity patterns A substantial body of work is aimed at reducing the $O(N^2)$ time complexity of the attention computation $A(\boldsymbol{Q}, \boldsymbol{K}, \boldsymbol{V}) = \text{softmax}(\frac{\boldsymbol{Q}\boldsymbol{K}^T}{\sqrt{d}})\boldsymbol{V}$ or $A(\boldsymbol{q}_i, \boldsymbol{K}, \boldsymbol{V})$ for each query vector q_i. The $N \times d$ matrices $\boldsymbol{Q}$, $\boldsymbol{K}$, and $\boldsymbol{V}$ represent the sequence of N *query*, *key*, and *value* vectors. A common approach is sparse attention, i.e., limiting the computation for each query to a small set of KV pairs $\boldsymbol{K}_{\mathcal{I}}$, $\boldsymbol{V}_{\mathcal{I}}$ at indices $\mathcal{I} \subseteq \{1, 2, ..., n\}$ (or $\mathcal{I}_i$ per q_i) of tokens believed to be important, reducing the overall complexity to $O(N|\mathcal{I}|)$. $\mathcal{I}_i$ can be simply determined by fixed patterns. A prominent example is sliding-window attention (SWA), that selects the most recent w tokens and discards KVs that fall outside the window during decoding. SWA is often combined with "global tokens", e.g., the first s tokens of the input (Beltagy et al., 2020; Ainslie et al., 2020; Zaheer et al., 2020), a.k.a. attention sink (Xiao et al., 2023). Despite simplicity and efficiency, fixed patterns are nevertheless input-agnostic and show suboptimal performance.

Token eviction Many works focus on more flexible strategies to identify tokens believed to be important for future attention, discarding less important k_j, v_j from KV cache and evicting them from any future $\mathcal{I}_i, i > j$ to maintain a memory budget $|\mathcal{I}_j| \leq S$. Because high historical attention score (i.e. more contribution to past outputs) often correlates with future importance, a common strategy is to retain tokens with top-K accumulated attention scores in history (Wang et al., 2021; Zhang et al., 2023; Jo & Shin, 2024), in a window (Liu et al., 2023; Zhao et al., 2024; Oren et al., 2024; Li et al., 2024), or to specific query tokens (Chen et al., 2024b; Kim et al., 2024). Notably, attention patterns vary by layer and head, which motivates non-uniform budget heuristics (Yang et al., 2024a; Cai et al., 2024; Qin et al., 2024b) or global budget allocation (Feng et al., 2025; Zhou et al., 2025). All these methods are training-free and primarily based on heuristics on attention scores. However, many tokens are *typically* important (e.g., infobox in Wikipedia articles) but without high local importance and attention score. Also, attention scores are not materialized in Flash Attention (FA) (Dao et al., 2022); extracting them adds implementation complexity and overheads.

A more flexible and potent alternative is learnable token eviction, a direction explored less extensively. An early attempt is ColT5, which selects tokens with a lightweight MLP router (Ainslie et al., 2023b), taking each token as input and hence not contexualized. Anagnostidis et al. (2023) train a layerwise bias on attention scores for token importance. Zeng et al. (2024) train a separate attention layer for LTE. Though fully contextualized, this involves heavy quadratic computation and is limited to prefilling stage. Very recently, several concurrent works explore LTE with per-token single-layer modules. Łańcucki et al. (2025) fine-tune LLMs with a global compression ratio constraint, which allows many layers to remain uncompressed and contradicts our $O(1)$ complexity goal. Piękos et al. (2025) train small scale LMs under predetermined cache budget, without considering layer and head variations. Shi et al. (2025) predict importance scores that are then multiplied with attention scores, which deviates from standard attention computation and leads to extra implementation complexity. As far as we are aware, this paper is the first to introduce a contextualized lightweight LTE variant.

Query-aware selection The token eviction mechanisms above attempt to identify tokens relevant to future queries without actually knowing them, which is inherently fallible. If we relax the space constraint and retain all past KVs, we can select the $\mathcal{I}_i = \text{select}(\boldsymbol{q}_i, \boldsymbol{K}, \boldsymbol{V})$ in a $\boldsymbol{q}_i$-aware manner, via an $O(N)$ but lightweight probing step before the actual attention. Early methods utilize online hashing and clustering (Kitaev et al., 2020; Roy et al., 2021), while many recent approaches highlight GPU-efficient blockwise approximation, exemplified by Mixture of Block Attention (MoBA) (Lu et al., 2025): input sequences are split into M-sized blocks, and keys in each block B_m are compressed into a block-level key $\boldsymbol{k}_m^B = \phi(\boldsymbol{k}_j), j \in B_m$, where ϕ is mean-pooling. Then $\text{score}(\boldsymbol{q}, B_m) = \langle \boldsymbol{q}, \boldsymbol{k}_m^B \rangle$ represents an approximated attention score at block level, and KVs in $\mathcal{I}_i^{(slc)} = \cup B_m$ for $m \in \text{top-K}(\text{score}(\boldsymbol{q}_i, B_m))$ are used for sparse attention. Intuitively, tokens in blocks with high attention score between $\boldsymbol{q}_i$ and mean-pooled keys are selected, and the whole network is then trained end-to-end. Given the probing results $\mathcal{I}_i^{(slc)}$, with only a small constant number MK of tokens attended, the actual attention time complexity is kept constant. We further adopt the stronger and more sophisticated Native Sparse Attention (NSA) (Yuan et al., 2025), which uses a weighted sum of sparse attention, block-level compressed attention, and SWA. Other learned variants directly predict block- or token-level attention scores (Gao et al., 2025; DeepSeek-AI, 2025), approximate attention scores in lower rank (Singhania et al., 2024; Tan et al., 2025), combine NSA branches (Zhao et al., 2025), or insert a "landmark" token per block as $\boldsymbol{k}^B$ (Mohtashami & Jaggi, 2023). Many other methods focus on training-free scenarios, and heuristically take block statistics such as mean-pooling as $\boldsymbol{k}^B$ (Tang et al., 2024; Chen et al., 2024a; Jiang et al., 2024; Xiao et al., 2024; Lu et al., 2024; Zhang et al., 2025; Lee et al., 2024); while we focus on exploring the trained scenario to best fit the model to sparse context.

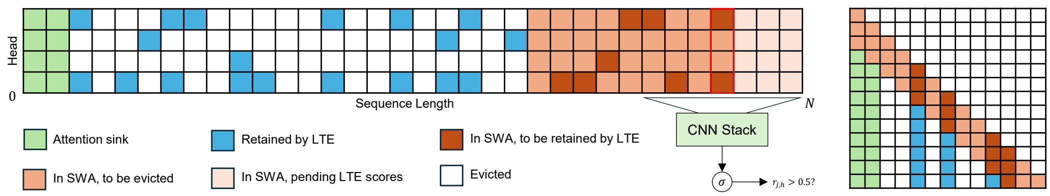

Figure 2: Left: Illustration of our KV eviction scheme, combining LTE, an attention sink (size $s = 2$), and SWA ($w = 12$), with 4 KV heads. A CNN stack is trained end-to-end to predict a per-token, per-head retention score $r_{j,h}$ to decide whether a KV-pair will be evicted when moved out of SWA. With recent KVs cached by SWA, the CNN can read short-range past and future context. r_j can be computed once the CNN receptive field is fully covered by SWA. Right: Resulting per-head A-plus-column sparsity pattern; colored tiles indicate selected tokens at each step.

2.2 Linear attention

Linear Attention (Katharopoulos et al., 2020) models offer strong alternatives to transformers that recurrently process and compress input context into fixed-size memory like classical RNNs, but use expanded matrix-valued memory and support parallel training through linear state transition. Later models including Mamba (Gu & Dao, 2024; Dao & Gu, 2024) and GLA (Yang et al., 2024c) enhance the compression by input-dependent gating to ignore unimportant inputs and better retain critical past memory. DeltaNet adopts delta rule updates for controlled removal of collided memory (Schlag et al., 2021; Yang et al., 2024d). Gated DeltaNet further uses forget gates for selective memory decay, and achieves state-of-the-art long-context retrieval performance. Hence we choose it as a strong baseline and the model backbone in our study. Beyond gating for better selectivity, other works focus on boosting expressiveness of limited memory via improved memory parameterization (Qin et al., 2024a; Peng et al., 2025) or a mixture of routed memories (Du et al., 2025). However, without direct access to past tokens, forgetfulness is inevitable for RNNs with limited memory.

2.3 Hybrid models

A straightforward remedy for the forgetfulness of linear attention is to put attention back. A small number of full attention layers can significantly improve retrieval performance (Lenz et al., 2025; Waleffe et al., 2024; MiniMax et al., 2025; Wang et al., 2025) but at increased computational demands. By contrast, we prioritize preserving time and space complexity by interleaving token mixers of intermediate costs. SE-Attn explores parameter efficient fine-tuning of attention layers in hybrid Mamba2 into query-aware sparse attention (Nunez et al., 2024), while laNSA utilizes the more sophisticated token mixer in a different pretraining scenario. Furthermore, it has been observed that interleaving SWA layers can markedly enhance linear attention performance (Ren et al., 2024; Zuo et al., 2024; Yang et al., 2024b; Arora et al., 2024b; Cabannes et al., 2025). Closer to laLTE, Ren et al. (2023) and Nguyen et al. (2024) study hybrid models that learn to select tokens as queries to perform SWA. These works nevertheless pursue aims distinct from ours: While SWA can be viewed as an eviction policy, it does not grant access to distant tokens, even when stacked together (Xiao, 2025). Most similar to our work, a concurrent work, LoLA (McDermott et al., 2025), keeps difficult past entries for vanilla linear attention in a side memory; another concurrent work, HAX, combines Mamba with hashing-based sparse attention and key selection, without considering trained query-aware sparsity and contextualized token eviction (Zhan et al., 2025).

3 Methods

3.1 Learnable Token Eviction

3.1.1 LTE module

We retain KV pairs using a per-token, per-head retention score $r_{j,h}$ predicted by the LTE module, and restrict attention to the retained KVs, SWA, and an attention sink. Concretely, for the decoding step i and head h, the index set is $\mathcal{I}_{i,h} = \{j : j \leq i, (j \leq s) \vee (r_{j,h} > 0.5) \vee (j \geq i - w)\}$, which induces an A+column-shaped attention pattern as shown in Figure 2. To determine retention accurately and efficiently, we anchor our LTE module design around four key properties:

1. **Fine-grained**: Patterns of attention maps vary significantly across layers and heads, some have sharp focus on a few tokens, some demonstrate recognizable local clusters, while others are rather dispersed. Therefore, instead of blockwise and layerwise decision, we choose to make an eviction decision of each KV pair on each head to capture head-specific patterns and make the best use of each head's own cache budget.

2. **Contextualized**: In addition to each token's own KV, we try to leverage local context including both past and future tokens to better understand each token and its eventual utility. Information from future tokens can not be captured by the causal/recurrent layers. Hence we use a 3-layer 1D CNN with kernel size=3, dilation=2, and a total receptive field $R = 13$ tokens to ingest local context from both sides. This "look-ahead" is made possible by SWA during decoding: recent KVs are stored in a window $w \gg R$ (e.g., with $w = 768$). We can defer computing r_j by $R//2$ steps until all necessary KVs within the receptive field are readily available. This computation can be even further deferred, since r_j is not needed as long as token j remains inside the window.

3. **Independent**: Inspired by ReMoE (Wang et al., 2024), we independently decide whether a token is important per se, without global scheduling such as picking the top-K tokens per head as in previous eviction methods. This reduces computational overheads, and more importantly leads to adaptivity, flexibility, and fine-grained budgeting compared to manually allocating the memory budget K per head.

4. **Lightweight**: We prioritize efficient and parallel inference, hence we choose to use a small 1D CNN module with only 3 layers and channel width halved at each layer, introducing roughly 1% extra parameters in our experiments.

Figure 2 sketches the full eviction scheme. The LTE module consumes the concatenation of key and value vectors per head, prior to rotary positional encodings (RoPE) (Su et al., 2024) for better position generalization. Convolutions are applied independently per head, but executed in parallel across heads using grouped 2D convolutions. Each convolution is followed by Swish activation and a dropout, and a final linear layer maps the per-head feature to scalar scores, also implemented by grouped convolution. Unlike ReMoE, we pass the scalar through a sigmoid to obtain $r_{j,h}$, which stabilizes training. We then binarize the scores with a 0.5 decision threshold. Inputs are zero-padded on the left, and thanks to the deferred computation with SWA, padding is not necessary on the right. The first $s = 4$ tokens are further forcefully retained as the attention sink. We can then simply leverage FlexAttention (Dong et al., 2024) for efficient sparse attention computation during training.

3.1.2 ADAPTIVE TRAINING

Training poses a challenge as LTE outputs only r scores to decide a discrete attention mask, hence the gradient cannot be directly back-propagated into LTE parameters. Instead, we leverage straight-through estimator by conceptually applying an all-one mask m to the value vectors so that $v'_{j,h} := v_{j,h} \cdot m_{j,h}$, and use gradients on $m_{j,h}$ as the surrogate gradient, That is to say,

$$\frac{\partial \mathcal{L}}{\partial r_{j,h}} := \frac{\partial \mathcal{L}}{\partial m_{j,h}} = \left\langle \boldsymbol{v}_{j,h},\ \frac{\partial \mathcal{L}}{\partial \boldsymbol{v}'_{j,h}} \right\rangle, \tag{1}$$

In this way, the gradient on $r_{j,h}$ is tied with the contribution to the loss of the underlying $v_{j,h}$ it controls. Stable end-to-end training is observed in practice under this scheme.

Some heads mainly focus on a few tokens, and their attention patterns are naturally sparsified by LTE without any additional constraints: On unimportant $v_{j,h}$, the all-one $m_{j,h}$ is pulled to zero by a negative gradient, which is copied to $r_{j,h}$. Some other heads have a dispersed attention pattern and benefit from an explicit sparsity prior to avoid excessive density. Inspired by ReMoE (Wang et al., 2024), we impose a dynamically adjusted L1-style penalty to encourage sparsity on retained tokens:

$$\mathcal{L}_{sparse} = \sum_h \lambda_h \sum_i \text{ReLU}(r_{i,h} - 0.5) \tag{2}$$

We target a cap $b = 512$ on the number of retained tokens, anticipating $c_h = \sum_i \mathbf{1}[r_{i,h} > 0.5] \leq b$. To adapt the model to this cap, we adjust λ_h with a feedback loop during training: increase when the moving average of c_h exceeds b, decrease when it falls below $0.95 \cdot b$, with an upper bound $\lambda_h \leq 1$ for stability. Empirically, many heads quickly drive λ_h toward zero, suggesting that LTE gradient

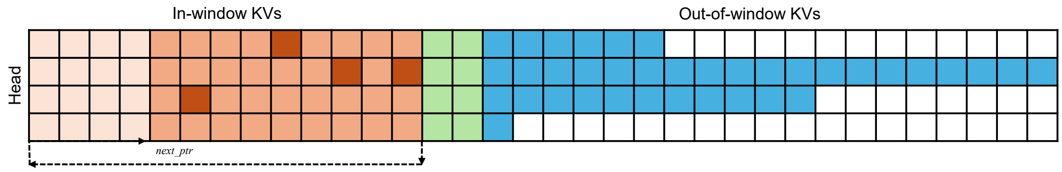

Figure 3: Illustration of the KV cache layout in LTE inference, coloring identical to Figure 2: a circular SWA segment for in-window KVs, and an out-of-window segment with capped capacity for LTE-retained KVs and the attention sink. LTE scoring is batched and deferred as late as possible.

signals alone suffice to induce adequate sparsity. In this way, the working memory capacity of each head is determined from end-to-end training using gradient signals on the values, without the need of handcrafted rules for budget allocation. More details are described in Appendix A.

3.1.3 Inference implementation

Capped capacity Our models are trained on fixed-length sequences ($N = 4096$ in our experiments). Although the number of retained tokens is regularized by the sparsity penalty, the actual computation depends on the retention score of each KV predicted from the input, and no hard cap is enforced as mentioned above. The case is different during inference as we expect guaranteed time and space complexity given arbitrary input of arbitrary length. Therefore we still have to resort to a global budget cap: the number of cached KVs out of window is restricted to at most b per head.

Cache prefilling We first allocate and fill a fixed-size cache as in Figure 3, consisting of two segments: one holds the last w KVs within the SWA window, and one gathers KVs already out-of-window but retained by LTE (or as the attention sink). This yields contiguous KVs amenable to tiled attention implementation, avoids memory management intricacies due to cache length imbalance between heads, and prevents worst-case regressions when a head is unusually dense. More specifically, since we store post-RoPE KVs, the physical locations in the cache are irrelevant and we can freely reorder or compact them. Hence we keep the SWA segment in a circular buffer with a `next_ptr` maintained, and compact out-of-window KVs into a contiguous span. Similar to Pagliardini et al. (2023), we leverage this compaction to achieve faster KV-sparse attention. If the capacity happens to exceed on certain head, we keep those with top-b scores. We then implement a Triton kernel for SWA+KV-sparse attention in the Flash Attention manner. Attention outputs of each query tile are computed in two stages: 1) SWA using tiles from original KVs; and 2) KV-sparse attention using tiles from the compacted cache. To avoid double counting, compacted KVs inside the window of each query are masked out, and tiles completely inside the window or breaking causality are skipped.

Cached decoding During decoding, when a new KV pair is pushed into the SWA buffer, the leftmost KV is popped out. If its $r > 0.5$, we will copy it to the next available slot of each head in the out-of-window segment. If the segment is also full, we will evict the lowest-score entry if it scores higher. A fused Triton kernel for this cache replacement process is implemented to obtain best efficiency. Now there is only one query token, and its relevant KVs are already compacted into a contiguous segment when the SWA segment is full. Hence we can simply utilize the existing highly-efficient `flash_attn_with_kvcache` API that supports `cache_seqlens` during decoding.

Lazy scoring We do not need to compute r_j immediately when the computation is possible (i.e. current n reaches $j + R//2$). Instead, we can defer and batchify LTE computation as long as all tokens within the receptive field remain in-window. Therefore, we trigger LTE scoring only when uncomputed LTE score r_j requires a token that is about to leave the window, i.e. $N - w = j - R//2$. We then rotate the buffer to move *next_ptr* to 0, and compute the LTE score of tokens from $N - w + R//2$ to $N - R//2$ altogether. Another problem is that LTE consumes pre-RoPE keys, while KV cache stores post-RoPE keys. Hence we need to undo RoPE on cached keys by applying the corresponding inverse rotation. With RoPE implemented using cached sinusoids, this is equivalent to applying RoPE on the keys with negated sine values at offset $N - w$. Using this trick, LTE scoring is carried out only every $W - R$ steps in parallel with minimal overheads.

3.2 Hybrid modeling

We use Gated DeltaNet as our baseline and backbone linear attention for laLTE and laNSA, thanks to its relatively strong performance on retrieval tasks. We apply a 1:1 interleaving of linear attention

Table 1: Results (perplexity and accuracy%) on short-context tasks. Best numbers are bolded.

Model	Wiki. ppl↓	LMB ppl↓	LMB acc↑	PIQA acc↑	Hella acc_n↑	Winog. acc↑	ArcE acc↑	ArcC acc_n↑	BoolQ acc↑	SciQ acc↑	SIQA acc↑	Avg.↑
0.4B Params., 10B Tokens												
GDN	27.56	36.06	31.9	66.4	39.1	50.9	56.8	25.9	58.4	71.0	37.5	48.7
GDN+SWA	26.99	33.87	33.7	66.4	39.6	51.5	56.4	27.9	60.2	72.4	38.5	49.6
laLTE	26.65	**30.03**	34.4	66.1	**40.2**	**52.8**	57.2	26.3	**61.8**	**75.6**	38.1	**50.3**
laNSA	26.84	36.19	33.2	66.1	39.9	51.8	**58.0**	**28.3**	61.6	73.1	37.7	50.0
GDN+Attn.	**26.61**	33.50	**34.6**	**66.5**	39.8	51.6	57.4	27.0	55.8	74.9	38.2	49.5
NSA	27.70	39.33	33.5	66.4	38.7	50.8	55.8	28.1	59.7	69.4	37.3	48.8
Transf.++	27.97	40.36	32.8	65.9	38.3	51.1	56.8	28.0	61.2	71.0	**38.6**	49.3
1.4B Params, 30B Tokens												
GDN	18.31	15.27	43.2	70.9	52.4	54.6	67.8	34.4	58.0	**83.9**	40.4	56.2
GDN+SWA	18.53	14.77	43.7	70.4	51.7	55.0	66.8	35.2	61.4	81.5	40.2	56.2
laLTE	**17.99**	14.79	43.9	71.1	**54.0**	**55.2**	**68.4**	**35.8**	60.6	83.4	39.4	**56.9**
laNSA	18.16	**14.73**	**44.9**	**71.2**	52.4	54.6	67.3	35.5	59.9	82.4	**40.8**	56.6
GDN+Attn.	18.68	16.10	43.2	69.6	51.0	53.5	66.3	32.8	53.8	81.5	40.1	54.6
NSA	19.53	18.48	42.2	68.8	50.1	53.0	65.6	34.0	59.1	80.5	39.1	54.7
Transf.++	20.68	20.17	39.1	68.1	47.2	52.0	63.8	33.5	**62.5**	80.5	38.8	54.0

and alternative token mixers layers, following the Gated DeltaNet + SWA design by (Yang et al., 2024b) known as Gated DeltaNet-H1 for a simplified comparison. We use $w = 1024$ for SWA and $w = 768$ for LTE. Although the LTE cache size is capped to $b = 512$, the observed per-head average retained counts is around 256, so their effective total memory capacity is comparable.

4 Empirical studies

We evaluate hybrid linear-attention models that interleave Gated DeltaNet (GDN) with various mixers from the hierarchy in Figure 1: sliding-window attention (GDN+SWA), learnable token eviction (laLTE), native sparse attention (laNSA), full-attention (GDN+Attn.), plus several ablated variants of LTE. We compare against pure GDN, NSA, and a strong transformer (Transf.++) following Gu & Dao (2024). Given limited computational resources available, we train 0.4B- and 1.4B-sized models on 10B and 30B tokens from FineWeb-Edu (Penedo et al., 2024) from scratch. We implement LTE layers, hybrid models, as well as inference kernels for LTE and NSA on top of Flash Linear Attention (Yang & Zhang, 2024). Below we leverage both short-context and long-context/retrieval-intensive benchmarks to assess models above and to demonstrate the effectiveness of our laLTE and laNSA approaches, and compare the prefilling and decoding efficiency. More details are available in Appendix A. Appendix B further provides an inspection on memory retention rate per layer.

4.1 Short-context benchmarks

We first evaluate zero-shot commonsense tasks using lm-evaluation-harness (Gao et al., 2024). With inputs generally shorter than the SWA window, we do not expect denser attention to confer any advantage. Performance should be broadly similar across models, and differences would not reflect capabilities of retrieving distant tokens. This is confirmed by results in Table 1. At 1.4B, GDN slightly outperforms Transformer++, while NSA and GDN+Attn. fall in between; the ordering is not consistent at 0.4B. GDN+SWA performs similar or slightly better, while adding LTE or NSA further yields small gains. Overall, our observations corroborate Yang et al. (2024b) that linear attention and hybrid models are comparable to or slightly better than transformers on these tasks, while our laLTE and laNSA training does not break short-context performance.

4.2 Retrieval-intensive benchmarks

We then assess on single needle-in-a-haystack (S-NIAH) from RULER (Hsieh et al., 2024) and the retrieval-intensive EVAPORATE suite (Arora et al., 2024b). S-NIAH spans tasks from retrieving numbers in synthetic texts (S1) to the rather challenging extraction of long UUIDs from real essays (S3). We evaluate contexts of 1K–4K, where models at our scale can attain non-trivial accuracy. EVAPORATE comprises realistic QA-style retrieval over passages up to 4K tokens, which is challenging for linear-attention models and is widely used in the area. With the demand to retrieve from distant past, we expect that hybrid models with more direct and accurate access to past tokens will have better performance, for which laLTE and laNSA have an edge when time and/or space

Table 2: Results (%) on recall-intensive EVAPORATE tasks. Best linear-time results are bolded.

Model	FDA	SWDE	NQ	SQUAD	TQA	DROP	Avg.
0.4B Params., 10B Tokens							
GDN	4.17	8.37	10.90	27.95	47.69	17.73	19.47
GDN+SWA	5.08	18.27	9.34	**34.52**	49.11	20.51	22.81
laLTE	15.52	19.26	**11.15**	32.98	48.99	**23.67**	25.26
laNSA	**17.33**	23.13	10.71	31.60	**49.17**	21.56	**25.58**
GDN+Attn.	29.31	26.19	11.53	30.53	46.92	18.69	27.19
NSA	7.99	**26.28**	10.01	33.91	47.16	19.74	24.18
Transf.++	28.58	26.46	11.09	31.94	46.68	17.30	27.01
1.4B Params., 30B Tokens							
GDN	20.96	25.83	**17.96**	35.02	56.46	20.75	29.50
GDN+SWA	26.68	27.99	16.53	41.09	58.95	**23.29**	32.42
laLTE	28.68	28.98	17.68	**43.00**	**60.60**	22.62	33.59
laNSA	**31.85**	**36.99**	17.20	38.07	58.06	20.60	**33.80**
GDN+Attn.	46.64	41.13	18.09	40.21	58.47	22.57	37.85
NSA	25.59	34.83	13.91	40.65	56.28	23.05	32.38
Transf.++	38.84	34.92	16.19	37.60	55.92	21.66	34.19

complexity is constrained. Nevertheless, we acknowledge that access to past tokens alone does not always translate into successful retrieval, higher computational complexity does not monotonically improve accuracy, and the actual outcomes can be task-dependent as observed by Wang et al. (2025).

Results in Table 3 and Table 2 confirm the effectiveness of hybrid models, laLTE and laNSA in particular. Specifically, pure GDN excels on S-NIAH; at 1.4B it approaches full transformers, echoing Yang et al. (2024b). However, GDN lags on the real EVAPORATE QA tasks. GDN+SWA considerably improves on it, but is much weaker on S-NIAH, especially beyond its 1K window size and at 1.4B. At the cost of efficiency, GDN+Attn. consistently achieves best EVAPORATE results across scales, surpassing full transformers, and performs comparably to full transformers and pure GDN on S-NIAH. As for our models, despite similar time and space complexity, laLTE outperforms pure GDN and GDN+SWA on both suites, and approaches GDN+Attn. on S-NIAH at 1.4B. This confirms laLTE's effectiveness and versatility for long-context retrieval under strict complexity budgets. With full history kept in memory, laNSA further attains the best results among linear-time models on EVAPORATE, approaches full transformers on 0.4B scale S-NIAH, and outperforms pure NSA. However it trails full transformer and GDN+Attn. on EVAPORATE and weaker on 1.4B S-NIAH, despite theoretical all-history access. It also has lower performance than laLTE on retrieval from shorter context (e.g. S3-1K), possibly due to conflict between different attention branches (Hu et al., 2025). Notably, NSA requires at least 16 group size for grouped-query attention. In our models with relatively small hidden sizes and head counts, NSA is rendered closer to multi-query attention with only 2 KV heads, which can be a performance bottleneck and a factor of instability.

Furthermore, we conduct a series of ablation studies to demonstrate the necessity of our LTE design compared to various alternative memory eviction approaches, which can be found in Appendix C. To summarize, all our results confirm the importance of direct and accurate access of historical context on retrieval-intensive tasks. From GDN, GDN+SWA/laLTE, laNSA, to GDN+Attn., more complete direct access to past tokens generally allows better long-context performance, though task-level outcomes vary. Particularly, laLTE highlights its strength despite rigorous time and space complexity constraints over GDN and GDN+SWA.

4.3 Efficiency

We then benchmark the actual computational efficiency of our LTE approach. All results are measured on a single H100-SXM GPU following our 1.4B configuration with batch size=32. Figure 4 (a) shows the runtime of prefilling kernels at different sequence lengths, compared with SWA and full attention using the highly-optimized FlashAttention-2 (Dao, 2024) kernels. As for LTE we use an artificial half-full out-of-window KV-cache, with 1024 tokens identical to the SWA window size. Our SWA+KV-sparse attention kernel exhibits a linear-growing runtime comparable to SWA, much faster than full attention. While LTE depends on real context for token eviction decision; many layers and heads retain only a few tokens, but the LTE scoring and caching introduce some addi-

Table 3: Results (%) on single needle-in-a-haystack benchmarks from the RULER suite. Numbers higher than 80% are bolded.

Model	S1-1K	S1-2K	S1-4K	S2-1K	S2-2K	S2-4K	S3-1K	S3-2K	S3-4K	Avg.
0.4B Params., 10B Tokens										
GDN	**99.8**	**92.8**	45.2	**100.0**	**94.8**	32.4	0.2	0.2	0.0	51.7
GDN+SWA	**100.0**	52.0	27.0	61.2	67.2	27.4	75.4	64.8	17.4	54.7
laLTE	**99.8**	**86.8**	36.4	**99.8**	74.2	25.0	**87.2**	42.8	17.8	63.3
laNSA	**100.0**	**100.0**	65.0	**88.6**	**100.0**	37.6	**97.6**	73.0	11.8	74.8
GDN+Attn.	**100.0**	**100.0**	77.0	**100.0**	**99.6**	52.6	**95.4**	**87.0**	12.2	80.4
NSA	**100.0**	**98.8**	46.6	**100.0**	**95.8**	36.4	**94.4**	64.6	18.6	72.8
Transf.++	**100.0**	**99.6**	50.8	**100.0**	**100.0**	55.4	62.8	71.4	35.8	75.1
1.4B Params, 30B Tokens										
GDN	**100.0**	**100.0**	**100.0**	**93.6**	**99.0**	**88.0**	**93.0**	**77.6**	49.0	88.9
GDN+SWA	**100.0**	52.0	29.6	**93.4**	**83.2**	34.2	**92.8**	57.0	19.4	62.4
laLTE	**100.0**	**99.2**	**95.0**	**100.0**	**98.0**	**81.4**	**85.4**	55.6	33.2	83.1
laNSA	**100.0**	**99.8**	57.8	**80.6**	**100.0**	68.2	60.4	**94.2**	34.0	77.2
GDN+Attn.	**99.8**	**100.0**	78.4	**100.0**	**100.0**	**90.6**	**83.8**	**71.0**	58.0	86.8
NSA	**100.0**	**93.8**	36.2	**100.0**	**99.4**	40.2	**97.8**	**78.6**	26.8	74.8
Transf.++	**100.0**	**100.0**	**98.4**	**100.0**	**100.0**	**85.6**	**89.4**	**84.2**	64.8	91.4

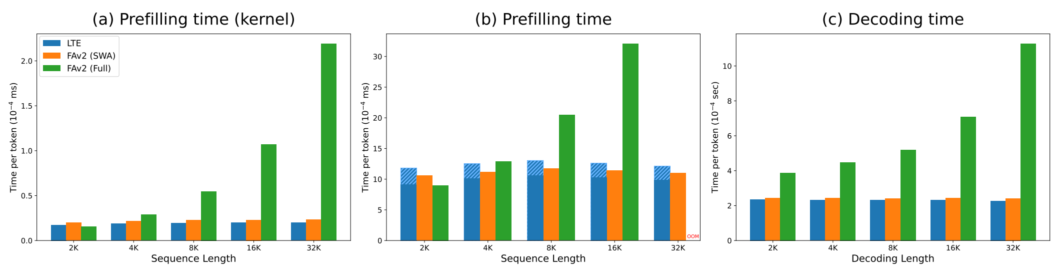

Figure 4: Comparing time costs between LTE and SWA or full attention with Flash Attention-2, on (a) attention kernels prefilling with artificial KVs; and hybrid layers with real inputs during (b) prefilling and (c) decoding with a 4K-token prompt, using our 1.4B models with batch size=32. Shadowed part of the LTE bar in (b) indicate time spent on scoring. Note that decoding takes much longer time than prefilling. Overall, LTE and SWA take similar time, much lower than full attention.

tional overheads. We therefore also measure the actual model prefilling and decoding time on real inputs, implemented based on Flash Linear Attention. To better isolate the speed of our targets, we only measure the time costs of hybrid layers (i.e. excluding other layers like GDN). Results shown in Figure 4 (b)-(c) similarly confirm the efficiency of our implementation. Scoring creates some overheads during prefilling (indicated by the shadowed part), which are largely compensated for by faster attention computation. Thanks to lazy scoring, the overhead becomes ignorable and LTE runs slightly faster than SWA during the decoding stage, which dominates inference time. It is further noteworthy that LTE uses a constant-size KV-cache equal to 1280 tokens, i.e. 2.5MiB per sample per layer under our 1.4B configuration. Compared to full KV cache under longer context, this allows larger batch sizes and higher token throughput given limited memory. Taken together, these results support adopting LTE in lieu of SWA for better performance without much efficiency degradation.

5 Conclusion

We study hybrid models with a series of token mixers to improve the retrieval capability of linear attention methods, trying to trade off between time/space complexity and accuracy. We further propose two approaches with different efficiency: laNSA to interleave query-aware native sparse attention of sub-quadratic time complexity when memory capacity permits, and laLTE for token eviction using a learnable contextualized module that maintains constant space complexity and supports an efficient decoding algorithm. Empirical results confirm the effectiveness of our approaches.

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A IMPLEMENTATION DETAILS

We train our models using Flame (Zhang & Yang, 2025) and refer to their training recipes, while adapted to our computational resources available. All the models are implemented based on Flash Linear Attention (FLA). Particularly, FLA implements the prefilling kernel for NSA, and we develop the decoding kernel based on it. It is noteworthy that NSA implemented by FLA is a simplified version that directly use mean pooling to produce the compressed $\boldsymbol{k}_m^B$ and $\boldsymbol{v}_m^B$, which may further limit its capability. We also develop the LTE prefilling kernel upon the FLA attention kernel. Both the 0.4B and 1.4B models have 24 layers, with hidden size = 1024 and 2048 respectively. As for GDN layers, we use 8 heads and expand_v = 1. As for attention layers, we set number of heads $H_q = H_{kv} = 16$ at 0.4B, and grouped-query attention (Ainslie et al., 2023a) with $H_q = 32, H_{kv} = 8$ at 1.4B. Similar configurations are also adopted by Wang et al. (2025). The exception is that NSA requires at least 16 group size due to hardware constraints, hence we use $H_q = 32, H_{kv} = 2$. Token embeddings are tied on 0.4B models. The LTE module consists of a 3-layer 1D grouped convolution stack, using kernel size = 3 and dilation = 2, yielding a receptive field $R = 13$. Each layer halves the channel width. Each layer is followed by a Swish activation and a dropout layer with $p = 0.1$. It then passes linear projection per head implemented as grouped convolution with kernel size = 1, and produces the retention score r after sigmoid. We implement directly with Conv2D to avoid tensor manipulation overheads. The number of groups equals the number of KV heads to process each head independently and efficiently. The LTE module will add $\sim$1.7% and $\sim$0.7% parameters on 0.4B and 1.4B models respectively.

We use the AdamW optimizer with learning rate = 3e-4 under cosine decay. At 0.4B/1.4B, we use global batch sizes of 0.5M/1.0M tokens, context length 4096, for 20480/30720 steps with 1024/512 warmup steps. As for LTE training, we initialize the regularization weight matrix λ to 1e-9, and update it via a feedback loop every $u = 32$ steps for stability, following the pseudocode given in Algorithm 1. The update is based on the exponential moving average $\bar{c}$ of the retention count c, with coefficient $\alpha_{ema} = 2/(1 + u/2)$ to approximate the average within the update cycle.

Algorithm 1 Pseudo-code for the LTE training loop.

$\lambda_{:} \leftarrow 10^{-9}$ ▷ Initialize λ matrix (shaped [#layers, #heads])
for step $\leftarrow 0$ to T **do** ▷ T is total training steps
    $r, c, \ell \leftarrow$ model(fetch_inputs()) ▷ Retention score, retention count, and LM loss
    $\ell \leftarrow \ell + \lambda \cdot \mathrm{ReLU}(r)$ ▷ Augment loss with L1 penalty on r
    optimizer.step()
    $\bar{c} \leftarrow \alpha_{ema} \cdot \bar{c} + (1 - \alpha_{ema}) \cdot c$ ▷ Exponential moving average of retained token counts
    if step $\bmod u = 0$ **then** ▷ Every u steps, update λ
        Synchronize $\bar{c}$ across devices
        $s \leftarrow \mathbf{1}(\bar{c} > L) - \mathbf{1}(\bar{c} < 0.95 \cdot L)$ ▷ Direction to adjust λ
        $\lambda \leftarrow \lambda \cdot \alpha^s$
        $\lambda \leftarrow \min(\lambda, 1.0)$ ▷ Clamp λ to at most 1.0
        $\lambda[\lambda < 10^{-9}] \leftarrow 0$ ▷ Eliminate near-zero values
        $\lambda[\bar{c} > L \;\&\; \lambda = 0] \leftarrow 10^{-9}$ ▷ Re-enable penalty when too many retained
    end if
end for

During LTE decoding, we directly use the `flash_attention_kv_cache` API by Flash Attention, which supports different past KV cache length per sample in a batch by passing in `cache_seqlens`. Since we have different cache length per head, we merge the dimensions for

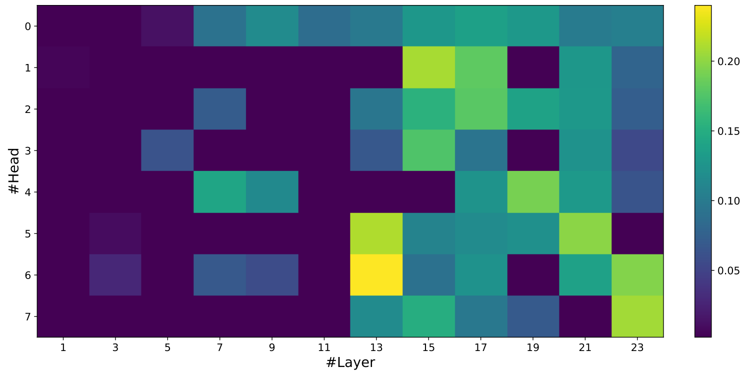

Figure 5: Average retention rate per head per layer produced by a laLTE model.

Table 4: Ablation results (%) on recall-intensive EVAPORATE tasks.

Model	FDA	SWDE	NQ	SQUAD	TQA	DROP	Avg.
0.4B Params., 10B Tokens							
laLTE	15.52	19.26	11.15	32.98	48.99	23.67	25.26
LTE-MLP	12.70	18.63	11.15	31.27	49.76	20.17	23.95
Pure LTE	14.25	18.72	7.44	33.31	46.15	19.45	23.22
1.4B Params., 30B Tokens							
laLTE	28.68	28.98	17.68	43.00	60.60	22.62	33.59
TOVA	20.96	30.78	13.49	40.18	58.59	22.62	31.10
Unif.+SWA	15.97	24.21	11.82	22.32	57.17	15.81	24.55

KV head and batch size, leaving a KV cache with the number of query heads equal to the GQA group size. As for evaluation, we use LM-eval-harness for short-context and RULER evaluations, and leverage the reference implementation by Arora et al. (2024c) for EVAPORATE benchmarks. We follow their input truncation scheme, but only truncate the input length to 4K tokens when it exceeds 4K to better reflect long-context performance. Inference is carried out using greedy decoding in a single run in FP32 for stability, except that GDN and Flash Attention kernels support FP16 only, hence we perform all token-mixing ops in FP16 for parity. For compute-cost measurements, we report the total elapsed time between the CUDA event pairs at layer entry after a full warm-up phase; for prefilling we average the middle three of five runs. Samples from the training dataset are used as inputs.

B Retention pattern

Figure 5 shows the average memory retention rate per head per layer from the out-of-SWA part of 16 samples drawn from the training data produced by our l.4B laLTE model. We find that retention rates are highly varied across layers and heads, while the higher layers are often denser. This is similar to findings in other works, e.g. Łańcucki et al. (2025), except that dense heads do not present in lowest layers; The interleaved GDN layers may have a role in it. The observation also supports the necessity of having a fine-grained cache budget different from head to head and from layer to layer.

C Ablation studies

We ablate laLTE against GDN+following token mixers with constant time and space complexity: **LTE-MLP** replaces CNN with a 2-layer MLP with no context access (specifically, no future access); **Pure LTE** is a pure transformer with all layers sparsified by LTE; **Unif.+SWA** retains out-of-

Table 5: Ablation results (%) on RULER single needle-in-a-haystack benchmarks. Numbers >80% are bolded.

Model	S1-1K	S1-2K	S1-4K	S2-1K	S2-2K	S2-4K	S3-1K	S3-2K	S3-4K	Avg.
0.4B Params., 10B Tokens										
laLTE	**99.8**	**86.8**	36.4	**99.8**	74.2	25.0	**87.2**	42.8	17.8	63.3
LTE-MLP	**95.6**	50.0	23.0	**100.0**	56.6	24.4	25.4	32.0	16.2	47.0
Pure LTE	**94.4**	37.8	21.4	74.2	54.6	22.6	**85.8**	37.6	15.0	49.3
1.4B Params, 30B Tokens										
laLTE	**100.0**	**99.2**	**95.0**	**100.0**	**98.0**	**81.4**	**85.4**	55.6	33.2	83.1
TOVA	**99.8**	**81.6**	41.0	**100.0**	11.4	8.0	**83.8**	0.0	0.8	47.4
Unif.+SWA	**94.2**	37.6	20.2	**100.0**	54.4	24.2	**84.8**	37.2	23.0	52.8

window tokens at a uniform stride; **TOVA** is one of the state-of-the-art eviction heuristics based on accumulated attention scores (Oren et al., 2024); the latter two are training-free. These two mechanisms are evaluated by applying them to pretrained GDN + Attn. for which use an identical 1024 total memory size. Results are given in Table 4 and Table 5. Although LTE-MLP and TOVA achieve moderate results on EVAPORATE, better than pure GDN, all of them underperform laLTE, especially on S-NIAH; while Unif.+SWA performs much worse than all other models. This confirms the importance of an accurate eviction rule, and underscores the effectiveness of our design of contextualized and learnable eviction. The observation is similar on Pure LTE, which supports our choice to combine sparse attention with linear attention to compensate for the evicted memory.